Review and Validation of Models for Photovoltaic Array Performance

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Abstract — In the last decade, photovoltaic energy has shown enormous growth both in individual and corporate level. The need for a reliable tool to predict energy production of the system has increased. The fundamental goal of this research is the review and validation of models for photovoltaic array performance. Numerous models have been used and enhanced over the years, the model used in this research work is the Five Parameter Model. This model uses the data provided by manufacturers to predict energy output taking into account certain cell parameters under any set of operating conditions. It’s also given emphasis on major influencing parameters in cell production, one of them being incident radiation. A model is presented in order to obtain effective incident radiation on the solar panel. Current and voltage measurements of photovoltaic panels for two different systems are supplied, one off grid system and one grid connected system. Both systems are modelled through Matlab/Simulink software, which allows for comparison between real and predicted data, verifying the model’s accuracy.

Keywords — solar energy, photovoltaic cell, 5 Parameter Model, irradiance, MPPT, IV curve

I. INTRODUCTION

This paper aims to describe the work developed in [1]. As the interest in photovoltaic (PV) energy grows, the need to reduce investment costs and improve the efficiency of the system also increases. Designers need a reliable tool to predict energy production of the system under any set of operating conditions, in order to decide whether or not to invest in this technology. Photovoltaic energy production depends on incident irradiance, cell temperature, incidence angle of irradiance and load resistance. Unfortunately, manufacturers give limited module data, mostly under Standard Test Conditions (STC), with an irradiance, cell temperature and air mass equal to 1000 watts per square meter, 25 degrees and 1.5 respectively. This set of conditions is very unlikely to be found in real situations.

The cell operating point is highly dependent on irradiance and cell temperature. The current produced by the cell has a proportional relationship with irradiance and voltage produced is mostly affected by the cell temperature. As the cell temperature increases, current remains the same, but voltage decreases, reducing the power output. Logically, this type of influence has to be taken into account when providing reliable prediction of energy production.

The model used in this thesis is the 5 Parameter Models. This model requires only data provided by the manufacturers (avoiding the need for additional tests) in order to achieve the necessary cell parameters to obtain power output.

The main goal of this study is to develop the referred model. Irradiance and cell temperature influence on cell parameters is accounted, with emphasis on effective incident irradiance.

Further in this research, the model is validated through data provided of two different systems. One of the systems is a Stand Alone PV System (which required modelling of the battery, controller and inverter) and the other is a Grid Connected PV System. The simulated results are analysed and compared with the real data provided, showing the model’s limitations. The data provided itself had some limitations, which will be mentioned.

II. EFFECTIVE IRRADIANCE

A. Irradiance Components

Incident irradiance is divided in three different components. Beam irradiance ($G_b$) is the irradiance received from the sun without having been scattered by the atmosphere. Diffuse irradiance ($G_d$) is the irradiance received from the sun after its direction has been changed upon being scattered by the atmosphere. Ground-reflected Irradiance (albedo) is only taking into account on tilted panels, being the irradiance that strikes the ground and it’s reflected on the panel surface. Its relevance depends on the nature of the surrounding ground.

Irradiance data in this research was treated in hourly periods. From now on, the letter I will be used when referring to irradiance over an hour period, (watt hour per square meter), with subscript b and d for beam and diffuse irradiation, subscript T is for tilted irradiation. When there’s no subscript T, the irradiation is on a horizontal plane.

Extraterrestrial Irradiation ($I_0$) is noted as the solar irradiation before crossing the earth’s atmosphere and it’s given by:

$$I_0 = \frac{12 \times 3600}{\pi} G_{sc} \varepsilon_0 \left[ \cos \theta \cos \delta \left( \sin \omega_2 - \frac{\pi (\omega_2 - \omega_1)}{180} \sin \theta \sin \delta \right) \right]$$

(II.1)

$G_{sc}$ is the solar constant and it’s equal to 1367 W/m². Variables $\omega_1$ and $\omega_2$ are the hour angles where $\omega_2 > \omega_1$, $\theta$ is the latitude of the PV system location, $\delta$ is the solar declination and $\varepsilon_0$ is the earth’s eccentricity correction factor.
B. Global Horizontal Irradiance

Irradiance measurements are usually acquired on a horizontal surface. In order to increase absorbed radiation, most PV systems are not mounted horizontally, thus, a model to convert horizontal irradiance to irradiance on an inclined surface is crucial. Knowledge of the surface tilt, surface azimuth angle, and coordinates of the installation is required.

