Pulse Propagation in Optical Fibers using the Moment Method

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***Abstract*—The scope of this paper is to use the semianalytic technique of the Moment Method to study pulse propagation in optical fibers. It is made an overview of pulse propagation in both the linear and non-linear regime, exploring the effects of Group Velocity Dispersion (GVD) and Self-Phase Modulation (SPM) as well as an analysis of bit rate dependencies. Both the Gaussian pulse and the hyperbolic-secant pulse are evaluated in the linear and non-linear regimes, analysing the evolution of their parameters using the Moment Method. Dispersion maps are also studied using the Moment Method so as to assess how different optical fiber characteristics influence the pulse parameters and propagation. Finally, the Moment Method the Moment Method for both the Gaussian and hyperbolic-secant pulses in the non-linear regime is analysed, being compared against the Split-Step Fourier Method (SSFM) for the same propagation conditions in order to evaluate the Moment Method merits and shortcomings.**

***Keywords*—Optical Fibers, Moment Method, semianalytic technique, GVD, SPM, chirp, soliton, SSFM, linear regime, non-linear regime, Gaussian pulse, hyperbolic-secant pulse, NLS, Dispersion Maps.**

I. Introduction

Optical fibers communication systems are systems that use optical fibers to transmit information. These systems have been being used on a global scale since 1980 and have revolutionized the telecommunications field [1]. The main limitation factors in optical fibers are attenuation and GVD. However, another factor which condition pulse propagation is the development of non-linear effects, most notably, SPM [9].

The study of pulse propagation can have three distinct approaches. We may consider the analytical approach which results in precise and closed solutions but is not always possible. Another approach is a pure numerical one, which despite its precision requires a considerable computational effort. Finally, we have the semi analytic approach which can be variational or the Moment Method. Although the Moment Method cannot offer, by itself, a solution for pulse propagation it enables us to consider a parametric space in order to gain physical insight of the pulse through its parameters. A simple example of the application of the Moment Method can be considered in [2][3]. It is my goal to show how one can use the Moment Method in different pulse shapes, propagation conditions and compare it with the SSFM in order to draw conclusions about its advantages and shortcomings as an alternative, versatile solution. Other applications of the Moment Method can be found in [10][11].

II. Pulse propagation in Optical Fibers

Considering the pulse propagation in the linear regime, according to [4] we have the equation that describes the pulse propagation in the linear regime as

$$\frac{\partial A}{\partial z}+\beta_1\frac{dA}{dt}+i\frac{1}{2}\beta_2\frac{d^2A}{dt^2}-\frac{1}{6}\beta_3\frac{d^3A}{dt^3}+\frac{\alpha}{2}A=0 \tag{1}$$

Where A is the envelope of the pulse, t represents time, z is the coordinate through which the pulse propagates in the fiber, β_1 is the inverse of the group velocity, β_2 is the GVD coefficient, β_3 is the high order dispersion coefficient and α is the attenuation coeffient.

Taking into account the process to obtain the equation which describes the pulse propagation in the non-linear regime in [5] we can write the Non-Linear Schrödinger Equation (NLS) as

$$i\frac{\partial u}{\partial \zeta}-\frac{1}{2}\operatorname{sgn}(\beta_2)\frac{\partial^2 u}{\partial \tau^2}+|u|^2u=0 \tag{2}$$

as long as fiber losses and high order dispersion effects are not taken into account and where u represents the normalized amplitude

$$u(\zeta,\tau)=NU(\zeta,\tau) \tag{3}$$

Where ζ is the normalized distance

$$\zeta=\frac{z}{L_D} \tag{4}$$

τ is the normalized time

$$\tau=\frac{t-\beta_1 z}{T_0}=\frac{T}{T_0} \tag{5}$$

and N can be deduced from the expression

$$N^2=\frac{L_D}{L_{NL}}=\gamma L_D P_0=\frac{T_0^2}{|\beta_2|}\gamma P_0 \tag{6}$$

Where L_D represents the dispersion length, L_{NL} represents the non-linear length, T_0 is the pulse width at the beginning of the fiber, γ is the non-linearity coefficient and P_o represents the input power.

The Moment Method was first used, in 1971, by Vlasov [6]. If we consider the NLS in the form

$$\frac{dU}{dz}+i\frac{\beta_2}{2}\frac{d^2U}{dT^2}-\bar{\gamma}|U|^2U=0 \tag{7}$$

Where $\bar{\gamma}=\gamma\exp[-\int_0^z\alpha(z)dz]$ is non-linear parameter which contains both the non-linear effects and the fiber losses. The

Moment Method can be used to solve (7) as long as one can assume the pulse retains a specific shape during its propagation although its amplitude, width and chirp can change in a continuous way. There are cases where this is valid, for example, a Gaussian pulse retains its shape in a linear dispersive medium as well as if the non-linear effects are of little significance. On the other hand, a pulse can also retain its shape even if the non-linear effects are strong as long as the dispersive effects are weak.

The concept behind the Moment Method is to address the optical pulse like a particle in which its energy E_p, RMS width σ_p and chirp C_p are related to $U(z,T)$ as

$$E_p = \int_{-\infty}^{\infty} |U|^2 \, dT \tag{8}$$

$$\sigma_p{}^2 = \frac{1}{E_p}\int_{-\infty}^{\infty} T^2 |U|^2 \, dT \tag{9}$$

$$C_p = \frac{i}{2E_p}\int_{-\infty}^{\infty} T\left(U^* \frac{\partial U}{\partial T} - U\frac{\partial U^*}{\partial T}\right) dT \tag{10}$$

As the pulse propagates inside the fiber, these three moments change. To evaluate how they evolve with *z* we differentiate the equations with respect to *z* and use the NLS. After some algebra, detailed in [7], we find

$$\frac{\partial E_p}{\partial z} = 0 \tag{11}$$

$$\frac{\partial C_p}{\partial z} = \frac{\beta_2}{E_p}\int_{-\infty}^{\infty}\left|\frac{\partial U}{\partial T}\right|^2 dT + \frac{\bar{\gamma}}{2E_p}\int_{-\infty}^{\infty}|U|^4\, dT \tag{12}$$

$$\frac{d\sigma_p}{dz} = \frac{\beta_2 C_p}{\sigma_p} \tag{13}$$

If we consider a Gaussian pulse with chirp, its ansatz may be described by

$$U(z,T) = a_p \exp\left[-\frac{1}{2}\left(1+iC_p\right)\left(\frac{T}{T_p}\right)^2 + i\Phi_p\right] \tag{14}$$

