Understanding the Hip Joint: a Morphological Study of the Articular Surfaces in Asymptomatic and Pathologic Conditions

Sara Raquel Pais Monteiro Pires
sara.pires@tecnico.ulisboa.pt

Thesis to obtain the Master of Science Degree in Biomedical Engineering
Supervisors: Professor Miguel Tavares da Silva* and Daniel Simões Lopes, PhD**
*Instituto Superior Técnico, Universidade de Lisboa
**Visualization and Intelligent Multimodal Interfaces Group, INESC-ID
October 2016

Abstract — Understanding the morphological and functional features characterizing a normal hip joint is critical and necessary for a more comprehensive definition of pathological presentations, such as femoroacetabular impingement (FAI) and hip dysplasia, and improved designs of prosthetic devices. MacConaill introduced the notion, based on anatomical observations, that the articular surfaces of synovial joints are better represented by ovoidal shapes, in comparison with the spherical one, which is very well-established within the orthopaedic community.

This work aims at testing MacConaill’s hypothesis by using a surface fitting framework to assess the goodness-of-fit of a set of implicitly represented sphere-like shapes to the articular surfaces of the femoral head and the acetabular cavity, whose anatomical and geometrical raw data were obtained from computed tomography (CT) and magnetic resonance imaging (MRI) data sets. The framework involved image segmentation with active contour methods, mesh smoothing and decimation, and surface fitting to point clouds performed with genetic algorithms.

The study included two stages: (i) application of the surface fitting procedure on an asymptomatic population of 11 subjects, using 10 canonical shapes; and (ii) comparison of asymptomatic and pathological hips, by fitting a sphere, an ellipsoid, and a tapered ellipsoid to three populations (asymptomatic, FAI and dysplasia) of 20 subjects each. The statistical analysis of the surface fitting errors for both studies revealed the superior approximation of non-spherical shapes, namely ovoids and tapered ellipsoids, to both articular joint surfaces, especially in the pathological cases.

Keywords: surface fitting, implicit surfaces, hip joint morphology, (super)quadrics, femoroacetabular impingement, hip dysplasia

1. INTRODUCTION

Diseases affecting one’s ability to move, namely to walk, have a considerable impact on one’s quality of life. Addressing these conditions becomes increasingly important given the progressive ageing that today’s societies are experiencing. It is especially critical to focus on illnesses which manifest relatively early in life, such as morphological conditions of the hip joint, where femoroacetabular impingement (FAI) and dysplasia can be included. These morphological variations of the anatomy of the hip joint have been suggested to help describe the lesion mechanism of the articular cartilage and progress towards osteoarthritis (OA) [1–4], which in addition to being painful diminishes the patient’s ability to walk, and ultimately requires surgical treatment.

It is clear that a more comprehensive effort should be put in the full understanding and description of the morphology of synovial joints, with particular emphasis on the hip joint, given the inconsistency between clinical practice and medical and computational evidence regarding the shape of articular joint surfaces. It has long been suggested that the articular surfaces of a normal, asymptomatic hip joint are only symmetric in a limited number of axes, presenting an ovoidal shape instead of a spherical one [5,6]. Several authors have conducted studies on the morphological aspects of the articular joint surfaces, by approximating spheres, rotational conchoids, and ellipsoids to the anatomical data of the femoral head and acetabular cavity [7-16]. Various prosthetic designs have also been under the scope of investigation [17-19].

However, the idea introduced by MacConaill was that the hip joint, along with other spheroidal joints, are most well represented by ovoidal shapes, i.e., they present egg-like appearance and morphological features which account for global geometrical aspects of a surface, such as axial asymmetry and non-homogeneous. To date, the single study accounting for this assumption was carried out by Lopes et al. [20], highlighting the need for better models of the hip joint arises, along with new sets of parameters that allow for clear and unambiguous classification and identification of the femoral head and acetabular cavity, regardless of the form.

Therefore, the main objective of this work consisted in conducting a morphological study on the articular surfaces of the hip joint, i.e., femoral head and acetabular cavity, in order to investigate two complementary questions:

1. What is the shape model that best portrays the geometrical characteristics of the articular surfaces of a normal hip joint?
2. How well defined is the morphological difference between asymptomatic hips and the conditions known as femoroacetabular impingement and hip dysplasia?

To this purpose, two separate studies were carried out: a morphological study of asymptomatic hip joints, and a comparison study between asymptomatic and pathological hip joints. Computed tomography (CT) and magnetic resonance imaging (MRI) data sets of the hip joint, for asymptomatic and pathological cases, respectively, were used to extract the anatomical and geometrical information to which smooth and mainly convex canonical surfaces
were adjusted, using a least-squares minimization approach solved by a genetic algorithm. The shapes that were to be fitted belonged to the large family of (super)quadrics [21,22] and include an increasing degree of complexity to account for as many variations as possible within the set of subjects being considered in the study. This surface fitting procedure allowed the comparison between the shapes’ goodness-of-fit to the articualr surfaces, which was assessed according to the Euclidean distance of each point in the point clouds to the optimally fitted surface of each shape model. Error–of-fit statistical analyses were also performed to provide a better understanding of the results.

2. METHODOLOGY

2.1. Shape Models

The surfaces that have the best potential to represent the macroscopic features exhibited by the articualr surfaces of the hip joint should be limited and closed, topologically similar to a sphere, and present second-degree continuity for most of the surface range and convexity. These relatively simple geometric features make them particularly suitable for analytical modeling purposes where deduction of geometric attributes such as normal and tangent vectors, curvature, and principal directions are important [23].

