Optimization of reinforcements in plan panels with standard beams using discrete optimization methods for stiffness criteria

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Abstract

The purpose of this work is to develop a computational model to optimize the reinforcement of plan panels with standard beams, using, for this, discrete optimization methods, subject to different loadings and boundary conditions, in order to increase its stiffness. For this, it is necessary to develop a computational model that allows determining the optimal location of these reinforcements, as well as the type of beams to be used, satisfying constraints of volume/weight of the reinforcements. Before designing the program, it was necessary to define the formulation of the problem of optimization, including the objective function (the total elastic strain energy), design variables and constraints. It was used ANSYS, Inc. for finite element analysis, the GLODS (global and local optimization using direct search) as optimization algorithm, and MATLAB, The MathWorks, Inc., to create an interface between them. This model has been tested for several sets of reinforcement beams, and it determined the best solutions in a wide range of case studies, which arose from two main groups of configurations (configurations in schemes of 24 and 32 positions of beams to be introduced). The results concluded that the discrete optimization program of 32 positions were, in general, more effective. However, there are areas where it is advantageous to use the discrete optimization program of 24 positions.
Keywords: Finite Element Method, Optimization Algorithm, Totology Optimization, Structural Analysis, Plate Theory

1. Introdution

The purpose of this work is to develop a computational model to optimize the reinforcement of plan panels with standard beams through discrete optimization methods. It is intended to reinforce a flat panel with beams of standard dimensions, subject to different loads and boundary conditions, in order to increase its stiffness. For this, it is necessary to develop a computational model that allows determining the optimal location of these reinforcements, as well as the type of beams to be used, satisfying constraints of volume/weight of the reinforcement.

2. Motivation

The process of design and manufacture of reinforcements systems in structures have been developed over the last few centuries. In the complex construction of most of metal structures of vehicles, both in the aerospace and naval and in the automobile industry, the reinforcement of plates is a constant need. The use of reinforcements on plates aims to increase their stiffness and, thus, to obtain an improvement of their mechanical characteristics. This technique can significantly reduce the amount of material used and reduce the weight/power ratio of each vehicle in question. The optimization techniques are widespread, have a wide range of applicability in various fields and they are only limited by the imagination or creativity of the engineers who use them. The creation of the desired design, which depends on the skills of the engineers, can sometimes lead to incorrect results in synthesizing complex systems. In order to improve this critical phase of the project, systems of computational methods are being developed to optimize the topology of reinforcements on plates, using numerical and analytical methods for structural analysis.[2],[5]

3. Objectives

To achieve the goals, it is necessary to find the most appropriate formulation for this study. By using the finite element software (ANSYS, Inc.), a computational model is built to simulate the structural analysis. Through an iterative software (MATLAB, The MathWorks, Inc.), a computational model is created to perform the optimization. Finally, it is necessary to create an interface between the structural part and numerical calculation to obtain the desired control and optimization.

4. Optimization

Optimization is a peculiar concept in humanity, who, by instinct and at any time, makes strategic decisions in order to take full advantage of the available resources without compromising the effectiveness of the work performed. In basic terms, optimization is a mathematical discipline which concerns the discovery of extremes (minimum and maximum) in numbers, functions or systems. The resolution of problems of global optimization is a challenging task, with additional difficulties when derivatives are not available for use. However, there are a number of practical applications in the real world where a derivative-free global optimization is required. The work of Custódio and Madeira was the first attempt of global optimization by pattern search (or direct search): GLODS (or global and local optimization using direct search) [4]. The direct search is a family of numerical methods of optimization which does not require calculation of derivatives. Therefore, it can be used in functions that are not continuous or differentiable [6]. GLODS is a new algorithm developed for single optimization, suitable for limited constraints, global optimization, and it's free from derivatives. By using directional direct search, the method switches between search step, where potentially good areas are located, and poll step, where previously localized regions are explored. This exploration is done through the launch of several standard search methods (pattern search), one in each region of interest. Differently from a multistart strategy, the several methods of pattern search are going to fuse between each one when they are sufficiently close to each other. The goal of GLODS is to eliminate the largest number of active pattern searches, such as the number of local minima, which easily would allow the location of the possible value of the global extreme.

5. Structural optimization

The structural optimization is completely related to the improvement of its structural and mechanical characteristics, while minimizing the material consumption and the final cost of the project.

