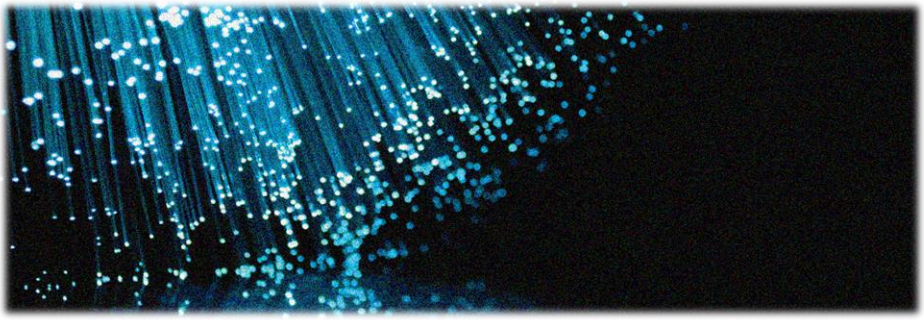

Study and optimization of core allocation in multi-core optical fibers

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Resumo

Multiplexagem por divisão de espaço (SDM) é vista como uma solução promissora para a iminente crise de escassez de capacidade. O crescimento exponencial do tráfego de rede que nos encaminha para esta crise criou a necessidade de sistemas ópticos de alta capacidade, que é onde as fibras homogéneas de múltiplos núcleos de modo único (SM-MCF) podem ser úteis.

Apresenta-se um método para estimar a interferência dentro de uma fibra de múltiplos núcleos (MCF), bem como várias configurações que visam minimizar a interferência entre núcleos (XT). Um método para escolher a melhor configuração para os núcleos é concebido e três fibras com diferentes diâmetros de revestimento ($C_d = 125, 260, 300\ \mu m$) são analisadas.

Keywords – Multiplexação por divisão de espaço (SDM), fibras de múltiplos núcleos (MCF), interferência entre núcleos (XT), alocação de núcleos, layout

Abstract

Space-division multiplexing (SDM) is regarded as a promising solution for the capacity crunch looming just around the corner. The exponential growth of network traffic that has us gravitating towards this crunch has created the need for high-capacity optical transmission systems, which is where homogeneous single-mode multi-core fibers (SM-MCF) step into the scene.

A method for the estimation of crosstalk inside a MCF is introduced, along with several layouts that seek to minimize the inter-core crosstalk (XT) amongst the cores. A method for choosing the best layout for the cores on a given MCF is devised and three fibers differing only in cladding diameter ($C_d = 125, 260, 300\ \mu m$) are analysed.

Keywords – Space-division multiplexing (SDM), multi-core fiber (MCF), inter-core crosstalk (XT), core allocation, layout

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Nomenclature

λ	*Wavelength*
Λ	*Core pitch*
a_1	*Core radius*
a_2	*Cladding radius*
a_3	*Trench radius*
wtr	*Trench width*
n_1	*Core refractive index*
n_2	*Cladding refractive index*
n_3	*Trench refractive index*
Δ_1	*Core-Cladding relative refractive index*
Δ_2	*Cladding-Trench relative refractive index*
R_b	*Bending Radius*
d_c	*Correlation Length*
C_d	*Cladding Diameter*
OCT	*Outer Cladding Thickness*
L	*Fiber Length*
XT	*Inter-core crosstalk*
$P(Z)$	*Power at the output of the interference core*
$P'(Z)$	*Power at the output of the reference core*
β_m	*Propagation Constant*
k_{mn}	*Mode coupling coefficient*
k'_{mn}	*Mode coupling coefficient for the trench-assisted case*
$\bar{h}_{mn}$	*Power coupling coefficient*
N	*Number of cores in the Layout*
M_1	*Number of cores in the Outer Ring*
M_2	*Number of cores in the second most outward ring*
M_3	*Number of cores in the third most outward ring*
r_1	*Radius of the Outer Circle*
r_2	*Radius of the second most outward circle*
r_3	*Radius of the third most outward circle*
Λ_1	*Distance between Outer Cores*
Λ_2	*Distance between cores on the second most outward circle*
Λ_3	*Distance between cores on the third most outward circle*
Λ_x	*Distance between cores in different rings*

Glossary

BER	Bit Error Rate
CMT	Coupled-Mode Theory
CPT	Coupled-Power Theory
DWDM	Dense Wavelength-Division Multiplexing
FDM	Frequency-Division Multiplexing
FTTH	Fiber to the Home
Inner Circle	Circumference containing nuclei closest to the center of the fiber
Inner Ring	Structure composed by nuclei laying on the Inner Circle
Inner Cores	Cores composing the Inner Ring
Layout Variations	Different spatial distributions of the cores in a layout as a function of N
Limiting Crosstalk	Highest crosstalk in the structure
MCF	Multi-Core Fiber
Middle Circle	Circumference containing nuclei in-between the Inner and Outer Circles
Middle Ring	Structure composed by nuclei laying on the Middle Circle
Middle Cores	Cores composing the Middle Ring
MMF	Multi-Mode Fiber
Neighbour	Any of the closest cores also laying on a circumference of the same radius
Outer Circle	Circumference containing nuclei the furthest from the center of the fiber
Outer Ring	Structure composed by nuclei laying on the Outer Circle
Outer Cores	Cores composing the Outer Ring
OSNR	Optical Signal-to-Noise Ratio
PAM	Pulse-Amplitude Modulation
PDM	Polarization-Division Modulation
QAM	Quadrature Amplitude Modulation
QPSK	Quadrature Phase-Shift Keying
SDM	Space-Division Multiplexing
SM-MCF	Single-Mode Multi-Core Fiber
SNR	Signal-to-Noise Ratio
WDM	Wavelength-Division Multiplexing

1 Introduction

1.1 Scope of the Work

Ever since the first “LO” message went through ARPANET’s first computer-to-computer link on October 29, 1969, the need for capacity in packet switching networks has never ceased to increase. While Neil Armstrong had just taken a step on the moon, Mankind stepped into the Information Age. The Revolution was underway, with individuals able to instantly access information and freely transferring it through leased telephone lines at a rate of 50kb/s.

By the end of 1973 there were thirty seven sites on the ARPANET [1], ranging from governmental entities such as NASA to private firms like Xerox PARC, where Ethernet technology had just been invented. This new technology quickly became prominent within corporations and institutions, providing data transfer rates up to 2.94 Mbit/s over coaxial cables.

Marveled by these new tools of communication, many engineers and inventors wondered about ways to translate what was going on inside companies and organizations to a wider geography, as the existing labyrinth of copper wires making up the telephone network lacked the pure transmission speed enjoyed by corporate LANs. The solution was just outside the door, literally.

For forty years the cable television industry had been mounting coaxial and fiber-optic cables through conduits and utility poles that ran straight to your door, with the single purpose of filling living rooms across America with even more television channels. Tens of thousands of kilometers of what this industry called plant, the network of coaxial and fiber optic cables, had been deployed to form this sleek, fast, multi-billion dollar data network that was surprisingly well-suited for data.

Cable-television companies didn’t take long to realize the innate communication abilities of the plant they’d built, and soon businesses were being offered T1 speed (1544 Mbit/s) communication services with the use of a cable modem. It wasn’t, however, until the late-1990s that cable modems became mainstream, when the majority of U.S cable systems were activated for bidirectional signaling.

Up until the 90s, optical fiber communications were only used in long-distance applications. Despite boasting much lower attenuation and interference than the existing copper wire, optical fibers were costly and complex to deploy, only fully exploited in big-data applications. As the use of the Internet exploded in the 1990s, the demand for such an infrastructure, capable of carrying heavy loads of digital data, led to the deployment of thousands of kilometers of fiber cable all around the world.

Optical fiber had come a long way. From carrying its first live telephone traffic at 6 Mbit/s back in 1977 across a California beach, to DWDM systems able of transmitting 100 wavelengths at 10Gbit/s each in the late 90’s, there were no doubts that optical fiber was the future.

Today, sixteen years later, it still is. Although fiber transmission capacity has been increasing 20% per year since 2000 [2], new solutions are required as the demand for data bandwidth is showing no signs of slowing down.

1.2 Motivation

In recent decades network traffic demand has been growing unceasingly, showing growth rates in-between 20% and 60% per year in this last decade [3]. Forecasts show this demand isn't likely to slow down the pace anytime soon, which is the result of a variety of factors.

The emergence of new technologies and applications, changing the rate at which we consume data, combined with the recent rise of machine-to-machine communications and the advent of the so called "Internet of Things", has caused demand to skyrocket to heights never before imagined by network engineers.

Despite technologies such as Dense Wavelength-Division Multiplexing (DWDM) and Coherent detection that allowed for a multiplicative increase of capacity in optical systems, engineers still struggle to cope with the exponential growth traffic demand is experimenting nowadays. Furthermore, current systems are quickly approaching the limit for the maximum amount of information that can be transmitted over a given channel [4], leading to the looming capacity crunch.

In order to overcome the capacity limits in the existing optical fiber communication infrastructure, increasing the spatial efficiency within the available fiber cross-section is the most effective solution. Multi-core fibers (MCFs), in the scope of space-division multiplexing (SDM), make up a promising solution to the aforementioned efficiency issue.

Performing SDM, with the use of uncoupled MCFs, consists of a simple and robust solution that doesn't require complex multiple-input multiple-output signal processing at the receiver side. The main issue and focus of this dissertation is, however, being able to increase the number of cores inside the fiber while keeping the inter-core crosstalk (XT) low.

Note that keeping the crosstalk low is important not only for allowing data to reach longer distances, but also for being able to use high multi-level modulation formats.

Different strategies have been employed to achieve this. The use of a trench, originally proposed to reduce the fiber bending loss in FTTH applications [5], has proven to be very effective for XT reduction in MCFs when applied to each core; making up the so called "trench-assisted" structures.

By making use of these trench-assisted structures, we will analyze the XT of different proposed core arrangements (layouts) in an attempt to minimize the crosstalk in the fiber. Finally, in a bid to maximize the fiber's capacity, we describe a method for spatially arranging identical cores inside a MCF using the layouts that were previously proposed.

1.3 Thesis Outline

The purpose of this dissertation is to describe a way of spatially arranging identical cores inside a multi-core fiber (MCF), in a bid to maximize its capacity by placing as many cores as possible inside. In an attempt to find an optimal solution, several layouts were proposed and analyzed from a crosstalk point of view, with the aid of a MatLab algorithm.

The thesis is structured as follows:

In Chapter 2, an explanation of the main concepts invoked throughout this dissertation is provided, from basic theory on MCF to the method used in the crosstalk estimation.

In Chapter 3, a practical description of the different proposed layouts is given. These descriptions feature explanations on how the layout was build and the crosstalk estimated, and include the necessary information on how to reproduce its geometry.

In Chapter 4, an illustration of the algorithm used in the crosstalk estimation is provided, followed by a brief validation in which the obtained results are proven consistent with those from a related paper.

In Chapter 5, a description of the problem is provided along with an available state-of-the-art solution. An alternative solution, subject of this thesis, is then exposed for each of the three case studies considered.

In chapter 6, the final conclusions and achievements are presented, as well as the potential improvements and future work.

1.4 Original Contributions

The main contributions of this work are:

- Development of a functional crosstalk estimation algorithm for a set of layouts with adjustable fiber parameters.
- Description of a method for selecting the most adequate layout for a MCF with a precise cladding diameter, envisioning crosstalk optimization.
- Proposal of commercially viable low-crosstalk solutions for the core distribution in MCF's.
- Performance optimization, by means of a low-crosstalk layout, of three MCF's varying only in size: $C_d = 125 / 260 / 300\ \mu m$.

2 Background

2.1 Space Division Multiplexing

Data transmission, either through copper or fiber, makes use of electromagnetic waves, which are governed by Maxwell's equations in a classical context. These equations describe an electromagnetic field that can vary across five physical dimensions, which can be used for modulation and multiplexing, as shown in figure 1.

Time Dimension

By creating a series of time slots and varying within each a single scalar quantity (such as the amplitude or phase) of the electromagnetic field according to a specific pattern allows the formation of communication symbols.

These communication symbols are then transmitted in temporal succession at a certain rate, carrying often more than one bit per symbol depending on the amplitude modulation employed.

Quadrature Dimension

Many communication systems modulate pulses onto an electromagnetic carrier wave whose frequency is much larger than the symbol rate. Take the example of WDM networks, which use optical carrier frequencies in the 193-THz regime for the optical carrier and optical amplification bandwidths in the 5-THz range; these systems boast a very small fractional bandwidth in which the electromagnetic field can be thought of having two independent components (a sine and a cosine component).

These two components, often names quadratures, can then be modulated to form a two-dimensional symbol alphabet, such as the quadrature amplitude modulation (QAM), which in its simplest form may be perceived as two quadrature-multiplexed PAM signals.

Frequency Dimension

Transmitting multiple communication signals in parallel on different carrier frequencies over the same transmission medium is known as frequency division multiplexing (FDM), or wavelength division multiplexing (WDM) in the optical communications context. Carrier frequencies require a spacing in-between them, which should be no less than the symbol rate used on each carrier so that the multiplexed QAM signals can be individually decodable without crosstalk.

On an inherently shared medium (e.g., the mobile wireless channel), the capacity scalability of FDM systems is typically limited by regulatory bandwidth constraints, while fundamental physical or engineering limitations set such limits on waveguides (e.g., coaxial, twisted-pair, or fiber cables).

Polarization Dimension

In some applications, such as coherent optical communications, the vector nature of electromagnetic waves may be exploited to simultaneously transmit two independent information streams on a set of two orthogonal polarizations. Polarization division multiplexing (PDM) doubles the transmission capacity compared to a single-polarization system; and with the introduction of correlation between symbols in the two polarizations the construction of four-dimensional modulation formats is made possible, which can be designed to optimize transmission performance at the cost of spectral efficiency [6].

Spatial Dimension

The spatial dimension is exploited by sending information over different parallel spatial paths. This entails a wide variety of techniques across many communications segments, ranging from data buses on printed circuit boards to more complex multi-antenna techniques in cellular wireless systems.

In recent years, optical communications research has focused on fibers with multiple parallel cores within a common cladding (MCFs) as well as on “few-mode fibers”, which support multiple independent spatial patterns of light (modes) across their core areas. A particular challenge with these systems, as well as with many other whether electrical or optical, is the presence of crosstalk among the parallel spatial paths. While in some applications this crosstalk can be dealt with interference cancellation and multiple-input-multiple-output digital signal processing techniques, this dissertation will exploit low-crosstalk waveguide designs.

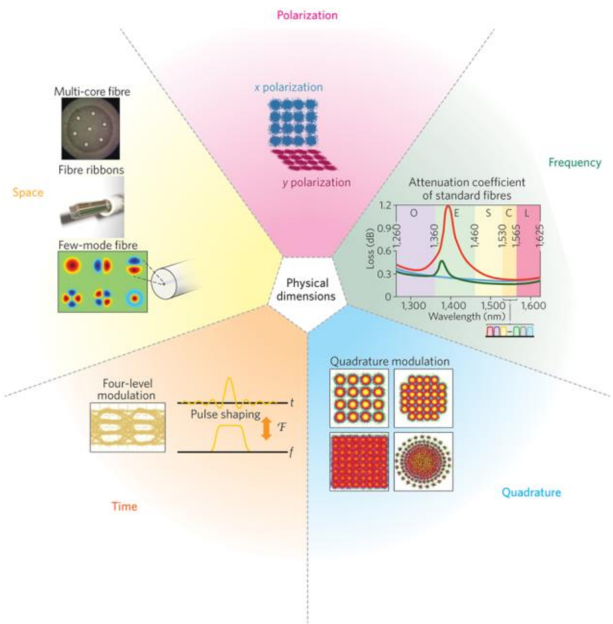

Figure 1: Physical dimensions for modulation and multiplexing of electromagnetic waves [3]

2.2 Optical Fibers

SM optical fibers are the leading transmission medium for optical communication systems all around the world. Using a high-index core surrounded by a low-index cladding, light can be guided through an optical fiber by means of multiple total internal reflections at the core-cladding boundary.

In most single-mode fibers and some multi-mode fibers the step-index profile is used, where the refractive index is uniformly distributed across the length of the core, facing a sharp decrease at the core-cladding interface as to guarantee a lower refractive index in the cladding. Most fibers have a low refractive index contrast ($\Delta \ll 1$), causing the electric field to leak and travel through the cladding, resulting in weakly guided fiber modes that can be simplified using linear polarization (LP) modes.

Multi-mode fibers, which are not in the scope of this dissertation, make use of the propagation modes to increase capacity by allowing several to be transmitted simultaneously. A MMF will generally boast a wider core diameter than its SMF counterpart, being used for short-distance communication links and for applications where high power must be transmitted

2.2.1 Multi-Core Fibers

First manufactured by Furakawa Electric in 1979, MCFs consist of a structure enclosing multiple cores in a single cladding.

Presently a hot topic for its promising potential in improving the efficiency of SDM, MCFs can be classified into coupled-type and uncoupled-type fibers. The first type makes use of several cores placed in such a way that allows cores to couple with each other. Much like in MMFs, coupled-type MCF make use of the propagation modes to perform spatial multiplexing without requiring the complex index profiles of advanced MMFs.

The latter, uncoupled-type MCFs, require each core to be properly arranged inside the fiber to keep the inter-core crosstalk low enough as to allow for long-distance transmission applications.

Other than properly arranging the cores inside an uncoupled-type MCF, some other strategies have been proposed to further reduce the crosstalk. A trench-assisted structure, with strong light confining capabilities, will be used in the scenarios to be studied. As demonstrated in section 4.2, a trench-assisted multi-core fiber (TA-MCF) will yield a lower inter-core crosstalk than a MCF with a step-index profile thanks to the existence of a low index trench layer that is able to reduce the overlap of electromagnetic fields between neighbouring cores.

The number of cores to be placed inside such a fiber depends on the design parameters taken into consideration.

For illustration purposes, a seven core trench-assisted MCF has been chosen to represent the main parameters considered when computing the crosstalk for different proposed core arrangements (or layouts).

