TÉCNICO LISBOA

# Application of the Simulated Annealing with Adaptive Local Neighborhood Search to the Tail Assignment Problem

The Case Study of TAP

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Thesis to obtain the Master of Science Degree in

**Industrial Engineering and Management**

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# Abstract

The airline industry is present in a strongly competitive market. In recent years, low-cost carriers have entered the market with new business models, forcing traditional airlines to decrease flights price. In an effort to maintain the profit margins, airlines are now seeking for strategies to improve their operational efficiency and therefore reduce the overall operating costs.

The main Portuguese carrier TAP is currently looking for opportunities to improve their current operations. TAP has identified their planning process, specifically the tail assignment phase, as one of their top priorities to improve efficiency. The objective in tail assignment is to define which aircraft should operate which flight. Furthermore, the currently used method at TAP to do the tail assignment does not consider the fact that different aircrafts have distinct operating costs. The creation of a solution approach that takes into consideration the individual characteristics of each tail, would enable the reduction of the operational.

The objective of this Masters Dissertation is to understand the current planning process at TAP, analyze potential limitations and present a simulated annealing algorithm with adaptive neighborhood search that minimizes the operational costs, while considering at the demand for each flight. Moreover, we analyze different scenarios that include the limitation of the utilization of each aircraft for a given schedule. Finally, we have also created three different algorithms than can generate an initial feasible solution in a short period of time.

**Keywords:** tail assigment, fleet assigment, airline operating costs, simulated annealing, adaptative neighborhood search, meta-heuristics

# Resumo

A indústria da aviação civil está inserida num mercado bastante competitivo. Nos últimos anos, companhias aéreas *low cost* entraram no mercado com novos modelos de negócio, forçando as companhias de bandeira a baixar o preço dos bilhetes vendidos. Estas empresas estão agora à procura de estratégias que permitam reduzir os custos operacionais, conseguindo assim manter as margens de lucro desejadas.

A principal companhia aérea Portuguesa TAP está agora à procura de novas oportunidades por forma a poder melhorar a eficiência das suas operações. A empresa identificou a fase de planeamento como uma das mais criticas a ser melhorada, mais especificamente a fase da alocação de aviões. O objetivo na alocação de aviões é definir qual é o avião que vai fazer um determinado voo. Atualmente, a alocação de aviões na TAP não tem em consideração os custos operacionais específicos de cada avião. A criação de um método de solução que tivesse em estes custos para cada avião, vai permitir uma alocação mais eficiente, onde os custos operacionais são menores.

O objetivo desta Dissertação de Mestrado é perceber atual processo de planeamento na TAP, analisando potencias limitações e propor um modelo de *simulated annealing* com procura local adaptativa que permita minimizar os custos operacionais, e ao mesmo tempo considerando a restrição da procura para cada voo. Adicionalmente, são analisados diferentes cenários onde são incluídos limites de utilização para cada avião. Por fim, foram criados três algoritmos diferentes que permitem gerar uma solução inicial num curto espaço de tempo.

**Palavras-chave:** alocação de aviões, alocação de frotas, custos operacionais, simulated annealing, método de procura local adaptativo, meta-heurísticas

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# List of Abbreviations and Acronyms

| | |
|---|---|
| **IATA** | Intrenational Air Transport Association |
| **DOO** | Day-of-Operation |
| **FAM** | Fleet Assigment Model |
| **LOF** | Line-of-Flight |
| **MILP** | Mixed Intiger Linear Programming |
| **OCC** | Operations Control Center |
| **DOC** | Direct Operating Cost |
| **IOC** | Indirect Operating Cost |
| **ASM** | Available Seat Mile |
| **BH** | Block Hour |
| **FH** | Flight Hour |
| **BH** | Business Unit |
| **IP** | Integer Programming |
| **MTOW** | Maximum Takeoff Weight |
| **INAC** | Instituto Nacional de Aviação Civil |
| **AGIFORS** | Airline Group of the International Federation of Operational Research Societies |
| **PCAA** | Portuguese Civil Aviation Authority |
| **GAMS** | General Algebraic Modeling System |
| **ICAO** | International Civil Aviation Organization |

# Chapter One

# Introduction

## 1.1 Problem Background and Motivation

The airline industry sector is one of the most competitive and dynamic markets in the world. Every year airline carriers appear with new offers, reduced prices and different market strategies. Furthermore, the recent growth of low-cost carriers has contributed to the increase of competition in this sector. Due to their different value proposition, low-cost airlines can offer cheaper flights when compared with traditional flag carriers like TAP (the main Portuguese air carrier). All these factors contributed to the reduction of profit margins across the industry (IATA 2014).

Another key factor, besides competition, that contributed largely to the low profit margins experienced in this sector is the increase of the jet fuel costs. These costs typically represent between 17% to 30% of airline total expenses (IATA 2015a). At same time, the growing concern with greenhouse gas emissions has led airlines to focus more in fuel efficiency strategies. IATA (International Air Transportation Association), defined 2020 as the year to change all airlines to carbon-neutral, where one of the most important topics was more efficient operations.

Driven by competitive pressure, low profit margins and environmental concerns, TAP is now seeking for new initiatives to increase operational efficiency and at the same time reduce costs. Presently, one of the key areas for TAP to reduce operational costs is the planning process phase, more specifically the tail assignment stage. The objective of this stage is to determine which aircraft should operate which flight. In TAP's current tail assignment method the only goal is to find a feasible solution that respects operational restrictions, not taking into account the different aircraft characteristics such as specific fuel consumption profiles. With an improved model where the more efficient aircrafts are assigned to highly fuel-intensive flights, TAP would be able to increase the planning efficiency and reduce costs.

Considering this, the main objective of the subsequent master thesis is to create a new tail assignment method where aircrafts are efficiently allocated to flights in order to reduce operational costs. Thus, for this study, is crucial to characterize the current planning process and also the individual characteristics of each aircraft in TAP's fleet.

## 1.2 Dissertation Objectives

The main objective for this work is to solve TAP's tail assignment problem, while considering the operational restrictions and the operational costs. For this study we will consider two different scenarios, the unbalanced aircraft utilization and the balanced aircraft utilization. Moreover, it is necessary to build an algorithm that generates an initial feasible solution that could be used by the simulated annealing as a first solution.

## 1.3 Dissertation Outline

This Dissertation is divided in six different chapters. In this first Chapter we describe the present environment of the airline industry, which motivated the problem being studied in this work. Additionally, two subchapters are dedicated to the methodology to be used and dissertation objectives.

In Chapter 2, is described the current planning process used at TAP, from the moment that flights are scheduled until day-of-operations. After, is given an overview of all operational costs with a special emphasis on the variable costs. To conclude this chapter it is analyzed the current tail assignment process, where we discuss the current limitations and possible improvements.

In Chapter 3, a literature review is carried to see existing best practices in airline planning process. A general summary of all the planning stages is followed by a deeper analysis on the tail assignment and maintenance routing phases. Then is explored all the solution methodologies and network representations used in the planning process.

In Chapter 4 we present the mathematical model that is used to solve the tail assignment problem. After, we give an extensive explanation of the structure used to program the algorithms. Finally, we describe the simulated annealing and the initial feasible solution algorithms.

In Chapter 5 we use the created algorithms to solve the tail assignment problem. This chapter includes the results for all algorithm created, one scenario where we consider a balanced tail assignment approach and finally is carried a sensitivity analysis to test the robustness of the results.

To conclude this dissertation, in Chapter 6 is presented the conclusions retrieved form this work and some directions for future work.

## 1.4 Methodology

In this subchapter is described the methodology that will be used in dissertation as guideline. Figure 1 shows the general structure that will be followed for the dissertation.

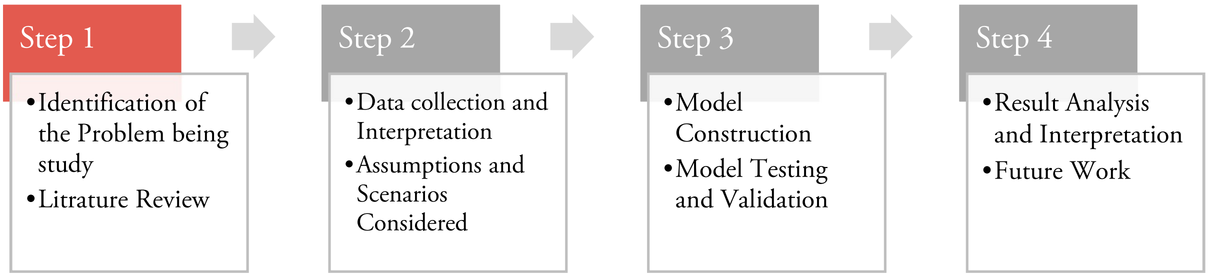


**Figure 1.** Methodology for this dissertation

1) **Step 1:** The first step starts, with the problem description and motivation, and then followed by a literature review to see current best practices in the airline planning process.

2) **Step 2:** In this step is carried an extensive data collection regarding TAP's fleet, maintenance obligations and flight operations. Then any assumptions and scenarios considered for this study are explained.

3) **Step 3:** This step starts with the design and development of the model that will be used to optimize the tail assignment phase. After this, the model is implemented in a software and is tested with real data to ensure that all restrictions are respected.

4) **Step 4:** In the final step, all the results obtained are analyzed and discussed. Furthermore, is used sensitivity analysis to study the relationship between the input data and the output results. This dissertation is then finished with all retrieved main conclusions and future work directions.

# Chapter Two

## Case Presentation

This chapter starts with a short description of TAP, followed by the description of the problem being studied. Then is presented the route network for all short and medium-haul destinations and the demand pattern for this flights, followed by the description of TAP's fleet where is detailed the number of aircrafts and some technical information for each model. Later, is given a comprehensive description of TAP's planning process and the complete explanation of the current operating costs sources. Finally, in the end of this chapter are discussed the limitations and possible improvements to the current tail assignment method.

### 2.1 Introduction to TAP

Created in 1945 by the hand of the General Humberto Delgado, TAP is a Portuguese company that operates commercial flights to Europe, Africa, North and South America. The main hub is in the Lisbon airport and the second one in Porto.

Presently, the airline has flights to 89 destinations in 34 different countries. Operating on average 2500 flights per week, with a fleet of 77 aircrafts where 61 are Airbus and the rest flight with the brand PGA (Portugália Airlines) and White, which are 100% owned by TAP, SGPS, S.A.

The main objective for TAP's operations is to assure a high quality service to every costumer, by providing the best in-flight experience and a wide-ranging choice of destinations. In 2016, 45% of the company was bought by a private consortium named Gateway. Although the airline is now managed by private shareholders, the values and mission remained the same.

TAP, SGPS, S.A owns several companies in different business areas. In this dissertation we will focus on TAP, S.A., more specifically on TAP Air Transportation (see Figure 2). This Business Unit (BU) handles all the commercial, operational and administrative part of the flight operations. TAP - maintenance and Engineering handles all the maintenance services necessary for keeping the airplanes in the air, and at the same time doing outsourcing for other airline companies. Finally, TAP services provide administrative support for the other two BU's.

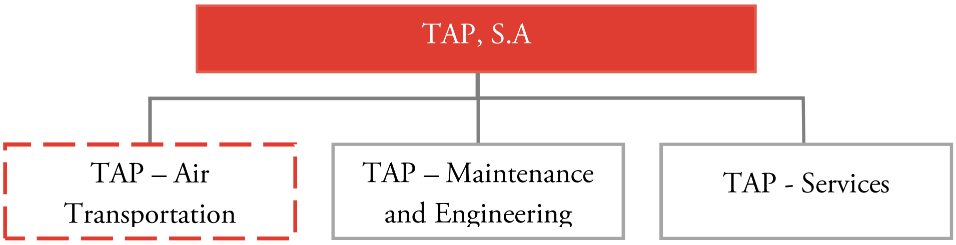


**Figure 2.** TAP, S.A Business units

One of the duties of the Air Transportation BU is to plan the airline operations, by defining the flight schedule, assigning crew members and aircrafts to each flight, being the last the subject matter for this work. Therefore, in the next sub-sections a detailed study on the planning department of this BU will take place.

## 2.2 Problem Description

The purpose of this work is to build a model that improves TAP's current aircraft allocation. The main objective it to reduce the operating costs and at the same time respect all the operational restrictions of this planning phase. For this, is essential to characterize TAP's operations by doing an extensive analysis on the planning process, fleet specifications and operational costs.

In this chapter an introduction to TAP's current planning process and the existing resources are described in order to understand the problem in better detail.

## 2.3 Route Network

As previously mentioned TAP offers short, medium and long-haul flights to 89 different destinations. It is important to note, that in this work is only explored the short and medium-haul operations.

The short-haul flights have on maximum 1h30min of flight time, where the longest flight is Lisbon-Oviedo and shortest one Lisbon-Faro with an average 45 minutes of flight time. In the medium-haul operation the flights have at maximum 6 hours, representing the flight from Lisbon to Moscow. In Figure 3 is represented all the flights within the short-haul and medium-haul operations.


**Figure 3.** Short and medium-haul flight connections (TAP, 2016)

TAP works with a spoke-hub network, where all the flights must depart and return to Lisbon or Porto hub. The main hub is located at the Lisbon airport and the secondary in Porto. The spoke-hub model increases the market coverage, as flights are directly or indirectly connected to the main Hub, enabling the maximization on the number of marketable destinations.

Furthermore, each flight leg is represented by an airport-pair, for example the flight Lisbon-London represents the outbound airport-pair, and London-Lisbon is other airport-pair representing the inbound flight. Table 1 lists all the airport-pairs for the short and medium-haul operations, coupled with the corresponding IATA codes.

**Table 1.** Short-haul and medium-haul airport-pairs

| Airport-Pairs | |
|---|---|
| Lisbon (**LIS**) | Porto Santo (**PXO**), Faro (**FAO**), Bologna (**BLQ**), Seville (**SVQ**), Zagreb (**ZAG**), Madrid (**MAD**), Porto (**OPO**), Málaga (**AGP**), Tangier (**TNG**), Coruña (**LCG**), Casablanca (**CMN**), Bilbao (**BIO**), Marrakech (**RAK**), Valencia (**VLC**), Funchal (**FNC**), Ranón (**OVD**), Barcelona (**BCN**), Bordeaux (**BOD**), Toulouse (**TLS**), Nantes (**NTE**), Algiers (**ALG**), Marseille (**MRS**), Lyon (**LYS**), Paris (**ORY**), Ponta Delgada (**PDL**), Nice (**NCE**), Geneva (**GVA**), Cagliari (**CAG**), London (**LGW and LHR**), Lajes (**TER**), Milan (**MXP**), Brussels (**BRU**), Luxembourg (**LUX**), Bergen Op Zoom (**WOE**), Manchester (**MAN**), Zurich (**ZRH**), Düsseldorf (**DUS**), Rome (**FCO**), Amsterdam (**AMS**), Venice (**VCE**), Munich (**MUC**), Frankfurt (**FRA**), Hannover (**HAJ**), Hamburg (**HAM**), Vienna (**VIE**), Prague (**PRG**), Berlin (**SXF**), Budapest (**BUD**), Copenhagen (**CPH**), Gothenburg (**GOT**), Warsaw (**WAW**), Dakar (**DKR**), Oslo (**OSL**), Bucharest (**OTP**), Praia (**RAI**), Espargos (**SID**), Rabil (**BVC**), Bissau (**OXB**), São Pedro (**VXE**), Stockholm (**ARN**), Helsinki (**HEL**), Moscow (**DME**), Accra (**ACC**) |
| Porto (**POR**) | Lisbon (**LIS**), Madrid (**MAD**), Barcelona (**BCN**), Funchal (**FNC**), Porto Santo (**PXO**), Vigo (**VGO**), Paris (**ORY**), Geneva (**GVA**), London (**LGW**), Luxembourg (**LUX**), Milan (**MXP**), Brussels (**BRU**), Torino (**TRN**), Zurich (**ZRH**), Amsterdam (**AMS**), Düsseldorf (**DUS**), Rome (**FCO**) |

Moreover, the number of short and medium-haul flights for the Airbus fleet are on average 1200 per week, with the regularity varying as a result of the demand behavior. Thus, there is some seasonality in flight sales, where the Christmas and summer months experience higher demands.

## 2.4 TAP Current Fleet

There are different aircrafts available for each flight operation category. For the short and medium-haul operations TAP haves three different models from the same manufacture: Airbus A319, Airbus A320 and the Airbus A320. For the long-haul operations the fleet is composed by the Airbus A330 and the Airbus A340. Table 2, shows a detailed picture of all the aircrafts currently available in TAP's fleet.

**Table 2.** TAP's current fleet

| Fleet | Manufacture | Variant | Nº Aircrafts |
|---|---|---|---|
| Short and Medium-Haul | Airbus | A319-111 | 16 |
| | | A319-112 | 5 |
| | | A320-214 | 19 |
| | | A321-211 | 3 |
| Long-Haul | Airbus | A330-202 | 4 |
| | | A330-203 | 2 |
| | | A330-223 | 8 |
| | | A340-312 | 4 |

**Short and medium-haul fleet**

This study will only focus on the short and medium-haul fleets. Thus, is important to characterize the three main differences between the aircraft models: seat capacities, engines and the Maximum Takeoff Weight (MTOW). The seat capacity, limits the flights that the aircraft can perform, flights with a higher demand need larger airplanes and flights with a lower demand smaller airplanes. The engines will influence the fuel consumption of each airplane, newer engines will consume less fuel than the older ones. Finally, the MTOW will have an impact on the operating costs of each aircraft, since this value is used to calculate the navigation and landing fees (explained in better detail in Section 2.6.2).

The MTOW is the maximum weight witch an aircraft may safely takeoff due to structural or power limits. This limit is normally defined by the manufacture and takes into account several components. The major components used to calculate the MTOW are: fuel (including reserve fuel), cargo, passengers, luggage and the empty weight of the aircraft.

In Table 3 is shown the MTOW's and the seat capacity for the short and medium-haul fleet.

**Table 3.** Short and Medium-haul fleet

| Aircraft Model | Tail | Engine Type | MTOW (Tonnes) | Seat capacity |
|---|---|---|---|---|
| A319-111 | CS-TT(X) – A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P | 2 x CFMI CFM56-5B5/3 | 68 | 144 |
| A319-112 | CS-TT(X) – Q, R, S, U, V | 2 x CFMI CFM56-5B6/P | 70 | 144 |
| A320-214 | CS-TQD, CS-TN(X) – G, H, I, J, K, L, W, M, N, P, W, X, Q, R, S, T, U, V | 2 x CFMI CFM56-5B4/P | 77 | 174 |
| A321-211 | CS-TJ(X) – E, F, G | 2 x CFMI CFM56-5B3/P | 89 | 216 |

## 2.5 Introduction to Airline Planning Process

The planning phase at TAP is a crucial part of the operations. In this process, is created a schedule where the crew, aircrafts, maintenance and flight legs are linked in a final master schedule. In this section, we give an overview of the current planning process of TAP. The planning process is very similar across all the airline industry, though some companies have variances to this procedure (Grosche 2009). Besides, is important to note that each part of the planning process is well defined in a time line, thus performed sequentially. Nevertheless, since this is a very dynamic business, the operations and decisions are not static and can be shifted in time if necessary.

There are three main areas in the planning process: flight schedule design, aircraft scheduling and crew scheduling. These phases are normally solved sequentially, and the result of an upstream planning stage is delivered into the next downstream planning stage. The flight schedule design is handled by both networking planning center and sales department, whereas the aircraft scheduling and crew scheduling are managed by the flight operation division.

As the day-of-operations (DOO) comes near, there are many things that can go wrong in the original plan. These, normally called disruptions, are handle by the operation control center in a process named disruption management. Amid the sources of disruptions are: late passengers or crew members, bad meteorological conditions, unforeseen maintenance tasks and cabin crew strikes. When these occur, the aircrafts or the crew must be reassigned or the flights must be postponed or cancelled. Currently, the disruptions are managed manually with the empirical knowledge of the workers.

This three main areas and the disruption management can be decomposed in sub sections as presented in Figure 4.

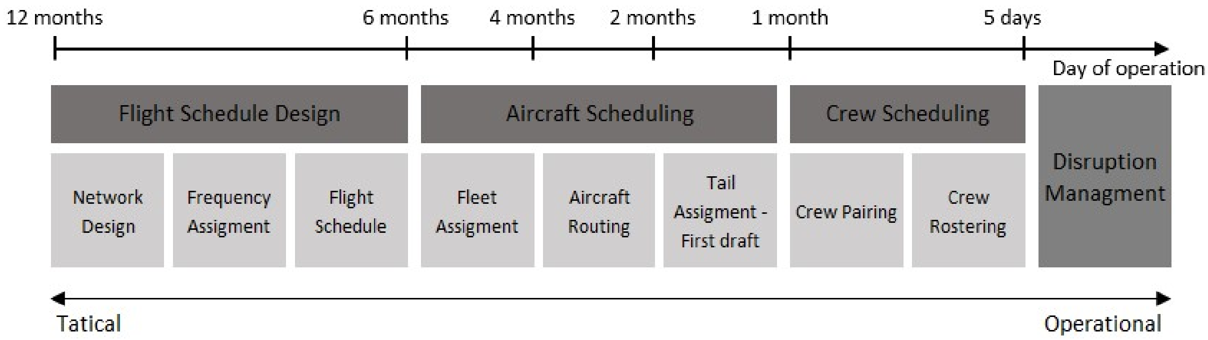


**Figure 4.** Time-line of TAP's current planning process at TAP

In the literature there are some articles that make a brief summary of all the planning stages in airline industry, where the most relevant are the ones written by Barnhart et al. (2003a), Barnhart and Cohn (2004), Klabjan (2003), Cohn and Lapp (2010) and more recently Yu (2012).

From Section 2.5.1 to Section 2.5.6 is explained in detail how is conducted each planning phase at TAP. The first stage is the flight schedule design that encompasses the network design, frequency assignment and flight schedule. Next, the aircraft scheduling phase is decomposed in fleet assignment, aircraft routing and tail assignment, which is the focus of this thesis. These phases are followed by the crew scheduling and finally the disruption management.

## 2.5.1 Flight Schedule Design

One of the crucial decisions for an airline company encompasses the schedule design. This stage can determine the future profits as well as the expected growth of the company, thus having a greatest impact on TAP final revenues. In this stage the company must choose in what markets they should be operating, the frequency and the exact departure times. This step involves defining the origin-destination airport pairs that should be connected with direct flights thus creating the route network that the airline desires to fulfill (network design), the number of flights that will be operated to a certain airport per week or month (frequency assignment) and at which day of the week and hour the flight would take place (flight schedule).

In this stage there are some restrictions that have to be considered such as the demand forecast for each route, total number of aircrafts available, crew available on a certain date and the number of slots available in each airport. Airport slot is defined by an authorization agreed with a coordinator (IATA) for a scheduled flight to use the airport infrastructure necessary to arrive and depart at a given date and time. The slots available in each airport for TAP operations, are defined at the International Air Transport Association (IATA) slot conference. This meeting happens in two distinct times of the year. The first conference is held in June, where the slots are assigned to each airline company for the winter period. The second meeting is in November, corresponding to the summer period. It is important to note that if in the former year an airline executed successfully at least 80% of the booked flights, then it is eligible to have same slots in next season. This enables the scheduling process to start before the IATA conference (IATA 2015b).

Bearing in mind all the aspects previously discussed, to find an optimal solution is necessary that all the components in the schedule design phase are considered, resulting in a model that would incorporate all stages. This is presently very difficult to do, due to the complexity and size of each sub-problem. However, solving a sub-problem leads to a sub-optimal solution, which is far from the overall optimum solution.

Another issue when building a schedule is the requirement of data that is normally only partially available. This includes the demand which is very difficult to predict exactly with one year of antecedence and the airport fees that are very difficult to estimate. For the reasons above mentioned, TAP usually constructs their schedule by adjusting the previous one. Moreover, most of these adjustments are driven by the fluctuations in demand when compared with the previous years.

### 2.5.2 Fleet assigment

In this planning stage the objective is to allocate specific fleet types (e.g. A319 and A320) with the concern of minimizing the overall costs. In order to minimize the total costs, we have to take in account the spill and operational costs. Spill costs correspond to the lost opportunity costs that appear when the demand surpasses the seat capacity of the aircraft, resulting in loss of potential sales (Barnhart et al. 2003a). The operational costs are related with the fuel efficiency of each fleet, maintenance costs and fees (Ozdemir et al. 2012). Additionally, the availability and the number of the aircrafts of a certain fleet should be taken in account in this planning phase.

At TAP this planning step is conducted by the Network Planning department. Currently they focus on minimizing the spill costs by matching the most suitable fleet type to each leg. This means that if aircrafts are assigned to a certain flight, whereas the capacity does not match with demand, there will be a loss of potential revenue. It can be noted that this stage only involves fleet types, and not a particular aircrafts (or tails). This decision of assigning each fleet is majorly driven by the historical demand data of similar periods. Since the demand matching is one of the most important aspects for TAP, mainly because the future revenues, this stage constitutes a vital part of TAP planning process.

### 2.5.3 Aircraft Routing

In the aircraft (or maintenance) routing the objective is to find sequences of flights known in industry as line-of-flights (LOFs). The LOFs are operated by a particular fleet and represent a sequence of flights from one airport to another, maintenance checks and the turnaround times. The turnaround time is defined by the amount of time required by an aircraft between the landing and takeoff. During this period the airplane is unloaded, loaded, cleaned, refueled, boarded and eye inspected. On average the turnaround time in TAP operations is 45 minutes, however this time can vary depending on each airport.

Furthermore, the starting airport must be the same at the end of the LOFs. The aircraft routing can be used only to find a feasible solution, or to find a solution that minimizes the operational costs. In Appendix 1 is an example of a LOFs for several TAP aircrafts, where it is noticeable the different flights, the maintenance checks and the turnaround times. In Appendix 1 is represented a LOF for a TAP operation.

The periodicity and the specifications of each maintenance checks is regulated by PCAA (Portuguese Civil Aviation Authority) for each fleet type. In Section 2.6.2 is explained in better detail each of this maintenance checks.

At TAP, this planning phase is handled by the Network Planning department, after the assignment of the slots at the IATA conference. With the help of the Maintenance and Engineering department, they shape the LOFs for each fleet for one week of operations. These, normally are acyclic since the schedule is not necessarily repeated for every single week.

### 2.5.4 Tail Assignment

In tail assignment, individual aircrafts or commonly named tails, are assigned to the pre-established LOFs. This process differs from the fleet assignment, because instead from being assigned fleet types, specific aircrafts (or tails) are assigned.

At TAP, this phase happens in one to two months before the flight takes place, and it is executed by the network planning department. This department only takes in consideration the feasibility of the solution and the match between the demand and the seat capacity of each tail. At this point of the planning phase, the network planning department already haves on hands all the information regarding maintenance checks (durations and schedule). This planning stage has a great impact in the overall profits, first because it is possible in this stage to assign the most fuel efficient tails to the busiest LOFs, the second reason is because aircrafts with less maintenance costs per hour can be allocated to LOFs with the longest flights.

### 2.5.5 Crew Scheduling

Crew scheduling encompasses two different processes. In the first process, named crew pairing, the Flight Operations department defines a sequence of flight legs to be performed by an unknown single crew. This stage takes into account all the obligatory labor rules and the feasibility of the final solution. In the second phase (named crew rostering), each crew member working at TAP is assigned to the pairings previously defined. The overall objective of the Flight operations department for these two phases, is to minimize the overall costs related to crew assignment.

The feasibility of this solution is defined by the crew that is able to operate a unique fleet. For instance, a pilot may be habilitated to operate an A320, but not an A340.

### 2.5.6 Disruption Management

Five days prior to the DOO, the master schedule is given to the Operations Control Center (OCC) department. Before this stage, the planning is mainly tactical (as we can see in Figure 4), where the objective is to allocate all the resources in a feasible and efficient manner. The disruption management phase deals with the operational aspects after the schedule completion. The main objectives at this stage are to prevent and recover from disruptions, such as, flight delays due to aircraft or crew unavailability or adverse meteorological conditions. Furthermore, the disruptions that happen in TAP operations, are handled with the empirical knowledge of the OCC team that seeks to diminish the effects of possible unforeseen incidents and bottlenecks.

## 2.6 Operational Costs

The main objective in this work is to build a model that can help TAP to reduce the operating costs, by efficiently allocating different aircrafts to each pre-schedule flight. Since aircrafts are heterogeneous, meaning different fuel consumptions, navigation costs, landing fees and others, the allocation of each was a direct impact on the final operating costs. TAP operating costs have a large percentage of the total costs, so even a small reduction can have a big impact on the final profit.

In the next subsection it will take place an identification and description of all the operating costs.

### 2.6.1 Operating Costs: General Classification

The operational costs can be separated in two different categories: direct operational costs (DOC) and indirect operational costs (IOC). The DOC, include all the costs that depend on the aircraft variant, for instance, costs of flight (crew, fuel, catering, etc.), maintenance, depreciation and navigation costs. Additionally, the IOC comprises, all the costs that stay unaffected with the change of an aircraft, such as, marketing, administration and sales costs. IOC are not directly related with flight operation, so changing the tail that will perform a certain flight will not have an impact on the final operational costs and so they will not be explored in this work.

