Multi-revolution Transfers in Orbital Mechanics

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Abstract

"Is it possible to improve the design of interplanetary trajectories by using multi-revolutions when comparing to the cases where only single-revolutions are used?" This is the question this thesis intends to answer by comparing solutions using just "standard" single-revolution transfers with those including multi-revolution transfers, where the spacecraft makes at least one complete revolution around the Sun before interacting with a planet. If the delta-V budget reduces, better payload to total mass ratios arise, which may improve future space missions. Optimization methods are used to optimize the delta-V budget of interplanetary transfers defined as multiple gravity assist (MGA) trajectory problems, including only impulsive manoeuvres when close to the planets. Three different cases are analysed to Mars, Mercury and Jupiter, respectively. Better solutions were found in terms of delta-V budget for all those cases by including multi-revolutions, specially in the mission to Jupiter where improvements were obtained not only in the delta-V and mass budgets, but also in the transfer's duration and departure epoch. So, the answer to the initial question is affirmative.

Keywords: interplanetary transfer, multi-revolutions, optimization, MGA problem, delta-V budget

1. Introduction

1.1. Motivation

In the last decades, mankind has witnessed the advent of space exploration. At this moment, all eight planets in the Solar System and also Pluto have been visited. Now, one of the major challenges is to send heavier and more capable instruments to better study the different bodies orbiting the Sun.

Multi-revolution transfers using high thrust propulsion may provide better trajectory solutions in terms of mass budget required when comparing to those with single-revolutions only. This is one of several strategies to achieve better ratios between payload mass and total mass of the spacecraft, which translates into an increasing of the payload mass that can be delivered to a certain target or a decreasing in the propellant needed to deliver a certain payload to its destination. However, it also affects other mission's aspects such as the launch/departure windows and its duration.

Single-revolution transfers are those in which the interplanetary arc connecting two bodies only completes a portion of a revolution around the Sun, meaning that the initial position of the orbit is only acquired once [1]. On the other hand, multi-revolution transfers are those where that initial position is repeated at least once, meaning that the satellite completes one or more complete revolutions in the same orbit before changing it by means of manoeuvres or encounters with other bodies such as planets or asteroids.

The reason for using multi-revolution transfers is the possibility of obtaining better geometry conditions between the different bodies in the trajectory, leading to a decrease in the delta-V and mass budgets. However, its use may bring a major drawback of considerably increasing the duration of the transfer to the target body. Thus, this scheme will only be used when analysing missions to inner planets/bodies, up to the Asteroid Belt between Mars and Jupiter, or in the initial segments of a multiple gravity assist trajectory to the outer planets such as Jupiter and so forth, which still occur in that inner region.

1.2. State of the Art

Since the first successful interplanetary mission, Mariner 2, to Venus in 1962, many probes have left the Earth heading to different targets in the Solar System. The great majority used high thrust propulsion to escape the Earth and arrive at their destination. From those high thrust missions, most of them, if not all, only used single-revolution transfers, even when using multiple gravity assists to decrease the delta-V budget.

Although knowledge of multi-revolutions exists for a long time, probably the same time as knowledge of single-revolutions, they are not commonly applied in missions, being merely treated as a theoretical result. The reason is the increase in transfer time required and the difficulties introduced in the trajectory design due to the increase in number variables/cases to analyse. Nevertheless, it may be advantageous to study those cases including multi-revolution transfers because of the possibility of improving the mission.

Another interesting option is low thrust (electric) propulsion where gases are accelerated by means of electrical heating and/or electromagnetic body forces which then accelerate the spacecraft [2]. Here, the goal is mainly attained by the higher values of specific impulse I_{sp} of the thrusters [3]. It is becoming more and more attractive because of its advantages, even if the duration increases due to the low thrust provided [3].

Unlike chemical (high thrust) propulsion, electrical propulsion systems require much more power to operate and use more complex systems. This electrical power can be obtained from solar energy whenever the spacecraft is at small distances from the Sun. However, for missions to the outer planets, where solar energy is small, nuclear power sources are required: nuclear reactors for heavy probes or devices heated by the decay of radioisotopes for lighter ones [4]. Also, for huge changes in velocity, such as those required to inject the probe into an interplanetary transfer (acceleration) or to inject it into an orbit around the target planet (deceleration), it is required thrust forces which are, usually, beyond the capability of these low thrusters.

These two solutions, multi-revolutions with high thrust propulsion and low thrust propulsion, compete to the yet "standard" high thrust single-revolution interplanetary trajectories. But, due to the requirements of better mass efficiency, their importance may increase, as already seen for the low thrust case with the increasing number of missions conducted. Lastly, despite presenting both the disadvantage of increasing the duration of the transfer, the multi-revolution option uses the simpler high thrust systems and does not need secondary thrust systems or the need to use nuclear propulsion, which still is a controversial topic for political and safety reasons [5]. So, if better solutions using high thrust multi-revolution transfers are found, comparing to the single-revolution ones, maybe this strategy will start to be used more often in the future.

2. The Multiple Gravity Assist Trajectory Optimization Problem

The complete motion of a spacecraft in an interplanetary transfer is very complex and can be modelled as an N-body problem with N being large enough for not allowing a complete analytic solution. Therefore, it is necessary to find simpler models allowing faster searches for solutions. These models should be simple enough so that solutions can be found, but accurate enough to make sure that those solutions obtained have physical meaning, not deviating that much from reality.

The multiple gravity assist (MGA) trajectory problem is an interplanetary mission design optimization problem to minimize the propellant and total masses required to deliver a specific payload mass to a planet/body or to maximize the payload mass that can be delivered regarding the capabilities of the launcher(s) used [6]. Here, the spacecraft leaves a departure body and travels to a target one. The trajectory can be direct or have N swing-bys along the way.