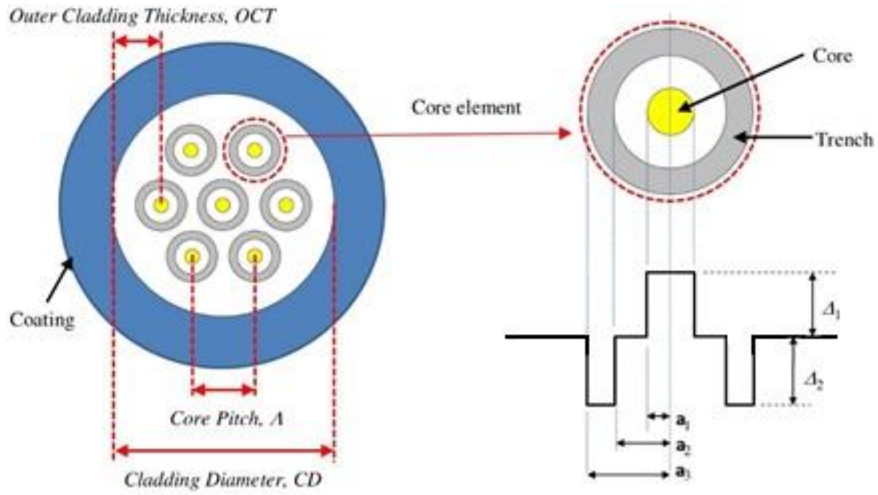

Figure 2: Main parameters of a multi-core fiber

Three different optical fibers will be considered in this work, all sharing the same set of parameters but for the Cladding Diameter (C_d). This parameter, often named Fiber Diameter and measuring the distance between two opposite edges of the fiber, will take a different value for each one of the three considered scenarios.

The Outer Cladding Thickness (OCT), the distance between an Outer Core and the edge of the Outer Cladding cannot be any smaller than 30 µm, as to minimize the micro-bending loss [7]. The distance between the two ends of the optical fiber, not represented in figure 2, is defined as Fiber Length (L).

The homogeneous cores placed inside the fiber, all sharing the same size and refractive indexes, will be enclosed by a trench that further confines light into the center of the core. The thickness of the trench (wtr) and of both Cladding and Core will be the same; making the radius of the Cladding (a_2) and Trench (a_3) two and three times larger than, respectively, the radius of the Core (a_1). The relative refractive indexes of the Core-Cladding (Δ_1) and Cladding-Trench (Δ_2) will be identical; and by setting a cladding refractive index (n_2) both the core refractive index (n_1) and the trench refractive index (n_3) can be deduced.

As for the distance that separates two cores, the core pitch (Λ), it will vary in function of the different proposed layouts and number of cores integrating them. This parameter, directly correlated with XT, will be key in balancing the crosstalk values inside the fiber.

Additionally, a minimum distance of 3 µm between trench edges must be ensured as to safeguard any contact between trenches when the fiber bends [8].

2.3 Crosstalk Estimation

Crosstalk, by definition, is the disturbance of a signal by the electric or magnetic field of another adjacent telecommunications signal.

Considering the way in which cores are densely packed inside a single cladding of a MCF, it shouldn't come as a surprise that crosstalk management is crucial when dealing with coupling and the consequent degradation of transmitted signals.

Crosstalk can be described by the following expression; where $P'(Z)$ is the power at the output of the reference core and $P(Z)$ is the power at the output of the interfering core.

$$XT(Z) = 10 \times log_{10} \frac{P'(Z)}{P(Z)} [dB] \tag{2.1}$$

Crosstalk mitigation can be a daunting task, but if not carried out, will prevent the fiber from achieving its maximum potential in terms of performance and capacity.

For crosstalk to be properly dealt with, an accurate method for its estimation in MCFs is required. Two methods exist for doing so: Coupled-Mode Theory (CMT) and Coupled-Power Theory (CPT).

Coupled-Mode Theory (CMT), a perturbational approach for the analysis of coupling in vibrational systems, takes into account the interference between optical modes from both waveguides when these are brought sufficiently close to each other. In those cases where the electromagnetic field distributions after mode coupling don't substantially differ from those previous to it, this method can be used to analyze the propagation characteristics of the waveguides.

While this method allows for an accurate estimation of the crosstalk while taking into account the twisting and bending effects that the fiber is subjected to, a large number of simulations are required to estimate the value of crosstalk.

Coupled-power theory, on the other hand, is based on the principle of measuring the amount of power that the signal being transmitted in one core is transferring to its neighbouring core. Unlike CMT, CPT is able to provide a fast and accurate estimation of inter-core crosstalk in MCFs by averaging the bending and twisting effects along the fiber using a predetermined correlation length d_c [9].

Making use of CPT, the crosstalk between two cores within a fiber with length L can be estimated as [10]:

$$XT = \tanh(\bar{h}_{mn}L) \tag{2.2}$$

Moreover, if the crosstalk is very small it can be approximated as [10]:

$$XT = \bar{h}_{mn}L \tag{2.3}$$

Where $\bar{h}_{mn}$ is the average PCC between core m and core n. For the case of homogeneous fibers with a small bending radius, the average PCC can be expressed as [10]:

$$\bar{h}_{mn} = \frac{2k_{mn}^2 R_b}{\beta_m \Lambda_{mn}} \tag{2.4}$$

Where R_b is the bending radius, Λ the core pitch and β_m the propagation constant. k_{mn}, the mode coupling coefficient, is given by:

$$k_{mn} = \frac{w\varepsilon_0 \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(N^2 - N_n^2)E_m^* \cdot E_m dxdy}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} u_c \cdot (E_m^* \times H_m + E_m \times H_m^*)dxdy} \tag{2.5}$$

Being w the angular frequency of the electromagnetic field, ε_0 the vacuum permittivity, $N^2(x,y)$ the refractive index distribution in the entire coupled region and N_n^2 the refractive index distribution of waveguide n.

The case we'll be considering is of a trench-assisted structure, in which three different refractive indexes can be identified: n_1 for the core, n_0 for the cladding and n_2 for the trench. The relative refractive index difference between core and cladding is Δ_1, while between trench and cladding is Δ_2.

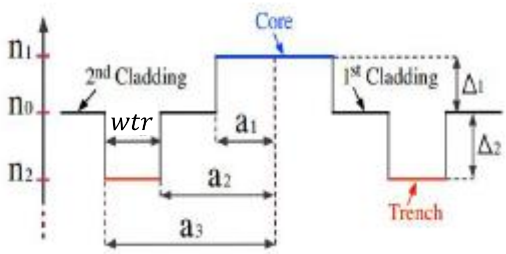

Figure 3: Refractive index profile and cross-sectional dimensions of trench-assisted structure

The core, cladding and trench radii relative to the center of the core are given by a_1, a_2 and a_3 respectively; wtr being the width of the trench.

For this trench-assisted case, and having under consideration that the size of the first cladding and trench are not infinitely large, the mode coupling coefficient can be estimated by the following analytical approach [10]:

$$k'_{mn} = \frac{\sqrt{T\,\Delta_1}}{a_1} \frac{U_1^2}{V_1^3 K_1^2(W_1)} \sqrt{\frac{\pi a_1}{W_1 \Lambda}} e^{[-\frac{W_1 \Lambda + 2(W_2 - W_1) wtr}{a_1}]} \tag{2.6}$$

Where $U_1^2 = a_1^2(k^2 n_1^2 - \beta^2)$, $W_1^2 = a_1^2(\beta^2 - k^2 n_0^2)$,$V_1 = k a_1 n_1 \times \sqrt{2\,\Delta_1}$, $W_2 = \sqrt{V_2^2 + W_1^2}$ and $V_2 = k a_1 \times \sqrt{n_0^2 - n_2^2}$. k represents the wavenumber, λ the wavelength of light in the vacuum and $K_1(W_1)$ the modified Bessel function of the 2^{nd} kind with 1^{st} order.

Lastly, having outlined a method to estimate the mode coupling coefficient between two cores (2.6), it is possible to estimate the crosstalk by replacing (2.4) on (2.3):

$$\mathrm{XT} = \frac{2 {k'_{mn}}^2 R_b}{\beta \Lambda} L \tag{2.7}$$

2.4 Crosstalk Constraints

In order to further increase transmission capacity, greater spectral efficiencies are sought by means of higher-level quadrature amplitude modulation (QAM) schemes. These modulations, along with the OSNR (Optical Signal-to-Noise Ratio) penalty they bring about, set some limitations for the maximum value of crosstalk allowed inside each core of a multi-core optical fiber.

Figure 4(c) illustrates the OSNR penalty as a function of the crosstalk. This penalty, obtained with the aid of a Monte Carlo simulation with 2^{17} symbols, represents the SNR per symbol required to achieve a bit error rate (BER) of 10^{-3} for ideal square 4-, 16-, 64- and 256-QAM constellations. Both Fig 4(a) and Fig 4(b) show the same 16-QAM constellation (open circles) with different interferer constellations (filled circles); the first figure featuring a 16-QAM interferer constellation in-phase with the signal and the latter a 16-QAM interferer constellation with a $45º$ rotation relative to the signal. As expected, the case with the rotated interferer is more strongly affected due to the minimal distance between symbols, as seen in figure 4(c).

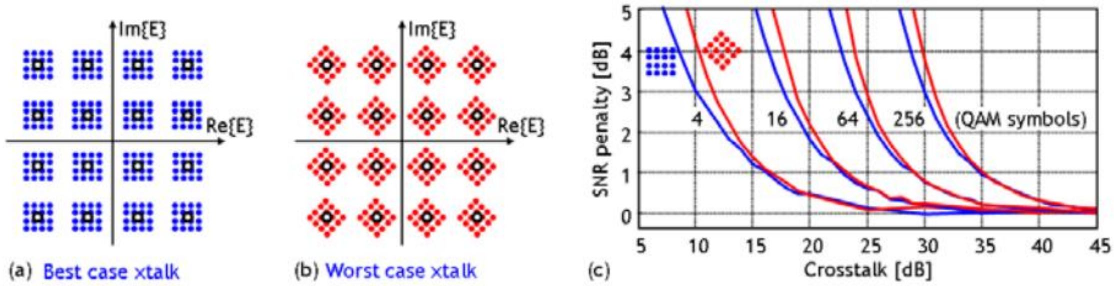

Figure 4: Impact of in-band crosstalk on QAM formats; (a, b): crosstalk models; (c): Monte Carlo simulations of crosstalk penalties for ideal, square 4-, 16-, 64-, and 256-QAM [11]

Three different optical fibers will be analyzed in this dissertation and for each case one must set a limit on the maximum amount of crosstalk tolerated by any core of the fiber. Two scenarios for an optical fiber link are conceived, sharing the same fiber length and differing in the chosen modulation format.

The first scenario, $1000\ km$ in length, will feature QPSK and won't tolerate more than $10\ dB$ of crosstalk in any of its cores (figure 4(c)). It will work with a $4\ dB$ OSNR penalty.

The second scenario, also $1000\ km$ in length, will feature 256-QAM and won't tolerate more than $30\ dB$ of crosstalk in any of its cores (figure 4(c)). This optical fiber link will also boast the same OSNR penalty as the previous.

Knowing that all the simulations to be presented in this dissertation were made for a fiber $100\ km$ in length, the value of crosstalk is always expressed as $dB/100km$. Considering that the two afore mentioned scenarios express the value of crosstalk for a signal having traveled a distance of 1000 km, there is the need to convert the maximum tolerable crosstalk limits from $dB/1000km$ to the $dB/100km$. Throughout this dissertation the crosstalk will be defined with the negative sign, hence the crosstalk limits of the two previous scenarios are -$10\ dB$ and -$30\ dB$, for the link using QPSK and 256-QAM respectively.

By taking (2.8), a formula that relates the unitary crosstalk with the crosstalk of a signal after traveling a certain fiber length (L); then replacing the values for the two fiber lengths in question (2.9):

$$XT(L) = XT(1\ km) + 10 \times \log(L) \tag{2.8}$$

$$\begin{cases} XT(100) = XT(1\ km) + 10 \times \log(100) \\ XT(1000) = XT(1\ km) + 10 \times \log(1000) \end{cases} \tag{2.9}$$

It is possible to determine the relation between the crosstalk of a signal that has traveled $100km$ and the crosstalk of that same signal after traveling $1000\ km$:

$$XT(100) = XT(1000) - 10 \tag{2.10}$$

The crosstalk tolerance of the two described optical links is therefore -$20\ dB$ for the link using QPSK modulation and -$40\ dB$ for the link using 256-QAM. These two different crosstalk tolerances will make up for two different designs within each one of the three different fibers to be studied in this dissertation.

3 Proposed Layouts

There are countless ways of organizing the cores inside a SM-MCF such as placing them in rings, hexagonally, or simply without any geometrical form at all. The proposed layouts will strive to balance the crosstalk across the cores, making it as low as possible for every single one in the structure. Given that three different scenarios will be studied, each with a different fiber diameter, all the proposed layouts share the possibility of enlargement (addition of cores), by making use of a precise geometry when designed. Furthermore, as the layouts needs to be commercially viable, it's highly convenient for them to be geometrically symmetric.

Given that the crosstalk is heavily influenced by the core pitch, the logical approach for when designing the layouts would be to maximize the distance between neighbouring cores. In the light of this reasoning, circle packing theory was considered. Although circle packing theory states that the densest packing of identical circles in a plane is the hexagonal lattice of the bee's honeycomb [12], many other geometries offer a denser packing for a specific number of circles to be packed. This being said, both these geometries and hexagonal placement will be considered. On an endnote, it is relevant to point out that unconsidered non-geometrical structures for the core allocation would sometimes produce better results for this problem of maximizing the distance between neighbouring cores.

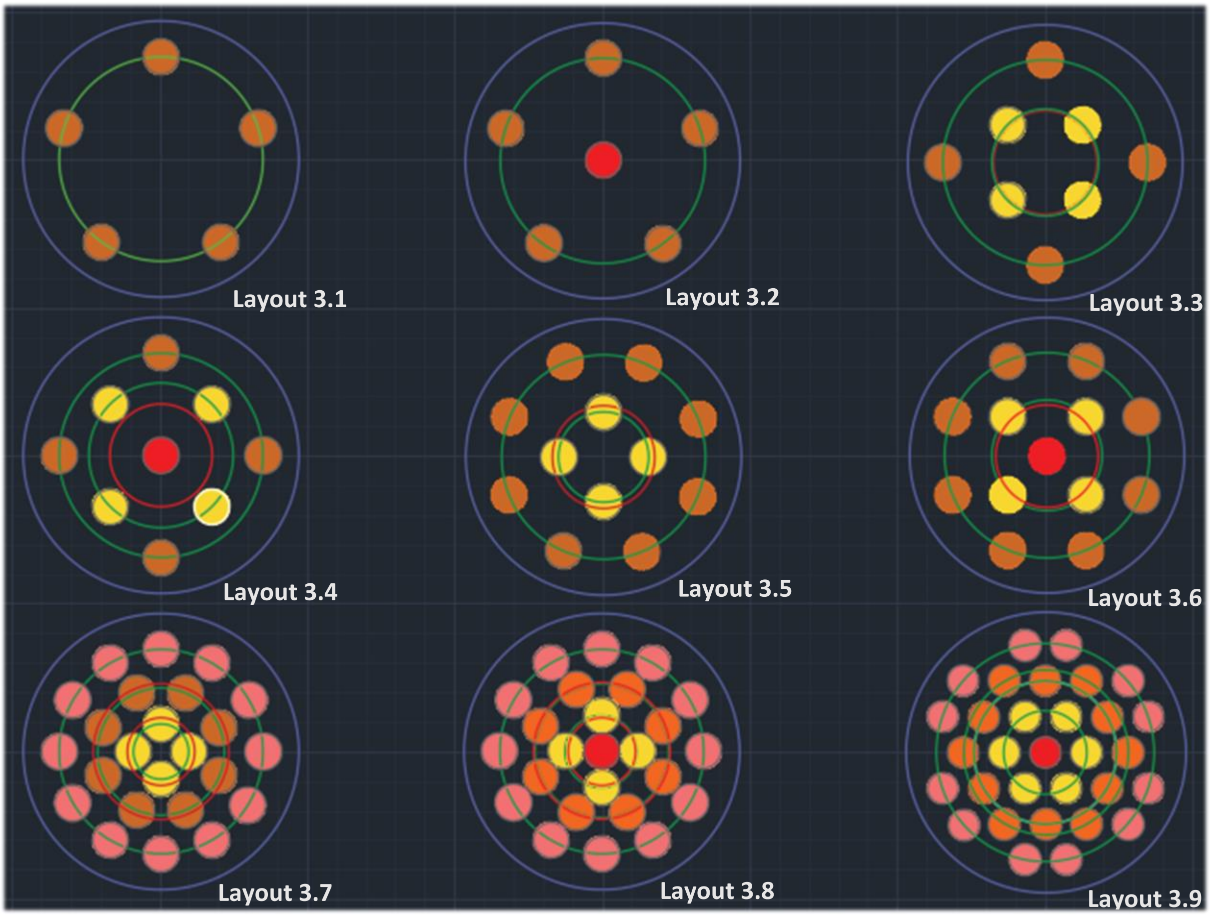

Figure 5: Proposed Layouts

3.1 One Ring

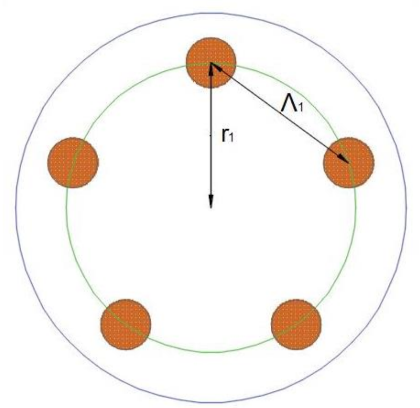

Figure 6: One Ring

Using the One Ring layout the cores are placed evenly spaced on a circle. The radius of the circle (r_1), which I will name Outer Circle, is made as big as possible as to guarantee the spacing between cores is the maximum allowed by the fiber parameters. A detailed explanation on how to obtain this circle is featured in Annex A.

