Optimal Location and Tuning of Power System Stabilizers

Joana Margarida Ribeiro de Oliveira (contact: joana.oliveira@ist.utl.pt), Instituto Superior Técnico, Portugal
Supervisor: José Manuel Dias Ferreira de Jesus, Instituto Superior Técnico, Portugal

Abstract—A linearized state-space model for a multimachine power system is formulated. This model, which accommodates the dynamics of synchronous machines, excitation systems and turbine-governors, is implemented in MATLAB and is used for small-disturbance stability studies. The stability studies performed in MATLAB are based on the analysis of the eigenvalues obtained from the state-space system’s matrix and the small-disturbances come in the form of unitary power increments at a load bus. The presented model is then extended to incorporate the dynamics of speed-input power system stabilizers. This paper addresses as well the problem of the most efficient location and tuning for the power system stabilizer in the modelled power systems whose stability is limited by the presence of poorly damped local mode and inter-area mode oscillations. Two approaches for the optimal location of the stabilizer are presented. The tuning of the stabilizer gain is performed in time domain by emulating the gain margin field test and the tuning of the time constants is performed in frequency domain, recurring to the use of Bode plots. The methodology described in this paper was successfully applied to three power systems with the results from these tests included in the latter part of the paper.

Index Terms—Dynamic Analysis, MATLAB, Power System Stabilizer, Small Signal Stability.

I. INTRODUCTION

This paper deals with a MATLAB-based power system small signal analysis program referred to as MaSSA. MaSSA is used to analyse the small-disturbance stability of power networks. However, MaSSA was not prepared to accommodate for the dynamics of power system stabilizers (PSS) in these networks.

The expansion of MaSSA in order to incorporate the dynamics of PSS is here documented. Furthermore, criteria for the optimal PSS location and tuning are established. With continuous adjustments on the PSS location and parameters, ultimately the MaSSA user can conclude about the most efficient PSS for a certain network.

Historically, most of the attention of power system stability was focused on transient stability but with the growth of interconnections, the use of new technologies and controls, the increased operation in highly stressed conditions and the renewable energy integration, different forms of system instability have emerged. In particular, there has been a tendency of power systems to exhibit oscillatory instability, narrowing the stability limits.

Power System Stabilizers were developed as a complementary control system that offsets the reductions on stability margins by providing damping to the small magnitude low frequency injurious oscillations. Its application and design has been the subject of much attention in the past decades and is extremely relevant nowadays, as power systems frequently operate close to their stability limits.

II. LITERATURE REVIEW

Firstly, the reason that motivates the PSS installation - Rotor Angle Stability - is reviewed.

Then, because the PSS acts by means of the AVR, a review on linearized excitation control of synchronous machine is presented.

The Heffron-Phillips machine representation was an important tool in the methodology and so it is here detailed.

A. Rotor Angle Stability

When synchronous machines are interconnected, their stator voltages and currents must have the same frequency and the rotor mechanical speed of each machine is synchronized to this frequency. Hence, the rotors of all interconnected synchronous machines must be in synchronism [1].

Stability is a matter of equilibrium between torques. In steady-state, there is equilibrium between the input mechanical torque and the output electrical torque of each machine, and so speed remains constant. In case of a disturbance this equilibrium is upset, resulting in acceleration (or deceleration) of the rotors.

The system’s stability depends on whether or not these deviations in rotors angle result in sufficient restoring torques. The change in electrical torque of a synchronous machine following a perturbation, \( \Delta T_e \), can be decomposed in two components [1], as shown in equation 1.

\[
\Delta T_e = T_S \Delta \delta + T_D \Delta \omega
\]

(1)

Where:

- Synchronizing torque component, \( T_S \Delta \delta \): is the component of torque change that is in phase with rotor angle perturbation. Lack of sufficient synchronizing torque will result in aperiodic or non-oscillatory instability, which manifests itself through an aperiodic drift in rotor angle.
- Damping torque component, \( T_D \Delta \omega \): is the component of torque change that is in phase with the speed perturbation. Lack of sufficient damping torque will result in oscillatory instability, which manifests itself through rotor oscillations of increasing amplitude, the so called electromechanical oscillations.

The aperiodic instability problem has been largely eliminated by use of continuously acting modern AVR. Nowadays,
small-disturbance rotor angle stability problems are mainly due to insufficient damping torque, giving rise to electromechanical oscillations.

Electromechanical oscillations are classified according to the interactions between the power system’s components that originates them. Two categories of electromechanical oscillations are relevant in this paper:

- **Inter-area Modes**
  With a typical frequency range of 0.2 to 0.5 Hz, inter-area modes result when an aggregate of machines in one area is swinging relative to an aggregate of machines in another area.
  This complex phenomenon involves many parts of the system with highly non-linear dynamic behaviour. The damping characteristic of the inter-area mode is dictated by the tie line strength, the nature of the loads and the power-flow through the interconnection.
  The operation of the system in the presence of a lightly damped inter-area mode is very difficult.

- **Local Modes**
  With a typical frequency range of 0.5 to 1.8 Hz, local modes result when a single machine is swinging relative to the rest of the system.
  They are usually a consequence of remote machines connected to a large system through weak, essentially radial transmission lines and are more pronounced when high response excitation systems are used.
  These are the most commonly encountered modes of oscillation.