2.1. Notation and Simplifications

The notation used here is the same as presented in the works of Izzo [1] and Addis et al. [7]. So, the state vector of the spacecraft is given by $\vec{x} = \begin{bmatrix} \vec{r} & \vec{v} & m \end{bmatrix}$, where $\vec{r}$ and $\vec{v}$ are, respectively, the position and velocity vectors of the spacecraft and m is its mass. Each component of that state vector is denoted as x_i. The positions and velocities of the celestial bodies are denoted by capital letters, $\vec{R}^i$ and $\vec{V}^i$, to distinguish from those of the probe. The departure body has index $i = 0$, the next N swing-by manoeuvres occur at the bodies with indexes $i = 1, 2, ..., N$ and the target body corresponds to the index $i = N + 1$.

Each phase of the interplanetary trajectory corresponds to the interplanetary leg where the spacecraft is. During those phases the state vector is represented by $\vec{x}^i$, being i the phase index. Further, each leg's index i corresponds to a transfer from the celestial body with index $i - 1$ to the one with index i.

The general form of the problem is very complex to handle and hardly any solution can be found or, at least, in a reasonable amount of computation time. For that reason, several simplifications are assumed in order to obtain a simpler yet representative model of that problem, whose solutions can be achieved much faster. The simplifications used to define the actual MGA problem that will be used here are [1]:

- Outside the spheres of influence of the planets the only external force acting on the spacecraft is due to the (point mass) gravitational attraction exerted by the Sun.
- The sequence of celestial bodies visited by the spacecraft as well as the number of multi-revolutions per leg must be provided by the user as an input, not being an optimization parameter.
- The interactions with the different celestial bodies (except the Sun) only occur inside their spheres of influence, being these reduced to single points in the heliocentric reference frame.
- The gravity assist manoeuvres are treated as instantaneous discontinuities in the state of the spacecraft. Therefore, the change in the state vector is $\Delta\vec{x}^i = \vec{x}_s^{i+1} - \vec{x}_f^i$ and $\Delta t^i = t_s^{i+1} - t_f^i = 0$.
- The only constraint in the swing-by trajectories around the planets is the minimum (safety) value allowed for the radius of periapsis of the approaching and departing hyperbolic arcs.
- All manoeuvres executed are assumed to be impulsive because of using high thrust propulsion, so they change the velocity of the spacecraft instantaneously. Also, thrusting is only executed at the beginning of the trajectory to put the spacecraft in a departing hyperbolic orbit, in the end to insert it in an orbit around the target planet (if rendezvous is required instead of just a flyby) and during the powered gravity assist manoeuvres at the intermediate bodies. Therefore, no Deep Space Manoeuvres (DSM) are considered, for the sole purpose of only treating the influence of multi-revolutions when comparing to single-revolutions.

2.2. Final Form of the MGA Problem

With the simplifications made, the simpler and final version of the MGA problem to be optimized can be given mathematically by

$$\begin{aligned} \text{Optimize:} \quad & \phi(\vec{p}) \\ \text{Subjected to:} \quad & \mathcal{G}(\vec{p}) \leq 0 \\ & r_p^i \geq \tilde{r}_p^i \ , \end{aligned} \tag{1}$$

where ϕ is the objective/cost function to be minimized, $\vec{p}$ the decision vector of the problem and $\mathcal{G}(\vec{p})$ represents the boundary conditions for the variables in that vector, usually their upper and lower bounds. The parameters r_p^i and $\tilde{r}_p^i$ are, respectively, the actual radius of periapsis at the different gravity assist bodies and its minimum safety value.

Figure 1 presents a sketch of the phases in the trajectory followed by the spacecraft from departure till arrival at its destination. Although displaying at least one swing-by, direct transfers are also possible by using $N = 0$. If there are N swing-bys in the trajectory, the number of interactions with celestial bodies is $N + 2$ and the number of interplanetary legs (phases of the trajectory) is $N + 1$. A more detailed explanation of crucial aspects of this problem is presented next.

Decision Vector

The decision vector $\vec{p}$ to be optimized contains only the departure epoch t^{dep} (in MJD2000) and the durations of the different legs T^i with $i = 1, 2, ..., N + 1$ (in days). This happens because it is assumed that the swing-by trajectories are designed to exactly match the incoming and outgoing hyperbolic orbits. So,

$$\vec{p} = \begin{bmatrix} t^{dep} & T^1 & T^2 & ... & T^{N+1} \end{bmatrix} \ . \tag{2}$$

Objective Function

After an impulsive manoeuvre, the total mass of the spacecraft decreases because of the fuel consumption. The final mass, after an impulse Δv is applied, can be obtained from Tsiolkovsky's rocket equation [8]:

$$m_f = m_s \exp\left(-\frac{\Delta v}{g_0 I_{sp}}\right) \ , \tag{3}$$

where $g_0 = 9.80665$ m/s^2 is the gravitational acceleration near the surface of the Earth and I_{sp} (in seconds) is the specific impulse of the thrusters.

Many times the main goal of the MGA problem is the minimization of the ratio between the total mass and the payload mass. Since no payload is expelled during the overall trajectory and the thrusting conditions remain the same (the specific impulse I_{sp} is constant), this is equivalent to the minimization of the the total delta-V budget of the mission due to the exponential behaviour present in Equation (3). So, the objective function to be minimized is

$$\phi(\vec{p}) = \Delta v_{total} = \Delta v^{dep} + \sum_{i=1}^{N} \Delta v^i + \Delta v^{arr} \ . \tag{4}$$

Encounter Epochs and Mission's Duration

The encounter epochs between the spacecraft and the different celestial bodies are crucial to find $\vec{R}^i$ and $\vec{V}^i$. They can be easily computed from

$$t^i = t^{dep} + \sum_{k=1}^{i} T^k \ , \tag{5}$$

where $t^{arr} = t^{N+1}$. Moreover, the total duration of the interplanetary trajectory (time of flight) is just the sum of the durations of all legs.

Departure from the Initial Body

The departure from the initial body can be treated in two different ways: 1) the launcher puts the spacecraft directly in its departing orbit or 2) the launcher puts the spacecraft in a parking orbit with semi-major axis a^{dep} and eccentricity e^{dep} and only then the spacecraft is injected into the departing orbit. In the first case, no impulsive manoeuvre is required from the spacecraft itself ($\Delta v^{dep} = 0$) but v_∞^{dep} becomes a constraint regarding the characteristics of the launcher used. The case considered in this work is the second one with an initial parking orbit.