Initially two cores are placed in the fiber laying on the circle that was previously described, distanced by a diameter of that same circle. More cores are then added to the fiber, one at a time, always ensuring equal spacing between a core and its neighbours until the physical limitations of the fiber prevent us from adding more cores.

Taking into consideration equation (2.7), the crosstalk calculation for each core will only take into account the interference from its two neighbours (3.1), which gives approximately the same crosstalk value as when taking all the cores into consideration. This approximation, also present in the crosstalk calculations of other Layouts, is explained in Annex B.
For the case when only have two cores incorporate the fiber ($N = 2$), the crosstalk calculation will be done differently as there's only one neighbour for each core.

$$XT_{Core} = 2 \times XT(\Lambda_1), \qquad Except\ for\ N = 2 \ \ where \ \ \ XT_{Core} = XT(\Lambda_1) \tag{3.1}$$

Relevant dimensions	Formula
Outer Ring radius	r_1
Distance between Outer Cores	$\Lambda_1 = 2r_1 \times sin(\frac{2\pi}{2N})$

Table 1: Relevant dimensions in "One Ring" Layout

3.2 One Ring with Central Core

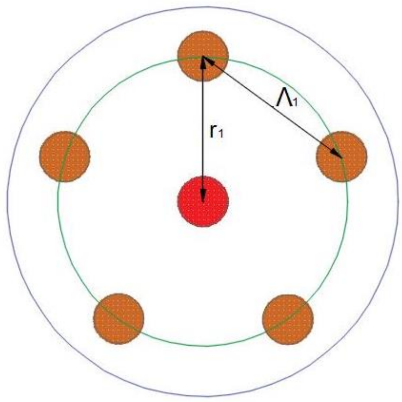

Figure 7: One Ring with Central Core

One Ring with Central Core is characterized by having a set of equally spaced cores laying on the Outer Circle and a core in the center of the fiber; somewhat similar to the "One Ring" layout but with an additional core in the center.

Initially a core is placed in the center of the fiber encircled by three other cores. More cores are then added to the fiber, one by one to the Outer Circle, always maintaining equal spacing between the cores in the Outer Circle until the physical limitations of the fiber prevent us from adding more cores.

Taking equation (2.7) and knowing the crosstalk of the cores laying on the Outer Circle will mainly come from the central core and its two neighbours, the crosstalk of an Outer Core is determined (3.3). The crosstalk calculation for the central core will take into account all the cores encircling it (3.2).

$$XT_{Central\ Core} = (N-1) \times XT(r_1\ ) \qquad (3.2)$$

$$XT_{Outer\ Core} = XT(r_1) + 2 \times XT(\Lambda_1) \qquad (3.3)$$

Relevant dimensions	Formula
Outer Ring radius	r_1
Distance between Outer Cores	$\Lambda_1 = 2r_1 \times sin(\frac{2\pi}{2(N-1)})$

Table 2: Relevant dimensions in "One Ring with Central Core" Layout

3.3 Two Rings

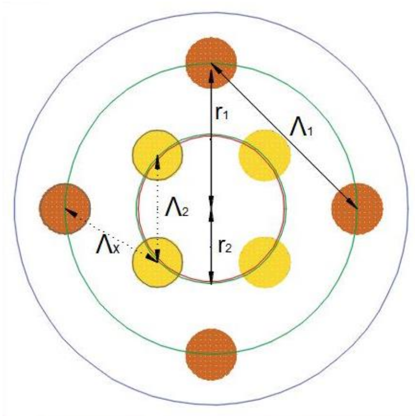

Figure 8: Two Rings

In the Two Rings layout the nuclei lay in two circumferences with different radiuses, Inner Circle and Outer Circle, each containing the same amount of nuclei. To the structure of nuclei laying on each circle I will refer as rings, with the ring boasting the larger radius (r_1) being the Outer Ring while the other, the Inner Ring, with its radius (r_2) varying in order to maintain a certain condition. When having two rings with the same number of nuclei we expect the distance between Inner Cores (Λ_2) to be smaller than the distance between Outer Cores (Λ_1), thus having the Inner Cores with the highest values of crosstalk.
In an attempt to balance the values of crosstalk, and knowing the distance between Inner Cores to be the smallest, we can move the Inner Cores further out until the distance between them is as big as the distance between two cores laying in different rings (Λ_x). The radius of the Inner Circle, containing the Inner Cores, will therefore be variable and dependent on the number of cores we decide to use in this layout. By doing this radius adjustment we are able to obtain the best possible crosstalk results for this kind of structure, as explained in Annex C.

Initially six cores are placed in the fiber, three in each ring in a way that maximizes the distance between cores in different rings. More cores are then added to the fiber, two at a time, increasing the number of cores in each ring one by one until the physical limitations of the fiber prevent us from adding more cores.

Making use of equation (2.7), it's possible to compute the crosstalk value of an Inner Core by considering both its two neighbours and two closest Outer Cores (3.4). Similarly, when computing the crosstalk value for an Outer Core, only its two neighbours and two closest Inner Cores are taken into consideration (3.5).

$$XT_{Inner\ Core} = 2 \times XT(\Lambda_2) + 2 \times XT(\Lambda_x) \tag{3.4}$$

$$XT_{Outer\ Core} = 2 \times XT(\Lambda_1) + 2 \times XT(\Lambda_x) \tag{3.5}$$

Relevant dimensions	**Formula**
Outer Ring radius	r_1
Distance between Outer Cores	$\Lambda_1 = 2r_1 \times sin(\frac{2\pi}{2M_1})$
Inner Ring radius	$r_2 = \frac{\left(2r_1 \times cos\left(\frac{2\pi}{2M_1}\right)\right) - \sqrt{\left(-2r_1 \times cos\left(\frac{2\pi}{2M_1}\right)\right)^2 - 4 \times \left(1 - 4sin^2\left(\frac{2\pi}{2M_1}\right)\right) \times (r_1{}^2)}}{2 \times \left(1 - 4sin^2\left(\frac{2\pi}{2M_1}\right)\right)}$
Distance between Inner Cores	$\Lambda_2 = 2r_2 \times sin(\frac{2\pi}{2M_2})$
Inner-Outer Core distance	$\Lambda_x = \Lambda_2$

Table 3: Relevant dimensions in "Two Rings" Layout

3.4 Two Rings with Central Core

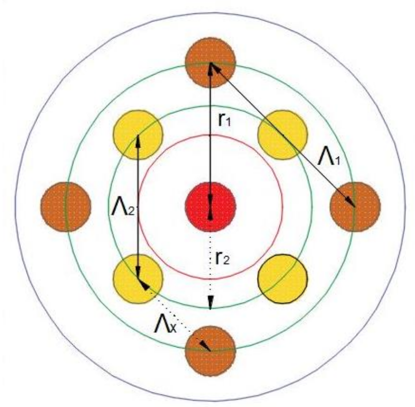

Figure 9: Two Rings with Central Core

Two Rings with Central Core distributes its cores in two rings, each containing the same number of cores, with an additional core present in the center. The Outer Ring, with the larger radius, lays at a distance r_1 from the center while the Inner Ring ranges r_2. The Inner Ring radius will be dependent on the number of cores, always bigger than half the Outer Ring radius, varying to ensure the distance from an Inner to the Central core remains the same as the distance from an Inner to the closest Outer Core. By doing this we expect to better balance the values of crosstalk when comparing to a similar structure in which the Inner Cores would be placed at half the Outer Ring radius. This procedure is explained in Annex D.

Initially seven cores are placed in the fiber, three in each ring in a way that maximizes the distance between cores in different rings, plus another core in the center. More cores are then added to the fiber, two at a time, increasing the number of cores in each ring one by one until the physical limitations of the fiber prevent us from adding more cores.

From equation (2.7), we compute the crosstalk value of an Inner Core by considering its two neighbours, two closest Outer Cores and the central core (3.7). Similarly, when computing the crosstalk of an Outer Core we only take into account its two neighbours, closest Inner Core and the Central Core (3.8). The crosstalk calculation for the central core will take into account all the cores encircling it (3.6).

$$XT_{Central\ Core} = M_1 \times XT(r_1) + M_2 \times XT(r_2) \tag{3.6}$$

$$XT_{Inner\ Core} = XT(r_2) + 2 \times XT(\Lambda_2) + 2 \times XT(\Lambda_x) \tag{3.7}$$

$$XT_{Outer\ Core} = XT(r_1\ ) + 2 \times XT(\Lambda_1) + 2 \times XT(\Lambda_x) \tag{3.8}$$

Relevant dimensions	Formula
Outer Ring radius	r_1
Distance between Outer Cores	$\Lambda_1 = 2r_1 \times sin(\frac{2\pi}{2M_1})$
Inner Ring radius	$r_2 = \frac{\frac{r_1}{2}}{cos(\frac{2\pi}{2M_1})}$
Distance between Inner Cores	$\Lambda_2 = 2r_2 \times sin(\frac{2\pi}{2M_2})$
Inner-Outer Core distance	$\Lambda_x = r_2$

Table 4: Relevant dimensions in "Two Rings with Central Core" Layout

3.5 Two Different Rings

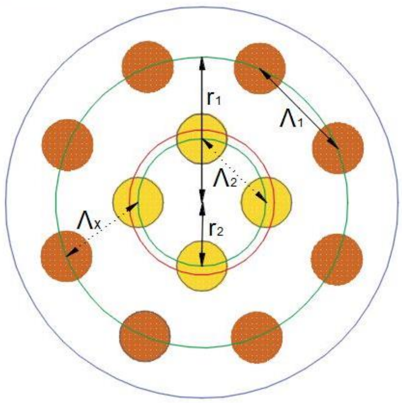

Figure 10: Two Different Rings

Two Different Rings organizes its cores in two rings, where the Inner Ring has half the number of cores than the Outer Ring. This new approach to the two ring layout comes as an attempt to more evenly balance the distance between neighbouring cores in both rings, in which the Outer Ring is placed at a distance r_1 from the center and the Inner Ring at a distance r_2.

Similarly to the “Two Ring” layout, the Inner Core Crosstalk is the limiting factor in this layout. In order to minimize the Inner Core Crosstalk, we adjust the Inner Ring radius as to ensure that the distance between Inner Ring neighbours (Λ_2) is the same as the distance between an Inner Core and the closest Outer Cores (Λ_x); in other words, we guarantee that an Inner Core is equally spaced to its closest neighbour and Outer Core. This procedure is explained in Annex E.

Initially nine cores are placed in the fiber, six in the Outer plus three in the Inner Ring in a way that maximizes the distance between cores in different rings. More cores are then added to the fiber, three at a time, increasing respectively by one and two at a time the number of cores in the Inner and Outer Ring until the physical limitations of the fiber prevent us from adding more cores.

Making use of equation (2.7), it’s possible to compute the crosstalk value of an Inner Core by considering its two neighbours and two closest Outer Cores (3.9). When computing the crosstalk of an Outer Core, its two neighbours and closest Inner Core are considered (3.10).

$$XT_{Inner\ Core} = 2 \times XT(\Lambda_2) + 2 \times XT(\Lambda_x) \qquad (3.9)$$

$$XT_{Outer\ Core} = 2 \times XT(\Lambda_1) + XT(\Lambda_x) \qquad (3.10)$$

Relevant dimensions	Formula
Outer Ring radius	r_1
Distance between Outer Cores	$\Lambda_1 = 2r_1 \times sin(\frac{2\pi}{2M_1})$
Inner Ring radius	$r_2 = \frac{\left(2r_1 \times cos\left(\frac{2\pi}{2M_1}\right)\right) - \sqrt{\left(-2r_1 \times cos\left(\frac{2\pi}{2M_1}\right)\right)^2 - 4\left(1 - 4sin^2\left(\frac{2\pi}{2M_2}\right)\right) \times (r_1{}^2)}}{2 \times \left(1 - 4sin^2\left(\frac{2\pi}{2M_2}\right)\right)}$
Distance between Inner Cores	$\Lambda_2 = 2r_2 \times sin(\frac{2\pi}{2M_2})$
Inner-Outer Core distance	$\Lambda_x = \Lambda_2$

Table 5: Relevant dimensions in “Two Different Rings” Layout

3.6 Two Different Rings with Central Core

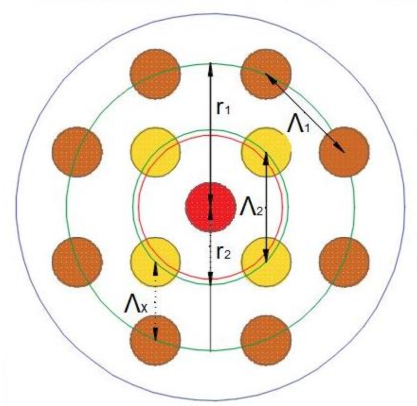

Figure 11: Two Different Rings with Central Core

Two Rings with Central Core organizes its cores in two rings plus another core in the center, where the Inner Ring has half the cores the Outer Ring has. The Outer Ring, with the larger radius, laying at a distance r_1 and the Inner Ring at a distance r_2 from the center.

The Inner Ring radius will be dependent on the number of cores, always bigger than half the Outer Ring radius, varying to ensure the distance from an Inner to the Central core remains the same as the distance from an Inner to the closest Outer Cores. By doing so we expect to better balance the values of Crosstalk when comparing to a similar structure in which the Inner Cores would be placed at half the Outer Ring radius. This procedure is explained in Annex F.

Initially seven cores are placed in the fiber, one in the center, four in the Outer and two in the Inner Ring in a way that maximizes the distance between cores in different rings. More cores are then added to the fiber, three at a time, increasing respectively by one and two at a time the number of cores in the Inner and Outer Ring until the physical limitations of the fiber prevent us from adding more cores.

Taking equation (2.7) into account, it's possible to compute the crosstalk value of an Inner Core by taking into consideration its two neighbours, two closest Outer Cores and the Central core (3.12). Similarly, when computing the crosstalk value for an Outer Core only its two neighbours, closest Inner Core and central core are considered (3.13). The crosstalk calculation for the central core will take into account all the cores encircling it (3.11).

$$XT_{Central\ Core} = M_1 \times XT(r_1\ ) + M_2 \times XT(r_2) \tag{3.11}$$

$$XT_{Inner\ Core} = XT(r_2) + 2 \times XT(\Lambda_2) + 2 \times XT(\Lambda_x) \tag{3.12}$$

$$XT_{Outer\ Core} = XT(r_1\ ) + 2 \times XT(\Lambda_1) + XT(\Lambda_x) \tag{3.13}$$

Relevant dimensions	Formula
Outer Ring radius	r_1
Distance between Outer Cores	$\Lambda_1 = 2r_1 \times sin(\frac{2\pi}{2M_1})$
Inner Ring radius	$r_2 = \frac{\frac{r_1}{2}}{cos(\frac{2\pi}{2M_1})}$
Distance between Inner Cores	$\Lambda_2 = 2r_2 \times sin(\frac{2\pi}{2M_2})$
Inner-Outer Core distance	$\Lambda_x = r_2$

Table 6: Relevant dimensions in "Two Different Rings with Central Core" Layout

3.7 Three Different Rings

Three Different Rings has its cores organized in three rings, where both the Inner and Middle Rings have, respectively, a third and two thirds of the cores composing Outer Ring. The Outer Ring, boasting the largest radius, lays at a distance r_1 from the center while the Inner Ring at a distance r_3 from the center. The ring that lays in-between these two goes by Middle Ring, and lays at a distance r_2 from the center of the layout.

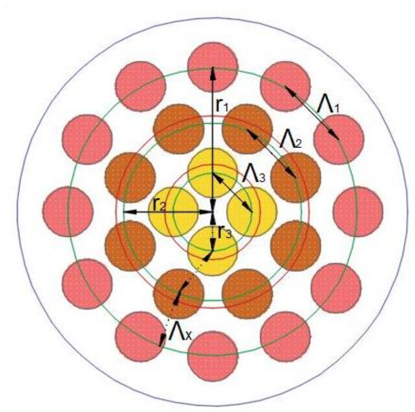

Figure 12: Three Different Rings

Both the Middle and Inner Ring radii will be dependent on the number of cores, changing in order to maintain the distance between Inner Ring neighbours the same as both the distance between an Inner Core and its closest Middle Core and between a Middle Core and the Outer Circle.

It is important to note that the distance between an Outer Core and its closest Middle Core will depend on the Outer Core we pick, which is why it is necessary to make the distance between Inner Ring neighbours the same as the distance between a Middle Core and the Outer Circle, to ensure this latter is either the same or smaller than the distance between a Middle and an Outer Core. This algorithm produces better crosstalk results when comparing to a similar structure where the rings are equally spaced, as explained in Annex G.

Initially eighteen cores are placed in the fiber, nine in the Outer Ring, six in the Middle Ring and three in the Inner Ring in a way that maximizes the distance between cores in different rings. More cores are then added to the fiber, six at a time, increasing respectively by one, two and three at a time the number of cores in the Inner, Middle, and Outer Ring until the physical limitations of the fiber prevent us from adding more cores.

From equation (2.7), we can compute the crosstalk of an Inner Core by taking into account both its neighbours and two closest Middle Cores (3.14). For the crosstalk of a Middle Core, its two neighbours are considered as well as its closest Inner and Outer Core (3.15). As for the crosstalk of an Outer Core, its two neighbours and closest Middle Core are taken into account (3.16).