In addition to the shape models which had already been addressed in previous studies, namely spheres, ellipsoids, and rotational conchoids [7,9-16,24], other quadric and superquadric surfaces were considered. Additionally to rotating an ellipsoidal shape around an axis, resulting in a rotational ellipsoidal shape, it is also possible to perform tapering deformations along the major axis of the elliptical cross-section [25]. The real-valued scalar function that implicitly represents the resulting surface in the canonical form is given by

\[ F_{SG}(x, y, z) = \left( \frac{x}{T_x z + 1} \right)^2 + \left( \frac{y}{T_y z + 1} \right)^2 + z^2 \]  

where \( T_x \) and \( T_y \) are the tapering values in the \( x \) and \( y \) directions, restricted between -1.0 and 1.0.

Spheres and ellipsoids can be generalized to their superquadric form, in which the fixed exponent seen is replaced by an arbitrary non-negative value equal or larger than 2. As the exponent increases, so does the squareness of the surface, giving it a more rectangular shape [26]. A specific case of superellipsoidal surfaces, was first introduced by Barr [21], which considers two distinct squareness parameters, instead of three. Its implicit representation in the canonical form can be seen in function (2).

\[ F_{SEB}(x, y, z) = \left[ x^{2/\varepsilon_1} + y^{2/\varepsilon_1} \right]^{\varepsilon_2} + z^{2/\varepsilon_2} \]  

where \( \varepsilon_1 \), \( \varepsilon_2 \) are the squareness parameters. These parameters are bound to the range \([0, 2]\), given the smoothness of the shapes being modeled in this work. The tapering deformation seen in function (1) can also be applied to function (2).

The highest level of generalization regarding the shape models here considered are (super)ovoids, specifically the ones introduced by Todd and Smart [22]. The superquadric form, implicitly represented in function (3) consists in applying the same exponentiation modification as in the case of superellipsoids.

\[ F_{SO}(x, y, z) = \left( \frac{x}{c_{0x} + c_{1x} z + c_{2x} z^2 + c_{3x} z^3} \right)^{2/\varepsilon_1} + \left( \frac{y}{c_{0y} + c_{1y} z + c_{2y} z^2 + c_{3y} z^3} \right)^{2/\varepsilon_1} + z^{2/\varepsilon_3} \]  

where \( \varepsilon_1 \), \( \varepsilon_2 \) and \( \varepsilon_3 \) are the squareness parameters, are bound to the range \([0, 1]\), since the exponents are real non-negative values for smooth convex shapes. \( c_{0x}, c_{1x}, c_{2x}, c_{3x}, c_{0y}, c_{1y}, c_{2y} \) and \( c_{3y} \) are the ovoidal shape coefficients. The zero- and first-degree coefficients \( c_{0x}, c_{1x}, c_{0y}, c_{1y} \) are restricted to the range \([0, 1]\), while the second and third degree coefficients \( c_{2x}, c_{3x}, c_{2y}, c_{3y} \) are limited to the interval \([-0.1, 0.1]\).

The effect of varying exponents for superellipsoids and superovoids can be seen in Figure 1.

\[ \varepsilon = 0.95 \hspace{1cm} \varepsilon = 0.70 \hspace{1cm} \varepsilon = 0.40 \]

Superellipsoids

Superovoids

Figure 1 - Canonical implicit surfaces of superellipsoids and superovoids with varying exponents.

The surfaces defined by expressions (1)-(3) are represented in their respective local systems, where the referential’s origin corresponds to the surfaces’ centre. For modeling purposes, however, it is important to guarantee the possibility of granting the surface any spatial configuration. This geometrical modification consists in applying affine transformations to the surface’s coordinate system, such as translation, rotation, and scaling, in order to convert local coordinates \( x \) into global coordinates \( x_g \). This transformation is expressed in equation (4):

\[ x = [x \hspace{1cm} y \hspace{1cm} z \hspace{1cm} 1]^T = \begin{bmatrix} R & D \end{bmatrix} \begin{bmatrix} 0 \hspace{1cm} 1 \end{bmatrix}^{-1} x_g \]  

where \( x \) and \( x_g \) are written in homogenous coordinates \( D \) is the scaling matrix, \( R \) is the rotation matrix, and \( t \) the translation vector corresponding to a position column vector that can be interpreted as a shift in the origin of the local reference frame. The scaling matrix \( D \) is a diagonal matrix containing the scaling parameters \( a, b \) and \( c \) along the local reference system axes \( x, y \) and \( z \).

The hierarchical connection between all the primitives is highlighted in Figure 2, which displays that through morphing operations applied to the sphere, such as rescaling, exponentiation, and asymmetrization, all the shapes considered in this work can be obtained. The orientation of the arrows is an indication of which shape models constitute generalizations and which are particular
cases within a given geometric primitive. For example, superovoids are a generalization of superellipsoids and ovoids, whereas ellipsoids are a particular case of both superellipsoids and tapered ellipsoids.