The first step is to obtain separate values for beam and diffuse components. Most developed models obtain diffuse fraction of horizontal irradiation, $d$, based on the hourly clearness index. The hourly clearness index, $k_t$, is given by the fraction of extraterrestrial irradiation that crosses the earth’s atmosphere.

$$k_t = \frac{l}{l_0}$$  \hspace{1cm} (II.2)

Boland, Ridley and Lauret [2] developed the BRL model for horizontal diffuse fraction, in which $d$ is obtained through:

$$d = \frac{1}{1 + e^{-5.0033 + 8.6025k_t}}$$  \hspace{1cm} (II.3)

Having calculated $d$, the diffuse horizontal irradiation is obtained, $I_d = I_x \times d$. Since horizontal irradiation only has beam and diffuse components, beam irradiation is very easily obtained as $I_b = I - I_d$.

C. Irradiance on a tilted surface

Irradiation on a tilted surface is given by:

$$I_T = I_{BT} + I_{DT} + I_B \left( \frac{1 - \cos \beta}{2} \right)$$  \hspace{1cm} (II.4)

Where $\beta$ is the slope of the PV panel, $\rho$ is reflection coefficient of the ground and when unknown it is usually assumed equal to 0.2.

Given its anisotropic behavior, Hay and Davies [3] developed a model for diffuse irradiation on a tilted surface. The model divides diffuse irradiation into two components: a circumsolar component coming directly from the direction of the sun, and an isotropic component coming from the entire celestial hemisphere defined respectively by:

$$I_{DT}^{\text{C}} = \frac{I_d k_1}{\cos \theta_{zs}} \max(0, \cos \theta)$$  \hspace{1cm} (II.5)

$$I_{DT}^{\text{I}} = I_d (1 - k_1) \left( \frac{1 + \cos \beta}{2} \right)$$  \hspace{1cm} (II.6)

$\theta_{zs}$ is the zenith angle, the angle of incidence of beam radiation on a horizontal surface and $\theta$ is the angle of incidence, defined as the angle between the beam radiation on a surface and the normal to that surface.

The variable $k_1 = I_b / l_0$ is the anisotropic coefficient and expresses the weighing of each component. Diffuse irradiation on a tilted surface is then obtained by adding its two components, $I_{DT}^{\text{C}}$ and $I_{DT}^{\text{I}}$. Beam irradiation on an inclined surface comes as:

$$I_{BT} = \frac{l_0}{\cos \theta_{zs}} \max(0, \cos \theta)$$  \hspace{1cm} (II.7)

Having calculated both diffuse and beam components and using II.8, the final value for irradiation on a tilted surface is achieved. Though the values where treated in hourly intervals, when accounting for irradiance influence on PV cell parameters, one has to use irradiance ($W/m^2$) and not irradiation ($Wh/m^2$) values, thus, assuming irradiance is constant during the hour period $G_T = I_T / 3600$.

D. Incidence Angle Modifier

The incidence angle modifier (IAM) accounts for the amount of irradiance that is not absorbed when the irradiance direction is not perpendicular to the plane of the array. As the incidence angle $\theta$ increases, the amount of radiation reflected from the cover also increases.

An approximation is made for the glass transmittance considering both reflective and absorption losses and is defined as the transmittance-absorptance product, $\tau \alpha$. In an angle normal to surface, the transmittance-absorptance product is given by:

$$\tau \alpha(0) = e^{-kL} \left[ 1 - \left( \frac{n_{glass} - n_{air}}{n_{glass} + n_{air}} \right)^2 \right]$$  \hspace{1cm} (II.9)

Where $K$ is the glazing extinction coefficient, assumed as $4 m^{-1}$ and $L$ is the glazing thickness, usually provided by the manufacturer. Refractive indexes for glass and air are set to 1.526 and 1 respectively. The value for $\tau \alpha$ on an incidence angle $\theta$ is found through:

$$\tau \alpha(\theta) = e^{-((KL) / \cos \theta_T)} \times \left[ 1 - \frac{1}{2} \left( \sin(\theta_T - \theta) \right)^2 + \left( \tan(\theta_T + \theta) \right)^2 \left( \tan(\theta_T - \theta) \right)^2 \right]$$  \hspace{1cm} (II.10)

The angle of refraction $\theta_T$ is determined from Snell’s law, $n_{air} \sin \theta = n_{glass} \sin \theta_T$.

