Building Subjective Probabilities for Health and Safety Risks with MACBETH

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Abstract

Within the framework of its activity, organizations adopt procedures for the prevention of risks that could jeopardize their goals. The Occupational Health and Safety Unit (OHSU) of the Regional Health Administration of Lisbon and the Tagus Valley (RHALVT) is responsible for supporting safety and health services at work, in the different health facilities and administrative offices belonging to RHALVT making use of a risk matrix based on classic procedures, while acknowledging the various difficulties associated with the use of this matrix. This work develops a support system for OHSU of the RHALVT to assess the likelihood of risks occurring in health and safety at work. This system is based on the design and implementation of a socio-technical process to build subjective probabilities based on qualitative judgments and in the MACBETH approach. The results obtained enable to support RHALVT in the evaluation of the probability of occupational hazards and seek to be used in the construction of a value risk matrix for the assessment of risks and risk mitigation in health and safety at work.

Key words: Subjective Probabilities, Risk Management, Safety and Health at Work, Risk Matrix, MACBETH.

1 Introduction

Due to the frequent occurrence of unanticipated events on a global scale, organizations have been adopting processes for the prevention of risks that could compromise their objectives. Risk managers, in particular, need quantitative or qualitative methods to analyse the relative importance of the different sources of risk, and this essential information is required in order to execute an adequate risk management. This topic is of special importance in the current context of scarce resources.

The risk of a health professional contracting work-related diseases is about 1.5 times greater than for all the other workers [1], so accidents at work and occupational diseases are a critical public health problem for all involved in the working environment, so the assessment of occupational hazards is the base for effective management so as to reduce these health and safety risks at work.

The Occupational Health and Safety Unit (OHSU) of the Regional Health Administration of Lisbon and the Tagus Valley (RHALVT) is responsible for assessing health and safety work risks for all the workers employed in any of RHALVT's establishments and makes use of a risk matrix based on classic procedures, while at the same time acknowledging the various difficulties associated with the use of this matrix.

Risk matrices are tables (or plots) that have categories of 'probability' for its rows (or columns) and categories of 'severity' for its columns (or rows) respectively [2] and they appear in literature under different designations – probability impact table [3], probability and impact matrix [4], probability impact grid [5] and risk maps [6]. Typically, in each risk matrix, a recommended level of risk, urgency, priority or management action is presented in each row-column pair; risks categorized with higher priority are given a higher managerial attention for treatment and mitigation [7].

Despite the use of risk matrices for risk assessment, recommended by international organizations namely the *Project Management Institute* [4], several studies have shown that the use of these matrices may bring about inconsistencies in risk management and that they do not respect important theoretical properties [7].

In this sense, it is necessary to support OHSU in the development of s support system to the assessment oh health and safety risks at work.

Looking at the improved design of risk matrices in use by RHALVT, we identified several challenges, particularly procedures to construct the probability and impact scale to be used in the value risk matrix. Thus, the motivation for conducting this thesis arises in response to one of the needs identified in the assessment and management process of occupational hazards at RHALVT, the difficulty being in estimating the probability of risks because the literature available in the area is inadequate. This is a generic problem because many other organizations have problems in defining probability in multiple contexts of risk assessment. Therefore, the aim of this dissertation is the development of a methodology to support the modelling and construction of subjective probabilities applied to risk assessment for safety and health at work.

This paper is structured as follows: Section 2 presents a brief review of related methodology. Section 3 contains the proposed and applied methodology. Section 4 shows the results of the methodology's application. Section 5 presents some discussion and section 6 the concluding remarks.

2 Literature Review

Considering the problem at hand, an extensive literature review was conducted and it shows that literature is rare in this area and does not provide methods for supporting RHALVT's problem in estimating risk probabilities. Online databases were consulted using queries as *risk management in health care, risk matrices, experts' probabilities, different formats for eliciting responses* and *modeling qualitative judgments in subjective probabilities.*

Despite its simplicity and vast usage, available studies indicate that the use of risk matrices compromise their feasibility, mainly in the probability assessment for different risks, particularly in cases where historical information is not available. Given the difficulties of organizations to elicit quantitative probabilities for risks because of the lack of historical information (or its use is inappropriate), risk matrices are used and the risks are typically categorized using qualitative scales (for example: low, medium and high). The use of probabilities in a discrete scale creates problems, as it contributes for discontinuity jumps in the risk matrix.