Power systems stabilizers are used to damp local mode and inter-area mode oscillations. In order to do so, it must compensate for the lack of damping torque that gives rise to these oscillations of concern.

**B. Excitation Control Effect on Machine Stability [2]**

The installation of Automatic Voltage Regulators (AVR) in generating units significantly contributes to improve monotonous stability at the cost of worsening oscillatory stability. Rephrasing, the AVR significantly contributes to improving synchronizing torque at the cost of worsening damping torque.

Furthermore, both the increase in synchronizing and the decrease in damping are proportional to the AVR gain $K_A$.

Thus, there is a conflicting problem: an AVR is a major help in providing synchronizing torque and curing that part of the stability problem. However, in doing so, it destroys the natural damping of the machine which is small to start with. Moreover, the AVR does not perform at its best potential because of the limitations imposed on $K_A$.

The solution is to provide extra damping through transient manipulation of the voltage reference of the AVR, $V_{ref}$, by means of an auxiliary stabilizing function. This stabilizing function is the PSS.

**C. Heffron-Phillips Representation**

The stability of synchronous machines under small perturbations is often studied recurring to the scenario of a single machine connected to an infinite bus through external impedance (SMIB), represented in Figure 1. The analysis of a SMIB can be extended to multimachine systems and is instrumental in understanding the requirements for stabilization of a perturbed machine, ergo the requirements for a PSS.

Figure 2 is a block diagram representation of a SMIB with a speed-based PSS installed, represented by the grey $G_{PSS}$ block. The constants $K_1$ to $K_6$ were firstly derived by Heffron and Phillips in [3] and so this SMIB representation is referred to as the Heffron-Phillips model.

The relations in the block diagram apply to a flux-decay machine with a field circuit in the d-axis without amortisseur effects and incorporates an AVR represented by the block $G_{AV/R}$. The coefficient $H$ is the machine’s inertia constant and $D$ is the coefficient that represents mechanical damping due to shaft motion. The physical nature and expressions of $K_1$ to $K_6$ are listed in equations 2 to 7.

- $K_1$: Influence of rotor angle on the electric torque with constant flux linkages in the direct axis.
  \[
  K_1 = \frac{\Delta T_e}{\Delta \delta} \bigg|_{E_q^0} \tag{2}
  \]

- $K_2$: Influence of q-axis electromotive force on the electric torque with constant rotor angle.
  \[
  K_2 = \frac{\Delta T_e}{\Delta E_q} \bigg|_{\delta} \tag{3}
  \]

- $K_3$: Factor between the d-axis synchronous reactance and the d-axis transient reactance. Equation 4 assumes null
external resistance $R_e$. Constant $K_3$ can be interpreted as the "machine’s gain".

$$K_3 = \frac{X'_d + X_e}{X_d + X_e}$$

(4)

- $K_4$: Influence of rotor angle on q-axis electromotive force, i.e. demagnetizing effect of a change in rotor angle.

$$K_4 = \frac{1}{\omega_s} \frac{\Delta E_{q}'}{\Delta \delta}$$

(5)

- $K_5$: Influence of rotor angle on terminal voltage with constant flux linkages in the direct axis.

$$K_5 = \frac{\Delta V}{\Delta \delta} \mid E_{q}'$$

(6)

- $K_6$: Influence of q-axis electromotive force on terminal voltage with constant rotor angle.

$$K_6 = \frac{\Delta V}{\Delta E_{q}'} \mid \delta$$

(7)

All quantities in the block diagram are normalized to pu\(^1\) except for rotor angle deviations $\Delta \delta$, which is in radians. The normalization of rotor angle velocity deviations $\Delta \omega$ is performed as in equations 8 and 9, where $\omega_s$ is called the synchronous speed.

$$\Delta \nu = \frac{\Delta \omega}{\omega_s}$$

(8)

$$\omega_s = 2\pi f_{\text{base}}$$

(9)

As mentioned, stability is a matter of equilibrium between torques. Equation 10 is called the the machine’s swing equation and relates speed changes with the torques applied on the shaft.

$$2H \Delta \nu = \Delta T_M - \Delta T_e$$

(10)

From Figure 2 it can be seen that the total electrical torque $\Delta T_e$ applied on the generator shaft results from three contributions:

- $\Delta T_{eD}^D$: The electrical torque contributed by the torque-speed loop. This speed loop represents the torque due to mechanical friction of the shaft.

- $\Delta T_{e\text{upper}}$: The electrical torque contributed by the upper torque-angle loop. This upper loop is known as the electromechanical oscillation loop because it represents the machine’s linearized rotor motion equation.

- $\Delta T_{e\text{lower}}$: The electrical torque contributed by the lower torque-angle loop. This lower loop represents the dynamics of the field winding of the machine as well as the dynamics of the AVR.

It is immediate to see that $\Delta T_{eD}^D$ is a purely damping torque because it has no component in phase with $\Delta \delta$. Likewise, $\Delta T_{e\text{upper}}$ is a purely synchronizing torque. However, $\Delta T_{e\text{lower}}$ is a function of the complex variable $s$ and so it will have both damping and synchronizing components.

The blue loop on the Heffron-Phillips representation in Figure 2 shows how, by installing a PSS, an input signal derived from rotor speed deviations $\Delta \omega$ is converted into a correcting stabilizing signal $\Delta V_e$ that is fed to the AVR, ultimately influencing the produced torque $\Delta T_{e\text{lower}}$ that is applied on the shaft.