5.1. Types of problems of structural optimization

The structural optimization problems are classified depending on their geometric feature. They can be classified into three classes: dimensional optimization, geometrical optimization and topology optimization. Subsequently, it is presented briefly the most comprehensive optimization in structural projects.

5.1.1 Topological optimization

In this method, the design variables are numerical parameters that can change the distribution of material throughout the structure in order to economize material in regions with reduced application thereof. There are two types of design variables, continuous or discrete; however, much of this work was performed with discrete variables. In a discrete case, e.g., a truss with cross-sectional areas of the bars as design variables, one can allow these can go down to zero, which makes it possible to remove the truss bars which do not make no effort, as shown in Figure 1.

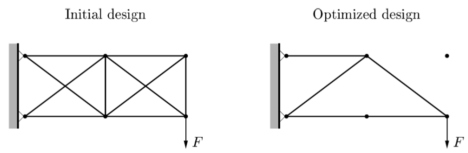


Figure 1: Topology optimization

6. Stages of the formulation of the optimization problem

The formulation of an optimization project aims the translation of the description of the problem in a well-defined math instruction. In most problems, it is used a procedure of tasks to be performed in each of the following stages [2]:

- 1st stage – objectives of the optimization problem;
- 2nd stage – data and information from the problem;
- 3rd stage – identification and definition of the design variables;
- 4th stage – identification of the objective function;
- 5th stage – identification of the constraints.

6.1. Objectives of the optimization problem

In this work, it is desired to reinforce a plan panel with beams, submitted to various loads and boundary conditions, in order to increase its stiffness. To maximize this stiffness it is used a computational model to determine the optimal location of these reinforcements, satisfying constraints of volume/weight of the reinforcement. To proceed with the analysis of the problem, a base model was created for the structure. This model consists of a square panel, shown in Figure 2.

As shown in Figure 2, the panel is 5 meters in side and its thickness is 2 centimeters. It was created in this form to obtain the greatest number of

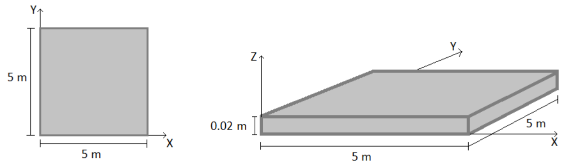


Figure 2: Basic panel/model of the structure

planes of symmetry of the structure, in particular the symmetry about the diagonals of the square. Therefore, one can take advantage of the symmetry in the creation of reinforcement when loads and constraints also symmetric are applied to this plan.

6.2. Data and information from the problem

A plate is a flat structure which has a thickness much smaller than the other dimensions. The plate can be referred to the mean surface which bisects the thickness at each point [7]. Many theories of plates have been developed since the late nineteenth century, but in engineering, two theories have been accepted and are widely used and they are the following:

- **Kirchhoff-Love**, used to determine stresses and strains in thin plates subject to applied loads and momenta.
- **Mindlin-Reissner**, an extension of the Kirchhoff-Love theory of plates and it is used to calculate strains and stresses on plates whose thickness is of the order of one tenth of the planar dimensions.

As the design developed in this work includes modeling a reinforcement in a thin plate, thereafter it is presented the most adequate theory for this analysis, which is in agreement with the plate element used, that is, the Kirchhoff-Love theory of plates.

6.3. Identification and definition of the design variables

Next step in the formulation process is to identify a set of design variables that describe the system.

This set of n design variables is commonly referred to as:

$$\mathbf{X} = (x_1, x_2, ..., x_n) \tag{1}$$

In this work, the design variables are the cross sections of the beams used to reinforce the panel. Square beam sections were created, as can be seen in Figure 3, and the design variables are, more precisely, the thickness of t section.

These beams of t thickness may have a thickness of "zero" and "ten" centimeters, that is, the beams to be placed as reinforcement in the panel can only

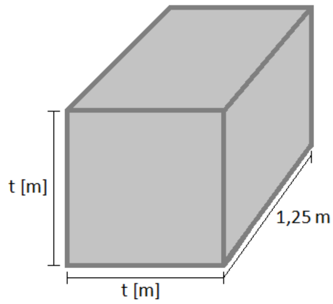


Figure 3: Shape of the beams/design variables.

be a measure of cross-section, which leads the optimization program to decide if it put or not the beam in the predetermined site.