Where a_p represents the amplitude of the pulse, C_p represents the chirp of the pulse, T_p represents the width of the pulse and Φ_p represents the phase of the pulse. All the four parameters are function of z. However since the phase Φ_p does not affect the other parameters it can be ignored. a_p relates with the pulse energy through

$$E_p = \sqrt{\pi}\, a_p{}^2 T_p \tag{15}$$

Since the energy E_p does not change with z it can be replaced by its initial value $E_0 = \sqrt{\pi} T_0$. On the other hand, $T_p = \sqrt{2}\sigma_p$. Using (12) and (13), the pulse width T_p and its chirp C_p may be described by the system of coupled differential equation, such as

$$\frac{dT_p}{dz} = \frac{\beta_2 C_p}{T_p} \tag{16}$$

$$\frac{dC_p}{dz} = \left(1 + C_p{}^2\right)\frac{\beta_2}{T_p{}^2} + \bar{\gamma} P_0 \frac{T_0}{\sqrt{2} T_p} \tag{17}$$

On the other hand, if an hyperbolic secant pulse is to be considered, its ansatz takes the form

$$U(z,T) = a_p \operatorname{sech}\left(\frac{T}{T_p}\right)\exp\left[-iC_p\left(\frac{T}{T_p}\right)^2 + i\Phi_p\right] \tag{18}$$

As with the Gaussian pulse all four parameters are a function of z and the phase Φ_p may be ignored as it does not affect the other parameters. As the energy E_p remains the same throughout z it can be replaced by its initial value, $E_0 = 2P_0T_0$. Repeating the same process used for the Gaussian pulse, using (18) in (12) and (13), the width T_p and the chirp C_p of the hyperbolic secant pulse are described by the following system of coupled differential equations

$$\frac{dT_p}{dz} = \frac{\beta_2 C_p}{T_p} \tag{19}$$

$$\frac{dC_p}{dz} = \left(\frac{4}{\pi^2} + C_p{}^2\right)\frac{\beta_2}{T_p{}^2} + 4\bar{\gamma} P_0 \frac{T_0}{\pi^2 T_p} \tag{20}$$

The bit rate of an optical system may be defined as the number of bits that conveyed per unit of time. Bit rate is limited by inter symbols interference, which in turn is related to pulse broadening caused by dispersion. As such, the study of bit rate is of high importance in optical systems [4]. Pulse broadening is related to different parameters such as the RMS width of the source, the initial width of the pulse, the GVD and, sometimes, high order dispersion.

The RMS width of the pulse is given by [8]

$$\sigma^2 = \left[\left\langle t^2\right\rangle - \left\langle t\right\rangle^2\right] \tag{21}$$

The broadening factor is defined in [18] as

$$\left(\frac{\sigma}{\sigma_0}\right)^2 = \left(1 + \frac{LC\beta_2}{2\sigma_0^2}\right)^2 + \left(\frac{\beta_2 L}{2\sigma_0^2}\right)^2 + (1+C^2)\left(\frac{L\beta_3}{4\sqrt{2}\sigma_0^3}\right)^2 \tag{22}$$

Considering T_B the period of a bit slot, the bit rate is $B = \frac{1}{T_B}$. In order to avoid inter symbol interference the following rule must be uphold

$$\sigma \le \frac{T_B}{4} = \frac{1}{4B} \Rightarrow B \le B_0 = \frac{1}{4\sigma} \tag{23}$$

Taking into consideration (22) and ignoring the high order dispersion it can be written

$$\sigma^2 = \sigma_0^2 + C\beta_2 L + (1+C^2)\left(\frac{\beta_2 L}{2\sigma_0}\right)^2 \tag{24}$$

The optimum value of σ_0 to minimize pulse broadening can be calculated as

$$\frac{d\sigma}{d\sigma_0} = 0 \Leftrightarrow \sigma_0^2 = \sqrt{1+C^2}\left(\frac{|\beta_2 L|}{2}\right) \tag{25}$$

Substituting (25) in (24) we get

$$\sigma^2 = |\beta_2| L\left[\operatorname{sgn}(\beta_2)C + \sqrt{1+C^2}\right] \tag{26}$$

Applying (26) in rule (23), the maximum value of the bit rate is defined as

$$B_0 = \frac{1}{4\sqrt{|\beta_2| L}\sqrt{C\,\mathrm{sgn}(\beta_2)+\sqrt{1+C^2}}} \quad (27)$$

However, analyzing equation (26) it can be concluded that for a given value of C, $\sigma < \sigma_0$. This may be observed in the following picture.

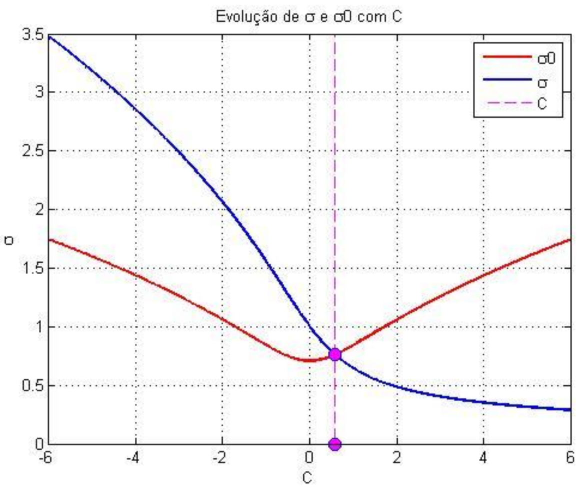

Fig. 1 Evolution of σ and σ_0 with chirp

Using equations (25) and (26) it's easy to conclude the intersection point happens at $C = \frac{1}{\sqrt{3}}$. From that point on, σ_0 takes higher values than σ. As such, rule (23) should be rewritten as

$$B \le B_0 = \frac{1}{4\sigma_0} \quad (28)$$

It is also interesting to analyse how the bit rate evolves in function of C, that behavior can be seen in the following picture

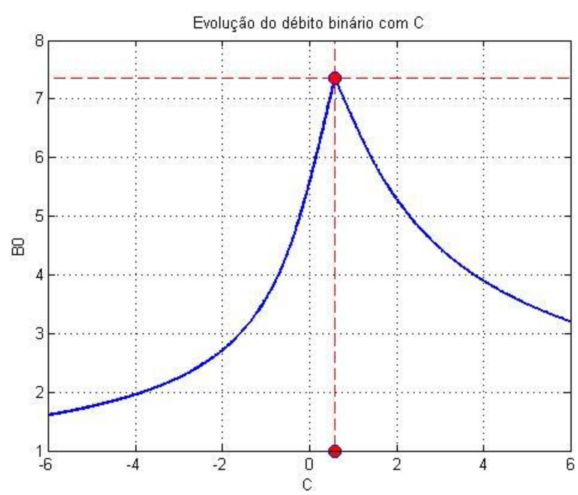

Fig. 2 Bit rate evolution with chirp

The maximum bit rate happens at $C = \frac{1}{\sqrt{3}}$, the intersection point between σ_0 and σ. The bit rate is maximum at that point because that is where the minimum of σ occurs. From that point on rule (28) should be considered instead of rule (21). As such, σ_0 will take values higher than the minimum of σ and the bit rate will decrease.

III. Linear Regime simulations using the moment method

The first part of this chapter will consider the Gaussian and hyperbolic secant pulses in constant dispersion fibers, analyzing the evolution of the pulse parameters in the linear regime.

