# Turbofan Engine Optimization for Low $NO_x$ Emissions

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# September 2016

#### Abstract

More stringent limits for pollutants emissions are imposed to accommodate the experienced growth in the world's aircraft fleet. The most regulated of these pollutants are the nitrogen oxides,  $NO_x$ . These represent a health and environmental hazard, when emitted at both low and high altitudes. To reduce emissions without reducing engines performance presents a difficult task for engine design engineering. In order to do so, early-stage design tools are used to compute fast and realistically accurate results. These tools include the formulation of the models under the 0D framework. Following this approach, a 0D model of a two-spool turbofan was formulated and validated by comparing the results with those obtained from commercially available software. A combustion model was also formulated under the 0D framework, and validated by comparing its results with those presented in the literature. Two turbo-emissions models were achieved, one by unifying both turbofan and combustion models, and the second by coupling the turbofan model with a semi-empirical  $NO_x$  predictions model. Both models were validated by comparing the results with available data from the literature. The second unified model was used under an optimization algorithm in order to compute optimized parameters with the objective of low  $NO_x$  emissions while maintaining a low specific fuel consumption. To better understand the effects of the optimization in the combustion mechanism, the optimized parameters were computed through the more detailed turbo-combustion model.

**Keywords:** Turbofan Engine Model 0D, Combustion Model 0D,  $NO_x$  Emissions, Parametric Design Optimization

### 1. Introduction

With an increasing interest in global interactions and communications, airline traveling and freight transport becomes a primary necessity for those who want to be in line with progress. Data collected by the Forecasting and Economic Analysis Support Group (FESG) of the International Civil Aviation Organization (ICAO) Committee on Aviation Environmental Protection (CAEP), foreseen a great increment in air traffic and environmental footprint in a 30-year time horizon. In Most Likely scenarios, which are used as a central forecast, data shows that the demand for this mean of transportation is expected to increase each year by an average of 4.9% for passenger traffic, and an average 5.2%for freight traffic for the next 15 years [1]. To accommodate the expected growth in air traffic, an increase of fleet size is inevitable. By 2040 it will be needed around 56000 new airliners to commercial passenger traffic, 6000 new aircraft to freight traffic and around 45000 will be needed to business passenger traffic. Combining both passenger and freight operations, it is expected that the number of flights worldwide triple by 2040.

With such an increase in fleet and operations, an increase in environmental concerns are also to be expected and even more stringent throughout the years to come. Aircraft engines currently run on fossil-fuel which it combustion emit various pollutants to the atmosphere, affecting local and regional air quality, as well as affecting its cleanest regions at high altitude under cruise conditions. Of particular importance to the environment are the emissions of Carbon Dioxide (CO<sub>2</sub>), Nitrogen Monoxide (NO) and Nitrogen Dioxide (NO<sub>2</sub>) (collectively known as Nitrogen Oxides (NO<sub>x</sub>)), Sulfur Dioxide (SO<sub>2</sub>) and Sulfate (SO<sub>4</sub>) (collectively know as Sulfur Oxides (SO<sub>x</sub>)), Unburned Hydrocarbons (UHC), and Particulate Matter (PM).

In special focus for this work is  $NO_x$  emissions from subsonic aircraft engines. When locally emitted, exposes great concern as it can lead to the formation of other pollutants, such as particulates and ground-level ozone, which are harmful to health. While when emitted at cruise altitudes, at the upper tropospheric layer, the production of  $NO_x$  leads to, the formation of ozone, by a photochemical processes, and the reduction in CH<sub>4</sub> concentration, which are both GHG. The estimated radioactive forcing of  $NO_x$  can be considered 100 to 130 times more powerful than CO2 when concerning global warming effects (100-130 Global Warming Potential (GWP)).

To reduce the aerospace industry pollutant emissions the CAEP, which is responsible for formulating new standards in aircraft noise and emissions regulations, established limits addressing local air quality in the vicinity of the airports based on a LTO cycle analysis. Those limits present to be more stringent for  $NO_x$  than for the rest of the trace species. For this reason, proper design tools are in demand in order to provide solutions for low  $NO_x$ emissions while increasing fuel efficiency.