To obtain de incidence angle modifier, $K_{\tau \alpha}$:

$$K_{\tau \alpha} = \frac{\tau \alpha(\theta)}{\tau \alpha(0)}$$  \hspace{1cm} (II.11)

Separate IAM are needed for diffuse, beam and ground-reflected irradiance. Average angles for isotropic diffuse and ground-reflected irradiance are provided as a function of the slope of the panel as determined by Brandemuehl and Beckman [4]. Circumsolar diffuse irradiance is treated as having the same incidence angle as beam irradiance, since it’s the diffuse irradiance coming from the direction of the sun. This method for estimating the IAM is used in all of the following results for the 5 parameter model.

E. Air Mass Modifier

Selective absorption by species in the atmosphere causes the spectral content of irradiance to change, changing the spectral
distribution of the radiation incident on the PV panel. King et al. [5] established an empirical relation to account for air mass:

$$\frac{M}{M_{\text{ref}}} = \sum_{i=0}^{4} a_i (AM)^i$$  \hspace{1cm} (II.12)

Where $AM = 1/\cos \theta_e$ is the air mass and constants $a_0$, $a_1$, $a_2$, $a_3$ and $a_4$ are different for each PV material and are provided by the Sandia National Laboratories [6].

Finally, the effective irradiance incident on an inclined surface is given by:

$$G_r = \alpha(0)M \left[ (G_{br} + G_{bt})K(\tau_a) + G_{bt}K(\tau_a) \right] + G_d \left( \frac{1 - \cos \beta}{2} \right)K(\tau_a)$$  \hspace{1cm} (II.13)

F. Solar Time

Solar Time is the time used in all of the sun-angle relationships, thus, when performing the calculations presented, one has to use solar time and not the local clock time. Conversion from local time to solar time requires two corrections.

First a longitude correction for the difference in longitude between the PV system’s meridian and the Greenwich meridian. The second correction is from the equation of time, which takes into account the perturbations in the earth’s rate of rotation which affects the time the sun crosses the PV system’s meridian.

Solar Time = Local Time + 4(LST − Long) + E  \hspace{1cm} (II.14)

Where LST is Local Standard Meridian, obtained by multiplying the time difference between local time and GMT by 15. Long is the PV system location’s longitude and E is the equation of time, expressed in minutes and equal to:

$$E = 229.2(0.000075 + 0.001868 \cos B - 0.032077 \sin B - 0.014615 \cos 2B - 0.04089 \sin 2B)$$  \hspace{1cm} (II.15)

B is given by 360($d_n - 1$)/365 and $d_n$ is the day of the year.

III. The 5 Parameter Model

A. Parameters on Standard Test Conditions

In an ideal model, the photovoltaic cell could be represented by the Light Current, $I_L$ in parallel with the diode. $I_L$ represents the current when photoelectric effect occurs. In the absence of light, the cell behaves as diode and doesn’t produce any current. However, given the non-ideal behavior of the cell, the parallel and series resistance are added to the model. $R_s$ stands for the structural resistances in the PV module and $R_{sh}$ denotes the leakage current in the p-n junction. According to the equivalent circuit shown in Figure 1, the PV module current-voltage relationship for a specified set of conditions is given by:

$$I = I_L - I_{0d} \left[ \exp \left( \frac{V + R_s I}{R_{sh}} \right) - 1 \right] - \frac{V + R_s I}{R_{sh}}$$  \hspace{1cm} (III.1)

Where $V_t$ is the cell thermal voltage:

$$V_t = \frac{kT_{\text{ref}}}{q}$$  \hspace{1cm} (III.2)

$k$ is the Boltzmann’s constant, $q$ is the electron charge and $T_{\text{ref}}$ if the temperature at STC in kelvins.

The 5 parameters of the model are the resistances $R_s$ and $R_{sh}$, the diode ideality factor $n$, light current $I_L$ and diode inverse saturation current $I_{0d}$. As mentioned before, the model only requires the knowledge of data provided by the manufacturer, most datasheets provide values for open circuit voltage $V_{oc}$, short circuit current $I_{sc}$, current and voltage at the maximum power point (MPP) $I_{mp}$ and $V_{mp}$ and the number of module cells $N_s$.