Considering the linear regime, $\bar{\gamma} = 0$. As such, equations (16) and (17), which describe the evolution of the pulse width and chirp for the Gaussian pulse, can be rewritten as

$$\frac{dT_p}{dz} = \frac{\beta_2 C_p}{T_p} \quad (29)$$

$$\frac{dC_p}{dz} = \left(1+C_p^{\,2}\right)\frac{\beta_2}{T_p^{\,2}} \quad (30)$$

If the pulse propagates in the anomalous region $(\beta_2 < 0)$ and that there is no chirp in the beginning of the fiber $(C_0 = 0)$ the evolution of the pulse chirp, C_p, and the broadening factor, $\eta = \frac{T_p}{T_0}$, can be observed in the following pictures

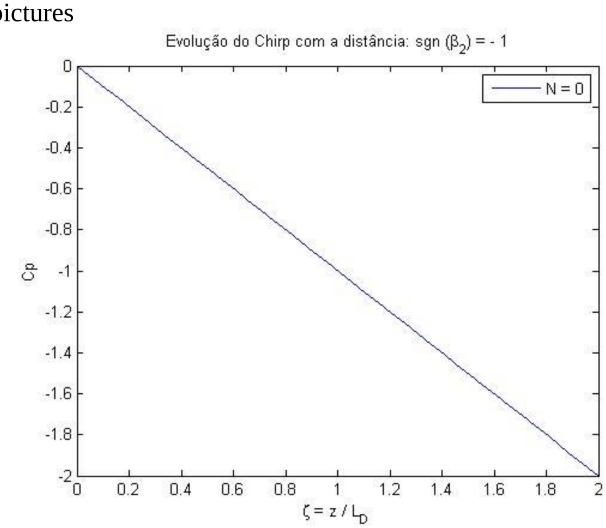

Fig. 3 Chirp evolution for the Gaussian pulse in the linear regime

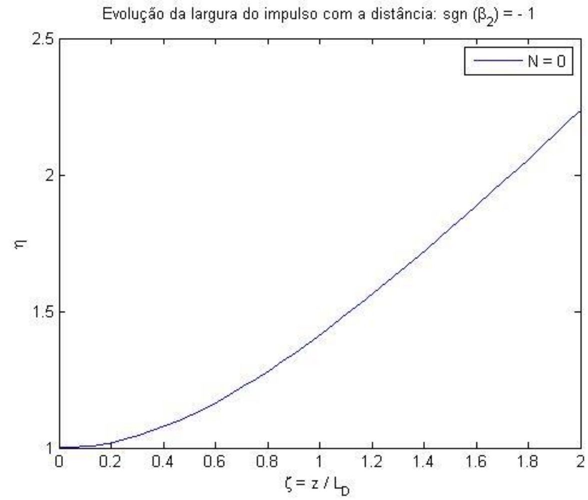

Fig. 4 Broadening factor evolution for the Gaussian pulse in the linear regime

It can be seen the pulse broadens fast and develops considerable chirp with a linear evolution. This happens because considering $\bar{\gamma} = 0$ only the first term of the right side of equation (17) is taken into account, resulting in equation (30). As such, seeing the anomalous region is being considered and the terms in equation (15) would have opposite signals, there is no term contributing with the non-linear effects to even out the fast development evolution of the chirp into negative values. This will influence the pulse broadening because considering the chirp develops negative values and considering $\beta_2 < 0$, then $\beta_2 C_p > 0$. So, considering (29), it is easy to conclude the pulse will broaden fast and proportionally to the development of the chirp.

Considering now the hyperbolic secant pulse and the linear regime, equations (17) and (18) can be rewritten as

$$\frac{dT_p}{dz} = \frac{\beta_2 C_p}{T_p} \tag{31}$$

$$\frac{dC_p}{dz} = \left(\frac{4}{\pi^2} + C_p{}^2 \right) \frac{\beta_2}{T_p{}^2} \tag{32}$$

The evolution of the parameters for the hyperbolic secant pulse can be found in the next pictures.

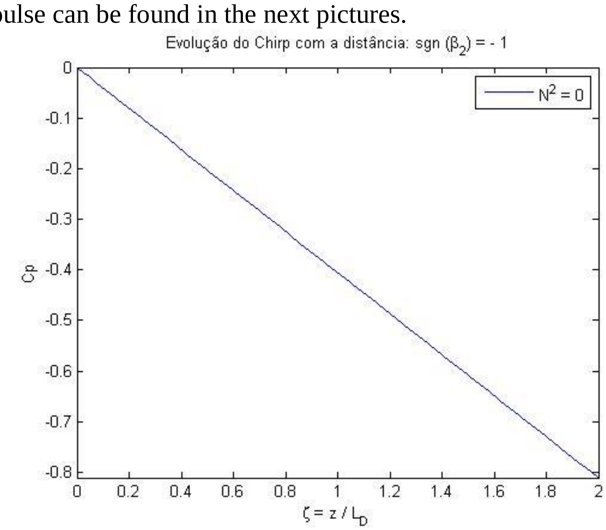

Fig. 5 Chirp evolution for the 'sech' pulse in the linear regime

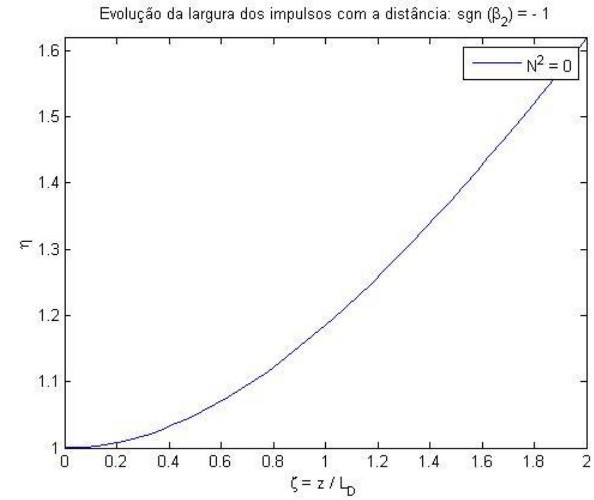

Fig. 6 Broadening factor evolution for the 'sech' pulse in the linear regime

Relatively to Fig. 5 we can see the pulse acquires a negative chirp. However, unlike the Gaussian pulse the evolution is not linear. In Fig. 6 there is a pulse broadening, consequence of the chirp acquired by the pulse. However, since the chirp values are smaller than in the Gaussian pulse also the pulse broadening is smaller than the one seen in Fig. 4.

The second part of this chapter will address fibers with variable dispersion. This means fibers with different segments, each one with different characteristics. These are the so called dispersion maps and are a powerful tool to overcome dispersion in optical fibers.

