Modelling of deterioration processes in ship structures through dynamic Bayesian networks

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ABSTRACT: The present paper studies the application of dynamic Bayesian Networks for modelling degradation processes in ship structures. The theoretical background on temporal models in general and Dynamic Bayesian Networks (DBN) in particular is presented. A DBN tool consisting of a Matlab code for performing inference on models is developed. The tool is then applied to study the variation on the probabilistic model of the ultimate strength of a ship plate under compression subjected to corrosion degradation. A series of simulated empirical measurements is used to increase the accuracy on the model predictions. The results are validated against those obtained in previous studies by Monte Carlo simulation. Finally, several parametric studies are carried out to investigate how changes in the empirical measurements, in the frequency of inspections, in the inference algorithm and in the prediction method for corrosion degradation affect the posterior probability distribution of the ultimate strength of the plate.

1 INTRODUCTION

In modern industrial processes dependability assessment and management (reliability, availability and maintainability) is a key aspect to ensure safety and to optimize performance. However, a number of difficulties are faced when assessing the dependability of systems:
- **Size and complexity of the system**: Meaning the number of variables involved and the number of dependencies among them. Modern models try to include interaction between technical, human, organizational and environmental elements and account for different modes of failure. This can lead to really intricate models [1];
- **Integration of qualitative information and quantitative knowledge**: Human agents are often unable to provide reliable quantitative estimations and act according to them, so often the information provided is of a qualitative nature. However, inference can only be performed in a systematic way if dealing with quantities. Thus it is necessary to find ways to quantify all this information and integrate it in the model [2];
- **Multi-state and continuous variables**: When dealing with hybrid models (containing continuous variables) it is necessary either to discretize them (normally implying variables with a large number of possible states) or use some inference algorithm that allows accounting for continuous probability distributions ([3] and [4]);
- **Uncertainties in the parameters estimations**: Frequently reliable methods for obtaining information are not available. Thus it is necessary to account for uncertainty in the measurements and the prediction models when assessing reliability [5];
- **Temporal aspects**: In many cases it is necessary to consider the temporal dimension. One of the key factors that can endanger the survival of the structure is the presence of degradation processes, which are typically time dependant. The usual approach is to look into a particular moment of the structure service life, and deal with the problem as if it was a static one, which is a simplification of the reality.

Some of the classical reliability methods (like First and Second Order Reliability methods, see [6] and [7]) might prove to be insufficient to deal with accuracy with such kind problems. There are a number of reliability and dependability methods, usually implying graphic representation of the models, developed to cope with all these requirements. Examples of these are the fault and event trees [8], Petri nets ([9],[10],[11]) or Bayesian networks [12]. To deal with temporal aspects a number of methods have been used, being examples of this Markov chain Monte-Carlo simulations or DET/DFT (Dynamic Event Trees and Dynamic Fault trees), see [13], and more recently Dynamic Bayesian networks.

During the last years a growing interest in both Bayesian Networks and Dynamic Bayesian Networks (DBNs) as a modelling technique for uncertainty
analysis in different domains, and particularly in dependability studies, has been observed [14].

Dynamic Bayesian, or Belief, Networks (DBN) are a particular type of Bayesian network especially conceived to represent the relations between variables at a given time and those same variables in past or future times. They can also be seen as a generalization of hidden Markov models (HMMs) and Kalmar filters (KFM), where BN tools are used to take advantage of sparseness in the temporal model. They were developed in the early 90s initially to be used in Artificial Intelligence (AI) applications.

DBN have been used for assessing reliability, degradation, maintenance plans and risk assessment problems ([15],[16],[17],[18], [19],[20]) DBNs have also been applied for studying deteriorating processes, such as fatigue crack growth [21] and pitting corrosion in pipes [22].

In the field of ship and offshore floating structures, Bayesian updating has been used by Garbatov and Soares [23]. However, the model used in this paper is not implemented as a graphic network, nor are the typical DNB inference algorithms used.

