GPS Heading and Pitch Estimation using Single-Frequency, Dual-Frequency or Wide-Lane Measurements

Rita Maria dos Santos Pereira

IT - Instituto de Telecomunicações
Instituto Superior Técnico, Technical University of Lisbon, Portugal
rita.santos.pereira@tecnico.ulisboa.pt

Abstract—The Global Positioning System allows to estimate the attitude of a vehicle by using multiple receivers with the antennas in fixed positions. The carrier phase measurements, with millimeter precision, are used for this purpose. However, these measurements have a number of integer cycles that is unknown, to which it is given the name of phase ambiguity. The LAMBDA method is currently acknowledged as the most efficient method for determining the phase ambiguities, mainly when dual-frequency measurements (L1 and L2) are used. Alternatively, the L1 and L2 frequencies can be combined to create a new set of measurements (wide-lane) with which the LAMBDA can also be applied. Using only L1 frequency, the LAMBDA is not enough to solve the ambiguities, so, the method known as the Ambiguity Filter is used to stabilize the solution obtained. With the phase ambiguities determined, the attitude of a vehicle can be estimated with high precision.

In this paper, the attitude is determined by using measurements from frequencies L1 and L2, wide-lane and only frequency L1, in order to compare these three options and to verify that the results obtained with L1 frequency can achieve the same precision as the other two options. Besides this, the Ambiguity Filter is studied more thoroughly, leading to the implementation of a new verification step within this filter. The presented results were obtained with field tests performed with two receivers, allowing the determination of two attitude angles (pitch and heading).

Keywords: GPS, attitude determination, phase integer ambiguity, double differences, Ambiguity Filter, LAMBDA method

I. INTRODUCTION

By having multiple GPS receivers with antennas placed with known positions in a vehicle body frame (rigid body) and by estimating their relative positioning vectors (the vector between two antennas is called baseline), it is possible to obtain the orientation of the rigid body known as attitude (heading, pitch and roll). With two antennas, only two attitude angles can be known, like heading and pitch.

A high precision GPS attitude estimation system can be developed based on RTK (Real-Time Kinematics) techniques with which it is possible to estimate the baselines. This is done by using the GPS measurements with the carrier cycles information (carrier phase measurements) which are more precise than the measured ranges from the satellites to the receiver (pseudorange or code measurements). Nonetheless, the phase measurements have the disadvantage of having an unknown integer number of cycles that is known as Integer Ambiguity, [1], [2].

The problem of solving the unknown integer ambiguities has been thoroughly studied for some years now and the most representative methods are reviewed in [3]. The LAMBDA method, [4], is known as the most efficient and high success rate method, [3], [5]–[7]. Nevertheless, the success rate of this method depends on the use of dual-frequency measurements (L1 and L2). With single-frequency measurements (L1) the results for the ambiguities are unstable. For this case, the Ambiguity Filter method uses the baseline length as constraint to stabilize the LAMBDA solution and to select the best one, as described in [8]–[13].

The main objective of this work is the implementation of a tool that allows attitude determination by using GPS measurements in single-frequency, dual-frequency or wide-lane, allowing a performance comparison between these three options.

As said before, when single-frequency measurements are used, the Ambiguity Filter makes use of the prior knowledge of the distance between the antennas (baseline length). If the baseline length is not known with precision, the Ambiguity Filter could fix an incorrect set of ambiguities as fixed solution and, in this case, the solution for the baseline and attitude angles starts to degrade. In this work the performance of the Ambiguity Filter is tested, leading to the implementation of a new verification step within this filter in order to improve the detection of false locks for the ambiguity solution.

Throughout this work only two antennas and two receivers were used because only two dual-frequency receivers were available in the development of this thesis. However, the system could be adapted to use more receivers without difficulty. In this work, it is expected to obtain a precision of $10^{-2}$ degrees for the attitude angles, with two antennas at approximately 12 meters apart, when the Ambiguity Filter is applied with single-frequency measurements, as shown in [11].

This paper have a structure as follows. In Section II the system that represents the problem in study is defined. In Section III the ambiguity resolution is explained. In Section IV the methods for baseline and attitude determination are
explained. In Section [VI] the practical perspective of this work is addressed. In Section [VII] the results are presented and discussed, leading to the conclusions in Section [VII].

II. SYSTEM DEFINITION

DGPS is a method to improve GPS performance in many applications that require an increased level of precision. To use this method, at least two GPS receivers are needed, which are separated by a given distance (baseline length). The formation of carrier phase (phase) and pseudorange (code) double differences (DD or $\nabla \Delta$) removes the majority of the error sources (clock offsets, ionospheric and tropospheric delay) and is essential for the baseline estimation. [1], [2].

A. Observables

The carrier phase taken by a receiver $k$ to a satellite $p$, in meters, is as follows,

$$\phi_k^p = \rho_k^p + \lambda_c N_k^p + c(t_k^p + t_k) + T_k^p - I_k^p + \zeta_k^p$$

where

- $\rho_k^p$ is the distance between the receiver $k$ and the satellite $p$ (meters),
- $\lambda_c$ is the carrier wavelength (meters),
- $N_k^p$ is the unknown integer ambiguity (cycles),
- $c$ is the speed of light in vacuum (meters per seconds),
- $t_k^p$ is the satellite clock offset (seconds),
- $t_k$ is the receiver clock offset (seconds),
- $T_k^p$ is the delay due to the effects of the troposphere (meters),
- $I_k^p$ is the advance due to the effects of the ionosphere (meters),
- $\zeta_k^p$ is unmodeled noise due to various sources (hardware, multipath) for carrier phase measurements (meters).