To start with we will consider different dispersion maps for the Gaussian pulse. For that analysis it is important to consider the relationship between the dispersion coefficient and the GVD given by

$$\beta_2 = -\frac{\lambda^2}{2\pi c} D \tag{33}$$

For a dispersion map with two segments and an average GVD of zero $\left(\overline{\beta_2} = 0\right)$ which mean there is total compensation of the dispersion we consider

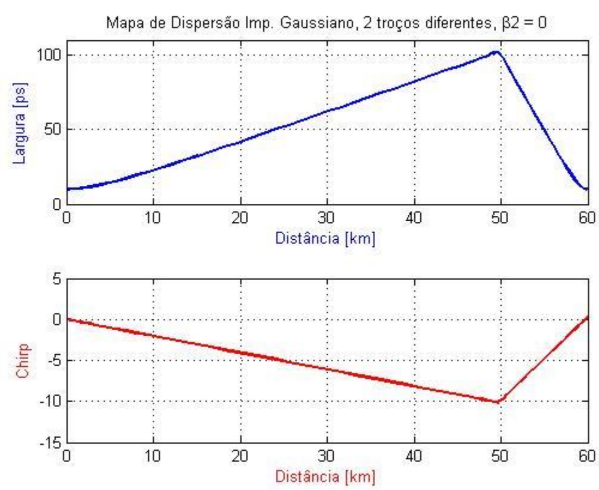

Fig. 7 Dispersion map for the Gaussian pulse with 2 segments

The first segment of the map is characterized by $L_1 = 50km$; $D_1 = 16\ ps/(km.nm)$ while the second segment is defined by $L_2 = 10km$; $D_2 = -80\ ps/(km.nm)$. Looking at Fig. 7 it easy to conclude this map obeys to the expression $D_1L_1 + D_2L_2 = 0$. As such, there is total compensation of dispersion which can be seen by the fact that after the 60km the pulse goes through it regains its initial width.

For the hyperbolic secant pulse and considering a dispersion map of 3 segments instead of 2 we can observe

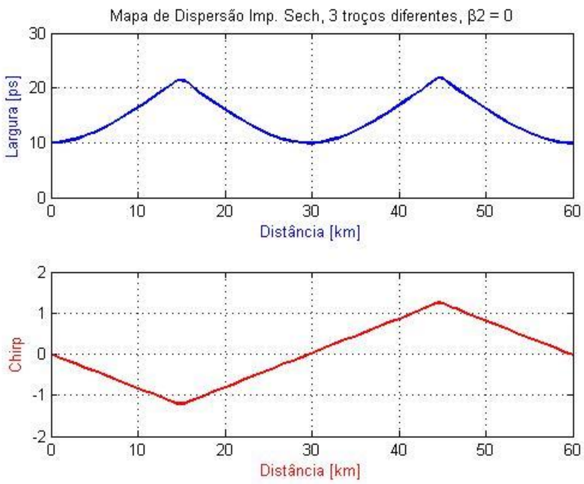

Fig. 8 Dispersion map for the 'sech' pulse with 3 segments

The first and third segments are characterized by $L_{1,3} = 15km$; $D_{1,3} = 16\ ps/(km.nm)$ while the second segment is carachterized by $L_2 = 30km$; $D_2 = -32\ ps/(km.nm)$.

IV. Non-Linear Regime: Gaussian Pulse

This chapter will evaluate the Gaussian pulse propagation as well as its parameters in the non-linear regime. It will consider the propagation of the Gaussian pulse for different values of $N^2 = \frac{L_D}{L_{NL}}$, where $L_{NL} = \frac{1}{\gamma P_0}$. The results obtained with the Moment Method will then be assessed against the SSFM to evaluate the Moment Method precision. Considering propagation in the anomalous zone $(\beta_2 < 0)$, an unchirped pulse at the beginning of the fiber $(C_0 = 0)$ and equations (14) and (15) the evolution of the pulse width T_p and the chirp C_p for different values of N^2 can be seen in the following pictures. It should be noted the pulse width is evaluated considering the broadening factor, $\eta = \frac{T_p}{T_0}$.

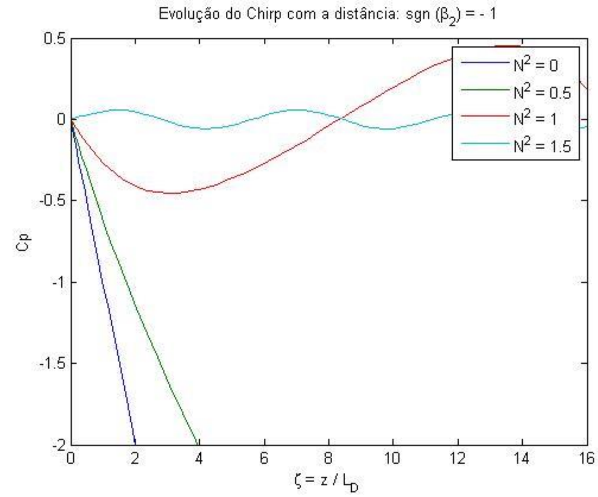

Fig. 9 Chirp evolution for the Gaussian pulse in the non-linear regime

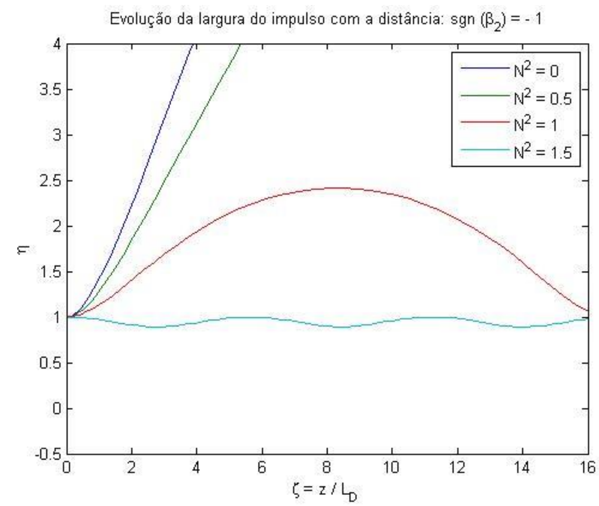

Fig. 10 Pulse broadening evolution for the Gaussian pulse in the non-linear regime

It can be seen that as N^2 increases (which mean the non-linear effects are stronger) the pulse broadens less and less, eventually compressing for $N^2 = 1.5$. This is a consequence of the fact that for higher values of N^2 the pulse gains less chirp, going as far as to acquire positive chirp for $N^2 = 1.5$. This behavior would be expected by analyzing equations (14) and (15). In the non-linear regime the Self-Phase Modulation (SPM) should be considered. In equation (15) the SPM effects (represented by the second term of the equation's right side) will compensate the dispersive effects in chirp evolution. This means that the bigger the non-linear component the bigger will the SPM is and as such the pulse will broaden less. Continuing analyzing equations (14) and (15) it can be inferred that for $N^2 = 1$ it is expected that at some point in the fiber the contributions of SPM and dispersion will null each other with the chirp taking on positive values, which will result in a pulse compression. That can be observed in Fig. 10 where for $N^2 = 1$, in $\zeta = 8$ the chirp takes positive values which results in a pulse compression.