6.4. Identification of the objective function

The criterion is usually called objective function, $f(X)$, and it needs to be maximized or minimized, depending on the requirements of the problem. To maximize the stiffness of a structure which is found in the elastic domain, the stiffness can be determined by minimizing the work done by the loads imposed on the structure when this is in its equilibrium state. This work is called ***compliance***, **C**, and, indirectly, the minimization of this measure allows minimizing the displacements caused by these loads. Therefore, one can consider the compliance, C, as an objective function, $F(X)$:

$$C = F(X) \tag{2}$$

The field displacements, u, associated with the equilibrium position of the structure can be obtained by the equation:

$$F = [K]u \tag{3}$$

where F is the vector of forces applied to the structure. With the vector of forces, F, and the field displacements, u, one can confirm the compliance, C, given by:

$$C = F^T u \tag{4}$$

On the other hand, the total elastic strain energy of the structure, U, is defined as:

$$U = \frac{1}{2} u^T [K] u \tag{5}$$

Combining equation (4), with equation (3), it's obtained the relationship between the compliance, C, with the total elastic strain energy of the structure, U, that is:

$$C = F^T u = u^T [K] u = 2U \tag{6}$$

where the compliance, C, is linearly related to the total elastic strain energy of the structure, U [3].

In a work with these features, both the compliance and elastic strain energy can be treated as the objective function. However, in carrying out this project, to optimize the reinforcement of the panel, the **total elastic strain energy of the structure**, **U**, was chosen as the objective function. Keeping this, it can be said that the equation for the objective function, $f(X)$, to be treated is given as:

$$f(X) = U \tag{7}$$

7. Computational model

The structural analysis is performed by the finite element software (ANSYS) and the optimization algorithm is the GLODS, which functions implemented in the MATLAB.

7.1. Construction of the panel model

In order to shape the panel according to the objectives of the optimization problem previously described in section 6.1, the panel was built in APDL code (designated ***codigo.txt***),so that it can access the data sent by MATLAB (*Constante.txt*)and the user can also change its dimensions if he wants. The dimensions are defined in the ***codigo.txt*** file, like width (**a**), length (**b**) and thickness (**espessura**).

The properties of the material used in the element are shown in Table 1.

Properties of the material	
Elastic modulus	290 GPa
Poisson's ratio	0.3

Table 1: Properties of the material used.

The mesh is created in order to generate quadrangular elements, and their refining was established in order to combine two important points, that is, to get a good solution in the finite element analysis (convergence of the solution), and, secondly, be a process of calculation not too heavy computationally.

7.1.1 Selection of the types of element

Analyzing the application of some types of elements, and the respective solutions envisaged, the type of elements chosen to be used for computational modeling of the panel and beam were, respectively, the *SHELL93* and *BEAM189* [1].

7.1.2 Defining panel reinforcements

For this study, it was designed two profiles of possible positions to put the beams, a profile of 24 and other of 32 beams, that were created so that the positions were evenly spaced and covering the entire panel area, as shown in Figure 4.

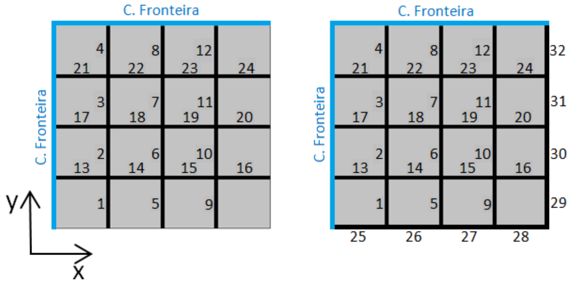


Figure 4: Scheme of 24 (left) and 32 variables (right).

Another interesting analysis for this study is to take advantage of the symmetry of the structure, in order to simplify the finite element program, by halving the number of design variables, as shown in Figure 5.

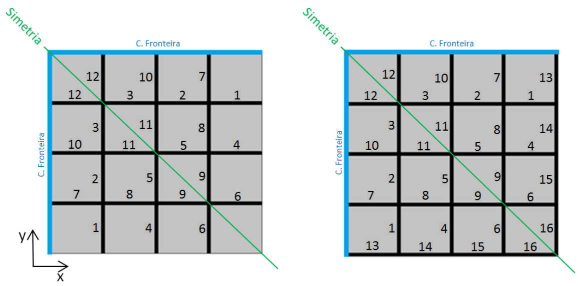


Figure 5: Scheme of 12 (left) and 16 variables (right).