It is important to note that due to the variable distance between different Middle Cores and their closest Outer Cores, when I refer to that distance I'm actually considering the distance between a Middle Core and the Outer Circle, thus always guaranteeing the former distance to be equal or greater than the one considered.

$$XT_{Inner\ Core} = 2 \times XT(\Lambda_3) + 2 \times XT(\Lambda_x) \tag{3.14}$$

$$XT_{Middle\ Core} = XT(\Lambda_x) + XT(\Lambda_x) + 2 \times XT(\Lambda_2) \tag{3.15}$$

$$XT_{Outer\ Core} = 2 \times XT(\Lambda_1) + XT(\Lambda_x) \tag{3.16}$$

Relevant dimensions	Formula
Outer Ring radius	r_1
Distance between Outer Cores	$\Lambda_1 = 2r_1 \times sin(\frac{2\pi}{2M_1})$
Inner Ring radius	$r_3 = \frac{2r_1 \times cos\left(\frac{2\pi}{2M_2}\right)+4r_1 \times sen\left(\frac{2\pi}{2M_3}\right)-\sqrt{\left(-2r_1 \times cos\left(\frac{2\pi}{2M_2}\right)-4r_1 \times sen\left(\frac{2\pi}{2M_3}\right)\right)^2-4\times\left(1+4\,sin\left(\frac{2\pi}{2M_3}\right)cos\left(\frac{2\pi}{2M_2}\right)\right)\times({r_1}^2)}}{2\times\left(1+4\,sin\left(\frac{2\pi}{2M_3}\right)cos\left(\frac{2\pi}{2M_2}\right)\right)}$
Distance between Inner Cores	$\Lambda_3 = 2r_3 \times sin(\frac{2\pi}{2M_3})$
Middle Ring radius	$r_2 = r_1 - \Lambda_3$
Distance between Middle Cores	$\Lambda_2 = 2r_2 \times sin(\frac{2\pi}{2M_2})$
Inner-Middle-Outer Core distance	$\Lambda_x = \Lambda_3$

Table 7: Relevant dimensions in "Three Different Rings" Layout

3.8 Three Different rings with Central Core

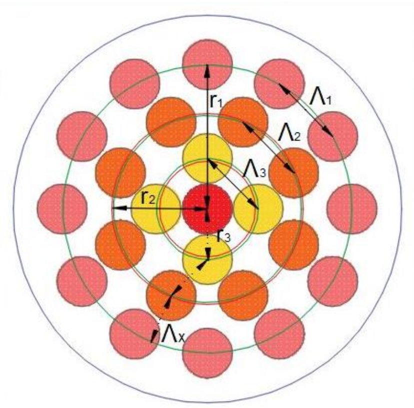

Figure 13: Three Different Rings with Central Core

Three Different Rings with Central Core is a layout in which the nuclei are organized in three rings and a central core, where both the Inner and Middle Rings have, respectively, a third and two thirds of the cores composing the Outer Ring. The Outer Ring, boasting the largest radius, lays at a distance r_1 from the center while the Middle and Inner Ring at a distance of, respectively, r_2 and r_3 from the center.

Both the Middle and Inner Ring radii will be dependent on the number of cores, varying in order to maintain constant the distance from the Central Core to an Inner Core, from an Inner Core to a Middle Core and from a Middle Core to the Outer Circle. It is important to note that the distance between an

Outer Core and its closest Middle Core will depend on the Outer Core we pick, which is why it is necessary to make the distance from the Central Core to an Inner Core the same as the distance between a Middle Core and the Outer Circle, to ensure this latter is either the same or smaller than the distance between a Middle and an Outer Core. This algorithm produces better crosstalk results when comparing to a similar structure in which the rings are equally spaced, as explained in Annex H.

Initially nineteen cores are placed in the fiber, one in the center, nine in the Outer Ring, six in the Middle Ring and three in the Inner Ring in a way that maximizes the distance between cores in different rings. More cores are then added to the fiber, six at a time, increasing respectively by one, two and three at a time the number of cores in the Inner, Middle, and Outer Ring until the physical limitations of the fiber prevent us from adding more cores.

Making use of equation (2.7), it's possible to determine the crosstalk of an Inner Core taking under consideration its two neighbours, two closest Middle Cores and the central core (3.18). For the crosstalk of a Middle Core, its two neighbours are considered as well as its closest Inner and Outer Cores (3.19). As for the crosstalk of an Outer Core, its two neighbours and closest Middle Core are taken into account (3.20). The crosstalk calculation for the central core will take into account all the cores encircling it (3.17). It is important to note that due to the variable distance between different Middle Cores and their closest Outer Cores, when I refer to that distance I'm actually considering the distance between a Middle Core and the Outer Circle, thus always guaranteeing the former distance to be equal or greater than the one considered.

$$XT_{Central\ Core} = M_1 \times XT(r_1\ ) + M_2 \times XT(r_2) + M_3 \times XT(r_3\ ) \tag{3.17}$$

$$XT_{Inner\ Core} = XT(r_3\ ) + 2 \times XT(\Lambda_3) + 2 \times XT(\Lambda_x) \tag{3.18}$$

$$XT_{Middle\ Core} = XT(\Lambda_x) + XT(\Lambda_x) + 2 \times XT(\Lambda_2) \tag{3.19}$$

$$XT_{Outer\ Core} = 2 \times XT(\Lambda_1) + XT(\Lambda_x) \tag{3.20}$$

Relevant dimensions	Formula
Outer Ring radius	r_1
Distance between Outer Cores	$\Lambda_1 = 2r_1 \times sin(\frac{2\pi}{2M_1})$
Middle Ring radius	$r_2 = \frac{r_1}{1+2cos(\frac{2\pi}{2M_2})}$
Distance between Middle Cores	$\Lambda_2 = 2r_2 \times sin(\frac{2\pi}{2M_2})$
Inner Ring radius	$r_3 = r_1 - r_2$
Distance between Inner Cores	$\Lambda_3 = 2r_3 \times sin(\frac{2\pi}{2M_3})$
Inner-Middle-Outer Core distance	$\Lambda_x = r_3$

Table 8: Relevant dimensions in "Three Different Rings with Central Core" Layout

3.9 Hexagonal Placement

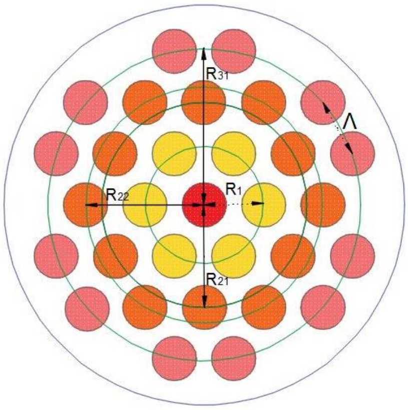

Figure 14: Hexagonal Placement

With Hexagonal Placement the cores are organized in different rings, all having a hexagonal shape. Different nuclei in the same ring can have different distances to the center, meaning that rings are not, unlike in previous layouts, characterized by having all its nuclei equidistant to the center. There's always a ways a central core present in this layout and the inter-core distance between any two adjacent cores is constant.

The way in which we add cores to this layout is more complex than one might think. Rings cannot be simply added to the fiber until the physical limitations of the fiber allow, since some cores in the same ring may differ in distance to the central core; meaning some cores may fit inside the fiber while others don't. For this reason, whenever cores are added to the fiber they all share the same radius; meaning rings are built in one, two, three and sometimes more phases.

Initially seven cores are placed in the fiber, as all the six cores composing the first ring have the same radius R_1. We then construct part of the second ring by adding those cores that are closest to the center (R_{21}) before being able to complete it by adding the remaining cores at a distance R_{22} from the center. Having the second ring in place, built in two iterations, we persist adding more cores a few at a time crafting only part each rings on each iteration until we can no longer fit any more cores in the fiber.
The colored hexagons in figure 15, in which the cores will be inscribed, mark the different iterations.

Relevant dimensions	Formula
$Core\ pitch$	Λ
$1^{st} Ring\ - Green$	$R_1 = \Lambda$
$2^{nd} Ring(1) - Blue$	$R_{21} = 2\Lambda \times cos(30)$
$2^{nd} Ring(2) - Red$	$R_{22} = 2\Lambda$
$3^{rd} Ring(1) - Cyan$	$R_{31} = \sqrt{7}\Lambda$
$3^{rd} Ring(2) - Yellow$	$R_{32} = 3\Lambda$
$4^{th} Ring(1) - Orange$	$R_{41} = 4\Lambda \times cos(30)$
$4^{th} Ring(2) - Pink$	$R_{42} = \sqrt{13}\Lambda$
$4^{th} Ring(3) - Light\ Green$	$R_{43} = 4\Lambda$

Table 9: Relevant dimensions in "Hexagonal Placement" Layout

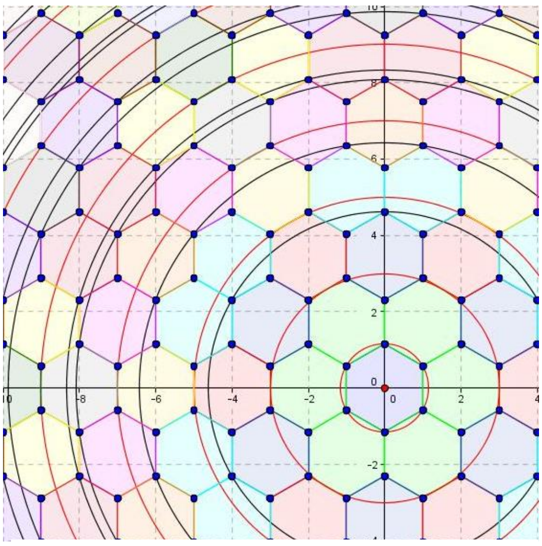
Figure 15: Different iterations in "Hexagonal Placement" construction process

Given that the inter-core distance is the same between any two adjacent cores in the structure, for the crosstalk calculation we only need to multiply the value of crosstalk between any two cores with the number of adjacent cores to it:

$$XT_{Given\ Core} = Nº Adjacent\ Cores \times XT(\Lambda) \tag{3.21}$$

4 Implementation

4.1 Numerical Model

A MatLab algorithm was used to determine and plot, for each proposed layout, the different crosstalk values as a function of the number of cores featured in it.

Figure 16 describes the use of this algorithm for the first theoretical model, the "One Ring" layout. In this model, the initial number of cores is two and the way to add more cores is one at a time.

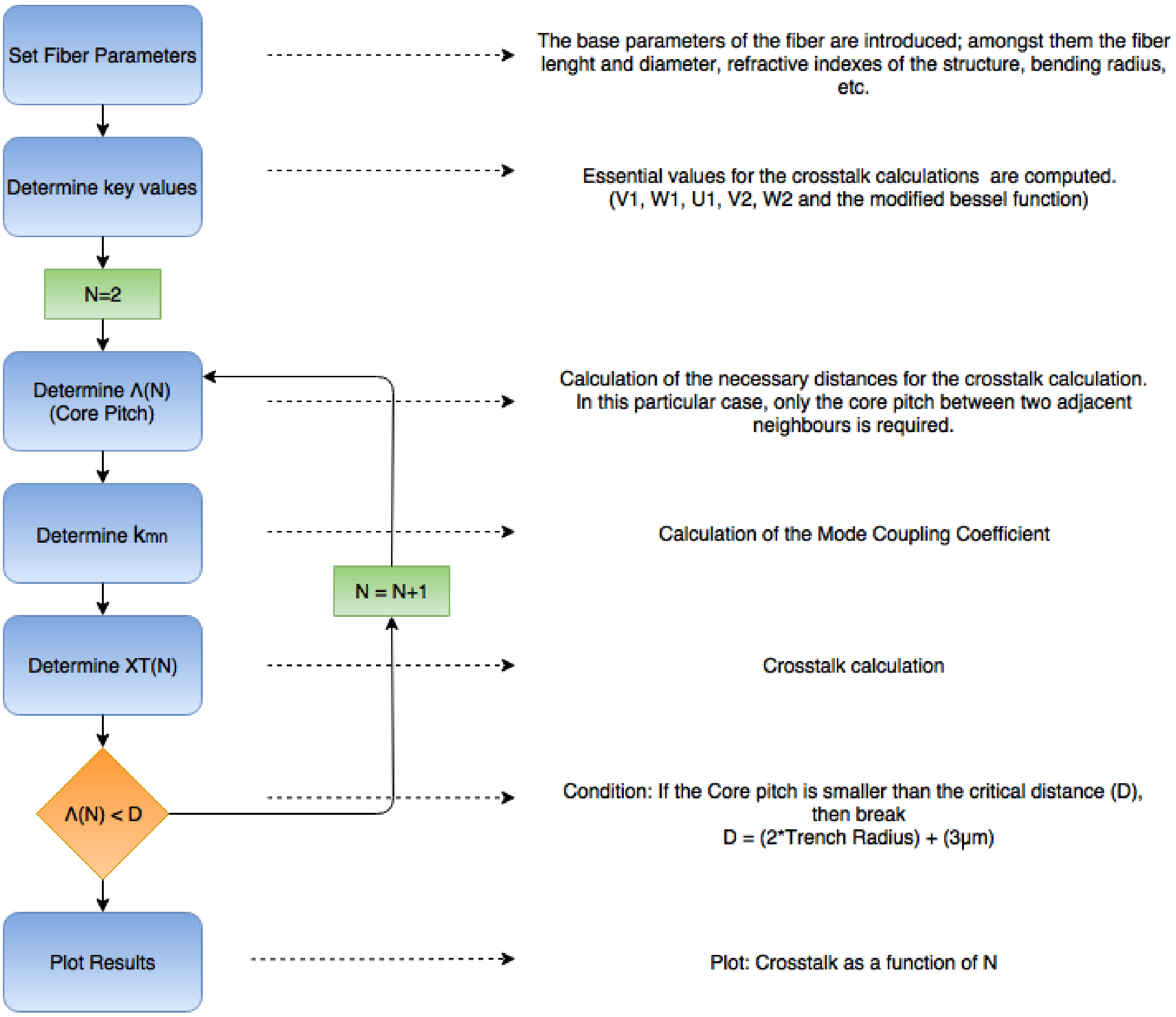

Figure 16: Crosstalk estimation algorithm's block diagram

4.2-Verification and Validation

Having developed an algorithm to predict the crosstalk in each core of a given layout, it is necessary to ensure the results it produces are in accordance with the results from other related papers. For doing so, a comparison with a 2014 paper [8] on homogeneous TA-MCF was made.

Plots of the crosstalk as a function of the core pitch were extracted from this paper; figure 17.

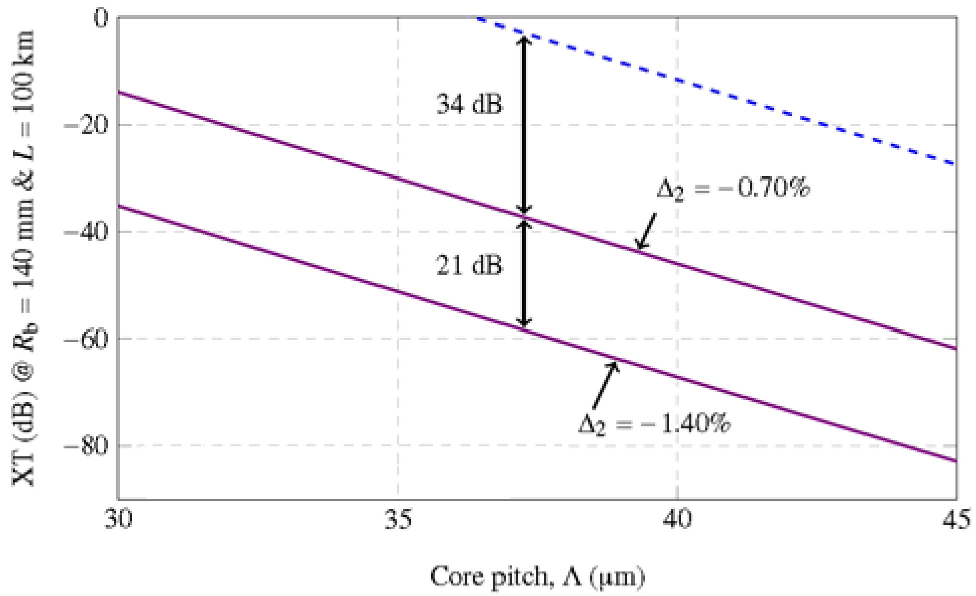

Figure 17: Crosstalk as a function of the core pitch extracted from [8] for three different scenarios

Three lines can be seen in figure 17; a dotted blue one for the step-index profile and two purple ones for the trench-assisted profile, which requires a slightly different calculation method. For the trench-assisted profile two cases with different relative refractive indexes are presented; one with $\Delta_2 = -0.7\%$ and another with $\Delta_2 = -1.4\%$ having 21 dB less crosstalk.

It becomes clear that the use of a trench drastically reduces the inter-core crosstalk. As the relative refractive index of the cladding-trench increases, the more confined light becomes in the cores and the less inter-core crosstalk is verified.

As for the crosstalk computing algorithm previously presented in this dissertation, its results are depicted in figure 18:

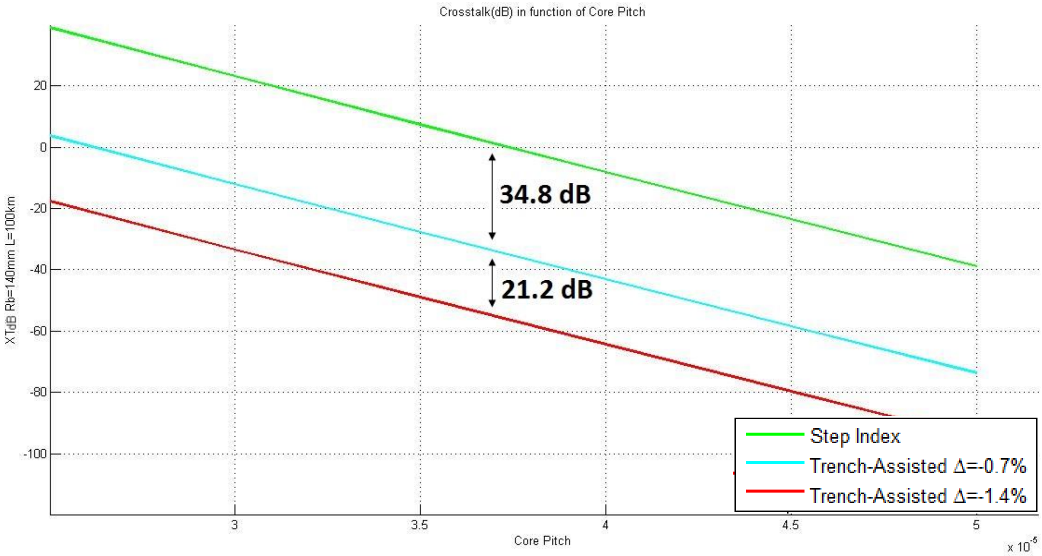

Figure 18: Crosstalk as a function of the core pitch generated by the section 4.1 algorithm for three different scenarios

Both simulations yield nearly identical results, with the gaps between plots differing less than 1 dB. Having proven the validity and accuracy of the proposed crosstalk calculation algorithm, it is safe to proceed and make use of it to analyze the crosstalk behavior of the proposed layouts.