Rebuilding the ansatz for the Gaussian pulse described in (12) for $N^2=1$, using the Moment Method and using the SSFM it can be obtained

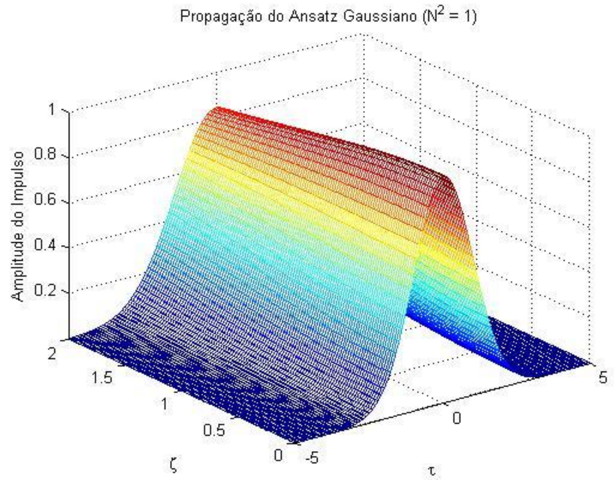

Fig. 11 Gaussian pulse - Moment Method, $N^2=1$

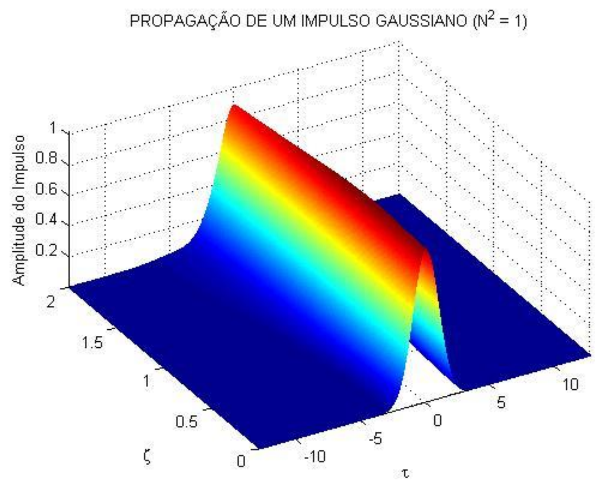

Fig. 12 Gaussian pulse - SSFM, $N^2=1$

Comparing both figures it can be concluded that for $N^2=1$ the Moment Method is a valid technique to study the propagation of a Gaussian pulse. The pulse shape is similar in both techniques and the amplitude value at the end of the fiber is of $a_p=0.8433$ for the Moment Method and $a_p=0.8894$ for the SSFM which is a good indicator of the validity of the Moment Method. Comparing this situation with the linear case there is less broadening of the pulse thanks to the non-linear present in the fiber.

For $N^2=0,5$, the Moment Method is also a valid technique to study the Gaussian pulse. The amplitude values at the end of the fiber are quite similar for both techniques and the pulse shape throughout the fiber is identical as it can be seen in the following pictures.

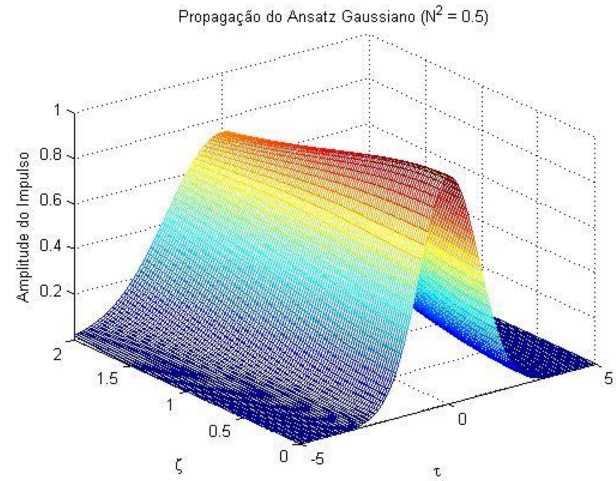

Fig. 13 Gaussian pulse- Moment Method, $N^2=0.5$

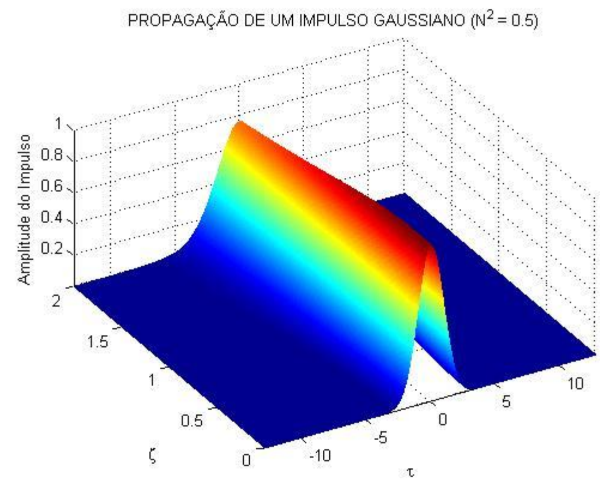

Fig. 14 Gaussian pulse - SSFM, $N^2=0.5$

Finally, for $N^2=1.5$ the confrontation between the Moment Method and the SSFM can be found in the below pictures

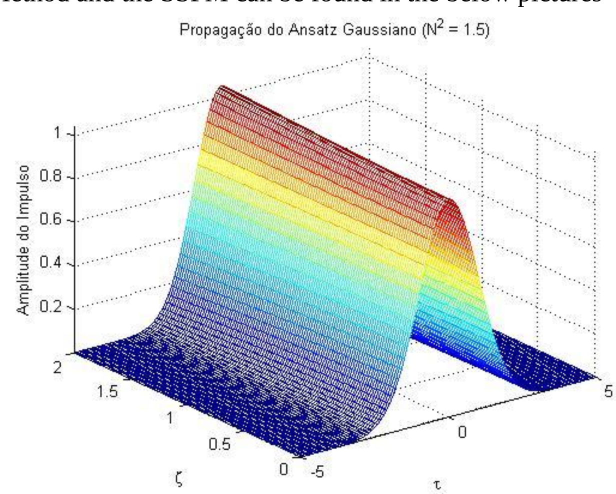

Fig. 15 Gaussian pulse - Moment Method, $N^2=1.5$

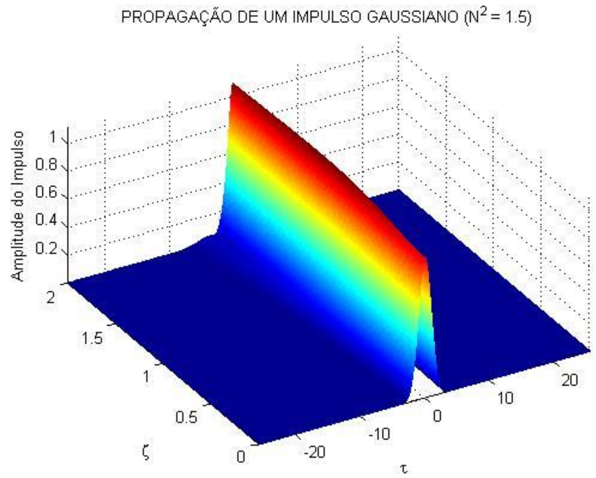

Fig. 16 Gaussian pulse - SSFM, $N^2 = 1.5$

In this situation the Moment Method proves even more reliable than in the previous situations with final value of amplitude being very similar between both techniques, with $a_p = 0.1044$ for the Moment Method and $a_p = 0.1091$ for the SSFM.