The DOC can be divided in to different types of costs: fixed and variable costs. The variable costs depend on the usage rate, which means the number and the duration of flights taken by a specific tail. On the other hand, the fixed costs do not depend on the utilization of a particular tail, thus they do not change with the allocation of different tails. Since this costs are the same, independently of the utilization of the aircrafts, they cannot be optimized. In Figure 5 we have a brief description of all the fixed and variable operating costs.

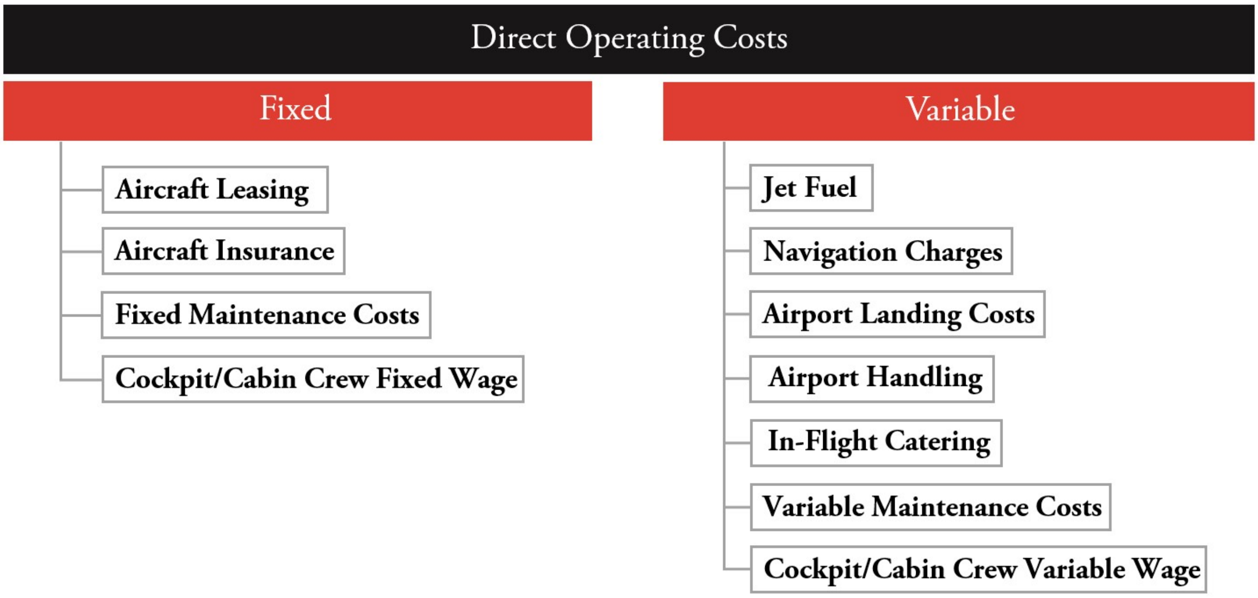


**Figure 5.** Division of TAP's Direct Operating Costs

In this study we distinguish each tail, in a sense that each aircraft as a unique profile of operational costs. This means that two different aircrafts operating the same flight will have different variable costs in the categories listed above.

Figure 6 shows the weight of each component of the variable DOC for the short and medium-haul fleet, totaling the costs from all the different aircraft variants. By examining the figure, is clear that the fuel cost has the biggest impact on the total variable DOC, representing 47.5% followed by maintenance costs.

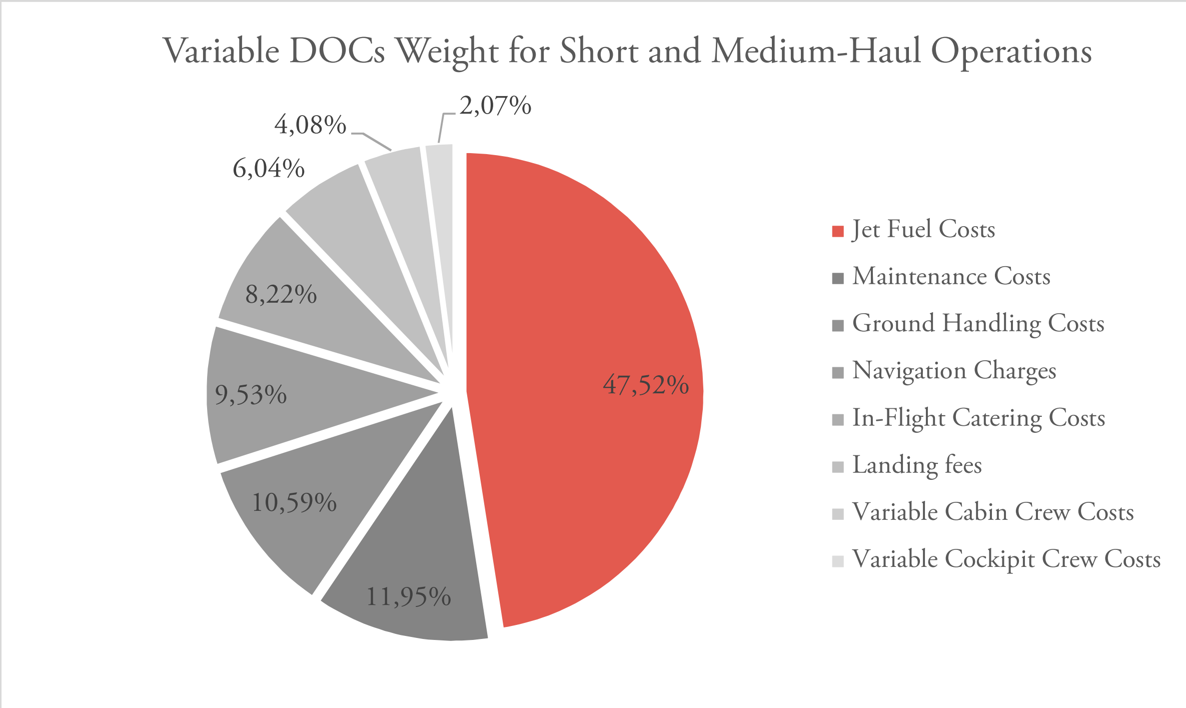


**Figure 6.** Variable DOCs weight for short and medium-haul operations (TAP, 2015)

In the airline industry the costs are usually calculated per Available Seat Mile (ASM), per Block Hour (BH) or per Flight Hour (FH). The cost per ASM represents the operating costs incurred by an airline to fly a single seat one mile. Normally, the lower the metric more efficient is the aircraft. The BH and FH are the industry standard measure for aircraft utilization. The BH is the amount of time that an a needs to perform a complete flight, from the moment that the aircraft closes the doors at the departure gate until the moment it opens the doors at the arrival gate. The flight hours represent only the time needed between takeoff and landing, excluding the taxi-in and taxi-out phases. In this study, TAP established that the costs should be calculated per BH.

## 2.6.2 Variable Direct Operating Costs

### Jet Fuel Costs

As mentioned above, the jet fuel costs represent a major fraction of TAP operational expense. Combined with the high volatility experienced by this commodity, especially in recent years, makes this component the most important for cost reduction. Moreover, with the increasing concern with climate change, there is a need to reduce greenhouse gases emitted by aircrafts. With an improved tail assignment, where the most fuel-efficient aircrafts are assigned to the longer flights and the least efficient to the shorter ones, TAP can reduce the fuel consumption as well the emission of greenhouse gases. Therefore, there is a great potential for TAP in having a better and more efficient tail assignment that would enable large savings in fuel costs.

Each tail has a unique consumption profile, meaning that each aircraft has unique characteristics that result in distinct performances. The factors with the largest influence in the efficiency of an aircraft are: age of the aircraft, weigh, flight level, meteorological conditions and engine performance.

In the short and medium-haul fleet, in terms of performance factors, the aircrafts are very similar. The main parameter that has the most impact in the aircraft performance is the engine. As the engines grow older the performance tends to decay due to mechanical wear, leading to a higher fuel consumption profile.

Thus, to have an efficient allocation of tails is necessary to identify the fuel consumption profile of the engines present in each tail. There are two ways to obtain the consumption profile, by the performance degradation factor or by historical fuel consumptions. The first method consists in comparing the actual performance of the engine, with the projected performance defined by the manufacture when the aircraft was first bought. As an example, if an aircraft has a degradation factor of 1.05, this indicates that the aircraft has an increased fuel consumption of 5% compared with a new one. In Appendix 2 is represented all the performance degradation factors for the short and medium-haul fleet.

Although this method enables a comprehensive and accurate measure for engine performance, it does not allow a direct comparison between aircraft variants and can be is only estimated when the engine goes into maintenance. The second method, historical fuel consumption, is obtained by doing an average of the last flights of a specific tail with the purpose of obtaining the fuel consumption per operated mile. This method allows the comparison between different aircraft variants and it is easier to keep up-to-date.

For this study the second method will be used to assess the different aircrafts.

**Navigation Charges and Airport Landing Fees**

The expenses of the European air traffic management services (operational, staff and infrastructure costs) are financed by the air navigation charges. These services are paid by the "user pays principle". In Europe, EUROCONTROL is the organization responsible for regulate and collect the navigation charges.

The European airspace is divided in charging zones, each zone having a single unit rate ($t_i$) that is charged to the aircrafts that overflown the airspace. According to EUROCONTROL (2016), the navigation charge of a given flight ($R$), is the sum of all the charges ($r_i$) generated in the different charging zones.

$$R = \sum_{n} r_i \tag{1}$$

Each individual navigation charge ($r_i$) is calculated by the product of the distance factor ($d_i$), the weight factor ($p$) and the unit rate ($t_i$).

$$r_i = d_i \times p \times t_i \tag{2}$$

The distance factor is equal to the distance overflown, in kilometers, within a specific charge zone. Finally, the weight factor is obtain by determining the square root of the division of the MTOW of the aircraft by fifty.

$$p = \sqrt{\frac{MTOW}{50}} \tag{3}$$

Concerning the airport landing fee (also charged by the "user pay principle"), these are paid when an aircraft lands at a particular airport. These fees can vary significantly across different airports, where the more congested airports charge the higher fees based on the supply and demand principle. They are also based on the MTOW of the aircraft, where the heavier is the aircraft the higher is the landing fee.

Given that each aircraft has a distinct MTOW and the navigation and landing costs are calculated based on this characteristic, these will be taken into consideration in this study.

**Cockpit and Cabin Crew Expenses**

The cockpit and crew expenses can be divided in two types of costs, fixed and variable. As explained above, the fixed part of the wages does not change with the alteration of the flights. The variable costs are mainly composed by commissions and allowances that depend mostly on the aircraft block time.

Since we are not considering reducing the block time in this study, and all crew is habilitated to operate all of TAP short and medium-haul fleet, the costs will not change with the allocation of different aircrafts to each flight. Therefore, the remuneration of the crew members will not be considered in this study.

**Airport Handling and In-Flight Catering Costs**

The airport handling groups all the activities and services performed in the aircraft during the turnaround time, more specifically the refuel, cargo load and unload, water refill and cleaning. In Portugal these services are provided by Groundforce, which is a company owned by TAP, in rest Europe this services are provided by a local company. The cost of the handling services is based on the type and MTOW of the aircraft.

In-flight catering is responsible of providing the meals, the on-board shopping and magazines. The cost driver here is the number of passengers that are on a specific flight.

As these two costs have a small weight on the final costs and they vary slightly from an aircraft to another, they will not be included in this work.

**Maintenance Costs**

Every aircraft in service must go through periodic maintenance inspections to attest its airworthiness. Each fleet has different inspection intervals along with specific inspection content. The maintenance plan is first defined by the manufacturer and TAP maintenance review board and then approved by the civil aviation authority, Instituto Nacional de Aviação Civil (INAC).

TAP Maintenance and Engineering division is responsible for defining in detail the intervals in which the maintenance tasks take place. If an aircraft surpasses the intervals, it is grounded until the task is completed. A grounded aircraft that is waiting for a maintenance service causes a high opportunity cost. Therefore, these requirements are extremely important restrictions in tail assignment.

There are four main factors that have an influence on maintenance services: Calendar, Flight Hours (FH), Flight Cycles (FC) – where each cycle represents a takeoff and landing – and unpredictable defects. The latter can be classified as irregular maintenance activities and result from unpredictable deficiencies through operation. An example of this is the repair of a broken overhead compartment. The calendar maintenances are not dependent on the usage of the aircraft, this type of services depend mostly on the airplane age, thus contributing only for fixed DOCs.

The systems and mechanisms that are subject to a usage-dependent wear are normally replaced based on flight hours. The components that are only stressed at particular moments are replaced based on flight cycles. For instance, the landing gear system is mostly used during takeoff and landing.

Table 4 summarizes all the regular inspection categories and their time intervals for TAP's short and medium-haul fleet.

**Table 4.** Maintenance Intervals for the short and medium-haul fleet

| Inspection Type | Tail | Tolerances |
|---|---|---|
| T | Before each flight | ------ |
| T1 | 36 Hours | 10% |
| T2 | 8 Days | |
| A | 750 FH<br>750 FC<br>4 Months | |
| C | 7500 FH<br>5000 FC<br>24 Months | 2 Months<br>500 FH |
| Structural | 10/12 Years<br>24000 FC | 3 Months<br>250 FC |
| Corrosion Prevention and Control Program (CPCP) | 10/12 Years | 3 Months |

Furthermore, is important to note that engine maintenance does not need aircraft immobilization. In order to reduce the ground-time, TAP owns several replacement engines that can be swapped at any moment. Moreover, only the maintenance activities enumerated formerly can be predictable and therefore have to be considered as a constraint in tail assignment. Additionally, for this study, calendar maintenances are considered flight activities pre-assigned to a specific tail. Maintenances dependent on

FH or FC will be considered as such. Finally, minor inspections are assumed to be completed during the turnaround time.

The maintenance costs are mainly composed by equipment costs and staff wages. This costs can expressed per BH and can be integrated in the model as a variable cost that depends on aircraft usage. Lastly, maintenances activities are normally performed in the Lisbon hub, thus is required that aircrafts needing maintenance services must be grounded at the Lisbon airport.

## 2.7 Current Tail assignment Limitations

The existing tail assignment implemented by TAP considers that all the aircrafts are homogeneous. This means that differences between the operational costs of each airplane are not taken into account. The only objective in the current model is to obtain a feasible solution that fulfils all the operational constraints and maintenance activities. So far the company has not been able to build a model that could represent all the differences between the available aircrafts. For that reason, TAP is not capable to generate accurate information about the costs and possible savings when assigning an aircraft to a specific flight.

Currently, TAP is using the compass software (Appendix 2) that only reaches a feasible solution, being incapable to obtain an optimal solution based on the operational costs of each airplane. Therefore, the software does not take into account different fuel consumptions or other tail-dependent costs like maintenance.

To summarize, the current tail assignment is limited and not able to give an optimal solution that minimizes the operational costs. TAP is now seeking for a model that would enable an increase in operation efficiency, since it recognizes that in reality each airplane is unique with distinct characteristics.

## 2.8 Chapter Conclusions

Presently, the short and medium-haul operations comprises 43 unique tails from three different aircraft models – A319-100, A320-200 and A321-200 – and a network that includes 69 different destinations. The operation is based on the hub-and-spoke configuration, where the main airport is in Lisbon and the secondary in Porto.

The current planning process at TAP is only focused at satisfying demand and commercial issues, respecting all the operational restrictions, connections, maintenances and unpredictable events. More specifically, in the tail assignment phase, the only objective is to find a feasible solution that matches the restrictions mentioned above. Thus, the airline does not makes distinction between different aircrafts and only considers tail assignment as a feasibility problem, not taking into account the different cost profiles of each tail .

Considering heterogeneity in TAP's fleet, where different fuel consumption efficiency, maintenance costs, navigation charges and landing fees are incorporated in an enhanced tail assignment model. This implementation would able to reduce the overall operating costs and also provide detailed information regarding the operational costs of each aircraft in a given period.

Given this information, the main purpose of this work is to improve the current tail assignment method. To do this, is necessary to build a mathematical model that considers all the selected operational costs and at the same time respects all the restrictions previously mentioned. After the construction of the model the objective is to analyze and implement the improvements in the current tail assignment process.

In the next Chapter we provide a brief literature review of the works published in this subject. This review will allow the reader to understand the tail assignment problem in better detail and also provide a literature summary of all the stages of the airline planning process.

# Chapter Three

## Literature Review

The airline industry has been continuously growing from the past thirty years. Much of this growth has been driven by the growing economy, the technological developments that have improved the safety and comfort of travelling by airplane and the market liberalization that has led to low-cost flights (ICAO 2006). In IATA (with 260 members) annual report is stated that in 2015 airlines will transport 3.5 billion customers and 50 million tonnes of cargo (IATA 2015a).

On the other hand, the increasing growth of the low-cost air carriers has led to slimmer profit margins, with an average profit of 7.85 € per passenger in Europe, forcing the traditional airlines to focus on improving the cost efficiency and customer satisfaction (IATA 2015a).

The increasing competition in the airline industry market has put more pressure on the management level to persistently reduce costs and increase revenues (Abdelghany and Abdelghany 2012). With market competition becoming stronger the need for solutions that enables cost reduction increases, leading to the implementation of optimization models (Shao 2013). Moreover, the operation research community has had a great influence on the operations of the present air transportation. Motivated by the highly dynamic environment in combination with the complex airline planning system, researchers are using advanced optimization methods to improve decision support systems and improving the overall airline operations (Yu 2012).

In this literature review, we summarize published studies on all airline planning phases, providing a greater attention to the tail assignment problem. We first provide a summary of the influence that operations research has had in the airline industry (Section 3.1). Section 3.2, presents an overview of all the phases involved in the airline planning process. In Section 3.3, we focus on the tail assignment and maintenance routing problem. Sections 3.4 and 3.5 describe the solution methodologies and network representations found in literature respectively. Finally, in section 3.6 the chapter ends with the main conclusions.

## 3.1 Airline Industry in Operations Research

Operation research has been in the airline industry for the past 60 years. The creation of the Airline Group of the International Federation of Operation Research Societies (AGISFORS) in 1961, has helped greatly the dissemination of knowledge in this field (Richter 1989, Barnhart and Talluri 1997). Furthermore, the continuously technology development associated with the discovery of new operation research models and techniques has helped the resolution of airline

Barnhart et al. (2003a) divides the contributions of operations research to the airline industry in three main themes:

- **Revenue Management**, or yield management, is related with the creation and management of service packages in order to maximize sales. The objective is to recognize different costumer value functions so the airlines can offer the most appropriate service package to each costumer segment (Chiang et al. 2006). Further information on this area can be found in Talluri and Van Ryzin (2006), or more recently in Belobaba et al. (2015).

- **Aviation Infrastructure** comprises the design and operation of airports, including the runways, taxiways, aircraft stands and passenger buildings, as also the air traffic flow management. Surveys for design and operation of airports can be found in Tošić (1992) and De Neufville and Odoni (2003). Kuchar and Yang (2000) developed a review of the operational research topics related with the air traffic control management.

- **Aircraft and Scheduling Planning,** or more commonly named airline planning process, is traditionally considered as a well-defined succession of activities that depend on established information flows. Furthermore, the feedback to upstream activities is very limited, given that an integrated planning process would be impossible to achieve with current computers (Clarke and Smith 2004). Nevertheless, these activities can be treated independently and have a major impact in the profitability and the quality of service, thus the optimization of each sub problem enables the airlines to become more competitive (Bielli et al. 2011). In the next section we give a detailed explanation of each planning phase, together with a brief summary of the published studies.

## 3.2 Airline Planning Process

The main goal in this process is to minimize the costs and at the same time create a robust schedule that reduces the chances of disruptions. Constructing such plan is difficult because of the various restrictions that have to be taken into account. These include capacity factors, maintenance obligations, crew management and airports limitations (Mariani 2015).

In Figure 7 is represented the different stages of the planning process in chronological order, being the first phase the schedule generation and the last the recovery planning.


**Figure 7.** Airline Planning Process (adapted from Gopalan and Talluri (1998b) and (Barnhart et al. 2003b))

Abdelghany and Abdelghany (2012) divide the planning phases in two main areas: tactical (long-medium term horizon) and operational (short term horizon). The tactical area comprises the first four steps, where the main focus is the efficient distribution of resources. The last phase, recovery planning, is classified as being an operational phase, where the goal is to reduce possible disruptions.

Although this planning process is one of the most used in literature, is not the standard design. Therefore, is normal to find different planning structures in literature. In addition, the airline industry have also different approaches in managing and creating the planning structure (Etschmaier and Mathaisel 1985).

There are two more differences that can be found in literature regarding each phase in the planning process, denomination and the start time. For instance, Barnhart et al. (2003b) names the first phase as "schedule generation", whereas Grönkvist (2005) calls the same phase as "timetable construction". Furthermore, the start time defined for each phase can differ from one study to another, and can be easily noticed when comparing a European airline to an American. In their study, Rexing et al. (2000) analyzes a US major carrier and characterizes the fleet assignment phase as starting ten weeks before the last phase, where Clausen et al. (2010) studying a European airline defines the same phase as starting five months before the last phase.

Nonetheless, is widely agreed that the downstream phase receives the data from the preceding phase, and as approaching the day of operation each phase becomes more specific (Mathaisel 1997). Moreover, as previously mentioned, the airline planning process is a large and complex problem that needs to be divided into sub problems due to computational intractability. Thus, the planning process is generated by solving each phase sequentially (Haouari et al. 2009).

### 3.2.1 Planning Phases Description

#### Schedule Generation

The schedule generation is the first phase in the airline planning process. At this stage the main priority is to maximize revenues considering the resources available (Erdmann et al. 2001). The objective is to define the airports that will be connected for a particular scheduled time period (Wu 2010).

The creation of the schedule depends mostly on strategic decisions comprising the choice of airports to operate, time and regularity of each flight. These decisions depend on several aspects including the macroeconomic data, future development of the market, airport landing fees and strategy of the airline (Bazargan 2012).

A comprehensive description of this planning phase can be found at Lohatepanont and Barnhart (2004).

#### Fleet Assignment

Afterward the schedule creation, each flight has to be assigned to an aircraft family type (fleet). This process is named fleet assignment. At this point, the main goal is to maximize the potential revenue by matching the aircraft seat capacity with the expected demand, taking into consideration the number of aircrafts available (Weide 2009, Liang and Chaovalitwongse 2012).

Furthermore, the fleet assignment is limited by the technical characteristics of each aircraft family, such as the flight range and necessary runway length. All these aspects mixed with the combinatorial nature of the problem (assign aircrafts to flights) makes it very complex to solve (El Moudani and Mora-Camino 2000). Further information on the fleet assignment problem can be found in Barnhart et al. (2009).

### Aircraft Scheduling

In the aircraft scheduling phase, the previously scheduled flights are allocated to individual aircrafts (or tails). This stage can be seen as a resource allocation scheduling problem, where specific resources are allocated to scheduled activities (Kilborn 2000). Differently from the fleet assignment problem where is only considered the scheduled flights, specific operational restrictions, particularly maintenance checks are integrated in this phase. At this stage, the managers need to take into account the different types of maintenances, with unique frequencies, periodicities and costs (Cordeau et al. 2001).

The highly dynamic and unpredictable environment experienced in the airline industry means that farther is the day of operations, more disruptions and planning failures will happen. Thus, tail assignment is normally done only a few days before the scheduled date of a flight (Grönkvist 2005). The downside is that schedule problems related with maintenances and other operational restrictions are identified very late. To overcome this, airlines use an additional planning stage intermediating the fleet and tail assignment. This stage is named as aircraft maintenance routing where sequences of flights are created (LOFs) without allocating a specific aircraft. The objective is the generation of generic LOFs such that sufficient maintenance occasions are provided to future allocated tails (Ruther et al. 2013). Since this planning stage is the main focus of this study, a detailed analysis is provided in Section 3.3.

### Crew Scheduling

The Crew scheduling process is analogous to aircraft scheduling and is as well divided into two distinct stages: crew paring and the crew assignment. In the first, anonymous crew work schedules (i.e. pairings) are generated, taking into account the labor laws and policies. In the crew assignment stage, individual crew members are assigned to the predefined work schedules. The main goal of crew scheduling is to find a feasible set of pairings that minimizes labor costs (Grönkvist 2005).

In the crew assignment stage the vacation requests, training periods and crew preferences must be considered when assigning the employees. Additionally, some airline companies let the crew members choose the preferred schedule, based on seniority level (Revelle and McGarity 1997). Kohl and Karisch (2004) provide a detailed review on the crew assignment problem.

**Disruption Management**

As soon as the crew assignment stage is completed, the planning process is finished. Though, unpredictable events can cause disruptions in the day-to-day operations. These events can be bad weather conditions, unavailable crew (e.g. sickness), airport delays or aircraft malfunctions (Argüello et al. 1997). The main task at this stage, is to overcome any disruption by creating a new feasible solution that minimizes delays and maximizes customer satisfaction (Kohl et al. 2007).

There are some measures that can be taken in order to minimize schedule disruptions. One of the possible solutions is to intentionally delay consecutive flights. In this way, the airline ensures that all the connecting passengers reach their final destination. Another solution is to increase the cruise speed on delayed flights. Furthermore, all the decisions made at this phase are normally close or in the day of operations, requiring a very fast execution (Revelle and McGarity 1997). Finally, a review presenting some of the models used in disruption management can be found in Clausen et al. (2010).

## 3.3 Aircraft Scheduling Literature

### 3.3.1 Tail Assignment

Differently from other planning stages, the tail assignment problem has not received much attention by the operation research community. Published studies are limited and fairly recent (Ruther et al. 2013, Başdere and Bilge 2014). The majority of published studies in this area do not consider the heterogeneity between aircrafts. The main concern in these studies is to find a feasible solution that respects the operational restrictions. Therefore, the individual specifications of each aircraft are seldom used in this planning phase (Gabteni and Grönkvist 2009).

Grönkvist (2005) develops a mathematical and constraint programing model that allocates aircrafts to flights for an operational period of several weeks. The operational restrictions considered in this model include: maintenance, night flying restrictions (i.e. curfews), turnaround times and preassigned activities. To solve this model, he uses a hybrid approach that combines column generation with local search heuristics. Finally, he applies the model in a real-word scenario helping an airline company to reduce disruptions and decrease aircraft leasing expenses.

A connection network is used by Sarac et al. (2006) to create a model that considers maintenance requirements, maintenance station availability and unpredictable events. The focus of this study is in the day-of-operations rather than long-term planning, where aircrafts needing maintenance end their flights at the maintenance station. The authors propose a modified branch-and-price algorithm to solve a network of 175 flight legs and 32 aircrafts, corresponding to a small airline company.

Afsar et al. (2006) propose a method that maximizes aircraft utilization before maintenance services, enabling regular maintenance checks for all the fleet. The model is applied for one week of planned schedule and all the maintenance services are previously scheduled. To solve this model they use a two-step heuristic approach that first prioritizes aircrafts that have to undergo a maintenance service and after assigns aircrafts without preassigned activities. In (Castagliola et al. 2009) the authors improve the solution method by using the simulated annealing approach. Furthermore, Başdere and Bilge (2014) uses the same idea on their model, but include maintenance station capacity as a constraint. To reach a solution they use the compressed annealing heuristic.

A tail assignment approach considering fuel consumption costs and environmental constraints is presented by Lapp and Wikenhauser (2012). In their study they use two approaches to assign aircrafts. In the first approach they assume that line-of-flights are already created and therefore the only task is to assign each aircraft to a specific LOF (i.e. line-based tail assignment). In the latter aircrafts are directly assigned to individual flights (i.e. flight-based tail assignment). It is important to note that maintenance services where not considered in both assumptions. After analyzing the results by the both methods the authors have concluded that flight-based tail assignment generates higher savings. However, this method involves substantial modifications to the planned schedule. To overcome this problem, the authors built a Pareto-curve that enables to analyze the tradeoff between savings and schedule modifications.

Dovica and Borndörfer (2014) use stochastic programing to develop a robust tail assignment model where generated schedules are more resilient to delays and disruptions. The stochastic approach uses historical data to create a probabilistic distribution that is used to reduce possible delays and thus save in operational costs. A column generation algorithm is then used to solve the stochastic model. Furthermore, Froyland et al. (2013) solves the same problem but with a two-phase algorithm that uses Benders decomposition and Pareto-optimal cuts. Yan and Kung (2015) also create a model for the robust tail assignment problem, where flight leg delays lie in a predetermined uncertainty set instead of following a probabilistic distribution.

Finally, in a more recent approach Maher et al. (2015) creates dynamic and iterative algorithm where the objective minimizes prescheduled maintenance misalignments and at the same time the optimizes gate assignments. This model is able to adjust the previously constructed LOFs to create feasible solutions that respect maintenance obligations by using a column generation algorithm. Despite considering all the required maintenance checks, the planning horizon in this study is only 3 days.

### 3.3.2 Aircraft maintenance routing

The aircraft maintenance routing has been continuously one of the most studied planning stages because of the maintenance constrains. Despite all the attention received most of the studies do not consider maintenance costs (Grönkvist 2005).