It is convenient to conceive the Heffron-Phillips model of Figure 2 as represented in Figure 3, valid for a speed-based PSS.

In this representation, the blue loop is a torque-speed loop through which the PSS acts on the generator, the exciter and the power (GEP) system. The torque resulting from this loop is thus the electrical torque produced solely by the PSS via modulation of the AVR. It is therefore referred to as stabilizing torque $\Delta T_{e\text{stab}}$ and the loop is called the stabilizing loop. All other sources of electrical torque are represented by $\Delta T_{e\text{other}}$.

From Figure 3 it can be seen that the transfer functions of GEP and of a speed-based PSS are as in equations 11 and 12, respectively.

$$G_{\text{GEP}}(s) = \frac{\Delta T_{e\text{stab}}}{\Delta V_s}$$

(11)

$$G_{\text{PSS}}(s) = \frac{\Delta V_e}{\Delta \nu}$$

(12)

Finally, the contribution of torque due to the stabilizer path, i.e the torque contributed solely by the PSS is as in equation 13.

$$\Delta T_{e\text{stab}} = G_{\text{PSS}}(s)G_{\text{GEP}}(s)\Delta \nu$$

(13)

III. METHODOLOGY

In broad terms, the methodology involves modelling the power system and the PSS, establishing criteria for the optimal PSS location and establishing criteria for the optimal PSS tuning.

A. Multimachine Power System Modelling [4]

A power system may be modelled by the non-linear equations 14 and 15. In this model, $x$ represents the state variables vector, $y$ the algebraic variables vector and $u$ the control variables vector.

$$\dot{x} = f(x, y, u)$$

(14)

$$0 = g(x, y)$$

(15)

Equation 14 consists of the differential equations of the dynamic elements of the network such as machines and respective controls. Equation 15 consists of the algebraic stator equations and the algebraic network equations.

\(^1\)MaSSA adopts $f_{\text{base}} = 60$ Hz and $S_{\text{base}} = 100$ MVA
Linearizing around an equilibrium point yields equations 16 to 19 which constitute the so called Differential Algebraic Equation model (DAE).

\[
\begin{align*}
\Delta \dot{x} &= A_1 \Delta x + B_1 \Delta I_g + B_2 \Delta V_g + E_1 \Delta u, \\
0 &= C_1 \Delta x + D_1 \Delta I_g + D_2 \Delta V_g, \\
0 &= C_2 \Delta x + D_3 \Delta I_g + D_4 \Delta V_g + D_5 \Delta V_l, \\
0 &= D_6 \Delta V_g + D_7 \Delta V_l.
\end{align*}
\]  

Equation 16 represents the differential equations of the dynamic elements of the power system and equation 17 the algebraic equations of the stators. Equations 18 and 19 represent the network equations for the generator buses and for the load buses, respectively.

Consider a power system with \( n \) buses and \( m \) generators. Bus 1 is the slack generator bus, buses 2 to \( m \) are the generator buses (referred to as PV buses) and buses \( m + 1 \) to \( n \) are the load buses (referred to as PQ buses). Then, the algebraic variables vectors are arranged as shown in equations 20 to 22.

\[
\begin{align*}
\Delta I_g &= [\Delta I_{d_1} \Delta I_{q_1} \ldots \Delta I_{d_m} \Delta I_{q_m}]^T, \\
\Delta V_g &= [\Delta \theta_1 \Delta V_1 \ldots \Delta \theta_m \Delta V_m]^T, \\
\Delta V_l &= [\Delta \theta_{m+1} \Delta V_{m+1} \ldots \Delta \theta_n \Delta V_n]^T.
\end{align*}
\]

Because the machine currents are not of interest in voltage stability and rotor angle stability studies, \( \Delta I_g \) is eliminated and the DAE set of four equations is reduced to three equations. This procedure will modify the coefficients matrices and so the reduced DAE is as written in equation 23.

\[
\begin{bmatrix}
\Delta \dot{x} \\
\Delta y
\end{bmatrix} =
\begin{bmatrix}
A & B \\
C & D_{11} & D_{12} \\
& D_{21} & J_{LF}
\end{bmatrix}
\begin{bmatrix}
\Delta x \\
\Delta y_a \\
\Delta y_b
\end{bmatrix} +
\begin{bmatrix}
E_1 \\
0 \\
0
\end{bmatrix} \Delta u.
\]

Equation 23 is implemented in MaSSA. It uses normalized shaft speed \( \nu \) as input signal and it contributes to the DAE with three more state variables \( \Delta x_{p1}, \Delta x_{p2} \) and \( \Delta V_s \).

**B. Power System Stabilizer**

Figure 4 is a block diagram of the adopted PSS structure implemented in MaSSA. It uses normalized shaft speed \( \Delta \nu \) as input signal and it contributes to the DAE with three more state variables \( \Delta x_{p1}, \Delta x_{p2} \) and \( \Delta V_s \).

![Fig. 4. Adopted structure of the implemented PSS.](image)

1) **Performance Objectives:** "The basic function of a power system stabilizer is to extend stability limits by modulating generator excitation to provide damping to the oscillations of synchronous machine rotors relative to one another." [5]

Such oscillations correspond to the aforementioned electromechanical local modes of oscillation together with inter-area modes of oscillation.