Interplanetary Legs (Arcs)

During the transfer between two celestial bodies, the spacecraft follows a Keplerian orbit governed by the gravitational force of the Sun. So, the i^{th} interplanetary arc of the trajectory is a solution of Lambert's problem for a transfer from body $i - 1$ to body i in T^i seconds, being mathematically described as

$$\begin{cases} \dot{\vec{r}}^i = \vec{v}^i \\ \dot{\vec{v}}^i = -\frac{\mu_{Sun}}{r^3}\vec{r} \\ \vec{r}_s^i = \vec{R}^i(t_s^i), \ \vec{r}_f^i = \vec{R}^f(t_f^i) \end{cases} . \tag{6}$$

The total number of solutions for that specific transfer is equal to $2(1 + 2M_{max}^i)$, where M_{max}^i is the maximum number of complete revolutions (multi-revolutions) the spacecraft can travel for the problem's conditions. Only posigrade orbits are considered since retrograde ones require huge delta-V budgets; this reduces the number of solutions in half. Therefore, the total number of solutions for the entire trajectory and with the same decision vector $\vec{p}$ is equal to $\Pi_{i=1}^{N+1}(1 + 2M_{max}^i)$.

Gravity Assist Manoeuvres

Each gravity assist, at a certain body and at time t^i, is considered as an instantaneous change in the state vector of the spacecraft. Since it is considered that $t_f^i = t_s^{i+1}$, then the position remains the same. The mass of the spacecraft only decreases if an impulsive manoeuvre is executed; otherwise, it remains constant. Finally, the velocity changes to match the initial velocity of the next heliocentric leg.

If the new velocity can be obtained by just varying the value of r_p^i, not being required to go bellow the safety value $\tilde{r}_p^i$, then the impulsive manoeuvre is only performed at the periapsis to match $||\vec{v}_\infty^{in}||$ and $||\vec{v}_\infty^{out}||$,

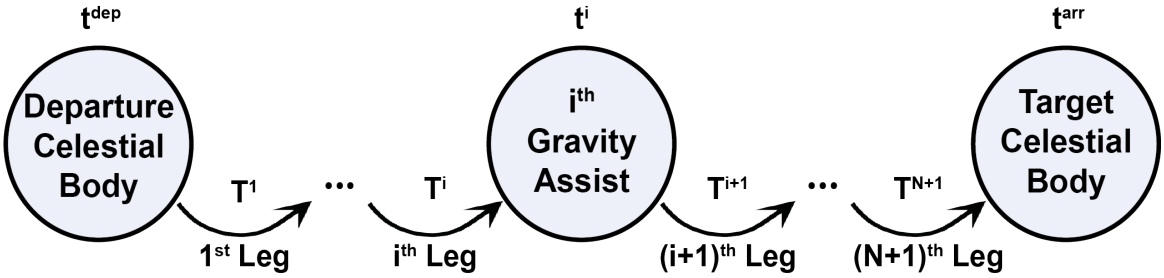

Figure 1: Sketch of the different phases in the multiple gravity assist trajectory flown by the spacecraft. The variables t^i represent the encounter epochs and T_i are the durations of the different interplanetary phases.

if necessary. However, if the change in direction is such that the value of r_p^i must be smaller than $\tilde{r}_p^i$, then it is used $r_p^i = \tilde{r}_p^i$ and an additional impulse is executed immediately after entering the sphere of influence of the planet or immediately before leaving it just to decrease the required turning angle α_{req} to α_{max} determined from $\tilde{r}_p$.

Arrival and Rendezvous with the Target Body

Once the spacecraft arrives at its destination, two options exist depending on the mission's requirements: 1) the spacecraft performs a flyby around that body or 2) it is inserted in a final orbit around that body with semi-major axis a^{arr} and eccentricity e^{arr}. In the first case, no impulsive manoeuvre is needed ($\Delta v^{arr} = 0$). However, the second case requires a deceleration of the spacecraft through an impulse opposite to the velocity's direction and with magnitude depending on that final orbit, being this the option considered here.

2.3. Optimization

Being this an optimization problem, it is necessary to choose which methods to employ to obtain the optimum solutions for that problem. Due to its complexity, it is not practical to use calculus based schemes which require to known not only the objective function itself but also its derivatives; therefore, stochastic and heuristic methods are used instead.

To obtain optimum solutions for the different cases studied, global optimization methods are applied. In Izzo [1], the author states that, from accumulated experience, the two best global heuristic optimization algorithms for the analysis of the MGA problem are Differential Evolution (DE) and Simulated Annealing with Adaptive Neighbourhood (SA-AN), being these the two global optimization methods used. In addition, Compass Search (CS) is employed for local optimization to refine the solutions obtained from the global optimization algorithms.

3. Implementation

To study the practical cases of interplanetary missions using both single-revolution and multi-revolution transfers, a python code is developed using the PyGMO and PyKEP toolboxes as a base of that implementation. That code consists of two different scripts: one where the entire multiple gravity assist trajectory problem is defined and another where optimization of that problem is made. Both scripts are explained in Sections 3.1 and 3.2, respectively.

3.1. Implementation of the MGA Problem

The MGA problem consists entirely of a class type variable with all the parameters, variables and functions/methods defined inside. It makes use of several functions already implemented in PyKEP such as Izzo's Lambert solver, functions to deal with epochs and with the ephemeris of the planets (computed using JPL's data). It also inherits from the *base* class as used by all PyGMO problems. The MGA class consists of ten functions which completely execute all that may be requested during the simulations performed. These methods use the PyKEP functions referred before as well as a function called *swingby_DV()* which computes the total delta-V required during a gravity assist manoeuvre. A more detailed explanation about each of those methods and the parameters used by them is presented next:

- *__init__()*: constructs an instance of the MGA class with initialization of the parameters relevant for the problem, which are:
  - *seq*: sequence of planets in the interplanetary transfer, each stored as an instance of a class containing the values for different planetary parameters: name, radius, safe radius, ephemeris, its gravitational constant and that of the Sun.
  - M: list containing the number of complete revolutions around the Sun for each interplanetary leg.
  - S: list with the specification of which Lambert solution to use (only relevant for multi-revolution cases where $S^i = 1$ refers to the solution x_1 and $S^i = 2$ to x_2).
  - *t_dep*: list with the departure window defined by the minimum and maximum departure epochs.
  - *T_legs*: list including two lists with the lower and upper bounds for the durations of the different interplanetary legs.
  - *rp_dep + e_dep*: radius of periapsis and eccentricity of parking orbit around the departure planet.
  - *rp_arr + e_arr*: radius of periapsis and eccentricity of insertion orbit around the target planet.