One of the first contributions to the aircraft maintenance routing problem can be found in Feo and Bard (1989). The authors create a model that integrates line-of-fights with cyclical maintenance services, and also considering maintenance station locations. A cost minimization multi-commodity flow network is formulated to represent the problem, and then solved with a two-phase heuristic approach. To test the model they use data from American Airlines fleet. The results show a significant cost reduction due to elimination of 5 maintenance stations. Gopalan and Talluri (1998a) propose a similar model where each aircraft is obliged to be grounded for maintenance inspections every three days.

Barnhart et al. (1998a) develop a linear programing (LP) routing model and uses a branch-and-bound algorithm to solve it. Furthermore, each node generated with the branch-and-bound algorithm is then solved with column generation. To test the concept, the authors used ten short-haul operation samples, with all having different number of total flights. For operations with few connections, up to 900, the model was fairly quick, still for schedules with a number of connections closer to 6000 the algorithm took nearly 10 hours to solve the problem

Sriram and Haghani (2003) propose a formulation for the aircraft routing that considers the tradeoff between aircraft re-assignment (i.e. changing LOFs) and allocation to maintenance services. The authors try to solve the model using the commercial solver CPLEX, however due to the dimension of the problem, they could not get a solution in a reasonable time frame. To overcome this, they propose a hybrid heuristic approach that comprises random search and depth search. The created heuristic is able to solve the problem in a reduced amount of time and with a final solution within 5% of the optimum value.

A mixed-integer programming approach is used by Liang et al. (2010) to construct a representation of an aircraft maintenance compact network. They build a model that forces aircrafts to be grounded for maintenance after a maximum number of flights. The main objective in this work is to create an improved method that solves the routing problem in the shortest period of time. The model is tested with a fleet of 70 aircrafts and 352 flight legs, reaching the optimal solution in 15 seconds. Nonetheless, besides the fast computations times, the authors have only considered in this study night maintenances which does not represent the typical operations on majority of airline companies. In Liang and Chaovalitwongse (2012), the authors improve the previous method proposed by the authors. The improved algorithm is able to find a solution within five minutes for a fleet size of 330 aircrafts and 5700 scheduled flights, corresponding to one of the largest airline fleets in the world.

## 3.4 Solution Methodologies

Was we have seen in the last sections, most of the studies use a high level algorithm to solve the created formulations. The most difficult step in solving these problems is to find a solution in a reasonable time frame. In literature, the solution methodologies used, vary from the use of exact methods (e.g. CPLEX) to heuristic approaches (i.e. simulated annealing) (Sriram and Haghani 2003, Bielli et al. 2011). The most used solution methodologies for the tail assignment and maintenance routing problem can be divided in four groups (Barnhart et al. 2003a, Klabjan 2003) :

- Constraint Programing
- Benders decomposition
- Branch-and-price (column search)
- Heuristics

From this list, we can conclude that exact methods have to be replaced by other methods that return faster but non-optimized solutions.

**Constraint Programing**

Constraint programing is a declarative programing method to formulate and solve problems that have variables with defined domains and constraints (Van Hentenryck 1995). The main focus of this method is to find feasible solutions rather than an optimal one. As a result, this method is able to generate a non-optimal solution in a very short amount of time. This method is mostly used in problems that need to find a feasible solution but not necessarily the best solution (Grönkvist 2005).

### Benders Decomposition

This multi-stage mathematical programing technique has firstly appeared in Benders (1962) and is mostly used to solve stochastic and mixed-integer nonlinear programming problems. This technique breaks down the original problem into a master problem (pure integer programing problem) and a subproblem (LP problem) that can be solved independently. In a first stage, the master problem is solved for a subset of decision variables. Afterwards, in a second stage subproblem the unsolved variables are determined, taking into account the previously calculated decision variables in the first stage. Finally, if the subproblem detects that values calculated in the first stage are infeasible, then additional constraints are added to the master problem and the process restarts until a solution is found. Thus, instead of solving a large problem, a sequence of small problems are solved, decreasing the computational requirements to solve the original problem (Kalvelagen 2002, Taskın 2010).

### Branch-and-Price with Column Generation

The branch-and-price is a hybrid method for solving large integer LP and MILP problems that combines branch-and-bound with column generation. The algorithm starts with a relaxed version of the original LP (linear programing) or MLP (mixed linear programing) problem. Then it is solved with the column generation algorithm at each node created by the branch-and-bound algorithm. The column generation algorithm is mostly used when the generated nodes have a large number of decision variables (columns), thus not being possible to determine with a traditional LP solver (e.g. CLPEX) (Barnhart et al. 1998a).

To check for the optimality of the solution a subproblem (i.e. pricing problem) is created where the dual LP is solved in order to isolate the new columns to enter the basis (i.e. set of columns). If any column is found, the LP is again optimized. When there are no more columns to enter in the basis and the LP does not respect the constraints another branching process happens. This process repeats until the optimal solution is found (Barnhart et al. 1998b).

### Heuristics

Heuristic is a solution strategy that is normally used when a problem with a discrete search-space is too complex (i.e. computationally intractable) to be solved by an exact approach within a reasonable practical time. In contrast to aforementioned exact methods, heuristics do not guarantee the globally optimal solution. Thus, with heuristics is possible to find a good quality solution using limited computer resources. Within the generated solutions, is probable that some are near the optimum, however there is no assurance of reaching the optimum solution (Osman and Kelly 2012). Heuristics can be also used to generate an initial feasible solution (e.g. Randomized greedy algorithms) or to seek another solution in a local neighborhood (e.g. local search). The most used heuristic algorithms are: simulated annealing, genetic algorithms, tabu search, particle swarm optimization, bee algorithms and ant algorithms Yang (2010).

Furthermore, some heuristic technics are used (to decrease the search-space) in combination with exact models to create hybrid models, enabling the formulation of all sorts of restrictions and conditions (Weide 2009).

## 3.5 Network Modeling Techniques

In order to build a mathematical model that represents the all the airline planning stages, one was to select the most suitable network representation. Moreover, each network can have a cyclical period where the planed schedule is repeated for a finite planning horizon, or instead a non-cyclical approach where there is a well-defined initial and final activities. In the literature, the two networks most often used for representing a flight schedule are: connection and time-line networks. (Sherali et al. 2006, Liang et al. 2011) .

The connection network is modeled as a directed graph where each node represents one activity (flight or maintenance) and an arc represents the connection between the outbound and inbound flights. Every two nodes are connected by an arc, when the outbound airport of a particular node matches the inbound airport of other node and the departure time of outbound flight is later than the arrival time of the inbound flight. Furthermore, to connect a maintenance node with a flight node we have to ensure that the outbound flight airport is in the same place as the maintenance station. When a connection satisfies all the above mention constraints the arc is named “legal” and it is represented in the graph. Otherwise, the connection is not included in the network and is called “illegal” (Barnhart et al. 1998a, Grönkvist 2005). In Figure 8 is represented a connection network for a generic schedule with 7 flight pairs.

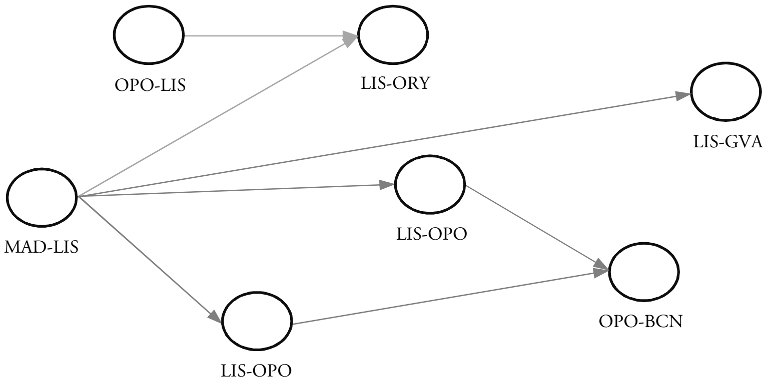


**Figure 8.** Connection network example

The time-line network, studied first by Hane et al. (1995), has a time-line for each airport or maintenance station, with nodes representing the arrival or departure of a specific flight (at an exact hour), and activities defined by arcs that connect inbound and outbound nodes. This network representation is smaller when compared with the connection network, although there is no differentiation amid individual connections, thus being a relaxed version of the connection network. For instance, it is impossible to distinguish different turnaround times between medium-haul and long-haul flights (Grönkvist 2005, Haouari et al. 2009) . Figure 9 shows a time-line network for a timetable that entails 3 airports, 2 aircrafts and 8 flights with a fixed one hour turnaround time between them.

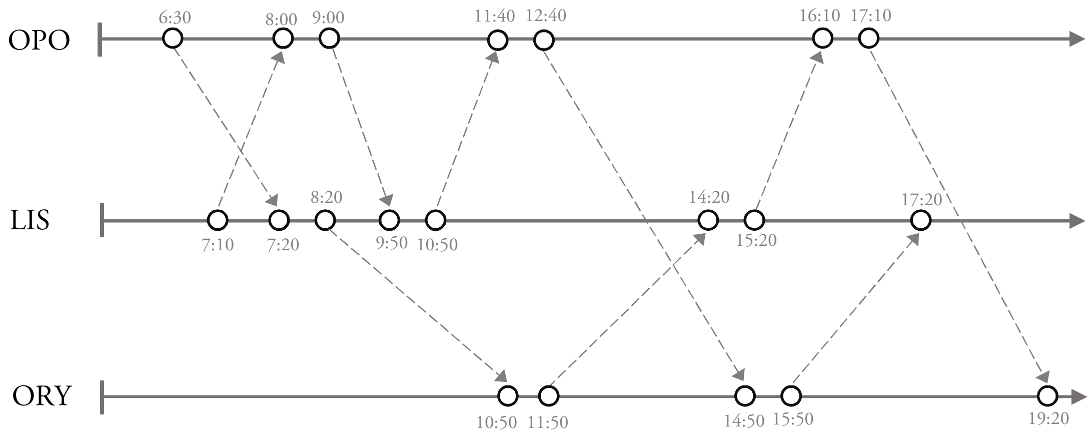


**Figure 9.** Time-line network example

## 3.6 Chapter Conclusions

In this chapter we have presented the key concepts regarding the airline planning process, with focus on the tail assignment, maintenance routing and solution methods. This literature review helped to make a bridge between the published studies in this area and TAP's current tail assignment. Furthermore, it will help to define the approach and the main path to be studied and applied in this dissertation.

As we have seen in this literature review, the tail assignment problem did not have received as much attention as the other planning phases. In the aircraft assignment phase, most studies are related with the maintenance routing problem, where the main motivation in to find a feasible line-of-flights plan without considering the heterogeneity between aircrafts. Furthermore, most studies only consider pre-scheduled maintenances, not considering usage dependent maintenances. It is important to note that only a few authors consider non-cyclic planning schedules, which is the type of schedule used by TAP.

Grönkvist (2005) is one of the few authors that considers both tail assignment and aircraft routing in the same model. In his study he includes all the maintenance obligations, but does not make a distinction between different aircrafts. Moreover, Lapp and Wikenhauser (2012) is one of the first studies to consider fleet heterogeneity, by including in their model different aircraft fuel consumption profiles. However, the authors have not considered any type of maintenance services in their tail assignment study.

Finally, it is noticeable in the literature that most of the studies regarding the tail assignment problem and maintenance routing are solved with a heuristic method instead of an exact method. The motivation to use heuristic approaches lies in the fact that the models being studied are usually very complex with a great number of decision variables. Furthermore, hybrid approaches are also many times used in literature, where authors combine a heuristic method with an exact method in order to achieve the optimum solution.

# CHAPTER FOUR

## Adopted Solution Method

To decide the solution methodology that will be applied in this study, we have used an adapted IP formulation from Ribeiro (2012) implemented on GAMS (General Algebraic Modelling System) and solved with CPLEX to test if we could get a solution in a reasonable amount of time. For this test we have considered approximately 1200 flights (corresponding one week of short-haul and medium-haul operations) and 42 tails. Additionally, this model considered fleet heterogeneity, where were included all the costs described in the case study, but not maintenance services. The results showed that the model was too complicated to be solved with a traditional exact solver, showing the error “out of memory”.

As we were not able to use an exact approach to solve the tail assignment problem for TAP’s short and medium-haul operations, we have decided to use a heuristic algorithm approach. As we have seen in literature, some authors have already used heuristic algorithms to optimize parts of the airlines planning process. For this work, we have choose to use the simulated annealing algorithm, as it was been widely use to solve network flow problems that have restrictions (Dowsland 2012). Furthermore, Wang and Yong (2010), used a simulated annealing algorithm to solve a fleet assignment problem - aircrafts are considered homogeneous within the same variant - with 280 weekly activities, 7 variants and 22 aircrafts, where they have obtained satisfactory results.

With this in mind,  the objective of this work is to solve the tail assigment problem that will help TAP allocate the short and medium-haul aircrafts to a large number of flights. The main model is mainly based on the optimization model built by Ribeiro (2012) and Lapp and Wikenhauser (2012) with some alterations.

In section 4.1, we present the mathematical model that was used to built our solution approach. Next, in Section 4.2 we decribe the simulated annealing and the initial feasible solution algorithms that were developed in this work. Finally, in Section 4.3, we retrive the main conclusions of this chapter.

## 4.1 Mathematical Model

In this section we present a mathematical model that can be used in the entire TAP operation, although in this work we will only apply it for the short and medium-haul fleet operations. The model respects different operational constraints such as maintenance, turnaround times and preassigned activities. The objective is to minimize the total direct operating costs (DOC) while considering all the operational restrictions. For this objective function we have considered the maintenance costs, fuel costs, landing charges, navigation rates and costs related with the not fulfillment of the demand. It is important to note that all these costs are flight and aircraft dependent at the same time. Additionally, the proposed model can only be used within a variable finite planning period.

The proposed model is based on the work of Ribeiro (2012) and Lapp and Wikenhauser (2012). The model is very similar to the one proposed by the authors with some differences and extensions. The main difference in our model is that we have considered that flights are aggregated whenever is possible. This means that activities that have only a single possible connection are merged, resulting in a single aggregated activity. The starting time and departure of the aggregated activity are given by the first activity, the total duration is given by the sum of the durations of each activity and finally, the arrival time and airport is given by the last activity. The DOCs of the aggregated activities equals to the sum of the costs of each activity. This modification is particularly effective for hub-and-spoke networks, which is the specific case of TAP.

In this type of networks, the aircraft that performs the outbound flight from the hub to the spoke, is the same that performs the returning flight from the spoke to the hub. Since there is only one aircraft at the spoke at the same time, is clear that the same aircraft has to carry out the inbound flight. In Figure 10 is represented the aggregation of three activities performed by a single TAP aircraft, where this three sequential flights are aggregated into one. As a result, this modification will enable to reduce the number of activities almost in half, which reduces greatly the complexity of the model being studied.

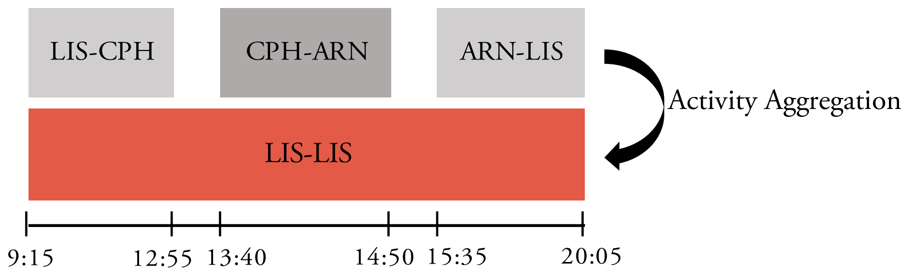


**Figure 10.** Aggregation of three flights, example for the tail CS-TNS

There are two extensions to the model proposed by Ribeiro (2012) and Lapp and Wikenhauser (2012). The first extension is the creation of a new term in the objective function, that penalizes (increases the cost) whenever a demand for a specific flight surpasses the aircraft seat capacity. This cost is calculated based on total amount of US$ in tickets that were not sold because of the airplane seat capacity. The second extension is also a term in the objective function that penalizes the objective function whenever an airplane exceeds the total utilization defined by the user.

### 4.1.1 Model Formulation

In this section we present the mathematical formulation that was used to structure the simulated annealing solution approach. We start by describing the indexes and sets, parameters and decision variables. Then we present the mathematical formulation and explain the objective function and the network flow constraints.

**Index and sets:** In this model there are six different types of sets that comprise the aircrafts, activities and airports. The index sets are the following:

- $a \in A$ – as the set of all the activities, that include all the flights and maintenance activities in a given finite schedule;
- $t \in T$ – as the set of all tails available for the schedule to be prepared;
- $s, ss \in S$ – as the set of all the airports contemplated in the activities defined in set $A$;
- $R$ – as the set of assignment of each activity $a$ to the respective inbound ($s$) and outbound airport ($ss$);
- $IC$ – as the set of all the activities $a$ that cannot be assigned to a given subset of tails $t$. For example, if activity 1 is a maintenance activity then only tail 1 is allowed to perform this activity, whereas the rest of the tails cannot be assigned to this activity;
- $UP$ – as the set of all the possible upstream activities for activity $a$. This set of activities must have the same departure airport as the arrival airport of the previous activity, and the departure time has to be higher than the arrival time of the previous activity;
- $DN$ – as the set of all the possible downstream activities for activity $a$. This set of activities must have the same arrival airport as the departure airport of the next activity, and the arrival time has to be lower than the departure time of the next activity;

- *AD* – as the set of all the activities *a* that have a higher demand than the seat capacity of tail *t*.

**Parameters:** This model has several different parameters, being most of them only used in the objective function. The parameters defined next are presented as independent, tail dependent and activity (and hub or spoke) dependent.

Independent parameters:

- *fcost* – jet fuel cost per kilogram;
- *putili* – utilization penalization factor.

Tail dependent parameters:

- $fconsump_t$ – fuel consumption in kilograms per block hour for a given tail *t*;
- $mtow_t$ – maximum takeoff weight in tonnes for a given tail *t*;
- $mcost_t$ – maintenance cost per BH for a given tail *t*;
- $tavail_t$ – initial availability time for a specific tail *t*;
- $seatcap_t$ – total number of seats for a given tail *t*;
- $maxutili_t$ –maximum utilization percentage for a specific tail *t*;
- $p_t$ –weight factor for a given tail *t*.

Activity (and hub or spoke) dependent parameters:

- $urate_{s,ss}$ – unit rate of the airport-pair *s, ss*;
- $dfactor_{s,ss}$ – distance between each airport-pair *s, ss*;
- $lcharge_{ss}$ – landing charge for the destination airport *ss*;
- $tmin_{ss}$ – minimum required turnaround time for each airport-pair *s, ss*;
- $tduration_a$ – total duration time in block hours of each activity *a*;
- $fpercent_a$ – percentage of the flight *a* block hours in the total block hours of the whole set A;
- $Tprice_a$ – average ticket price activity for activity *a*;

- *fdemand*$_a$ – total number of tickets sold or forecasted for activity $a$.

**Decision variables:** The model has three distinct binary variables, which are the following:

- $w_{a,t}$ – is 1 if the activity $a$ is the first activity to be assigned to tail $t$ and 0 otherwise;
- $x_{a,t}$ – is 1 if the activity $a$ is assigned to tail $t$ and 0 otherwise;
- $y_{a,t}$ – is 1 if the activity $a$ is the last activity to be assigned to tail $t$ and 0 otherwise;

Now, the mathematical formulation can be written as a multi-commodity flow model:

$$
\begin{aligned}
min \sum_{a\epsilon A:R}\sum_{t\epsilon T}\left[fcost \times fconsump_t \times tduration_a \times \left(\sum_{a\in A:DN} x_{a,t} + w_{a,t} + y_{a,t}\right)\right] + \\
\sum_{a\epsilon A:R}\sum_{s,ss\in S}\sum_{t\epsilon T}\left[p_t \times urate_{s,ss} \times dfactor_{s,ss} \times \left(\sum_{a\in A:DN} x_{a,t} + w_{a,t} + y_{a,t}\right)\right] + \\
\sum_{a\epsilon A:R}\sum_{ss\in S}\sum_{t\epsilon T}\left[mtow_t \times lcharge_{ss} \times \left(\sum_{a\in A:DN} x_{a,t} + w_{a,t} + y_{a,t}\right)\right] + \\
\sum_{a\epsilon A:R}\sum_{t\epsilon T}\left[mcost_t \times tduration_a \times \left(\sum_{a\in A:DN} x_{a,t} + w_{a,t} + y_{a,t}\right)\right] + \\
\sum_{a\epsilon A:R:AC}\sum_{t\epsilon T}\left[(fdemand_a - seatcap_t) \times tprice_a \times \left(\sum_{a\in A:DN} x_{a,t} + w_{a,t} + y_{a,t}\right)\right] + \\
\sum_{a\epsilon A:R:AC}\sum_{t\epsilon T}\left[putili \times (fpercent - maxutili_t) \times \left(\sum_{a\in A:DN} x_{a,t} + w_{a,t} + y_{a,t}\right)\right]
\end{aligned} \tag{4.1}
$$

Subject to:

$$\sum_{a,aa\epsilon A} w_{a,t} \leq 1, \forall t \in T \tag{4.2}$$

$$\sum_{a\in A:DN} \left(x_{a,aa,t} + y_{a,t}\right) = 1, \forall t \in T \tag{4.3}$$

$$\sum_{a,aa\in A:UP} x_{a,aa,t} + w_{a,t} = \sum_{a,aa\in A:DN} x_{a,aa,t} + y_{a,t}, \forall t \in T \tag{4.4}$$

$$w_{a,t} \times tavail_t \leq tavail_t, \forall t \in T \wedge a, aa \in A \tag{4.5}$$

$$w_{a,t}, x_{a,aa,t}, y_{a,t} \in \{0,1\}, \forall t \in T \wedge a, aa \in A \tag{4.6}$$

**Objective function:** Equation 4.1 represents the objective function of the mathematical model, and has 6 different terms. The first 4 terms are related with the minimization of the DOCs and the 2 last terms are penalizations related with tail utilization and demand fulfillment. In the first term is calculated the total fuel consumption of the schedule, which is dependent of the current jet fuel price, the consumption profile of each tail and the duration of each flight. The second term is related with the navigation costs that are dependent on the distance between the departure and arrival airport, the MTOW of the airplane and the specific rate for each connection. The third term gives the total cost of the landing charges. This value is calculated based on the MTOW of the airplane and the landing charge at the arrival airport. In the fourth term is calculated the total maintenance cost that depends on the average maintenance cost per block hour and the number of block hours per flight. The fifth term is a penalization factor that increases the cost of the solution, whenever the demand for a specific flight is higher than the seat capacity of the aircraft. This penalization is based on the average ticket price for each flight. The sixth and final term is also a penalization factor, but related with the utilization of a specific tail. This term depends on the number of BHs assigned for a specific aircraft, the maximum utilization (BHs) defined for each aircraft and the penalization cost factor that is defined by the user. The more the aircraft exceeds the maximum utilization defined by the user, the higher is the penalization. For instance, is the maximum utilization is defined as 30 BHs and an aircraft is assigned to 35 BHs, the aircraft will be penalized for the extra 5 BHs.

**Constraints:** Constraint 4.2 ensures that each aircraft has no more than one first activity assigned to it (if $w_{a,t}$ is equal to zero, it means that the aircraft is not used in on the schedule being study). Constraint 4.3 guarantees that every activity as one and only one aircraft allocated to that activity. The network flow balance constraint (4.4) forces each aircraft to follow a feasible sequence of activities. Constraint 4.5 ensures that a specific tail is only assigned to an activity after becoming available at the beginning of the schedule (some tails might have assignments resulting from a previous schedule or being unavailable due to maintenance). Finally, constraint 4.6 defines the decision variables domain.

## 4.2 Solution Approach Framework

As we have described in the conclusions of Chapter 3, in order to decide the solution methodology for the tail assignment problem of the short and medium-haul TAP operation, we have tested the model developed by Ribeiro (2012) for an entire week of the short and medium-haul operation. As a result, we found that the model was to large be solved with a commercial solver and in a regular computer. With this, and after a literature review, we decided to build a simulated annealing model and three different heuristic models for generating an initial feasible solution. In this section, we explain all the steps that were made to build the solution algorithms.

In Section 4.2.1 is explained the general structure of the solution approach, where is described all the functionalities and features. In the next Section (4.2.2) is described all the auxiliary functions created to assist all the developed algorithms in this work. Section 4.2.3 presents the three different models created to generate an initial feasible solution. Finally, in Section 4.2.4 is explained the simulated annealing optimization approach used for this work.

### 4.2.1 Solution Approach Model Structure

In one of our meetings with the fleet management department of TAP, we have discussed what would be the best solution approach to develop the solution model. It that appointment it was defined that the solution approach that would be used in this work would need to satisfy the following three conditions:

1. Any software used in the solution approach should already be owned by TAP;
2. Easy to implement it as a management system, and with easy input and output data communication with current management systems;
3. Previous knowledge, by someone at TAP, of the solution approach that would enable future alterations and enhancements of the code.

There were two software that matched the requirements of condition 1: Microsoft Excel with VBA and R. For the easiness of use and software familiarity of the author, we decided to use Microsoft Excel 2013 to build our solution approach.

The model developed in Excel has two different types of worksheets: the input sheets and the output sheets. In the input sheets the user inserts all the data needed to run the model, in the same data structure as the output of TAP's Compass software. The output sheets gives the user all the information generated by the program that will assist the decision maker throughout the tail assignment process.

In Figure 11 is represented all the sheets created for this model, where the input sheets are represented in green and the output sheets in red.


**Figure 11**. Model input (green) and output (red) sheets

Following, we explain the data and structure for the different input and output sheets.

### Input Worksheets (Structure and Data)

There are four different worksheets were the user can change the data required for running the model. In the first sheet, Index, the user can change all of the independent parameters that were previously described in Section 4.1.1. The Original Schedule sheet receives the information about the time scheduling and the aircraft that was originally assigned for a specific activity (flight or maintenance). These data are extracted from TAP's Compass software without any modification. Table 5 shows the input data of the sheet that is equal to the output data of TAP's software, except for the pair type column that will be explained in the output worksheet section.

**Table 5.** Output data from TAP's Compass Software for the first four flights of 2016 (extracted from Excel)

| Flight Date | Flight Number | Carrier | From Airp | To Airp | Aircraft Model | Aircraft Reg | Departure | Arraival |
|---|---|---|---|---|---|---|---|---|
| 2016.01.01 00:00 | 1532 | TP | RAI | LIS | 320 | CSTNW | 2016.01.01 01:55 | 2016.01.01 06:00 |
| 2016.01.01 00:00 | 1480 | TP | DKR | LIS | 321 | CSTJE | 2016.01.01 02:00 | 2016.01.01 06:00 |
| 2016.01.01 00:00 | 1548 | TP | SID | LIS | 320 | CSTNQ | 2016.01.01 02:00 | 2016.01.01 05:50 |

The Aircraft Data sheet and the Flight Data sheets receive all the parameters that are aircraft and flight dependent respectively. In the Aircraft Data sheet the user defines all the parameters individually for each tail, as shown is Table 6.

**Table 6.** Input of the Aircraft Data sheet for the tails CS-TTA and CS-TTB (extracted from Excel)

| Tail | Fuel Consumption (kg/h) | MTOW (tonnes) | Nº Seats | Weight factor | Maintenance Cost (US$/BH) |
|---|---|---|---|---|---|
| CSTTA | 2157 | 68 | 132 | 1,17 | 160 |
| CSTTB | 2147 | 68 | 132 | 1,17 | 213 |

For the Flight Data sheet, the information is organized by the flight number, which represents a specific airport-pair (e.g. LIS-OPO). In Table 7, we can see the input data for this sheet, where in each flight

**Table 7.** Input of the Flight Data sheet data for the flights 352 and 567 (extracted from Excel)

| Flight Nº | Airport Pair | Distance Factor (km) | Duration (hours) | Demand | Landing Charge (US$/tonne) | Unit Rate (US$) |
|---|---|---|---|---|---|---|
| 352 | LIS-LHR | 13 | 2,7 | 117 | 7,14 | 45 |
| 567 | HAM-LIS | 20 | 3,4 | 105 | 6,45 | 41 |

number can be defined the flight dependent parameters.

### Output Worksheets (Structure and Data)

The output worksheets are divided in three types: the pre-optimization sheets, the optimization sheet and the post-optimization sheets. In the pre-optimization sheets we included the Optimized Schedule, Cost Matrices, Global Cost Matrix and the Original Matrix.

The Optimized Schedule worksheet aggregates all the flights and gives the initial availability time and airport for each aircraft, which were first inserted in the Original Schedule sheet. To make the aggregation of all the flights and give the initial conditions for each aircraft, we had to build three VBA functions named organize(), unique() and generate() that are run in this sequence. The first function, organize(), writes the type of airport-pair for each flight in the sheet Original Schedule, as shown in Table 5. For instance, if the flight pair is RAI-LIS the function returns T_LIS, meaning to Lisbon. In Table 8 is represented all the codes for each type of flight.