However, despite the qualitative levels of subjective probabilities associated with risk matrices being relatively easy to evaluate, certain technical and behavioural inconsistencies associated with numerical probabilities to verbal expressions have been mentioned in literature. When questioned directly by numerical probability values, it is possible to reduce certain ambiguities. However, different people have a tendency in attributing probabilities to different numbers of the same expression [8]. Studies reported in literature [9] demonstrated that in contrast to the probabilities of precise numerical values on a scale between zero and one, the verbal probabilities, which are *unlikely* or *almost certain*, tend to have imprecise meanings [9]. Another fact which was also observed by Erev et al. (1990) shows that the majority of people prefer to express verbal probabilities, but to receive them numerically [10]. In this section, particular emphasis is given in to the work of Wallsten et al. (1993), which explains the motivation for the choice of verbal probability judgments in the context of this work. One of the main motivations for the choice of verbal probability terms is to avoid the problem of "false precision", which is often associated with numerical probability statements [11]. The use of subjective probabilities has the advantage of being more intuitive for the specialists, and they are often preferred by individuals for communicating uncertainty since they are more conducive to transmitting the former [12]. Therefore, it is in this context that the motivation arises for the choice of subjective probabilities in the design of risk matrices.

Techniques to elicit information can typically be classified as either direct, where the information is asked from the expert directly, or indirect, where the information is generated without the expert necessarily generating the probability explicitly [11]. Regarding the direct method, *questionnaires* are perhaps the most common form of elicitation but its format can substantially affect the expert's response and their degree [11]. Concerning the indirect methods, the standard techniques are the *betting method*, the *equivalent lottery method* and *pairwise comparisons*. The first approach consists of adjusting amounts of money of two alternatives until the expert is indifferent about betting for or against the occurrence of the event of interest [13]. After knowing the amount of the two alternatives we can compute the expert probability for the event of interest. *Equivalent lottery method* consist of adjusting the value of one parameter until the experts is indifference between two lotteries. The final value of the parameter is the expert assessment of the probability of the event [11]. *Pairwise comparison* is based on the elicitation from the decision maker of relative or pairwise judgments of the importance of the different attributes of interest. From these pairwise judgments, a priority ordering of the probability of the events can be derived [11].

The first two techniques have the disadvantage of being time consuming and pairwise comparison showed to be the method that best fits the problem that we want to address.

Multicriteria Decision Analysis (MCDA) provides a set of methods based on theoretical foundations that can be used to correct some weaknesses in risk matrices identified in the literature [14]. Given this the use of MCDA approach can be helpful in this context. In order to build quantitative (subjective) probabilities using qualitative judgments from the Decision Maker, following previous theoretical literature building probabilities using qualitative information, we will explore how MACBETH can be used to build quantitative probabilities with pairwise comparisons.

3 Methodological Framework

This paper proposes a support system for the definition of probabilities applied to the assessment of health and safety risks at work. This system is built from the modelling and construction of a subjective probability scale, by using qualitative judgments by decision-makers, and from the creation of a system of rules to define risk probability in the construction of risk matrices. Thus, to enable the assessment and management of any type of risk, the system we propose to develop will be comprised of two components: (1) the construction of a subjective probability scale based on MACBETH approach to a representative group of risks and which is intended to be used within the system and (2) a system of rules that may be used by decision-makers for the quantification of a new risk that is not on the database.

The motivation for the choice of methodology MACBETH (Measuring Attractiveness by a Categorical Evaluation Technique) resides in the manner of questioning used by this methodology which can be useful to associate qualitative probability levels to the same numerical values and this is done by asking the evaluated group verbal judgment about the difference in probability between the various risk events. So the construction of the probability scale involves defining a new protocol that adapts the use of the MACBETH approach to the modelling context of subjective probability, which is in line with some of the previous studies. The steps involved in the construction of this methodology were a result of a sociotechnical approach and included: (1) Background preparation; (2) Motivation; (3) relevant aspects in computing probabilities that you would like to see improved; (4) definition of a probability elicitation protocol that adapts the use of the MACBETH approach to the modelling context of subjective probabilities; (4) the construction of a probability scale (technical component) by using the MACBETH methodology in conjunction with a protocol based on question types; (5) validation of the scale obtained; (6) drawing up a system of rules for assessing the likelihood of new risks and (7) validation of this system of rules (Figure 1).