Providing damping translates into increasing damping torque, i.e. increasing the component of electrical torque that is in phase with rotor speed deviations \( \Delta \omega \) (that is to say \( \Delta \nu \)).

Modulating generator excitation consists, in the PSS case, on superposing on the voltage error signal of the AVR an auxiliary and transient stabilizing signal. In the case of this paper, the stabilizing signal is \( \Delta V_s \), derived from the normalized rotor speed deviation \( \Delta \nu \).

Hence, the objective of the PSS is to contribute with a torque \( T_{e,stab} \) that provides damping to the system’s local mode and inter-area mode oscillations.

2) **Power System Stabilizer Modelling:** Equations 30 to 32 are the linearized equations that define the dynamics of a PSS with the structure of Figure 4. The component \( \Delta \omega \) depends on the chosen machine’s model in which the PSS is installed.

\[
\begin{align*}
\Delta V_s &= -\frac{1}{T_4} \Delta V_s + \frac{T_3}{T_4} \Delta x_{p2} + \frac{1}{T_4} \Delta x_{p2}, \\
\Delta x_{p2} &= -\frac{1}{T_2} \Delta x_{p2} + \frac{T_1}{T_2} \Delta x_{p1} + \frac{1}{T_2} \Delta x_{p1}, \\
\Delta x_{p1} &= \frac{K_{PSS}}{\omega_s} \Delta \omega - \frac{1}{T_W} \Delta x_{p1}.
\end{align*}
\]

3) **Integration in the DAE:** Because the PSS is characterized only by differential equations, the only equation of the DAE that is extended when including the PSS dynamics is equation 16. Since including a PSS in a network does not influence the static behaviour of the system, it is only natural that the algebraic equations of the DAE remain unchanged. Furthermore, the DAE matrix \( A_1 \) is modified to incorporate the stabilizing voltage signal \( \Delta V_s \) in the dynamics of the AVR.

Performing \( \Delta I_g \) elimination and then \( \Delta y_a \) and \( \Delta y_b \) elimination on the new extended DAE, the new system matrix \( A_{sys}^{PSS} \) is obtained and from it the new eigenvalues are computed.
C. Dynamic Analysis of the Power System Model

Figure 5 is a flowchart of the dynamic analysis performed in MaSSA each time the system is subjected to a small disturbance. The disturbances come in the form of increments of 1 MW at a chosen load bus.

![Flowchart of the dynamic analysis performed in MaSSA.](Image)

The system reaches oscillatory instability when a pair of complex eigenvalues of the state-space system matrix $A_{sys}$ cross to the right-half-complex-plane (RHCP). This phenomenon is called the Hopf Bifurcation.

D. Power System Stabilizer Location

Two indicators are used together to conclude on the optimal machine to install the PSS.

1) Power Transfer Capability: Recalling that the oscillations that the PSS is intended to damp are undesirable because they limit the system’s power transfer capability then the optimal machine to install a PSS will be the machine that improves the system’s power transfer the most.

A dynamic analysis of the system is performed for each possible location of the PSS and the machine that reaches instability for the heaviest load at the perturbed bus is the machine elected as the optimal site for the PSS.

2) Optimum PSS Location Index: The Optimum PSS Location Index (OPLI) is a technique adapted from [6] that indicates the optimal site for the PSS as the machine with highest OPLI, defined in equation 33.

$$OPLI = \left| \frac{G_{AVR}(\lambda') - G_{AVR}(\lambda^0)}{G_{PSS}(\lambda')} \right|$$  \hspace{1cm} (33)

The characters $\lambda^0$ and $\lambda'$ are respectively the critical swing mode before and after the inclusion of the PSS. The transfer functions $G_{AVR}$ and $G_{PSS}$ are defined in equations 34 and 35.

$$G_{AVR}(s) = \frac{\Delta E_{fd}(s)}{\Delta V_{s}(s)}$$  \hspace{1cm} (34)

$$G_{PSS}(s) = \frac{\Delta V_{s}(s)}{\Delta V_{ref}}$$  \hspace{1cm} (35)

To identify the critical swing modes a swing mode identification index termed as swing-loop participation ratio $\rho_i$ defined in equation 36 is introduced.

$$\rho_i = \frac{p_{\Delta \delta,i} + \sum p_{\Delta \omega,i}}{\sum p_{k,i}}$$  \hspace{1cm} (36)

The relative participation of a state variable $k$ in mode $i$ is called the participation factor $p_{k,i}$. Thus, for each mode $i$, $\rho_i$ is essentially a ratio of the mechanical state variables ($\Delta \delta$ and $\Delta \omega$) and the total state variables that participate in that mode. The modes that satisfy $\rho_i > 0.5$ are considered electromechanical swing modes, and from these, the one with the highest $\rho_i$ is considered to be the critical swing mode, i.e. it will be $\lambda_0$ (before PSS inclusion) or $\lambda'$ (after PSS inclusion).

E. Power System Stabilizer Tuning

The tuning of $K_{PSS}$ is performed in time domain and the tuning of the remainder parameters is performed in frequency domain.

1) Washout Filter Tuning: The purpose of the washout filter is to ensure that the PSS does not interfere with the normal operation of the system, that is, with the steady-state system.

This is accomplished with a high-pass filter with a time constant $T_W$ such that the filter is approximately unitary for the range of frequencies in which it is desired the PSS to perform. In general, this is satisfied by setting $T_W = 10$ sec.