The different situations can be organized in the following table

	Amplitude, a_p, at the end of the fiber ($\zeta = 2$)	
	Moment Method	SSFM
$N = 0$	0,6686	0,6684
$N^2 = 0.5$	0,7361	0,7554
$N = 1$	0,8433	0,8894
$N^2 = 1.5$	1,044	1,091

Table. 1 Moment Method and SSFM comparison for the Gaussian pulse

From the above table it can be concluded that the Moment Method is a valid technique for realistic situations where the non-linear effects are not that intense but it starts to lose validity when they become more significant. This is because the Moment Method assumes the pulse shape doesn't change during the pulse propagation, something the non-linear effects cause to happen. As such, the Moment Method does not realistically represent a pulse in those conditions because it acts like a straitjacket for the pulse.

V. Non-Linear Regime: Hyperbolic Secant Pulse

This chapter will evaluate the Gaussian pulse propagation as well as its parameters in the non-linear regime, considering different values of N^2 and assessing the results of the Moment Method against those of the SSFM. Considering propagation in the anomalous zone $(\beta_2 < 0)$, an unchirped pulse at the beginning of the fiber $(C_0 = 0)$ and equations (17) and (18) the evolution of the pulse width T_p and the chirp C_p can be seen in the following pictures. It should be noted the pulse width is evaluated considering the broadening factor.

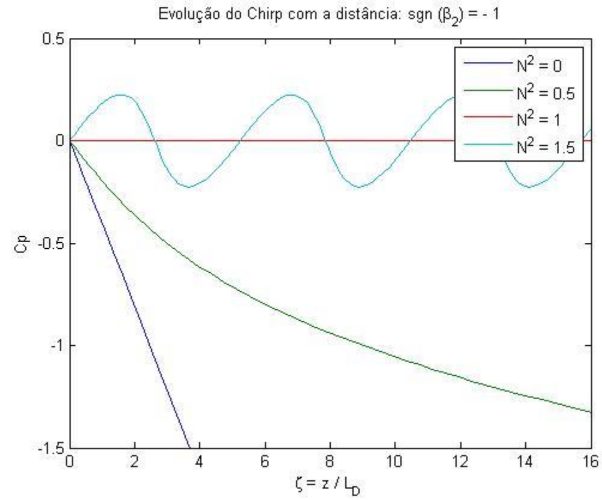

Fig. 17 Chirp evolution for the 'sech' pulse in the non-linear regime

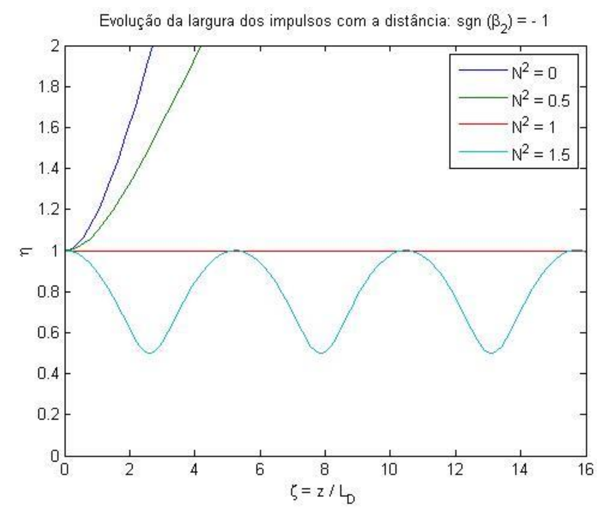

Fig. 18 Pulse broadening evolution for the 'sech' pulse in the non-linear regime

For $N^2 = 0$ and $N^2 = 0.5$ the chirp and broadening factor evolution is similar to the one observed for the Gaussian pulse although the chirp takes smaller values which result in less broadening of the pulse. However for $N^2 \geq 1$ bigger changes are seen in the pulse behavior when compared to the Gaussian pulse.

For $N^2 = 1$ the chirp remains 0 throughout the fiber. Since there is no chirp variation there is also no variation in the pulse width and as such the broadening factor remains 1. This behavior is similar to the behavior of the fundamental soliton [5].

For $N^2 = 1.5$ the chirp and broadening factor behavior is similar to the one in the Gaussian pulse, however with some changes. The hyperbolic secant pulse develops a higher positive chirp value than the Gaussian pulse which will result in a bigger pulse compression. This behavior is relevant because it is opposed to what is perceived for $N^2 < 1$ where the chirp values and broadening factors are smaller than in the Gaussian pulse. The explanation for this is in the comparison between equations (15) and (18). When the coefficients of both equations are taken into account there is a bigger discrepancy between the SPM and the GVD contributions in the Gaussian pulse than in the hyperbolic secant pulse where the contributions of both phenomenon are equal as it can be seen by the pulse behavior for $N^2 = 1$. For the Gaussian pulse the term which translates the GVD has more significant contribution than the one which translates the SPM. This results in a bigger broadening of the pulse for $N^2 < 1$ and a smaller compression of the pulses for $N^2 > 1$ as it can be seen when comparing Fig. 10 and 18. This way, for the 'sech' pulse the GVD will not influence the impulse in such an expressive way as in the Gaussian pulse, as such the pulse will develop less chirp and the pulse will broaden less. On the other hand, for $N^2 > 1$ the opposite happens. As GVD as a smaller contribution it won't compensate the influence of the SPM, resulting of the non-linear effects which in turn will result in higher chirp values and bigger pulse compression. Physically, this behavior is in agreement with the expected behavior of the 'sech' pulse which for higher values of non-linearity it will approach solitons of higher order.

Next, similar to chapter IV an assessment of the Moment Method against the SSFM will be made for the hyperbolic secant pulses in different non-linear situations.