The objective of the present work lies on the optimization of the design parameters of a turbofan in order to reduce  $NO_x$  formation at the high temperatures achieved in the primary zone of the combustor. In order to compute such optimization it is necessary to develop parametric models of the turbofan and combustion phenomena. The thermodynamic model of a turbofan should be developed under a 0D framework and provide realistic results. The validation of the model should be taken care by the comparison of the obtained results with those obtained from a recognized commercial software, and by comparing the obtain performance parameters with those retrieved from ICAO data bank. A chemical reaction model of the primary zone needs to be formulated in order to assess the  $NO_x$  formation mechanism. The primary zone combustion model will be submitted to validation by comparing the obtained emissions results with data from ICAO emissions data bank. Lastly, an optimization algorithm will be paired with the turbofan model in order to achieve optimized design parameters for low  $NO_x$  emissions.

# 2. Turbofan Model

A two-spool turbofan model is proposed in order to perform a parametric cycle analysis of the engine. The model follows the guidelines of a 0D framework providing the simplicity necessary to the optimization process which will take place further in chapter 5. An overview of each core component of the engine is carried out while expressing the equations of the undergoing processes of which the working fluid is subjected at each station. Reference of the stage numbering within the turbofan is presented as follows in figure 1:



Figure 1: Turbofan stage numbering.

Input design parameters have to be defined a priori in order to evaluate the parametric cycle. Such inputs are represented by two blocks. The first block, referring to design choices for bypass ratio, B, fan pressure ratio, FPR, compressor pressure ratio, CPR, and operating conditions. While the second block, refers to the physical limitations of technological designs, materials thermal resistance, atmospheric air conditions and fuel proprieties. Data referring to these limitations, is retrieved from literature. Performance parameters such as the specific thrust,  $\Psi$ , and the specific fuel consumption, SFC, are evaluated for a number of different input design parameters. Specific thrust and fuel consumption are defined as follows:

$$\Psi = \frac{T_{bypass}}{\dot{m}g} + \frac{T_{core}}{\dot{m}g} \tag{1}$$

$$\frac{T_{bypass}}{\dot{m}g} = \frac{\dot{m}_b(u_8 - u_0)}{\dot{m}g} + \frac{A_8(p_8 - p_0)}{\dot{m}g} \quad (2)$$

$$\frac{T_{core}}{\dot{m}g} = \frac{\dot{m}_c(u_e - u_0)}{\dot{m}g} + \frac{A_e(p_e - p_0)}{\dot{m}g} \qquad (3)$$

$$SFC = \frac{\dot{m}_f g}{T_{bypass} + T_{core}} \tag{4}$$

$$SFC = \frac{1}{B+1} \frac{1}{\Psi} \frac{c_{pg} T_{t5} - c_p T_{T_{t4}}}{\eta_b H V_{fuel} - c_{pg} T_{t5}}$$
(5)

#### 2.1. Model Results and Validation

The proposed thermodynamic model of the turbofan is validated by comparing the computed results with those obtained using the GasTurb<sup>®</sup> software [2]. The turbofan performance is analyzed through the computation of  $\Psi$  and SFC, with the variation of design parameters around their baseline values. The analysis was carried out for a design point at an operating altitude of H = 10000m, and a flight Mach number of  $M_0 = 0.8$ .

# Baseline:

 $\begin{array}{ll} B=5 & \pi_d=0.95 & \eta_b=0.95 \\ FPR=1.6 & \eta_{pf}=0.85 & \pi_b=0.95 \\ CPR=18.75 & \eta_{pc}=0.85 & \eta_{pn}=0.999 \\ TIT=1600K & \eta_{pt}=0.85 & \eta_m=0.999 \\ c_p=1005 \; J/(kgK) & c_{pg}=1140 \; J/(kgK) \\ HV_{fuel}=43.4 \; kJ/kg & \gamma_g=1.33 & \gamma_g=1.4 \end{array}$ 

Figures 2 and 3 express the variation of performance parameters SFC and  $\Psi$ , respectively, as function of the bypass ratio for the conditions above referred.



Figure 2: Specific Fuel Consumption versus Bypass Ratio.



Figure 3: Specific Thrust versus Bypass Ratio.

The obtained values from both the proposed model and the GasTurb<sup>(R)</sup> software, show good agreement in the computed performance parameters for the range of evaluated bypass ratios. The computed SFC present a mean relative error of 2.2%, while the computed  $\Psi$  present a mean relative error of 10.2%. Despite the difference presented, results show the same trend as those from GasTurb<sup>(R)</sup>.