**Table 8.** Airport Pair Codes

| Airport Pair | Airport Pair Code |
|---|---|
| XXX-LIS | T_LIS |
| LIS-XXX | F_LIS |
| XXX-OPO | T_OPO |
| OPO-XXX | F_OPO |
| OPO-LIS | OPO_LIS |
| LIS-OPO | LIS_OPO |
| XXX-XXX | CONECT |
| LIS-LIS | MAINT |

After the identification of all the flights, the unique() function writes all the unique aircrafts in the selected schedule into the Optimized Schedule sheet. Finally, the generate() function aggregates the flights based on the Airport Pair code and the original tail that was assigned to each flight in the original schedule. The output of the generate() function creates two distinct tables. One with all the aggregated flights that contains among other information, the departure and arrival airports and the starting and end times of each activity. In the second table, is given the initial conditions for each aircraft (starting airport and initial time availability). It is important to note that data created in this sheet will be further used by the optimization algorithm. In Appendix 3 is represented the aggregated schedule and the initial aircraft conditions for the first flights of 2016 (short and medium-haul operation).

The Cost Matrices worksheet is composed by 5 cost matrices, representing all the terms in the objective function minus the utilization factor. The matrices have $m$ columns and $n$ lines where the $m$ represents the set of all aircrafts that are available for the schedule being studied, and the *n* represents the set of all aggregated activities for the selected schedule. Furthermore, all aircrafts have an explicit cost for each activity, even if the aircraft cannot perform a given activity (when the activity is a maintenance service the cost for all matrices is zero). The costs of the activities that are not possible to be done by a specific aircraft are further ignored by the optimization model. In order to build the 5 matrices, it was created the costmatrices() function that identifies all the flights within each aggregated activity, and calculates the cost for each aircraft. In Appendix 4 is represented the fuel cost matrix for the first five days of 2016 for the short and medium-haul fleet. Moreover, the Global Matrix sheet is simply a matrix, in which we have the sum of all 5 matrices.

In the Original Matrix worksheet is created a matrix, based on the original TAP tail assignment, where each and every single aggregated activity has one aircraft assigned. For example, if tail CS-TNW is the airplane assigned to activity 1, then it will appear in the respective cell the number 1 and nothing in the remaining of the column. The matrix is built using the Excel vlookup() function, that searches in the Optimized Schedule sheet for the tails performing each flight. This matrix will be used further to compare the original solution with the improved one. In Appendix 5 is represented the original schedule matrix for the first 40 activities of 2016.

The optimization sheet SA is where the main models (initial solution and simulated annealing algorithms) are implemented. The structure of the sheet is similar with the Original Matrix sheet, with some additions. In Appendix 6 is shown the matrix with the additional data. In the top rows above the list of activities, from the top to the bottom we have: the maintenance row that indicates if the activity is pre-assigned maintenance job or flight, the percentage of each flight in the total schedule (calculated based on the BHs), the departure and landing airports (where 1 represents the Lisbon airport and 2 the Porto airport) and finally we have the departure and arrival time for the specific activity. From the leftmost columns to the right is represented: the objective function for each aircraft, the utilization based on the current assignment, the initial availability time and airport, and lastly the maximum utilization allowed for each aircraft.

To conclude, we have created 3 post-optimization sheets (Charts, Gantt chart and SA Schedule), that will help the decision maker analyze and compare the new with the original tail assignment. The Charts sheet gives all type of charts and information comparing the new solution with the old one. The Gantt Chart sheet is pretty much self-explicative, in the sheet is created a Gantt chart for the new solution. Finally, the SA Schedule sheet is the output list of activities in the same structure as the input list of activities of TAP's Compass software. All this 3 sheets are constructed with Excel built-in charts and formulas.

### 4.2.2 Algorithms Auxiliary Functions

To assist the simulated annealing and the initial feasible solution models, three auxiliary function were created: availability(), availability2() and assigntail(). The objective of the function availability(), is to evaluate which aircraft is available for a specific activity at a given moment. The input of the function is a number that corresponds to the activity number we want to analyze, and the output is a list of all the aircrafts available to perform that flight and the cost of each one for the selected activity. In Table 9 is shown the output for activity number 3 (corresponding to the third activity of 2016). In this table we can see that we have a total of 3 aircrafts that are able to perform the selected activity. Additionally, the aircrafts are ordered from the lowest cost to the highest.

**Table 9.** Output of the function availability for the activity 3

| Aircraft | Total Cost |
|---|---|
| CSTQD | 8852,84 |
| CSTJE | 9130,83 |
| CSTJF | 9915,982 |

The Pseudo-code for the availability() function is shown in Table 10. In a cycle (2), we check the availability (5, 10) for each airplane and write the airplane and the respective in the output table (15).

**Table 10.** Pseudo-code for the availability() function

| | Availability() function Pseudo-code |
|---|---|
| 1 | **For** activity a |
| 2 | **While** n is lower or equal than the total number of aircrafts |
| 3 | **For** aircraft n |
| 4 | Find the activity before performed by the same aircraft |
| 5 | Check if the departure and the arrival airports match and if the end time of the previous activity is lower than the start time of the selected activity |
| 6 | **If** one of the above conditions is not met **then** |
| 7 | Availability is equal to False |
| 8 | **End if** |
| 9 | **If** availability **is not** False **then** |
| 10 | Check if the arrival and the departure airports match and if the start time of the next activity is higher than the end time of the selected activity |
| 11 | **If** one of the above conditions is not met **then** |
| 12 | Availability is equal to False |
| 13 | **End if** |
| 14 | **End if** |
| 15 | **If** availability is True **then** |
| 16 | Write the aircraft and the associated cost and short by the lowest to highest cost |
| 17 | **End if** |

The availability2() function is very similar with the availability() function. Instead of giving the available aircrafts for a single activity, availability2() returns the number of the activity that is not allowing the specific aircraft perform the flight. This function is used when there is no available aircraft to do a specific activity. For example, if we are trying to assign an aircraft to activity 1, but there is no available aircraft to perform that activity, we can call the function availability2(1). The function will return the number of the activity, that when deleted, will allow the assignment of an aircraft to activity 1.

To conclude the auxiliary functions, assigntail() picks one of the aircrafts from the output of the availability() function and assigns it to the selected activity. The choice of the aircraft to be assigned can be purely random, partially random and greedy. When purely random, the function selects randomly one of the aircrafts available to perform the activity. Partially random, the function selects randomly between the n aircrafts with the lowest cost, where n is a number defined by the user. Finally, if greedy is chosen, the aircraft with the lowest cost available is allocated.

### 4.2.3 Initial Feasible Solution Algorithms

In order to generate an initial feasible solution three models were created: First-in, First-out (FIFO), Last-in, First-out (LIFO) and Random assignment model. Each one of this methods can be purely random, partially random or greedy, depending on what is chosen in the assigntail() function.

**First-in, First-out Model**

The FIFO algorithm is the simplest of all three. The method starts in activity 1, and assigns based on the method chosen (e.g. greedy) the aircraft. In a cycle, the algorithm goes through every single activity, assigning an airplane to each one. This process is repeated until the final activity is reached. When the algorithm encounters a pre-assigned maintenance activity, the activity is skipped without assigning an aircraft, this way we guarantee that the activity is performed by the right aircraft. The main advantage of this model is the simplicity which also keeps the time needed to run this algorithm exceptionally low (solving time results in Section 5.2.2).

**Last-in, First-out Model**

Just like FIFO algorithm, LIFO assigns each activity sequentially. The main difference between the two is the assignment order. The LIFO method assigns each activity from the last to the first. Differently, from the FIFO method were the only restrictions to be satisfied are the ones from previously assigned activities, in the LIFO method we have activities that were previously assigned and the initial conditions of each aircraft. In FIFO algorithm there is always an aircraft available for each activity, on the other way in LIFO, sometimes there is no aircraft available to be assigned to a specific activity. For these situations, we have created the availability2() function. When there is no aircraft available to perform an activity, LIFO algorithm calls availability2() that gives the list of activities that can be changed to another aircraft. By changing the airplane performing one of this activities we are able to assign an aircraft to the first activity. This method is slower than the FIFO model, because of the infeasibilities that are found during the process.

**Random Assignment Model**

This model, unlike the other two models, assigns each activity randomly without a specific order. The activities are only visited once during the algorithm, unless there is an infeasibility problem. In this case, similarly in what happens with the LIFO model the algorithm tries to find activities where the assigned tails can be changed to different ones, in order to solve the infeasibility. This assigning method has very similar solving times to the LIFO algorithm.

### 4.2.4 Simulated Annealing Algorithm

Simulated annealing, firstly introduced by Kirkpatrick (1983), is a random-search meta-heuristic that tries to mimic the natural cooling process of metals, into a minimum energy crystalline structure. The process starts with an initial feasible solution and progressively reaches improved solutions by randomly changing the previous solutions. The process is initiated with a very high “temperature” which makes it very likely to accept worst solutions. The “temperature” then starts to decrease, according with an annealing schedule, and the probability of accepting worst solutions is lower. This accepting method avoids the solution to becoming trapped in a local optimum, which is the main advantage of this method when compared with others. Furthermore, this method is particularly suited to solve large combinatorial problems, which is the case of the tail assignment problem.

The simulated annealing process was three distinct phases in order to find the best solution for a given schedule: Initialization, Iterative process and Stopping. In the initialization process is defined all the simulated annealing parameters and is created an initial solution. In the iterative process are created new solutions that are evaluated and that can be accepted or not. In the last phase, if the stopping criteria are meet than the model stops and returns the last reached solution.

In Figure 12 is explained the algorithm procedures used for this work. Moreover, we have divided the method into 6 main points as represented in the diagram. After the figure, is given a brief explanation about each phase.

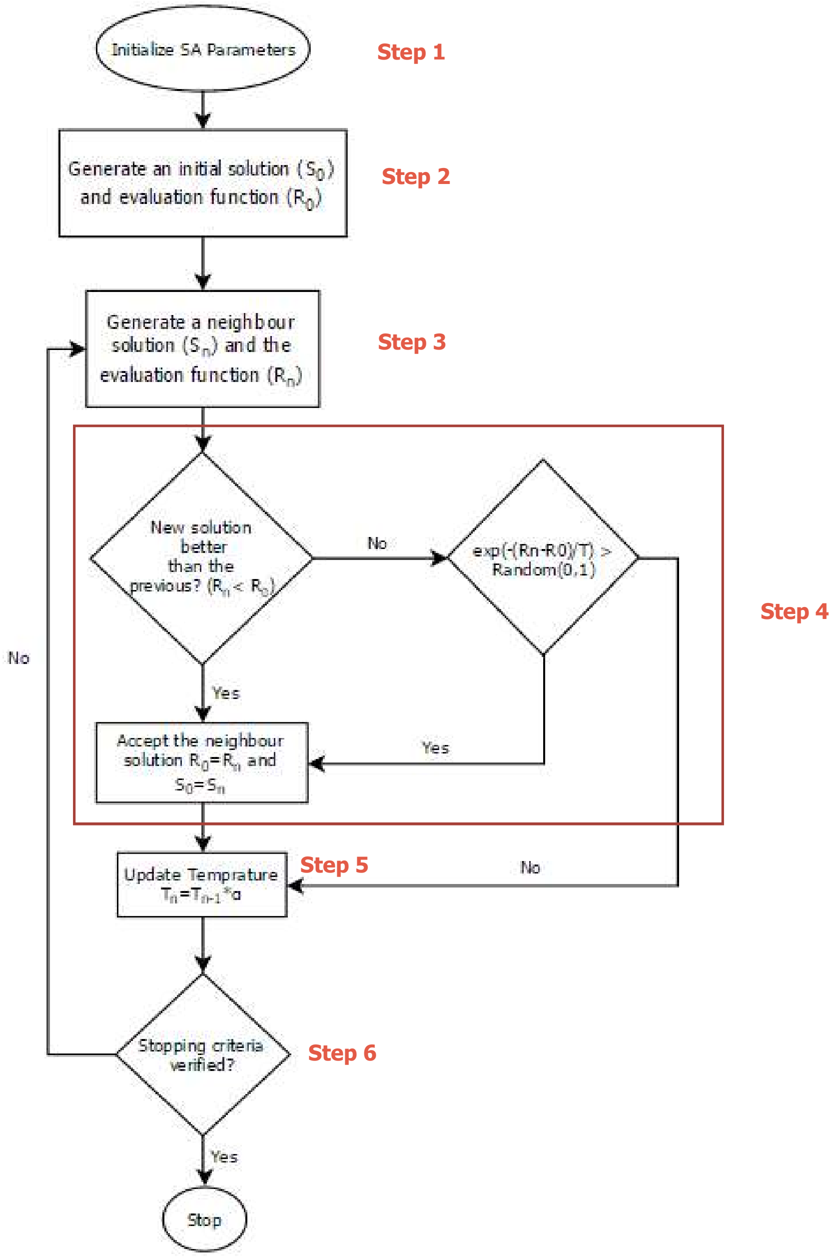


**Figure 12.** Simulated annealing algorithm flow diagram

1. The first procedure is definition of the initial parameters that will be used during the algorithm. These include, the initial temperature, the annealing schedule (number of iterations at the same temperature) and the stopping criteria (minimum temperature);

2. In this step the algorithm generates an initial feasible solution, using one of the models described in Section 4.2.3, and uses the objective function to evaluate the initial feasible solution;

3. Here is defined a new solution, based on an adaptive local neighborhood search algorithm that is explained further in Section 4.2.5, and it is evaluated based on the objective function;

4. The new solution is compared with the previous accepted solution. If the new solution was a lower cost than the previous, this solution is automatically accepted. When the neighbour solution is worst (higher cost), the probability of acceptance is defined by Bolzmann machine function with $P_{acceptance} = e^{-\frac{(NewSolution - PreviousSolut\quad)}{Temprature}}$ (5). To accept the new solution a random number between [0, 1] is generated. If $P_{acceptance} > Randomnumber$ the new solution is accepted, otherwise the neighbour solution is rejected. As the temperature decreases is less likely to be accepted worst solutions.

5. After a defined number of iterations the temperature decreases by a cooling constant $\alpha$, where temperature is defined by the formula: $T_n = T_{n-1} \times \alpha$ (6). This enables the solution to escape the potential local optimum in the beginning of the algorithm (higher temperatures), and to fix the solution at the end of the algorithm (lower temperatures). The temperature is updated after a number of iterations defined by the user. For instance, if the number defined is 200 the temperature will only be updated after 200 iterations.

6. The model stops after $n$ consecutive temperature levels (the value for $n$ is defined in Section 5.2.2.1) wherein the difference between the actual solution and the best solution found during the process is equal or inferior than 0.01% of the value of the best solution, $(R_0 - R_{best}) \leq 0.0001 \times R_{best}$ (7). This criteria keeps the algorithm running until the probability of accepting new is very low. This stopping method enables the stabilization of the solution value, which is very important due to the combinatorial extension of the problem. If the stopping criteria is met the algorithm stops, returning the current solution, if not, is initiated a new iteration.

All the parameters including the initial temperature, the number of iterations before temperature updating and the cooling constant ($\alpha$) are analyzed and defined in Section 5.2.2.1.

## 4.2.5 Adaptive Neighborhood Local Search

The adaptive neighborhood local search is an improvement of the traditional neighborhood local search (Pisinger and Ropke 2007). In the traditional search method, is used only a single search algorithm to generate a possible neighborhood solution throughout the entire optimization process. Though, it is very difficult to define the best suited method for any size of the problem. It is also easy to understand that the performance of one method can vary during the process. For instance, some methods could be better for the first iterations of the simulated annealing algorithm, where most of the solutions are accepted, and others could be more suited to the end of the algorithm where the probability of acceptance is lower.

The adaptive search enables the use of different search methods within the same optimization algorithm. Each method is chosen randomly, where the probability of being selected depends on the performance of previous iterations. All of the methods start with an equal probability of being selected. The new probabilities are calculated at the same time of the simulated annealing temperature readjustment. These new probabilities are defined by calculating the total amount that a specific method contributed to the objective function divided by the time spent on that method (8). These ratios are then compared with each other and the higher is the ratio, the higher is the probability of method to be chosen in future iterations.

$$r_{improvement}\ (n) = \frac{Total\ improvementof\ method\ n}{Total\ time\ spent\ on\ method\ n} \tag{8}$$

After the ratio comparison between all search methods, the probability is defined based on predefined values. For example, if we have two search methods (method 1 and method 2) that start with the same probability of being chosen (50%), after the first temperature level the method 1 has a higher ratio of improvement than method 2. In this case, the probability of method 1 to be selected will be higher than method 2. The new probabilities are defined by the decision maker, for instance method 1 will now have a probability of being selected of 75% and method 2 only 25%. This method of defining the new probabilities guarantees that there is never a scenario of assigning a null probability to a specific method. The probabilities defined for the solution approach created are discussed in Section 5.2.2.1.

For this work we have created two different local search methods: activity change and Line-of-activities change. Following, we explain each one of this methods by giving a description and an application example.

### Activity Change Method

The activity change is the simplest method among the two. This method randomly searches an activity and tries to change also randomly the aircraft that is currently assigned. If the selected activity has only one aircraft available to perform that activity, the model continuous the search until is found one activity with two or more available aircrafts. For instance, giving that the model selects the first activity of 2016 which is currently performed by the tail CS-TTR. There are 1 aircrafts available to perform this activity. The model chooses the only available aircraft (i.e. CT-TTN) and swaps the aircrafts. In Figure 13 is represented the swap of activity 1 (OPO-ORY-OPO) between the tail CS-TTR and tail CT-TTN.

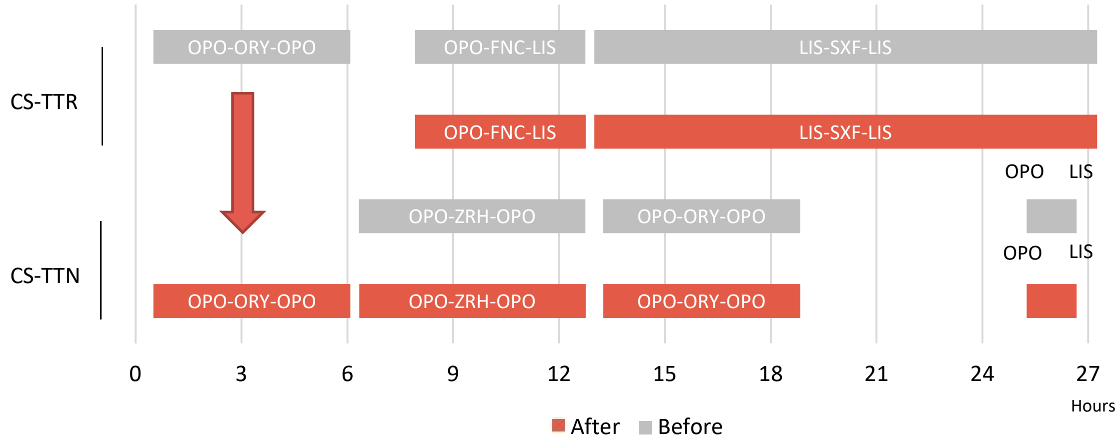


**Figure 13.** Activity Change between the tails CS-TTR and CS-TTN

### Line-of-Activities Change Method

This method is consists in swapping the line-of-activities of two aircrafts at a given point in schedule. The method starts by searching randomly for an activity with at least one available aircraft, besides the currently assigned aircraft, to perform the activity. The aircrafts selected to swap the line-of-activities must not have a maintenance activity after the selected activity, otherwise the method starts the search again. When this two conditions are met, the line-of-activities from the selected activity until the end of the planning horizon are swapped between the two aircrafts. In Figure 14**.** Line-of-activities change between the tails CS-TTR and CS-TTN is represented the swap of the lines-of-activities between tail CS-TTR and tail CS-TTN.

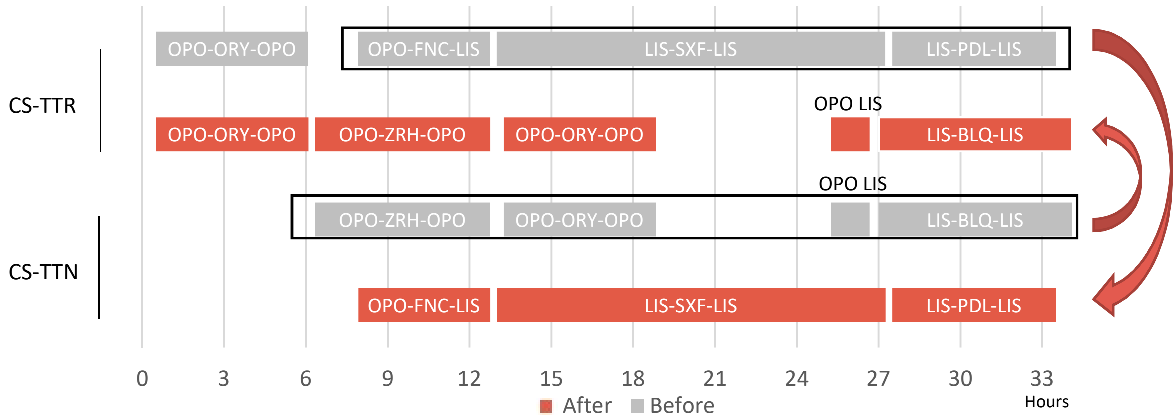


**Figure 14.** Line-of-activities change between the tails CS-TTR and CS-TTN

## 4.3 Chapter Conclusions

The Simulated annealing model presented in this chapter was built with the purpose of assining eficiently tails to the scheduled flights. The creation of the model is inspired in the mathematical formulation proposed by Ribeiro (2012) and Lapp and Wikenhauser (2012), although the approach to the problem is completely different. As we are dealing with larger number of flghts and aircrafts, it was intractable to solve the problem troughtout a mathematical solver. Additionally, we have constructed a adaptive local neghberhood search that enables to reach better solutions in shorter times. This model will enable TAP to reach an optimized solution for the short and medium-haul operations in a reasonable time frame.

Furthemore, we have created three distinct methods to generate an initial feasible solutions. This methods can give a good solution if the decision maker needs a faster approach to the problem. Troughout, the next chapter we will analyse different scenarios and compare the different models with distinct schedules. Nevertheless, the next chapter starts with the data collection that will be further used to test the models and the description of the diffent scenarios that will be analysed.

# CHAPTER FIVE

## Case Resolution

In the previous chapters, we have given a contextualization of the tail assignment problem, by making the characterization of TAP's current operations and short and medium-haul fleets. We have seen the current limitations of this process and after we have made a research to see the best practices regarding all the planning phases, with a greater focus on the main subject of this work (tail assignment phase). Finally, we have explain our approach to tackle this problem, where we have created a simulated annealing model with an adaptive local search method that will provide optimized solutions for the tail assignment process that will minimize the overall DOCs.

In consonance with the work developed in the aforementioned chapters, in this chapter we will present the results obtained for all the algorithms developed during this work. In order to test the algorithms, in Section 5.1, is detailed all of the data necessary as input for the developed models and the different schedules that will be used further to assess the models. Next, in Section 5.2.2, is analyzed the three different models created to generate an initial feasible solution. Section 5.2.3, presents the results for the main model, considering different scenarios and configurations. Finally, Section 5.2.3.4, is given a sensitivity analysis where we evaluate the robustness of the results achieved.

## 5.1 Data Collection and Analysis

In this section we present all the data required as input for the built models. This data collection and analysis is divided in three different sections: independent data, aircraft related data and activity related data. The same structure was used to explain the model formulation in Section 4.2.

### 5.1.1 Independent Related Data

The independent data consist in all the information that do not change between the assignment of an aircraft or activity. There are only two types of data used in this model that have this characteristics, the jet fuel price and utilization penalization factor.

**Jet Fuel Price**

The jet fuel price considered for this work is 0,427 US$/Kg. The source for this value has the IATA web site that was consulted in September 2016[1]. Further in this chapter, we carry a sensitivity analysis where we increase in 25% and a decrease in 10% the fuel price.

**Utilization Penalization Factor**

The utilization penalization factor is a value defined by the decision maker to penalize the objective function, whenever that an aircraft exceeds the maximum defined utilization. This value has to be big enough, in order to have a meaningful impact in the objective function. This cost increases linearly from the time that an aircraft surpasses the defined value of utilization. For example, if we define a penalization value of $10000 and a maximum utilization of 5% the cost of assigning a tail that surpasses this value would increase as represented in Figure 15. As we can observe in the chart, the higher is the usage percentage the higher will be the penalization in the objective function.

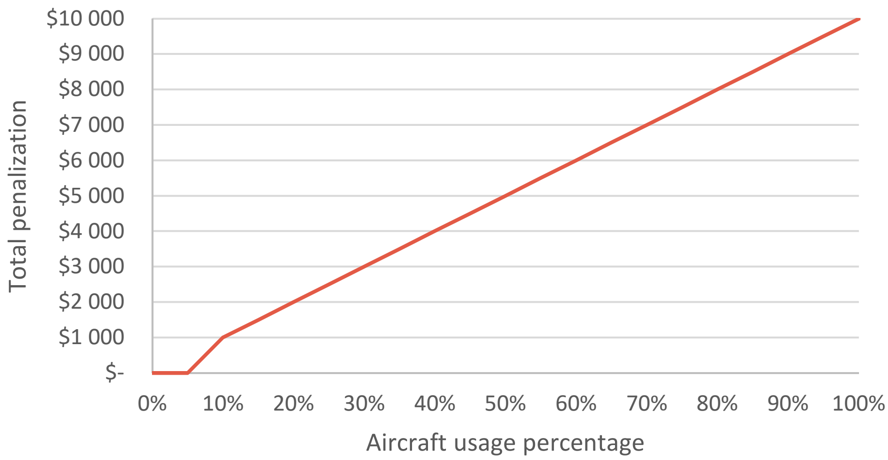


**Figure 15.** Penalization factor increase, for a penalization value of $ 10000 and a maximum utilization of 5%

Furthermore, the penalization factor value, is further analyzed in Section 5.2.2.3.

## 5.1.2 Aircraft Related Data

As described in Section 2.4, for this work we will take into account all of the 43 Airbus aircrafts that are part of the short and medium-haul operations. Where 21 are from the variant A319, 19 are A320

[1] Source: http://www.iata.org/publications/economics/fuel-monitor/Pages/price-analysis.aspx consulted at 23 of September 2016.

and 3 are A321. For the aircrafts, the input data is: fuel consumption, maintenance costs, number of seats and the MTOW.

**Fuel Consumption**

For the fuel consumption we have considered the historical data from the beginning of 2015 until September of 2016. The value (in Kg per block hour) has calculated by making the average fuel consumption for each aircraft in each flight. In Table 11 is represented the average, minimum and maximum consumption by variant for the 42 short and medium-haul aircrafts. As we can see, the higher is the number of the variant, the higher is the fuel consumption. This is mainly related with the size of the airplane, the A321has 200 seats and the A319 has only 132 seats. There is approximately an increase of 26% in fuel consumption between the A319 variant and the A321.

**Table 11.** Fuel consumption statistics for the short and medium-haul fleet

| | Aircraft Variant | | |
|---|---|---|---|
| | **A319** | **A320** | **A321** |
| Maximum Consumption (kg/BH) | 2,149 | 2,332 | 2,727 |
| Average Consumption (kg/BH) | 2,172 | 2,290 | 2,735 |
| Minimum Consumption (kg/BH) | 2,096 | 2,266 | 2,712 |

**Maintenance Costs**

The maintenance cost represents the sum of all costs related with maintenance services done in a specific aircraft. At the time of this study, the maintenance costs for the short and medium-haul fleet where not yet calculated by TAP. With this in mind, we have decided to make an extrapolation of the engines maintenance values considered by Ribeiro (2012) in her model. Although, it would be better to utilize the full cost of the maintenance services, the engines maintenance cost represent more than 60% of the total maintenance costs, having the biggest impact on the aircraft overall maintenance cost.

To extrapolate the data from the study, we have considered that the maintenance cost increases linearly with the size of the aircraft (i.e. number of seats). To calculate the values for each variant, we have used the engine maintenance value for the A330 (453 $US/h) and total number of seats (268). This gives a value of 1.7 $US/h/Seat. Multiplying this value by the number of seats in each short and medium-haul variant we have: 224.4 $US/BH for the A319, 275.4 $US/BH and 340 $US/BH. The maintenance cost has great impact in TAP's overall results. For this reason, we carry a sensitivity analysis in Section 5.2.2.4 to analyze the impact of the increase or decrease of this cost in the DOCs.

### Number of Seats

The number of seats in each variant will have a direct impact on the revenues in each flight. Despite the existence of different seat categories (economical and business class), for this study it was not considered this distinction between this seats. For the variant A319 were considered 132 seats, for the A320 162 seats and for the A321 200 seats.

### MTOW

The MTOW is a certified value that is equal for every aircraft within a specific variant. With this value it is possible to calculate the weight factor ($= \sqrt{MTOW/50}$). The MTOW (used for the calculation of the landing charges) and the weight factor (used for the calculation of the navigation costs) for each variant are represented in Table 12.

**Table 12.** MOTW and weight factor for the short and medium-haul fleet

| | Aircraft Variant | | |
|---|---|---|---|
| | **A319** | **A320** | **A321** |
| MTOW (tonnes) | 68 | 77 | 89 |
| Weight Factor | 1.17 | 1.24 | 1.33 |

## 5.1.3 Activity Related Data

As mention in previous chapter, for this study is only considered the short and medium-haul operations that comprises all the flights in Europe and North Africa. The number of activities in this type of operation has a significant variation between the high season period and the low season period. For the short and medium-haul operation, in the low season we have an average of 1200 flights per week and for the high season 1450, resulting in an increase of 21%. Throughout this section, we will look into the following parameters: distance factor, unit rate, minimum turnaround time, landing charge, ticket price and flight demand.