- *set_sequence()*: sets the sequence *seq* of planets as well as the number of complete revolutions M and the Lambert solution to use in each leg S.
- *set_departure_window()*: sets the departure window *t_dep* for the mission.
- *set_parking_orbit()*: sets the parking orbit around the departure planet: *rp_dep* and *e_dep*.
- *set_insertion_orbit()*: sets the insertion orbit around the target planet: *rp_arr* and *e_arr*.
- *human_readable_extra()*: presents general information about the problem to the user.
- *_objfun_impl()*: implicit objective function used later by PyGMO's optimization algorithms.
- *detailed_info()*: displays complete information about the trajectory for a specific decision vector $\vec{p}$.
- *plot()*: plots the trajectory for that instance of MGA problem and for a specific decision vector $\vec{p}$.

The last three functions use the same procedure to obtain the trajectory and respective results from a specific decision vector. The difference is that the first returns Δv_{total}, while the second and third ones do not return anything, displaying instead information about the trajectory obtained and the plot of that trajectory, respectively. The steps to obtain the delta-V budget from a decision vector $\vec{p}$ are the following:

1. Obtain the encounter epochs t^i between the spacecraft and the planets from the decision vector.
2. Compute, for each encounter, the state vector of the planet in the heliocentric frame from its ephemeris.
3. Compute the Lambert solution for the first leg. Determine the probe's heliocentric velocities at the start and end of that arc according to M^1 and S^1.
4. Determine $\vec{v}_\infty^{dep}$, from the velocity at the beginning of the first arc and the velocity of the departure planet, and the first impulse Δv^{dep}.
5. Cycle to compute the remaining legs and impulses required for $N > 0$:
   - 5.1. Compute the incoming hyperbolic excess velocity at the i^{th} swing-by, $\vec{v}_\infty^{in,i}$, from the velocity at the end of the i^{th} leg and the velocity of the i^{th} planet during the encounter.
   - 5.2. Compute the $(i+1)^{th}$ Lambert solution between the i^{th} and $(i+1)^{th}$ planets. Determine the probe's heliocentric velocities at the start and end of that arc according to M^{i+1} and S^{i+1}.
   - 5.3. Compute the outgoing hyperbolic excess velocity at the i^{th} swing-by, $\vec{v}_\infty^{out,i}$, from the velocity at the start of the $(i+1)^{th}$ leg and the velocity of the i^{th} planet during the encounter.
   - 5.4. Obtain the impulse Δv^i required during the gravity assist to match the previous and next interplanetary legs.
6. Determine $\vec{v}_\infty^{arr}$, from the velocity at the end of the last leg and the velocity of the target/arrival planet, and the final impulse Δv^{arr}.
7. Sum all the impulses required, as presented in Equation (4), to obtain the total delta-V budget for this specific solution.

The problem when dealing with multi-revolutions is that there is always a minimum duration for a certain number of complete revolutions to be possible. A simple way for solving that problem is just letting the decision vector to have any values inside, within the boundaries of its parameters, and as soon as an invalid solution is obtained (M^i_{max} smaller than the required M^i), the function stops and returns a value for Δv_{total} equal to 10^{100}. This means that a solution so bad is rapidly excluded by the optimization algorithms.

3.2. Implementation of the Optimization Process

This optimization process is implemented in a python script containing the *main* function of the code and consisting of a sequence of steps to be executed whenever a simulation is performed:

1. Import the required functions/methods from PyGMO and PyKEP and the MGA class previously created and defined in a different script.
2. Define the different instances for the planets in the Solar System using their planetary data and respective ephemeris obtained from JPL's data.
3. Define the MGA class instance to study by defining the planetary sequence, multi-revolutions to use or not (and which Lambert solutions to use), the departure window, the boundaries for the legs' durations and the parking and insertion orbits.
4. Choose between optimizing the MGA problem or to analyse a specific solution for that problem:
   - 4.1. If optimization is intended, the algorithms are defined and the optimization of the function defined in *_objfun_impl()* takes place with the display of detailed information of the optimum solution found in the end.
   - 4.2. If a specific decision vector is provided to be analysed, detailed information is presented using the *detailed_info()* method of the MGA class and the plot of the trajectory is displayed using the *plot()* function.

Referring now to the optimization process itself, it consists of the same strategy used for all other problems in PyGMO and which can be described as follows: create an *archipelago* with several *islands*, which may be connected between them or not, and each of those islands contains a *population* of several *individuals* where a single optimization algorithm acts independently. This strategy takes advantage of the parallel optimization capability of modern computers, decreasing a lot the overall optimization time.

Here, an archipelago with seven islands is created and arranged in *rim* topology as shown in Figure 2. It consists of a ring of external islands connected only to their neighbours and to the central one which is connected to all. The external islands use global optimization algorithms (Differential Evolution and Simulated Annealing with Adaptive Neighbourhood), while the inner island uses local optimization (Compass Search).

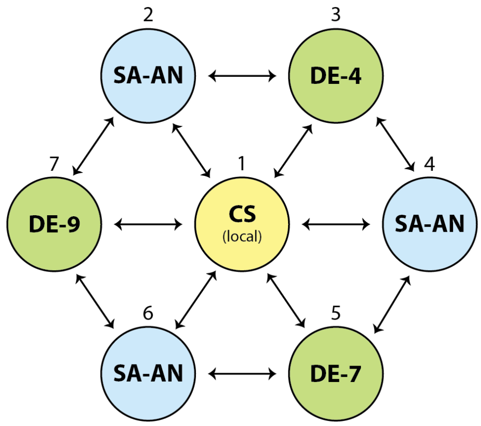

Figure 2: Archipelago for optimization with seven islands in rim topology. The external islands use global optimization and the inner one uses local optimization.