The proposed turbofan model also performs the computation of the specific fuel consumption using the inputs retrieved from the ICAO Data Bank [3]. Such inputs are the overall compression ratio, bypass ratio and date when first tested. In this case, the results were computed using the data of the twospool engines from General Electric engines only, thus providing the necessary consistency of manufacturer design preferences. Under the limitation of the design data available, the fan pressure ratio is assumed to be 1.7 throughout the analysis. This analysis refer for take-off conditions, with  $M_0 = 0$ and H = 0m. To compute the specific fuel consumption the fuel mass flow,  $\dot{m}_f$ , and engine maximum thrust,  $T_N$ , relative to take-off conditions are retrieved from ICAO data bank, and equation (4) is directly applied. In order to accommodate the technological advancements made throughout the years, it is defined four levels of technology regarding component efficiencies, pressure drops and thermal resistance [4]. Using the dates when first tested, levels of technology can be attributed to each engine. Figure 4 presents the comparison between estimated values and those retrieved from ICAO data bank.



Figure 4: Turbofan model vs ICAO data: SFC as function of overall pressure ratio.

Considering the limited information provided for the conducted analysis, the proposed model proves to yield reasonable results throughout the range of engines analyzed. As can be denoted, the proposed model tends to over estimate the specific fuel consumption, although relative error decrease with the increase of the overall pressure ratio, achieving good correlation towards the high pressure ratios zone. The mean relative error of 14.2% proves a reasonable correlation with the real data. It is also important to denote that the underestimation presented when the results were compared with those of GasTurb<sup>(R)</sup> software was not of particular importance as the proposed model actually tends to slightly over estimate the results when faced with real data.

### 3. Combustion Model

 $NO_x$  formation is usually defined by the thermal fixation of atmospheric nitrogen, Thermal  $NO_x$ ; by the reaction of atmospheric nitrogen with a free hydrocarbon radical, Prompt NO; by organically bound nitrogen, present in certain fuels, which is

readily oxidized, Fuel  $NO_x$ ; or by reacting with  $N_2O$  which is formed in at high pressure conditions,  $N_2O$  pathway. Fuel  $NO_x$  are most relevant, and the predominant source of  $NO_x$  formation, when burning fuels with bounded nitrogen, as in the case of coal. Prompt  $NO_x$  formation occurs at low temperature, fuel-rich conditions and short residence times. Fuel  $NO_x$  and Prompt  $NO_x$  are not going to be assessed in this work, since their contribution proved to be negligible at the conditions of this study, which focus on fuel-lean conditions at high temperature and pressure where the fuel burnt don't have any bounded nitrogen. Although the  $N_2O$  pathway present a not negligible contribution to the global  $NO_x$  formation, it will not be assessed in this work.

Since the majority of the produced  $NO_x$  are emitted as NO, followed by the conversion in  $NO_2$  by reacting with atmospheric oxygen when in contact with the atmospheric air at the exhaust, this work will only assess the formation of NO in the combustor.

In the primary zone of combustion at high temperatures, thermal  $NO_x$  formation derive from the thermal fixation of atmospheric nitrogen through the overall oxidation reaction expressed below:

$$\frac{1}{2}N_2 + \frac{1}{2}O_2 \rightleftharpoons NO \tag{6}$$

This reaction is highly endothermic  $[\Delta h_r^{\circ}(298) = 90.0 \ kJmol^{-1}]$ , thus resulting that chemical equilibrium is just attained at very high temperatures present at near stoichiometric combustion. When temperatures are not sufficiently high the equilibrium concentration of NO decreases rapidly since its exponentially dependent of the temperature. The direct reaction of nitrogen with oxygen is to slow to be accounted for NO formation, however free oxygen atoms from O<sub>2</sub> dissociation or radical attack on O<sub>2</sub> are present and react readily with the N<sub>2</sub> molecules thus beginning a chain reaction mechanism, first postulated by Zeldovich in 1947, and thus called the extended Zeldovich mechanism.

1) 
$$N_2 + O \rightleftharpoons NO + N$$
 (7)

2) 
$$N + O_2 \rightleftharpoons NO + O$$
 (8)

$$3) \qquad N + OH \rightleftharpoons NO + H \tag{9}$$

A combustion model was formulated in order to achieve a comprehensive simulation of the reaction mechanism occurring inside a gas turbine combustor. Seeing that the thermal contribution through the Zeldovich reaction mechanism accounts for the majority of the  $NO_x$  formation, the proposed model uses combustion kinetic analysis while also considering the dissociation effects in order to provide the estimations.