### Distance Factor

The distance factor is a parameter used for the calculation of the navigation costs. This value, represents the one hundredth of the great circle distance between two points in a given charging zone, where each charging zone has a delimited airspace and a specific cost associated (EUROCONTROL 2016). The route used by each aircraft is only defined in the day-of-operations. Based on this fact, is impossible to know a priori what would be the exact navigation cost. For this reason, in this study, we have considered the distance factor as the great circle distance between the departure and arrival airports. This is a good approximation, has the sum of all the great circle distances overflown within each charging zone is similar with the value of the great circle distance between an airport-pair. As an example, the great circle distance between the airport-pair LIS-OPO is 275.5 km and the corresponding distance factor is 2,755 km.

**Unit Rate**

The unit rate, calculated by TAP, is the quotient of the average navigation costs (between two different airports) by the weight and distance factors. This means that every airport-pair considered in this model as a different unit rate.  To simplify, these unit rates represent the charges that each aircraft was to pay by overflying different charge zones for a specific leg.

At the time of this study, TAP did not have the values for the short and medium-haul operation. Because of this, and as this rate is independent of any other data, we have assigned randomly the unit rates of the long-haul operation used in Ribeiro (2012) study to the short and medium-haul operations. The average unit rate for all the airport-pairs is 54.3 $US, where the maximum value is 59.7 $US and the minimum is 50.3 $US.

**Minimum Turnaround Time**

As previously mentioned, the minimum turnaround time represents the minimum time required by an aircraft between two activities. This value depends on many operational variables (handling, check-in, aircraft cleaning, inspections, etc.) that normally have a specific planned time to be performed, but due to several different reasons can have a delay. This makes it impossible to know in advance the exact time that an airplane is going to be on the ground between the arrival and departure activities. As a simplification, TAP defined that for the short and medium-haul operations this time is on average 45 minutes. With this, for this model, is used the minimum 45 minutes turnaround time between each flight.

**Landing Charge**

The landing charges represent the costs associated to the utilization of a specific the airport. These costs are defined by the government or by private companies that explore the airports. For this model we use the data from latest ICAO report publicly available (ICAO 2010). As this data is from the year 2010, we have used an average inflation rate of 1.52% to update the values to 2016. From the landing charges retrieved from the various airports, the average charge is 12.7 US$/MTOW.

### Ticket Price

The ticket price for each flight is used in this model as a penalization cost. The idea is to input the loss of the tickets that are not sold, due to bad matching between aircrafts and demand, into the objective function. Moreover, the number of seats occupied in an aircraft have a small impact on the total operating cost. Besides the catering and some administrative services, the other costs operating costs do not change with the number of passengers. This means that is essential for TAP to sell the maximum number of tickets possible. If the aircrafts are assigned properly, taking into account the demand for the flight, there is a potential increase in revenues.

For this study we have made some assumptions to determine the ticket price for each flight, as the historical average ticket price were not available at the time of this study. To calculate the average ticket price for a given flight, we have divided the average DOCs for that flight with the average number of seats for the fleet being studied (for the short and medium-haul fleet is 150 seats). The calculated value was then multiplied by a factor of 1.4 that represents the IOCs and the operational profit margin. As an example, the average ticket price, calculated with the formula described above, for a flight between Lisbon and Rome, costs 92$US.

### Flight Demand

As previously mentioned, the flight demand data enables the model to assess the exceeding demand for a certain flight and aircraft. For the results in this work, we have used historical data for the number of seats occupied for a given activity. If we instead of using past schedules, used future schedules to assess the solution approach, the input data should be the forecasted demand.

## 5.2 Results and Evaluation

In this section we present the results and all the scenarios considered for the solution approach. To assess the performance of our approach, we use different schedules with different planning horizons. Furthermore, a sensitivity analysis is carried where is changed some of the parameters first considered.

The results for the proposed solution approach designed in Microsoft Excel 2013 64-bits were obtained using an Intel® Core™ i5-3317U CPU at 1.70GHz with 2 cores and 4 logical processors, and with 8.00 GB RAM. It is important to note, that VBA for excel is a single-threaded application, meaning that all the functions written in VBA only use one processor at the time instead of the 4 available.

### 5.2.1 Schedules and Scenarios Considered

As mention in Case Presentation Section, TAP operation is cyclical, this means that there is none or a very small difference between the schedules for one week of operation in the same season. Given that evidence, there is no need to use several different schedule weeks. To test the solution approach, four different schedules where considered, where two of them represent the low season period and the other two the high season period. For each season period it was considered two schedule lengths: 5 days and 7 days. In Table 13 is represented all the schedules used during this study, where is described various components of each schedule. Is worth mentioning, that although the number of aircrafts for the fleet being studied are 43, in some schedule periods there are airplanes that are not available during that interval due to long maintenance inspections or other restriction.

**Table 13.** Scenarios considered for this study

| Season Period | Instance | Schedule Period | | Nº of Activities | | Nº of Available Aircrafts | Block Hours | Original Solution Cost |
|---|---|---|---|---|---|---|---|---|
| | | Start Time | Start Time | Single | Aggregated | | | |
| **Low Season** | LS5 | 01/03/16 | 05/03/16 | 788 | 395 | 41 | 1936 | $ 3,935,410 |
| | LS7 | 06/03/16 | 12/01/16 | 1171 | 592 | 42 | 2925 | $ 5,874,500 |
| **High Season** | HS5 | 01/07/16 | 05/07/16 | 1011 | 489 | 43 | 2397 | $ 4,840,198 |
| | HS7 | 01/07/16 | 07/07/16 | 1421 | 699 | 43 | 3430 | $ 6,921,284 |

By analyzing the schedules, we can see that in the low season the number of activities is significantly lower. In the high season, a schedule of 5 days as almost many flights than a schedule of 7 days in the low season. Also, the number of aircrafts available in the low season is inferior than the number in the high season. This happens because TAP takes advantage of the low demand periods to anticipate long maintenance services.

Furthermore, in Figure 16, is represented the results and analysis structure for this chapter with all the scenarios considered for this work. In the first step we assess the initial feasible solution models where we compare the three algorithms. After, in the second step, we analyze the constructed solution approach considering two different scenarios, and a sensitivity analysis where we change the value of the fuel cost and maintenance cost to see how robust the solutions are. In the first scenario, we do not consider maximum usage percentage for each aircraft, meaning that there is no usage penalization. For the second scenario we have considered an evenly usage percentage where aircrafts are penalized if this value is exceeded.

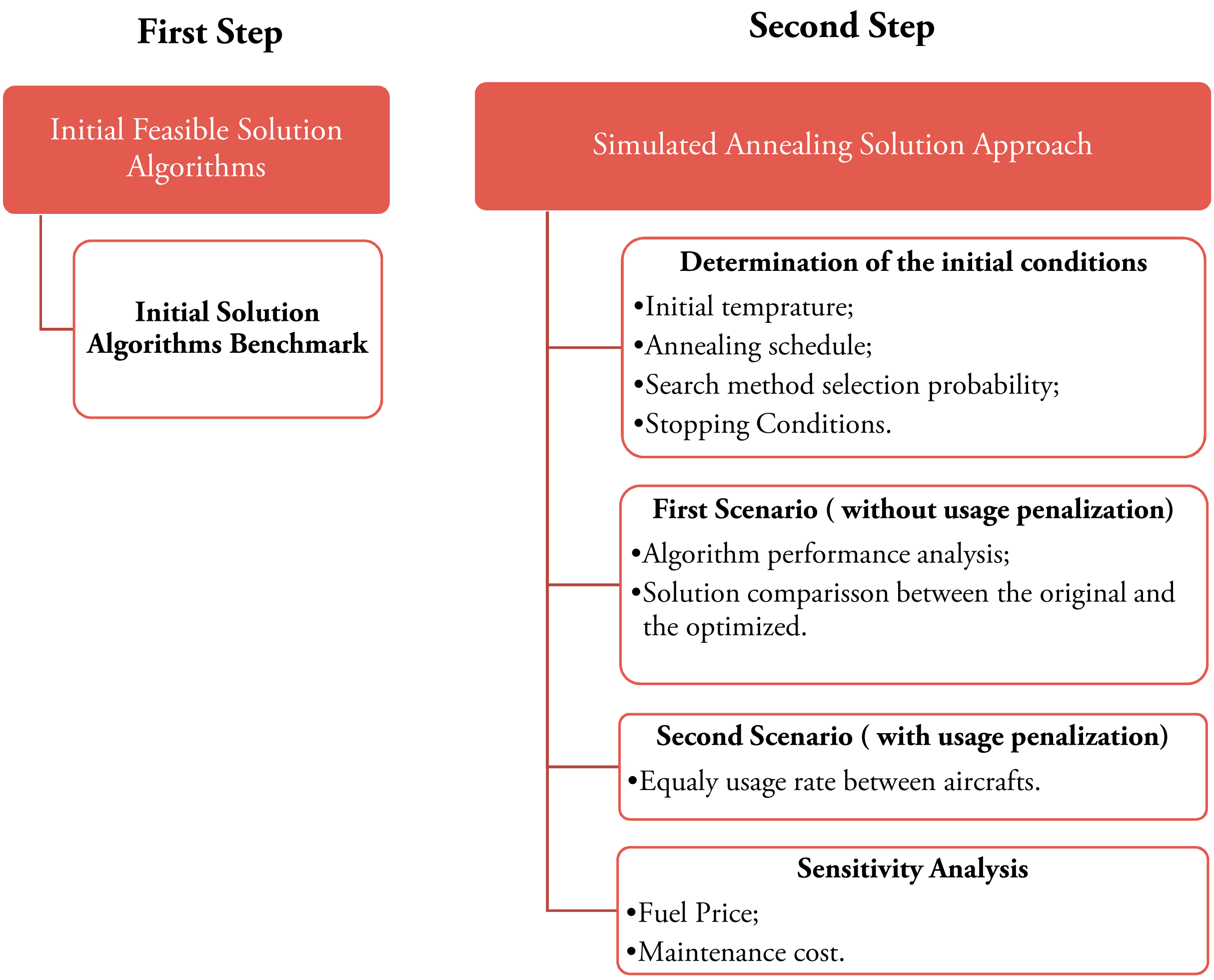


**Figure 16.** Analysis structure and scenarios considered in this study

### 5.2.2 Initial Feasible Solution Algorithms Benchmark

In this section we compare the three different initial solution algorithms, FIFO, LIFO and random assignment with the three different aircraft assignment methods, greedy, partially greedy and random assignment. For the partially greedy algorithm, we have defined that the aircraft to perform a specific activity, is chosen randomly among the 5 less expensive aircrafts.

To assess the different methods, we have used the HS7 schedule, as it represents the usual scheduling period used by TAP in the most demanding period (higher number of activities). In Table 14, is shown the total cost, the solving time and the comparison with the original solution.

**Table 14.** Benchmark between the initial feasible solution algorithms

| | | Total Cost (USD) | Comparison With the Original Solution | Solving Time (s) |
|---|---|---|---|---|
| Original Solution | | $ 6,921,284 | - | - |
| FIFO | Greedy | $ 6,881,295 | $ -39,989 | 10,4 |
| | Partially Greedy | $ 6,901,352 | $ -19,932 | 12,6 |
| | Random | $ 6,939,340 | $ 18,056 | 11,5 |
| LIFO | Greedy | $ 6,892,450 | $ -28,834 | 253,3 |
| | Partially Greedy | $ 6,905,341 | $ -15,943 | 248,4 |
| | Random | $ 6,942,152 | $ 20,868 | 235,4 |
| Random Assignment | Greedy | $ 6,890,340 | $ -30,944 | 256,7 |
| | Partially Greedy | $ 6,906,721 | $ -14,563 | 242,7 |
| | Random | $ 6,945,700 | $ 24,416 | 232,6 |

By analyzing the table results, we can see that in all cases the greedy and partially greedy allocation solutions have smaller costs than the original solution. On the other way around, the random allocation solutions have a slightly higher cost than the original solution. The difference between the random solutions and TAP original solution is the demand penalization. The only concern in the original schedule is to meet the demand, whereas in the random solutions the aircrafts are allocated randomly without any criteria.

The solving time is very different between the FIFO and the other two algorithms, being the FIFO more than 3 minutes faster than the average solving time of LIFO and the Random assignment. This difference of solving times is mainly related with the recovery phase of the LIFO and Random assignment algorithm. In contrast to FIFO algorithm, where there is always an aircraft to be assigned to a specific activity, in the other methods, sometimes there is no aircraft available to be assigned. In this cases the algorithms have to change the already assigned activities in order to find a feasible solution. This process takes a significant amount of time to be completed, which makes the LIFO and the Random assignment algorithms much slower than the FIFO algorithm.

Furthermore, the FIFO algorithm with the greedy assignment method returns the best solution in the shorter amount of time, when compared with the other algorithms. The solution for this algorithm costs less $9,045, than the second best solution (Random assignment with greedy allocation). In Figure 17, is represented the comparison of each component of the objective function, between the FIFO solution with greedy assignment and the original solution.

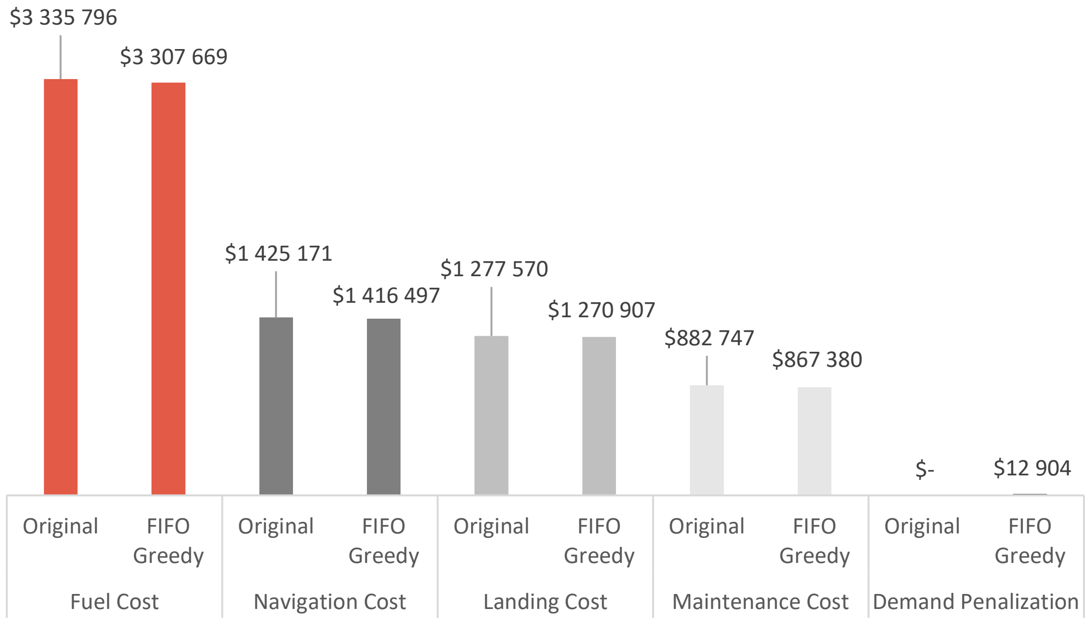


**Figure 17.** Breakdown of the operational costs for the original and FIFO with greedy assignment

The FIFO greedy solution shows lower costs for 4 out of the total 5 components of the operational costs. The largest difference is in the fuel costs where the original solution is $ 28,127 more expensive than the FIFO approach. On the other hand, the FIFO solution does not meet the entire demand for this given schedule, resulting in a penalization for the final solution. Nevertheless, the sum of the savings for all 4 operational components compensates the loss in unsold tickets.

In Figure 18, is represented the comparison of the utilization in BHs by variant for the original and FIFO greedy solutions. As shown in the chart, there is decrease in the utilization of the A321 and A320 variants and subsequently an increase in the utilization of the variant A319. This transition in the utilization of the variants, is mainly related with the total fuel cost. The fuel cost represents the largest part of the total costs, accounting for approximately 48% of the total costs. If the aircrafts with lower fuel consumption have more BHs assigned, and the ones with higher fuel consumption less BHs assigned, the total fuel costs will be lower. As we have seen in Section 5.1.2 the A319 variant has the lowest fuel consumption of all three variants, thus justifying the greater utilization in the FIFO greedy schedule.

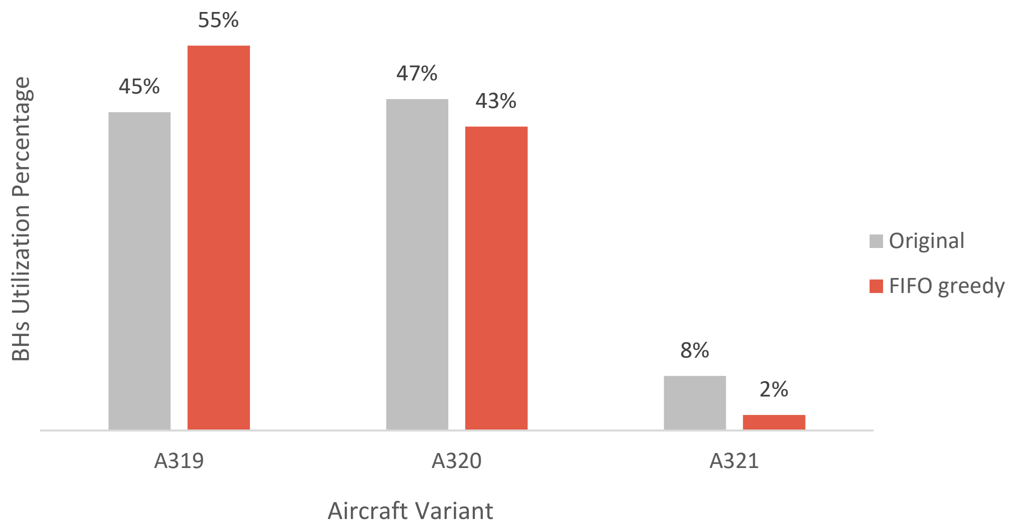


**Figure 18.** Comparison of the utilization percentage for each variant, between the Original Schedule and FIFO greedy solution schedule

To conclude, the FIFO greedy algorithm provides a good solution with approximately 40 thousand dollars in savings when compared with the original solution. Furthermore, the solving time for this algorithm is very short, making it a good a first comparison with the original solution, or even be used by TAP if there is no time to run the simulated annealing algorithm.

### 5.2.3 Simulated Annealing Performance Evaluation

In this subsection we analyze the performance of the algorithm created for TAP to solve the tail assignment problem. We start by analyzing and defining the initial parameters of the model (i.e. initial temperature, stopping criteria, annealing schedule and search method selection probability) that are used later to generate an optimal solution. After, in Section 5.2.2.2, we analyze the output results and performance of the model for the scenario without aircrafts utilization limits, and compare with the original solution. In Section 5.2.2.3, we present the scenario where the airplanes are penalized, if exceed the maximum defined use percentage. Finally, in Section 5.2.2.4, we test the robustness of the model, by making a sensitivity analysis.

### 5.2.3.1 Determination and Assessment of the Initial Parameters

It is extremely important to define well the initial parameters for the simulated annealing algorithm, as they have a great influence on the overall performance of the algorithm. In this section we analyze and then define the initial parameters that will be used to assess our solution approach in the next sections. We start by analyzing the initial temperature, then the stopping criteria and the annealing schedule, and finally the search method selection probability. Finally, we use the schedule HS7 for the all the analysis carried in this section.

#### Initial Temperature

A suitable initial temperature *T*, will enable the algorithm to accept worst solutions, but not all. Accepting wort solutions will allow the model to escape local optimum solutions. A very high temperature will affect negatively the solving time, as the algorithm will take much more time to reach the stopping criteria. On the other hand, a too low temperature will not enable the model to escape local optimums. Furthermore, we have defined two distinct neighborhood search methods with different output magnitude values. This means that if we want define the same probability of acceptance for both methods, we have to use two distinct temperatures.

There are some research papers that propose a formulation to define the initial temperature. For this work we use the approach of Franco Busetti (2003), which defines that "a suitable initial temperature *T* is one that results in an average increase of acceptance probability $p$ of 0.8". This means that a solution that increases the value of the objective function will have a probability of 80% to be accepted.

To define the initial temperature for both local search methods, we have made a total of 10,000 iterations for each method separately, where we listed the all the positive ΔE ($New\ solution - Old\ solution$) values. Using the formula $T = \frac{\Delta \mathrm{E}}{\ln(p)}$ (9) we have calculated for each ΔE the initial temperature needed for an acceptance probability of 80%. Finally, we made an average of all temperatures calculated. For the activity change local search method we have calculated an initial temperature of 3,700 and for the line-of activities method we have defined an initial temperature of 5,500.

**Stopping criteria**

The stopping criteria must be one that stops the model when little or no improvements occur in the objective function. As mentioned in Section 4.2.4, we have defined the model will stop at *n* consecutive temperature levels, wherein the difference between the actual solution and the best solution found during the process is equal or inferior than 0.001% of the best solution. To define the number of consecutive temperature levels *n* before stopping the algorithm, we have made a test where we defined stopping criteria as 50,000 iterations. Then, for each consecutive level *n* reached (e.g. 1, 2, 3, *n*+1) the algorithm registered the number of the iteration. In Figure 19, is shown the evolution of the objective function and the iteration number where *n* consecutive levels (5, 10, 15, 20, 25 and 30) where reached.

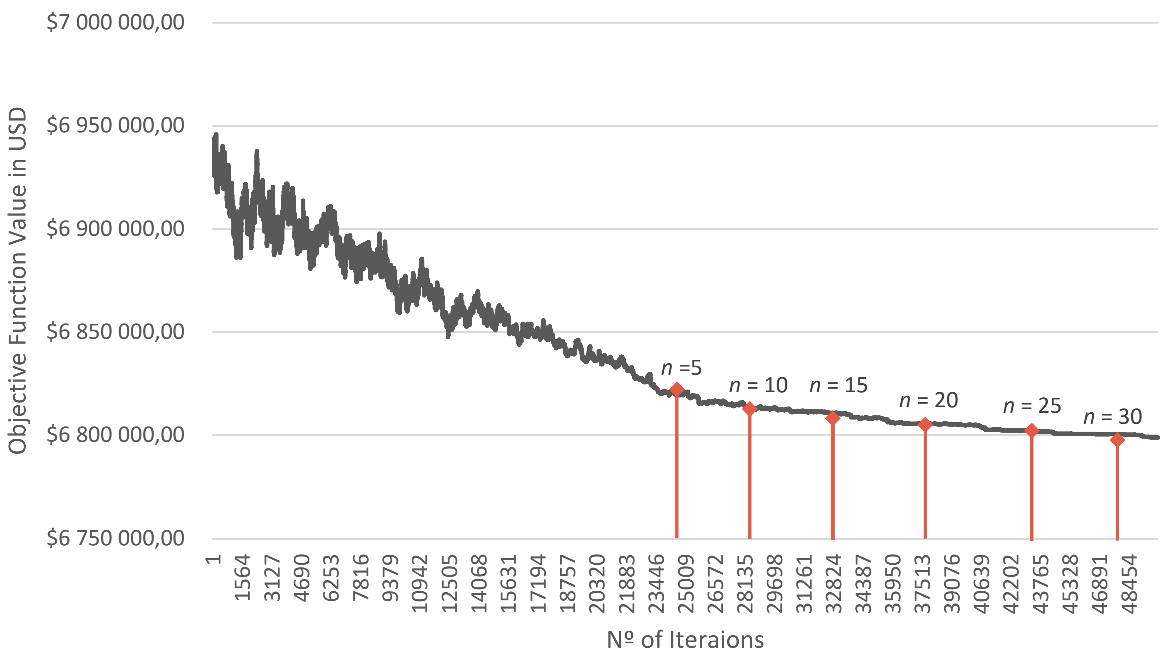


**Figure 19.** Objective function evolution and number of consecutive levels *n* where difference between the actual solution and the best solution found during the process is equal or inferior than 0.001% of the best solution

By analyzing the graph we can see that when *n* is equal to 20 (iteration 37,899) the objective function solution starts to stabilize. After this point, the solution values experience much less variance than the values before. The temperature at this level is 24 for the activity change method and 36 for the line-of-activities method. At this temperatures, the probability of accepting worst solutions is very small.

Although, the 20 consecutive levels provide a good stopping point, we have decided to use in this work 23 levels as a "safety margin".

**Annealing Schedule**

The annealing schedule is a critical factor for the simulated annealing process. If the annealing schedule is too slow it can be very computationally expensive, on the other hand, if it is too fast, the algorithm can be stuck more easily in a local optimum. In Section 4.2.4, we have defined the annealing schedule structure for this model, where the temperature is reduced after 200 iterations by a factor of $\alpha$. To define the $\alpha$ factor we have tested the solution approach for 4 different values of $\alpha$: 0.9, 0.95, 0.975 and 0.99. In Table 15, is represented for the four test instances, the final objective function value, the number of iterations made the solving time, and the comparison between the results.

**Table 15.** Algorithm results for $\alpha$ equal to 0.90, 0.95, 0.975 and 0.99

| $\alpha$ | Objective Function Value in USD | Nº of Iterations | Solving Time (s) | Comparison with the Best Solutions ($\alpha$=0.99 and $\alpha$=0.90) | |
|---|---|---|---|---|---|
| | | | | Obj. Func. | Solving Time |
| **0.90** | $ 6,839,295 | 11,201 | 305,6 | + 0,5% | - |
| **0.95** | $ 6,824,625 | 21,201 | 593,25 | +0,3% | + 94,4% |
| **0.975** | $ 6,808,400 | 35,401 | 912.8 | +0,1% | + 199% |
| **0.99** | $ 6,803,383 | 71,347 | 1,876.6 | - | + 536,9% |

As is can be observed in Table 15, the best solution is found when $\alpha$ is equal to 0.99. However, the algorithm, takes a great amount of time (approximately 30 minutes) to reach this solution. At the other end, when $\alpha$ takes the value of 0.9 the solution is reached much faster, only taking around 5 minutes, nevertheless this solution is $ 35,912 more expensive than the best solution. Although it seems a small value when compared with the total cost, this value multiplied by the 51 weeks of the year represents a total of $1,867,424 in potential savings.

For our solution approach, we have decided to define $\alpha$ as 0.975. This cooling factor represents the better tradeoff between the solving time and the objective function value. The value for the objective function is only 0.1% worst then the best solution, and the total solving time is 15 minutes shorter.

**Local Search Method Selection Probability**

As mention in previous chapter, we have defined an adaptive local neighborhood search for the simulated annealing model. For this local search model, two distinct search methods where methods were created. This methods are chosen by the algorithm, based on their past performance. This means that a method with better past performance, has a higher probability of being chosen for future iterations. For this work we have tested three different levels of probabilities. Additionally, the probabilities are adjusted at the same time as the temperature levels, 200 iterations.

In Table 16 , is represented the three levels of probabilities with the respective objective function value, solving time and the number of iterations made for each search method.

**Table 16.** Probability levels to select the local search method

| Probability of the Local Search Method to be Chosen | | Objective Function Value in USD | Solving Time (s) | Nº of Iterations for each Method Used | |
|---|---|---|---|---|---|
| Best Past Performance | Worst Past Performance | | | Activity Change | Line-of-Activities Change |
| **65%** | **35%** | $ 6,832,069 | 916,3 | 22,023 | 14,456 |
| **75%** | **25%** | $ 6,813,625 | 912,8 | 23,494 | 11,907 |
| **85%** | **15%** | $ 6,820,623 | 887,8 | 25,489 | 8,653 |

By analyzing the table, we can see that all of the probability levels have about the same solving time, being the level 85%-15% the one with the lower solving time. The best value for the objective function is obtained when using the 75%-25% level. Additionally, we can see that the activity change local search method is selected more times for this schedule. For this work we have selected the 75%-25% percentage, due to the smaller objective function cost.

### 5.2.3.2 Simulated Annealing Algorithm Evaluation

To test the performance of the created simulated annealing algorithm, we used all the schedules defined in Section 5.2.1 In Table 17, is represented the comparison between the original solution with the optimal one and the solving time for each one of the tested schedules.

**Table 17.** Simulated annealing results for the scenarios considered in Section 5.2.1

| Test Instance | Objective Function Value in USD | | Savings In USD | Savings Percentage | Solving Time (s) |
|---|---|---|---|---|---|
| | Simulated Annealing Solution | Original Solution | | | |
| **LS5** | $ 3,835,448 | $ 3,899,627 | $ 64,180 | 1,7% | 647,7 |
| **HS5** | $ 4,750,756 | $ 4,840,198 | $ 89,442 | 1,8% | 699,5 |
| **LS7** | $ 5,874,500 | $ 5,808,761 | $ 65,740 | 1,1% | 811,9 |
| **HS7** | $ 6,810,761 | $ 6,912,284 | $ 110,707 | 1,6% | 1033,04 |

As we can see in Table 17, when we compare the two five days schedules (LS5 and HS5), it is noticeable that the savings in the HS5 are higher than in LS5. This happens because there is a higher amount of flights in the high season period, making it possible to create more saving opportunities. On the other of hand, the savings percentages are very similar, with a slightly higher percentage savings for the HS5 schedule. As we have more activities in HS5 the margin for savings is greater when compared with the LS5. In terms of computational results, the LS5 is 51 seconds faster than the HS5, this can be explained by the lower number of activities and aircrafts when compared the HS5 schedule.