A phase single difference (SD) can be created with a measurement from another receiver ($m$), for the same instant:

$$\Delta \phi_{km}^p = \phi_k^p - \phi_m^p = \Delta \rho_{km}^p + \lambda_c \Delta N_{km}^p + c \Delta t_{km} + \Delta \zeta_{km}^p$$

where

- $\rho_{km}^p$ is the distance between the two receivers $k$ and $m$ (meters),
- $\lambda_c$ is the carrier wavelength (meters),
- $N_{km}^p$ is the unknown integer ambiguity (cycles),
- $c$ is the speed of light in vacuum (meters per seconds),
- $t_{km}$ is the satellite clock offset (seconds),
- $t_k$ is the receiver clock offset (seconds),
- $T_{km}^p$ is the delay due to the effects of the troposphere (meters),
- $I_{km}^p$ is the advance due to the effects of the ionosphere (meters),
- $\zeta_{km}^p$ is unmodeled noise due to various sources (hardware, multipath) for carrier phase measurements (meters).

A phase single difference (SD) can be created with a measurement from another receiver ($m$) by subtracting both phase measurements, $\phi_k^p$ and $\phi_m^p$, for the same instant:

$$\Delta \phi_{km}^p = \phi_k^p - \phi_m^p = \Delta \rho_{km}^p + \lambda_c \Delta N_{km}^p + c \Delta t_{km} + \Delta \zeta_{km}^p$$

Forming a SD leads to the cancellation of both the satellite clock offset and the atmospheric effects (troposphere and ionosphere) since these are assumed to be equal for both receivers (for baselines with less than 50 kilometers, [1]).

Within the same logic, a double difference can be generated by using an additional satellite ($q$) and by subtracting the single differences of the two satellites ($\Delta \phi_{km}^p$ and $\Delta \phi_{km}^q$), as shown below

$$\nabla \Delta \rho_{km}^{pq} = \Delta \phi_{km}^p - \Delta \phi_{km}^q = \nabla \Delta \rho_{km}^p + \lambda_c \nabla \Delta N_{km}^p + \nabla \Delta \zeta_{km}^p$$

where $\xi$ is unmodeled noise due to various sources (hardware, multipath), in meters, but for pseudorange measurements.

Code double differences are unambiguous, however, they are noisier than phase double differences. The error in code DDs has a standard deviation with meter level and the error in phase DDs has a standard deviation with centimeter level, [1], [2]. This is the motivation for using the ambiguous phase measurements.

B. Baseline Vector and Double Differences

After defining the double differences, it is crucial to relate the DDs to the baseline (vector between the antennas of two receivers). Since the baseline (with meter level) is orders of magnitude shorter than the distance to a satellite (satellite orbit at a mean distance of 20, 200 km), the propagation paths between a satellite and the antennas are assumed to be parallel, [1]. The range of the single difference can be written as the inner product between the baseline and the LoS (Line of Sight) unit vector (direction cosine), as shown below

$$\Delta \rho_{km}^p = b \cdot e^p$$

where $b$ is the baseline between the two antennas of the receivers $k$ and $m$, and $e^p$ and $e^q$ are the LoS unit vectors for satellites $p$ and $q$, respectively, which are considered approximately equal for both receivers ($e_k^p \approx e_m^p \approx e^p$ and $e_k^q \approx e_m^q \approx e^q$). The determination of the direction cosines depends on the calculated satellite position and the estimated receiver’s position.

Thus, the double difference would have the form

$$\nabla \Delta \rho_{km}^{pq} = b \cdot (e^p - e^q) = b \cdot e^{pq}$$

which allows the formulation of code and phase DDs using the baseline and the direction cosines.

C. Observation Model

It is possible to create a system to define the problem in study by using equations (3) and (4) and combining them with equation (2). When only single-frequency measurements are used and for a case of a GPS constellation with $n$ satellites, the linear equation system has $2(n - 1)$ equations and can be reduced as follows

$$y_{[2(n-1)\times 1]} = B_{[2(n-1)\times 3][3\times 1]} + A_{[2(n-1)\times (n-1)][n-1]\times 1] + E_{[2(n-1)\times 1]}$$

where the dimensions of all matrices are indicated and,

- $y$ is the vector with the measured double differences (for both phase and code measurements),
- $B$ is the matrix for the baseline vector, containing the differenced unit vectors,
- $b$ is the vector with the baseline coordinates,
- $A$ is the matrix for the integer ambiguities, containing the carrier wavelength,
- $a$ is the vector with the integer phase ambiguities,
- $E$ is the vector of unmodeled effects and measurement noise.
The augmented system has enough observations to determine all states (integer ambiguities and baseline coordinates) when the satellite constellation has, at least, four satellites ($n \geq 4$), so the full rank of the augmented system matrix be equal to the dimension of the state vector. The augmented system matrix is given by $H = [A \ B]$, with dimension $(2(n - 1)) \times ((n - 1) + 3)$. The corresponding augmented state vector is $x = [ \alpha^T \ b^T]^T$, with dimension $((n - 1) + 3) \times 1$.

A dual-frequency receiver would provide another $2(\!n - 1)$ equations from the measurements at L2. With the conjunction of L1 and L2 measurements the new linear system is identical to the system presented in equation (7), but with a total of $(4(n - 1))$ equations. In this case the dimension of the augmented system matrix $H$ is $(4(n - 1)) \times (2(n - 1) + 3)$ and the dimension of the corresponding state vector $x$ is $(2(n - 1) + 3) \times 1$.