5 Results

5.1 Problem Description

Single-mode optical fiber transmission systems are rapidly approaching its fundamental limit, reaching capacities up to roughly 100Tb/s per fiber [13] by employing various state-of-the-art multiplexing techniques (figure 19).

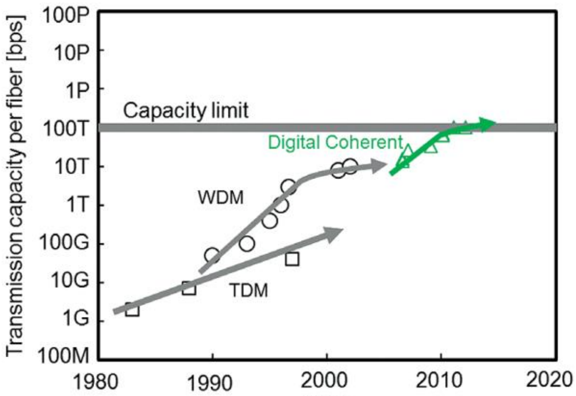

Figure 19: Evolution of Optical Fiber Transmission Capacity

In order to overcome the impending capacity crunch we turn to space-division multiplexing, more precisely single-mode multi-core optical fibers, as a solution to overcome these capacity limits.

Fibers containing multiple cores with reasonable values of crosstalk will achieve larger capacities than those with only one. This being said, a method to spatially set up the cores and minimize their crosstalk is sought.

In accordance with the fiber parameters mentioned in table 10, several layouts were proposed and their crosstalk results generated by means of a MatLab simulation. An analysis of these results will now help determine the number of cores that should be placed inside the fiber to maximize its capacity.

Three different scenarios will be considered and analyzed. One in which the cladding diameter is 260 μm, another in which it is 300 μm and one last in which it is 125 μm.

Parameter	Unit	Value
C_d	$[\mu m]$	*125, 260, 300*
OCT	$[\mu m]$	*30*
L	$[km]$	*100*
a_1	$[\mu m]$	4.5
a_3/a_1	--	3
a_2/a_1	--	2
wtr/a_1	--	1
n_1	--	*1.4551*
Δ_1	*%*	*0.35*
Δ_2	*%*	*0.35*
λ	$[nm]$	*1550*
R_b	$[mm]$	*140*

Table 10: Structural Parameters of the considered TA-MCF

5.2 Baseline Solution

When trying to maximize the number of cores inside a fiber, several are the possible layouts. This section will focus on a particular one [14], published on the 16th of May 2016, that makes use of an hexagonal structure to distribute thirty-one homogeneous trench-assisted cores inside a $230\ \mu m$ fiber (figure 20).

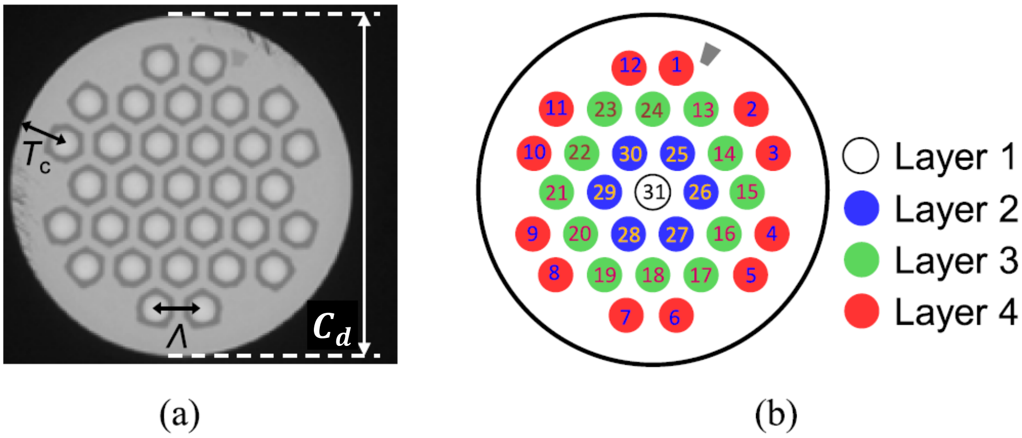

Figure 20: (a): A fabricated homogeneous 31-core fiber; (b): The definition of layer and core numbers [14]

This layout, presenting a high core-count homogeneous MCF solution, was employed on a fiber with different parameters from those used in this dissertation's simulations. In order to allow for a fair comparison to be made between this layout and those previously proposed in this dissertation, a new simulation is required.

Considering the same fiber parameters as those used in all simulations throughout this dissertation (Table 10) and choosing $260\ \mu m$ as the cladding diameter, it is possible to obtain the crosstalk values of each core in the layout (figure 21).

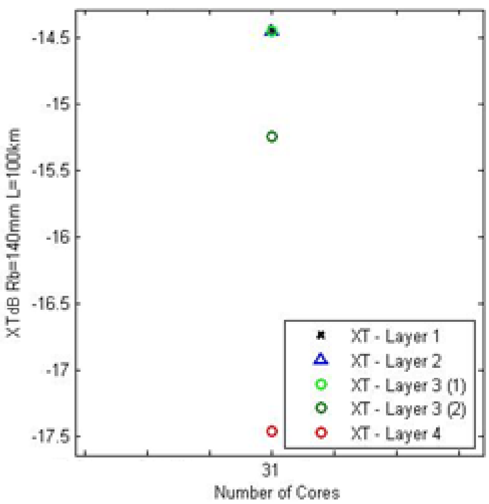

Figure 21: Crosstalk of different layers – 31 Core Hexagonal layout from [14]

Despite the appearance, figure 21 show this layout has a very balanced distribution amongst the cores, which is a very good indicator that it is suited for the placement of this many cores in a fiber.

Given that the distance between any two given cores in the layout is constant, the amount of crosstalk present on each will be proportional to the number of adjacent cores. Having said so, it is now clear that the cores in layer four have the smallest value of crosstalk, followed by those in layer three that only have five neighbours. The remaining cores, in layers one, two and three, share the highest value of crosstalk as all of them are surrounded by six cores.

5.3 Enhanced Solution

Providing an optimal solution for the problem of core allocation in MCF's is an elusive task. Many layouts were proposed for the core distribution inside the fiber but only after a thorough study of the crosstalk behavior in each is it possible to conclude which should be chosen.

Three different cases will be studied separately, each featuring an optical fiber with a well-defined set of parameters. Cladding diameter, the only parameter varying between cases, will be $260\ \mu m$ in the first one for it's the highest value manufacturers are currently able to achieve.

The proposed layouts will then be used to spatially distribute the cores in the fiber, and with the help of MatLab, their crosstalk results compared.

After having figured out what the best layout is for the first case, two more will be studied. One case featuring a fiber $300\ \mu m$ in cladding diameter, as deemed feasible in a near future, and another $125\ \mu m$ much like most single mode fibers employed nowadays.

5.3.1 First Case: $CD = 260\ \mu m$

In order to determine which layout suits best for a given number of cores, it is necessary to analyze the behavior of crosstalk as a function of the number of cores for all the proposed layouts. In the following analysis the filled circles represent the values of crosstalk for a given number of cores whereas the solid lines illustrates the trend described by a given set of crosstalk values.

One Ring

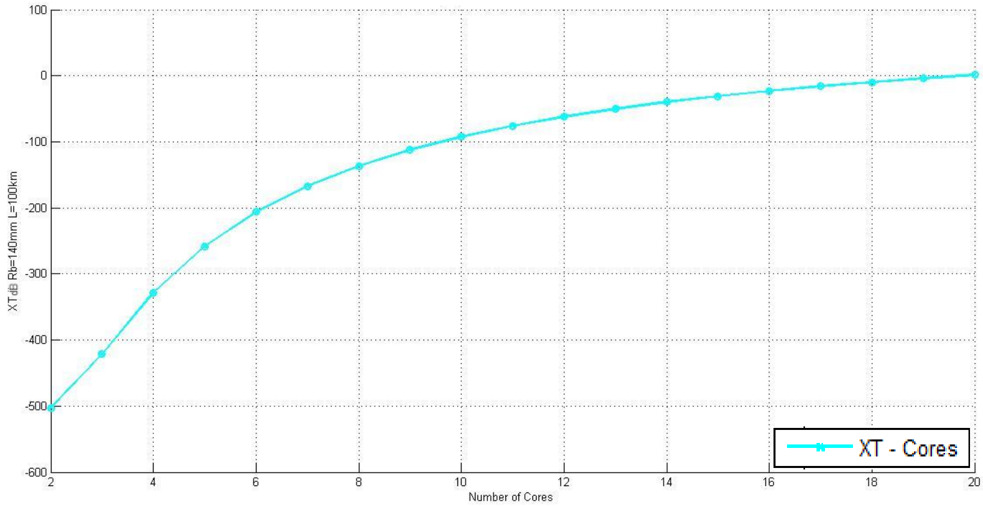

Figure 22: Crosstalk vs Number of cores- One Ring

The crosstalk of a core, expressed in dB, increases as more and more cores integrate the structure, being inversely proportional to the inter-core distance. We can see that when placing the cores this way no more than twenty cores can fit inside the fiber.

One Ring with Central Core

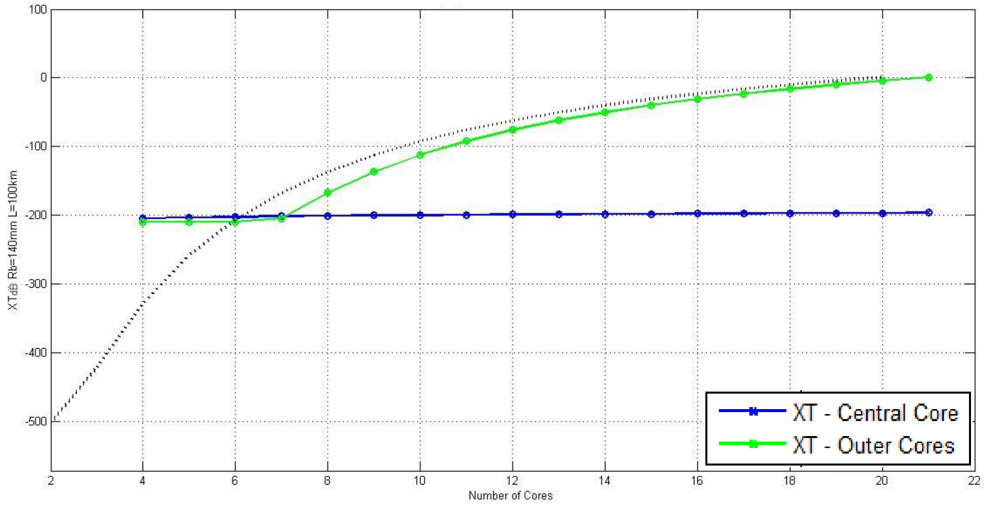

Figure 23: Crosstalk vs Number of cores - One Ring with Central Core

Using a dotted line we plot the best results so far obtained from previous layouts, so that a comparison can be made with the current one. As only “One Ring” layout has been previously presented, the dotted line will be coincident with its results.

The plots in color represent the different values of crosstalk for this layout. Initially the crosstalk of the central core is approximately the same as of its neighbours. However, as more cores are added to the layout, the closer the Outer Cores become to each other resulting in a drastic increase of crosstalk in the Outer Cores. Given that the crosstalk of the central core comes from its interaction with the Outer Cores and that these always remain at the same distance from it, the crosstalk of the central core will only slightly increase as more cores integrate that Outer Ring.

We can see that this layout outperforms the previous whenever seven or more cores integrate the fiber.

Two Rings

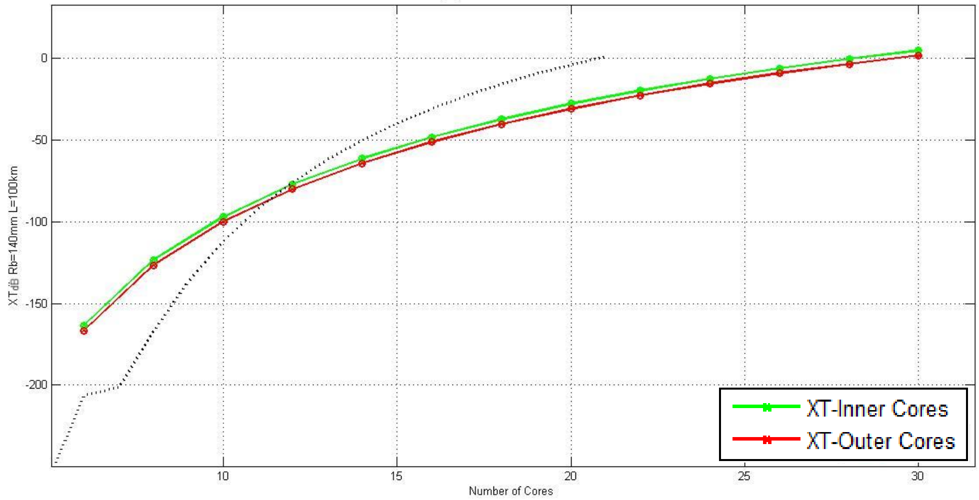

Figure 24: Crosstalk vs Number of cores - Two Rings

The two lines in color represent the crosstalk of a core in each ring, while the dotted line gives us the best results we've so far achieved. By analyzing the figure it's clear this layout outperforms the previous for twelve or more cores.

The crosstalk in both rings is very similar for any given number of cores, which was made possible by having adjusted the radius of the Inner Ring (see Annex C).

Two Rings with Central Core

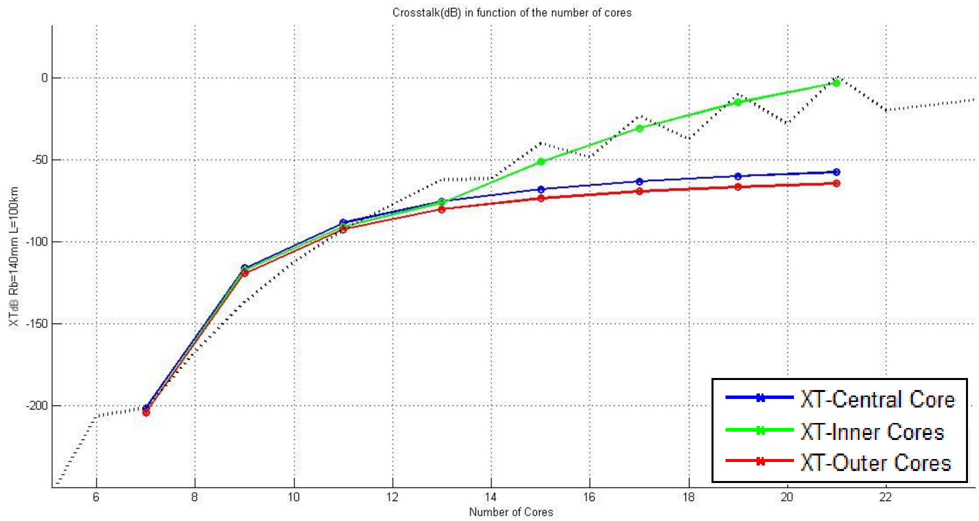

Figure 25: Crosstalk vs Number of cores - Two Rings with Central Core

Three plots are required to analyze this layout's performance, each representing a different type of core in the structure. We can see that this layout is better than any previous for 13, 15, 17, 19 and 21 cores.

When this layout comprises seven cores, it assumes the same structure as the "One Ring with Central Core", thus rendering exactly the same crosstalk results. This occurrence is due to the crosstalk minimization algorithm employed when adjusting the Inner Ring radius (see annex D); its attempt to place the Inner cores at the same distance to the Central and Outer Cores causes this core arrangement.

Two Different Rings

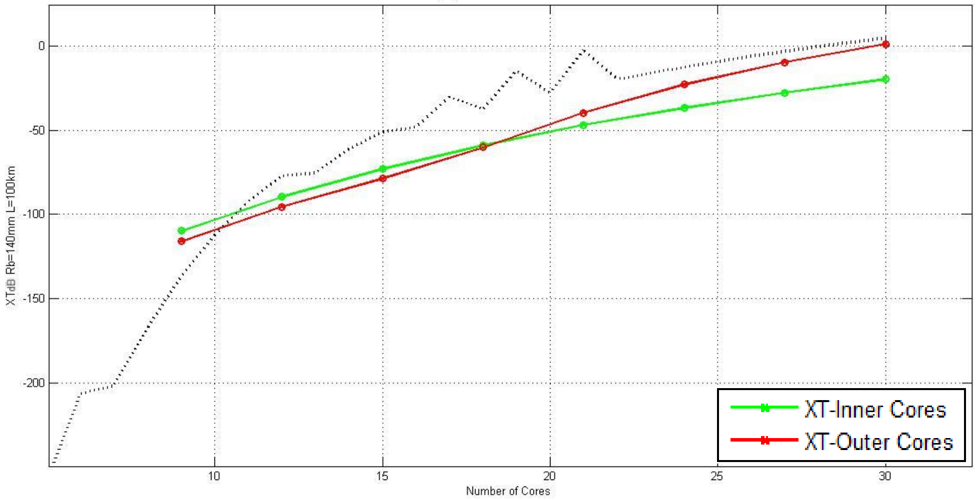

Figure 26: Crosstalk vs Number of cores - Two Different Rings

The crosstalk of the Inner and Outer Cores is represented by the green and red lines, respectively. It has a balanced distribution as both lines don't drift too far apart.