Starting with $N^2 = 1$,

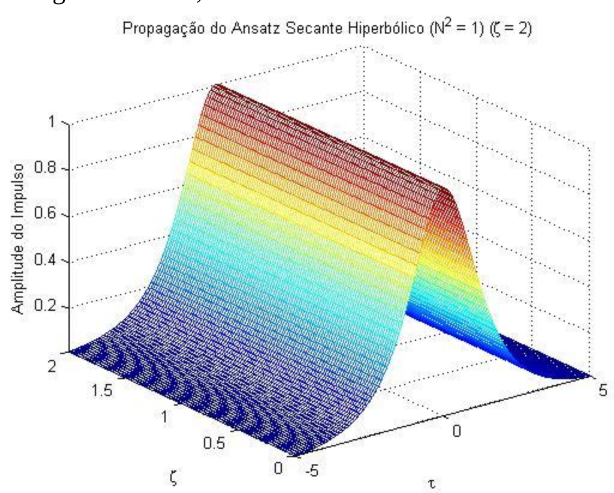

Fig. 19 Sech pulse - Moment Method, $N^2 = 1$

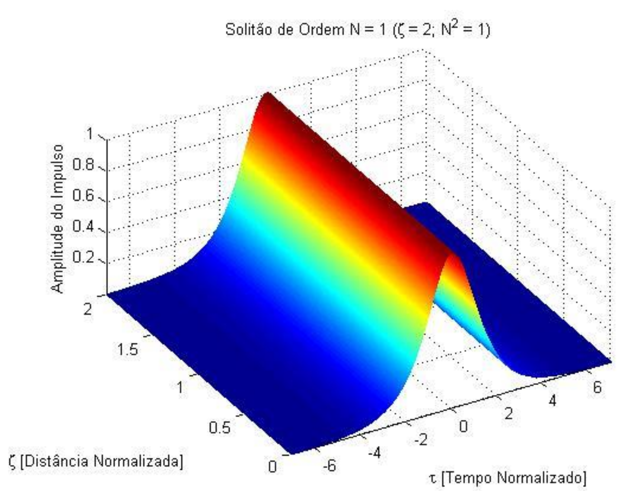

Fig. 20 Sech pulse - SSFM, $N^2 = 1$

For this situation the Moment Method is a perfect solution as it completely replicates the pulse as it is obtained using the SSFM. The pulse behaves as a fundamental soliton, keeping its amplitude, shape and width throughout the fiber.

For $N^2 = 0.5$

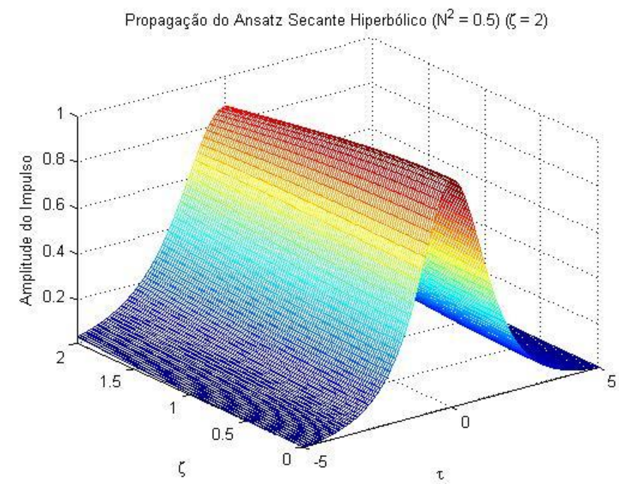

Fig. 20 Sech pulse - Moment Method, $N^2 = 0.5$

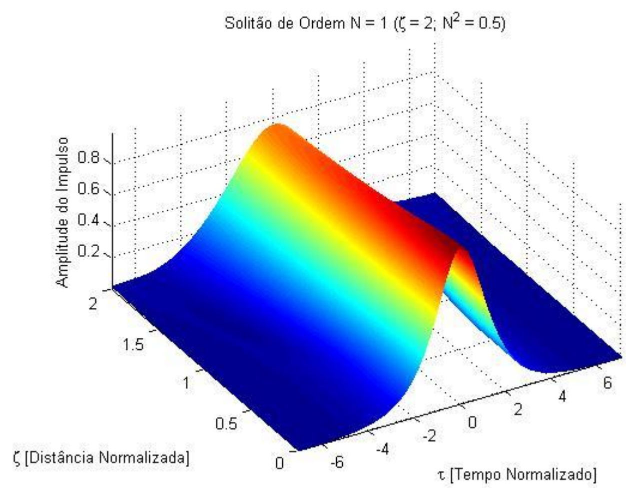

Fig. 21 Sech pulse - SSFM, $N^2 = 0.5$

It can be seen that for this case the Moment Method also provides a valid solution as the pulse evolution is quite similar when using both the Moment Method and the SSFM.

Finally, for $N^2 = 1.5$ it is to be expected to observe a pulse compression as well as a periodic behavior as per Fig. 17. As such, in order to observe the periodic behavior the propagation distance of the simulation will be longer than the one used in the previous situations.