Eq. III.1 is evaluated at three major points, the short circuit point where $V = 0$, III.3, the open circuit point where $I = 0$, III.4, and the maximum power point where $V = V_{mp}$ and $I = I_{mp}$, III.5.

$$I_{sc} = I_L - I_{0d} \left[ \exp \left( \frac{R_s I_{sc}}{N_s V_t n} \right) - 1 \right] - \frac{R_s I_{sc}}{R_{sh}}$$  \hspace{1cm} (III.3)

$$0 = I_L - I_{0d} \left[ \exp \left( \frac{V_{oc}}{N_s V_t n} \right) - 1 \right] - \frac{V_{oc}}{R_{sh}}$$  \hspace{1cm} (III.4)

$$I_{mp} = I_L - I_{0d} \left[ \exp \left( \frac{V_{mp} + R_s I_{mp}}{N_s V_t n} \right) - 1 \right] - \frac{V_{mp} + R_s I_{mp}}{R_{sh}}$$  \hspace{1cm} (III.5)

Equations III.3, III.4 and III.5 are three independent equations with five unknown variables, thus, two more equations are necessary. Proposed equations are:

$$\frac{\partial P}{\partial V} \bigg|_{V=V_{mp}} = 0$$  \hspace{1cm} (III.6)

$$\frac{\partial I}{\partial V} \bigg|_{V=0} = -\frac{1}{R_{sh}}$$  \hspace{1cm} (III.7)

The model proposed by Hejri et al. [7] resides in reducing the proposed system at three independent equations with three unknown variables. The need to reduce the system is due to the very small value $I_{0d}$ which may cause the system to converge
to a non-desirable solution. Therefore, after some manipulation of the system, the three following equations are presented:

\[
0 = \frac{I_{mp}}{V_{mp}} - \frac{1}{\gamma V_t} \left(1 - R_s \frac{I_{mp}}{V_{mp}}\right) \times \left(-\frac{V_{oc} + (R_s + R_{sh})I_{sc}}{R_{sh}}\right) \\
\times \exp\left(\frac{V_{mp} - V_{oc} + R_s I_{mp}}{\gamma V_t}\right) - \frac{1}{R_{sh}} \left(1 - R_s \frac{I_{mp}}{V_{mp}}\right)
\] (III.8)

\[
0 = -I_{mp} \frac{1 + R_s}{R_{sh}} + \frac{-V_{oc} + (R_s + R_{sh})I_{sc}}{R_{sh}} \\
\times \left[1 - \exp\left(\frac{V_{mp} - V_{oc} + R_s I_{mp}}{\gamma V_t}\right)\right] + \frac{V_{oc} - V_{mp}}{R_{sh}}
\] (III.9)

\[
0 = -\frac{R_s}{R_{sh}} + \frac{R_{sh} - R_s}{\gamma V_t} \\
\times \left(-\frac{V_{oc} + (R_s + R_{sh})I_{sc}}{R_{sh}}\right) \exp\left(\frac{R_s I_{sc} - V_{oc}}{\gamma V_t}\right)
\] (III.10)

For numerical solution of Equations III.8-III.10 with three unknown variables \(R_s\), \(R_{sh}\) and \(\gamma\) (defined as \(N_x \times n\)), a good estimation for the initial point is needed, equations III.11-III.13 are proposed:

\[
\gamma = \frac{2V_{mp} - V_{oc}}{V_t \ln\left((I_{sc} - I_{mp})/I_{sc}\right) + I_{mp}/(I_{sc} - I_{mp})}
\] (III.11)

\[
R_{sh} = \sqrt{\frac{R_s}{V_{sc}/V_t} \exp\left((R_s I_{sc} - V_{oc})/\gamma V_t\right)}
\] (III.12)

\[
R_s = \frac{V_m}{I_m} - \frac{(2V_m - V_{oc})/(I_{sc} - I_m)}{\ln((I_{sc} - I_m)/I_{sc}) + I_m/(I_{sc} - I_m)}
\] (III.13)

In some cases, in which the PV module has very small series resistance, \(R_s\), equation III.13 results in and unrealistic negative value. In these cases, the initial value for \(R_s\) is set to zero and initial value for \(R_{sh}\) and \(\gamma\) is obtained through:

\[
\gamma = \frac{V_m - V_{oc}}{V_t (1 - I_m/I_{sc})}
\] (III.14)

\[
R_{sh} = \frac{I_m}{V_m} - \frac{I_{sc} - I_m}{V_m - V_{oc}} \ln\left(1 - \frac{I_m}{I_{sc}}\right)
\] (III.15)

Having solved the system of equations III.8-III.10, two parameters remain to be determined, \(I_L\) and \(I_{od}\).