The present work deals particularly with the problem of corrosion on marine structures. It covers both theoretical aspects of reliability analysis and temporal probabilistic models as well as one practical example. First the theoretical foundations of the DBN model are presented. Bayesian probabilistic reasoning over time, general inference in this kind of models and some of the most common existing methods to build a temporal model are discussed and a tool for performing inference on models is developed. The basic aspects of DBN construction and inference are presented. Afterwards a practical case study dealing with the ultimate strength analysis of a ship plate under compression subjected to corrosion degradation is developed. This covers the construction of the model and graphic representation through a DBN, the simulation of sets of empirical data and inference on the model. Parametric studied are carried out for different parameters and model assumptions and the results are compared with the ones obtained in a previous work through a FOSM (First Order Second Moment) analysis and Monte Carlo simulation [24]. A sensitivity analysis on the model variables is also carried out.

2 THEORETICAL ASPECTS OF DBN

2.1 Probabilistic reasoning over time

Quite often uncertainty modelling is done in the context of a static world, in which every random variable considered has a fixed value over time. Although, this does not reflect the real situations, for many cases the changes in the variables can be assumed to take place slow enough so that can be taken as constant for the time window considered. However, this cannot be assumed for most of the problems due to their highly dynamic aspects.

Although time is a continuous dimension, it is possible to look at time dependent problems as a series of stationary states, of snapshots, each one describing the problem at a particular time. At each time step there will be a set of random variables, that might be observable (evidence variables, $E_t$, being $e_t$ the set of observations at time $t$) or not (state variables, $X_t$, being $x_t$ the set for time $t$). Within the time span between snapshots, the variables are assumed to remain static.

To avoid unbounded computational complexity two assumptions are necessary: to consider the process to be both stationary and a Markov chain. The first implies that, thought the variables might change along time, the laws that govern the relation between them remain constant and thus it is only necessary to define a number of conditional probability tables equal to the number of variables in a single slice. The second implies that current states depend only on a finite number of previous slices, not the whole historical.

Particularly for the present work, a first-order Markov chain has been considered. Also the parents of evidence variables have been restricted to state variables at the present time slice. These two conditions can be expressed as:

\[ P(X_t | X_{0:t-1}) = P(X_t | X_{t-1}) \]  
\[ P(E_t | X_{0:t-1}, E_{0:t-1}) = P(E_t | X_t) \]

$P(X_t | X_{t-1})$ is a probability distribution describing how the state evolves along time and will be known from now on as the transition model. $P(E_t | X_t)$ describes the ability to obtain empirical evidence. It will be known from now on as the sensor model.

Finally, to completely define the joint distribution, it is necessary to specify a prior probability, $P(X_0)$, which accounts for the beliefs regarding the world before observations actually start. Taking all this into account, the joint probability distribution of all variables for a finite time, $t$, will be:

\[ P(X_{0:t}, E_{1:t}) = P(X_0) \cdot \prod_{i=1}^{t} P(X_i | X_{i-1}) P(E_i | X_i) \]  

2.2 Inference in temporal models

Once the structure of a generic temporal model is established, there are different inference tasks that can be performed, depending on which information is desired to be obtained from the model. Some typical operations include filtering (probability of a variable at the current state, given all the evidence collected), smoothing (probability at a past time slice, given all the evidence collected until present time), prediction (probability of a variable in a future time slice, using
evidence until present time), most likely explanation (to compute the most likely sequence of states that might have generated the evidence) and model learning (to use the DBN framework to adjust the parameters of the transition model). Only the first two have been considered for the present work.

Filtered distribution of a given state variable at a time \( t+1 \) can be calculated as:

\[
P(X_{t+1}|e_{1:t+1}) = \alpha P(e_{t+1}|X_{t+1}) \sum_{x_t} P(X_{t+1}|x_t)P(x_t|e_{1:t})
\]  

\[
(4)
\]

Within the summation the second term is obtained from the state filtered distribution at time \( t \). Thus, the filtering operation can be seen as a recursive call that unwraps along time in chronological order. This algorithm is known as \textit{forward} operation:

\[
f_{1:t+1} = aFORWARD(f_{1:t}, e_{t+1})
\]

\[
(5)
\]