III. AMBIGUITY RESOLUTION

A floating point solution for the ambiguities can be found resorting to a Weighted Least Squares (WLS) algorithm to solve the system (7). This solution minimizes the residual error:

$$\min_{a,b} \| y - Aa - Bb \|^2_W$$

where $W$ is the weight matrix and $\| \cdot \|^2_W = (\cdot)^T W (\cdot)$. Nonetheless, the float solution does not lead to an accurate solution for the baseline vector (and attitude angles) because the correct ambiguities are integers (fixed solution). The float solution for the baseline vector ($b$) is recognized as the most efficient to determine the phase DDs to center the phase DDs, limiting the size of the integer ambiguity, $\delta b$.

Despite that, to find the correct integer solution, it is necessary to apply search techniques. In this paper the search method LAMBDA was used, which resorts to the float solution for the baseline vector (and attitude angles) because the integer ambiguity solutions are unacceptably unstable, this will be shown in the Results (Section VII). The candidates outputted from the LAMBDA method do not correspond with the descending order of accuracy. For that reason, it was necessary to apply search techniques. In this paper, an additional step, called Verification Step, is added to the Ambiguity Filter in order to improve the detection of false locks for the solution of the ambiguities.

1) Validation Step

In the validation step, it is defined a limit for the baseline length error ($\delta b$), leading to the exclusion of all candidates that do not fulfill the following condition:

$$\delta b = |\hat{b} - l| \leq 0.1 m.$$  \hspace{1cm} (9)

where $\hat{b}$ is the baseline length solution obtained with the candidate given by the LAMBDA method ($\hat{a}$) that needs to be tested and $l$ is the known baseline length.

It is assumed that the baseline length is precisely known ($l$), however, there are errors that must be accounted for. For this reason, the threshold for the error is usually set to 10 centimeters, [8]–[13].

2) Selection Step

In the selection step, the remaining candidates given by the LAMBDA are sorted according to the available tests. The three tests that can be employed for merit assignment are the Baseline Length Constraint and the Residual Ratio (presented in [8], [10], [11]) and the Up Coordinate Constraint (proposed in [9], [12], [13]).

Then, two of these tests are combined to create a metric that allows the selection of the best candidate for the ambiguities, in the current time epoch.

a) Baseline length constraint

The error of the estimated baseline is calculated by applying the equation (9) implemented in the validation step:

$$\delta b = |\hat{b} - l|.$$  \hspace{1cm} (10)

b) Residual Ratio

The residual ratio is calculated as shown below

$$\| V \|^2 = V^T (cov(\Delta \phi))^{-1} V$$  \hspace{1cm} (11)

where $\Delta \phi$ are the phase double differences, $cov(\cdot)$ is the corresponding covariance matrix and $V$ is the phase residual vector, given by

$$V = B\hat{b} + A\hat{a} - \nabla \Delta \phi.$$  \hspace{1cm} (12)

The phase residual vector $V$, in equation (12), is calculated by making the difference between the estimated phase DDs (calculated with each ambiguity set $\hat{a}$) and the corresponding baseline $\hat{b}$ and the measured phase DDs ($\nabla \Delta \phi$).

c) Up Coordinate constraint

The Up coordinate error can be calculated when the vehicle is stable during the initialization of the Ambiguity Filter and is given by

$$\delta u = |\hat{u}(\hat{a}) - u_{real}|.$$  \hspace{1cm} (13)
where \( \hat{u}(\hat{a}) \) is the estimated baseline Up coordinate (for each candidate given by LAMBDA, \( \hat{a} \), that needs to be tested) and \( u_{\text{real}} \) is the real baseline Up coordinate which is calculated by making the difference between the altitude of the two antennas.

d) Candidate Selection

For each one of the tests, all of the candidates are sorted in ascending order of the correspondent error, which corresponds with the descending order of merit. As defined in \( [9], [12], [13] \), the merit is assigned (for each test) to each one of the candidates according to the sorted position \( i \), as follows

\[
M_i = \frac{1}{i}. \tag{14}
\]

The candidates are sorted according to a certain metric that combines two of the tests defined, by summing the merit given to each candidate for both tests. The metrics used in this paper, to obtain the results presented in Section \( [V] \) are:

1) the Residual Ratio plus the Baseline Length Constraint (Metric 1).
2) the Baseline Length Constraint plus the Up Coordinate Constraint (Metric 2).

Using one of the metrics above and according definition of merit given, the candidate set with higher merit is chosen as the final solution for the current time epoch.

3) Stabilization Step

The two previous steps improve the success rate of the ambiguity solution. Despite this, there is still some noise that influences the solution and, its accuracy is still not acceptable. In order to stabilize the filter solution on the correct ambiguity set, the first step is to save all ambiguity sets chosen as the final solution in each one of the epochs by the Ambiguity Filter. So, when an ambiguity set was solution in 50 different time epochs (50 seconds), this ambiguity set is chosen as fixed solution for the subsequent epochs, as explained in \( [8] \).

3) The Proposed Verification Step

When using carrier phase measurements, it is possible to achieve results for the baseline vector with millimeter precision, however, this only happens when the solution for the ambiguities is correctly determined. If the Ambiguity Filter fixes an incorrect set of ambiguities, the solution for the baseline (and attitude angles) starts to degrade, which may not be immediately noticeable. An example of these two situations is shown in Figure 1, revealing the difference for the baseline length solution when the ambiguities are correctly and incorrectly fixed.