Considering the 0D approach of the combustion model, the reaction zone can be modeled taking into account the following inputs: the entry conditions of the pressurized air in the combustor, represented by the stagnation temperature and pressure after the compressor stage; the equivalence ratio,  $\Phi$ ; and the residence time in the reaction zone of the combustor. The reaction mechanism proposed to combustion equilibrium is represented by the following reactions:

$$CO_2 \rightleftharpoons CO + \frac{1}{2}O_2$$
 (10)

$$H_2 O \rightleftharpoons H_2 + \frac{1}{2}O_2 \tag{11}$$

$$\frac{1}{2}N_2 + \frac{1}{2}O_2 \rightleftharpoons NO \tag{12}$$

$$\frac{1}{2}N_2 \rightleftharpoons N \tag{13}$$

$$\frac{1}{2}O_2 \rightleftharpoons O \tag{14}$$

$$\frac{1}{2}H_2O + \frac{1}{4}O_2 \rightleftharpoons OH \tag{15}$$

$$\frac{1}{2}H_2O \rightleftharpoons H + \frac{1}{4}O_2 \tag{16}$$

Under the following assumptions that O, H and OH radicals are present at their equilibrium concentrations, that the N rate of formation is considered to be in a quasi-steady state, and that NO formation takes place after the combustion reaction at constant adiabatic flame temperature, the formation rate of nitric oxide can be expressed as follows:

$$R_{NO} = k_{+1}[N_2][O] - k_{-1}[N][NO] + k_{+2}[N][O_2] - k_{-2}[NO][O] + k_{+3}[N][OH] - k_{-3}[NO][H]$$
(17)

where k represent the rate constants for the Zeldovich mechanism of the respective subscripted direct and inverse reactions.

### 3.1. Results and Model Validation

The proposed combustion model is validated by comparing the computed results with reference values taken from [5] under the same primary zone inlet conditions. The mole fractions of the major products of combustion are presented, as well the mole fraction of the nitrogen oxide. Results are computed for the combustion reaction of  $CH_{1.88}$  at 10 atm and 560 K, while also considering the dissociation processes of H<sub>2</sub>O and CO<sub>2</sub>, thus yielding the mole fractions of H<sub>2</sub> and CO.

Entities	Reference	Model
Mole fractions		
$N_2$	0.7510	0.7479
$\rm CO_2$	0.1081	0.1061
$O_2$	0.0397	0.0405
$H_2O$	0.1020	0.1009
CO		$1.584 \times 10^{-3}$
$H_2$		$2.931 \times 10^{-4}$
NO	$1.180 \times 10^{-3}$	$1.027 \times 10^{-3}$
$T_f K$	2304.0	2293.7

Table 1: Comparison of results for a incomplete combustion with  $NO_x$  formation.

where  $T_f$  denotes the adiabatic flame temperature. The model results for the oxidation reaction with dissociation effects don't differ much from the former values in the four main combustion products, N<sub>2</sub>, CO<sub>2</sub>, O<sub>2</sub> and H<sub>2</sub>O. As expected, the adiabatic flame temperature decreases. Even if small, the decrease in flame temperature by the endothermic dissociation reactions led to a significant decrease in NO formation. This proves the high sensibility of NO formation to variations of temperature, as mentioned before.

# 4. Turbo-Emissions Model

In order to compute and optimize the design parameters in an early stage of a turbofan engine design, both turbofan and combustion process have been paired to give an overall understanding of the complete engine performance, regarding pollutant formation and specific fuel consumption.

Two models were assembled to produce estimates for pollutant emission. The first unified model, pairs the turbofan model produced and validated at chapter 2 and the combustion model with dissociation effects and NO formation described and validated at chapter 3. The second unified model, consists in the turbofan model coupled with the NO<sub>x</sub> emission prevision made through the semi-empirical expressions derived by Rizk & Mongia [6].

For comparison purposes, the ICAO engine data bank will be used as source of the parametric inputs, such as overall pressure ratio, by-pass ratio and fuel composition, and also used to retrieve the the  $NO_x$  emission index to compare with the obtained results. The same set of engines selected in chapter 2.1, are also used now as reference.

Other conditions must also be taken into account so the values can be compared. For instance, as the reference values for  $\text{EI}_{NO_x}$  retrieved from the data bank refer to take-off conditions, therefore the same conditions have to be applied to the models.