The social component, used to evaluate risks, to structure the problem and discuss the system of rules, involves an interactive process between the group of decision-makers and the facilitator during the various stages of the methodology. The author of this thesis has assumed the role of facilitator, moderating social processes namely one interview,

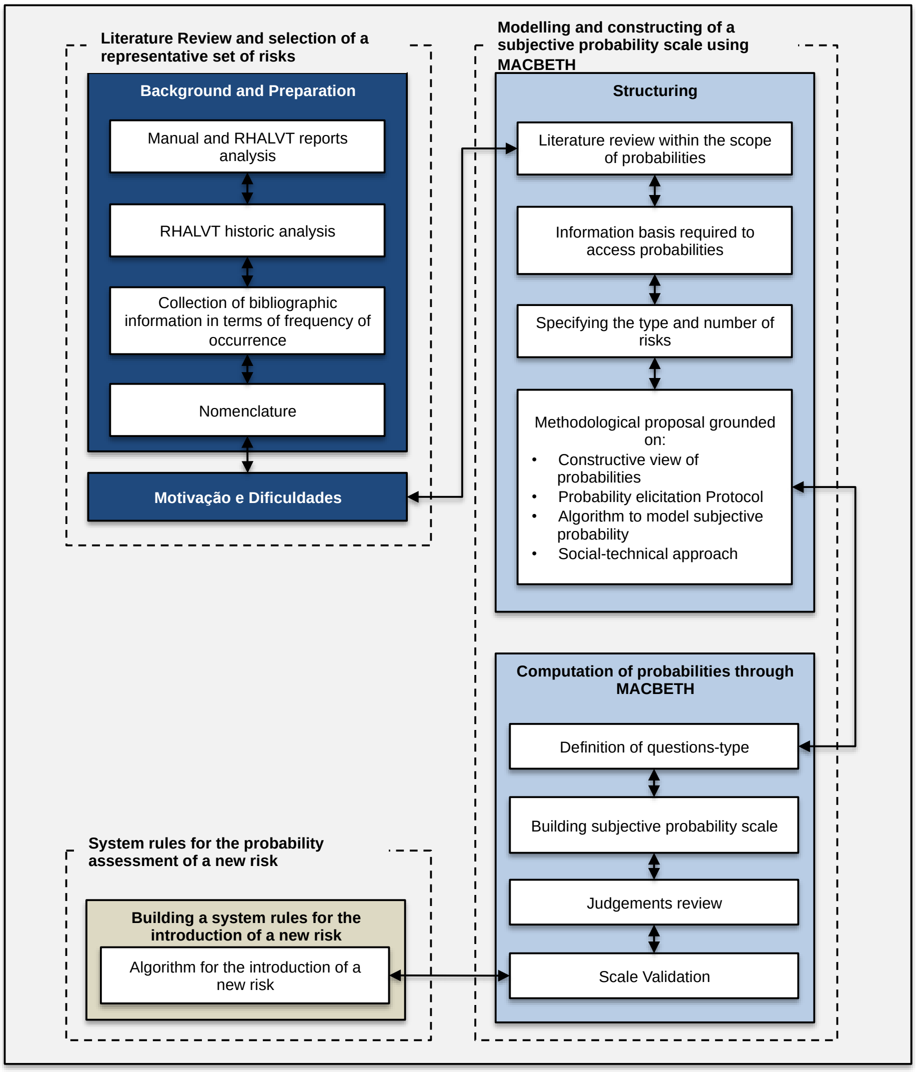

Figure 1: Conceptual framework for building a procedure to assist the choice of probability for health and safety risks. Solid arrows represent a sequential relation.

one motivation session and two decision conferences. The first meeting consisted of an interview to the group of decision-makers in order to identify problems and look for opportunities to improve the evaluation process of the probability of occupational hazards occurring in the workplace.