The proposed verification step is the main contribution of this work and was added to the Ambiguity Filter in order to improve the detection of incorrect solutions for the phase integer ambiguities. As previously explained, the Ambiguity Filter is dependent on the prior knowledge of the baseline length which will lead to wrong solutions when this value is not precisely known. With this verification step for detecting wrong solutions for the ambiguities, it is also possible to decrease the sensitivity of the Ambiguity Filter to the precision to which the baseline length has to be known.

All of the measurements from the available satellites, for which the ambiguities are known, are used to estimate the baseline vector. The verification step explores the redundant measurements, above the required minimum, that are used to estimate the baseline.

Over several tests, it was found that, when a satellite was lost at a given time and the set of ambiguities was incorrectly fixed, it was more likely the occurrence of a large jump in the baseline length solution, coincident with the moment when the satellite was lost. This does not happen when the set of ambiguities is correctly fixed. These two cases can be seen in Figure 1 where the satellite was lost around epoch 1300 seconds.

The observation of this behavior inspired the development of this additional verification step for the Ambiguity Filter, which was implemented according to the following guidelines. Whenever the solution for the ambiguities is considered fixed (stabilization step) it is tested if the baseline length obtained (with \( n \) satellites) suffers any sudden change by removing one or two satellites (solution obtained with \((n-1)\) or \((n-2)\) satellites), if enough redundant data is available. If it is verified the existence of a jump in the baseline length solution, typically above 5 millimeters, the correspondent solution for the phase ambiguities ceases to be considered fixed and the respective count for the stabilization step returns to 1. If redundant data is available this detection is immediate and thus the degradation that usually follows a false lock is not experienced.

B. Wide-Lane

For receivers that can track GPS satellites on both L1 and L2 frequencies, it is possible to create wide-lane measurements by combining the two frequencies, which is an alternative to use both L1 and L2 measurements in all of the methods previously described. The wide-lane frequency is given by

\[
f_{WL} = f_{L1} - f_{L2} = 1,575.42 - 1,227.6 = 347.82 \text{ MHz}. \tag{15}\]

The wide-lane phase measurement between the receiver \( k \) and the satellite \( p \) is:

\[
\phi_{k}[WL] = \phi_{k}[L1] - \phi_{k}[L2] \tag{16}\]

which must be used in conjunction with the narrow-lane pseudorange measurement given by

\[
PR_{k}[NL] = \frac{f_{L1}PR_{k}[L1] + f_{L2}PR_{k}[L2]}{f_{L1} + f_{L2}}. \tag{17}\]
The code and phase double differences formation, previously described in Section [III-A], is directly applicable to these new measurements and all of the methods applied for the L1 case, are also applicable to the wide-lane case. By using wide-lane measurements the carrier phase integer ambiguities are easily determined since the ionospheric delay is almost eliminated, however, the disadvantage of using wide-lane is an increased noise level, [1]. For the wide-lane case, a validation step could have been created to apply after the LAMBDA method, instead, the Ambiguity Filter was used, which was already implemented.

After having the fixed solution for the phase ambiguities (by using the Ambiguity Filter described in Section III-A), it is useful to revert to single-frequency measurements, since the noise is reduced and the signal strength of L1 C/A code is greater than the L2 P(Y) code, [1].

The phase double differences measurements from equation (5) can be expanded and rearranged as follows,

\[ \nabla \Delta \rho_{km[L1]}^p - \lambda_{c[L1]} \nabla \Delta N_{km[L1]}^p = \nabla \Delta \rho_{km[L1]} = \nabla \Delta \sigma_{km[WL]}^p - \lambda_{c[WL]} \nabla \Delta N_{km[WL]}^p. \]  

(18)

The recovery to the L1 integer ambiguities is given by,

\[ \nabla \Delta N_{km[L1]}^p = \frac{\nabla \Delta \sigma_{km[L1]}^p - \nabla \Delta \sigma_{km[WL]}^p + \lambda_{c[WL]} \nabla \Delta N_{km[WL]}^p}{\lambda_{c[L1]}}. \]  

(19)

Usually, the results from equation (19) are very close to integers so, the correct solution is given by the rounding to the nearest integer value. As explained in [1], extra care is needed because when there is enough noise the wrong ambiguity can be selected if the nearest integer is chosen.

IV. BASELINE AND ATTITUDE ESTIMATION

A. Baseline Estimation

One essential step to do, after the LAMBDA method and after the Ambiguity Filter (for single-frequency), is the estimation of the baseline fixed solution. The baseline can be estimated by adjusting the baseline float solution, \( \hat{b} \), resorting to the fixed integer ambiguities, [7], [16], [17].

After the fixed solution for the integer ambiguities (\( \hat{a} \)) has been determined, the baseline float solution (\( \hat{b} \)) is adjusted by using the residual (\( \hat{a} - \hat{a} \)). This will provide the least-squares estimate for the fixed baseline solution:

\[ \hat{b} = \hat{b} - Q_{ba} Q_{a\hat{a}}^{-1} (\hat{a} - \hat{a}). \]  

(20)

In equation (20), it is possible to see that the difference between the float and fixed baseline solutions (\( \hat{b} \) and \( \hat{b} \)) relies on the difference between the two ambiguity solutions (\( \hat{a} \) and \( \hat{a} \)).

If there are confidence that the integer ambiguity solution is correctly determined (if it can be assumed to be deterministic), [7], [16], [17], the covariance matrix of the baseline fixed solution is calculated by

\[ Q_b = Q_{b\hat{b}} Q_{a\hat{a}}^{-1} Q_{a\hat{b}}. \]  

(21)

As can be seen from the previous equation, the covariance matrix of the baseline fixed solution is smaller than the covariance matrix of the baseline float solution (\( Q_b < Q_{\hat{b}} \)) and, for this reason, the baseline float solution is less accurate than the baseline fixed solution. Although, the result might be the opposite if the ambiguities are incorrectly fixed.