![Graph illustrative of the origin of each shape model.](image)

**Figure 2** - Graph illustrative of the origin of each shape model.

### 2.2. Image-based Anatomical Modeling

The geometric modeling pipeline used to study the spheroidal articular surfaces of the hip joint is described in Figure 3. It takes as input collections of both CT and MRI data sets, where the former correspond to asymptomatic hips and the latter to hips presenting FAI or dysplasia. The duality of the this work required two separate populations, from which consent to use their medical images was acquired. The first was composed by 11 subjects with ages between 21 and 39 yr (27.5±5.6 years, 5 males and 6 females) whose hips were scanned using CT imaging. Ten of these scans (512×512 acquisition matrix, in-plane x and y resolutions = 0.216–0.264 mm, slice thickness = 0.70–1.0 mm, and 241-357 slices) can be taken from the Musculoskeletal Research Laboratories at the University of Utah [28]. To these ten scans, the image set of one subject’s hip was added (512×512 acquisition matrix, in-plane resolution = 0.664×0.664 mm, slice thickness = 1.5 mm, and 356 slices), having the scanning been performed by a Philips MX 8000 IDT 16 (Philips Medical Systems, Eindhoven, The Netherlands). This image set can be found in OsiriX’s DICOM sample image sets website under the name of PELVIX [29].

The second comprised three sub-populations:

a) Asymptomatic: 20 individuals with ages between 18 and 45 yr (32.9±8.5 years, 9 males and 11 females). CT scans of the entire pelvis and both femurs (512×512 acquisition matrix, in-plane x and y resolutions = 0.602–0.869 mm, slice thickness = 1.5–2 mm, 262–929 slices) were taken using different CT scanning models by Siemens a Siemens Emotion 16 (Siemens Healthineers, Germany).

b) FAI: 20 subjects with ages between 21 and 53 yr (38.9±6.8 years, 13 males and 7 females). MRI scans of the pathological side of the hip (224–256) acquisition matrix, in-plane x and y resolutions = 0.703–0.804 mm, slice thickness = 0.7–0.8 mm, 96–128 slices) were taken using a T1-weighted fluid-attenuated sequence performed by a Siemens MAGNETOM® 3T Verio (Siemens Healthineers, Germany).

c) Dysplasia: 20 individuals with ages between 14 and 49 yr (34.0±9.8 years, 6 males and 14 females). MRI scans of the pathological side of the hip were taken using a T1-weighted fluid-attenuated sequence ((224–256) acquisition matrix, in-plane x and y resolutions = 0.703–0.804 mm, slice thickness = 0.7–0.8 mm, 96–128 slices) performed by a Siemens MAGNETOM® 3T Verio (manufactured by Siemens Healthineers, Germany).

Image data sets undergo a segmentation process to extract the articular surfaces of the hip joint, which consists in outlining the smooth cortical bone boundaries displaying a close homogenous curvature. To perform this task, a semiautomatic approach relying on active contour evolution was used, which the definition of a global threshold to the images so the algorithm can determine the probability of each belonging to the structure of interest [27].

From the segmentation task results a triangular 3-D surface mesh by the application of a marching cubes algorithm to the data corresponding to the acetabular cavity and the femoral head [29]. However, the generated model presents some undesired geometric features, such as a stair-like aspect and an extremely large number of vertices and edges that introduce redundancy and do not add value to the model. To address the first issue, a Laplacian filter was applied to the isosurface, maintaining its topology, and smoothing the vertices’ position relatively to their neighbours. The repositioning of a single node \( p_i \) from position \( \mathbf{x}_i \) to position \( \mathbf{x}_{i+1} \) respected the following expression:

\[
\mathbf{x}_{i+1} = \mathbf{x}_i + \lambda \sum_{j=1}^{n} (\mathbf{x}_j - \mathbf{x}_i)
\]

where \( \mathbf{x}_j \) are the positions of the \( n \) adjacent nodes \( p_j \) connected to \( p_i \) and \( \lambda \) is the parameter responsible for determining the level with which the smoothing operation is performed. \( \lambda \) is specified by the user and the higher its value, the stronger the smoothing effect [30]. A decimation filter was also applied to the 3-D model, in order to reduce the total number of triangles in the mesh [31]. The areas of the 3-D surface mesh corresponding to the anatomical relevant structures were identified and manually delimited, and the edges and faces were deleted until only the vertices remained, generating a point cloud containing the same high-resolution geometric measurements found in the 3-D surface model.

Given that articular surfaces are well represented by their global features, only a fraction of the point cloud was maintained and given as input to the surface fitting process. This downsampling procedure increases the point cloud’s homogeneity by uniformly selecting a set of points that is representative of the original point cloud for both the femoral head (1216.7±219.4) and the acetabular cavity (1195.9±21.1).

### 2.3. Surface Fitting

Approximating the object model that is the best representative of a point cloud requires model fitting algorithms capable of estimating the model parameters. Here, the purpose was to compute the parameters that
define the geometry of the surface from the unstructured point cloud, for each of the shape models and following
the hierarchy of non-linear surface evolution described in Figure 2. This process occurred by direct fitting of an
implicit surface to a point cloud and constituted a non-linear optimization problem with simple boundary
constraints based on the least-squares method (LSM), which minimizes the square sum of a predefined error-offit (EOF) [23].

For a point cloud with \( n \in \mathbb{N} \) points in Cartesian space belonging to the outer cortical bone surface of the hip
joint, the vector of geometric parameters \( \lambda \in \mathbb{R}^m \), where \( m \in \mathbb{N} \) is the number of parameters characterizing a given
implicit surface, which minimizes the EOF objective function, \( EOF(\lambda) \), was determined. This objective
function is defined as the square sum of residual function \( f \) for each point \( i = \{1, ..., n\} \), where \( f \) is the difference
between the shape model function and the \( i \)-th point datum, as depicted in expression (6):

\[
\min_{\lambda} EOF(\lambda) = \min_{\lambda} \sum_{i=1}^{n} f^2(x_g, y_g, z_g; \lambda)
= \min_{\lambda} \sum_{i=1}^{n} (1 - F_i(x_g, y_g, z_g; \lambda))^2
\]  