The smoothed distribution is obtained by considering separately the evidence up to a given time in the past, \( k \), and the evidence from \( k \) to the current time, \( t \):

\[
P(X_k|e_{1:t}) = \alpha P(X_k|e_{1:k})P(e_{k+1:t}|X_k)
\]

\[
P(X_k|e_{1:t}) = af_{1:k}b_{k+1:t}
\]

\[
(6)
\]

\[
(7)
\]

\( b_{k+1:t} \) is known as the backwards operation, which is calculated also recursively, but starting by the present state and going to the past direction:

\[
b_{k+1:t} = \text{BACKWARD}(b_{k+2:t}, e_{k+2})
\]

\[
b_{k+1:t} = P(e_{k+1:t}|X_k) = \sum_{x_k} P(e_{k+1:t}|x_{k+1})P(x_{k+1}|P(x_{t+k}|X_k)
\]

\[
(8)
\]

\[
(9)
\]

2.3 Common temporal models

The general framework for temporal models that has been presented has been applied to develop different methods, some of which are:

- Hidden Markov Model (HMM): An HMM is a temporal model where there is one single discrete state variable at each time slice. It can deal also with problems with several state variables; however, they need to be joined in a single combined variable that contains tuples with all the possible values of the original variables. When dealing with problems containing a large number of variables, as in some problems related to structural reliability, this approach can become quite inconvenient.

- Kalman filter: Used when dealing with continuous variables and noisy observations over time, typically for tracking purposes. Kalman filter algorithm assumes the transition and sensor models as Gaussian distributions. Given these conditions, posterior state variable distributions will be Gaussian distributed as well. This is important because models with continuous or hybrid (continuous and discrete) variables tend to generate state distributions whose representation grows without bound as times goes on. There are few exceptions to this tendency and Gaussian distributions are one of them. This turns out to be a reasonable assumption in many occasions, since the error in observations is often normally distributed.

When the Gaussian assumption is no longer acceptable, it is possible to use an extended Kalman filter (EKF) to overcome the problem. In systems presenting high non-linearity in their transitions, a switching Kalman filter can be used.

- Dynamic Bayesian Networks: A DBN is a Bayesian Network where variables are ordered chronologically. It is possible to see HMM and Kalman filters as particular instances of a DBN. The great advantages of DBNs are that their flexibility to represent any kind of system, as well as the possibility to account for the sparseness in the temporal probability model, reducing computational time and space.

2.4 DBN construction

It is necessary to specify only three kinds of information to construct a DBN: a prior distribution over the state variables, a transition model and the sensor model. Once all these are set, it is possible to completely define graphically the whole network just by representing two time slices: one at time zero, before the evidence starts and another one containing the first piece of evidence.

General rules for regular BN manual construction can be used to define the prior distribution. First it is necessary to identify all the state variables involved and the causal relations among them. Nodes are added one by one, drawing directional links have from any node previously added which might have a dependency with the new variable. Although variables can be added in any order and will represent faithfully the system, doing it following the causality sequence tends to get simpler networks. According to convention nodes are drawn from top to bottom.

Once time zero slice is built, it is enough to replicate it to obtain the state variables at time one. This will be done by convention on the right side of slice zero. Next step is to establish the links between slices. This is done by considering which circumstances (variables) of the past do affect the present state of the problem. Finally, it is necessary to include the evidence nodes in the slice corresponding to time one. Typically, the evidences are taken as children of state variables. Figure 1 presents a simple example of a DBN graphic representation.
2.5 DBN inference

There are several ways of performing inference in Bayesian networks. The most straightforward is called enumeration and consists of summing out the terms of the joint distribution. The distribution of a query variable, \( X \), within a model with \( n \) non-query state variables, \( Y \), and a number of evidence variables, \( e \), can be written as:

\[
P(X|e) = a P(X,e) = a \sum_{Y} P(X,e,y)
\]

Developing the summation and factorizing:

\[
P(X|e) = a \sum_{y_1} P(y_1) \cdot \sum_{y_{n-1}} P(y_{n-1}|y_n) \cdot \ldots \cdot \sum_{y_1} P(X,e|y_1,y_2,\ldots,y_n)
\]

However, there are a number of algorithms that try to avoid repeating the same operations several times. The best known is the variable elimination algorithm, which has been used in the present work. Variable elimination takes equation 10 and starts calculating it by the right side, performing the summations and storing them as factors, so that they are not calculated again.