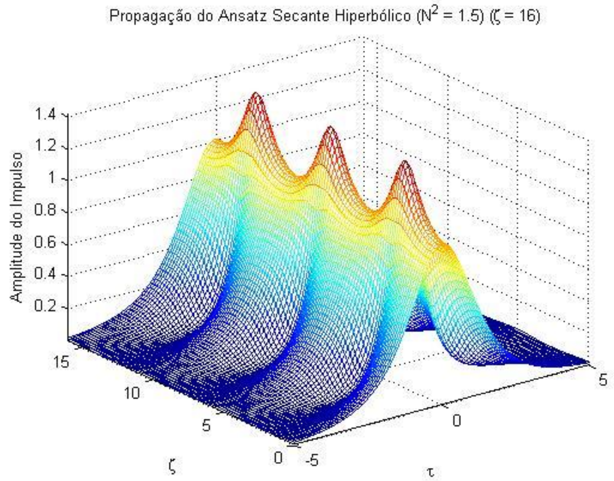

Fig. 22 Sech pulse - Moment Method, $N^2 = 1.5$

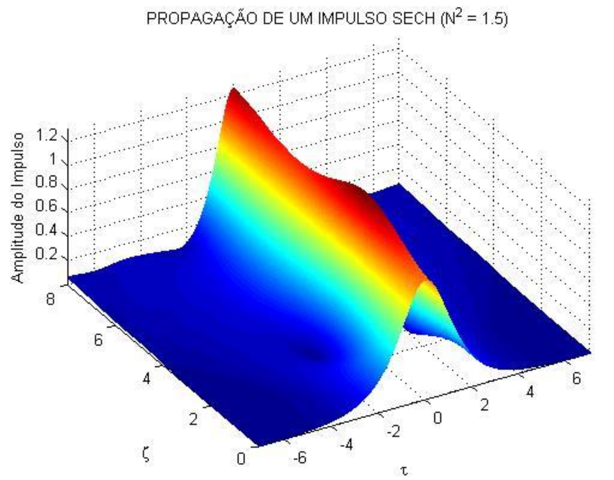

Fig. 23 Sech pulse - SSFM, $N^2 = 1.5$

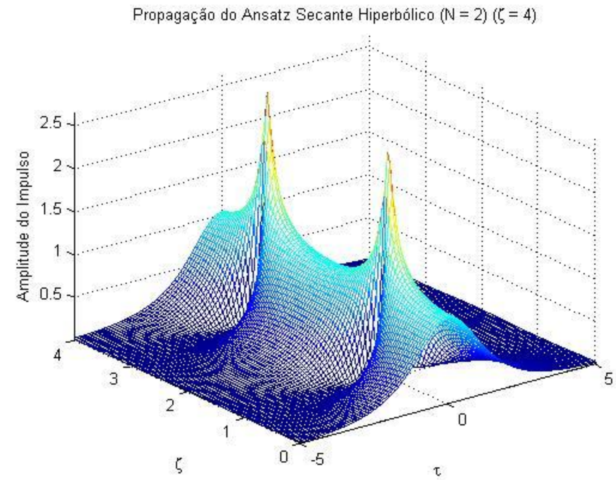

Fig. 24 Sech pulse - Moment Method, $N^2 = 4$

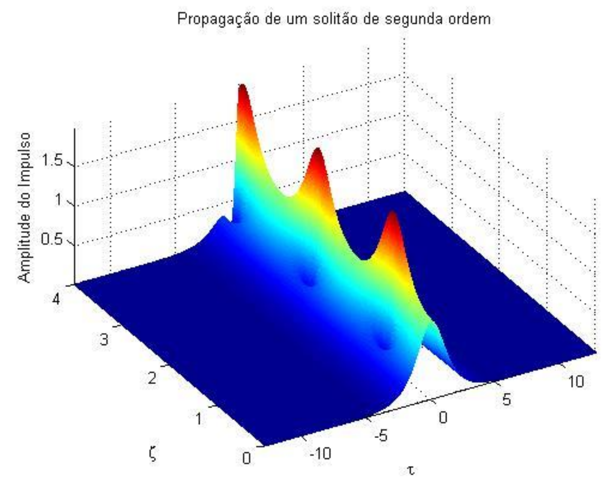

Fig. 25 Sech pulse - SSFM, $N^2 = 4$

Analysing Fig. 22 the periodic behavior observed in Fig. 17 can be confirmed, with the peaks of Fig. 22, where the is a pulse compression correspond to the minimums of Fig. 17. This behavior is also starting to approach that of the second order soliton. It is to be noted that the propagation distance in both simulations is different. This happens because using a propagation distance of $\zeta = 16$ with the SSFM is not realistic as it takes too much time to run. Even with $\zeta = 8$ the computation time was too high and unpractical. This situation is a practical example where the Moment Method provides advantages against the SSFM as it takes less computing effort. However some discrepancies between the two methods start to show as even though the peaks of the pulse occur in the same places of the fiber, their amplitude values are somewhat different.

In order to better explore the discrepancies that are starting to show between the Moment Method and the SSFM simulations using both methods for $N^2 = 4$ were run, in order to simulate a second order soliton.

Although the hyperbolic secant pulse evolution using the Moment Method has a similar behavior to that of a second order soliton the fact is both the period and peak amplitude of both pulses are quite different. This agrees with the conclusion of chapter IV that for high values of non-linearity the Moment Method does not hold true, losing precision as the non-linearity increases.

The comparison between the Moment Method and the SSFM is best illustrated in the below table

	Amplitude, a_p, at the end of the fiber ($\zeta = 2$)	
	Moment Method	SSFM
$N = 0$	0,7175	0,7855
$N^2 = 0.5$	0,8668	0,8243
$N = 1$	1	1
$N^2 = 1.5$	1,265	1,278

Table. 2 Moment Method and SSFM comparison for the ‘sech’ pulse

Analysing the table it can be verified that the Moment Method, in general, agrees with the SSFM. An interesting inference is to observe that when the non-linear effects are less significant the agreement between both techniques is smaller, increasing as the non-linear effects increase. As such, for values close to $N=1$ the Moment Method is a very valid solution, however for $N=2$ the Moment Method loses its validity as the non-linear effects are too intense. Comparing with Table. 1 it can be concluded that when the non-linear effects are weaker and the pulse is propagating near the linear regime the Gaussian ansatz provides more accurate solutions. However, when transitioning to the non-linear regime the ansatz of the hyperbolic secant pulse becomes more accurate than the Gaussian one. This phenomenon is related to the appearance of solitons in optical fibers which are better described by an hyperbolic secant shaped pulse.

VI. Conclusion

The main goal of this paper was to study pulse propagation in optical fibers using the Moment Method, addressing both its advantages and disadvantages, comparing it with analytical and numerical solution, like the Split-Step Fourier Method.

The main conclusion drawn from this paper is that the main advantage of the Moment Method against a numerical solution is that we can observe which specific parameter is affecting the pulse behavior. As it was pointed out throughout the paper a pulse is described by different parameters. While some have no influence on the way the pulse propagates some may be responsible for the pulse broadening or compression, for its change in amplitude or even its period. Using the Moment Method there is the possibility to only manipulate one of those parameters at a time and analyse what effect it has on the pulse. If we think of the pulse as being controlled by a control panel with different buttons, each parameter would be a button. Switching different buttons, different aspects of the pulse may be controlled which provides a profound qualitative analysis of the pulse, achieving a better physical perception of the pulse as opposed to the brute force method of a numerical solution. Besides, it was possible to conclude that a numerical method is vastly more demanding in regards to computing time, taking significantly more time to run than the Moment Method, while producing similar outputs. So, even though the SSFM is more precise there are situations where its use is not justified as the Moment Method provides accurate and faster solutions.

It can also be concluded that the GVD and the non-linear effects have a big influence on pulse behavior. If in the linear regime the GVD influence is absolute SPM does not exist as the non-linear effects raise their significance there starts to exist a balance between the SPM and the GVD. As such, while the GVD causes the pulses to broaden the SPM challenges this effect causing pulse compression. This dynamic is intimately connected with the development of chirp and its analysis and maintenance is of high importance in optical communication systems.

After extensive analysis of the Gaussian and hyperbolic secant pulse it can be concluded both pulses translate a fair representation of a pulse propagating in an optical fiber, however each pulse shape better represents this in different propagation conditions. While in the linear regime the Gaussian pulse translates a pulse in an optical fiber in a fairly accurate fashion when the non-linear effects start to appear and become more significant, accumulating over distance the Gaussian pulse evolves towards an hyperbolic secant shape, a behavior linked with the appearance of solitons. As such, in non-linear propagation conditions, the hyperbolic secant pulse better translates an optical pulse. These statements are supported by the comparison between the Moment Method and the SSFM as when considering the Gaussian pulse there is a bigger agreement between the Moment Method and the SSFM in the linear regime while when considering the hyperbolic secant pulse the biggest agreement appears for $N=1$.

Finally, it can be conclude that for very high non-linear effects the Moment Method loses precision. This happens because when using the Moment Method a pulse shape has to be assumed and that pulse shape does not change during propagation. However the non-linear effects cause changes in the pulse shape that when the non-linear effects are very high cause the Moment Method to lose validity. This means the Moment Method is a good solution when the non-linear effects do not dominate the pulse propagation in an optical fiber. However this is not exactly a disadvantage as in real propagation conditions the non-linear effects are never that significant nor do they surpass the limits in which the Moment Method is valid, making it a valid and versatile solution in most cases.

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