(6)

under the restriction

\[
1 \leq \lambda \leq u
\]  

(7)

where \( F \) is the implicit surface representation of the a
given shape model and \( \mathbf{l}, \mathbf{u} \in \mathbb{R}^m \) are the lower and upper
bound column vectors, respectively, setting the limits for the
solution presented in \( \lambda \). In addition to the parameters
needed to define each shape model, such as curvature and,
in the case of ovoids, asymmetry, vector \( \lambda \) also includes
the rotation and translation factors used in the affine
transformations. It is, therefore, a vector of global anatomical information. Table 1 summarizes the different
shape models used in the studies and the vector of geometric parameters associated to each of them. For the
shape models that include squareness parameters, the
exponents are represented as \( \gamma = \gamma_1, \gamma_2 \), meaning that \( \gamma_1, \gamma_2 \) and \( \gamma_3 \) are restricted to the range \([2, +\infty[/\].

The minimization problem was solved by a Genetic
Algorithm (GA), a metaheuristic method based on
concepts of natural selection, genetics and the biological
evolution process [32]. GAs use previous information to
direct the search into the region of better performance
within the search space, the same way that in nature,
genetic information is stored and perpetuated according to
its fitness to the conditions of the environment. This way,
the principle of survival of the fittest is used to produce
better approximations to an optimal solution.

The resulting approximations of the sphere and ellipsoid shapes constituted a good starting point in the search for
the best affine parameters, i.e., dimensions, relative
position and spatial orientation. For the remaining
geometric primitives, the optimal solutions obtained from the fitting process of the shape models hierarchically
linked to them were used as initial approximations. For instance, superellipsoid, tapered ellipsoids, and ovoid
fittings were initiated with resource to the optimal ellipsoidal parameters. In turn, the optimal parameters
found for ovoids and superellipsoids were used as the
initial approximation for superovoids.

The surface fitting optimizer took advantage of the
Genetic Algorithm and Direct Search Toolbox\textsuperscript{TM}, available
in MATLAB\textsuperscript{®}, and the code was run on an Intel\textsuperscript{®} Core\textsuperscript{TM}
i5 processor 2.4 GHz and 5 GB of RAM.

2.4. Surface Fitting Error Analyses

The comparison of goodness-of-fit between different primitives was performed based on the Euclidean distance.
It was measured between the fitted surface and each point
in the point cloud. This distance, however, was only equal
to the physical distance in the spherical case, which is the
reason why it is referred to as the pseudoEuclidean
distance instead. The minimum distance between each
point of the point cloud and the optimally fitted surface
used the signed Euclidean distance, \( \text{SED} \), and was
computed as:

\[
\min_{x_{OS}} SED(x_{OS}; x_{OP}) = \min_{x_{OS}} \| F(x_{OP}) - x_{OP} \|_2
\]  

(8)

and must respect the non-linear equality constraint

\[
F(\lambda^*) = 1
\]  

(9)

where \( x_{OS} \in \mathbb{R}^3 \) is the point belonging to the fitted
surface and \( x_{OP} \in \mathcal{P}C \) is the point from the point cloud
which can lie inside, outside or on top of the surface. \( x_{OS} \)
is in the neighbourhood of \( x_{OP} \), within a tolerance vector
\( e \), which takes the form \( e = e[1 \ 1 \ 1]^T \), with \( e \) much
smaller than the axial dimensions of the surface.

\[
x_{OP} - e \leq x_{OS} \leq x_{OP} + e
\]  

(10)

\( \text{sign}(\cdot) \) is the sign function, \( d_{PS} \in \mathbb{R}^3 \) represents the
distance vector between point \( P \) of the point cloud and the
iterated surface point \( S \). \( F \) is the implicit surface
representation for each of the geometric primitives, and \( \lambda^* \)
is the vector of geometric parameters characterizing the
optimally fitted surface.

The analysis of the surface fitting errors had two
components. The first constituted a qualitative analysis, where the approximation of both point cloud and
the respective fitted surface was visually inspected. The second component was of a quantitative nature and relied
on the surface fitting errors measured as the signed
Euclidean distances and the first-order statistics associated
with them. These included mean error, standard deviation,
minimum error and maximum error, and root mean square
(RMS) error, adjusted they were in terms of dispersion and
central tendency.

3. RESULTS AND DISCUSSION

3.1. Morphological Study of Asymptomatic Hip Joints

A. FEMORAL HEAD

The results from the surface fitting process are shown for
subject 11, in Figure 4. The reduced point cloud of subject
11’s femoral head is coloured as a function of
Figure 3 - Sequence of computational applications used for anatomical and geometric information extraction and modeling of spheroidal articular surfaces of the hip joint. White boxes represent the file formats used as input in the software tools referenced in the blue boxes. Examples of each step of the methodology pipeline are available on the right.

Table 1 - Vector of geometric parameters for all shape models considered and respective number of degrees of freedom, given by the total number of surface parameters, m.