Computational complexity using the variable elimination algorithm depends strongly on the structure of the network itself. If the network conforms a polytree, time and space complexity are linear with the size of the network. When talking about the size, it is meant the total number of entries at the conditional probability tables. If the network is not a polytree (like Figure 1), complexity using variable elimination might vary and can get as bad as with regular enumeration. For a network of \( n \) variables with \( m \) possible states, it would the order of magnitude of \( O(m^n) \).

Inference in DBNs is performed by “unrolling” the network to a size where it can account for all the evidences up to date. A graphical representation of the unrolling can be seen in Figure 2.

Then inference can be performed as in a regular BN. When performing filtering, results for each time sliced are stored (as a factor, according to the variable elimination algorithm) and used to update the predictions in future time slices, according to equation 4. The procedure for smoothing is analog, but starting by the present slice and going backwards. By doing so, both filtering and smoothing can be performed with constant computational time and space for each slice.

As more complex DBNs are constructed, it might become unfeasible from a computational point of view to obtain exact results. There are a number of approximate inference algorithms that can be used to get around the problem. In the present work an algorithm from the particle-filtering family has been used.

According to this algorithm, a set of samples is generated for the slice zero, using state variables distributions for performing uniform sampling. Using the transition model, it is possible to obtain a new sample for slice one according to each sample on slice zero. A weight is assigned to each sample according to the evidence on slice one, and the weighted resampling with substitution is performed. This new set of samples is passed forward to the next slice using the transition model again, re-starting the loop. The final set of samples remains unweighted and can be taken as the joint probability distribution itself.

3 APPLICATION OF DBN TO STRENGTH ASSESSMENT OF CORRODED PLATES.

3.1 Case study description

A DBN has been used for assessing the ultimate strength of a corroded steel plate with random initial distortions and random material and geometrical properties. These dimensions and properties of the plate have been chosen in accordance with a previous paper by Teixeira et al. [25].

Semi-empirical design equations are used for predicting strength. More particularly, the formula proposed by Guedes Soares [26] has been selected. In this work residual stresses in the plate have not been considered, thus the plate strength is given by:

\[
\Phi_{\text{GS}} = \frac{\sigma_u}{\sigma_y} = \{1.08\Phi_{\text{F}}\}{1 - (0.626 - 0.121\lambda)\delta_0} \quad \text{for } \lambda > 1
\]

where \( \delta_0 \) stands for initial distortions, \( \sigma_y \) stands for yield stress and \( \Phi_{\text{F}} \) is the expression proposed by Faulkner [27] to predict the strength of a plate with average level of initial imperfections.

Seven unobservable random variables plus the evidence variable for measured corrosion are consid-
ered. Six of them are time invariant, describing the initial characteristics of the plate: plate breadth \( b \), thickness \( t_o \), initial distortions \( \delta_o \), yield stress \( \sigma_y \) and Young modulus \( E \). The seventh is the corrosion wastage \( w \), which varies with time. All the random variables are defined using continuous probability distributions and discretized afterwards. The discretization is done taking a domain and dividing it into intervals of equal width along the variable axis. The domain to be used is defined considering the area under the probability curve associated to that domain. For each variable an area of 0.999 has been considered and it has been selected so that the tails excluded have equal areas.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Distribution</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>n. of classes</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b )</td>
<td>mm</td>
<td>Normal</td>
<td>850</td>
<td>1.65</td>
<td>301</td>
</tr>
<tr>
<td>( \delta_o )</td>
<td>-</td>
<td>Lognormal</td>
<td>0.1</td>
<td>0.06</td>
<td>301</td>
</tr>
<tr>
<td>( t_o )</td>
<td>mm</td>
<td>Normal</td>
<td>18</td>
<td>0.22</td>
<td>301</td>
</tr>
<tr>
<td>( \sigma_y )</td>
<td>MPa</td>
<td>Lognormal</td>
<td>269</td>
<td>21.52</td>
<td>301</td>
</tr>
<tr>
<td>( E )</td>
<td>MPa</td>
<td>Lognormal</td>
<td>206,000</td>
<td>20,600</td>
<td>301</td>
</tr>
<tr>
<td>( w(t) )</td>
<td>mm</td>
<td>Weibull</td>
<td>Varying</td>
<td>Varying</td>
<td>301</td>
</tr>
</tbody>
</table>

In the DBN, all variables are parents of the ultimate strength and corrosion wastage is also a parent of measured corrosion. All variables, excluding the ultimate strength, transit to themselves from one slice to the other. The graphic representation of the DBN model constructed is presented in Figure 3.