2) Gain Tuning: The optimal PSS gain $K_{PSS}^{optimal}$ may be set based on the PSS gain that leads the system to instability $K_{PSS}^{instability}$.

For a speed input PSS it is of general agreement that the optimum PSS gain for a particular lead-lag setting is consistently about one-third of the PSS gain that leads the overall system to instability [7], as formulated in equation 37.

$$K_{PSS}^{optimal} = \frac{1}{3} K_{PSS}^{instability}$$  \hspace{1cm} (37)

A typical approach to determining $K_{PSS}^{instability}$ is with a field test commonly referred to as the gain margin test which consists on slowly and continuously increasing $K_{PSS}$ until an instability is observed. This instability is characterized by growing oscillations of the signals that are being monitored [7].

This procedure is emulated in MaSSA, which has a "Perform a Gain Margin Test" functionality in which the user can monitor the machine’s speed $\omega(t)$ for oscillations each time the user changes $K_{PSS}$.

3) Phase Compensation Stage Tuning: Remembering that the goal of the PSS is to produce a component of torque in phase with speed deviations $\Delta \omega$ (that is to say $\Delta \nu$), the tuning criterion for the time constants $T_1$, $T_2$, $T_3$ and $T_4$ is that they should make the phase characteristic of $\Delta \nu$ as close to zero as possible for the range of frequencies of concern.

This is achieved by adjusting the PSS transfer function $G_{PSS}(s) = \frac{\Delta V_s}{\Delta V_{ref}}$ in a way that it compensates for the phase-lag of the GEP transfer function $G_{GEP}(s) = \frac{\Delta V_{ref}}{\Delta V_{ref}}$. 

IV. Numerical Results

Three power systems whose stabilities are improved by PSS inclusion are studied.

A. 3-Machine 9-Bus System

In this network the generators are modelled as flux-decay machines regulated by fast, high gain static AVR. The system’s data was taken from [4] and is illustrated in Figure 6.

The disturbances are applied to the load at bus 5, which at nominal operation is defined by \( P_{L_5} = 1.25 \text{ pu} \).

1) Analysis Before PSS Inclusion: A dynamic analysis of the system is performed and the resulting eigenvalues, as well as respective frequencies and damping ratios\(^2\), for the nominal operation are presented in Table I. The system is unstable because mode \( \lambda_{12} \) is in the RHCP. This mode’s frequency hints that it should be a local mode of oscillation.

Figure 7 shows, for each machine, the time responses of the rotor’s angle and velocity when a step signal is imposed in the AVR reference of machine 2. The observed instability occurs in the form of increasing angular swings, leading to loss of synchronism and a progressive increase in angular separation between rotors. The rotors are continuously accelerated-decelerated and oscillate with an increasing amplitude: these are the undamped electromechanical oscillations.

\(^2\)Damping ratio: \( \xi = \frac{\text{Real}(\lambda)}{\sqrt{\text{Real}^2(\lambda) + \text{Imag}^2(\lambda)}} \)

![Fig. 6. 3-Machine 9-Bus system, reprinted from [6].](image)

![Fig. 7. \( \delta(t) \) and \( \omega(t) \) for a step in \( V_{ref2} \), with no PSS installed.](image)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Eigenvalue</th>
<th>Frequency [Hz]</th>
<th>Damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_{1,2} )</td>
<td>0.1718 ± j9.0447</td>
<td>1.440</td>
<td>-0.019</td>
</tr>
<tr>
<td>( \lambda_3 )</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \lambda_{4,5} )</td>
<td>-0.0460 ± j13.2499</td>
<td>2.109</td>
<td>0.003</td>
</tr>
<tr>
<td>( \lambda_{6,7} )</td>
<td>0.1975</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \lambda_{7,8} )</td>
<td>-2.7568 ± j7.9476</td>
<td>1.265</td>
<td>0.328</td>
</tr>
<tr>
<td>( \lambda_{9,10} )</td>
<td>-2.7066 ± j11.8356</td>
<td>1.884</td>
<td>0.228</td>
</tr>
<tr>
<td>( \lambda_{11,12} )</td>
<td>-2.8396 ± j8.2657</td>
<td>1.316</td>
<td>0.325</td>
</tr>
</tbody>
</table>

Table I

TABLE I
3-MACHINE 9-BUS SYSTEM EIGENVALUES BEFORE PSS INCLUSION.

Thus, the 3-Machine 9-Bus system is an adequate candidate for PSS installation because its stability margins are significantly reduced due to the presence of undamped electromechanical oscillations that limit the system’s power transfer.

2) Analysis After PSS Inclusion: A dynamic analysis of the system with a single-stage PSS with washout filter installed at machine 2 is performed and the resulting eigenvalues for the nominal operation are presented in Table II. The chosen PSS parameters are in Table III. There are no eigenvalues in the RHCP and thus the system is now stable.

Figures 7 shows, for each machine, the time responses of the rotor’s angle and velocity when a step signal is imposed in the AVR reference of machine 2. The observed instability occurs in the form of increasing angular swings, leading to loss of synchronism and a progressive increase in angular separation between rotors. The rotors are continuously accelerated-decelerated and oscillate with an increasing amplitude: these are the undamped electromechanical oscillations.

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Thus, the 3-Machine 9-Bus system is an adequate candidate for PSS installation because its stability margins are significantly reduced due to the presence of undamped electromechanical oscillations that limit the system’s power transfer.