\[
I_{od} = \frac{I_{sc} - V_{oc} - R_s I_{sc}}{R_{sh}} \exp\left(\frac{V_{oc}}{\gamma V_t}\right) - \exp\left(\frac{R_s I_{sc}}{\gamma V_t}\right)
\] (III.16)

\[
I_L = I_{od} \left[\exp\left(\frac{V_{oc}}{\gamma V_t}\right) - 1\right] - \frac{V_{oc}}{R_{sh}}
\] (III.17)

All five parameters of the model can be calculated with the previous equations. However, standard test conditions are rarely found on real environments, thus, equations considering the influence of the irradiance and cell temperature on the parameters are in need.

**B. Dependence of parameters on operating conditions**

Both irradiance and cell temperature have an effect on cell parameters. The cell temperature is dependent on ambient temperature and effective irradiance accordingly to:

\[
T = T_{amb} + \frac{NOCT - 20}{800} G
\] (III.18)

\(T\) is the cell temperature, \(T_{amb}\) is the ambient temperature, \(G\) is the effective irradiance on the panel surface (calculated as shown in the section II.C) and the Nominal Operating Cell Temperature (NOCT) is temperature at an irradiance of 800 \(W/m^2\), ambient temperature of 20\(^\circ\)C and wind velocity equal to 1 \(m/s\).

The cell thermal voltage depends on temperature linearly:

\[
V_t = \frac{V_{t,ref} T}{T_{ref}}
\] (III.19)

The subscript ref specifies parameters on reference conditions obtained in the previous section. The material band gap \(E_g\) exhibits a small temperature dependence and its reference value for silicon cells is 1.12 eV

\[
E_g = \left(1 - 0.000267(T - T_{ref})\right) E_{g,ref}
\] (III.20)

As presented by Messenger and Ventre [8] the diode reverse saturation current, \(I_{od}\) depends on cell temperature, material band gap and cell thermal voltage according to:

\[
I_{od} = I_{od,ref} \left(\frac{T}{T_{ref}}\right)^3 \exp\left[\frac{1}{n} \frac{E_{g,ref} - E_g}{V_t}\right]
\] (III.21)

Open circuit voltage \(V_{oc}\) and light current \(I_L\) depend simultaneously on cell temperature and irradiance. For start, it would only be considered the effect of temperature as given by:

\[
I_{LT} = \frac{M}{M_{ref}} \left(I_{L,ref} + \alpha_{isc}(T - T_{ref})\right)
\] (III.22)

\[
V_{oc,T} = V_{oc,ref} + k_v(T - T_{ref})
\] (III.23)

\(\alpha_{isc}\) and \(k_v\) are the temperature coefficients for the short circuit current and open circuit voltage respectively, both values are given by the manufacturers. The short circuit current is a
linear function of the irradiance:

\[ I_{sc} = I_{sc,ref} \frac{G}{G_{ref}} \quad (\text{III.24}) \]

The shunt resistance is inversely proportional to absorbed radiation:

\[ R_{sh} = R_{sh,ref} \frac{G_{ref}}{G} \quad (\text{III.25}) \]

To complete the model, the effect of irradiance on \( V_{oc} \) and \( I_L \) remains to be accounted. \( I_L \), same as \( I_{sc} \) is a linear function of irradiance:

\[ I_L = I_{L,T} \frac{G}{G_{ref}} \quad (\text{III.26}) \]

The equation for open circuit voltage dependence on irradiance requires an iterative calculation method for which the initial value proposed is the one obtained in equation III.23

\[ V_{oc} = \gamma V_{ref} \ln \left( \frac{I_{mp} R_{sh} - V_{oc}}{R_{sh} I_{oc}} \right) \quad (\text{III.27}) \]

### C. Maximum Power Point

Under determined set of operating conditions for the PV cell, there is a unique point in its IV Curve when the product of voltage versus current is maximum.