This layout is the preferred one in almost all its variations, outperforming all the previous from twelve cores onwards.

Two Different Rings with Central Core

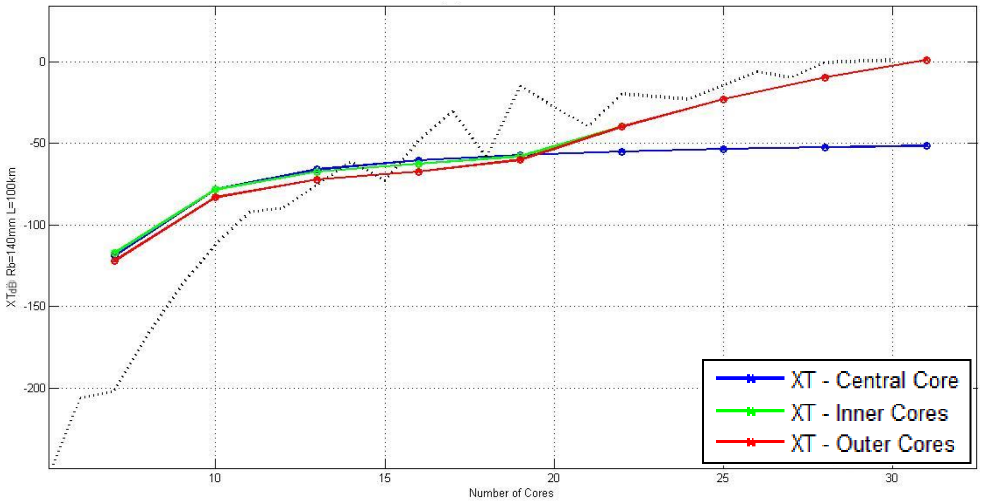

Figure 27: Crosstalk vs Number of cores - Two Different Rings with Central Core

This layout shows promise, as the majority of the results it yields are visibly better than those previously obtained. Despite its unbalanced appearance, it provides great results outperforming any previous layout from sixteen cores and onwards.

Three Different Rings

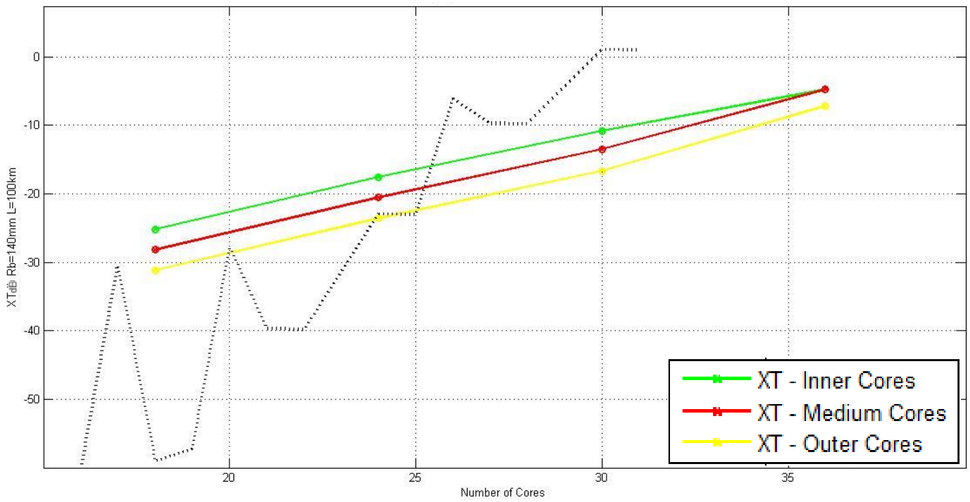

Figure 28: Crosstalk vs Number of cores - Three Different Rings

Using three different rings is only preferred for thirty and thirty-six cores, though both variations already yield a higher crosstalk value than the one deemed reasonable. This layout, although not viable for the $260\ \mu m$ case, shows promise for use in fibers with a bigger cladding diameter as given that all its cores yield a visibly balanced crosstalk.

Three Different Rings with Central Core

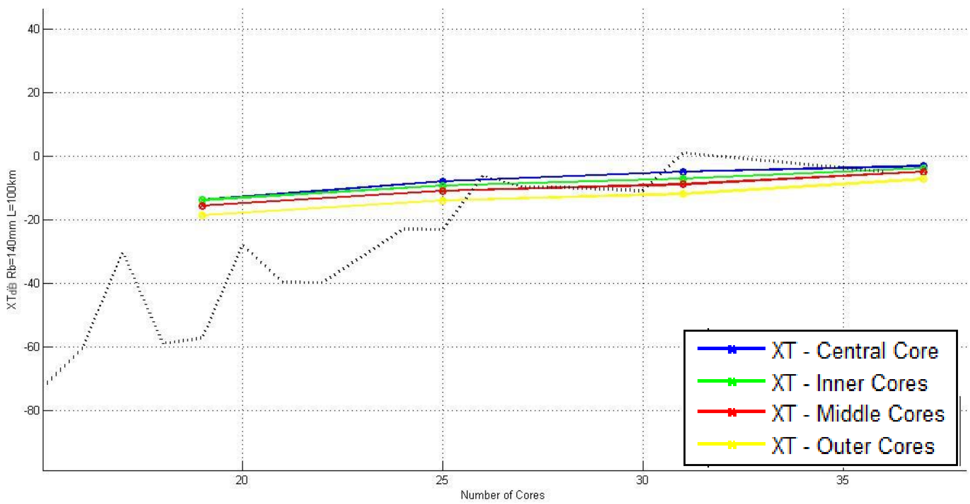

Figure 29: Crosstalk vs Number of cores – Three Different Rings with Central Core

This layout has little place on a $260\ \mu m$ optical fiber, as its crosstalk values are considerably high. When fitting thirty-one and thirty-seven cores it performs better than any of the previous layouts, although these core geometries are only expected to yield practical results for larger optical fibers.

Hexagonal Placement

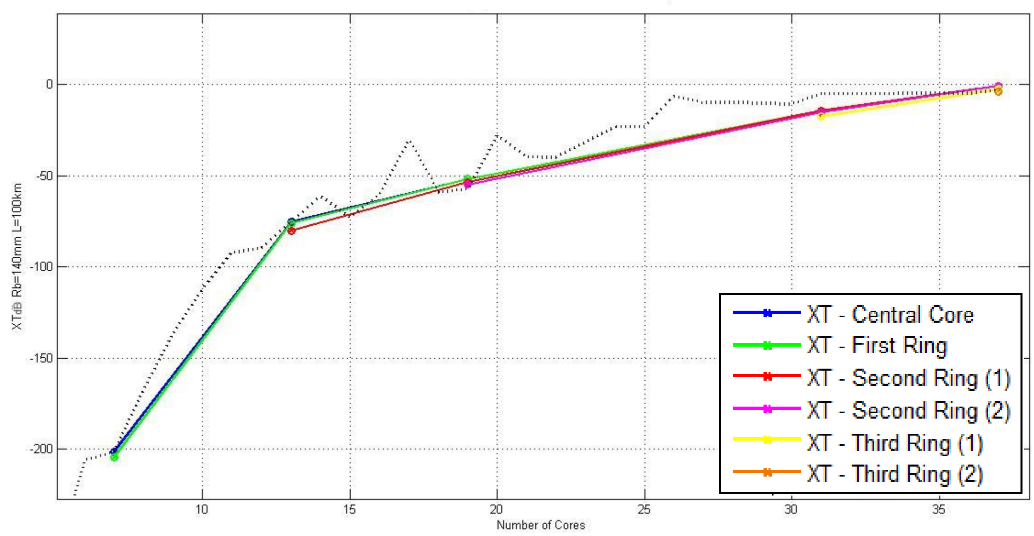

Figure 30: Crosstalk vs Number of cores – Hexagonal Placement

This layout consistently provides good results. Placing the cores according to a hexagonal structure guarantees extremely balanced values of crosstalk independently of the number of cores we decide to place inside the fiber.

When fitting seven and thirteen cores inside the fiber, this layout assumes the same structure as in "One Ring with Central Core" and "Two Rings with Central Core", respectively, rendering the same crosstalk results. For nineteen cores it is slightly worse than "Two Different Rings with Central Core" layout, and for thirty one it's better than any of the previous.

Finally, having obtained the crosstalk results for all proposed layouts, it is now possible to make a comparison between them. In order to do so, a more encompassing way of looking at the obtained results is provided (figure 31).

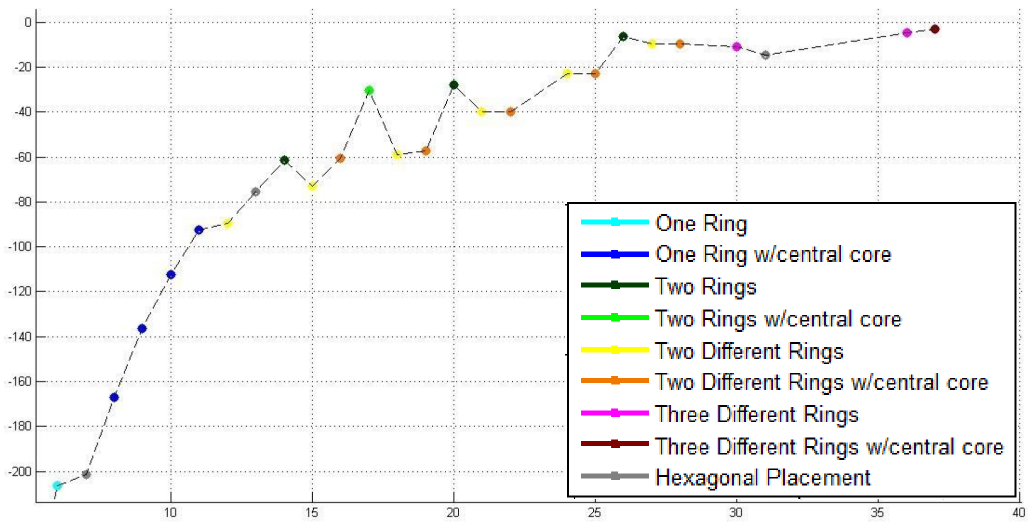

Figure 31: Crosstalk vs Number of cores – Overview for C_d = 260 µm

The dots represent the best crosstalk results obtained for a given number of cores. Their color identifies the layout employed to achieve this crosstalk value. It's of importance to recall that these values stand for the crosstalk of those core performing worse in each layout; the highest value of crosstalk in the layout. Given that we want to maximize the fiber capacity while complying with the crosstalk limits of the two scenarios proposed in section 2.4, it's time to determine how many cores can the fiber shelter in each..

If we want to use QPSK over a distance of $1000\ km$ with a $4\ dB$ OSNR penalty we cannot tolerate more than -$20\ dB$ of crosstalk in any core of the fiber. In this case, "Two Different Rings with a Central Core" should be the chosen layout as with no other is it possible to fit as many cores (twenty-five) in the fiber (figure 31).

This layout would be composed of sixteen cores in the Outer Ring and eight in the Inner Ring, all with approximately the same value of crosstalk: -$23\ dB$ (figure 32).

Only the central core in this layout would yield a crosstalk value considerably different from the other cores, of approximately -$53dB$.

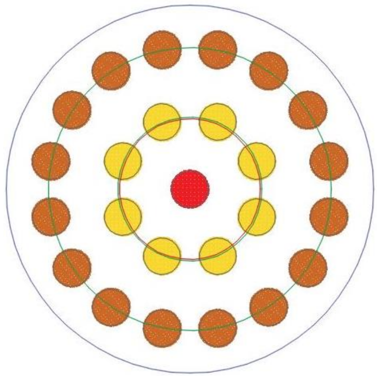

Figure 32: Two Different Rings - 25 Cores – C_d = 260 μm

If we want to use 256-QAM over a distance of $100\ km$ with a $4\ dB$ OSNR penalty, we cannot tolerate more than $-40\ dB$ of crosstalk in any core of the fiber. In this case, we should also use the "Two Different Rings with a central core" layout but this time only featuring twenty-two cores (figure 31).

This layout would be composed by fourteen cores in the Outer Ring and seven in the Inner Ring, all with approximately -$40\ dB$ of crosstalk (figure 33).

The central core, boasting -$55dB$ of crosstalk, is still considerably different than the crosstalk values of the rest of the cores in the structure, much like in the previous link using QPSK modulation.

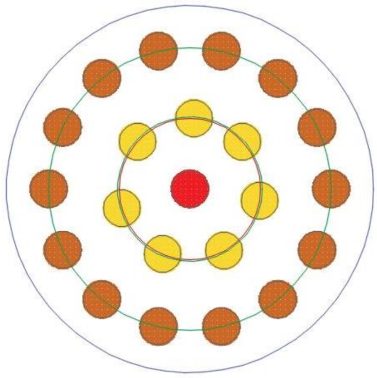

Figure 33: Two Different Rings - 22 Cores – C_d = 260 μm

Having chosen the adequate layouts for each scenario, a couple of remarks regarding the Baseline Solution must be made. The hexagonal distribution of thirty-one cores (as suggested in the Baseline Solution) is indeed a great way of minimizing the crosstalk, proving to be the best available solution for the placement of that exact number of cores. Unfortunately, placing that exact number of cores with the afore mentioned parameters (Table 10) on a fiber $260\ \mu m$ in diameter produces higher crosstalk values than those deemed tolerable. For that reason, and without the intent to undervalue this very effective layout, new ones had to be sought as to assure compliance with the different crosstalk tolerances.

5.3.2 Second Case: $CD = 300\ \mu m$

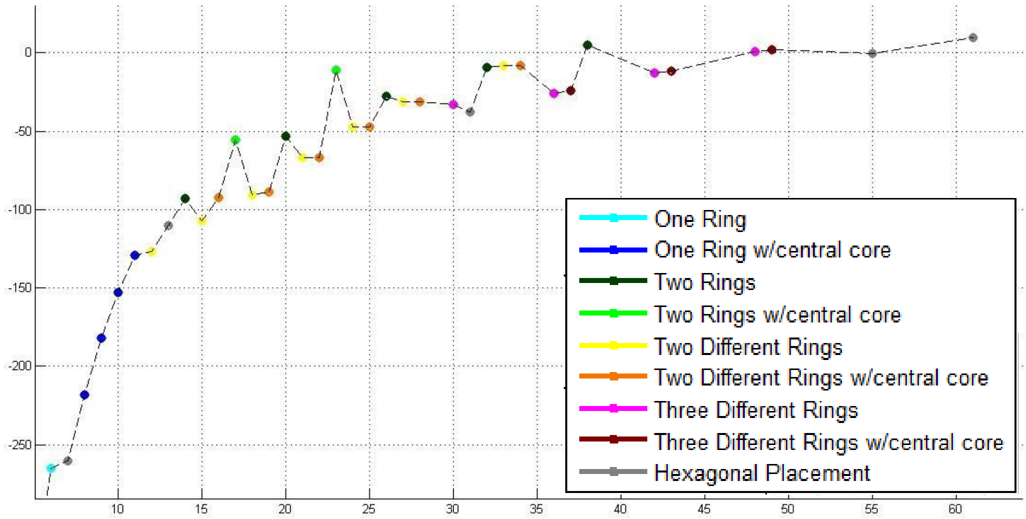

Figure 34: Crosstalk vs Number of cores – Overview for C_d = 300 µm

Making use of a fiber with a larger cladding diameter allows for more cores to be placed inside the fiber, which directly correlates with an increase in capacity. In order to choose the best core arrangement for the two proposed scenarios featuring different modulations, various simulations were conducted. Knowing the crosstalk tolerances for each scenario and having a figure summarizing the best crosstalk results obtained for this fiber $300\ \mu m$ in diameter (figure 34), we can proceed to draw some conclusions.

For the optical fiber link using QPSK that could only tolerate crosstalk values up to $-20\ dB$, "Three Rings with Central Core" is the layout yielding the best results. By choosing this layout, it is possible to fit thirty-seven cores in the fiber while complying with the crosstalk limit previously set for this link (figure 36). It is interesting to mention that this layout outperforms "Hexagonal Placement" when trying to fit thirty-seven cores in the fiber, which is a pretty good indication that this way of arranging this many cores is nearly optimal given that placing the cores hexagonally always renders nearly optimal results.

The crosstalk in all the cores will be extremely balanced, never greater than $-24\ dB$, as seen in figure 35.

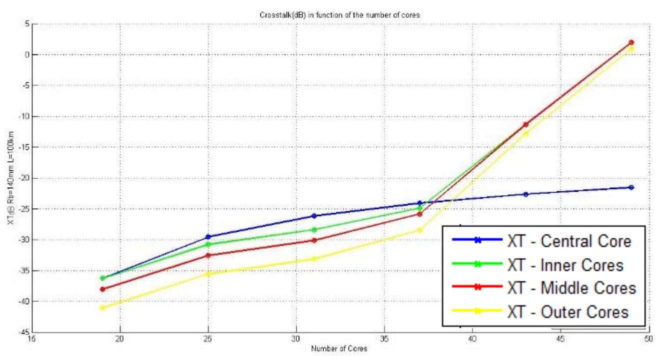

Figure 36: Crosstalk vs Number of cores – Three Different Rings (C_d =300 µm)

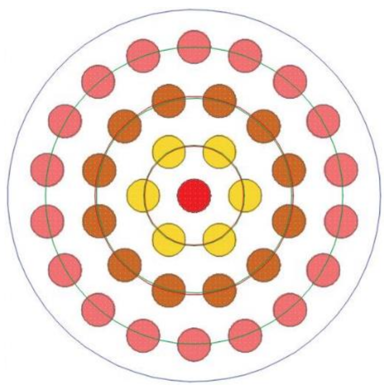

Figure 35: Three Different Rings - 37 Cores – C_d = 300 µm

As for the other scenario, featuring 256-QAM with a mere -$40\ dB$ limit for the maximum tolerable crosstalk, “Two Rings with Central Core” should be the chosen layout (figure 38).