Table II

<table>
<thead>
<tr>
<th>Mode</th>
<th>Eigenvalue</th>
<th>Frequency [Hz]</th>
<th>Damping</th>
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<tbody>
<tr>
<td>( \lambda_1 )</td>
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<td>-</td>
</tr>
<tr>
<td>( \lambda_2 )</td>
<td>-0.1109</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \lambda_3 )</td>
<td>-0.1790</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \lambda_{4,5} )</td>
<td>-0.8785 ± j12.6080</td>
<td>2.007</td>
<td>0.070</td>
</tr>
<tr>
<td>( \lambda_{6,7} )</td>
<td>-0.9392 ± j15.8188</td>
<td>2.518</td>
<td>0.059</td>
</tr>
<tr>
<td>( \lambda_{8,9} )</td>
<td>-1.3992 ± j6.3476</td>
<td>1.010</td>
<td>0.215</td>
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<tr>
<td>( \lambda_{10,11} )</td>
<td>-1.8944 ± j9.6291</td>
<td>1.533</td>
<td>0.193</td>
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<tr>
<td>( \lambda_{12,13} )</td>
<td>-2.7206 ± j7.8608</td>
<td>1.253</td>
<td>0.327</td>
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<tr>
<td>( \lambda_{14} )</td>
<td>-100.84</td>
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Table III

<table>
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<th>( KPSS )</th>
<th>( TW ) [sec]</th>
<th>( T1 ) [sec]</th>
<th>( T2 ) [sec]</th>
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<tbody>
<tr>
<td>10</td>
<td>10</td>
<td>0.5</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table III

CHOSEN PSS PARAMETERS FOR THE 3-MACHINE 9-BUS SYSTEM.
limits. According to the power transfer capability criterion, the optimal site for the PSS is at machine 2.

Table V shows that OPLI corroborates that the optimal site for PSS inclusion is at machine 2, that the second optimal site is at machine 3 and that the worst is at machine 1.

### TABLE V

<table>
<thead>
<tr>
<th>OPLI COMPUTED FOR THE 3-MACHINE 9-BUS SYSTEM.</th>
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<tbody>
<tr>
<td>PSS at Machine 1</td>
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<tr>
<td>OPLI</td>
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</table>

4) **PSS Tuning:** Table VI shows the frequency response of the washout filter transfer function $G_W(s)$. In steady-state operation $G_W(s)$ is null, not allowing the PSS to interfere with the normal, stable operation of the power system. When the oscillations of concern $\lambda_{1,2}$ and $\lambda_{4,5}$ arise, $G_W(s)$ becomes nearly unitary, activating the PSS.

Figure 9 shows the Bode plots of three transfer functions, obtained from MaSSA:

- the GEP system for machine 2 in red, $\frac{\Delta T_{e,\text{power}}}{\Delta V_{\text{ref},2}}(s)$;
- the PSS system in green, $\frac{\Delta V_s}{\Delta V_{\text{ref},2}}(s)$;
- the system resulting from the series of the two above in blue, $\frac{\Delta T_{\text{stab}}}{\Delta V_{\text{ref},2}}(s)$.

The torque contributed solely by action of the PSS, $\Delta T_{\text{stab}}$, should be in phase with speed deviations $\Delta \omega$. To achieve this, the phase characteristic of $\frac{\Delta T_{\text{stab}}}{\Delta V_{\text{ref},2}}(s)$ should be as close to zero as possible in the frequency range of the oscillations of concern.

Looking at the blue Bode, indeed the chosen $T_1$ and $T_2$ PSS values manage to keep the phase close to zero (more precisely, not exceeding $\pm 20^\circ$ from zero) from around $\omega = 9.4 \text{ rad/sec}$ to $\omega = 12.2 \text{ rad/sec}$.

A gain margin test at machine 2 is simulated with MaSSA. Figure 10 shows that the system is stable with $K_{PSS} = 40$ and

Figure 11 shows that the system is unstable with $K_{PSS} = 41$.

Considering $K_{PSS} = 41$ close enough to the $K_{\text{instability}}$ limit, the optimal gain is thus calculated as in equation 38.

$$K_{\text{optimal}} = \frac{1}{3} \approx 13.67$$  \hspace{1cm} (38)

The adopted $K_{PSS} = 10$ is close enough to $K_{\text{optimal}}$ to yield similar performance and is therefore considered to be equally satisfying.

### TABLE VI

<table>
<thead>
<tr>
<th>Washout filter behaviour for steady-state operation and for the oscillations of concern.</th>
</tr>
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<tbody>
<tr>
<td>$\lambda_{1,2}$</td>
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</tbody>
</table>

$G_W(\lambda)$
B. Two-Area System

Figure 12 represents the four-generator ten-bus two-area system, which is operating with area 1 exporting 153.28 MW to area 2. The areas, consisting of two coupled generating units, are linked by three tie lines. The generating units consist on a two-axis machine with no saturation model regulated by a fast, high gain static AVR. The system’s data was taken from [8].

The disturbances are applied to the load at bus 10, which at nominal operation is defined by $P_{L10} = 15.75 \text{ pu}$. 

1) Effect of installing a PSS: A dynamic analysis of the system is performed and the system becomes unstable at $P_{L10}^{\text{instability}} = 23.30 \text{ pu}$, which represents an improvement of 45%, compared to the system without PSS.