<table>
<thead>
<tr>
<th>Shape model</th>
<th>λ</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>( \lambda_S = [a, t_1, t_2, t_3] )</td>
<td>4</td>
</tr>
<tr>
<td>E</td>
<td>( \lambda_E = [a, b, c, t_1, t_2, t_3, \phi, \theta, \psi] )</td>
<td>9</td>
</tr>
<tr>
<td>SE</td>
<td>( \lambda_{SE} = [a, b, c, y_1, y_2, y_3, t_1, t_2, t_3, \phi, \theta, \psi] )</td>
<td>12</td>
</tr>
<tr>
<td>O</td>
<td>( \lambda_O = [a, b, c, c_{12x}, c_{22x}, c_{32x}, c_{33x}, c_{23x}, c_{23y}, c_{23z}, t_1, t_2, t_3, \phi, \theta, \psi] )</td>
<td>17</td>
</tr>
<tr>
<td>SO</td>
<td>( \lambda_{SO} = [a, b, c, y_1, y_2, y_3, c_{12x}, c_{22x}, c_{32x}, c_{33x}, c_{23x}, c_{23y}, c_{23z}, t_1, t_2, t_3, \phi, \theta, \psi] )</td>
<td>20</td>
</tr>
<tr>
<td>RE</td>
<td>( \lambda_{RE} = [a, b, c, t_1, t_2, t_3, \phi, \theta, \psi] )</td>
<td>8</td>
</tr>
<tr>
<td>TE</td>
<td>( \lambda_{TE} = [a, b, c, y_1, y_2, t_1, t_2, t_3, \phi, \theta, \psi] )</td>
<td>11</td>
</tr>
<tr>
<td>SEB</td>
<td>( \lambda_{SEB} = [a, b, c, y_1, y_2, t_1, t_2, t_3, \phi, \theta, \psi] )</td>
<td>11</td>
</tr>
<tr>
<td>TSEB</td>
<td>( \lambda_{TSEB} = [a, b, c, y_1, y_2, t_1, t_2, t_3, \phi, \theta, \psi] )</td>
<td>13</td>
</tr>
<tr>
<td>RC</td>
<td>( \lambda_{RC} = [a, b, t_1, t_2, t_3, \phi, \theta, \psi] )</td>
<td>8</td>
</tr>
</tbody>
</table>
The geometric properties endowing asphericity and higher geometric modeling freedom to the shape models originating from to the sphere are not clearly pronounced in the set of surfaces represented below, even though the non-spherical primitives allow for a better fit to the femoral head point cloud, as demonstrated by the higher number of grayscale points in these surfaces' adjustments.

The quantification of the differences between all shape models and assessment of their goodness-of-fit was performed using first-order statistics for all geometric primitives and for the population as a whole, as depicted in Table 2. From there, it is possible to establish the following relationship between the diverse shape models, according to increasing goodness-of-fit:

\[ O < SO < TE < TSEB < SEB < SE < E < RE < RC < S \]

These relationships express a division between (super)ellipsoids and (super)ovoids. On the other hand, they demonstrate that the augment in geometric complexity provided by the squareness parameters in superquadric surfaces does not lead to cost-efficient analyses, given that superovoid and superellipsoid shapes presented worse fitting results than their quadric homologous, ovoids and tapered ellipsoids.

<table>
<thead>
<tr>
<th></th>
<th>S</th>
<th>RE</th>
<th>E</th>
<th>SE</th>
<th>SEB</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>μ</strong></td>
<td>0.643</td>
<td>0.565</td>
<td>0.544</td>
<td>0.526</td>
<td>0.484</td>
</tr>
<tr>
<td><strong>σ</strong></td>
<td>0.521</td>
<td>0.497</td>
<td>0.481</td>
<td>0.476</td>
<td>0.465</td>
</tr>
<tr>
<td>Min</td>
<td>-3.464</td>
<td>-2.715</td>
<td>-2.616</td>
<td>-2.671</td>
<td>-2.604</td>
</tr>
<tr>
<td>Max</td>
<td>2.376</td>
<td>3.464</td>
<td>3.464</td>
<td>3.463</td>
<td>3.463</td>
</tr>
<tr>
<td>RMS</td>
<td>0.827</td>
<td>0.752</td>
<td>0.726</td>
<td>0.710</td>
<td>0.671</td>
</tr>
</tbody>
</table>

Additionally, the significance of the statistical analyses was substantiated with a paired Student’s t-test with statistical significance set at \( p < 0.05 \). This showed that the fitting errors of the sphere were significantly different from all other shapes, with \( p \approx 0.000 \). The difference between fitting errors was also significantly different for the following pairs: O and E (\( p = 0.011 \)); RE and SE (\( p = 0.020 \)); RE and O (\( p = 0.003 \)); RC and O (\( p = 0.047 \)); RE and SO (\( p = 0.029 \)); TE and RE (\( p = 0.000 \)); SEB and RE (\( p = 0.000 \)); TSEB and RE (\( p = 0.000 \)); RC and TE (\( p = 0.004 \)); RC and SEB (\( p = 0.007 \)); and RC and TSEB (\( p = 0.005 \)). All the remaining pairs presented non-significant differences in surface fitting errors (\( p > 0.05 \)).

The surface parameters for each geometric primitive are described in Table 3. Mean surface dimensions are very similar, being the largest difference observable between sphere and superovoid. Ovoidal shapes were the largest of the set. Generally speaking, dimensions along the local x axis were slightly larger, suggesting some eccentricity in this direction, with exception of Barr’s tapered superellipsoid and the ovoid. The exponents values of both superellipsoid and superovoid did not differ greatly the ones seen in the quadratic surfaces from which they originate.

### B. ACETABULAR CAVITY

The qualitative assessment of the goodness-of-fit of each shape to the given point clouds showed that the optimally fitted surfaces resultant from the surface fitting algorithm approximate well to the downsampled point clouds given as input to the optimizer, as shown in Figure 5. Unlike the femoral cases, the differences between the geometric features of shape are more easily distinguishable when acetabular point clouds are adjusted. As the fitting proceeds to more non-spherical shapes, the approximation of the point clouds improves drastically, especially when we move closer to the acetabular rim.