This layout perfectly balances the crosstalk in both rings while leaving the central core with a slightly different crosstalk. Cores in both rings will have -$47\ dB$ of crosstalk, whereas the central core will only have -$84.5\ dB$ (figure 37).

It’s relevant to mention that if the crosstalk limit for this scenario was slightly lower, -$37.7\ dB$, it would be possible to accommodate thirty-one cores hexagonally.

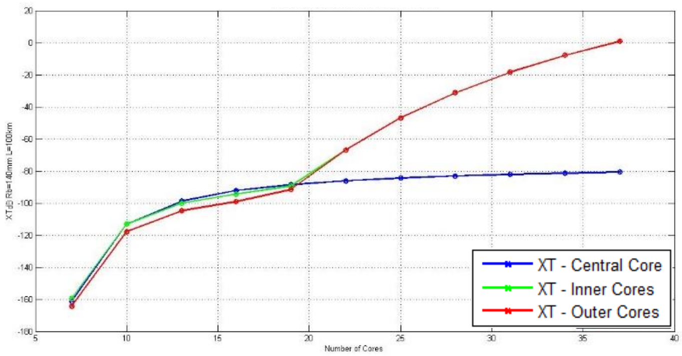

Figure 37: Two Different Rings - 25 Cores – C_d = 300 µm

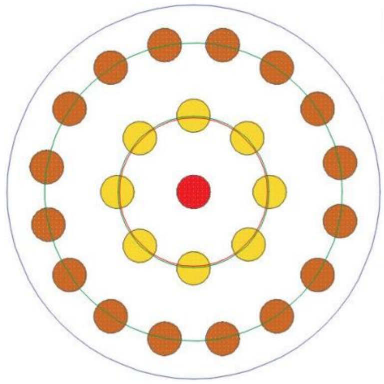

Figure 38: Two Different Rings - 25 Cores – C_d = 300 µm

5.3.3 Third Case: $CD = 125\ \mu m$

For a fiber as small as this one only two layouts are deemed relevant, their crosstalk results can be seen in figure 39 along with the dashed line that follows their trend.5

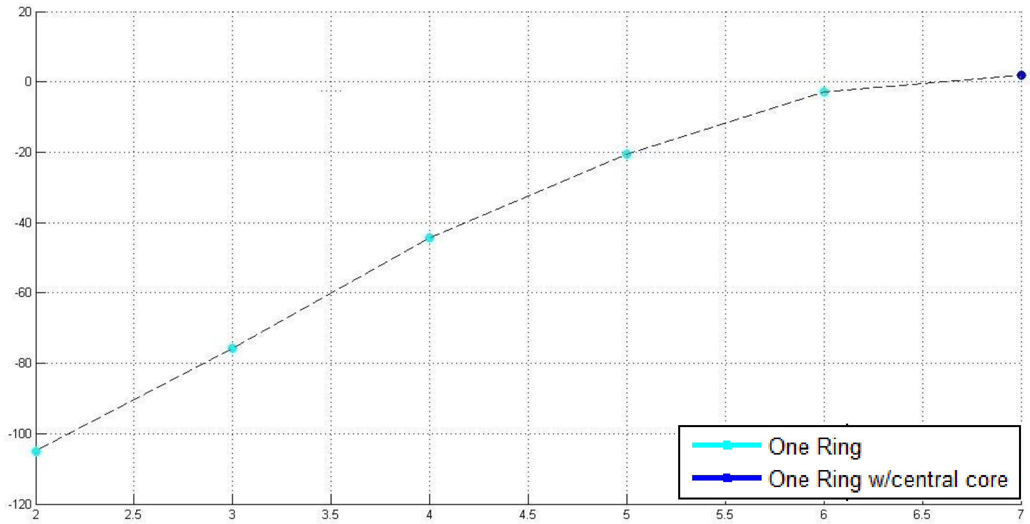

Figure 38: Crosstalk vs Number of cores – Overview for C_d = 125 μm

For a fiber using QPSK with a limit of -$20\ dB$, "One Ring" layout should be employed with five cores each having a crosstalk of approximately -$20.5\ dB$.

For a fiber using 256-QAM with a limit of -$40\ dB$, "One Ring" should also be the chosen layout as with four cores the crosstalk would be of -$44.5\ dB$ in each.

Both these structures, one for the link using QPSK (figure 41) and another for the link using QAM-256 (figure 40) can be seen below:

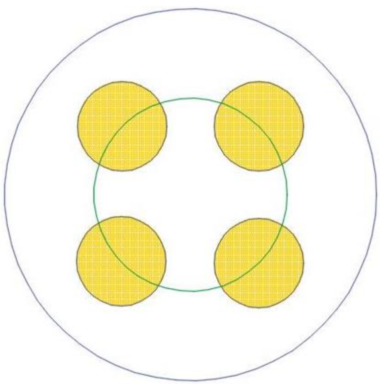

Figure 39: One Ring - 4 Cores – C_d = 125 μm

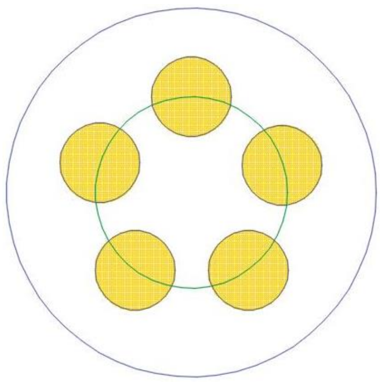

Figure 40: One Ring - 5 Cores – C_d = 125 μm

6 Conclusions and Future Work

A method for maximizing the throughput of a SM-MCF was proposed, describing how to best spatially arrange the cores inside three different fibers ($C_d = 125, 260, 300\ \mu m$).

A crosstalk estimation algorithm was designed, and the nine layouts that were proposed for doing this spatial arrangement of the cores were analyzed from a crosstalk perspective. From this analysis the best layouts for placing any given number of cores in the fiber were determined.

Two $1000\ km$ long optical fiber links using different modulations were given realistic crosstalk limits of -$10dB$ and -$30dB$ and, in accordance with these pre-established crosstalk limits, were attributed a layout for the spatial distribution of their cores. A different layout was attributed to both optical fiber links for each one of the three different fibers studied. For the fiber most intensively analyzed in this dissertation, the $260\ \mu m$ fiber, twenty-five cores could be placed in the fiber when using QPSK whereas only twenty-two when using 256-QAM; in both cases making use of the Two Rings with Central Core layout for distributing the cores. On the biggest fiber, $300\ \mu m$ in cladding diameter, 37 cores were able to fit inside using a QPSK modulation whereas only 25 using 256-QAM modulation. As for the smallest fiber, $125\ \mu m$ in cladding diameter, “One Ring” layout should always be the chosen layout for arranging the cores with either modulation.

Comparing with state-of-the-art solutions for this problem of core allocation, namely the hexagonal placement of thirty-one cores [14] considered in section 5.2, we conclude that the method for core allocation devised in this dissertation is functionally sound given its results are well-aligned with available cutting-edge solutions.

Consequently to the developed work, several suggestions for future work are presented:

- Development of a crosstalk optimization algorithm for heterogeneous MCF’s and few-mode fibers that, based on the layouts proposed in this dissertation, is able to determine which one best suits a fiber with a specified cladding diameter.
- Integration of a parameter optimization algorithm capable of further reducing the crosstalk of the cores following the choice of a layout.
- Development of a crosstalk estimation algorithm for spatial multiplexers/demultiplexers that, making use of the layouts as well as the method for choosing them presented in this dissertation, is able to minimize the crosstalk in these devices.

7 References

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8 Appendix

A – Method to determine the radius of the Outer Circle

Given a fiber diameter, and considering that no core can be closer than 30 μm to the edge of the outer cladding, we determine the radius of the circumference containing the center of the cores furthest from the center; the Outer Core.

Knowing the fiber radius to be:

$$Fiber\ radius = \frac{C_d}{2} \tag{8.1}$$

We know we can place the cores in the Outer Circle in a way that their core remains 30 µm away from the edge of the fiber:

$$r = Fiber\ radius - 30\mu m \tag{8.2}$$

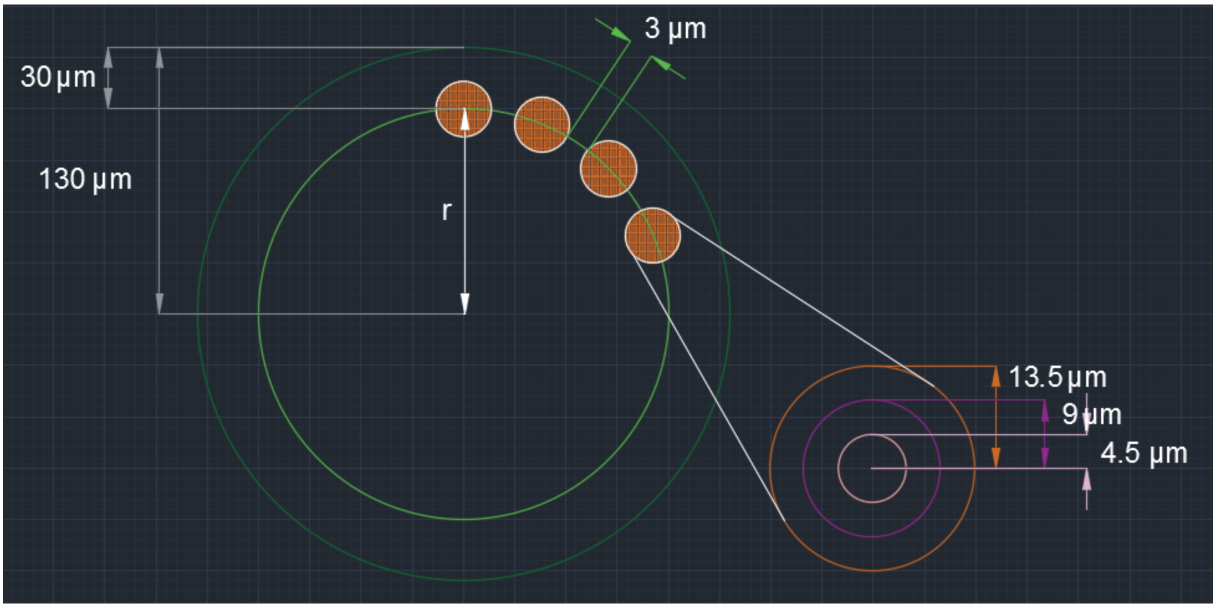

Figure 41: Fiber parameters representation for a 260 µm fiber

B – Crosstalk Approximation

For a given core, the crosstalk is the undesirable interference that every other core produces on its signal. Every single core, other than the reference one, must be taken into account in order to properly compute the crosstalk.

Knowing that the interference between cores (XT) quickly decays with a slight increase of the distance between them (Λ) at an approximate rate of 31 dB for each µm, it might make some sense to look into some kind of simplification.

The considered approximation for the crosstalk calculation meant only considering the interference from those cores closest to the reference core. This procedure is illustrated in figure 42, in which the green arrows represent the cores that were considered in the crosstalk calculation while the red arrows represent those that weren't. By not taking into account all the cores in the structure we're able to greatly simplify the crosstalk calculation task.

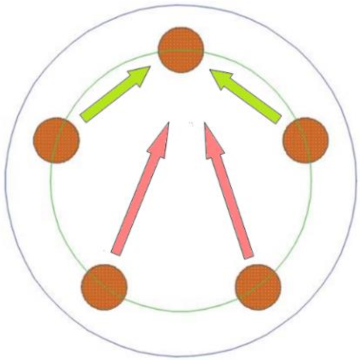

Figure 42: Approximation in One Ring

A comparison between the results obtained using this approximation and the results when taking into consideration all the cores in the structure can be seen in figure 44.

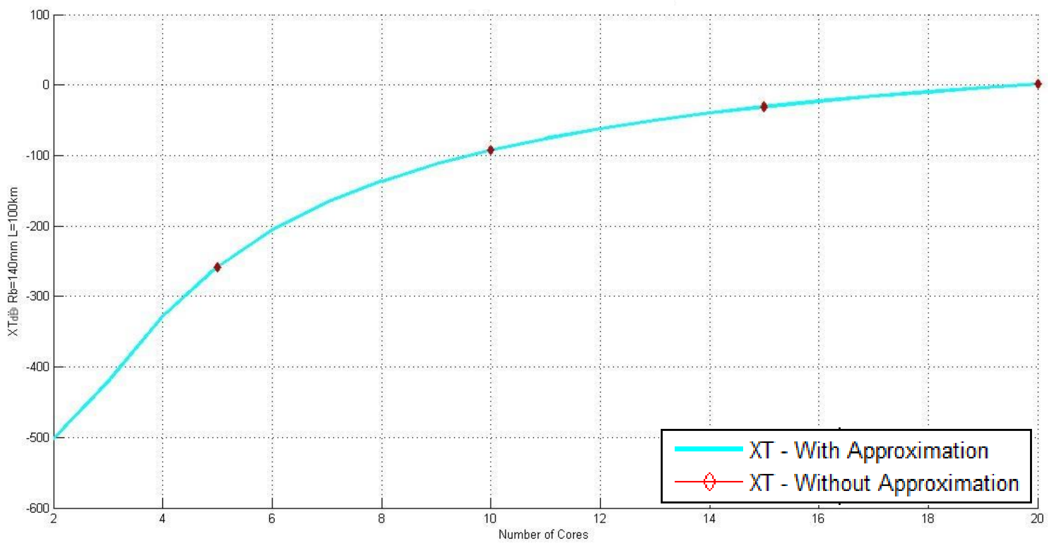

Figure 43: Crosstalk vs Number of cores – One Ring with approximation vs One Ring without approximation

For the same reasons that led to the use of this approximation, only for a few cases were the calculations made while taking into account all the cores in the layout.

It is clear that the use of this approximation has nearly no impact on the crosstalk calculation. Out of these four cases, it was for twenty cores that the crosstalk values differed the most from those featuring the approximation, varying $0.198716 \times 10^{-8} dB$.

C – Method for adjusting the Inner Ring radius in layout 3.3

In order to minimize the Inner Core crosstalk, we adjust the radius of the Inner Ring in a way that guarantees that the distance between an Inner Core and its neighbours is the same as the distance between an Inner Core and its closest Outer Core (8.3).

Note that both the Inner and Outer Rings contain the same number of cores in each ring; $M_1 = M_2$.

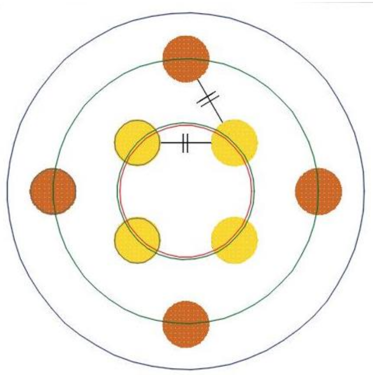

Figure 44: Two Rings adjustment

$$\begin{cases} \Lambda_2 = 2r_2 \times \sin(\frac{2\pi}{2M_2}) \\ \Lambda_x = r_1^2 + r_2^2 - (2r_1 * r_2 \times \cos\left(\frac{2\pi}{2M_1}\right)) \end{cases} \tag{8.3}$$

$$r_2 = \frac{\left(2r_1 \times \cos\left(\frac{2\pi}{2M_1}\right)\right) - \sqrt{\left(-2r_1 \times \cos\left(\frac{2\pi}{2M_1}\right)\right)^2 - 4 \times \left(1 - 4sin^2\left(\frac{2\pi}{2M_1}\right)\right) \times ({r_1}^2)}}{2 \times \left(1 - 4sin^2\left(\frac{2\pi}{2M_1}\right)\right)} \tag{8.4}$$

We should note that having the Inner Cores at this radius is optimal, since a larger radius would bring them closer to the Outer Cores while a smaller one would bring them closer to their neighbours.

By setting this value as the radius, instead of having the Inner Ring with half the radius of the Outer Ring, we are able to bring the Inner Core crosstalk to a minimum at the expense of slightly increasing the Outer Core Crosstalk.

Being the Inner Core crosstalk the highest across all variations of this layout, we will refer to it as the limiting crosstalk. Given that this limiting crosstalk was brought to a minimum, we can state there is no adjustment to the structure that would improve the overall crosstalk.

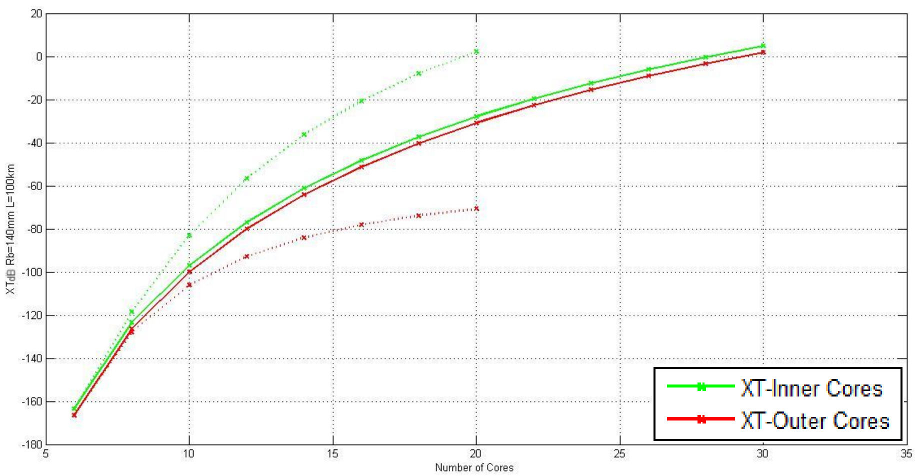

Figure 45: Crosstalk vs Number of cores – Two Rings with (solid line) and without (dotted line) adjustment

D - Method for adjusting the Inner Ring radius in layout 3.4

In order to reduce the Inner Core Crosstalk, we place the Inner Cores at a distance greater than half the Outer Ring radius, where the Inner Cores lay equidistant both to the Central and Outer Cores (8.5).

$$r_2 = \frac{\frac{r_1}{2}}{\cos\left(\frac{2\pi}{2M_1}\right)} \tag{8.5}$$

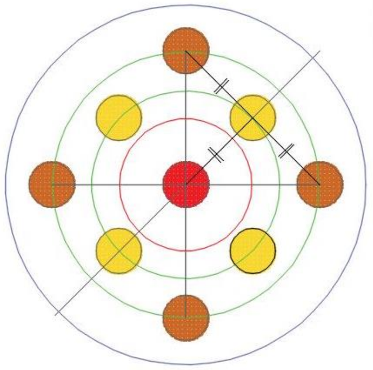

Figure 46: Two Rings with Central Core adjustment

By choosing this value for the radius, instead of having the Inner Ring with half the radius of the Outer Ring, we are able to reduce both the Central and Inner Core crosstalk at the expense of slightly increasing the Outer Core Crosstalk.