Table X shows the resulting eigenvalues at the nominal operation, after installing the PSS. The original oscillation of concern $\lambda_{3,4}$ corresponds now to the significantly more damped oscillating mode $\lambda_{5,6} = -0.1986 \pm j4.6711$ with a damping ratio of $\xi = 0.042$, which is circa ten times higher than the original damping ratio $\xi = 0.004$.

2) Tie Line Effect: Weakly connected power systems are particularly prone to inter-area oscillations. Hence, the number of tie lines connecting the two areas influences the systems stability. Table XI shows how $P_{L10}^{\text{instability}}$ changes with the number of tie lines, with and without the PSS.

A dynamic analysis is performed considering a two-stage PSS with washout filter installed at machine 3, with parameters shown in Table IX. The system becomes now unstable at $P_{L10}^{\text{instability}} = 16.26 \text{ pu}$, which represents an improvement of 45%, compared to the system without PSS.

Table X shows the resulting eigenvalues at the nominal operation, after installing the PSS. The original oscillation of concern $\lambda_{3,4}$ corresponds now to the significantly more damped oscillating mode $\lambda_{5,6} = -0.1986 \pm j4.6711$ with a damping ratio of $\xi = 0.042$, which is circa ten times higher than the original damping ratio $\xi = 0.004$.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Eigenvalue</th>
<th>Frequency [Hz]</th>
<th>Damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\lambda_{3,4}$</td>
<td>$-0.0162 \pm j4.6075$</td>
<td>0.7333</td>
<td>+0.004</td>
</tr>
<tr>
<td>$\lambda_{5,6}$</td>
<td>$-0.7765 \pm j7.4899$</td>
<td>1.1920</td>
<td>+0.103</td>
</tr>
<tr>
<td>$\lambda_{7,8}$</td>
<td>$-0.7848 \pm j6.8119$</td>
<td>1.0841</td>
<td>+0.114</td>
</tr>
<tr>
<td>$\lambda_9$</td>
<td>$-4.3111$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\lambda_{10,11}$</td>
<td>$-4.4273 \pm j17.3543$</td>
<td>2.7620</td>
<td>+0.247</td>
</tr>
<tr>
<td>$\lambda_{12,13}$</td>
<td>$-4.5311 \pm j0.0856$</td>
<td>0.0136</td>
<td>+1.000</td>
</tr>
<tr>
<td>$\lambda_{14}$</td>
<td>$-4.6827$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\lambda_{15,16}$</td>
<td>$-5.0989 \pm j11.6640$</td>
<td>1.8564</td>
<td>+0.401</td>
</tr>
<tr>
<td>$\lambda_{17,18}$</td>
<td>$-5.2733 \pm j7.3091$</td>
<td>1.1633</td>
<td>+0.585</td>
</tr>
<tr>
<td>$\lambda_{19,20}$</td>
<td>$-5.3341 \pm j6.9898$</td>
<td>1.1125</td>
<td>+0.607</td>
</tr>
</tbody>
</table>

The amount of exported power impacts the systems stability because heavy...
power transfers between areas contribute to the rise of inter-area oscillations.

Figure 13 shows, for different power transfer scenarios, the amount of incremented power at load bus 10 that leads the system (with a PSS installed at machine 3) to instability.

The lower the power transfer between areas, the heavier loading conditions the system can endure. This effect becomes even more evident with the loss of lines.

C. 10-Machine 39-Bus System

This system is commonly known as the New England Power System and its data was taken from [9]. The generating units consist on a two-axis machine with no saturation model.

The disturbances are applied to the load at bus 12, which at nominal operation is defined by \( P_{L_{12}} = 2.06 \text{ pu} \).

1) Effect of installing a PSS: A dynamic analysis of the system is performed and the system is unstable at the nominal operation, with \( P_{L_{12}}^{\text{instability}} = 2.06 \text{ pu} \). Since the power-flow only diverges at \( P_{L_{12}} = 18.24 \text{ pu} \) there is much potential for improvement by PSS inclusion.

The system is characterized by fifty eigenvalues, twenty of them corresponding to the ten swing modes presented in Table XII. Mode \( \lambda_{1,2} = +0.1057 \pm j6.7007 \) is in the RHCP, with negative damping ratio.

A dynamic analysis is performed considering a single-stage PSS with washout filter installed at machine 5, with parameters shown in Table XIII. The system becomes now unstable at \( P_{L_{12}}^{\text{instability}} = 13.04 \text{ pu} \). Table XIV shows the swing modes after the PSS is installed at machine 5, for the nominal operation case.

2) Effect of installing multiple PSS: For larger scale networks connecting several generating units it may be advantageous to include more than one PSS. Table XV shows how \( P_{L_{12}}^{\text{instability}} \) changes with the number of installed PSS. Without the aid of PSS, corresponding to the first row of Table XV, the system is unstable at the nominal operation. The second row shows that installing a PSS at machine 5 significantly extends the system’s stability limit. Up to five PSS, all of them identical and characterized by the parameters in Table XIII, are successively installed, each time increasing \( P_{L_{12}}^{\text{instability}} \). However, these successive increases of \( P_{L_{12}}^{\text{instability}} \) are not as substantial when compared with the first PSS at machine 5. The investment of multiple PSS may not be economically attractive.