Some design parameters are susceptible to variation when applied to the models, such as the conditions at flame front in the primary zone of the combustor, namely equivalence ratio,  $\Phi$ , and the residence time of the gases at combustion temperature,  $t_i$ . In the case of the model paired with Rizk & Mongia s expressions, the primary zone temperature, which is an explicit variable, is also susceptible to variation. Since they are strongly correlated with the proprietary designs of the combustors and are not expressed in the reviewed literature, values for this entities are estimated and explained as follows.

The values for the equivalence ratio at the primary zone are assumed to be stoichiometric to simulated the takeoff conditions. This tends to corroborate with the semi-empirical approach of the Rizk & Mongia model, yielding good results. However, the more theoretical approach of the model developed in this thesis lead to an overestimation of the NO emissions when presented with  $\Phi_{pz} = 1$ . Therefore, the values for equivalence ratio considered to the developed combustion model, range from  $\Phi_{pz} = 0.6$  to  $\Phi_{pz} = 0.8$ , in order to account for technological advancements made by the manufacturers to decrease  $NO_x$  emissions. The values for the residence time at flame front are also of great importance, therefor values of  $t_i = 0.8ms$  to  $t_i = 1ms$  are used as recommended by Odgers and Kretschmer's [7] and Flagan [5]. Lastly, the values for temperature variation in the primary zone range from  $\Delta T_{pz} = 1500k$ to  $\Delta T_{pz} = 1650K$ , this values are then used in  $T_{pz} = T_{i,c} + \Delta T_{pz}$  in order to compute the primary zone temperature.

# 4.1. Model Results and Discussion

The first analysis is made using the proposed combustion model paired with the two-spool turbofan model. As previously explained, the values of  $\Phi_{pz}$ and  $t_i$  are selected from the defined ranges. For the first set of engines the selected values are,  $\Phi_{pz} = 0.7$ and  $t_i = 1ms$ . The results are presented in figure 5. The marked line defines the equality of the obtained results with the extracted results from the emissions data bank.



Figure 5: Results comparison with data from ICAO emissions data bank.

By analyzing the computed results, it can be denoted a good correlation for the majority of the compared values. However, for high compression ratios the model tend to over estimate. This happens, due to the exponential relation between the rate of NO formation and combustion temperature, thus the high temperatures resulting from the high pressure compression ratios tend to be avoided by the manufacturers.

The second analysis focus upon the comparison of the Rizk & Mongia model paired with the turbofan model. As stated before, for this model is used the selected set of values,  $\Phi_{pz} = 1$ ,  $t_i = 1ms$  and a primary zone temperature variation,  $\Delta T_{pz} = 1625K$ .



Figure 6: Rizk & Mongia results comparison.

The Rizk & Mongia model also provide a good estimation over the wide range of engines compared. As in the case of the proposed model, this model also tends to over estimate in the high compression region, which are justified as it were before.

#### 5. Optimization

One of the goals of the present work is to achieve optimized design parameters in order to minimize  $NO_x$  emissions within the range of current technology levels. To do so, a large number of variables have to be computed to find the optimal solution. It is also important to account that a single-objective oriented approach is not enough for the purpose of early stage design of a gas turbine. It is then also important to minimize fuel consumption while minimizing  $NO_x$  emissions, turning this optimization problem even more complex.

To address such problem it was selected Genetic Algorithms as a tool for the optimization process. These algorithms are based on the natural selection process that mimics biological evolution, where when given a random selection of initial parameters (i.e. individuals) the algorithm converges to the optimal solution of the objective function. The convergence is tackled through a series of cross-over and mutation of the individuals that result in a new set of values, called a new generation, until a set of optimal parameters is achieved. Genetic Algorithms are also favorable for this type of optimization problems as they converge to a global maximum or minimum better than the more classical derivative approach, and can handle constrained or unconstrained problem as well as Multi-Objective problems like the one present in this work. It is then needed to establish the objective functions, or fitness functions, and the set of parameters that are going to be varied in order to proceed with the algorithm.

Although a  $NO_x$  emissions model has been developed in Chapter 3, it proved to be too computationally expensive to use for optimization purposes. Instead it will be used the expressions derived from the model formulated by Rizk & Mongia [6], that were revised in Chapter 1, and were paired with the turbofan model in chapter 4. However, the proposed model will be used later, providing a more comprehensive analysis of the effects of the optimized parameters in the combustion process.