The second meeting served to identify the basic information to be taken into account in calculating the probability of occupational hazards, and the following meetings consisted of decision conferences, where the modelling and building process of the subjective probability scale was carried out and validated.

The methodology proposed is ground on theoretical foundations as (a) the concept of probability judgments anchored on Savage's personal theory of probability [15]; (b) a constructive view of probabilities that helps to counteract the many biases that may affect intuitive judgements of uncertainty [16]; (c) deriving subjective probabilities from paired comparisons of events in terms of their likelihood (several authors have used it in the framework of AHP's paired comparisons process) [17]; and (d) the measurement of subjective probabilities as absolute probabilities.

3.1 Building a subjective probability scale

3.1.1. Proposed Methodology

Initially, it was necessary to conduct a "Background and Preparation" phase [18] in order to define the problem by identifying its variables.

This stage in the preparatory work was based on a survey with evidence about health and safety risks at work, which allowed the selection of a set of representative risks used to assess the building of the probability scale and to develop an Excel database with a comprehensive list of events and information obtained from literature regarding the frequency of occurrence.

After the second reunion, where some issues were highlighted with the aim of understanding the important factors in the computation of probabilities, and to create incentives for the decision-makers to express certain points of view, we proceeded with structuring the design for the subjective probability scale. This stage involves defining the relevant aspects for the computation of probabilities, as well as its constructivist view and the definition of a protocol for eliciting probabilities. With particular attention to the present case study, the first step in the design process is to ensure that the decision-maker has a clear understanding of the state or event upon which the assessment is being conducted, taking into account the conditional factors that may influence the value of the probability calculation in question [11, 13, 19]. Thus, it is important to ensure that events are clearly well-defined in order to obtain a common understanding by all decision-makers and to avoid ambiguity during the next stage of valuation probabilities [19].

After due reflection and discussion about the set of factors which are necessary to describe the risk in a clear and concise manner and not allow arbitrariness, in the process of evaluating probability, we concluded that the evaluation of the latter must be stipulated taking into account the five aspects: (a) take into consideration the source of risk and the respective consequence; (b) the exposure to a source of risk, (c) the effectiveness of security measures; (d) the job of the worker (e) a baseline (the average worker's characteristics). As workers may have distinct special features (younger or older workers, pregnant workers amongst others) that influence the probability assessment process, it became necessary to create a hypothetical average worker to serve as a basis of analysis for the description of the risk (Table 1).

Table 1: Average worker's characteristics that forms the baseline to the evaluation of the worker exposed

Average worker's characteristics	Description
Age	ARLVT's workers average age (48 years old)
Individual characteristics/ vulnerability	Not obese, no history of psychological disorders and chronic illnesses and taking medication that may increase their susceptibility
Knowledge of best practices	Yes
Experience in the role	Yes – Worker with formation

According to French et al. (2009), the construction of a protocol elicitation can help an analyst to challenge the judgments of decision-makers in a delicate way, by helping them build judgments that are coherent with their beliefs [19].

The methodology that we proposed to develop seeks to establish a measurement of subjective probabilities as absolute probabilities. Thus, rather than facilitate comparisons between risk events, which is more compatible with the concept of relative probability (i.e. the probability of occurrence of a particular risk event being greater, the same or less than the probability of occurrence of another event) the basis of elicitation proposed consists of qualitative judgments about the difference between the probability of a given event risk and an "impossible event" and the difference between the probability of that same event and a "certain event".

The issues we want to address follow the line of questioning that risk managers (or experts in risk assessment) are used to responding to when they have to define the qualitative levels of risk probability. So after a full risk characterization, we wanted the group of decision-makers to be confronted with the following questions: (a) "How likely is it for risk event x to happen?”. The first question posed intends to measure the difference in probability between event x and the impossible event and (b) "How unlikely is event x to happen?". On the other hand, this question aims to determine the probability of difference between an accurate event and event x. Microsoft PowerPoint worked as a support system, with the "slideshows" in MS PowerPoint mode and the questions posed were done in an interactive and sequential manner to the decision-makers, then the answers were recorded in real time using the ActiveX textbox tool.