3.2 Corrosion wastage: simulated data

The non-linear time dependent corrosion model proposed by Guedes Soares and Garbatov [28] has been used for all predictions regarding corrosion wastage. This model has been applied to model the time dependent corrosion wastage of bulk carriers ([25],[24],[29]) and of deck plates of tankers [30]. Particularly, for the present work, the following formula has been used, in which the model parameters, \( d_\infty = 1.07, \tau_\text{c} = 5, \tau_\text{e} = 1.64 \), were derived in[24],

\[
d(t) = d_\infty \cdot \left(1 - e^{-(t-\tau_\text{c})/\tau_\text{e}}\right) \quad t > \tau_\text{c}
\]

\[
d(t) = 0 \quad t < \tau_\text{c}
\]

Three different sets of experimental data are generated (see Figure 4). Measurements are generated one by one, in chronological order. Each measurement is obtained adding to the previous value the difference predicted by the time dependent corrosion model. Then a random deviation from the predicted value is added.

- **Similar values**: The curve follows the time dependent corrosion model (Eq. 13) with a \( \pm 10\% \) deviation on each time slice. Corrosion starts at some random point between the 5\(^{\text{th}}\) and 8\(^{\text{th}}\) year \( (\tau_\text{c}) \).
- **Lower values**: The curve follows the time dependent corrosion model (Eq. 13) with a deviation from -20\% to 0\% on each time slice. Corrosion starts at some random point between the 6\(^{\text{th}}\) and 9\(^{\text{th}}\) year.
- **Higher values**: The curve follows the time dependent corrosion model (Eq. 13) with a deviation from 0\% to +20\% on each time slice. Corrosion starts at some random point between the 4\(^{\text{th}}\) and 7\(^{\text{th}}\) year.

3.3 Corrosion wastage: inference

For time independent variables, the transition model states that the conditional probability tables remain the same at each time slice. However, this is not the case for the corrosion wastage. Therefore, it is necessary to define a transition function, a probability distribution for slice zero and a sensor model. Prior distributions at each time slice are calculated using the time dependent corrosion model (Eq. 13) as well. Corrosion is assumed to follow a Weibull dis-
tribution with mean value obtained from Eq.13, and coefficient of variation given by [24].

\[ \text{COV} = -0.0237 \cdot t + 1.1016 \]  \hfill (14)

Since corrosion is not linear, it is not possible to define a single transition function for all time slices. However it can be easily define for any time. The year where corrosion is firstly detected is stored and used as the \( \tau_{c*} \) parameter of Eq.13. The mean value for the following time step is obtained adding to the present value the increment of the function presented in Eq. 13. It is necessary to correct the difference between the theoretical starting year for corrosion and the observed one:

\[ w(t + 1) = w(t) + \Delta d(t - \tau_{c*} + 5 + 1) \]  \hfill (15)

It is necessary to define a prior corrosion distribution for time slice 0 (the year before corrosion is spotted for the first time). It is assumed that there is a probability of 0.9 that corrosion did actually start on the year it is first observed, thus the prior distribution would be a probability of 0 for any corrosion depth different from zero. However, there is a chance of having corrosion before it was detected. A probability of 0.1 is assigned to this possibility, assuming in that case that the prior distribution would be that corresponding to the first year after the protective coating disappeared according to the time dependent corrosion model (Eq. 13).

The sensor model used assumes that the only difference between the measured corrosion depth and the actual one comes from a measuring error. This is assumed to be normal distributed, with a mean of 0 and a standard deviation of 0.2 millimeters. Figure 5 shows how filtering and smoothing transform the prior corrosion distribution. Posterior distributions present a much lower dispersion of probability.