B. Attitude Estimation

1) Body Frame and Euler Angles

Placing the receivers with the antennas in specific positions on the rigid body is the first step for attitude estimation, so the baselines are known in the body-frame coordinate system. For a case of an aircraft, the +x axis is pointing in the direction of the movement (nose of the aircraft), the +z axis is pointing down and the +y axis is such as to form a right-handed coordinate system. The body-frame coordinate system is fixed to the rigid body, in other words, it follows the body movement. Therefore, the position of the antennas is constant and known within the body-frame system.

The Euler angles relate the orientation of the body-frame coordinate system with the local coordinate system NED (North-East-Down). The attitude of a vehicle is given by the Euler angles, pitch (\( \theta \)), roll (\( \alpha \)) and heading (\( \psi \)). As shown in Figure 2, the pitch angle depicts the rotation about the y axis, the roll angle depicts the rotation about the x axis and the heading depicts the rotation about the z axis.

![Euler angles](Figure 2)

(a) Pitch  (b) Roll  (c) Heading

Fig. 2. Euler angles

2) Attitude (One Baseline)

To estimate the attitude of a vehicle with two receivers, the antennas are usually placed longitudinally in the vehicle (x axis in the body-frame), so the baseline vector is in the direction of the movement. The baseline vector should be converted to a local coordinate system, which is useful to have a better perception of the baseline.

For a single-baseline setup, it is possible to estimate the Euler angles heading (\( \psi \)) and pitch (\( \theta \)) from the definition of azimuth and elevation, respectively. This is done by resorting to the local baseline coordinates (\( b_n, b_e, b_d \)) (NED - North, East, Down), as shown below,

\[ \psi = \tan^{-1} \left( \frac{b_e}{b_n} \right) \]  

(22)

\[ \theta = -\tan^{-1} \left( \frac{b_d}{\sqrt{b_e^2 + b_n^2}} \right) \]  

(23)

where \( \psi \in [-\pi, \pi] \) and \( \theta \in [-\pi/2, \pi/2] \). Due to the range of \( \psi \), the signs of \( b_e \) and \( b_n \) should be taken in consideration, in order to place \( \psi \) in the right quadrant.
V. SOLUTION IMPLEMENTATION

To attain the proposed objectives, at least two receivers with raw data output capability are needed. Two dual-frequency receivers from Ashtech were used: ZXW-Sensor and ProFlex 500, both of them available at the IT GNSS Monitoring Station. Since these two receivers are the only dual-frequency receivers available, the problem in study was a single-baseline case.

To evaluate the performance of the methods previously described, under the best available conditions, static trials were conducted in the IT GNSS Monitoring Station at Instituto Superior Técnico (IST), University of Lisbon. The antennas are placed at the rooftop of the Northern Tower in the university campus and the details of this set-up are fully explained in [18].

Since, only two dual-frequency receivers are available, the problem in study is a single-baseline case. The distance between the antennas is 12.47 meters, thereby the baseline length is precisely known. Between the two receivers, the ZXW-Sensor was chosen as the reference antenna, so the baseline vector is pointing North. The expected heading is $-4^\circ$ degrees. The baseline up coordinate is expected to be zero (meters) because the antennas are placed at the same altitude. As such, the expected value for the pitch is zero degrees.

VI. RESULTS

A. Float Solution

A WLS estimator is applied with single-frequency, dual-frequency and wide-lane measurements. The corresponding results are shown in Figure 3 and the performance, mean value ($\mu$) and standard deviation ($\sigma$), of the float solution is summarized in Table I.

<table>
<thead>
<tr>
<th></th>
<th>Single Frequency</th>
<th>Dual Frequency</th>
<th>Wide-Lane</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heading  $^\circ$</td>
<td>$\mu$</td>
<td>-6.96769</td>
<td>-5.33597</td>
</tr>
<tr>
<td></td>
<td>$\sigma$</td>
<td>1.89753</td>
<td>1.82479</td>
</tr>
<tr>
<td>Pitch $^\circ$</td>
<td>$\mu$</td>
<td>-8.77316</td>
<td>-3.88675</td>
</tr>
<tr>
<td></td>
<td>$\sigma$</td>
<td>4.37230</td>
<td>4.62128</td>
</tr>
</tbody>
</table>

As can be seen, the obtained solution for single-frequency differs from the other two, this was expected because only the L1 measurements are used to obtain the solution. Despite this, the float solution estimated with a WLS algorithm (for single-frequency, dual-frequency and wide-lane) does not fulfill the intended objectives, as expected: the solution is noisy and unstable.

B. Smooth Solution

After creating the smoothed-code measurements, the smooth solution is estimated by using a WLS algorithm. This method is applied with single-frequency, dual-frequency and wide-lane measurements, the corresponding results are shown in Figure 4 and the performance of the smooth solution is summarized in Table II.

<table>
<thead>
<tr>
<th></th>
<th>Single Frequency</th>
<th>Dual Frequency</th>
<th>Wide-Lane</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heading  $^\circ$</td>
<td>$\mu$</td>
<td>-6.91838</td>
<td>-5.30982</td>
</tr>
<tr>
<td></td>
<td>$\sigma$</td>
<td>0.94202</td>
<td>0.48363</td>
</tr>
<tr>
<td>Pitch $^\circ$</td>
<td>$\mu$</td>
<td>-8.90254</td>
<td>-4.04151</td>
</tr>
<tr>
<td></td>
<td>$\sigma$</td>
<td>1.98083</td>
<td>1.21096</td>
</tr>
</tbody>
</table>

Just like the float solution, the smooth solution obtained with single-frequency measurements is the one that is more distinct from the other two cases. The smooth solution (for single-frequency, dual-frequency and wide-lane) is an improvement relatively to the float solution, since it uses a smoothed version of the code measurements, however, the obtained smooth solutions neither fulfill the intended objectives: the solution is less noisy but is also unstable.