To answer these two questions, the decision-makers have to bear in mind that the difference between the probability of an accurate event and the impossible event is inevitably one, and that the answer is given by using a qualitative scale that mediates the difference in probability between risk events.

To this end, we adapted the scale proposed by MACBETH for measuring the differences in probability: (1) zero, (2) Very weakly probable, (3) Weakly probable, (4) Moderately probable, (5) Strongly probable (6) Very strongly probable and (7) Extremely probable.

Adapting the MACBETH algorithm probability context, the M-MACBETH allows the purpose of a numerical scale consistent with the qualitative judgements previously performed:

$$\textit{Minimize: } p\ (x_+)$$
$$\textit{Subject to:}$$
$$p(x_-) = 0$$
$$\forall (x,y) \in C_0 : p(x) - p(y) = 0$$
$$\forall (x,y) \in C_i U \ldots U C_s \ with\ i,s \in \{1,2,3,4,5,6\}\ and\ i \leq s : p(x) - p(y) \geq i$$
$$\forall (x,y) \in C_i U \ldots U C_s\ e\ \forall (w,z) \in C_{i'} U \ldots U C_{s'},\ with\ i,s,l',s' \in \{1,2,3,4,5,6\},\ i \leq s,\ i' \leq s'\ e\ i > s' : p(x) - p(y) \geq p(w) - p(z) + i - s'$$

Where X represent a finit set of risk events, $p\ (x)$ the score of element x, $C_k, k = 0 \ldots 6$ the seven MACBETH categories of difference in probability, x_+ and x_- the elements of X with the bigger and lower probability, respectively, x and y two elements of X such that x is at least as probable as y, and $\forall (x,y) \in C_i U \ldots U C_s\ (i,s = 1 \ldots 6\ with\ i < s)$ a MACBETH judgment of difference in attractiveness between x and y expressed by a subset of categories from C_i to C_s (in cases of judgmental hesitation or disagreement).

After having discussed the basic information for the calculation of probabilities, it was necessary to understand and identify the risks that should be taken into consideration when building the probability scale. The survey and identification of risks was based on cross-referencing information between the main risks found in the risk map reports of RHALVT and the risks found in the literature, so as to cover the entire range of probability by selecting risks of various graduations which can be seen on RHALVT´s current scale with the following markings (improbable, probable, fairly probable and very probable).

After identifying the risks to be pondered in a subsequent analysis, we proceeded with their full description considering in each case the source of the risk, the effectiveness of security measures which were implemented and the job of the worker in question, bearing in mind the individual characteristics of an average worker present in table 1.

3.1.2. Applied Methodology

Once the structuring phase was completed, we proceeded to determine the probability of each risk and then the design of the respective probability scale. As an example, the facilitator confronted the group of decision-makers with issues such as: (a) "How likely is the risk of Posture Extreme events to happen?" and (b) "How unlikely is the risk of Extreme Posture events to happen?" characterized by the available information is represented in Table 2.

These types of questions have been done for all risk events, always with a complete characterization of the risk as a basis for analysis, allowing the completion of the first row and last column of the MACBETH matrix.

Table 2: Full characterization of the risk *Posture Extreme*

Risk	Source of risk	Consequence	Exposure to source of risk	Effectiveness of security measures	Job of the worker
Posture Extreme	Without ergonomic conditions	Pain and muscle-skeletal injuries	Routine 8hours/day	Absent	Professionals who spent many hours sitting in their workplace