Regarding the quantitative analysis of the differences between shapes and respective goodness-of-fit to acetabular point clouds, the same first-order statistical measures previously seen for the femoral case are displayed in Table 4.
The comparison between the RMS of the surface fitting errors for the 10 different shapes results in the following order of goodness-of-fit:

\[ TE = TSEB < SO < O < SEB < SE < E < RE < RC < S \]

When cross-checked with the comparison drawn for the femoral case, a pattern can be clearly identified: sphere and sphere-like surfaces do not adjust as well to the two articular surfaces in question as the remaining smooth and convex surfaces non-linearly related to the sphere. Also, the RMS values of the surface fitting errors of the femoral head were lower than the ones observed for the acetabular cavity, which emphasizes the notion that the femoral head is more spherical than the acetabulum.

Table 4 - Surface fitting errors statistical analysis of the acetabular cavity for each shape model and the whole population present in the study. All metrics are represented in millimeters (mm). (S - Sphere; RE - Rotational Ellipsoid; E - Ellipsoid; SE - Superellipsoid; SEB - Barr’s Superellipsoid; O - Ovoid; SO - Superovoid; TE - Tapered Ellipsoid; TSEB - Barr’s Tapered Superellipsoid; RC - Rotational Conchoid).

The lack of shape model match between the articular surfaces is frequently described in the orthopaedic com-
-munity as “incongruity” and it implies a difference in contact area between the two surfaces dependent on the applied stress/load on the joint. The existence of this incongruity generates space between the two articular surfaces, which is thought to be a way of distributing load and protecting the cartilage from undue stress while giving synovial fluid access for lubrication and nutrition of the joint [33].

The surface parameters for each geometric primitive are described in Table 6. The dimension parameters for all shape models are larger than the ones observed for the femoral head, which was expected given the space and cartilage found in between the two articular surfaces. The values corresponding to the exponents of both superellipsoid and superovoid are extremely close to the quadratic values, despite the maximum of 2.62 exhibited by superellipsoid for $\gamma_2$. However, both surfaces proposed by Barr (1981) obtained higher values for their exponents than the superquadrics mentioned before. Barr’s superellipsoid and tapered superellipsoid had a maximum of 3.00 for $\gamma_3$ and displayed mean values of 2.41 and 2.34 for the same parameter, respectively.

Similarly to what had been done in the femoral case, a paired Student’s t-test was used to classify the significance of the differences between fitting errors of all shape models. Statistical significance was once again set at $p < 0.05$. The pairs which demonstrated significant results are summarized in Table 5, where X corresponds to non-significant values.

Table 5 - Statistical significance of the differences between fitting errors for all shape models, using a paired Student’s t-test, with statistical significance set at $p < 0.05$. (S - Sphere; RE - Rotational Ellipsoid; E - Ellipsoid; SE - Superellipsoid; SEB - Barr’s Superellipsoid; O - Ovoid; SO - Superovoid; TE - Tapered Ellipsoid; TSEB - Barr’s Tapered Superellipsoid; RC - Rotational Conchoid).

Table 6 - Surface parameters for each geometric primitive for the acetabular cavity. Each value considers the 11 subjects in the study. Parameters $a, b, c, \gamma_1, \gamma_2$ and $\gamma_3$ are given in millimeters (mm), and rotations $\varphi, \theta$ and $\psi$ in radians. (S - Sphere; RE - Rotational Ellipsoid; E - Ellipsoid; SE - Superellipsoid; SEB - Barr’s Superellipsoid; O - Ovoid; SO - Superovoid; TE - Tapered Ellipsoid; TSEB - Barr’s Tapered Superellipsoid; RC - Rotational Conchoid).

3.2. Comparison between Asymptomatic and Pathological Hip Joints

A. FEMORAL HEAD

The goodness-of-fit of the estimated surfaces for subject 1 of each population considered in this study is displayed in Figure 6. Comparatively to what had been seen for the femoral case of the first study, the approximation of different geometric primitives to point clouds corresponding to femoral head data generates optimally fitted surfaces visually very close to spheres.

The quantitative component of the assessment of the femoral results underwent included the statistical analysis for the combination of all fitting errors for each condition, which is demonstrated in Table 7. The RMS values of the fitting errors for the asymptomatic population revealed to be slightly higher when compared to the previous study. This might be due to the larger size of the population considered here. The RMS values follow the same pattern as seen before, i.e., the sphere fits worse than its non-linearly linked surfaces. The FAL presenting population
obtained the lowest values of RMS of the surface fitting errors for both the sphere and tapered ellipsoid models.