C. LAMBDA Solution

The LAMBDA method is applied with single-frequency, dual-frequency and wide-lane measurements. The corresponding results are shown in Figure 5 and the performance of the LAMBDA solution is summarized in Table III.

The results obtained with the LAMBDA for single-frequency measurements are identical to the ones obtained with the float solution: noisy and unstable. For wide-lane measurements, the results are far more stable (despite for...
some cases when the wrong solution is selected) because by combining the measurements for frequencies L1 and L2 the search for the correct set of ambiguities becomes more efficient as fewer wavelength need to be searched. The best solution for the LAMBDA method is obtained with dual-frequency measurements (only a few wrong solutions are selected) because more observables are used to find the solution which makes the search process for the ambiguity set more efficient.

D. Final Solution

For single-frequency measurements, the Ambiguity Filter method (Section III-A) is applied, with an input value of 12.47 meters for the true baseline length, to the output of the LAMBDA. For dual-frequency measurements, a validation step is applied after the LAMBDA method to ensure the correctness of the solution. For wide-lane measurements, the Ambiguity Filter is also used after the LAMBDA, followed by

As can be seen, the mean values for the heading and pitch are close to the expected values (−4 degrees for heading and zero degrees for pitch). The results obtained with single-frequency (except at the beginning where the solution has not yet been stabilized) are similar to the ones obtained with wide-lane. This was expected because, after finding the correct ambiguity set for wide-lane, only the L1 measurements are used to calculate the final solution.

Nevertheless, the results obtained with the Ambiguity Filter for single-frequency measurements have the same level of precision as those obtained for both wide-lane and dual-frequency measurements. The final solutions have a precision with an order of magnitude of $10^{-3}$ degrees for the heading and $10^{-2}$ degrees for

<table>
<thead>
<tr>
<th></th>
<th>Single Frequency</th>
<th>Dual Frequency</th>
<th>Wide-Lane</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heading [°]</td>
<td>µ</td>
<td>σ</td>
<td></td>
</tr>
<tr>
<td></td>
<td>−6.91833</td>
<td>2.21341</td>
<td>2.13727</td>
</tr>
<tr>
<td>Pitch [°]</td>
<td>µ</td>
<td>σ</td>
<td></td>
</tr>
<tr>
<td></td>
<td>−8.68033</td>
<td>5.22547</td>
<td>5.9469</td>
</tr>
</tbody>
</table>
the pitch. It was expected that the pitch angle obtained would have less precision because the results for the baseline Up coordinate have less precision when compared to horizontal precision. This can be seen in Table V. These baseline results also confirm the expected value close to zero for the baseline Up coordinate.

![Fig. 6. Final Solution](image)

**TABLE V**

**Baseline (ENU coordinates) final solution performance**

<table>
<thead>
<tr>
<th></th>
<th>Single Frequency</th>
<th>Dual Frequency</th>
<th>Wide-Lane</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b_{\text{East}}) [m] (\mu)</td>
<td>-0.88177</td>
<td>-0.88294</td>
<td>-0.88169</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>0.00183</td>
<td>0.00191</td>
<td>0.00182</td>
</tr>
<tr>
<td>(b_{\text{North}}) [m] (\mu)</td>
<td>12.44095</td>
<td>12.44092</td>
<td>12.44090</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>0.00208</td>
<td>0.00147</td>
<td>0.00207</td>
</tr>
<tr>
<td>(b_{\text{U,p}}) [m] (\mu)</td>
<td>0.02557</td>
<td>0.01649</td>
<td>0.02579</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>0.00486</td>
<td>0.00388</td>
<td>0.00484</td>
</tr>
</tbody>
</table>

**E. Analysis of the Ambiguity Filter Method**

The Ambiguity Filter was developed to be used with single-frequency measurements, which has the advantage of using low-cost receivers and this is an alternative to the more expensive alternative of dual-frequency receivers. For this very reason, the Ambiguity Filter was tested more thoroughly. An aspect that influences the results from the Ambiguity Filter is the input of the prior knowledge of the baseline length, which the filter uses in the ambiguity resolution process, unlike the LAMBDA method. So, the performance of the Ambiguity Filter is sensitive to the precision with which the baseline length is known.

To analyze the sensitivity of the Ambiguity Filter (without the verification step) to the precision with which the baseline length has to be known, twelve different data acquisitions were made leading to twelve data sets acquired under the same conditions, in different days and hours of the day, over a period of three months (September-November, 2015). For each data set the Ambiguity Filter (using only single-frequency measurements) was tested with different input values for the baseline length, starting at its true value (12.47 meters). Then, the limits within which the solution obtained with the Ambiguity Filter remained correct were registered (in centimeters). This analysis were performed for the Metrics 1 and 2 (described in Section II-A). The summary of results obtained from these tests is shown in Table VI.

Comparing both metrics, it is possible to see that the results obtained are not always the same. The values highlighted in the Table VI show the cases when one of the metrics had a better performance than the other (comparing for the same data set). The Metric 2 shows an advantage (7 highlights against 5), although, these results are not sufficient to conclude that there is a clear advantage in using the Metric 2 in detriment of Metric 1.

