Design Optimization of Variable Stiffness Bi-Stable Composites using Curvilinear Fibers

Luísa Barbosa
luisa.barbosa@ist.utl.pt
Instituto Superior Técnico, Lisboa, Portugal
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Abstract

The current study addresses the design of bi-stable variable stiffness composites for morphing aircraft using curvilinear fibers. The design philosophy consists of tailoring out-of-plane displacements using variable stiffness bi-stable laminates in order to optimize the performance of aircraft control surfaces. The study has practical relevance to the aircraft industry because it enables the performance optimization and thus potentially the reduction of aerodynamic drag and fuel consumption, which is a topic of current interest in the area of greening aircraft. The main contribution of the present work lies in the development of an optimization framework for bi-stable laminate design based on the integration and coupling of analysis tools (ABAQUS and Matlab) with an optimizer and integrator software (ModelCenter). Two methods to design a variable stiffness laminate are presented in this thesis: a discrete patch design and a continuous design that use a curvilinear function to describe the fiber paths. These two techniques are integrated into a unified design approach that ensures continuity of fiber over the planform of the composite structure and, in doing so, may impart additional structural strength due to the load path continuity. Two different configurations of cantilevered composite laminates geometries (rectangular and trapezoidal) have been studied for illustration and evaluation purposes and the final outcomes include the prediction of the cured shapes and their stability characteristics.

Keywords: Bi-stable laminates design, Variable stiffness composites, Optimization

1. Introduction

Recent years have seen a growing interest in conformal shape adaptation, or morphing, as means of significant performance enhancement, particularly in the aerospace domain. This morphing or shape adaptable structural systems change shape or state in order to change their operating characteristics or as a response to changes in the environmental conditions.

Composite materials made of orthotropic layers can develop a residual stress field when subjected to a thermal field that varies with time \[1\]. The thermal stresses are caused by the mismatch of coefficients of thermal expansion along fibers directions. If the material is unsymmetrically, bending and twisting moments are generated within the laminated structure which results in significant out-of-plane displacements. This happens during the curing process of a composite structure when the material is heated up to achieve a desired degree and subsequently cooled down to the room temperature. These multistable or bistable structures are good candidates to be used as morphing structures because of their ability to remain in natural equilibrium after a shape change occurs without the need for a continuous power supply \[2\]. Various theoretical and experimental studies \[3, 4, 5, 6, 7, 8\] have been carried out in order to understand and predict the behavior of the bi-stable composite structures.

The recent developments introduce the Non-Conventional Laminate concepts (NCLs) that enable the improvement of the mechanical properties and/or weight of the structure. It can be achieved by modifying the stacking sequence within the bounds of conventional design rules and is referred to as laminate tailoring. Laminate tailoring can be taken further by allowing fiber angle variation within a ply. This variation in fiber orientation angle results in spatially variable stiffness properties and is therefore referred to as variable stiffness (VS) laminates \[9\]. Studies have shown improvements in mechanical performance for variable stiffness laminates in comparison to traditional ones \[9, 10, 11, 12\]. This stiffness variation may be discrete, by defining several different patches within a laminate, or continuous, by varying the fiber angle orientation continuously within a plys domain. Continuous stiffness variation is essentially a gen-
eralization of discrete stiffness variation, which still stems from the more traditional method of defining laminates using a fixed stacking sequence.

These advanced composite laminates may be manufactured with Tow Placement Technology. Fiber placement systems [13] have become widely used to manufacture complex aerospace composite structures [13] which resulted in associated reductions in touch labor time, material wastage and part counts. Thus, the tow-steering technology enables to locally tailor fiber orientations across the planform of the laminate, giving the designers the opportunity to develop optimum laminates for target applications. A literature survey on tow placement technology and variable stiffness panels is given in [15].

The substantial increase in structural efficiency possible when using variable stiffness laminates and the lack of available design tools motivated the development of a design optimization routine for variable stiffness bi-stable structures.

The obtained solution provides the designer with the fiber angles distribution best satisfying the desired structural performance requirements. In this work the focus is to find the optimal fibers angles distribution that maximize the relative deflections of the two stable states of a bi-stable composite.

2. Background

2.1. Multistable Composites

Unsymmetric laminates that are cured at an elevated temperature deform severely as they are cooled to their service temperature. Under certain conditions the thermal warping can lead to two stable states. These deformations are due to the mismatch in the coefficients of thermal expansion $\alpha_i$ along different fibers directions. This means that when a composite material made of orthotropic layers is subjected to a temperature difference, it will experience different strains $\varepsilon_i$ along each direction $i$:

$$\varepsilon_i = \alpha_i \times \Delta T$$

Thus, when the laminate is cooling down from the autoclave cure temperature to a service temperature, say, room temperature, the layers with different orientation angles will tend to have different strains. As the layers cannot freely stretch relative to each other, the laminate will develop a curvature that best accommodates the conflicting individual strains and that minimizes the residual stresses associated with the inability of each layer to stretch by the amount given by equation (1).