For the 7 days schedule period, like in the 5 days schedule, the high season period has the higher absolute and relative value for the savings, when compared with the low season period. Additionally, Schedule LS7 shows the lower savings between all of the 4 schedules. Finally, we can see that, the solving time for the LS7 schedule is 238 seconds faster (4 minutes) then the HS7 schedule.

If we compare the solving times between the 5 day schedules and the 7 days schedules, it is observable that the variance amid the solving times is relatively small. For the high season schedule, the difference between the 5 and 7 days is 417 seconds (approximately 7 minutes), and for the low season the difference is even smaller, only 164.2 seconds ( approximately 2 minutes and 30 seconds) between the two solutions.

After this brief introduction, we carry a more extensive analysis for each schedule, starting by the low season period, and after the high season period.

**Low Season – 5 Days (41 Aircrafts Available)**

In Figure 20, is represented the evolution of the objective function for the schedule being analyzed (LS5). As we can see in the graph, the solution for this schedule took 39,801 iteration, where 23,948 iterations (60%) were made using the line-of-activities change method, and the other 15,852 iterations (40%) were made using the activity change method. The simulated annealing algorithm starts with a random solution, generated by the FIFO algorithm with random assignment, which costs $ 3,957,585 and ends with the optimal solution of $3,835,448. Finally, the algorithm took approximately 11 minutes to reach a solution.

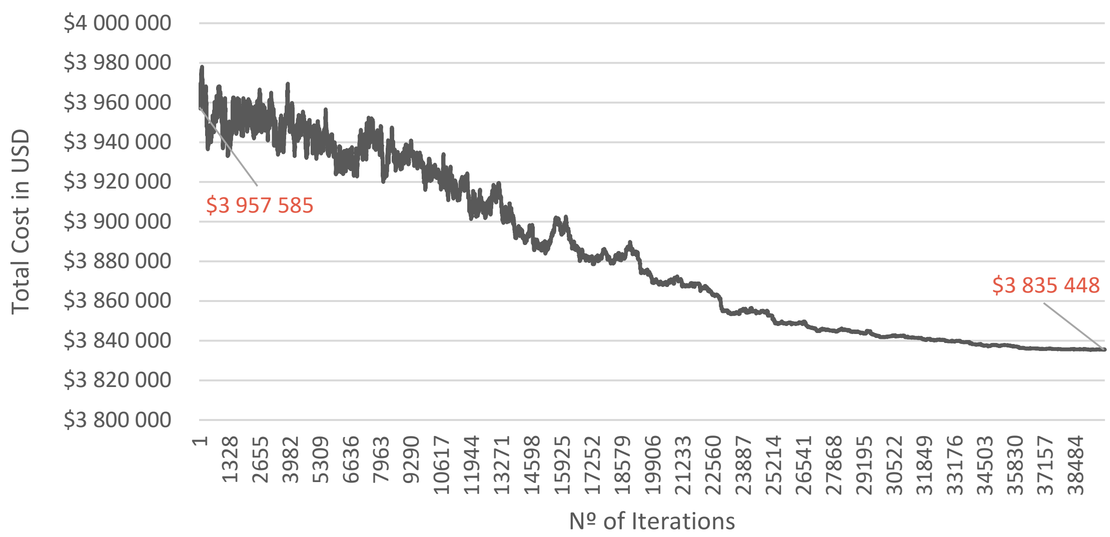


**Figure 20.** Evolution of the objective function for the LS5 schedule

As we seen before in Section 5.2.2 Figure 18 for the FIFO algorithm with greedy assignment, the A319 aircrafts are preferred (mainly due to the lower fuel consumption profile) than the other two variants, for an optimal solution. For this schedule, as shown in Figure 21, we can see the average block time per variant is higher for the A319, representing a total of 56,5 BHs per aircraft. This value represents 18% more in average BHs when compared with the original schedule. On the other side, the A321 is used less often, due to the high fuel consumption profile, leading to a reduction of utilization in 147%.

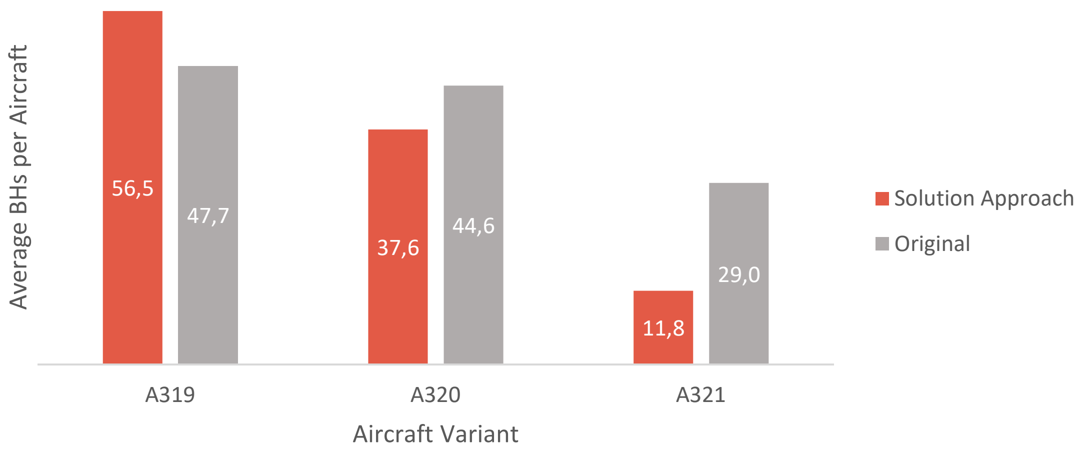


**Figure 22.** Comparison between the average BHs per variant for the LS5

If we compare each component of the operational costs, as shown in Figure 22, the highest savings are obtained by reducing the total fuel costs, which represents a total of $ 25,558 in savings. The second highest saving is in the maintenance costs, where the decrease is $12,782. This happens because the A319 variant is the one with the lower maintenance costs per BH. Furthermore, is noticeable that all components of the operational costs have a reduction in the solution approach tail assignment, making this more efficient in all aspects when compared with the original solution.

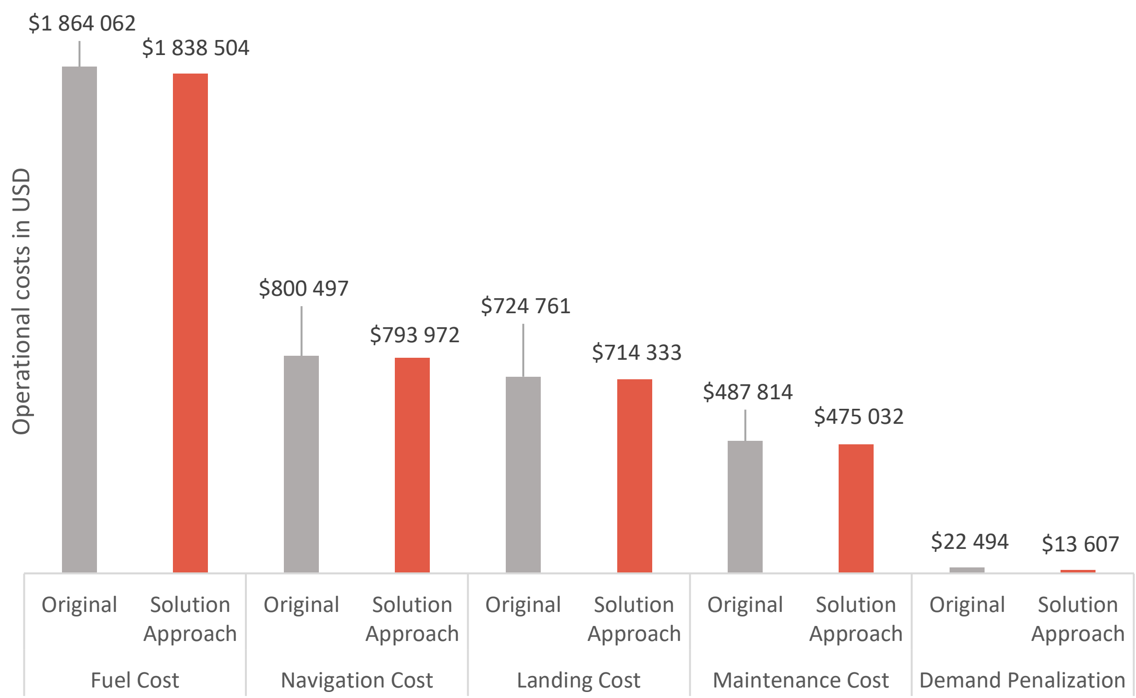


**Figure 21.** LS5 operational costs breakdown for the original and solution approach tail assignment

**High Season – 5 Days (43 Aircrafts Available)**

For this schedule, the number of iterations needed for reaching a solution was 36,601, taking less 3,000 iterations when compared with the low season schedule. From this iterations, 73% where performed with the line-of-activities change method and the other 27% were performed with the change activity method. Comparing the two solutions, we can see that for the HS5 schedule, the line-of-activities method was used more times than in schedule HS7. In Figure 23, it is observable that the total cost stabilizes around 31,000 iterations and ends with a total cost of $ 4,750,756. Furthermore, the algorithm starts with a random solution costing $ 4,848,701.

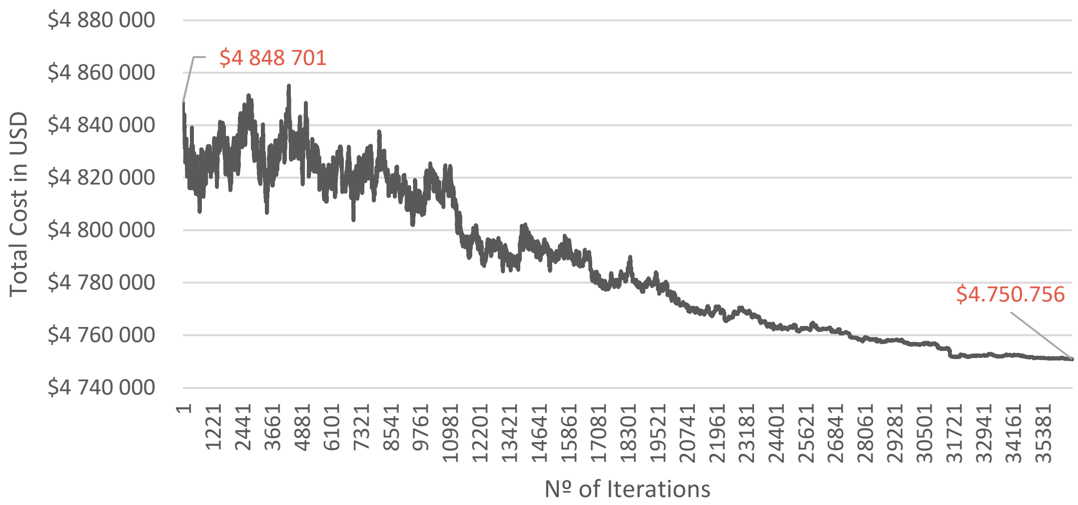


**Figure 23.** Evolution of the objective function for the HS5 schedule

For the total average BHs for each variant, we can observe in Figure 24 that TAPs original tail assignment has an evenly distributed BHs between all three variants. For the solution approach tail assignment, the number of BHs per tail is significantly lower for the A321 variant, with an increase in the utilization of the A319 fleet.

As explained in the previous schedule, this change of variants enables to achieve more efficiency in terms of fuel consumption. As the A321 is a bigger aircraft, the fuel consumption is higher when compared with the smaller A319. If for a given activity, we do not need the seat capacity of the A321, it is more profitable to the A321 fleet aircrafts to be on the ground and the A319 to perform the activities.

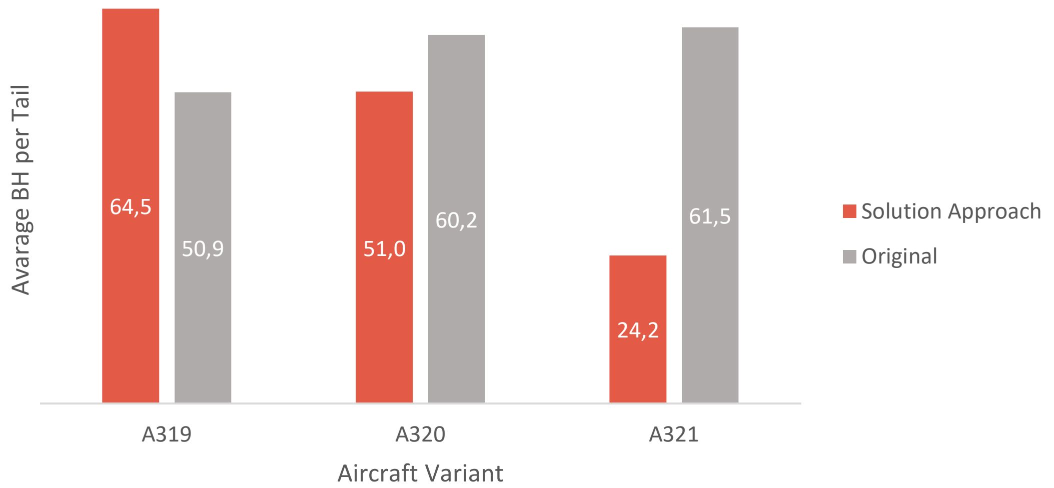


**Figure 24.** Comparison between the average BHs per variant for the HS5

In terms of operational costs, the savings structure is very similar with the LS5 schedule. The fuel costs represent 46.7% of the total savings, being the operational component with the higher savings. In second place, the navigation costs represent 12.7% of the total savings. Moreover, this schedule shows the highest saving opportunity, with a total 1.8% of savings when compared with the original solution.

**Low Season – 7 Days (41 Aircrafts Available)**

For the 7 days low season schedule, it was required 37,001 iterations to reach the optimal total cost of $ 5,810,681, where 23,082 iterations were made using the line-of-activities change method and the rest 3,918 were made using the activity change method. In Figure 25, is represented chart with the starting point of the solution ($ 6,014,230) and the progress made until the final solution is reached.

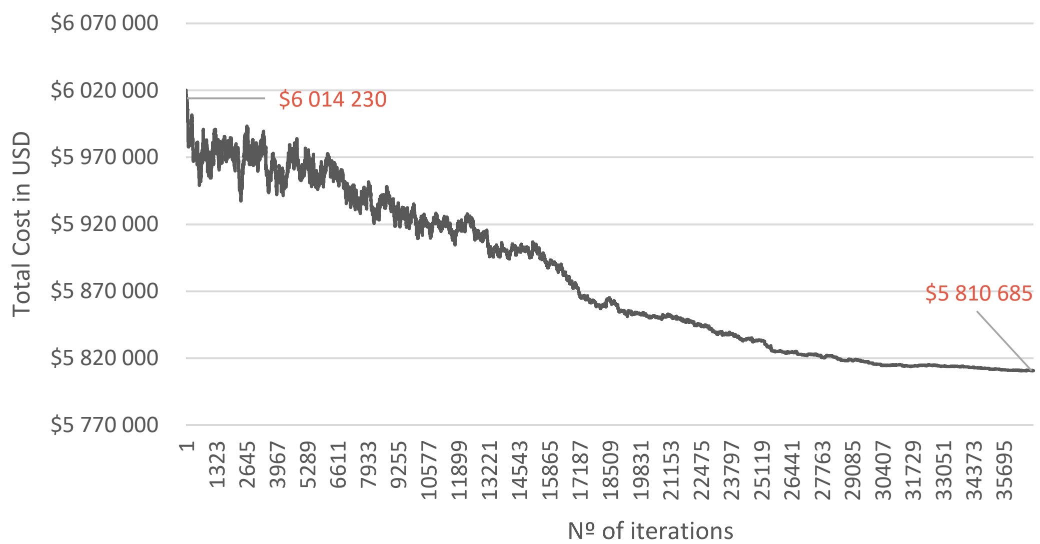


**Figure 25.** Evolution of the objective function for the LS7 schedule

For the average total BHs per variant, we can see in that similarly of what have been discussed in the previous schedules, the aircrafts from the variant A319 in the optimized tail assignment are used more times to the detriment of the A320 and A321. For this schedule, the original TAPs tail assignment has the total block hours distributed evenly for all the three variants.

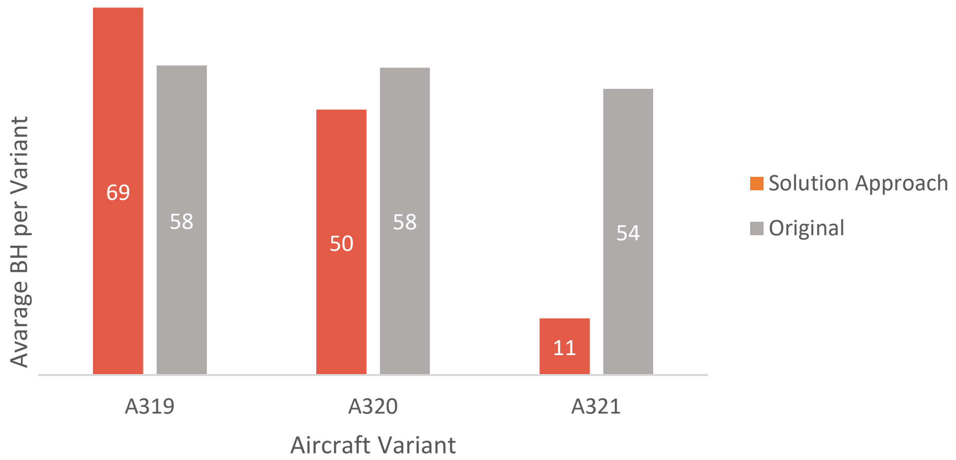


**Figure 26.** Comparison between the average BHs per variant for the LS7

In the breakdown down structure of the total operating costs, shown in Figure 27, we can see that as in the other schedules the fuel costs represent the highest savings ($ 39,037), when we compare the original solution with the provided solution approach. For this schedule, the second highest saving is for the maintenance cost that represents a total amount of $ 19,698 in savings. Furthermore, this schedule has the lower savings opportunity amid all four schedules, with only 1.1% of savings when compared with the original solution.

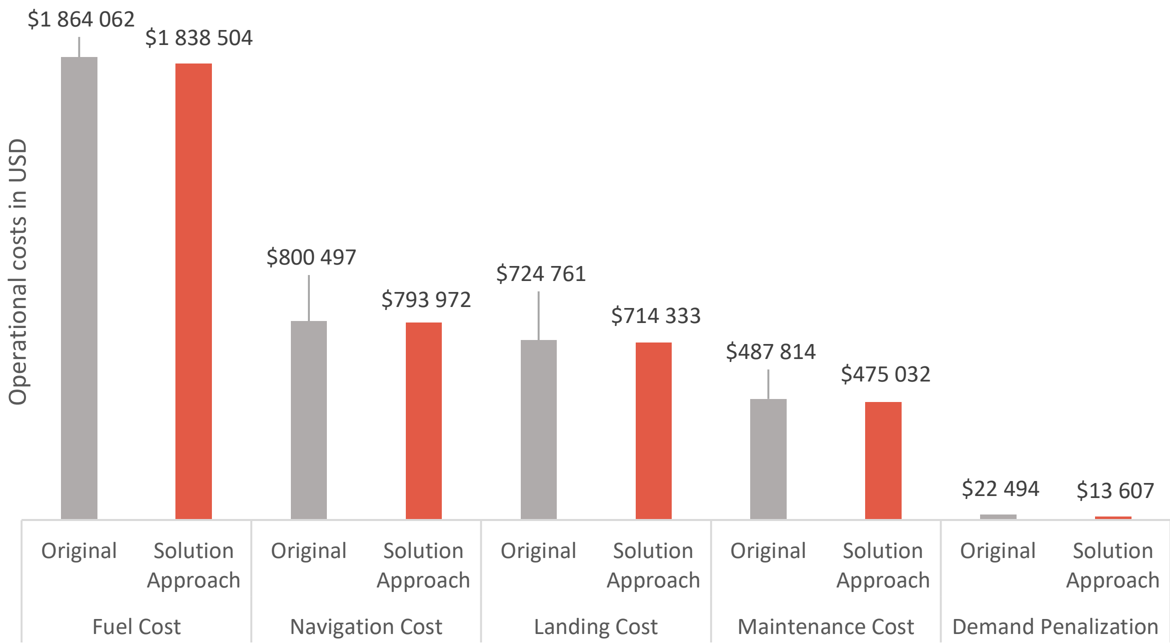


**Figure 27.** LS7 operational costs breakdown for the original and solution approach tail assignment

**High Season – 7 Days (41 Aircrafts Available)**

This schedule represents the most demanding schedule of all four, containing the highest number of activities (1421) and total block hours (3430 BHs). This explains the higher solving time and the higher number of iterations until the final optimal solution is reached. In Figure 28, is represented the evolution of the objective function for the HS7 schedule. The algorithm starts by generating a random solution that costs $ 6,914,263, and the after 34,401 iterations the algorithm stops with an optimized solution of $ 6,810,761. For this schedule, in 75% of the iterations the algorithm used the line-of-activities change method and 25% the activity change method.

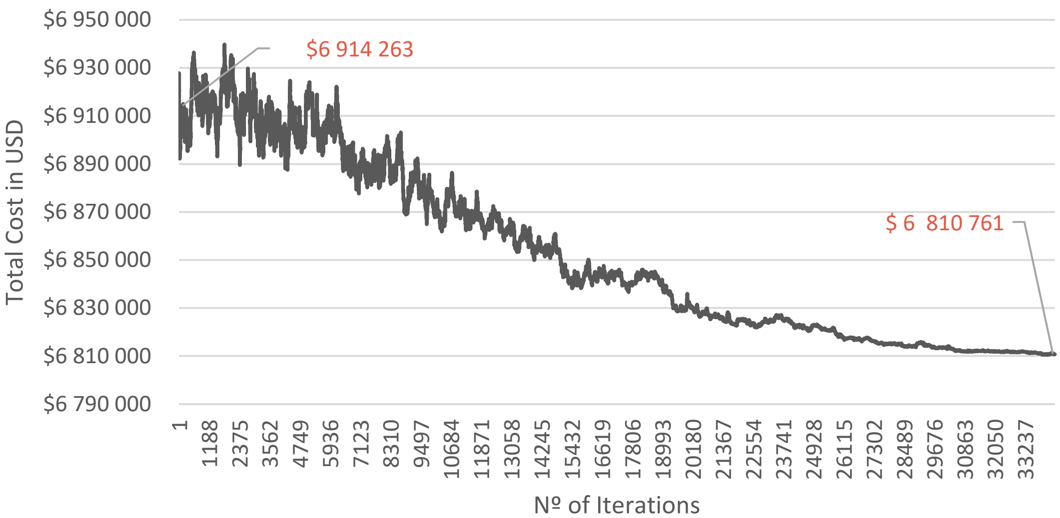


**Figure 28.** Evolution of the objective function for the LS7 schedule

Furthermore, the average utilization of the aircrafts for this schedule is very similar to the ones studied previously. In the original tail assignment, there is a utilization of 44%, 48% and 7%, representing the fleets A319, A320 and A321 respectively. In the solution approach, the utilization in the same order is: 54%, 42% and 4%. For the operating costs, the largest difference among the two solutions is the fuel cost, followed by the maintenance costs. The savings of this two components sum up $ 80,787 representing 73% of the total savings.

To conclude, the solution approach developed in this work, has proven throughout the presented schedules that is able to reduce significantly the operating costs. The algorithm was able to achieve better results for the high season periods, where there is a higher density of activities for the same period. Furthermore, the solving times are fairly small, considering that we are dealing with a large number of activities and aircrafts.

#### 5.2.3.3 Simulated Annealing Algorithm Considering a Balanced Aircraft Utilization

In this scenario we study the impacts to the final solution, when we define a penalization that imposes a balanced aircraft utilization. This scenario is important for TAP, as some of the leasing contracts and specific maintenance services require an evenly utilization of the aircrafts. In order to simplify this analysis, we have considered only the LS7 schedule, as this is the most demanding schedule in terms of activities and aircrafts. Furthermore, for the penalization cost factor we have considered $ 5,000,000 (this cost was calculated by trial and error). This cost is considered by the algorithm, every time that an aircraft surpasses the utilization limit. As an example, if an aircraft that has a defined maximum utilization of 2% of the total BHs in a schedule, if this value of utilization is exceeded, the aircraft receives a penalization that is directly proportional to the new utilization percentage.

During the testes made to the solution approach with balanced aircraft utilization, we have noticed that the convergence of the solutions was much slower, when compared with the solution approach without aircraft balancing. Taking this into consideration this fact, we had to decrease the initial temperature by 90% and manually stop the algorithm after 125,000 iterations. This alterations have made the algorithm to stop earlier than it should, but still the model has presented a good solution when compared with the original TAPs solution. In Table 18, is represented the comparison between the results obtained with the balanced aircraft utilization, the unbalance aircraft utilization and the original solution.

**Table 18.** Comparison between the original solution, unbalanced aircraft solution and balanced aircraft solution

| Test Instance | Original Solution in USD | Simulated Annealing Solution (unbalanced aircraft utilization) in USD | Simulated Annealing Solution (balanced aircraft utilization) in USD | Solving Time (s) |
|---|---|---|---|---|
| **LS5** | $ 6,912,284 | $ 6,810,761 | $ 6,902,229 | 4,312.4 |

The final savings for this solution approach, were much lower when compared with the original solution. For this solution we only get a total savings of $ 10,055 when compared with the original solution, where 43% of this savings are from the reductions of the fuel costs. Moreover, the solving time has quite high, around 71 minutes, due to the low convergence of the solutions.

When we compare the utilization of the aircrafts between all the solutions, as shown in Figure 29, we can see that in the balanced aircraft utilization approach, the usage between all three variants is much more balanced than in the solution approach with unbalanced aircraft utilization. This factor reduces the possibility of savings, as the A319 variant is assigned to less BHs.

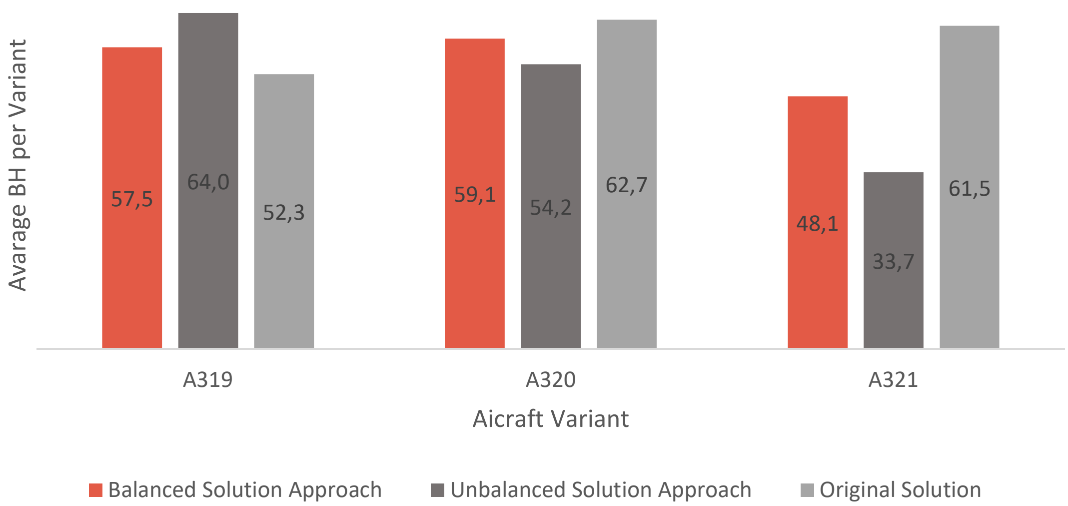


**Figure 29.** Average BH per Variant (comparison between the original solution, the unbalanced solution approach and balanced solution approach

### 5.2.3.4 Sensitivity Analysis

In this sensitivity analysis we test the robustness of the solutions, by changing the fuel cost price and the maintenance costs per aircraft. We choose this two parameters, as they represent in three out of the four schedules studied at Section 5.2.2.1 the costs with higher savings impact. This sensitivity analysis will also enable to understand how the optimal solution varies, when the parameters are changed. For this analysis we use the schedules HS7 and LS7 as they represent normal planning time and at TAP and cover the two seasonal periods. We start by analyzing the fuel costs and after the maintenance costs.

**Fuel Cost Sensitivity Analysis**

As seen in Section 5.2.2.1, the fuel costs represents the largest operational cost, and at the same time, the biggest opportunity for saving. Recently, we have seen a great decrease in total fuel price costs, helping airline companies to save millions of dollars in this commodity. As the probability of an increase in this cost is higher than a decrease (due to the already low price), we will analyze an increase of this cost by 25% and a decrease of 10%. In Table 19, is shown the results for the variation of the fuel price for the LS7 and HS7 schedules.