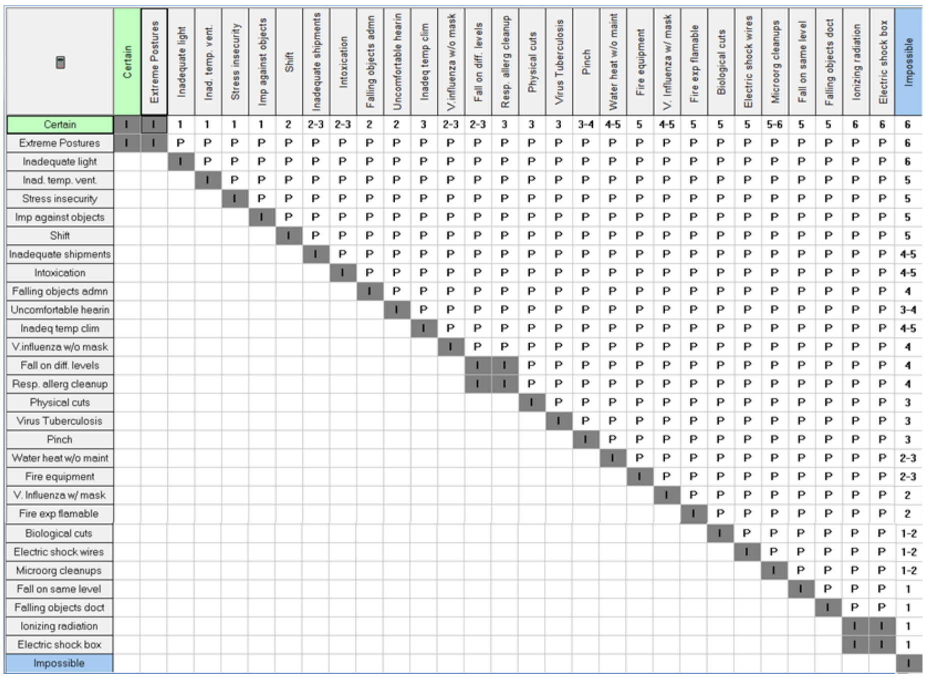

Figure 2: MACBETH matrix for the representative set of risks

It should be noted that often the decision-makers hesitated between two semantic categories, but this can be circumvented since the M-MACBETH software allows you to enter more than one category in the same cell of the matrix.

Once the first row and the last column of the judgment matrix are filled in, the facilitator simplified the integration process of qualitative judgments by making use of the transitivity process offered by the software. This feature allows you to make use of the information available from previous questions, in order to identify possible ordinal paired comparisons that they entail and fill in with a "P" the judgments of the respective cell matrix, suggesting that the risk event in the respective line is more likely to happen than the risk event of the respective column. The decision-makers should evaluate this classification and if the group does not agree with it, at least one of the two judgments carried out has to be revised. Certain risk events had the same probability category for both of the questions that were asked. Since there is no condition imposed on the MACBETH algorithm for judgments of the same category, it became necessary to conduct additional trials between each of these pairs of risk events.

During the completion of the matrix, the software detected some inconsistencies between the judgments issued. For each case, we discussed the problem and addressed the suggestions provided by the software, and then adopted the ones where there was a consensus in the group of decision-makers. In the second decision conference, the opinions expressed were reviewed and the validation of the probability scale was obtained.

3.2 Development of the System of Rules

In a first analysis, we only considered one risk test group and the intention is to build a generic system to assess any type of risk and not just those that are present on the probability scale obtained, therefore it became useful in a second phase to develop a system of rules to assess the likelihood of a new risk. In this sense, the methodology presented in this section aims to show the logic behind the construction of rules for calculating the probability of a new risk, making use of the description of reference risks previously considered, and from the literature found in the existing area.

There is a variety of variables that influence the probability of the risk materializing. Just a slight difference in the intensity of one of the relevant factors in the process of evaluating the probability of risk is enough to influence the possibility of risk occurring. Thus, the logical construction of the system of rules is to adjust the probability of the reference risk, according to pros (positive factors) and cons (negative factors) in order to obtain the probability of a similar risk or even a new risk that is not on the database. Taking into consideration the pros and cons is very important, as these can greatly affect the quantitative performance [20] of the probability of risk.

4 Results of the Applied Methodology

4.1 Subjective Probability Scale

Once the MACBETH matrix is completely filled in and the consistency of judgments issued verified, the software proposed a scale of subjective probability that assigns to each risk event a numerical value of probability.

The decision-makers were asked to validate the scale obtained, by comparing the size range of intervals between the probabilities of risk events and the order of the respective scale, and some adjustments had to be made. Figure 3 shows the scale validated by the decision-makers.