The optimization process can be viewed like an evolutionary scheme as seen in nature where a sequence of two steps is repeated as many times as desired: first each population from each island evolves independently and completely according to the algorithm defined there, then migration occurs between the different islands in the archipelago according to a certain migration scheme (and only between those connected to each other). Every island sends their best solutions found to their neighbours, replacing the worst solutions in them so that a constant population size is maintained.

The only thing left to discuss are the control parameters used for the optimization algorithms in the different islands. Each method has its own set of parameters, as shown in Table 1, having them been chosen based on tests performed using PyGMO's *racing* function which evaluates different optimization methods.

4. Results and Discussion

To see if introducing multi-revolutions in interplanetary transfers leads to improvements in the mission, it is necessary to analyse different cases and discuss the results obtained. The first one is a mission to Mars, since it is still an attractive destination. Then, a mission to Mercury is analysed and, unlike the previous case, this intends to go to a planet closer to the Sun. The third and most complex case is that of a mission to Jupiter, where it is only feasible to use multi-revolutions in the initial legs of the trajectory.

All possible cases within each of the three missions are optimized 5 times and in each one the archipelago is allowed to evolved 10 times so that the actual optimum

Table 1: Sets of parameters for the algorithms used in the archipelago displayed in Figure 2.

Island	Algorithm	Population Size	Control Parameters
1	CS (local)	500 (each)	$max\_eval = 1000$ $\Delta_{start} = 0.01$ $\Delta_{stop} = 0.00001$ $R = 0.5$
2, 4, 6	SA-AN (global)		$T_0 = 10$ $N_S = 5$ $N_T = 20$ $r_T = 0.9$
3, 5, 7	DE (global)		$G_{max} = 500$ $CR = 0.9$ $F = 0.5$ $variant = 4, 7, 9$

solutions are most likely to have been found. Moreover, to have comparable results, the same departure window from Earth is used, starting on January 1, 2020 and ending on December 31, 2024 (5 years span). Other optimization variables have their own boundaries defined according to the number of multi-revolutions per interplanetary leg as well as the planets from and to which the spacecraft travels.

4.1. Interplanetary Transfer to Mars

A space mission from Earth to Mars is the first practical case studied. Only direct transfers to the red planet are considered, meaning that no gravity assist manoeuvres are employed ($N = 0$) and leading to a single interplanetary leg connecting both planets. So, $\Delta v_{total} = \Delta v^{dep} + \Delta v^{arr}$. The initial parking orbit around the Earth is circular ($e = 0$) and with an altitude of 200 km. The final insertion orbit around Mars is also circular but at 300 km of altitude.

This particular MGA problem has two optimization variables: the departure epoch t^{dep} (inserted in Universal Time Coordinated and converted to MJD2000) and the duration T^1 of the transfer. Their boundaries are presented in Table 2. The maximum number of complete revolutions considered is $M^1_{max} = 2$.

Table 2: Boundaries for the optimization variables for a direct transfer to Mars.

Boundaries: Earth→Mars transfer

Variable		Lower Bound	Upper Bound
t^{dep} [UTC]		2020-Jan-01	2024-Dec-31
T^1 [days]	$M^1 = 0$	100	600
	$M^1 = 1$	300	1000
	$M^1 = 2$	500	1400

Five possible cases exist for this direct transfer to Mars: the single-revolution case ($M^1 = 0$), two for $M^1 = 1$ and other two for $M^1 = 2$. The best solutions found for each of the five cases are shown in Table 3.

Table 3: Results obtained for a direct transfer to Mars. Vector $\vec{M}$ contains the number of complete revolutions per leg and the vector $\vec{S}$ the Lambert solution per leg for the multi-revolution case.

Results: Earth→Mars transfer

$\vec{M}$	$\vec{S}$	Departure Epoch [UTC]	Duration [years]	Δv_{total} [m/s]
0	–	2024-Sep-30	0.91079	5724.7
1	1	2021-Nov-23	2.18041	5884.3
	2	2024-Jan-31	1.87970	6979.6
2	1	2023-Oct-16	3.81765	5679.8
	2	2021-May-01	3.12049	6095.7

Discussion of the Results

With the results obtained from the simulations made, some conclusions about this particular interplanetary mission can be retrieved, being the following aspects worth mention:

Only one multi-revolution case improved Δv_{total} with respect to the single-revolution case. This solution, with $M^1 = 2$ and $S^1 = 1$, saves 44.9 m/s in delta-V but increases the duration from less than 1 year to almost 4 years. So, the savings in Δv_{total} do not compensate the increase in transfer time.

Despite seeming that the best solution for this mission is the single-revolution transfer, it can only leave the Earth by the end of 2024, reaching Mars in mid 2025. One way to solve this problem is an earlier mission using one complete revolution ($M^1 = 1$ and $S^1 = 1$). Despite increasing Δv_{total} in 159.6 m/s and the duration in almost 1 year, the departure is anticipated in 3 years to November 2021 and the arrival at Mars also occurs sooner, at the beginning of 2024.

If the departure and arrival epochs have less importance in the mission design than the delta-V budget, then the single-revolution case should be used. However, if the departure from Earth and arrival at Mars must occur as soon as possible, then the multi-revolution case with $M^1 = 1$ and $S^1 = 1$ is an interesting option to use.

4.2. Interplanetary Transfer to Mercury

An interplanetary mission to Mercury is the second case analysed. Two different planetary sequences are considered now: a direct transfer from Earth to Mercury and another with a swing-by at Venus; therefore, the overall delta-V budget of the mission is composed of either two or three impulses. In addition, the spacecraft is initially in a circular parking orbit at 200 km of altitude and is inserted into a circular orbit around Mercury at an altitude of 300 km.

The first planetary sequence studied is a direct transfer. Therefore, the trajectory consists of a single leg and the number of optimization variables in the decision vector is two: the departure epoch t^{dep} and the duration T^1 of the transfer to Mercury. The boundaries for those variables are presented in Table 4.

Table 4: Boundaries for the optimization variables for a direct transfer to Mercury.