**Table 7 - Surface fitting errors statistical analysis of the femoral head for each shape model and each population.**

<table>
<thead>
<tr>
<th>Surface fitting error for all 20 subjects</th>
<th>S</th>
<th>E</th>
<th>TE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asymptomatic</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>μ</td>
<td>0.653</td>
<td>0.591</td>
<td>0.538</td>
</tr>
<tr>
<td>σ</td>
<td>0.575</td>
<td>0.544</td>
<td>0.530</td>
</tr>
<tr>
<td>Min</td>
<td>-3.464</td>
<td>-2.857</td>
<td>-2.831</td>
</tr>
<tr>
<td>Max</td>
<td>1.592</td>
<td>3.464</td>
<td>3.464</td>
</tr>
<tr>
<td>RMS</td>
<td>0.870</td>
<td>0.803</td>
<td>0.748</td>
</tr>
<tr>
<td>FAI</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>μ</td>
<td>0.521</td>
<td>0.458</td>
<td>0.414</td>
</tr>
<tr>
<td>σ</td>
<td>0.476</td>
<td>0.445</td>
<td>0.418</td>
</tr>
<tr>
<td>Min</td>
<td>-3.464</td>
<td>-3.426</td>
<td>-2.874</td>
</tr>
<tr>
<td>Max</td>
<td>-2.7×10⁻⁵</td>
<td>2.787</td>
<td>2.892</td>
</tr>
<tr>
<td>RMS</td>
<td>0.706</td>
<td>0.639</td>
<td>0.589</td>
</tr>
<tr>
<td>Hip dysplasia</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>μ</td>
<td>0.578</td>
<td>0.459</td>
<td>0.449</td>
</tr>
<tr>
<td>σ</td>
<td>0.485</td>
<td>0.434</td>
<td>0.430</td>
</tr>
<tr>
<td>Min</td>
<td>-2.850</td>
<td>-2.806</td>
<td>-2.698</td>
</tr>
<tr>
<td>Max</td>
<td>2.500</td>
<td>2.776</td>
<td>2.621</td>
</tr>
<tr>
<td>RMS</td>
<td>0.755</td>
<td>0.632</td>
<td>0.622</td>
</tr>
</tbody>
</table>

Surface dimension parameters were very similar, not only between shapes but also between conditions, being the dimension parameter along the z axis of tapered ellipsoids the highest of the three. Tapering parameters were, on average, very close to zero, having the asymptomatic population presented both the minimum and maximum for Tz, -0.34 and 0.50, respectively.

The paired Student’s t-test used to corroborate the significance of the statistical analysis revealed that the differences between the surface fitting parameters of all pairs of shapes for all conditions were significant, with p-value<0.05: sphere and ellipsoid (p = 0.000 for all conditions); sphere and tapered ellipsoid (p = 0.000 for asymptomatic subjects, and p = 0.005 for subjects presenting FAI or dysplasia).

**B. ACETABULAR CAVITY**

The initial assessment of the general goodness-of-fit of the shape models to the acetabular point clouds corresponding to subject 1 of each condition is performed through visual inspection, being the optimally fitted surfaces for each geometric primitive exhibited in Figure 7.

**Table 8 - Surface fitting errors statistical analysis of the acetabular cavity for each shape model and each population.**

<table>
<thead>
<tr>
<th>Surface fitting error for all 20 subjects</th>
<th>S</th>
<th>E</th>
<th>TE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asymptomatic</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>μ</td>
<td>0.789</td>
<td>0.640</td>
<td>0.611</td>
</tr>
<tr>
<td>σ</td>
<td>0.552</td>
<td>0.508</td>
<td>0.491</td>
</tr>
<tr>
<td>Min</td>
<td>-3.464</td>
<td>-3.458</td>
<td>-3.449</td>
</tr>
<tr>
<td>Max</td>
<td>-3.0×10⁻⁵</td>
<td>2.789</td>
<td>2.943</td>
</tr>
<tr>
<td>RMS</td>
<td>0.963</td>
<td>0.817</td>
<td>0.784</td>
</tr>
<tr>
<td>FAI</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>μ</td>
<td>0.742</td>
<td>0.606</td>
<td>0.570</td>
</tr>
<tr>
<td>σ</td>
<td>0.524</td>
<td>0.479</td>
<td>0.467</td>
</tr>
<tr>
<td>Min</td>
<td>-3.240</td>
<td>-2.980</td>
<td>-2.726</td>
</tr>
<tr>
<td>Max</td>
<td>2.301</td>
<td>2.612</td>
<td>2.600</td>
</tr>
<tr>
<td>RMS</td>
<td>0.909</td>
<td>0.772</td>
<td>0.737</td>
</tr>
<tr>
<td>Hip dysplasia</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>μ</td>
<td>0.740</td>
<td>0.602</td>
<td>0.580</td>
</tr>
<tr>
<td>σ</td>
<td>0.536</td>
<td>0.488</td>
<td>0.485</td>
</tr>
<tr>
<td>Min</td>
<td>-3.432</td>
<td>-2.921</td>
<td>-3.001</td>
</tr>
<tr>
<td>Max</td>
<td>0.834</td>
<td>2.734</td>
<td>2.605</td>
</tr>
<tr>
<td>RMS</td>
<td>0.913</td>
<td>0.775</td>
<td>0.756</td>
</tr>
</tbody>
</table>

The analysis of Figure 7 shows that the point clouds representing the acetabular cavity are much better approximated by primitives with less spherical geometric features, being the tapered ellipsoid the one which seems to be most well-adjusted. As for the quantitative analysis of the surface fitting errors regarding the adjustment of the three shape models here considered to the acetabular point clouds, Table 8 presents first-order statistical measures of the surface fitting errors for the whole population of each condition.
A careful look into the RMS values shows that dysplastic hips revealed better approximation to all the geometric primitives than the two remaining populations. In the acetabular case, the population for which the lowest fitting errors were obtained was the FAI diagnosed one, which is compliant with the results already seen for the femoral case. Similarly, the shape that fitted the point clouds better for all sets of subjects was the tapered ellipsoid. The good approximation between the dysplastic population and a more egg-looking shape was expected since hip dysplasia is reported as having a pear-like appearance.