7.1.3 Boundary conditions

In the computer program built in the ***codigo.txt*** file, it is created the boundary condition, which is based on having the panel simply supported on two consecutive sides, as shown in Figure 6.

7.1.4 Applied loads

Three different loadings were performed, the first (**F1**) in the central point of the panel, the second (**F2**) deflected downward and to the right, and the third (**F3**) on the edge of the panel, all applied to the symmetry axis of the structure, as one can see illustrated in Figure 7.

All these loadings carried out have the same applied force (**F=5000 N**), because when applied at different locations, the reinforcement obtained will also be different.

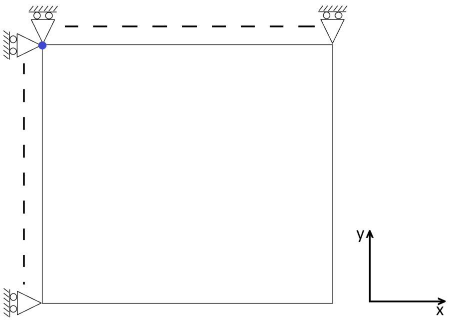


Figure 6: Boundary conditions of the panel.

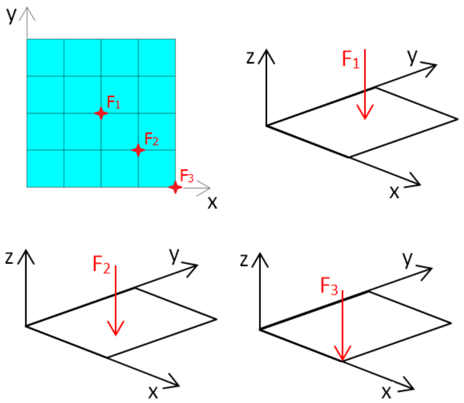


Figure 7: Boundary conditions of the panel.

8. Computational program

Throughout this work, it was necessary to divide the project into different study cases; however, all these converge to the same optimization method although they follow a generation of results, which individually are going to be adapting to their distinct limitations. For a better understanding of the various cases studied and the way they are organized in this work, it was used the construction of diagrams that are shown in Figures 8 to 9.

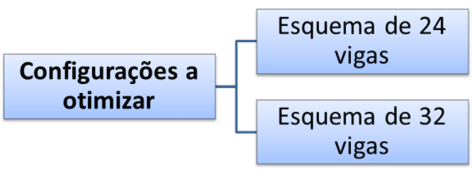


Figure 8: Diagram of schemes to analysis.

In the diagram of Figure 8 is shown the main concept of this work. It is divided into two subgroups of assessments; the first consists in the group of configurations that allows to optimize the reinforcement without placing beams on the edge of the panel (scheme of 24 beams), and the second is the group of configurations that allows beams in the end of the panel (scheme of 32 beams).

In Figure 9 is shown a continuation of the previous diagram, in which this diagram is representative of both schemes (24 and 32 beams).

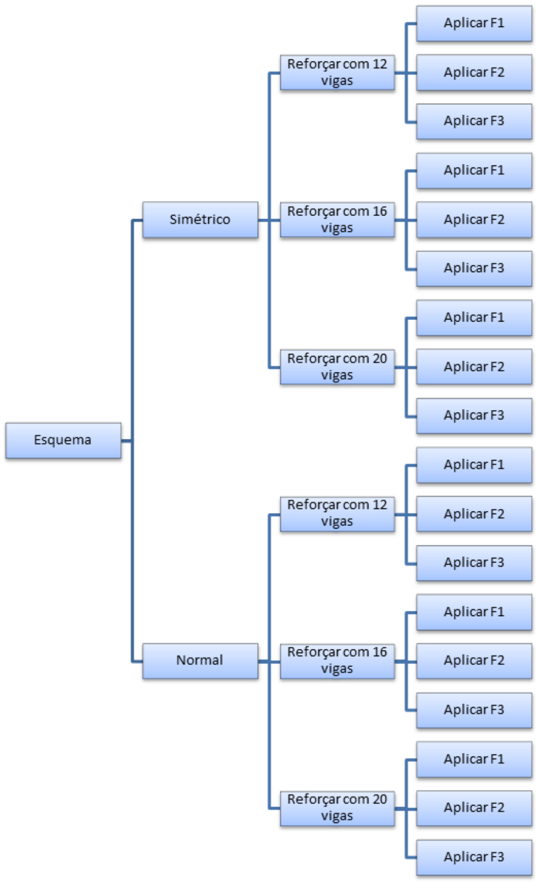


Figure 9: Representative diagram of the analysis performed on each scheme.