After observing and analysing the scale together with the group of decision-makers, they agreed with all the values except for two scores corresponding to risk events intoxication and improper loading. So we turned back to the MACBETH judgments matrix and after reviewing the brief description of each risk, all members of the group agreed to swap the order of these two judgments.

Since the interval between the fire risk probability (due to the use of equipment with inflammable materials) and the injury risk probability (i.e. fingers trapped while handling equipment) was significantly high (0.29 - 0.42), it became useful to introduce a new risk in the range of moderate probability. Through the analysis of risk maps found in the reports by RHALVT, we selected the biohazard contamination of water (due to water heaters without proper maintenance) and turned to confront the decision-makers with both types of questions.

The judgments made by the decision-makers were inserted in the MACBETH matrix. Therefore overall, the final scores obtained for each risk event identified with the qualitative degree of probability in the previously assigned risk maps and they indicated the decision-makers' preferences.

4.2 System of Rules

Once the scale of subjective probability was designed, we proceeded with the logic construction of the system of rules for evaluating the probability of a new risk, which is represented in the flowchart in figure 4 obtained and validated in the second conference decision.

In short, the purpose of this last stage of the proposed methodology is to provide a basis for analysis (taking into account the historical events and literature obtained) to calculate the probability of new risks.

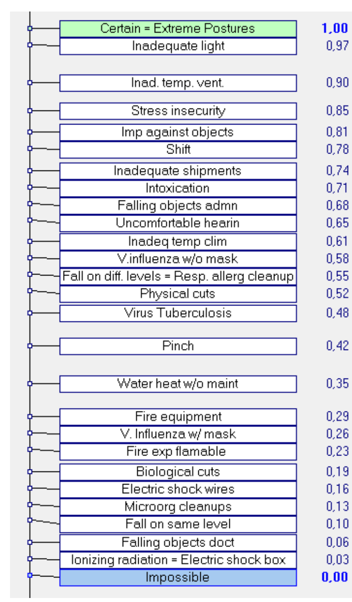

Figure 3: Subjective probability scale validated by the group of decision makers.