Boundaries: Earth→Mercury transfer

Variable		Lower Bound	Upper Bound
t^{dep} [UTC]		2020-Jan-01	2024-Dec-31
T^1 [days]	$M^1 = 0$	50	300
	$M^1 = 1$	150	600
	$M^1 = 2$	200	900

Once again, there are five solutions to consider for this direct transfer: the single-revolution case ($M^1 = 0$) and two for $M^1 = 1$ and for $M^1 = 2$, whose best solutions found are presented in Table 5.

Table 5: Results obtained for a direct transfer to Mercury. Vector $\vec{M}$ contains the number of complete revolutions per leg and the vector $\vec{S}$ the Lambert solution per leg for the multi-revolution case.

Results: Earth→Mercury transfer

$\vec{M}$	$\vec{S}$	Departure Epoch [UTC]	Duration [years]	Δv_{total} [m/s]
0	–	2024-May-09	0.32474	13532.1
1	1	2023-May-11	0.83575	12930.6
	2	2024-May-12	0.78159	13596.6
2	1	2022-May-11	1.34887	12771.6
	2	2022-May-17	1.33080	13125.4

The second trajectory analysed includes a swing-by at Venus before reaching Mercury. Therefore, it consists now of two interplanetary legs, leading to a three-dimensional problem to be optimized and whose variables are the departure epoch t^{dep}, the duration T^1 of the first leg from Earth to Venus and the duration T^2 of the second leg from Venus to Mercury. The boundaries for these variables are presented in Table 6.

Table 6: Boundaries for the optimization variables for a transfer to Mercury with swing-by at Venus.

Boundaries: Earth→Venus→Mercury transfer

Variable		Lower Bound	Upper Bound
t^{dep} [UTC]		2020-Jan-01	2024-Dec-31
T^1 [days]	$M^1 = 0$	50	600
	$M^1 = 1$	200	800
	$M^1 = 2$	300	1000
T^2 [days]	$M^1 = 0$	30	400
	$M^1 = 1$	100	600
	$M^1 = 2$	200	800

Having two interplanetary legs means a greater number of cases to study. Moreover, the sum of the complete revolutions around the Sun for both legs must be less or equal to 2, i.e., $M^1 + M^2 \leq 2$. So, thirteen cases exist, whose best solutions found are presented in Table 7, being them the single-revolution one, two for each of the cases where $\vec{M}$ is $(1,0)$, $(2,0)$, $(0,1)$ or $(0,2)$ and the final four solutions with $\vec{M} = (1,1)$.

Table 7: Results obtained for a transfer to Mercury with swing-by at Venus. Vector $\vec{M}$ contains the number of complete revolutions per leg and the vector $\vec{S}$ the Lambert solution for the multi-revolution case.

Results: Earth→Venus→Mercury transfer

$\vec{M}$	$\vec{S}$	Departure Epoch [UTC]	Duration [years]	Δv_{total} [m/s]
0,0	–,–	2020-Aug-02	0.75477	10754.5
1,0	1,–	2023-Jul-03	1.40654	9811.0
	2,–	2024-Jul-15	1.19069	13096.9
2,0	1,–	2020-Jun-15	2.05996	10164.1
	2,–	2022-Jul-14	1.99481	11483.4
0,1	–,1	2023-Jul-20	1.11738	10633.9
	–,2	2024-Dec-29	0.83224	10285.0
0,2	–,1	2021-Dec-04	1.63607	10694.3
	–,2	2024-Dec-31	1.29719	10940.6
1,1	1,1	2020-Jun-27	1.77535	10080.8
	1,2	2023-Feb-24	1.51357	11063.7
	2,1	2021-Jun-19	1.61133	11303.7
	2,2	2023-Mar-12	1.47913	11443.1

Discussion of the Results

With the results obtained from the simulations made, some conclusions about this particular interplanetary mission can be retrieved, being the following aspects worth mention:

The delta-V budget for direct transfers is very high and only one multi-revolution case led to a worst Δv_{total} when comparing to the single-revolution one.

The best direct transfer solution is that in which the spacecraft makes two complete revolutions before arriving at Mercury ($M^1 = 2$ and $S^1 = 1$). Despite increasing the duration in 1 year, it decreases Δv_{total} in 760.5 m/s to 12771.6 m/s and anticipates the departure from Earth in 2 years from May 2024 to May 2022.

The use of a swing-by at Venus decreases the delta-V budget required for the transfer and the results obtained show that all cases except one lead to better results even when comparing to the best solution found before. The following three cases are those which present the most interesting results.

The single-revolution case corresponds to the fastest transfer and, with a departure epoch at the beginning of the departure window, it is the solution with the earliest arrival at Mercury. But, it is not the most advantageous case in terms of Δv_{total}.

The solution with minimum Δv_{total} is that with one complete revolution from Earth to Venus and a single-revolution between Venus and Mercury, $\vec{M} = (1,0)$ and $\vec{S} = (1,-)$. This saves 943.5 m/s in Δv_{total} with respect to the previous case. However, this solution increases the transfer time in almost 8 months and delays the departure in almost 3 years, leading to an arrival at Mercury almost 3.5 years later.

The last case comes from a compromise between departure epoch, duration and delta-V budget and that solution involves one complete revolution in both legs ($M^1 = M^2 = 1$ and $S^1 = S^2 = 1$). With respect to the single-revolution case, it saves 673.7 m/s at the cost of increasing the transfer time in 1 year and delaying the arrival at Mercury in 11 months.

The use of a gravity assist at Venus is advantageous to the mission's design. Also, if the arrival epoch is the most important parameter, the solution to be used should be the single-revolution case. If Δv_{total} is also a relevant parameter, then the transfer with one complete revolution in each leg may be the best one.

4.3. Interplanetary Transfer to Jupiter

Jupiter, along with its moons, is one of the most attractive planets to explore. Only the interplanetary transfer from Earth to Jupiter is considered and not the trajectory upon reaching the Jovian system. The same parking orbit is used and the spacecraft is injected into a highly elliptical orbit with $e = 0.9$ and periapsis at an altitude of 600000 km upon reaching Jupiter.