On a mathematical approach, this point is where the power derivative in terms of voltage is zero as shown in equation III.6. Most PV systems possess a charge controller with a Maximum Power Point Tracker (MPPT), whose main goal is to guarantee that the module operates in this particular point, therefore, its output it’s the maximum power. In order to obtain maximum power \( P_{max} \), the maximum voltage and maximum current are calculated separately recurring to the system of equations described as:

\[ I_{mp} = I_{sc} - \left( \frac{V_{mp} + R_{L} I_{mp} - R_{L} I_{sc}}{R_{sh}} \right) - \left( \frac{V_{oc} - R_{L} I_{sc}}{R_{sh}} \right) \times \exp \left( \frac{V_{mp} + R_{L} I_{mp} - V_{oc}}{\gamma V_{t}} \right) \quad (\text{III.28}) \]

\[ \frac{\partial I}{\partial V} = V_{mp} \left( \frac{\text{num}}{\text{den}} \right) + I_{mp} \quad (\text{III.29}) \]

\[ \text{num} = -1 - \left( \frac{R_{sh} I_{sc} - V_{oc} + R_{L} I_{sc}}{\gamma V_{t} R_{sh}} \right) \times \exp \left( \frac{V_{mp} + R_{L} I_{mp} - V_{oc}}{\gamma V_{t}} \right) \quad (\text{III.30}) \]

\[ \text{den} = 1 + \frac{R_{L}}{R_{sh}} + \frac{R_{L} R_{sh}}{\gamma V_{t} R_{sh}} \times \exp \left( \frac{V_{mp} + R_{L} I_{mp} - V_{oc}}{\gamma V_{t}} \right) \quad (\text{III.31}) \]

### IV. Off-Grid System

Off-grid systems are often used in remote locations where utility service isn’t reliable or doesn’t exist at all. The system’s main components are an energy storage (mostly batteries), charge controllers to monitor current flow to and from the batteries, an inverter to supply the AC loads and the PV generator itself.

The data provided belongs to a system located in Lisbon Tech. The battery bank is composed of 8 LiFePO4 batteries, a battery management system (BMS), a charge controller, the inverter and an AC Load consisting of a fridge, a TV and a Lamp. The PV generator is directly coupled with the battery bank, meaning the PV modules aren’t operating on their MPP.

Irradiance and temperature measurements were taken from the meteorological station within the campus. The data was treated in hourly intervals, for example, irradiance at noon is considered constant and is acquired through the mean value between 11:30 and 12:30. The pyranometer is installed horizontally, thus, the steps provided to convert GHI to irradiance in a tilted surface are necessary.

One detail about the system needs to be consider, unfortunately, due to the system’s location, shading effects become evident around 14:00/15:00, therefore, data obtain after these hours will not be considered.

### A. The Battery Model

Hanser et al [9] [1994] developed a model for a Stand Alone PV System in simulink with the main purpose to establish a library of simple mathematical models for each individual element of the system, namely the PV modules, batteries, controller, inverter and load. This model was studied and implemented in this research.

The Kinetic Battery Model (KiBaM) presented by Manwell & McGowan [9] was used to model the battery and doesn’t require extra voltage or current measurements. The model is composed of two major parts describing a capacity model and a voltage model.

The basis of the capacity model is the assumption that some of the capacity in the battery is immediately available for the load and the rest is chemically bound. It’s easier to imagine as if capacity is held in two tanks:

- Tank 1 with width \( c \), containing the available capacity \( q_1 \);
- Tank 2 with width \( 1-c \), containing the bound capacity \( q_2 \);

These two tanks are separated by a conductance \( k \), denoting the rate at which the available charge is filled from the chemical bound storage. The process by which bound charge becomes available is proportional to difference in the head of two tanks \( (h_1 = q_1/c \text{ and } h_2 = q_2/(1-c)) \). The model is characterized by three constants \( k, c \) e \( q_{max} \) (\( q_{max} \) denotes the maximum battery capacity when discharging at a very slow rate, \( I \equiv 0 \)) through the system of equations:
\[
\begin{aligned}
\frac{dq_1}{dt} &= -I - k(1 - c)q_1 + kcq_2 \\
\frac{dq_2}{dt} &= k(1 - c)q_1 - kcq_2
\end{aligned}
\] (IV.1)

Where constant \( k \) is defined as \( k = k^1/c(1 - c) \) Following the model footsteps, the three constants are acquired and given by:

\[
\begin{aligned}
c &= 0.8523 \\
q_{\text{max}} &= 156.9632 \text{ Ah} \\
k &= 2.3394 \text{ h}^{-1}
\end{aligned}
\] (IV.2)

The Voltage Model provides the magnitude of the terminal voltage affected by different depths of charging and discharging. In the KiBaM model it is assumed that the battery terminal voltage, \( E \), varies with the state of charge and current as follows:

\[
\begin{aligned}
E &= E_0 + AX + \frac{CX}{D - X}
\end{aligned}
\] (IV.3)

- \( E_0 \) is the extrapolated voltage at zero current for a fully charged battery;
- \( A \) represents the initial linear variation of internal battery voltage with the state of charge;
- \( C \) and \( D \) are the parameters reflecting the sharpness of end-of-discharge voltage.
- \( X \) is the normalized capacity removed from the battery at a given discharge current, \( X = q_{\text{max}} q_{\text{out}}/q_{\text{max}}(I) \):

\( q_{\text{max}}(I) \) is the capacity of the battery at given discharge current \( I \) and \( q_{\text{out}} \) is the amount of charge that has been removed at a certain point of the discharge curve.