Up until thirteen cores the limiting crosstalk is the central Core's, meaning we might be able to produce slightly better results by placing the Inner Cores still further away from the central core, breaking this layout's equidistance condition and increasing both the Inner and Outer Core crosstalk. From thirteen cores onwards we could hardly achieve better results as the limiting crosstalk is the Inner Core's, and it already sits at a minimum or very close to it.

Note: For seven cores we come across an interesting result in which every single crosstalk value is lower than those from the equidistant rings scenario due to the structure morphing into a "One Ring" layout, thus fulfilling the condition of equidistance to both Outer and central cores.

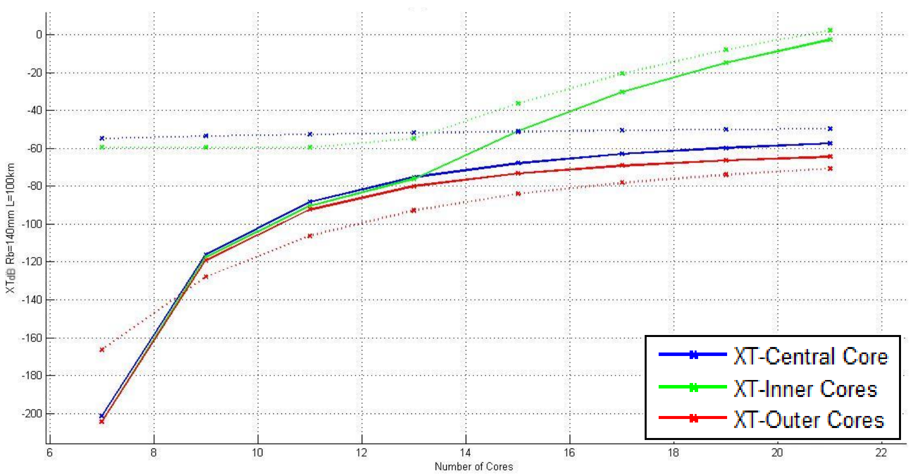

Figure 47: Crosstalk vs Number of cores – Two Rings with Central Core with (solid line) and without (dotted line) adjustment

E - Method for adjusting the Inner Ring radius in layout 3.5

In order to minimize the Inner Core crosstalk, we adjust the radius of the Inner Ring in a way that guarantees that the distance between an Inner Core and its neighbours is the same as the distance between an Inner Core and its closest Outer Core (8.6).

$$\begin{cases} \Lambda_2 = 2r_2 \times \sin(\frac{2\pi}{2M_2}) \\ \Lambda_x = r_1^2 + r_2^2 - (2r_1 \times r_2 \times \cos\left(\frac{2\pi}{2M_1}\right)) \end{cases} \quad (8.6)$$

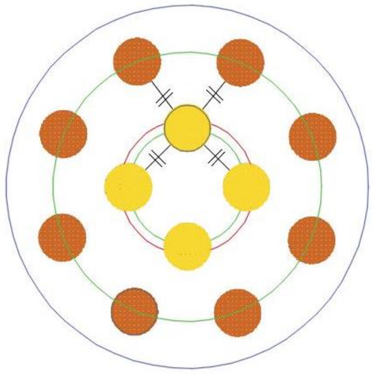

Figure 48: Two Different Rings Adjustment

$$r_2 = \frac{\left(2r_1 \times \cos\left(\frac{2\pi}{2M_1}\right)\right) - \sqrt{\left(-2r_1 \times \cos\left(\frac{2\pi}{2M_1}\right)\right)^2 - 4\left(1 - 4sin^2\left(\frac{2\pi}{2M_2}\right)\right) \times ({r_1}^2)}}{2 \times \left(1 - 4sin^2\left(\frac{2\pi}{2M_2}\right)\right)} \quad (8.7)$$

We should note that having the Inner Cores at this radius is optimal, since a larger radius would bring them closer to the Outer Cores while a smaller one would bring closer to their neighbours. By setting this value as the radius, instead of having the Inner Ring with half the radius of the Outer Ring, we are always able to reduce the Inner Core crosstalk.

In those cases where the number of cores present in the structure is fifteen or less, the new radius will be smaller than half the radius of the Outer Cores, which will reduce both Inner and Outer Core Crosstalk when compared to the case in which the rings are equally spaced. In those cases where the number of Cores is higher than fifteen cores, the new radius will be greater than half the radius of the Outer Cores; which will slightly increase the Outer Core crosstalk while reducing the Inner Core's.

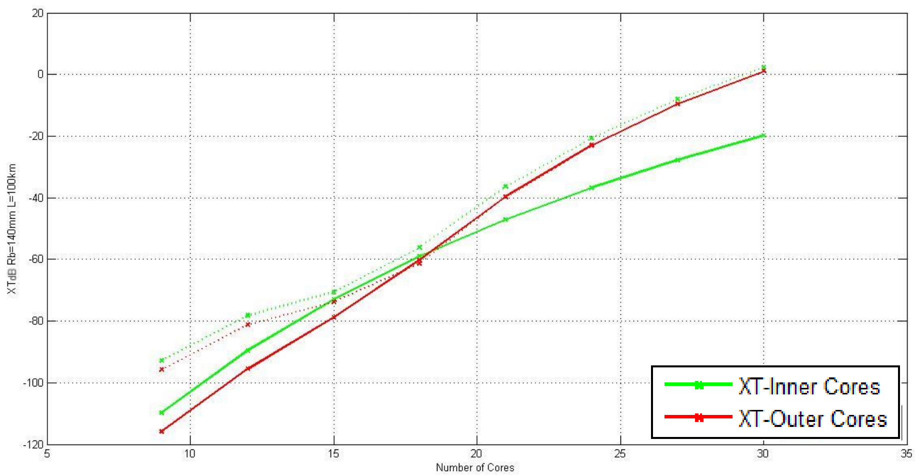

Figure 49: Crosstalk vs Number of cores – Two Different Rings with (solid line) and without (dotted line) adjustment

Given that up until eighteen cores the limiting crosstalk is the Inner Cores', and knowing a crosstalk minimization algorithm was employed for those cores, we can state there is no adjustment to the structure that would improve the overall crosstalk up until eighteen cores. For nineteen and more cores the Outer Core crosstalk is the limiting one, and although it always takes smaller values than those of the case in which the rings are equally spaced, there is still little room for improvement.

F – Method for adjusting the Inner Ring radius in layout 3.6

In order to reduce the Inner Core Crosstalk, we place the Inner Cores at a distance greater than half the Outer Ring radius, where the Inner Cores lay equidistant both to the Central and Outer Cores (8.8).

$$r_2 = \frac{\frac{r_1}{2}}{\cos\left(\frac{2\pi}{2M_1}\right)} \tag{8.8}$$

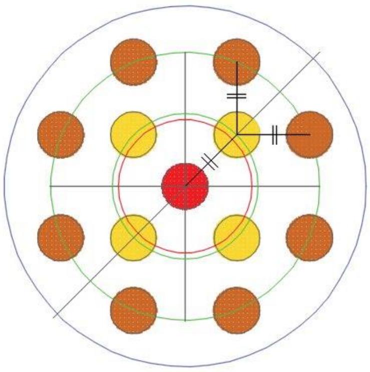

Figure 50: Two Different Rings with Central Core adjustment

By choosing this value for the radius, instead of having the Inner Ring with half the radius of the Outer Ring, we are able to reduce both the Central and Inner Core crosstalk at the expense of slightly increasing the Outer Core Crosstalk.

Up until nineteen cores the limiting crosstalk is the central Core's, meaning we might have been able to produce slightly better results by placing the Inner Cores further away from the central core. From nineteen core onwards, the limiting crosstalk of this layout is Inner Core's; and since this radius adjustment envisioned to reduce the Inner Core crosstalk as much as possible, we can hardly achieve better results.

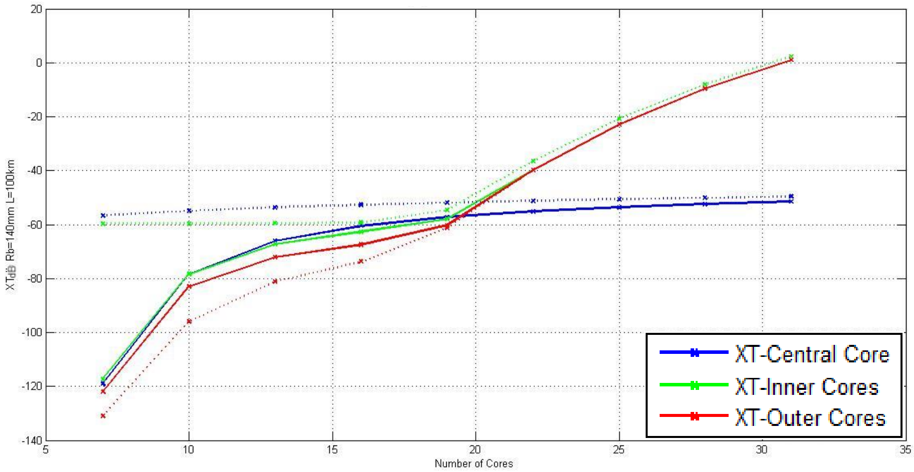

Figure 51: Crosstalk vs Number of cores – Two Different Rings with Central Core with (solid line) and without (dotted line) adjustment

G – Method for adjusting both the Inner and Middle Ring radii in layout 3.7

In order to reduce the levels of crosstalk all across this layout we proceed to determine new values for the radius of the Inner and Middle Rings. The values for the radii ensure the distance between Inner Ring neighbours is the same as both the distance from an Inner Core to a Medium Core, and from a Medium Core to the Outer Circle (8.9).

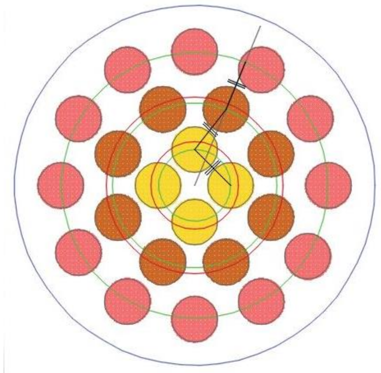

Figure 52: Three Different Rings adjustment

$$\begin{cases} \Lambda_3 = 2r_3 \times \sin(\dfrac{2\pi}{2M_3}) \\ \Lambda_x = r_2^2 + r_3^2 - (2r_2 \times r_3 \times \cos(\dfrac{2\pi}{2M_2}) \\ d = r_1 - r_2 \end{cases} \quad (8.9)$$

$$r_3 = \frac{2r_1 \times \cos\left(\frac{2\pi}{2M_2}\right) + 4r_1 \times \mathrm{sen}\left(\frac{2\pi}{2M_3}\right) - \sqrt{\left(-2r_1 \times \cos\left(\frac{2\pi}{2M_2}\right) - 4r_1 \times \mathrm{sen}\left(\frac{2\pi}{2M_3}\right)\right)^2 - 4 \times \left(1 + 4\sin\left(\frac{2\pi}{2M_3}\right)\cos\left(\frac{2\pi}{2M_2}\right)\right) \times (r_1{}^2)}}{2 \times \left(1 + 4\sin\left(\frac{2\pi}{2M_3}\right)\cos\left(\frac{2\pi}{2M_2}\right)\right)} \quad (8.10)$$

It is important to note that the distance between an Outer Core and its closest Middle Core will depend on the Outer Core we pick, which is why it is necessary make the distance between Inner Ring neighbours the same as the distance between a Middle Core and the Outer Circle, to ensure this latter is either the same or smaller than the distance between a Middle and an Outer Core.

By setting these values as the radii, instead of having the Inner and Middle Rings laying at, respectively, one third and two thirds of the Outer Ring radius, we are able to reduce the crosstalk of both Outer and Medium Cores while increasing the Inner Cores'.

Although this layout produces great results across all variations of itself, there is a minor room for improvement given that we can still reduce the value of the limiting crosstalk. This crosstalk, the Inner Cores', can be slightly reduced by increasing the radius of both Inner and Medium Rings, which would the other hand increase both Outer and Medium core crosstalk values. This extra crosstalk adjustment wasn't performed as it brings little benefit while requiring an individual adjustment for both rings for each variation of this layout.

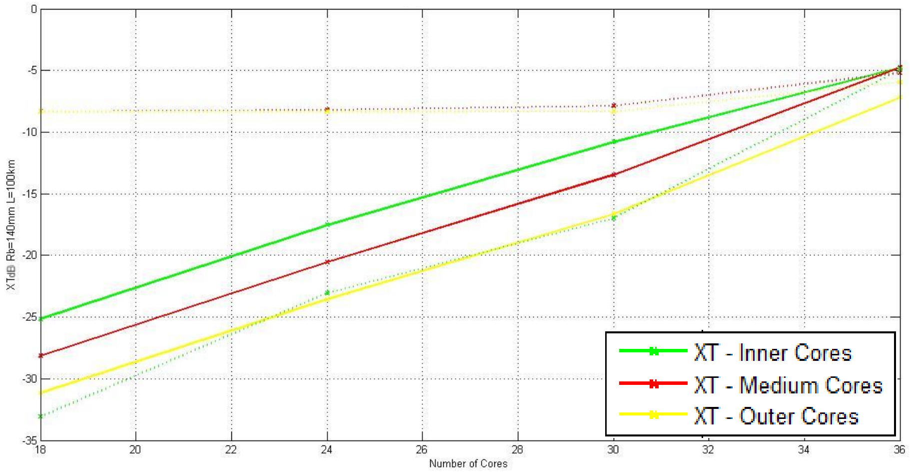

Figure 53: Crosstalk vs Number of cores – Three Different Rings with (solid line) and without (dotted line) adjustment

H –Method for adjusting both the Inner and Middle Ring radii in layout 3.8

In order to balance the levels of crosstalk in this layout, we proceed to determine new values for the radius of the Inner and Middle Rings. The values for the radii ensure the distance from the Central Core to an Inner Core is the same as both the distance from an Inner Core to a Middle Core and from a Middle Core to the Outer Circle (8.11).

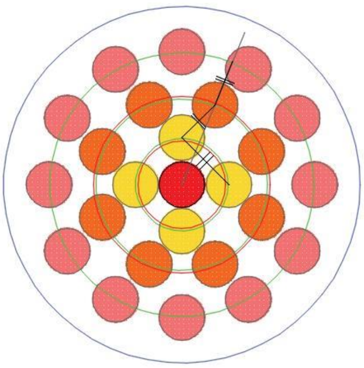

Figure 54: Three Different Rings with Central Core Adjustment

$$\begin{cases} r_3 = \dfrac{r_2}{2\cos(\frac{2\pi}{2M_2})} \\ r_1 = r_3 + r_2 \end{cases} \tag{8.11}$$

$$r_2 = \frac{r_1}{1 + 2cos(\frac{2\pi}{2M_2})} \tag{8.12}$$

It is important to note that the distance between an Outer Core and its closest Middle Core will depend on the Outer Core we pick, which is why it is necessary make the distance between a Central and an Inner Core the same as the distance between a Middle Core and the Outer Circle, to ensure this latter is either the same or smaller than the distance between a Middle and an Outer Core.

By setting these values as the radii, instead of having the rings equally spaced, we are able to reduce all the values of crosstalk.

Although this layout produces great results across all variations of itself, there is still room for improvement given that we can still reduce the value of the limiting crosstalk at the expense of increasing the others, this is valid both when the limiting is the central core's and Inner Core's. Although this extra crosstalk adjustment can be achieved choosing different values for the radii, it wasn't performed as it brings little benefit while requiring an individual adjustment for each ring for each variation of this layout.

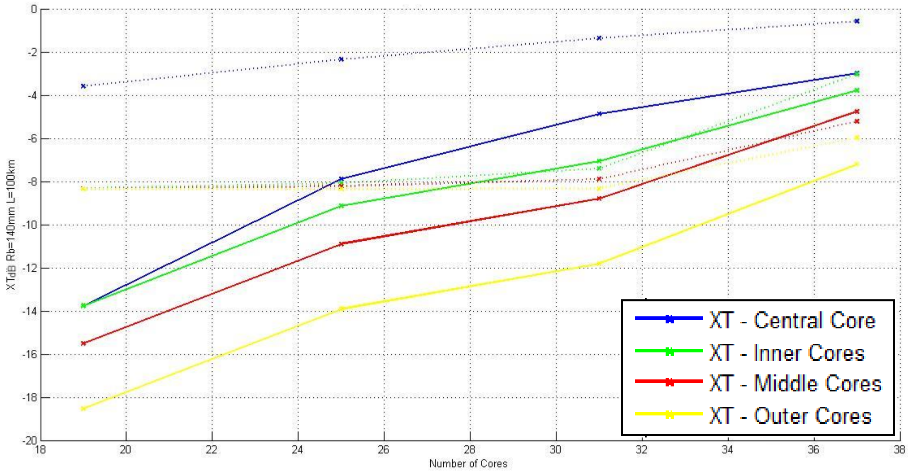

Figure 55: Crosstalk vs Number of cores – Three Different Rings with Central Core with (solid line) and without (dotted line) adjustment