### 3.4 Results

Results obtained through the DBN model, without using empirical data, are now compared with those obtained in [58] using Monte Carlo Simulation. This is done by comparing the probability distributions of the intact plate ultimate strength, on year 10. In Figure 6 both distributions are plotted, together with a normal distribution fitted to the Monte Carlo simulation results.

The probabilistic characteristics of the ultimate strength obtained through the DBN and Monte Carlo simulation methods are presented in Table 2. These show good correspondence in mean value and distribution shape. The differences in standard deviation can be attributed to the use of a transition model in the DBN framework, in opposition to the Monte Carlo simulation, which is blind to previous system states.

<table>
<thead>
<tr>
<th></th>
<th>DBN</th>
<th>MCS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>222.6</td>
<td>222.1</td>
</tr>
<tr>
<td><strong>Standard Deviation</strong></td>
<td>21.5</td>
<td>17.3</td>
</tr>
<tr>
<td><strong>COV</strong></td>
<td>0.10</td>
<td>0.08</td>
</tr>
<tr>
<td><strong>Skewness</strong></td>
<td>0.17</td>
<td>-0.02</td>
</tr>
<tr>
<td><strong>Kurtosis</strong></td>
<td>3.02</td>
<td>3.28</td>
</tr>
</tbody>
</table>

The analysis of the mean values of the ultimate strength obtained through the DBN algorithms shows that they have an averaging effect between theoretical predictions and predictions made considering empirical data as deterministically true (see Figure 7). Given the homogeneity of empirical data and that corrosion wastage is time dependent, there are not significant differences between filtered and smoothed results.
Much more significantly, the COV is reduced greatly when empirical data is taken into account. Looking into the 5% percentile for the ultimate strength (see Figure 8), it can be seen that filtered and smoothed results present larger values. Figure 5 was obtained using a set of empirical data with larger levels of corrosion than predicted by the time dependent corrosion model (Eq. 13). Still, it would be possible to get more optimistic predictions for safety levels than neglecting the empirical data.

To further test the software developed, the algorithm is run using different sets of empirical data. As lower, similar and higher levels of corrosion than expected are considered, the DBN results react accordingly (see Figure 9).

Initially yearly inspections were considered for the simulated empirical data. However, it is also important to study how larger intervals of time between inspections would affect the accuracy of the results. Therefore, the initial results where compared to those obtained for 3 and 5 years between inspections. Although differences are noticeable, they are not significant, and the results converge to a same value as time increases. This can be observed both in Figure 10 (for the mean values and Figure 11 (for the 5% percentile).

However, it should be reminded that the sensor model chosen had great influence on the results obtained. If results from inspections are considered to be less reliable, larger differences can be seen when changing the time between inspections.

A particle-filtering approximate inference algorithm has been run as well. Although not necessary
for the present practical case study due to its simplicity, it is desired to test the capabilities of the algorithm by comparing the results with those obtained through exact inference. The number of samples was taken as 20 times larger than the number of categories assigned to each variable. With these conditions, the running time of the approximate inference algorithm is 54% lower than that of the exact inference algorithm. The results obtained are similar to those predicted by exact inference, as the differences never exceed 0.5% for the average ultimate strength (see Figure 12 and Figure 13). However, with such a short number of samples, the 5% percentile values can be misleading and produce outlier in the time series.

The dynamic Bayesian network can be very useful when the model used for predicting degradation is not trustworthy. The framework is capable of “correct” bad predictions using the data obtained from measurements. If measurements and predictions are somehow similar, then the algorithm will have an averaging effect. However, if the prediction model deviates considerably from the empirical data, the resulting posteriori distribution will tend to resemble the measured degradation. For results to be reliable in this case, it is necessary to perform enough empirical measurements with a method that can be considered as accurate.

To test this, calculations were performed considering a linear corrosion model, while keeping the non-linear simulated measurements. The linear model is built assuming that corrosion will progress linearly and that by the end of the time series (thirty years) it will present the same depth of wastage as predicted by the time dependent corrosion model (Eq. 13). On Figure 14 results obtained through the model are compared to those produced by the original non-linear one. It can be seen some divergence in the results for the first years after corrosion starts to take place. However, as times goes on results from both models converge.