Since a great number of possible planetary sequences targeting Jupiter exists, it is first studied those from direct transfers ($N = 0$) up to those with three swing-bys ($N = 3$) considering only single-revolutions. Moreover, the only planets where gravity assists can be made are Venus, Earth and Mars. The departure window is the same as before and each leg's duration T^i must lie within 50 and 2000 days. the results obtained for the 40 possible cases are presented in Figure 3.

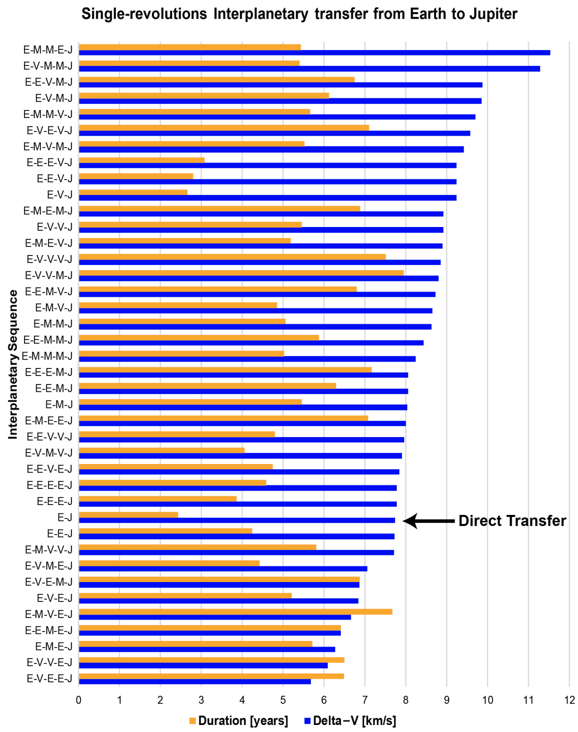

Figure 3: Delta-V and duration for best solutions found for a mission to Jupiter (J) involving single-revolutions from direct transfers to those with three swing-bys at Venus (V), Earth (E) and Mars (M).

It was found previously that better solutions in terms of Δv_{total} were obtained with the inclusion of multi-revolutions. So, it is expected here that at least one case with $M^i > 0$ also leads to a better solution when comparing to the corresponding single-revolution case. Therefore, the Venus-Earth-Earth Gravity Assist trajectory is considered to include multi-revolutions since it presents the minimum delta-V budget in Figure 3. If a better solution is found, then it is necessarily better than all single-revolutions cases, proving that using multi-revolutions produce better solutions than using only single-revolutions.

This sequence, involving one swing-by at Venus and two on Earth, has four segments and five optimization variables: the departure epoch t^{dep} and the durations of those four legs T^i, with $i = 1, 2, 3, 4$. The boundaries for all those variables can be seen in Table 8. It is considered that the first three legs, within the inner Solar System, can have 0, 1 or 2 complete revolutions, while the last one can only be a single-revolution transfer to go to the outer Solar System and reach Jupiter.

Table 8: Boundaries for the optimization variables for a transfer to Jupiter with VEEGA trajectory.

Boundaries: Earth→VEEGA→Jupiter

Variable		Lower Bound	Upper Bound
t^{dep} [UTC]		2020-Jan-01	2024-Dec-31
T^i [days] $i = 1, 2, 3$	$M^i = 0$	50	1000
	$M^i = 1$	200	1300
	$M^i = 2$	400	1600
T^4 [days]	$M^4 = 0$	500	2000

With four interplanetary legs, this is the most complex trajectory analysed here, having a great number of cases to study. Once again, the sum of all multi-revolutions must be less or equal to 2, $\sum_{i=1}^{4} M^i \leq 2$. This means that 25 cases exist, whose best solutions found are presented in Table 9. These cases are to the single-revolution one, two for each case where $\vec{M}$ is equal to $(1, 0, 0, 0)$, $(2, 0, 0, 0)$, $(0, 1, 0, 0)$, $(0, 2, 0, 0)$, $(0, 0, 1, 0)$ or $(0, 0, 2, 0)$ and four for each case where $\vec{M}$ is equal to $(1, 1, 0, 0)$, $(1, 0, 1, 0)$ or $(0, 1, 1, 0)$.

Discussion of the Results

With the results obtained from the simulations made, some conclusions about this particular interplanetary mission can be retrieved, being the following aspects worth mention:

When designing an interplanetary mission to the outer planets, gravity assist manoeuvres are employed to reduce the delta-V budget to make the mission feasible. Nonetheless, it is clear, from Figure 3, that the direct transfer is the fastest one.

From the 39 non-direct transfer cases studied, 29 of them led to worse minimum solutions in terms of Δv_{total} than the direct transfer, taking even more time to achieve Jupiter. On the other side, 10 of those cases decreased the delta-V budget required, increasing the duration of the transfer as well.

Table 9: Results obtained for a transfer to Jupiter with VEEGA trajectory. Vector $\vec{M}$ contains the number of complete revolutions per leg and the vector $\vec{S}$ the Lambert solution for the multi-revolution case.

Results: Earth→VEEGA→Jupiter transfer

$\vec{M}$	$\vec{S}$	Departure Epoch [UTC]	Duration [years]	Δv_{total} [m/s]
0,0,0,0	–,–,–,–	2023-May-18	6.49128	5686.9
1,0,0,0	1,–,–,–	2021-Mar-14	8.17927	5642.6
	2,–,–,–	2023-Oct-20	8.55666	9669.8
2,0,0,0	1,–,–,–	2021-Apr-03	8.65739	5734.1
	2,–,–,–	2020-Mar-21	6.80626	5920.1
0,1,0,0	–,1,–,–	2020-Mar-21	9.93585	6070.4
	–,2,–,–	2023-Apr-23	6.62904	5222.1
0,2,0,0	–,1,–,–	2023-May-23	5.73760	11761.1
	–,2,–,–	2020-Mar-07	7.09794	5029.3
0,0,1,0	–,–,1,–	2024-May-31	6.74195	7654.3
	–,–,2,–	2020-Mar-10	6.01515	5029.4
0,0,2,0	–,–,1,–	2024-Jun-08	7.31540	8108.9
	–,–,2,–	2023-May-12	8.56813	5091.3
1,1,0,0	1,1,–,–	2020-Oct-24	8.18729	10927.1
	1,2,–,–	2024-Jun-30	7.12222	5417.2
	2,1,–,–	2023-Apr-23	9.08791	9949.7
	2,2,–,–	2021-Jun-06	7.10061	5101.1
1,0,1,0	1,–,1,–	2021-Mar-26	8.31972	5791.8
	1,–,2,–	2021-May-18	7.18781	5054.0
	2,–,1,–	2023-Oct-13	7.41312	9003.4
	2,–,2,–	2021-May-24	7.18474	5039.8
0,1,1,0	–,1,1,–	2020-Apr-19	8.37979	6321.7
	–,1,2,–	2020-Mar-29	7.90757	5486.5
	–,2,1,–	2023-Apr-09	7.72436	5967.6
	–,2,2,–	2021-Oct-21	6.11077	7943.8