**Table 19.** Sensitivity analysis results for the fuel cost (Schedules LS7 and HS7)

| Fuel Cost Variation | | -10% | 0% | +25% |
|---|---|---|---|---|
| **LS7** | Original solution Cost in USD | $ 5,591,587 | $ 5,874,500 | $ 6,581,785 |
| | Solution Approach Cost in USD | $ 5,550,086 | $ 5,808,761 | $ 6,509,024 |
| | Savings in USD | **$ 41,501** | $ 65,740 | **$ 72,760** |
| **HS7** | Original solution Cost in USD | $ 6,587,705 | $ 6,912,284 | $ 7,755,233 |
| | Solution Approach Cost in USD | $ 6,481,848 | $ 6,810,761 | $ 7,625,693 |
| | Savings in USD | **$ 105,857** | $ 110,707 | **$ 129,541** |

As we can observe in Table 19, when there is an increase in the fuel price, the total potential savings increases as well. The increase of 25% for the fuel cost, represents in average increment for both schedules of approximately 14%, where the schedule HS7 shows the biggest increase in savings.

On the other hand, a decrease of 10% in the fuel price, has a great impact on the total savings for schedule LS7, representing a decrease in potential savings of approximately 37%.

**Maintenance Cost Sensitivity Analysis**

As mentioned in Section 5.1.2. , this costs where obtained by doing an extrapolation, based on the aircraft size, of the data obtained in the work of Ribeiro (2012). As this numbers could be a little different from the actual maintenance cost, and as they represent the second major possibility of savings, we have decided to assess the sensibility of the algorithm to this cost. For the analysis, we have considered a positive and a negative variance of 20% of the maintenance cost per BH. In Table 20, is represented the results obtain for this analysis.

**Table 20.** Sensitivity analysis results for the maintenance cost (Schedules LS7 and HS7)

| Maintenance Cost Variation | | -20% | 0% | +20% |
|---|---|---|---|---|
| **LS7** | Original solution Cost in USD | $ 6,603,495 | $ 5,874,500 | $ 6,023,174 |
| | Solution Approach Cost in USD | $ 6,570,777 | $ 5,808,761 | $ 5,947,78 |
| | Savings in USD | **$ 32,718** | $ 65,740 | **$ 75,796** |
| **HS7** | Original solution Cost in USD | $ 6,744,735 | $ 6,912,284 | $ 7,097.834 |
| | Solution Approach Cost in USD | $ 6,656,125 | $ 6,810,761 | $ 6,978,615 |
| | Savings in USD | **$ 88,609** | $ 110,707 | **$ 119,219** |

As expected, by analyzing Table 20, we can see that in most of the cases the differences between the variation of the fuel costs and for the maintenance costs are lower for the last one. Is interesting to note, that the decrease of the maintenance cost price was a higher impact than the increase in the same percentage. The biggest variance is in schedule LS7, where the decrease in potential savings is approximately 50%. Furthermore, when we increase the maintenance price for both schedules, we can see that the saving opportunity increases.

# Chapter Six

# Conclusion

In the last chapter of this work, we present the retrived conclusions of our study in Section 6.1 and after, in Section 6.2, we give some indications for future work possibilities.

## 6.1 Summary

The current airline industry market is facing now new challenges, forcing the traditional airline companies to be more efficient in their operations. New and improved operational strategies are now being adopt, in order to decrease the operational costs, thus improving the profit margin.

In this work, we have studied in detail one of the operational phases at TAP, the tail assignment process. This process consists in assigning individual aircrafts to previous defined activities, with the purpose of creating new LOFs. Currently, TAP only considers operational constraints and demand factors in their tail assignment process. Furthermore, this work was developed for the tail assignment of short-and-medium haul operations, as it is TAPs biggest operation in terms of aircrafts and flights. Nevertheless, all the algorithms developed during this work can be also used for the long-haul operations.

Most of the works in literature portray the tail assignment process as a pure feasibility model, where is considered the aircrafts as homogenous. This study proposes a new approach, where the aircrafts are assigned based on their individual operational costs and demand factors, and at the same time respecting the operational constraints. To do this, we have developed a comprehensive simulated annealing algorithm with an adaptive neighborhood local search method. The objective of this algorithm is to minimize the sum of the fuel, navigation, landing and maintenance costs, and the same time respecting the demand for each flight.

Additionally, to the simulated annealing algorithm, we have developed three distinct algorithms to create an initial feasible solution. These, are used to generate an initial solution for the simulated annealing algorithm, or also can be used to generate a good solution in a shorter amount of time. In Section 5.2.2, we have tested the three algorithms for a high season schedule of 7 days. It was shown that using FIFO algorithm with the greedy assignment method TAP would be able to save around $40,000 for a typical high season week.

For the results of the simulated annealing algorithm, we have seen that for all the schedules tested in this work, the algorithm was able to provide a good solution, with an average percentage of savings of 1,55%. Also, the solution approach developed, has showed relatively short solving times. The shortest solving time, approximately 11 minutes, was obtained for the HS5 schedule and the longest solving time, approximately 17 minutes, for the HS7.

Furthermore, we have observed that in terms of absolute value, the high season weeks represent a higher possibility of savings. As TAP schedules for the short and medium-haul operations are mostly cyclical, where the schedules repeat every week. If we extrapolate the Savings of HS7 to the 3 months of higher demand there could be a possible savings of $ 1,328,484. On the other hand, on the 9 low season months of the year, the savings using the solution approach tail assignment would represent $ 2,310,480. If we sum these two numbers we have $ 3,638,964, which represents the total saving opportunity for TAP, by doing an efficient tail assignment.

In a second scenario, we have tested the solution approach with a balanced aircraft utilization. The idea as to attribute a penalization when an aircraft exceeded a user defined limit utilization. By analyzing the results, we have seen that is possible to have a more balanced aircraft utilization and at the same time a small reduction in the total operational costs. Furthermore, this alteration to the model should be improved in future work, since the savings achieved in this scenario were very small when compared with the solution approach without balanced utilization of the aircrafts, and the solving time has too high.

Finally, to test the robustness of the model, we have carried a sensitivity analysis. For this analysis, we have tested different values for the fuel cost and maintenance cost. We have seen, that when we increase both prices, there is a bigger saving opportunity. On the other hand, when we decrease the prices, the saving opportunity is lower. For all the scenarios tested, the solution approach as given good savings results when compared with the original solution.

To conclude this section, is noteworthy to mention that algorithm is a final product and is ready to be implemented in TAP's short, medium and long-haul operations.

## 6.2 Future work

There are still some things that can be done to improve the presented tail assignment solution approach. In this final section we give some indications of what could be the future improvements and additions to the developed algorithm.

**Distinction between the Economic Class and Business Class**

For the algorithm developed in this study, we did not make the distinction between the economical class and the business class, since it was not provided the data related with the number of seats and the average ticket price for each class. The distinction between each class would provide to the decision maker a better and more accurate solution. Furthermore, is important to note, that the developed structure for the solution approach permits an easily implementation of this feature.

**Robust Tail Assignment Approach**

A further improvement to the tail assignment problem, is the development of a model that creates robust schedules. The idea here is the creation of a schedule that is more robust to activity delays. As widely known, one of the biggest problems in the airline industry is the flight delays. A robust schedule, where activities with a high probability of delay are assigned to aircrafts that have a slack time after and before the activity, would enable to reduce the total number of delays.

**Tail Assignment Problem as a Profitability Problem**

In this work we have considered the tail assignment problem as a minimization problem, where we have proved that is possible to reduce the current operating costs for the short-and-medium haul operations at TAP. Another way to view the tail assignment problem, is to consider the maximization of the profit, instead of the minimization of the operational costs. To create an accurate model that maximizes the profit, it would be necessary to develop a more precise function to calculate the revenues. The price of each ticket sold at TAP is dependent of several different variables, and calculated based on a complex algorithm. Nevertheless, a profitability model would enable TAP to know when the savings in the operational costs offset the tickets profit.

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## Appendix 1. LOFs represented in Compass software

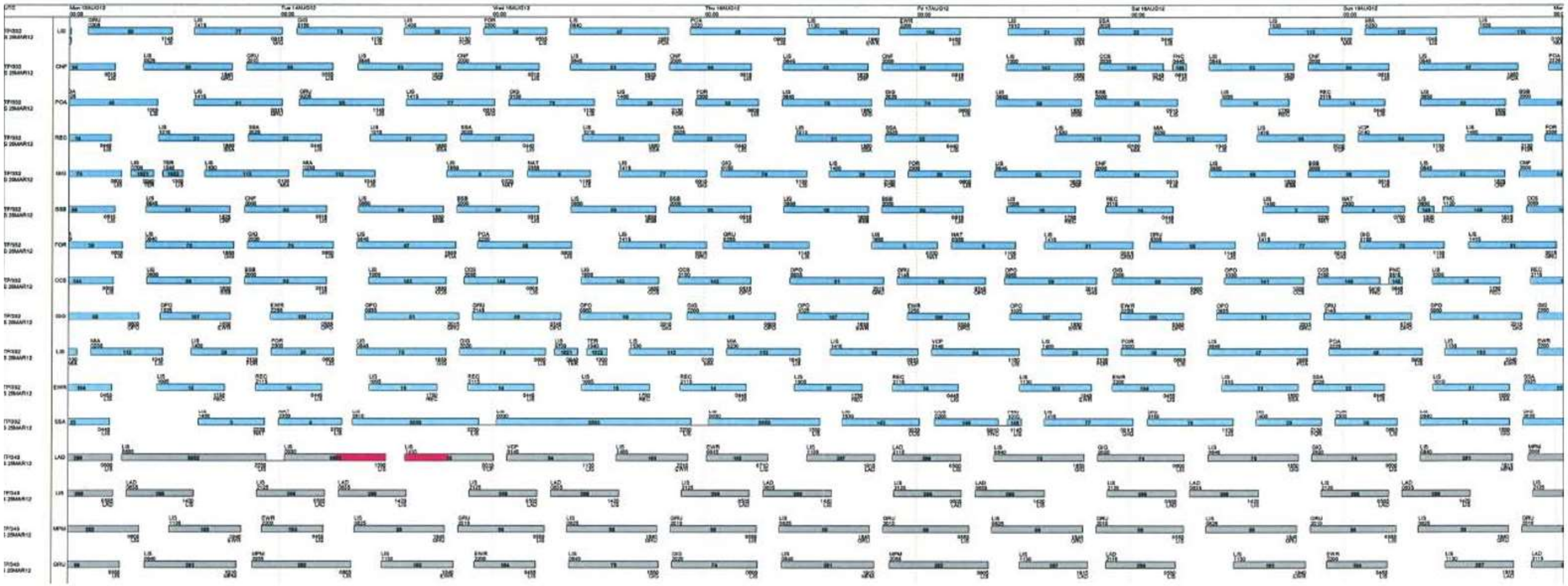

**Figure 30.** LOFs represented in Compass software

# Appendix 2. Performance degradation factors for TAP's Fleet

**TAP Portugal Fleet Report**

*October 2015 - TA/Fleet Management*

TAP PORTUGAL | A STAR ALLIANCE MEMBER

| Fleet | Tail | MSN | MTOW | Age | Monthly FC | Performance | Usage | Fuel Burn | IFE CR | Tech CNL | Engines | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Narrow Body / A319 | A319 Total | | | 16,5 | | | | | | | | |
| | CSTTA | 750 | 68,0 ton | 18,1 | 128 | 6% | 36% | 48,6 | 2 | 0 | CFM56-5B5P/779379 | CFM56-5B53/699621 |
| | CSTTB | 755 | 68,0 ton | 18,1 | 140 | 6,5% | 39% | 48,3 | 3 | 0 | CFM56-5B5P/575425 | CFM56-5B5P/575437 |
| | CSTTC | 763 | 68,0 ton | 18 | 148 | 5% | 41% | 48 | 0 | 0 | CFM56-5B5P/779202 | CFM56-5B5P/779554 |
| | CSTTD | 790 | 68,0 ton | 17,8 | 145 | 4,5% | 41% | 48,2 | 0 | 0 | CFM56-5B5P/575423 | CFM56-5B5P/779736 |
| | CSTTE | 821 | 68,0 ton | 17,7 | 138 | 6% | 39% | 48,7 | 1 | 2 | CFM56-5B5P/779354 | CFM56-5B5P/779478 |
| | CSTTF | 837 | 68,0 ton | 17,6 | 146 | 4,5% | 40% | 48,5 | 0 | 0 | CFM56-5B5P/779619 | CFM56-5B53/699517 |
| | CSTTG | 906 | 68,0 ton | 17,2 | 127 | 5,5% | 36% | 48,5 | 3 | 3 | CFM56-5B5P/779421 | CFM56-5B5P/779425 |
| | CSTTH | 917 | 68,0 ton | 17,1 | 133 | 7,5% | 37% | 49 | 0 | 1 | CFM56-5B5P/575432 | CFM56-5B5P/779809 |
| | CSTTI | 933 | 68,0 ton | 17,1 | 136 | 5% | 39% | 48,2 | 0 | 0 | CFM56-5B5P/779468 | CFM56-5B5P/779737 |
| | CSTTJ | 979 | 68,0 ton | 16,8 | 136 | 6% | 38% | 48,6 | 3 | 2 | CFM56-5B5P/575433 | CFM56-5B5P/779335 |
| | CSTTK | 1034 | 68,0 ton | 16,5 | 143 | 5% | 40% | 48,4 | 2 | 1 | CFM56-5B5P/575317 | CFM56-5B5P/779641 |
| | CSTTL | 1100 | 68,0 ton | 16,2 | 144 | 6% | 41% | 48,5 | 3 | 4 | CFM56-5B5P/779638 | CFM56-5B5P/779677 |
| | CSTTM | 1106 | 68,0 ton | 16,2 | 147 | 4% | 41% | 47,8 | 0 | 1 | CFM56-5B5P/779620 | CFM56-5B5P/779713 |
| | CSTTN | 1120 | 68,0 ton | 16,2 | 145 | 5% | 41% | 48,2 | 1 | 0 | CFM56-5B5P/779419 | CFM56-5B5P/779688 |
| | CSTTO | 1127 | 68,0 ton | 16,1 | 131 | 5,5% | 37% | 47,7 | 2 | 3 | CFM56-5B5P/779624 | CFM56-5B5P/779897 |
| | CSTTP | 1165 | 68,0 ton | 16 | 129 | 5% | 36% | 48,6 | 2 | 1 | CFM56-5B5P/575439 | CFM56-5B5P/779270 |
| | CSTTQ | 629 | 70,0 ton | 19,2 | 148 | 4,5% | 42% | 48,6 | 5 | 1 | CFM56-5B6P/575421 | CFM56-5B6P/779642 |
| | CSTTR | 1756 | 70,0 ton | 13,6 | 146 | 5% | 42% | 48 | 5 | 2 | CFM56-5B6P/779426 | CFM56-5B6P/779821 |
| | CSTTS | 1765 | 70,0 ton | 13,6 | 143 | 5,5% | 41% | 49 | 1 | 2 | CFM56-5B6P/779371 | CFM56-5B6P/779643 |
| | CSTTU | 1668 | 70,0 ton | 13,7 | 133 | 7% | 38% | 49,5 | 4 | 1 | CFM56-5B6P/575336 | CFM56-5B6P/575341 |
| | CSTTV | 1718 | 70,0 ton | 13,7 | 137 | 3% | 39% | 47,8 | 2 | 1 | CFM56-5B6P/575287 | CFM56-5B6P/575326 |
| A320 | A320 Total | | | 11,9 | | | | | | | | |
| | CSTMW | 1667 | 77,0 ton | 14,1 | 128 | 4,5% | 46% | 47,8 | 7 | 2 | CFM56-5B4P/575316 | CFM56-5B4P/779898 |
| | CSTNG | 945 | 73,5 ton | 17 | 129 | 4,5% | 41% | 47,6 | 0 | 0 | CFM56-5B4P/779336 | CFM56-5B42P/779958 |
| | CSTNH | 960 | 73,5 ton | 16,9 | 125 | 5,5% | 42% | 47,9 | 0 | 0 | CFM56-5B4P/779353 | CFM56-5B4P/779467 |
| | CSTNI | 982 | 73,5 ton | 16,7 | 138 | 5% | 44% | 48,3 | 7 | 0 | CFM56-5B4P/779439 | CFM56-5B4P/779639 |
| | CSTNJ | 1181 | 73,5 ton | 15,9 | 130 | 4% | 42% | 47,4 | 1 | 2 | CFM56-5B4P/575377 | CFM56-5B4P/779320 |
| | CSTNK | 1206 | 73,5 ton | 15,8 | 129 | 4% | 42% | 47,8 | 2 | 1 | CFM56-5B4P/575376 | CFM56-5B4P/779453 |
| | CSTNL | 1231 | 73,5 ton | 15,7 | 128 | 4,5% | 42% | 48,2 | 1 | 2 | CFM56-5B4P/779324 | CFM56-5B42P/779462 |
| | CSTNM | 1799 | 73,5 ton | 13,7 | 137 | 5% | 44% | 47,9 | 1 | 0 | CFM56-5B4P/779327 | CFM56-5B4P/779450 |
| | CSTNN | 1816 | 73,5 ton | 13,6 | 130 | 5% | 44% | 47,1 | 2 | 1 | CFM56-5B4P/779438 | CFM56-5B4P/779625 |
| | CSTNP | 2178 | 77,0 ton | 11,8 | 132 | 5% | 43% | 47,9 | 8 | 7 | CFM56-5B4P/575773 | CFM56-5B4P/575774 |
| | CSTNQ | 3769 | 77,0 ton | 6,9 | 139 | 3% | 50% | 46,8 | 3 | 0 | CFM56-5B42P/779473 | CFM56-5B43/699191 |
| | CSTNR | 3883 | 77,0 ton | 6,7 | 140 | 2,5% | 49% | 46,8 | 3 | 0 | CFM56-5B43/699338 | CFM56-5B43/699339 |
| | CSTNS | 4021 | 77,0 ton | 6,3 | 138 | 2,5% | 49% | 47,1 | 7 | 0 | CFM56-5B4P/779481 | CFM56-5B43/699515 |
| | CSTNT | 4095 | 77,0 ton | 6,2 | 137 | 4% | 50% | 46,9 | 4 | 0 | CFM56-5B4P/779378 | CFM56-5B43/699618 |
| | CSTNU | 4106 | 77,0 ton | 6,1 | 135 | 2% | 48% | 46,1 | 0 | 2 | CFM56-5B43/699627 | CFM56-5B43/699639 |
| | CSTNV | 4145 | 77,0 ton | 6,1 | 136 | 3% | 48% | 46,7 | 1 | 0 | CFM56-5B43/699688 | CFM56-5B43/699691 |
| | CSTNW | 2792 | 77,0 ton | 9,6 | 129 | 4% | 46% | 48,3 | 6 | 0 | CFM56-5B4P/577612 | |
| | CSTNX | 2822 | 77,0 ton | 9,6 | 126 | 2,5% | 46% | 47,2 | 6 | 2 | CFM56-5B4P/577644 | CFM56-5B4P/577645 |
| | CSTQD | 870 | 77,0 ton | 16,9 | 132 | 4,5% | 47% | 47,3 | 9 | 1 | CFM56-5B4P/779553 | |
| A321 | A321 Total | | | 14,7 | | | | | | | | |
| | CSTJE | 1307 | 89,0 ton | 15,4 | 121 | 5% | 40% | 47,3 | 2 | 2 | CFM56-5B3P/577601 | CFM56-5B3P/779519 |
| | CSTJF | 1399 | 89,0 ton | 15 | 128 | 4,5% | 43% | 47,1 | 1 | 0 | CFM56-5B3P/575424 | CFM56-5B33/699190 |
| | CSTJG | 1713 | 89,0 ton | 13,8 | 124 | 5% | 41% | 46,9 | 2 | 1 | CFM56-5B3P/779372 | CFM56-5B3P/779712 |

**Figure 31.** Performance degradation factors for TAP's Fleet

# Appendix 3. Aggregated Schedule and Aircrafts initial condition for the first 45 flights of 2016

| Aggregate Schedule | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| From Airp | Between | To Airp | Aircraft Reg | Departure | Arraival | Activity numbers | | | Flight Numbers | | | Activity | Tstart (hrs) | Tend (hrs) |
| OPO | DIRECT | LIS | CSTNK | 16.01.01 05:50 | 16.01.01 06:45 | 15 | | | 1921 | | | a1 | 0,00 | 1,42 |
| OPO | ORY | OPO | CSTTN | 16.01.01 06:20 | 16.01.01 11:25 | 18 | | 48 | 452 | | 451 | a2 | 0,50 | 6,08 |
| LIS | ORY | LIS | CSTNQ | 16.01.01 06:50 | 16.01.01 12:35 | 19 | | 58 | 434 | | 443 | a3 | 1,00 | 7,25 |
| LIS | VCE | LIS | CSTTF | 16.01.01 06:50 | 16.01.01 13:50 | 20 | | 65 | 866 | | 867 | a4 | 1,00 | 8,50 |
| LIS | FCO | LIS | CSTTK | 16.01.01 06:55 | 16.01.01 13:45 | 21 | | 61 | 844 | | 831 | a5 | 1,08 | 8,42 |
| LIS | AMS | LIS | CSTNN | 16.01.01 07:00 | 16.01.01 13:50 | 22 | | 68 | 664 | | 663 | a6 | 1,17 | 8,50 |
| LIS | ZRH | LIS | CSTNU | 16.01.01 07:00 | 16.01.01 13:30 | 23 | | 62 | 932 | | 937 | a7 | 1,17 | 8,17 |
| LIS | MAD | LIS | CSTTM | 16.01.01 07:00 | 16.01.01 10:25 | 24 | | 46 | 1024 | | 1011 | a8 | 1,17 | 5,08 |
| LIS | SXF | LIS | CSTNV | 16.01.01 07:05 | 16.01.01 15:00 | 26 | | 72 | 536 | | 537 | a9 | 1,25 | 9,67 |
| LIS | MAN | LIS | CSTQD | 16.01.01 07:05 | 16.01.01 13:35 | 25 | | 66 | 326 | | 321 | a10 | 1,25 | 8,25 |
| LIS | BCN | LIS | CSTTE | 16.01.01 07:05 | 16.01.01 11:45 | 28 | | 54 | 1050 | | 1043 | a11 | 1,25 | 6,42 |
| OPO | DIRECT | LIS | CSTTI | 16.01.01 07:05 | 16.01.01 08:00 | 31 | | | 1925 | | | a12 | 1,25 | 2,67 |
| LIS | BRU | LIS | CSTTP | 16.01.01 07:05 | 16.01.01 13:25 | 27 | | 64 | 616 | | 617 | a13 | 1,25 | 8,08 |
| OPO | FNC | LIS | CSTTS | 16.01.01 07:05 | 16.01.01 11:25 | 30 | | 55 | 1711 | | 1680 | a14 | 1,25 | 6,08 |
| LIS | LGW | LIS | CSTTU | 16.01.01 07:15 | 16.01.01 13:20 | 32 | | 63 | 348 | | 339 | a15 | 1,42 | 8,00 |
| LIS | MXP | LIS | CSTNH | 16.01.01 07:20 | 16.01.01 13:40 | 33 | | 69 | 808 | | 803 | a16 | 1,50 | 8,33 |
| LIS | GVA | LIS | CSTNG | 16.01.01 07:30 | 16.01.01 13:20 | 34 | | 67 | 956 | | 951 | a17 | 1,67 | 8,00 |
| LIS | HAM | LIS | CSTTQ | 16.01.01 07:40 | 16.01.01 15:15 | 35 | | 77 | 568 | | 569 | a18 | 1,83 | 9,92 |
| LIS | FNC | LIS | CSTJE | 16.01.01 08:00 | 16.01.01 12:10 | 37 | | 60 | 1671 | | 1682 | a19 | 2,17 | 6,83 |
| LIS | DUS | LIS | CSTNT | 16.01.01 08:00 | 16.01.01 14:45 | 36 | | 76 | 544 | | 541 | a20 | 2,17 | 9,42 |
| LIS | MUC | LIS | CSTNP | 16.01.01 08:10 | 16.01.01 15:15 | 38 | | 78 | 552 | | 557 | a21 | 2,33 | 9,92 |
| LIS | FRA | LIS | CSTTG | 16.01.01 08:25 | 16.01.01 15:25 | 40 | | 82 | 578 | | 583 | a22 | 2,58 | 10,08 |
| LIS | LHR | LIS | CSTJG | 16.01.01 08:35 | 16.01.01 15:05 | 41 | | 83 | 354 | | 379 | a23 | 2,75 | 9,75 |
| LIS | FNC | OPO | CSTTH | 16.01.01 08:40 | 16.01.01 13:00 | 42 | | 70 | 9328 | | 3852 | a24 | 2,83 | 7,67 |
| LIS | BLQ | LIS | CSTTI | 16.01.01 08:50 | 16.01.01 15:25 | 43 | | 84 | 872 | | 871 | a25 | 3,00 | 10,08 |
| LIS | OSL | LIS | CSTNL | 16.01.01 08:55 | 16.01.01 17:55 | 44 | | 99 | 762 | | 769 | a26 | 3,08 | 12,58 |
| LIS | ORY | LIS | CSTNK | 16.01.01 09:00 | 16.01.01 14:40 | 45 | | 81 | 442 | | 433 | a27 | 3,17 | 9,33 |
| LIS | WAW | LIS | CSTTB | 16.01.01 09:10 | 16.01.01 18:10 | 47 | | 101 | 1264 | | 1263 | a28 | 3,33 | 12,83 |
| LIS | CPH | LIS | CSTNS | 16.01.01 09:15 | 16.01.01 20:05 | 50 | 98 | 117 | 2924 | 2924 | 2924 | a29 | 3,42 | 14,75 |
| LIS | MAD | LIS | CSTTJ | 16.01.01 09:15 | 16.01.01 12:10 | 49 | | 73 | 1028 | | 1025 | a30 | 3,42 | 6,83 |
| LIS | PDL | LIS | CSTNI | 16.01.01 09:20 | 16.01.01 14:50 | 51 | | 87 | 1865 | | 1860 | a31 | 3,50 | 9,50 |
| LIS | DIRECT | OPO | CSTTA | 16.01.01 09:35 | 16.01.01 10:25 | 52 | | | 1926 | | | a32 | 3,75 | 5,08 |
| OPO | GVA | OPO | CSTTL | 16.01.01 09:45 | 16.01.01 15:00 | 53 | | 88 | 938 | | 939 | a33 | 3,92 | 9,67 |
| LIS | LHR | LIS | CSTTD | 16.01.01 09:50 | 16.01.01 16:20 | 56 | | 97 | 352 | | 359 | a34 | 4,00 | 11,00 |
| LIS | FAO | LIS | CSTTV | 16.01.01 09:50 | 16.01.01 12:05 | 57 | | 74 | 1907 | | 1908 | a35 | 4,00 | 6,75 |
| LIS | FNC | LIS | CSTTC | 16.01.01 10:20 | 16.01.01 14:30 | 59 | | 91 | 1685 | | 1672 | a36 | 4,50 | 9,17 |
| OPO | DIRECT | LIS | CSTTA | 16.01.01 11:10 | 16.01.01 12:05 | 71 | | | 1929 | | | a37 | 5,33 | 6,75 |
| LIS | FCO | LIS | CSTNW | 16.01.01 11:30 | 16.01.01 18:20 | 75 | | 113 | 838 | | 833 | a38 | 5,67 | 13,00 |
| LIS | PRG | LIS | CSTNR | 16.01.01 12:10 | 16.01.01 19:50 | 80 | | 127 | 1300 | | 1309 | a39 | 6,33 | 14,50 |
| OPO | ZRH | OPO | CSTTN | 16.01.01 12:10 | 16.01.01 18:05 | 79 | | 115 | 924 | | 917 | a40 | 6,33 | 12,75 |
| LIS | FNC | OPO | CSTTO | 16.01.01 12:25 | 16.01.01 16:50 | 85 | | 112 | 1683 | | 1720 | a41 | 6,58 | 11,50 |
| LIS | VCE | LIS | CSTTM | 16.01.01 12:30 | 16.01.01 19:25 | 86 | | 124 | 864 | | 859 | a42 | 6,67 | 14,08 |
| LIS | LUX | LIS | CSTTJ | 16.01.01 12:50 | 16.01.01 19:10 | 90 | | 126 | 694 | | 693 | a43 | 7,00 | 13,83 |
| LIS | ORY | LIS | CSTTS | 16.01.01 12:50 | 16.01.01 18:30 | 89 | | 120 | 436 | | 431 | a44 | 7,00 | 13,17 |
| LIS | AMS | LIS | CSTTA | 16.01.01 13:00 | 16.01.01 19:50 | 92 | | 132 | 662 | | 661 | a45 | 7,17 | 14,50 |

| Aircrafts Initial Condition | | | |
|---|---|---|---|
| Unique tails | Initial avail time | Tavailable (hrs) | Initial Airport |
| CSTNW | 42370,25 | 0,67 | LIS |
| CSTJE | 42370,25 | 0,67 | LIS |
| CSTNQ | 42370,24 | 0,50 | LIS |
| CSTNR | 42370,36 | 3,25 | LIS |
| CSTNS | 42370,35 | 3,17 | LIS |
| CSTTA | 42370,36 | 3,25 | LIS |
| CSTNJ | 42370,35 | 3,17 | LIS |
| CSTNL | 42370,28 | 1,50 | LIS |
| CSTTV | 42370,36 | 3,25 | LIS |
| CSTTD | 42370,35 | 3,08 | LIS |
| CSTTO | 42370,35 | 3,17 | LIS |
| CSTTB | 42370,34 | 2,92 | LIS |
| CSTTJ | 42370,34 | 2,83 | LIS |
| CSTTG | 42370,32 | 2,33 | LIS |
| CSTNK | | 0,00 | OPO |
| CSTTQ | 42370,28 | 1,50 | LIS |
| CSTNI | 42370,36 | 3,25 | LIS |
| CSTTN | | 0,00 | OPO |
| CSTTF | | 0,00 | LIS |
| CSTTK | | 0,00 | LIS |
| CSTNN | | 0,00 | LIS |
| CSTNU | | 0,00 | LIS |
| CSTTM | | 0,00 | LIS |
| CSTQD | | 0,00 | LIS |
| CSTNV | | 0,00 | LIS |
| CSTTP | | 0,00 | LIS |
| CSTTE | | 0,00 | LIS |
| CSTTL | 42370,38 | 3,67 | OPO |
| CSTTS | | 0,00 | OPO |
| CSTTI | | 0,00 | OPO |
| CSTTU | | 0,00 | LIS |
| CSTNH | | 0,00 | LIS |
| CSTNG | | 0,00 | LIS |
| CSTNT | | 0,00 | LIS |
| CSTNP | | 0,00 | LIS |
| CSTNM | 42370,45 | 5,58 | LIS |
| CSTJG | | 0,00 | LIS |
| CSTTH | | 0,00 | LIS |
| CSTTC | | 0,00 | LIS |
| CSTNX | | 0,00 | LIS |
| CSTJF | | 0,00 | LIS |
| CSTTR | | 0,00 | OPO |