3.5 Sensitivity analysis

A FOSM sensitivity analysis has been performed on the model. This would provide indications on possible improvements in efficiency by determining which variables are more relevant towards the final outcome. Based on sensitivity information, it is possible to make a more detailed discretization for those variables having a higher sensitivity index.

Sensitivity indices are obtained for every time slice, as they vary slightly from one year to another. Table 3 presents the range of the sensitivity indices accounting for the whole time series. It can be seen that the most important variables are Young modulus, initial distortions in the plate and the yield stress of the material. Although initial thickness and corrosion wastage do not seem to be as relevant as these three variables, it is still important enough to be included in the model. Plate width presents a very low sensitivity index, thus allowing for a much less detailed discretization of the variable, or even for neglecting it as a stochastic variable.
### 4 CONCLUSIONS & RECOMMENDATIONS

There is no doubt that DBNs are a powerful and flexible tool capable of dealing with very complex models and provide accurate results. When comparing DBNs to other methods applied to build temporal models, it is obvious that they offer the most versatile and probably the most efficient approach.

However, there are reasonable doubts about the capability of the method to perform exact inference using a practical amount of computational time and memory. Since computational complexity is conditioned by the network size and structure, it has to be considered on a case-by-case basis. If necessary, there are algorithms designed to perform approximate inference on dynamic Bayesian networks. It is interesting to note that most of these algorithms actually belong to the family of Monte Carlo methods, just adapted to the DBN framework. Thus, dynamic Bayesian networks could be seen as a tool to represent graphically complex problems and to find optimized ways to build the probabilistic models and perform inference on them.

Regarding the case study, the results obtained are similar to results obtained in previous studies using Monte Carlo simulation, and they respond as expected to changes in the model parameters. Also, the cases study has shown the utility of including empirical data into the reliability model, as it provides more optimistic predictions regarding the ultimate strength of the plates.

There is however room for possible improvements, by:

- **Defining a more realistic sensor model:** The sensor model used in the present work is rather simple, thus having a limited capability to represent corrosion inspections in a realistic way. The present model can account exclusively for tolerances in the measurements. However, neither particulars of the inspection method nor the possibility of measuring points with lower wastage than the average in plates with non-uniform corrosion, nor the chance of taking a completely erroneous measurement.

- **Accounting for variable environmental and operational conditions:** The present work does not take into account changes in environmental or operational conditions affecting corrosion. However, this does not represent reality faithfully. Although corrosion model adopted provides an average wastage over the whole plate life, a more accurate model should be able to account for its variability. Garbatov et al. [31] have discussed the possibility to adapt the time-dependent corrosion model to account for changes in the operational and environmental conditions of tanker ships. They have proposed that changes in the conditions affecting corrosion can be modelled through multiplicative factors modifying the corrosion model.

- **Maintenance modelling:** Maintenance aspects have been left out of the present work. However, a model dealing with real life problems has to account for maintaining actions carried out on the structures. This can be easily done within the DBN model.

- **Including other forms of degradation:** The most obvious example would be fatigue cracks, as the most relevant cause of ultimate strength loss as a marine structure ages. A comprehensive DBN model trying to assess overall structural reliability should necessarily include a crack growth model and integrate the results of inspections.

- **Yield stress empirical measurements:** Once the DBN model is built and the algorithm implemented, it is possible to take advantage of it by introducing new empirical data. One possible improvement is to include periodical measurements of the yield stress of the plate. Reducing uncertainty, it would be possible to make more optimistic predictions, especially considering the high sensitivity index obtained for the yield stress through the FOSM sensitivity analysis. This could be a way to compensate for the reduction of ultimate strength associated to corrosion wastage, which particularly important in life extension studies of ageing structural systems.

- **Computational optimization of the model:** Little attention has been placed to computational optimization of the code developed for the present work. However, more complex DBN models might require some computational optimization. Paying more attention to the discretization of the continuous variable could reduce significantly computational efforts while keeping accuracy.

### REFERENCES