The voltage model is characterized by the four constants, \( E_0, A, C \) and \( D \) which were calculated as the method describes and are given by:

\[
\begin{aligned}
A &= -0.0006 \text{ V/Ah} \\
C &= -0.023 \text{ V} \\
D &= 155.9978 \text{ Ah} \\
E_0 &= 3.32 \text{ V}
\end{aligned}
\] (IV.4)

B. System's Model

As mentioned the system is composed by five major components. The battery model was displayed in the previous section, now the model for the remaining components will be briefly explained.

The charge controller is used to manage energy flow to PV system, batteries and load by analysing the battery’s voltage and knowing the maximum and minimum values accepted. It has two main operating modes: the normal operating mode, when battery voltage is between its maximum and minimum values and the overcharge/over-discharge modes, which occur when the battery voltage reaches critical values. Thus the charge controller model receives terminal voltage measurements at each step and decides whether or not to break the circuit on the PV side or the load side.

The inverter is responsible for the conversion of the DC current originated from the PV panel to AC current necessary to supply AC loads. The main goal of the inverter is to keep on the AC side the voltage constant at the rated voltage 230 V and to convert the input power into the output power with the best possible efficiency, \( \eta \). Since the load’s power factor \( \cos \varphi \), and current \( I_{\text{AC}} \), are provided, the DC current required by the inverter from the DC side is given by:

\[
I_{\text{DC}} = \frac{V_{\text{AC}}I_{\text{AC}}\cos \varphi}{\eta V_{\text{DC}}}
\] (IV.5)

The PV model block receives information on the parameters calculated through the presented 5 Parameter Model along with the voltage from the batteries, allowing for current determination.

C. Results Analysis

In order to validate the previous model, two sets of data were analysed:

- December 6th of 2014. Difference between local time and solar time is 28 minutes. Battery initial voltage is 24.079 V;
- March 18th of 2015. Difference between local time and solar time is 47 minutes. Battery initial voltage is 26.297 V;

The first step was to obtain tilted irradiance and cell temperature hourly values. It was a typical day for solar irradiance on December 6th, as seen in Figure IV-1.

Figure IV-1 - Irradiance profile on tilted plane for December 6th

On the March 18th, diffuse fraction of solar irradiance became prominent around midday as shown in Figure IV-2.
It was given particular emphasis on the battery behaviour as predicted by the KiBaM model. Since the PV Voltage is imposed by the battery voltage, a wrongfully implementation of the KiBaM model could result in significant errors on the PV output. Curves for both the PV and battery voltage can be seen in Figure IV-3.

The KiBaM model gives better results when separate constants of the voltage model are obtained for charge and discharge. Regardless, since the manufacturer doesn’t provide battery charge tests, the constants acquired through the discharge curve were used in both scenarios. It’s perceptible that parameter C was the biggest influence on the differences observed in December 6th, originating a simulated charge curve more ‘flatten’ than the real one. This can be explained by the voltage initial value for this day - lower than the value for March 18th – which results in parameter C having a higher influence on the curve. However, in terms of percentage error, both days had very small values, ranging from 0.1% to 3%.

PV Voltage is always higher than the battery voltage since voltage drops were observed due to the resistance of the cables connecting both components.

<table>
<thead>
<tr>
<th>Percentage Error of $I_{pv}$ (%)</th>
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<tbody>
<tr>
<td>December 6th</td>
</tr>
<tr>
<td>7h</td>
</tr>
<tr>
<td>8h</td>
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<tr>
<td>9h</td>
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<tr>
<td>10h</td>
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<td>11h</td>
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<tr>
<td>12h</td>
</tr>
<tr>
<td>13h</td>
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<tr>
<td>14.97</td>
</tr>
</tbody>
</table>

Errors observed for the PV Current calculations were mostly due to some limitations on the provided data. The data acquisition systems for the PV system and values of irradiance/temperature had different sample times of one minute and 10 to 15 minutes respectively. This difference has a bigger impact on cloudy days and in the early day hours where irradiance is increasing rapidly, causing the PV System acquisition to obtain data for irradiances that are not measured, - namely very low irradiances on cloudy days where the PV Current can reach zero.