In this study, for each case presented, it was selected three quantities of beams to apply as reinforcement of the panel, ie, one can:

- apply 12 beams;
- apply 16 beams;
- or apply 20 beams.

These three enumerations were chosen in order to be able to implement them in any of the schemes submitted, and then to be able to compare the results obtained from the implemented schemes.

With these forces (F_1, F_2 e F_3), introduced in the plane of symmetry of each structure, it is obtained a design divided in 36 study cases where in each one was applied the same model of computational optimization.

8.1. Symmetrical schemes

For the treatment of the results of symmetric schemes, it was built a secondary program that aims to select the best and the worst set of solutions in each evaluated scheme.

In Figures 10 and 11, , it is shown, above each scheme, the amount of **elastic strain energy of the structure, U**, represented here by the variable **f** [N.m].], value that was used as objective function and that goes to follow the remaining results from here on out, thus serving as a comparison.

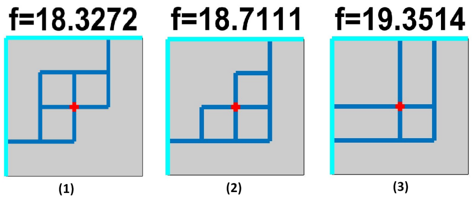


Figure 10: Representative schemes of the best evaluation.

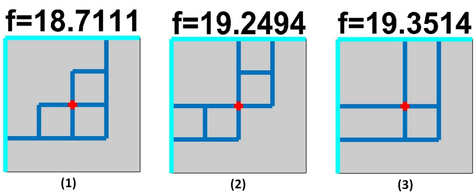


Figure 11: Representative schemes of the worst evaluation.

It is noted that, in these solutions that were found, there are equal schemes and with the same amount of elastic strain energy, **f**. This is due to the fact of the evaluations find common local minima among them, but this is not certain that all meet the global minimum, as happened in the same example. Therefore, it can be concluded that it is important to make various analysis to the same model in the study to make sure that one gets the best possible reinforcement configuration.

8.2. Normal (or non-symmetric) schemes

In this program, one does not get folders with the best and the worst groups of scheme solutions under evaluation, because the program is constructed to check if from the best configuration achieved in symmetric schemes it can improve to a non-symmetrical solution.

As an example, Figure 12 as shows the solutions obtained in a reinforcement constituted by 12 beams in the normal (not symmetrical) scheme of 24 positions and load applied to the center of the panel (F_1).

As one can see, the program found no better solution than that found in the program shown above, however, it was found two non-symmetric solutions that even being worse than the symmetric solutions,

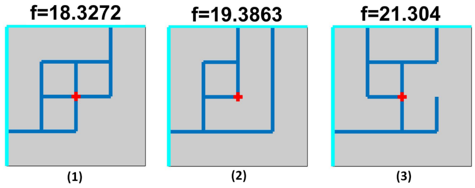


Figure 12: Representative solutions of normal (not symmetrical) schemes.

they possibly would give an acceptable reinforcement.

9. Analysis and discussion of results

9.1. Results of the schemes with 24 positions

Figure 13 illustrates succinctly the various cases implemented in 24-position schemes, where are presented the loads applied to the panel; however, these loads were used individually in relation to each other. They are performed together in order to remember the positions where they were placed, as highlighted above.

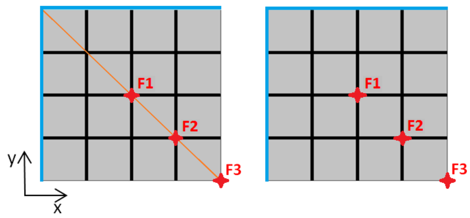


Figure 13: Symmetrical scheme and normal scheme of 24 positions, with loads represented.

For better visualization and comparison of results obtained in the schemes mentioned above, it was built the Table 2.