The design parameters which are not part of the optimization are constant throughout this process and present the same values of those presented under 'baseline' in chapter 2.1.

#### 5.1. Single-Objective Optimization

To achieve a set of optimized parameters in order to minimize  $NO_x$  emissions a Single-Objective optimization is set as a first approach. It is defined the following objective function which will be subject to optimize:

$$EI_{NO_{\pi}total} = f(FPR, CPR, ti, \Phi)$$
(18)

After defining the constrain limits of the design parameters, the optimization proceed using Matlab<sup> $\mathbb{R}$ </sup> software [8], yielding the results presented as follows.



Figure 7: Single-Objective Optimization process.

As can be seen in figure 7 the algorithm has converged to an optimal fitness solution after about 50 generations. The optimal values of the parameters that were found by the algorithm are presented in the following table 2.

Table 2: Single-Objective Optimization results.

Parameters	Optimized values	
FPR	1.0032	
CPR	10.0016	
Residence time in ms $(t_i)$	0.1038	
Equivalence Ratio $(\Phi)$	0.7051	
EI <sub>NO<sub>m</sub>total</sub>	0.0034	

The results demonstrate that for an optimal solution on  $NO_x$  emissions only, the design parameters would have to be kept at minimum values. These results were expected since that the lower compression of the core air implicates a lower temperature at the inlet of the combustor and therefore a lower temperature of combustion, the short residence time implicates that the combustion gases do not stay too much time at high temperatures and a low equivalence ratio yield lower flame temperatures, thus resulting in low  $NO_x$  emissions.

## 5.2. Multi-Objective Optimization

As mentioned before, the sole objective of minimizing the  $NO_x$  emissions is not enough for design purposes. Instead a Multi-objective approach is the norm, while trying to also minimize fuel consumption. This approach requires to define a second objective function so a compromise can be made in the selection of the optimal design parameters. Also, with the addition of a new objective function there is an increase in the number of parameters to consider since fuel consumption is also function of B and TIT that were omitted in the previous optimization.

$$EI_{NO_{\tau}total} = f(FPR, CPR, ti, \Phi)$$
 (19a)

$$SFC = f(FPR, CPR, ti, \Phi, TIT, B)$$
 (19b)

The following results for the  $NO_x$  emissions and Specific Fuel Consumption were obtained using Matlab's Optimization Toolbox originating a Pareto front where a decision upon the trade-offs between the two objective function can be made.



Figure 8: Multi-Objective Optimization Pareto front.

From the obtained optimized values, one can select sets of values that correspond to minimal fuel consumption, minimal  $NO_x$  emissions and a compromise between both. The choice of the compromised optimization is tackled by the analysis of the pareto front, where the optimal couple of SFC and  $EI_{NO_x}$  should be chosen from the set of values closer to the origin of the plot, where each objective can be simultaneously presented closer to their minimum without a significant variation of the other. These sets of values are presented in the following table.

Table 3: Multi-Objective Optimization results.

Parameters	SFC	$\mathbf{EI}_{NO_x}$	Optimal
FPR	1.3238	1.0838	1.3880
CPR	48.6066	10.1085	15.0505
$t_i$ in $ms$	0.1126	0.1009	0.1074
$\Phi$	0.7056	0.7009	0.7021
TIT in $K$	1616.1	1622.2	1620.5
В	10.9235	10.7567	10.9183
SFC in $N/Nh$	0.2458	0.6869	0.3346
$\mathrm{EI}_{NO_x}$	6.8414	0.0125	0.1124

The results can be analyzed by reviewing the values in table 3. A minimum SFC is obtained by high compression ratios yielding high temperatures in the combustor, thus increasing thermal and combustion efficiency, but increasing  $NO_x$ . For the minimum  $NO_x$  emissions, on the other hand, is achieved by low compression ratio, thus yielding low temperatures in the combustor. Optimal values for SFC and  $NO_x$  can be achieved by compromising on the optimized parameters. The overall compression ratio is kept in the middle of the values for minimum SFC and  $EI_{NO_x}$ , although FPR increases favoring the thrust generated by the bypass flow while alleviating the compression made by the compressor stage, thus resulting in better SFC and  $NO_x$  emissions. Residence time and Equivalence ratio are kept at a minimum so that low values of  $NO_x$  emissions can be achieved, denoting the high sensitivity of  $NO_x$  formation to this parameters. The TIT is also kept at a minimum resulting in lower SFC. In concordance with the values for FPR, the values for the Bypass ratio are also kept at a maximum providing a higher percentage of air to go through the bypass flow, thus favoring the thrust generated from the fan and decreasing SFC.