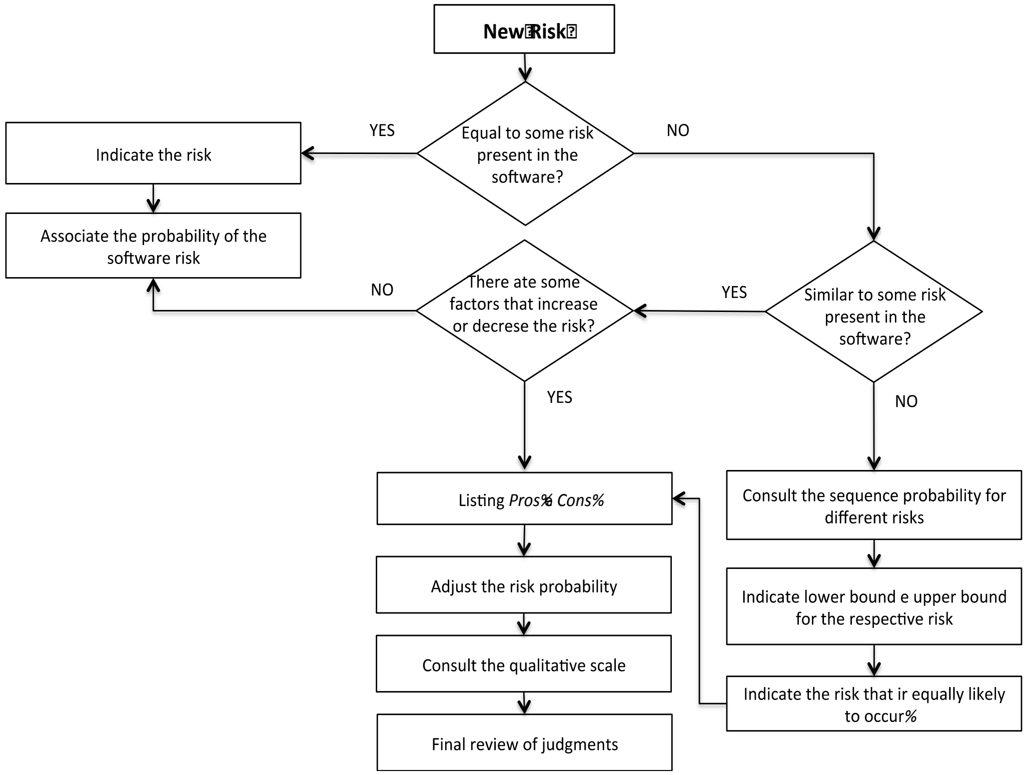

Figure 4: Logic construction of the system of rules for evaluating the probability of a new risk.

5 Discussion

The described methodology intended to support RHALVT in the probability assessment for health and safety risks, by addressing MACBETH approach, in a rational and transparent way, throughout an interactive consulting path.

The probability numerical scale obtained presents certain advantages for the group of decision makers. One of the advantages is the fact that it decreases the ambiguity present in the process of evaluating the probability of health and safety risks since unlike verbal probability that tends to have imprecise meanings, the numerical probability has a precise value scale between zero and one.

Given that risk assessment usually requires numerical information for subsequent calculations, verbal approaches have the key disadvantage of requiring translation if they are to be used for subsequent analysis. The quantification of probability by pairwise comparisons will allow OHSU to perform an analysis more precise and less ambiguous when assessing the likelihood of occupational hazards.

The results obtained are in line with the expected and the subjective probability scale and the algorithm flowchart for the system rules are being implemented into a software's value risk matrix that will be used by OHSU to assess health and safety risks.

Regarding the construction of the subjective probability scale by MACBETH, this approach request qualitative judgment rather than quantitative to measure the difference in likelihood between pairs of risks events. This makes the procedure simpler and intuitive for de decision maker since a set of studies demonstrate a strong preference to express probabilities in a verbal form. Therefore, the use of MACBETH becomes very attractive in the context of eliciting probabilities from the experts since it allows to obtain numerical scales consistent with the qualitative judgments performed, bypassing the need for individuals to express the probability in terms of frequency. Although this methodology has proved to be very advantageous in the context of elicitation probabilities from the experts, there were also some limitations associated with it.

One of the weaknesses points, from the point of view of the decision maker was the fact that the decision conferences have been very long and time consuming. As some studies have pointed out, individuals tend to employ certain "easy-to-use strategies" when faced with uncertainty situations. This type of strategy is called heuristic and may have been employed in this work, reflecting a limitation of the methodology since it leads to the practice of biased judgments. During the process of eliciting subjective probability, some elements of the group evaluator may have resorted to the heuristic "availability" by remembering some recent experience, which may have influenced the outcome of the analysis. However, despite these limitations and the fact that MACBETH wasn't known by the elements of the group of decision makers, the group surprisingly admired its overall performance.

6 Conclusions and future work

The developed system indented to support OHSU of RHALVT in the definition of probabilities applied to the assessment of health and safety risks at work. This system was built from the modelling and construction of a subjective probability scale, by using qualitative judgments by decision-makers, and from the creation of a system of rules to define risk probability in the construction of risk matrices.

The results proved to be satisfactory, given the uncertainty considered and the limitation of the proposed methodology and have enabled to support OHSU in assessing the probability of occupational risks, as the software that is being developed includes general information about the risk events analyzed, the probability associated with each and the integration of the logic construction of the system of rules for evaluating the probability of a new risk.

Taking into account what was written before, it's important to mention some suggestions for future developments. The results obtained might be used for the construction of a value risk matrix for the assessment and mitigation of health and safety risks at work.

Thus, the proposed generic methodology should be further validated in the context of a value risk matrix created in the project IRIS. The validation can be done as the software is built, initially with arbitrary data and later with representative real-world data. In this way, it will be possible to obtain insights to adjust the process proposed through the detection of possible inconsistencies that have not been detected so far.

The development of the support system created will provide a probability historical background that had not been performed so far. This will make the system more comprehensive as the introduction of news risks through the system of rules provide a more complete data base that will facilitate future evaluations.

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