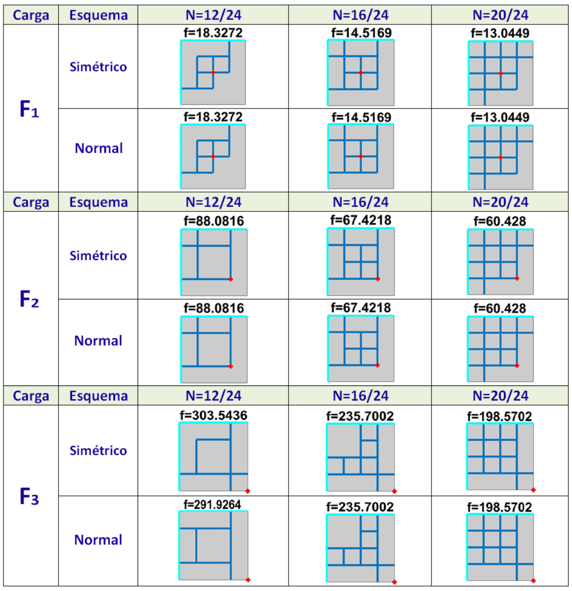
The **N** is the variable that corresponds to the number of beams to be introduced on each reinforcement, that is, $\mathbf{N = 12/24}$, means that one wants to reinforce the panel by introducing 12 beams in a total of 24 available positions.

9.2. Results of the schemes with 32 positions

As the presentation of previous results, here it was also presented a demonstrative scheme of the cases studied. The scheme is represented in Figure 14 and briefly it shows the schemes of 32 positions with three types of loadings applied to the panel.

For each of the cases illustrated in Figure 14,, the optimization of the objective function and the creation of the corresponding reinforcing structure were successfully obtained. For better presentation of them, they are also entered in Table 3as happened in previous cases.

Table 3 is organized in the same manner as the table exposed in the previous subchapter, so as to



Carga	Esquema	N=12/24	N=16/24	N=20/24
F_1	Simétrico	f=18.3272	f=14.5169	f=13.0449
	Normal	f=18.3272	f=14.5169	f=13.0449
Carga	**Esquema**	**N=12/24**	**N=16/24**	**N=20/24**
F_2	Simétrico	f=88.0816	f=67.4218	f=60.428
	Normal	f=88.0816	f=67.4218	f=60.428
Carga	**Esquema**	**N=12/24**	**N=16/24**	**N=20/24**
F_3	Simétrico	f=303.5436	f=235.7002	f=198.5702
	Normal	f=291.9264	f=235.7002	f=198.5702

Table 2: Configurations of the solutions of the schemes of 24 beams.

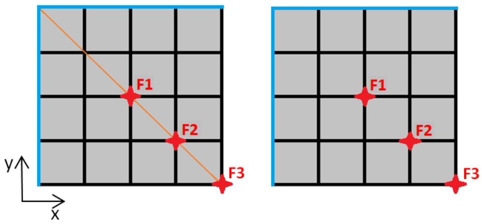
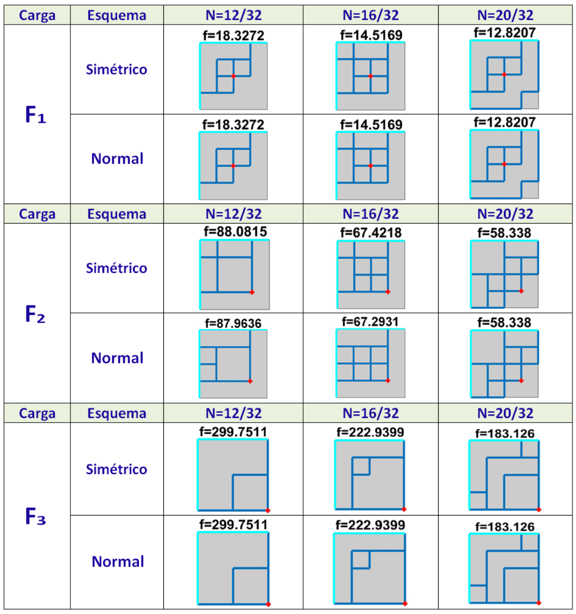


Figure 14: Symmetrical scheme and normal schedule of 32 positions, with loads represented.

be possible to see clearly all of the similarities and differences between them.