5.3. Analysis of Optimized Engine with the proposed Combustion Model

The results obtained in the previous chapter, were derived from the a less computationally demanding Rizk & Mongia model. This provided a less time consuming optimization, although, it also provided a less extent analysis of the combustion process. For this matter, the proposed combustion model can now be used to provide the extra data which could not be extracted by the semi-empirical model.

Applying the optimized parameters from table 3 to the developed combustion model, yields the results that can be examined in the following table.

Table 4: Multi-Objective Optimization results applied to the proposed model.

SFC	$\mathbf{El}_{NO_x}$	Optimal
91967	94157	93810
89654	89873	89862
460.6	111.3	176.5
2890.3	563.2	977.2
4606.6	1668.5	2336.4
119.3	17.5	32.4
2609.2	6.8	55.5
0.2458	0.6869	0.3346
59.4007	0.1558	1.2645
	91967 89654 460.6 2890.3 4606.6 119.3 2609.2 0.2458 59.4007	$\begin{array}{c cccc} \mathbf{SFC} & \mathbf{EI}_{NO_x} \\ \hline \\ 91967 & 94157 \\ 89654 & 89873 \\ 460.6 & 111.3 \\ 2890.3 & 563.2 \\ 4606.6 & 1668.5 \\ 119.3 & 17.5 \\ 2609.2 & 6.8 \\ \hline \\ 0.2458 & 0.6869 \\ 59.4007 & 0.1558 \\ \end{array}$

Although it is beyond the scope of this thesis to analyze the formation of trace species other then  $NO_x$ , the equilibrium concentrations of the several radicals and other trace species are obtain as result of the calculations made in the model.

For instance, the equilibrium concentration of the carbon monoxide is given by the made calculations, while computing the combustion process. For the incomplete reaction, CO is formed by the dissociation of  $CO_2$  in the high temperatures at the flame. By resorting to the literature [5], it is verified that for the range of residence times and equivalence ratio used throughout the analysis, the equilibrium condition is achieved for CO.

Although this analysis do not provide the complete review of the CO formation process, it is important to retrieve the maximum information from the expressed results, including the contributions of the equilibrium CO in emissions. In practice, CO emissions arises from incomplete combustion and are found to be much higher than predicted from the equilibrium analysis. Nevertheless, analyzing the results in table 3 provide a good qualitative insight of the influence which a low  $NO_x$  focused design analysis have on CO equilibrium concentration. Thus, it can be noted that low specific fuel consumption oriented design optimization yield higher CO equilibrium concentrations than the low  $NO_x$ approach. This is justified by the high temperatures, achieved by the high compression imposed by the SFC optimized parameters, which promote the incomplete combustion effects. Analogously, and as noted in the results and discussion of the multiobjective optimization, the NO concentration also increases from the low  $NO_x$  optimized parameters to the low SFC optimized parameters. Of major importance, is to denoted that the formation of the  $NO_x$  increases by a factor of 400 when exchanging from low NOx setup to a low SFC setup, while CO equilibrium concentrations just increase by a factor of 5.

### 6. Conclusions

In the present thesis the objectives proposed in Chapter 1 were accomplished with success. A complete model of a two-spool turbofan was formulated and validated with good agreement with both commercial software results and real data retrieved from ICAO engine data bank. Furthermore a theoretical model was formulated for the primary zone and  $NO_x$  combustion mechanism, and validated for throughout the formulation process with very good results. The proposed turbofan model was then paired with both the proposed combustion model and semi-empirical model available from the literature, namely Rizk & Mongia model, yielding very good results when compared with data retrieved from ICAO emissions data bank. Because it shown more computationally demanding the he optimization process was carried out by the conjoint model of the turbofan and the Rizk & Mongia model instead of using the proposed combustion model. Nevertheless, the optimization process led to realistic results for the optimized performance parameters. The more computationally demanding combustion model was then used with the achieved optimized performance parameters, in order to compute the  $NO_x$  emissions. With a stronger theoretical background, the model provided additional information about the formation of trace species other than  $NO_r$ .

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