The best solution obtained using only single-revolutions is that of a VEEGA trajectory which departs from Earth, executes one swing-by at Venus and then two on Earth before flying to Jupiter. Comparing with the direct transfer, the use of these three gravity assist manoeuvres saves 2051.2 m/s in Δv_{total}, which translates into great savings in propellant mass and probably in the total cost of the mission. However, the duration of the transfer increases from 2.43 years to 6.49 years, which is still quite reasonable for missions to the outer planets in the Solar System.

With the introduction of multi-revolutions to the VEEGA trajectory, two very interesting results were obtained regarding the single-revolution case: firstly, smaller values of Δv_{total} were obtained for several cases and, secondly, some cases produced smaller transfer times for the optimum solutions found, something that was not expected at the beginning of this work.

The best solution obtained with multi-revolutions is better than the single-revolution case in all four aspects of the mission: departure epoch, transfer time, arrival epoch and delta-V budget. This solution only uses one multi-revolution in the trajectory, between both swing-bys on Earth ($M^1 = M^2 = M^4 = 0$ and $M^3 = 1$ with $S^3 = 2$). The advantages of this solution are:

- Departure Epoch: Earliest departure from Earth (beginning of 2020), while the single-revolution transfer only departs in mid 2023.
- Transfer time: Decreases in 6 months the duration, taking only 6 years to arrive at Jupiter.
- Arrival at Jupiter: Earliest arrival at Jupiter, reaching its destination 3 years and 8 months before the arriving there by a spacecraft flying the single-revolution transfer.
- Delta-V budget: One of the minimum values obtained for Δv_{total}, leading to savings of 657.5 m/s with respect to the single-revolution VEEGA transfer, making it a very attractive option.

5. Conclusions

5.1. Achievements and Final Remarks

The starting point for this thesis can be described as the question: *is it possible to improve the design of an interplanetary trajectory by using multi-revolutions when comparing to the cases where only single-revolutions are used?* To answer it, a code was developed to optimize multiple gravity assist trajectories with both single-revolution and multi-revolution transfers. That code was then used to study three interplanetary missions to Mars, Mercury and Jupiter. The results obtained pointed to an affirmative answer.

The most amazing consequence of this outcome is that using the same high thrust propulsion systems in addition to multi-revolutions can decrease the delta-V budget of a mission when comparing to the "standard" single-revolution high thrust case, especially when swing-bys are employed. This decrease in Δv_{total} translates into improvements in the mass budget for the spacecraft, with the possibility of sending a certain payload to some planet, comet or asteroid with less propellant, reducing the overall cost of the mission.

Nonetheless, the inclusion of multi-revolutions leads most of the times to increasings in the duration of the transfer which, in turn, increases the cost of the mission and may even nullify the savings gained from the lower delta-V and mass budgets. However, it may happen that not only does Δv_{total} decrease but also the transfer time, as seen in the mission to Jupiter. Further, it may bring better departure epochs to leave the Earth and to earlier arrivals at the target planet, which may be a fundamental criterion in the mission's design.

5.2. Future Work

All the objectives proposed for this thesis were met. Nonetheless, some future work can still be done in this topic of high thrust multi-revolution transfers for an even stronger relevance in the design of future space missions. The following aspects can be considered as the next steps in the study of this topic:

- Inclusion of more complex and more complete trajectory models for interplanetary transfers, namely, with deep space manoeuvres and aerogravity assists.
- Test this strategy for other target planets, comets and asteroids to show how comprehensive it can be.
- Take the best multi-revolution solutions and adapt them to the complete model, analysed by numerical integration of the equations of motion including other perturbing forces affecting the trajectory.

Finally, as low thrust propulsion has seen its importance recognized in the last two decades, multi-revolutions should also start to be considered more often in the design of future space missions since they may provide better solutions than other transfer types.

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References

[1] Dario Izzo. Global Optimization and Space Pruning for Spacecraft Trajectory Design. In *Spacecraft Trajectory Optimization*, chapter 7, pages 178–199. Cambridge University Press, 2010.

[2] Robert G. Jahn. *Physics of Electric Propulsion*. McGraw-Hill, first edition, 1968.

[3] Karel F. Wakker. *Fundamentals of Astrodynamics*. Institutional Repository Library Delft University of Technology, January 2015.

[4] Edgar Y. Choueiri. New Dawn for Electric Rockets. *Scientific American*, 300:58–65, February 2009.

[5] Robert G. Jahn and Edgar Y. Choueiri. Electric Propulsion. *Encyclopedia of Physical Science and Technology*, 5, 2002.

[6] D. Izzo, V. M. Becerra, D. R. Myatt, S.J. Nasuto, and J. M. Bishop. Search space pruning and global optimisation of multiple gravity assist spacecraft trajectories. *Journal of Global Optimization*, 38(2):283–296, May 2007.

[7] B. Addis, A. Cassioli, M. Locatelli, and F. Schoen. Global Optimization for the Design of Space Trajectories.

[8] J. W. Cornelisse, H. F. R. Schöyer, and K. F. Wakker. *Rocket Propulsion and Spaceflight Dynamics*. Pitman Publishing Limited, first edition, 1979.