V. GRID CONNECTED SYSTEM

Grid Connected System are systems where the energy produced by the PV System is either for immediate consumption or it’s delivered to the electricity grid. During nights, the energy necessary to satisfy demand is bought back from the grid.

The analyzed data belongs to a system located in Faculdade de Ciência da Universidade de Lisboa. The irradiance on the tilted surface is provided, simplifying the calculations. Temperature values are once more obtained from the meteorological station in the Lisbon Tech. Values for DC current and voltage for 3 parallel strings of 23 modules each are also provided. Data was treated equally to the one in the
previous system, assuming irradiance and temperature constant on hourly periods. Three sets of data were observed:
- May 16th of 2015;
- April 30th of 2015;
- March 22nd of 2015.

The irradiance profiles for the day of May 16th and April 30th were very similar. As expected, cell temperatures were higher on May 16th.

In Table 2 are presented the percentage errors for both days. Some important facts were taken by observing this values and by comparing simulated and real PV Current. PV current simulated for early day hours is always higher than the real one, resulting in higher errors on PV Power. Between 12:00 and 17:00, the error decreases significantly, exhibiting very acceptable values. In the later hour of the day, 18:00, the error reaches absurdly higher values, and the simulated current is much lower than the real one.

To clarify the values shown in Table 2, the position of the PV modules along with the meteorological station that provides the irradiance values are shown in Figure V-II.

The meteorological station is at ground level and the PV modules are on top of buildings 15 to 20 meters high. Major conclusions were taken by observing Figure V-II:
- In the morning, due to the reflected irradiance caused by the building in blue, measured irradiance is overvalued, causing the simulated PV Current to have higher values than the real current.
- In the late afternoon, the station is shaded by the building in blue and in this case the measured irradiance is undervalued, causing the simulated PV Current to have lower values than the real current.

Subsequently, the day of March 22nd was studied. Similar to the previous tests, the first step was to obtain tilted irradiance and cell temperature for the hourly intervals. The irradiance profile for this day differs from the other two days, having a higher percentage of diffuse irradiance.
Table 3 - Percentage errors for the Power output in March 22nd.

<table>
<thead>
<tr>
<th>Percentage Error of $P_{pv}$ (%)</th>
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<tbody>
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<td>8h</td>
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<td>17h</td>
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<td>18h</td>
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</table>

There is an interesting fact to be noted on the above table. The error value for 18:00 is much lower than the ones presented in Table 2. The fact that this day had a bigger percentage of diffuse fraction actually caused the pyranometer to have a lower measurement error. Irradiance values for 18:00 on March 22nd are effectively lower meaning the shading effect of the blue building doesn’t cause such discrepancy between the irradiance on the panels and the irradiance on the meteorological station.

VI. CONCLUSIONS

The 5 Parameter Model used in this research work predicts energy production of the PV Cell using only data provided by the manufacturers. This study determines that the 5 Parameter Model is overall a great tool and its method is very simple when compared with other models such as King et al that requires additional tests.

For an isolated system, this type of predictions requires a lot of extra work, which will subsequently cause superior errors on the results, namely through the battery model, which was initially designed for lead-acid battery banks. This system main obstacle was the data acquisition systems. It would have been of interest to have the same rate sample for irradiance and PV output data. On the grid-connected system the model for the cell was more accurate. The most notable comparisons could be made between 11:00 and 17:00 when the difference in positions of the meteorological stations and the PV Models didn’t show great impact.

Due to the both systems observations, it is concluded that to have a good prediction of energy output it’s of extremely importance to obtain a correct irradiance profile for the location.

To increase the model’s accuracy De Soto [11] suggested that manufacturers should provide two sets of data at two different reference conditions. It is also suggested that they provide the value for the slope at the short circuit current, which will allow for direct calculation of the shunt resistance.

Another point of interest is to account for the cell temperature dependence on wind, and not just on ambient temperature and irradiance. In windy locations, cell temperature is expected to decrease, causing the output power to increase. The non-accountancy of wind could cause over prediction of PV rated power, therefore causing unnecessary costs.

References