For some data sets the bounds found have good values, like data set 2 (for Metric 2), data set 4 (for Metric 2) and data set 10 (for Metrics 1). However, some of the results were not as good which is shown by the values in bold (bounds of \(\pm 1 - 3\) cm) in Table VI. These differences between the results show that, in some cases, the Ambiguity Filter is highly dependent on the input value for the baseline length, which needs to be known with millimeter precision. With this, it is possible to conclude that the Ambiguity Filter algorithm needs improvements to become more flexible and less sensitive to the precision of the prior knowledge of the baseline length.

If the baseline length is not precisely known, the Ambiguity Filter may fix a wrong set of ambiguities as fixed solution. This will compromise the accuracy of the baseline solution (Figure 1) and, as a result of this, the attitude solution will also be compromised. To improve the detection of false locks for the ambiguity solution, a new verification step within the Ambiguity Filter was developed (Section II-A).

To verify if improvements in the results can be obtained with the implementation of the new verification step in the Ambiguity Filter, the same twelve data sets previously analyzed (Table VI) are tested again but, now, with the new step implemented in the filter. The summary of this new analysis is presented in Table VII.

Comparing Table VI and Table VII it can be seen that the results obtained without and with the verification step are not the same.

For some data sets, the bounds have good values (like data set 10), however, some other results are not as good (bounds of \(\pm 1 - 3\) cm) (emphasized in bold). Looking at the worst results in Table VI there are six worst results for Metric 1 and five for Metric 2. After the implementation of the verification step (Table VII), the number of worst results was reduced to three
for both metrics. For each data set, the results were compared between both metrics and the best ones were highlighted (lower and upper bounds). When a data set does not have two highlighted values (for two bounds), it means that the results obtained were the same for both metrics. In Table VI, Metric 2 shows an advantage (7 highlights) over Metric 1 (5 highlights), as said before. On the other hand, after the implementation of the new verification step (Table VII), Metric 1 shows an advantage (7 highlights) over Metric 2 (4 highlights). Although the results show an improvement with the implementation of the verification step, it is not possible to conclude that there is a clear advantage of one metric over the other.

Finally, when comparing the results from Table VI and Table VII, it can be seen that the implementation of the verification step in the Ambiguity Filter decreases the filter sensitivity to the precision to which the baseline length has to be known. From a total of 24 results for each metric, 10 results were not improved (marked with an asterisk), however, this does not always happen for the same combination of data set and bound in both metrics. Additionally, it was not observed any degradation of the results when the new verification step is used.

The results above show that the new verification step, implemented in the Ambiguity Filter, improves the detection of false locks for the ambiguities, contributing also to decrease the sensitivity of the Ambiguity Filter to the input of the baseline length.

VII. Conclusions

The main objective of this work was the implementation of a tool that allows attitude estimation (in the context of this work, heading and pitch) by using either single-frequency measurements, dual-frequency measurements or wide-lane measurements from GPS receivers. This objective was proposed to allow a performance comparison between these three options, more specifically to validate the results obtained with the Ambiguity Filter for single-frequency (L1 measurements), comparatively with the more expensive alternative of using

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Date</th>
<th>Time of Week [sec]</th>
<th>Number of Satellites</th>
<th>Bounds for Metric 1 [cm]</th>
<th>Bounds for Metric 2 [cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Lower</td>
<td>Upper</td>
</tr>
<tr>
<td>1</td>
<td>22 Sep 2015</td>
<td>205688 to 207868</td>
<td>6</td>
<td>−6</td>
<td>+1</td>
</tr>
<tr>
<td>2</td>
<td>22 Sep 2015</td>
<td>216339 to 218588</td>
<td>10</td>
<td>−8</td>
<td>+3</td>
</tr>
<tr>
<td>3</td>
<td>22 Sep 2015</td>
<td>223236 to 225438</td>
<td>9/8</td>
<td>−4</td>
<td>+5</td>
</tr>
<tr>
<td>4</td>
<td>20 Oct 2015</td>
<td>205385 to 207565</td>
<td>8</td>
<td>−2</td>
<td>+10</td>
</tr>
<tr>
<td>5</td>
<td>20 Oct 2015</td>
<td>209078 to 211264</td>
<td>9</td>
<td>−4</td>
<td>+7</td>
</tr>
<tr>
<td>6</td>
<td>20 Oct 2015</td>
<td>212604 to 214784</td>
<td>7</td>
<td>−6</td>
<td>+5</td>
</tr>
<tr>
<td>7</td>
<td>20 Oct 2015</td>
<td>223256 to 225436</td>
<td>10</td>
<td>−4</td>
<td>+6</td>
</tr>
<tr>
<td>8</td>
<td>20 Nov 2015</td>
<td>468128 to 470308</td>
<td>10</td>
<td>−7</td>
<td>+1</td>
</tr>
<tr>
<td>9</td>
<td>20 Nov 2015</td>
<td>471680 to 473860</td>
<td>9/8</td>
<td>−8</td>
<td>+2</td>
</tr>
<tr>
<td>10</td>
<td>20 Nov 2015</td>
<td>475280 to 477460</td>
<td>9</td>
<td>−8</td>
<td>+9</td>
</tr>
<tr>
<td>11</td>
<td>20 Nov 2015</td>
<td>478817 to 481119</td>
<td>9</td>
<td>−5</td>
<td>+6</td>
</tr>
<tr>
<td>12</td>
<td>20 Nov 2015</td>
<td>483140 to 485320</td>
<td>7</td>
<td>−2</td>
<td>+7</td>
</tr>
</tbody>
</table>