V. Conclusions

The main conclusion withdrawn from the developed work is that installing a PSS is not a sufficient condition for improved system stability. In fact, a PSS is only as good as its tuning and location. A PSS installed in a machine that is not the main responsible for instability will have no significant effect

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**TABLE XII**

<table>
<thead>
<tr>
<th>Mode</th>
<th>Eigenvalue</th>
<th>Frequency [Hz]</th>
<th>Damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_{1,2} )</td>
<td>+0.1057 ± j6.7007</td>
<td>1.066</td>
<td>−0.016</td>
</tr>
<tr>
<td>( \lambda_{3,6} )</td>
<td>−0.0698 ± j4.2339</td>
<td>0.674</td>
<td>+0.016</td>
</tr>
<tr>
<td>( \lambda_{7,8} )</td>
<td>−0.1107 ± j6.0373</td>
<td>0.961</td>
<td>+0.018</td>
</tr>
<tr>
<td>( \lambda_{10,11} )</td>
<td>−0.1832 ± j8.1952</td>
<td>1.304</td>
<td>+0.022</td>
</tr>
<tr>
<td>( \lambda_{12,13} )</td>
<td>−0.2909 ± j7.2184</td>
<td>1.149</td>
<td>+0.040</td>
</tr>
<tr>
<td>( \lambda_{14,15} )</td>
<td>−0.2969 ± j6.7839</td>
<td>1.080</td>
<td>+0.044</td>
</tr>
<tr>
<td>( \lambda_{16,17} )</td>
<td>−0.3025 ± j8.4671</td>
<td>1.348</td>
<td>+0.036</td>
</tr>
<tr>
<td>( \lambda_{18,19} )</td>
<td>−0.3111 ± j8.8605</td>
<td>1.410</td>
<td>+0.035</td>
</tr>
<tr>
<td>( \lambda_{30,31} )</td>
<td>−1.1736 ± j7.8354</td>
<td>1.247</td>
<td>+0.148</td>
</tr>
<tr>
<td>( \lambda_{37,38} )</td>
<td>−5.4553 ± j1.0909</td>
<td>0.174</td>
<td>+0.981</td>
</tr>
</tbody>
</table>

---

**TABLE XIII**


<table>
<thead>
<tr>
<th>( K_{PSS} )</th>
<th>( T_W ) [sec]</th>
<th>( T_1 ) [sec]</th>
<th>( T_2 ) [sec]</th>
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<tbody>
<tr>
<td>20</td>
<td>10</td>
<td>0.001</td>
<td>0.010</td>
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</table>

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**TABLE XIV**


<table>
<thead>
<tr>
<th>Mode</th>
<th>Eigenvalue</th>
<th>Frequency [Hz]</th>
<th>Damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_{1,4} )</td>
<td>−0.0294 ± j8.5875</td>
<td>1.367</td>
<td>+0.003</td>
</tr>
<tr>
<td>( \lambda_{3,6} )</td>
<td>−0.0886 ± j7.2757</td>
<td>1.158</td>
<td>+0.012</td>
</tr>
<tr>
<td>( \lambda_{7,9} )</td>
<td>−0.1173 ± j4.2706</td>
<td>0.680</td>
<td>+0.027</td>
</tr>
<tr>
<td>( \lambda_{10,11} )</td>
<td>−0.1220 ± j6.0491</td>
<td>0.963</td>
<td>+0.020</td>
</tr>
<tr>
<td>( \lambda_{13,14} )</td>
<td>−0.2809 ± j6.7833</td>
<td>1.080</td>
<td>+0.041</td>
</tr>
<tr>
<td>( \lambda_{15,16} )</td>
<td>−0.2937 ± j7.2147</td>
<td>1.148</td>
<td>+0.041</td>
</tr>
<tr>
<td>( \lambda_{17,18} )</td>
<td>−0.3118 ± j8.8639</td>
<td>1.411</td>
<td>+0.035</td>
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<tr>
<td>( \lambda_{19,20} )</td>
<td>−0.3138 ± j8.4745</td>
<td>1.349</td>
<td>+0.037</td>
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<tr>
<td>( \lambda_{31,32} )</td>
<td>−1.1757 ± j7.8310</td>
<td>1.246</td>
<td>+0.148</td>
</tr>
<tr>
<td>( \lambda_{38,39} )</td>
<td>−5.3823 ± j1.0741</td>
<td>0.171</td>
<td>+0.981</td>
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</tbody>
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**TABLE XV**

Effect of including multiple PSS on system’s stability.

<table>
<thead>
<tr>
<th>( F_{L_{12}}^{\text{instability}} )</th>
<th>1</th>
<th>2</th>
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</table>

\( F_{L_{12}}^{\text{instability}} = 13.04 \text{ pu} \).
in improving stability margins. Likewise, a poorly tuned PSS may even be detrimental to the system’s performance.

It was also seen that installing multiple PSS may not be economically attractive, depending on the improvement that they introduce on power transfer capability.

A two-area system was used to demonstrate that the strength of the interconnection between generating areas as well as the amount of power transfer between them influence the system’s oscillatory stability. This was a relevant analysis because interconnections are a growing trend in power systems, due to reliability and economical reasons.

PSS design by adaptive control is an unexplored, but with promising potential, topic in this paper. As future work it is suggested to determine in which conditions the effort of adaptive control may be justified and if so, which would be the best approach/method to implement it in MaSSA.

REFERENCES