9.3. Comparison of results between the schemes of 24 and 32 positions

- For load F_1;
  - It is concluded that, between the two schemes, only there were differences in outcomes of optimization using 20 beams of reinforcement. This is because the load F_1 is applied to the center of the panel, which leads the program of 32 positions to use only to the positions of the edge of the panel when the center is already partly filled.
  - The improvement found in the schema of 32 positions is not significant. However it is a better solution, and presents a more dispersed shape of structure, which can benefit other structure characteristics here not studied, as vibration modes, etc.



Carga	Esquema	N=12/32	N=16/32	N=20/32
F_1	Simétrico	f=18.3272	f=14.5169	f=12.8207
	Normal	f=18.3272	f=14.5169	f=12.8207
Carga	**Esquema**	**N=12/32**	**N=16/32**	**N=20/32**
F_2	Simétrico	f=88.0815	f=67.4218	f=58.338
	Normal	f=87.9636	f=67.2931	f=58.338
Carga	**Esquema**	**N=12/32**	**N=16/32**	**N=20/32**
F_3	Simétrico	f=299.7511	f=222.9399	f=183.126
	Normal	f=299.7511	f=222.9399	f=183.126

Table 3: Configurations of the solutions of the schemes of 32 beams.

- For load F_2;
  - It was noticed a difference that occurred in the optimization which used 20 beams of reinforcement, where it was gotten an improvement with the implementation of positions on the panel edge. It was created an equally symmetrical structure but with a slight gain in the objective function.
  - Another difference occurred in the optimization in normal schemes of 12 and 16 beams, i.e., in the normal schemes of 24 positions, there were no changes compared to symmetric schemes, but in normal schemes, with 32 positions, it was achieved a slight improvement. This relates to the location of local minima obtained in the evaluation of the structure as well as with the number of design variables. Therefore, when evaluating the same analysis in both schemes (24 and 32 positions), in a single evaluation, the scheme of 32 variables improved, while that of 24 remained equal. To be able to achieve the same results in the schemes of 24 variables, one should repeat the computational evaluation few more times.
- For load F_3;
  - It was the only case in which all the solutions obtained in the schemes of 32 posi-

tions gave rise to different solutions compared to the schemes of 24 positions. It is easy to understand why this event. As already mentioned, this was due to the fact that the load is applied to a vertex of the panel, where schemas of 24 beams do not have access to the placement of these beams.

– In nearly all evaluations in schemes of 32 positions, there was improvement of the objective function in view of the analysis carried out in the schemes of 24 variables, except in the normal scheme of N=12/24 in which the solution obtained was not won. As in the previous example (load F_2), here also should make some more evaluations, and it could possibly be found the same solution in the normal scheme of 32 variables.

It is noted that all not symmetrical results presented here have exactly the same solution of objective function in the inverted configuration of reinforcement relative to the axis of symmetry of the panel. As example, in Figure 15 is shown the two solutions for N=12/24 in normal schemes using load F_3 applied.

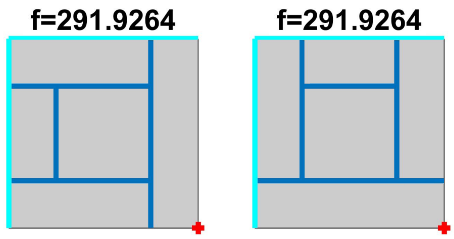


Figure 15: Solutions with the same objective function but with inverted configurations relative to the axis of symmetry of the panel.

10. Conclusions

The main conclusions are:

- Programs designed to 32-position schemas were generally more effective than those for 24-position schemes. However, the expended computation time was, in average, higher. However, for the applied loads F1 and F2, the program corresponding to the 24-position scheme has some advantage when compared to the 32-position scheme. The conclusion should be that, taking into account the processing time, in cases where the loads are applied in areas closest to the center of the panel, it is advisable to carry out the study with discrete optimization program of 24 positions.

- When evaluating the group of the best solutions obtained at each evaluation, it can be concluded that solutions that appear after the best solution have such good results such as the best, but sometimes with configurations of reinforcement visibly quite different, which can lead to better adaptation by these configurations to some projects where the panels will be applied. Another advantage in some of these solutions is the simplification of the reinforcementconstruction.

- Looking in a general way for all solutions obtained in the tables 2, 3, it can be concluded that only having as a starting point to maximize the stiffness and minimize the relation energy/weight of the structure, the best program to be used is the discrete optimization program of 32 positions.

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