TABLE VI
EXPERIMENTAL OFFSET BOUNDS FOR THE BASELINE LENGTH WITHOUT VERIFICATION STEP ON THE AMBIGUITY FILTER

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Date</th>
<th>Time of Week [sec]</th>
<th>Number of Satellites</th>
<th>Bounds for Metric 1 [cm]</th>
<th>Bounds for Metric 2 [cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Lower</td>
<td>Upper</td>
</tr>
<tr>
<td>1</td>
<td>22 Sep 2015</td>
<td>205688 to 207868</td>
<td>6</td>
<td>−6*</td>
<td>+3</td>
</tr>
<tr>
<td>2</td>
<td>22 Sep 2015</td>
<td>216339 to 218588</td>
<td>10</td>
<td>−9</td>
<td>+9</td>
</tr>
<tr>
<td>3</td>
<td>22 Sep 2015</td>
<td>223236 to 225438</td>
<td>9/8</td>
<td>−9</td>
<td>+5*</td>
</tr>
<tr>
<td>4</td>
<td>20 Oct 2015</td>
<td>205385 to 207565</td>
<td>8</td>
<td>−9</td>
<td>+10*</td>
</tr>
<tr>
<td>5</td>
<td>20 Oct 2015</td>
<td>209078 to 211264</td>
<td>9</td>
<td>−4*</td>
<td>+9</td>
</tr>
<tr>
<td>6</td>
<td>20 Oct 2015</td>
<td>212604 to 214784</td>
<td>7</td>
<td>−9</td>
<td>+9</td>
</tr>
<tr>
<td>7</td>
<td>20 Oct 2015</td>
<td>223256 to 225436</td>
<td>10</td>
<td>−5</td>
<td>+6*</td>
</tr>
<tr>
<td>8</td>
<td>20 Nov 2015</td>
<td>468128 to 470308</td>
<td>10</td>
<td>−7*</td>
<td>+1*</td>
</tr>
<tr>
<td>9</td>
<td>20 Nov 2015</td>
<td>471680 to 473860</td>
<td>9/8</td>
<td>−8*</td>
<td>+8</td>
</tr>
<tr>
<td>10</td>
<td>20 Nov 2015</td>
<td>475280 to 477460</td>
<td>9</td>
<td>−10</td>
<td>+9*</td>
</tr>
<tr>
<td>11</td>
<td>20 Nov 2015</td>
<td>478817 to 481119</td>
<td>9</td>
<td>−8</td>
<td>+9</td>
</tr>
<tr>
<td>12</td>
<td>20 Nov 2015</td>
<td>483140 to 485320</td>
<td>7</td>
<td>−2*</td>
<td>+8</td>
</tr>
</tbody>
</table>
dual-frequency receivers (L1 plus L2 or wide-lane measurements). Based on the results presented in Section VI it is possible to say this objective was achieved.

The results achieved for single-frequency, dual-frequency and wide-lane were compared for each one of the implemented methods. For the float and the smooth solutions it is possible to see the difference between the three cases, mostly for the smooth solution, however, the precision attained for the attitude angles with these methods was not the desire precision for the final solution. For the LAMBDA solution, the differences between the three cases are more visible: for single-frequency the results are very unstable and unreliable, for dual-frequency and wide-lane the results are significantly more stable but the correct solution is not always chosen. Lastly, the final solutions (obtained with a baseline length of 12.47 meters) have a baseline with millimeter precision and a precision with an order of magnitude of $10^{-3}$ degrees for the heading and $10^{-2}$ degrees for the pitch. These precisions were obtained for either single-frequency, dual-frequency and wide-lane which allows to conclude that the proposed objectives were achieved: using the Ambiguity Filter for single-frequency measurements, it is possible to obtain equivalent precisions to those obtained with the LAMBDA method alone (complemented with a validation step to correct the few wrong solutions) for dual-frequency and wide-lane. The results with better precision were still the ones that used dual-frequency measurements.

Using only single-frequency measurements has the advantage of allowing the utilization of low-cost receivers, therefore, the Ambiguity Filter was the subject of further analysis. An issue that was considered particularly important in Ambiguity Filter was the dependence of this method with the input of known baseline length.

Twelve different data sets, acquired during the period of three months, were tested with different input values for the baseline length with the objective of analyzing the sensitivity of the Ambiguity Filter (with the original three steps) to the precision of the prior knowledge of the baseline length. Despite some good values for the offsets from the true baseline length (such as $\pm 8$ centimeters), some other values were not as good ($\pm 1 - 3$ centimeters). These results allow to conclude that, although it is possible to find the ambiguities for carrier phase measurements with only single-frequency measurements by using the Ambiguity Filter after the LAMBDA method, this filter is highly sensitive to the precision to which the baseline length is known.

In order to improve the detection of false locks provided by the Ambiguity Filter, it was implemented a new verification step within this filter. To test this new implementation, the same twelve data sets previously tested were analyzed using different input values for the baseline length, with the new verification step within the Ambiguity Filter. After reviewing the results obtained (with and without the verification step), several conclusions can be drawn. The number of worst results (bounds of $\pm 1 - 3$ centimeters) is reduced after the implementation of the verification step in the Ambiguity Filter. It is also possible to verify that from a total of 24 results (for each metric) 14 results were improved with the new implementation. Additionally, it was not observed any degradation of the results when using the new verification step. The results obtained show that the new verification step, implemented in the Ambiguity Filter, improves the detection of false locks for the ambiguities, contributing also to decrease the sensitivity of the Ambiguity Filter to the input of the baseline length.

REFERENCES