The order in which the goodness-of-fit of each shape improves for all three conditions and both articular joint surfaces is

\[ TE < E < S \]

and surface fitting errors are increasingly lower for populations in both the femoral head and acetabular cavity cases in the following order:

\[ FAI < Dysplasia < Asymptomatic \]

Regarding the statistical significance of the differences in surface fitting errors for the acetabular cavity, in the asymptomatic case, these were significant for the pairs S - E and E - TE, both with a p-value of 0.000. The pair E - TE presented a p = 0.609. For the population presenting FAI, the pairs which presented statistical significance were S - E and S - TE, with p = 0.000, and E - TE with p = 0.011. Finally, subjects with dysplastic hips revealed statistical significance between the surface fitting errors of two pairs of shape models, S - E and S - TE, with p = 0.000. The pair of shapes E - TE exhibited p = 0.123.

The shape parameters for all primitives and conditions presented a much higher level of variation between the surface dimensions than in the femoral case, with a maximum value of 4.54 mm for the surface dimension along the local z axis between the sphere and the tapered ellipsoid fitted to the dysplastic population. With the exception of the ellipsoid fitted to the asymptomatic point clouds, ellipsoids and tapered ellipsoids reveal some eccentricity along the aforementioned local axis, given the somewhat higher values observed for parameters c. Tapering parameters exhibited in this case were marginally superior to the ones obtained for the femoral case. Along the local x axis, the minimum tapering value was -0.50, while maximum values of 0.50 were obtained for both \( T_x \) and \( T_y \).

4. CONCLUSIONS AND FUTURE WORK

In this work, a surface fitting framework was applied to smooth surfaces with the intention of studying their ability to describe the articular surfaces of the hip joint, in both asymptomatic and pathological conditions, and explore the idea introduced by MacConaill’s that this surfaces present an ovoidal shape. After analyzing 71 subjects, it is fair to say that the surface fitting framework used here and based on the work previously carried out by Lopes et al. [20] is a helpful computer-aided orthopaedic surgery (CAOS) tool. Morphological variability and complexity associated with the raw data were taken into account by the use of a genetic algorithm to solve the non-linear surface fitting minimization problem. Furthermore, the process of three-dimensional reconstruction of the model from which the point clouds are extracted is applicable not only to computer tomography but also to any imaging technique which produces three-dimensional data, such as the magnetic resonance imaging modality.

Both morphological studies presented here revealed the adjustment of the spherical shape to the two articular surfaces to be the worst among the hierarchy of shape models, regardless of the clinical presentation of the joint. Ovoidal shapes, on the other hand, approximated well not only to the femoral head but also to the acetabular cavity, thus validating MacConaill’s assumption for synovial joint classification. Results regarding the geometric parameters of the best-approximated surfaces and the surface fitting errors were considered reliable, given that their order of magnitude was identical to results from other morphological studies of the hip joint.

This work allowed the comparison of two conditions, revealing that, in general, articular surfaces of hip joints presenting femoroacetabular impingement approximated better to the tapered ellipsoidal shape than dysplastic hips.

Moreover, this framework provides professionals within the orthopaedic community with a non-invasive tool to measure subject-specific parameters characterizing both the morphology and mechanical function of the joint, which is useful in the monitoring of joint anatomical malformations through time. Not only can this provide more insight into the development of these malformations and the link between them and the evolution towards osteoarthritis, but it can also help physicians decide on the best course of treatment.

In addition to improving the treatment of hip joint conditions, the acceptance of synovial joints as having an ovoidal shape can contribute to the design of enhanced artificial joints.

Morpho-functional simulations using ovoidal and tapered ellipsoidal shapes as representations of the femoral head and acetabular cavity, respectively, have not been performed to this point. Given that these were the two shapes which presented best fitting results for the articular surfaces of the hip joint, the next step in understanding their true adjustment to both normal and pathological anatomical features of the latter is the estimation of the contact forces and moments, and the articular pressure patterns that result from the application of loads and stresses onto these surfaces, using either multibody modeling or finite element modeling. The estimated forces, moments and pressure patterns must then be compared to experimental data representing true physiological values, so that the hypothesis that ovoidal prosthetic devices conduct to better mechanical outputs can be either validated or refuted.

AKNOWLEDGEMENTS

I acknowledge that I was financially supported by national funds through the Portuguese Foundation for Science and
NOMENCLATURE

\( a, b, c \) – Shape coefficients along the \( x_A, y_A, z_A \) directions
\( c_{12}, c_{21}, c_{22}, c_{32}, c_{33}, c_{42}, c_{43}, c_{52}, c_{53} \) – Ovoidal shape coefficients along the \( x, y, z \) directions
\( D \) – Scaling matrix
\( d_{PS} \) – Distance vector between points P and S
\( E/O \) – Error-of-fit objective function
\( e \) – Tolerance vector
\( F_{TE} \) – Inside-outside function of a tapered ellipsoid
\( F_{TS} \) – Inside-outside function of a Barr’s superellipsoid
\( f \) – Residual of error-fit objective function
\( i \) – Lower bound column vector
\( m \) – Number of geometric parameters
\( n \) – Size of the point cloud
\( R \) – Rotation matrix
\( SED(\cdot) \) – Signed Euclidean distance objective function
\( t \) – Translation vector
\( u \) – Upper bound column vector
\( x_P \) – Vector point in local coordinates
\( x_P, y_P, z_P \) – Coordinates described in the local reference system
\( x, y, z \) – Coordinates described in the local reference system
\( \lambda \) – Smoothness parameter
\( \lambda' \) – Optimal surface parameters vector
\( \mu \) – Mean value
\( \sigma \) – Standard deviation
\( \phi, \theta, \psi \) – Rotation angles along \( x, y \) and \( z \) axes
\( e_1, e_2, e_3 \) – Squareness parameters
\( f_{12}, f_{23}, f_{31} \) – Real non-negative exponents
\( \| I \|_2 \) – Euclidean norm or 2-norm

REFERENCES