**Figure 32.** Aggregated Schedule and Aircrafts initial condition for the first 45 flights of 2016

# Appendix 4. Fuel Cost Matrix for the first 15 activities of 2016

Fuel Costs

| | a1 | a2 | a3 | a4 | a5 | a6 | a7 | a8 | a9 | a10 | a11 | a12 | a13 | a14 | a15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CSTNW | $ 803,84 | $ 4 119,68 | $ 4 823,04 | $ 5 928,32 | $ 5 928,32 | $ 5 827,84 | $ 5 526,40 | $ 2 512,00 | $ 6 933,12 | $ 5 526,40 | $ 3 617,28 | $ 803,84 | $ 5 425,92 | $ 3 516,80 | $ 5 224,96 |
| CSTJE | $ 740,01 | $ 3 792,55 | $ 4 440,06 | $ 5 457,58 | $ 5 457,58 | $ 5 365,08 | $ 5 087,57 | $ 2 312,53 | $ 6 382,59 | $ 5 087,57 | $ 3 330,05 | $ 740,01 | $ 4 995,07 | $ 3 237,55 | $ 4 810,07 |
| CSTNQ | $ 734,89 | $ 3 766,31 | $ 4 409,34 | $ 5 419,82 | $ 5 419,82 | $ 5 327,96 | $ 5 052,37 | $ 2 296,53 | $ 6 338,43 | $ 5 052,37 | $ 3 307,01 | $ 734,89 | $ 4 960,51 | $ 3 215,15 | $ 4 776,79 |
| CSTNR | $ 741,38 | $ 3 799,55 | $ 4 448,26 | $ 5 467,65 | $ 5 467,65 | $ 5 374,98 | $ 5 096,96 | $ 2 316,80 | $ 6 394,37 | $ 5 096,96 | $ 3 336,19 | $ 741,38 | $ 5 004,29 | $ 3 243,52 | $ 4 818,94 |
| CSTNS | $ 732,50 | $ 3 754,07 | $ 4 395,01 | $ 5 402,20 | $ 5 402,20 | $ 5 310,63 | $ 5 035,95 | $ 2 289,07 | $ 6 317,82 | $ 5 035,95 | $ 3 296,26 | $ 732,50 | $ 4 944,38 | $ 3 204,69 | $ 4 761,26 |
| CSTTA | $ 925,70 | $ 4 744,19 | $ 5 554,18 | $ 6 827,01 | $ 6 827,01 | $ 6 711,30 | $ 6 364,16 | $ 2 892,80 | $ 7 984,13 | $ 6 364,16 | $ 4 165,63 | $ 925,70 | $ 6 248,45 | $ 4 049,92 | $ 6 017,02 |
| CSTNJ | $ 801,79 | $ 4 109,18 | $ 4 810,75 | $ 5 913,22 | $ 5 913,22 | $ 5 812,99 | $ 5 512,32 | $ 2 505,60 | $ 6 915,46 | $ 5 512,32 | $ 3 608,06 | $ 801,79 | $ 5 412,10 | $ 3 507,84 | $ 5 211,65 |
| CSTNL | $ 794,97 | $ 4 074,20 | $ 4 769,79 | $ 5 862,87 | $ 5 862,87 | $ 5 763,50 | $ 5 465,39 | $ 2 484,27 | $ 6 856,58 | $ 5 465,39 | $ 3 577,34 | $ 794,97 | $ 5 366,02 | $ 3 477,97 | $ 5 167,27 |
| CSTTV | $ 725,33 | $ 3 717,33 | $ 4 352,00 | $ 5 349,33 | $ 5 349,33 | $ 5 258,67 | $ 4 986,67 | $ 2 266,67 | $ 6 256,00 | $ 4 986,67 | $ 3 264,00 | $ 725,33 | $ 4 896,00 | $ 3 173,33 | $ 4 714,67 |
| CSTTD | $ 793,60 | $ 4 067,20 | $ 4 761,60 | $ 5 852,80 | $ 5 852,80 | $ 5 753,60 | $ 5 456,00 | $ 2 480,00 | $ 6 844,80 | $ 5 456,00 | $ 3 571,20 | $ 793,60 | $ 5 356,80 | $ 3 472,00 | $ 5 158,40 |
| CSTTO | $ 730,45 | $ 3 743,57 | $ 4 382,72 | $ 5 387,09 | $ 5 387,09 | $ 5 295,79 | $ 5 021,87 | $ 2 282,67 | $ 6 300,16 | $ 5 021,87 | $ 3 287,04 | $ 730,45 | $ 4 930,56 | $ 3 195,73 | $ 4 747,95 |
| CSTTB | $ 738,65 | $ 3 785,56 | $ 4 431,87 | $ 5 447,51 | $ 5 447,51 | $ 5 355,18 | $ 5 078,19 | $ 2 308,27 | $ 6 370,82 | $ 5 078,19 | $ 3 323,90 | $ 738,65 | $ 4 985,86 | $ 3 231,57 | $ 4 801,19 |
| CSTTJ | $ 735,91 | $ 3 771,56 | $ 4 415,49 | $ 5 427,37 | $ 5 427,37 | $ 5 335,38 | $ 5 059,41 | $ 2 299,73 | $ 6 347,26 | $ 5 059,41 | $ 3 311,62 | $ 735,91 | $ 4 967,42 | $ 3 219,63 | $ 4 783,45 |
| CSTTG | $ 788,48 | $ 4 040,96 | $ 4 730,88 | $ 5 815,04 | $ 5 815,04 | $ 5 716,48 | $ 5 420,80 | $ 2 464,00 | $ 6 800,64 | $ 5 420,80 | $ 3 548,16 | $ 788,48 | $ 5 322,24 | $ 3 449,60 | $ 5 125,12 |
| CSTNK | $ 734,55 | $ 3 764,57 | $ 4 407,30 | $ 5 417,30 | $ 5 417,30 | $ 5 325,48 | $ 5 050,03 | $ 2 295,47 | $ 6 335,49 | $ 5 050,03 | $ 3 305,47 | $ 734,55 | $ 4 958,21 | $ 3 213,65 | $ 4 774,57 |
| CSTTQ | $ 803,50 | $ 4 117,93 | $ 4 820,99 | $ 5 925,80 | $ 5 925,80 | $ 5 825,37 | $ 5 524,05 | $ 2 510,93 | $ 6 930,18 | $ 5 524,05 | $ 3 615,74 | $ 803,50 | $ 5 423,62 | $ 3 515,31 | $ 5 222,74 |
| CSTNI | $ 732,84 | $ 3 755,82 | $ 4 397,06 | $ 5 404,71 | $ 5 404,71 | $ 5 313,11 | $ 5 038,29 | $ 2 290,13 | $ 6 320,77 | $ 5 038,29 | $ 3 297,79 | $ 732,84 | $ 4 946,69 | $ 3 206,19 | $ 4 763,48 |
| CSTTN | $ 735,57 | $ 3 769,81 | $ 4 413,44 | $ 5 424,85 | $ 5 424,85 | $ 5 332,91 | $ 5 057,07 | $ 2 298,67 | $ 6 344,32 | $ 5 057,07 | $ 3 310,08 | $ 735,57 | $ 4 965,12 | $ 3 218,13 | $ 4 781,23 |
| CSTTF | $ 734,89 | $ 3 766,31 | $ 4 409,34 | $ 5 419,82 | $ 5 419,82 | $ 5 327,96 | $ 5 052,37 | $ 2 296,53 | $ 6 338,43 | $ 5 052,37 | $ 3 307,01 | $ 734,89 | $ 4 960,51 | $ 3 215,15 | $ 4 776,79 |
| CSTTK | $ 789,50 | $ 4 046,21 | $ 4 737,02 | $ 5 822,59 | $ 5 822,59 | $ 5 723,90 | $ 5 427,84 | $ 2 467,20 | $ 6 809,47 | $ 5 427,84 | $ 3 552,77 | $ 789,50 | $ 5 329,15 | $ 3 454,08 | $ 5 131,78 |
| CSTNN | $ 800,77 | $ 4 103,94 | $ 4 804,61 | $ 5 905,66 | $ 5 905,66 | $ 5 805,57 | $ 5 505,28 | $ 2 502,40 | $ 6 906,62 | $ 5 505,28 | $ 3 603,46 | $ 800,77 | $ 5 405,18 | $ 3 503,36 | $ 5 204,99 |
| CSTNU | $ 736,26 | $ 3 773,31 | $ 4 417,54 | $ 5 429,89 | $ 5 429,89 | $ 5 337,86 | $ 5 061,76 | $ 2 300,80 | $ 6 350,21 | $ 5 061,76 | $ 3 313,15 | $ 736,26 | $ 4 969,73 | $ 3 221,12 | $ 4 785,66 |
| CSTTM | $ 792,58 | $ 4 061,95 | $ 4 755,46 | $ 5 845,25 | $ 5 845,25 | $ 5 746,18 | $ 5 448,96 | $ 2 476,80 | $ 6 835,97 | $ 5 448,96 | $ 3 566,59 | $ 792,58 | $ 5 349,89 | $ 3 467,52 | $ 5 151,74 |
| CSTQD | $ 715,43 | $ 3 666,60 | $ 4 292,61 | $ 5 276,33 | $ 5 276,33 | $ 5 186,90 | $ 4 918,61 | $ 2 235,73 | $ 6 170,62 | $ 4 918,61 | $ 3 219,46 | $ 715,43 | $ 4 829,18 | $ 3 130,03 | $ 4 650,33 |
| CSTNV | $ 807,25 | $ 4 137,17 | $ 4 843,52 | $ 5 953,49 | $ 5 953,49 | $ 5 852,59 | $ 5 549,87 | $ 2 522,67 | $ 6 962,56 | $ 5 549,87 | $ 3 632,64 | $ 807,25 | $ 5 448,96 | $ 3 531,73 | $ 5 247,15 |
| CSTTP | $ 794,62 | $ 4 072,45 | $ 4 767,74 | $ 5 860,35 | $ 5 860,35 | $ 5 761,02 | $ 5 463,04 | $ 2 483,20 | $ 6 853,63 | $ 5 463,04 | $ 3 575,81 | $ 794,62 | $ 5 363,71 | $ 3 476,48 | $ 5 165,06 |
| CSTTE | $ 792,23 | $ 4 060,20 | $ 4 753,41 | $ 5 842,73 | $ 5 842,73 | $ 5 743,70 | $ 5 446,61 | $ 2 475,73 | $ 6 833,02 | $ 5 446,61 | $ 3 565,06 | $ 792,23 | $ 5 347,58 | $ 3 466,03 | $ 5 149,53 |
| CSTTL | $ 793,60 | $ 4 067,20 | $ 4 761,60 | $ 5 852,80 | $ 5 852,80 | $ 5 753,60 | $ 5 456,00 | $ 2 480,00 | $ 6 844,80 | $ 5 456,00 | $ 3 571,20 | $ 793,60 | $ 5 356,80 | $ 3 472,00 | $ 5 158,40 |
| CSTTS | $ 730,11 | $ 3 741,82 | $ 4 380,67 | $ 5 384,58 | $ 5 384,58 | $ 5 293,31 | $ 5 019,52 | $ 2 281,60 | $ 6 297,22 | $ 5 019,52 | $ 3 285,50 | $ 730,11 | $ 4 928,26 | $ 3 194,24 | $ 4 745,73 |
| CSTTI | $ 738,65 | $ 3 785,56 | $ 4 431,87 | $ 5 447,51 | $ 5 447,51 | $ 5 355,18 | $ 5 078,19 | $ 2 308,27 | $ 6 370,82 | $ 5 078,19 | $ 3 323,90 | $ 738,65 | $ 4 985,86 | $ 3 231,57 | $ 4 801,19 |
| CSTTU | $ 801,45 | $ 4 107,43 | $ 4 808,70 | $ 5 910,70 | $ 5 910,70 | $ 5 810,52 | $ 5 509,97 | $ 2 504,53 | $ 6 912,51 | $ 5 509,97 | $ 3 606,53 | $ 801,45 | $ 5 409,79 | $ 3 506,35 | $ 5 209,43 |
| CSTNH | $ 793,60 | $ 4 067,20 | $ 4 761,60 | $ 5 852,80 | $ 5 852,80 | $ 5 753,60 | $ 5 456,00 | $ 2 480,00 | $ 6 844,80 | $ 5 456,00 | $ 3 571,20 | $ 793,60 | $ 5 356,80 | $ 3 472,00 | $ 5 158,40 |
| CSTNG | $ 933,55 | $ 4 784,43 | $ 5 601,28 | $ 6 884,91 | $ 6 884,91 | $ 6 768,21 | $ 6 418,13 | $ 2 917,33 | $ 8 051,84 | $ 6 418,13 | $ 4 200,96 | $ 933,55 | $ 6 301,44 | $ 4 084,27 | $ 6 068,05 |
| CSTNT | $ 781,65 | $ 4 005,97 | $ 4 689,92 | $ 5 764,69 | $ 5 764,69 | $ 5 666,99 | $ 5 373,87 | $ 2 442,67 | $ 6 741,76 | $ 5 373,87 | $ 3 517,44 | $ 781,65 | $ 5 276,16 | $ 3 419,73 | $ 5 080,75 |
| CSTNP | $ 737,62 | $ 3 780,31 | $ 4 425,73 | $ 5 439,96 | $ 5 439,96 | $ 5 347,75 | $ 5 071,15 | $ 2 305,07 | $ 6 361,98 | $ 5 071,15 | $ 3 319,30 | $ 737,62 | $ 4 978,94 | $ 3 227,09 | $ 4 794,54 |
| CSTNM | $ 726,70 | $ 3 724,33 | $ 4 360,19 | $ 5 359,40 | $ 5 359,40 | $ 5 268,57 | $ 4 996,05 | $ 2 270,93 | $ 6 267,78 | $ 4 996,05 | $ 3 270,14 | $ 726,70 | $ 4 905,22 | $ 3 179,31 | $ 4 723,54 |
| CSTJG | $ 728,75 | $ 3 734,83 | $ 4 372,48 | $ 5 374,51 | $ 5 374,51 | $ 5 283,41 | $ 5 010,13 | $ 2 277,33 | $ 6 285,44 | $ 5 010,13 | $ 3 279,36 | $ 728,75 | $ 4 919,04 | $ 3 188,27 | $ 4 736,85 |
| CSTTH | $ 736,26 | $ 3 773,31 | $ 4 417,54 | $ 5 429,89 | $ 5 429,89 | $ 5 337,86 | $ 5 061,76 | $ 2 300,80 | $ 6 350,21 | $ 5 061,76 | $ 3 313,15 | $ 736,26 | $ 4 969,73 | $ 3 221,12 | $ 4 785,66 |
| CSTTC | $ 790,53 | $ 4 051,46 | $ 4 743,17 | $ 5 830,14 | $ 5 830,14 | $ 5 731,33 | $ 5 434,88 | $ 2 470,40 | $ 6 818,30 | $ 5 434,88 | $ 3 557,38 | $ 790,53 | $ 5 336,06 | $ 3 458,56 | $ 5 138,43 |
| CSTNX | $ 736,26 | $ 3 773,31 | $ 4 417,54 | $ 5 429,89 | $ 5 429,89 | $ 5 337,86 | $ 5 061,76 | $ 2 300,80 | $ 6 350,21 | $ 5 061,76 | $ 3 313,15 | $ 736,26 | $ 4 969,73 | $ 3 221,12 | $ 4 785,66 |
| CSTJF | $ 790,53 | $ 4 051,46 | $ 4 743,17 | $ 5 830,14 | $ 5 830,14 | $ 5 731,33 | $ 5 434,88 | $ 2 470,40 | $ 6 818,30 | $ 5 434,88 | $ 3 557,38 | $ 790,53 | $ 5 336,06 | $ 3 458,56 | $ 5 138,43 |
| CSTTR | $ 933,21 | $ 4 782,68 | $ 5 599,23 | $ 6 882,39 | $ 6 882,39 | $ 6 765,74 | $ 6 415,79 | $ 2 916,27 | $ 8 048,90 | $ 6 415,79 | $ 4 199,42 | $ 933,21 | $ 6 299,14 | $ 4 082,77 | $ 6 065,83 |

**Figure 33.** Fuel Cost Matrix for the first 15 activities of 2016

# Appendix 5. Original assignment matrix for the first 40 activities of 2016

| | a1 | a2 | a3 | a4 | a5 | a6 | a7 | a8 | a9 | a10 | a11 | a12 | a13 | a14 | a15 | a16 | a17 | a18 | a19 | a20 | a21 | a22 | a23 | a24 | a25 | a26 | a27 | a28 | a29 | a30 | a31 | a32 | a33 | a34 | a35 | a36 | a37 | a38 | a39 | a40 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CSTQD | | | | | | | | | | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| CSTTQ | | | | | | | | | | | | | | | | | | 1 | | | | | | | | | | | | | | | | | | | | | | |
| CSTTU | | | | | | | | | | | | | | | 1 | | | | | | | | | | | | | | | | | | | | | | | | | |
| CSTTE | | | | | | | | | | | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| CSTTD | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 1 | | | | | | |
| CSTJG | | | | | | | | | | | | | | | | | | | | | | | 1 | | | | | | | | | | | | | | | | | |
| CSTNH | | | | | | | | | | | | | | | | 1 | | | | | | | | | | | | | | | | | | | | | | | | |
| CSTNK | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | 1 | | | | | | | | | | | | | |
| CSTTC | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 1 | | | | |
| CSTNM | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| CSTTG | | | | | | | | | | | | | | | | | | | | | | 1 | | | | | | | | | | | | | | | | | | |
| CSTTS | | | | | | | | | | | | | | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | |
| CSTTP | | | | | | | | | | | | | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| CSTNQ | | | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| CSTTI | | | | | | | | | | | | 1 | | | | | | | | | | | | | 1 | | | | | | | | | | | | | | | |
| CSTNI | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 1 | | | | | | | | | |
| CSTTB | | | | | | | | | | | | | | | | | | | | | | | | | | | | 1 | | | | | | | | | | | | |
| CSTTN | | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 1 |
| CSTTL | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 1 | | | | | | | |
| CSTNV | | | | | | | | | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| CSTNL | | | | | | | | | | | | | | | | | | | | | | | | | | 1 | | | | | | | | | | | | | | |
| CSTTA | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 1 | | | | | 1 | | | |
| CSTNT | | | | | | | | | | | | | | | | | | | | 1 | | | | | | | | | | | | | | | | | | | | |
| CSTTV | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 1 | | | | | |
| CSTNP | | | | | | | | | | | | | | | | | | | | | 1 | | | | | | | | | | | | | | | | | | | |
| CSTNG | | | | | | | | | | | | | | | | | 1 | | | | | | | | | | | | | | | | | | | | | | | |
| CSTNJ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| CSTNN | | | | | | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| CSTTR | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| CSTTJ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 1 | | | | | | | | | | |
| CSTNW | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 1 | | |
| CSTNS | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 1 | | | | | | | | | | | |
| CSTJE | | | | | | | | | | | | | | | | | | | 1 | | | | | | | | | | | | | | | | | | | | | |
| CSTNU | | | | | | | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| CSTTK | | | | | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| CSTTO | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| CSTTM | | | | | | | | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| CSTTF | | | | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| CSTNX | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| CSTTH | | | | | | | | | | | | | | | | | | | | | | | | 1 | | | | | | | | | | | | | | | | |
| CSTNR | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 1 | |
| CSTJF | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

**Figure 34.** Original assignment matrix for the first 40 activities of 2016

# Appendix 6. Simulated annealing matrix for the first 20 flights of 2016

| | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Maintenance | | | | | | | | | | | | | | | | | | | | |
| | | | | | Flight Percentage | 0,0004 | 0,0016 | 0,0017 | 0,0021 | 0,0021 | 0,0021 | 0,0020 | 0,0011 | 0,0024 | 0,0020 | 0,0014 | 0,0004 | 0,0019 | 0,0014 | 0,0018 | 0,0019 | 0,0018 | 0,0023 | 0,0013 | 0,0020 |
| | | | | | From Airport | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 1 | 2 | 1 | 1 | 1 | 1 | 1 | 1 |
| | | | | | To Airport | 1 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | | | | | Flight Start Time | 0,00 | 0,50 | 1,00 | 1,00 | 1,08 | 1,17 | 1,17 | 1,17 | 1,25 | 1,25 | 1,25 | 1,25 | 1,25 | 1,25 | 1,42 | 1,50 | 1,67 | 1,83 | 2,17 | 2,17 |
| | | | | | Duration | 1,42 | 5,58 | 6,25 | 7,50 | 7,33 | 7,33 | 7,00 | 3,92 | 8,42 | 7,00 | 5,17 | 1,42 | 6,83 | 4,83 | 6,58 | 6,83 | 6,33 | 8,08 | 4,67 | 7,25 |
| | | | | | Flight End Time | 1,42 | 6,08 | 7,25 | 8,50 | 8,42 | 8,50 | 8,17 | 5,08 | 9,67 | 8,25 | 6,42 | 2,67 | 8,08 | 6,08 | 8,00 | 8,33 | 8,00 | 9,92 | 6,83 | 9,42 |
| Aircraft Cost With Demand | Utilization | Initial Availability | Starting Airport | Max utilization | | a1 | a2 | a3 | a4 | a5 | a6 | a7 | a8 | a9 | a10 | a11 | a12 | a13 | a14 | a15 | a16 | a17 | a18 | a19 | a20 |
| 66953,4 | 1,907% | 0,00 | 1 | 100% | CSTQD | | | | | | | | | | | | | | | | | | | | |
| 108769,5173 | 2,531% | 1,50 | 1 | 100% | CSTTQ | | | | | | | | | | | | | | | | | | | | |
| 132445,3933 | 2,865% | 0,00 | 1 | 100% | CSTTU | | | | | | | | | | | | | | | 1 | | | | | |
| 105599,786 | 2,496% | 0,00 | 1 | 100% | CSTTE | | | | | | | | | | | 1 | | | | | | | | | |
| 87222,30933 | 2,510% | 3,08 | 1 | 100% | CSTTD | | | | | | | | | | | | | | | | | | | | |
| 50581,86 | 1,165% | 0,00 | 1 | 100% | CSTJG | | | | | | | | | | | | | | | | | | | | |
| 91289,108 | 1,970% | 0,00 | 1 | 100% | CSTNH | | | | | | | | | | | | | | | | 1 | | | | |
| 95215,488 | 2,317% | 0,00 | 2 | 100% | CSTNK | 1 | | | | | | | | | | | | | | | | | | | |
| 116578,72 | 2,487% | 0,00 | 1 | 100% | CSTTC | | | | | | 1 | | | | | | | | | | | | | | |
| 107528,76 | 2,515% | 5,58 | 1 | 100% | CSTNM | | | | | | | | | | | | | | | | | | | | |
| 109505,64 | 2,538% | 2,33 | 1 | 100% | CSTTG | | | | | | | | | | | | | | | | | | | | |
| 107837,54 | 2,634% | 0,00 | 2 | 100% | CSTTS | | | | | | | | | | | | | | 1 | | | | | | |
| 117448,234 | 2,615% | 0,00 | 1 | 100% | CSTTP | | | | | | | | | | | | | 1 | | | | | | | |
| 92696,12 | 2,748% | 0,50 | 1 | 100% | CSTNQ | | | | | | | | | 1 | | | | | | | | | | | |
| 105902,826 | 2,140% | 0,00 | 2 | 100% | CSTTI | | | | | | | | | | | | 1 | | | | | | | | |
| 86547,69867 | 2,482% | 3,25 | 1 | 100% | CSTNI | | | | | | | | | | | | | | | | | | | | |
| 106000,7973 | 2,820% | 2,92 | 1 | 100% | CSTTB | | | | | | | | | | | | | | | | | | | | |
| 118136,3167 | 2,636% | 0,00 | 2 | 100% | CSTTN | | | | | | | | | | | | | | | | | | | | |
| 110794,5207 | 2,387% | 3,67 | 2 | 100% | CSTTL | | | | | | | | | | | | | | | | | | | | |
| 96616,112 | 2,263% | 0,00 | 1 | 100% | CSTNV | | | | | | | | | | | | | | | | | | 1 | | |
| 104481,168 | 2,226% | 1,50 | 1 | 100% | CSTNL | | | | | | | | | | | | | | | | | | | | |
| 105197,924 | 2,592% | 3,25 | 1 | 100% | CSTTA | | | | | | | | | | | | | | | | | | | | |
| 113775,856 | 2,844% | 0,00 | 1 | 100% | CSTNT | | | | | | | | | | | | | | | | | | | 1 | |
| 117666,7827 | 2,664% | 3,25 | 1 | 100% | CSTTV | | | | | | | | | | | | | | | | | | | | |
| 86161,32 | 2,063% | 0,00 | 1 | 100% | CSTNP | | | | | | | | | | | | | | | | | | | | |
| 90221,424 | 2,459% | 0,00 | 1 | 100% | CSTNG | | | | | | | | | | | | | | | | | 1 | | | |
| 105484,3747 | 2,837% | 3,17 | 1 | 100% | CSTNJ | | | | | | | | | | | | | | | | | | | | |
| 89016,6 | 2,552% | 0,00 | 1 | 100% | CSTNN | | | | | | | 1 | | | | | | | | | | | | | |
| 105287,552 | 2,790% | 0,00 | 2 | 100% | CSTTR | | 1 | | | | | | | | | | | | | | | | | | |
| 96807,19333 | 2,429% | 2,83 | 1 | 100% | CSTTJ | | | | | | | | | | | | | | | | | | | | |
| 107158,9387 | 2,587% | 0,67 | 1 | 100% | CSTNW | | | | | | | | | | 1 | | | | | | | | | | |
| 125501,84 | 2,709% | 3,17 | 1 | 100% | CSTNS | | | | | | | | | | | | | | | | | | | | |
| 72561,26333 | 1,639% | 0,67 | 1 | 100% | CSTJE | | | | | | | | | | | | | | | | | | | | |
| 91643,36667 | 2,235% | 0,00 | 1 | 100% | CSTNU | | | | | | | | | | | | | | | | | | | | 1 |
| 93022,43333 | 2,212% | 0,00 | 1 | 100% | CSTTK | | | | | | | | 1 | | | | | | | | | | | | |
| 97535,086 | 2,123% | 3,17 | 1 | 100% | CSTTO | | | | | | | | | | | | | | | | | | | | |
| 106200,4233 | 2,559% | 0,00 | 1 | 100% | CSTTM | | | | 1 | | | | | | | | | | | | | | | | |
| 106886,198 | 2,678% | 0,00 | 1 | 100% | CSTTF | | | 1 | | | | | | | | | | | | | | | | | |
| 100559,184 | 2,161% | 0,00 | 1 | 100% | CSTNX | | | | | 1 | | | | | | | | | | | | | | | |
| 107613,884 | 2,531% | 0,00 | 1 | 100% | CSTTH | | | | | | | | | | | | | | | | | | | | |
| 85616,752 | 2,366% | 3,25 | 1 | 100% | CSTNR | | | | | | | | | | | | | | | | | | | | |
| 37079,67533 | 0,716% | 0,00 | 1 | 100% | CSTJF | | | | | | | | | | | | | | | | | | | | |
| | | | | | | 8 | 29 | 38 | 37 | 39 | 9 | 28 | 35 | 14 | 31 | 4 | 15 | 13 | 12 | 3 | 7 | 26 | 20 | 23 | 34 |

**Figure 35.** Simulated annealing matrix for the first 20 flights of 2016