As the temperature change relative to the curing temperature increases, the magnitude of the laminates curvature also increases. If the temperature change exceeds a certain threshold there is no curvature of the laminate that can accommodate the conflicting thermal strains of the various layers. In that case, the influence of one of these conflicting thermal strains prevails and the laminate acquires a curvature dominated by the thermal strains of one of the layers. However, significant residual stresses will be present and if an appropriate load, referred to as the snap-through force, is applied it will acquire a curvature dominated by the thermal strains of those other layers.

The possibility of snapping (bifurcating) from one configuration to another by means of an actuation system expands the range of possible application of these type of structures as the actuator is only required to provide energy during the snap-though process and not to maintain a configuration since both are stable (Mattioni et al., 2005, 2006 [16, 17]).

3. Problem formulation

As already said, this work aims to achieve a blended bi-stable laminate so that the performance of the structure can be maximized. To investigate that key parameters for laminate design need to be first defined.

1. Define the laminate geometry and its dimensions. Various geometries were tested: rectangular and trapezoidal laminates.

2. Define the boundary conditions of the structural problem. In this study, laminates arranged in a cantilever configuration are studied. To achieve this, a mixed lay-up showing symmetrical and unsymmetrical stacking sequences is used. The rectangular laminate is shown in figure [1].

3. Choose the laminate modeling philosophy that present the blending concept used to represent the fiber orientations of the continuous tow-paths.

4. Implement the tow-steering compatibility constraint. Here, the tow-steering compatibility constraint is introduced in the design methodology.

5. Define the design objective function that addresses the problem considered in this thesis by defining an appropriate objective function.

In this work, 4-ply thick laminates are considered within the top two plies having variable angle tow-steered configurations and the bottom two plies subjected to 0°fiber orientation over entire region. The non-bistable region, which represents the major structure near the perimeter of bi-stable section, has 0°fiber direction in each ply.
3.1. Laminate Modeling Philosophy

The focus of the modeling philosophy is to develop an efficient design for a variable stiffness bi-stable laminate, such that the advantages offered by the tow-steering techniques and composites materials are fully exploited.

The tow-steering technology uses equipment such as the Tajima TMLH-101 \[18\] to locally tailor fiber orientations across the planform of the laminate.

Tow-steered laminates with curved fiber paths can be designed to develop bistability in the structure along with ensuring fiber continuity within the wider structure, thereby structurally integrating the bi-stable section with the remaining structure.

However, allowing the use of variable stiffness laminates with arbitrary angles over the entire laminate significantly increase the design space due to the large number of associated design variables.

A cellular based patch design approach was adopted. This consist in decompose the laminate into several patches/regions of straight-fibers to locally and discretely represent the fiber orientations of continuous tow paths generated by the tow-steering technique. The angle assigned to a region (cell) corresponds to the tow direction (averaged) within this region.

Figure 2 presents the blending concept, where the area enclosed by the edges BB', CC' are representative of the bistable region that will be optimized and the area enclosed by the edges AA', BB' represents the non-bistable region.

The bistable region has a cellular grid structure comprising 100 identical square cells (10(number of rows) x 10(number of columns)).

The cellular grid structure is decomposed into two sets of 5 x 10 grids. The top grid is used for exploiting the possible combination of fiber angles while the bottom grid assumes anti-symmetric angles in the corresponding cells. In doing so, a symmetric deformation can be achieved.

The fiber angles $\theta_{(R,C)}$ assigned to the cells in the top 5 x 10 grid are limited to possible values from the set \{0°, 15°, 30°, 45°, 60°, 75°, 90°\}.

3.2. Tow-steering Compatibility Constraint

A tow-steering compatibility constraint was introduced in the design methodology to depict the continuous nature of the tow-steered construction in a cellular structure \[19\].

This constraint demands that each cell in the structure has fiber orientations not varying by an amount greater that $\theta_{adjoining}$ from its adjoining cells.

This compatibility constrain can be expressed by the Inequality,

$$| \theta_{(R,C)} - \theta_{(U,V)} | \leq \theta_{adjoining} \tag{2}$$

where $U = R - 1, R, R + 1; V = C - 1, C, C + 1$; $R$ and $C$ are the number of the rows and columns, respectively.

Figure 3 schematically explain the meaning of inequality (2), the red cell is the one considered for constraint verification and the yellow cells are the adjoining cells for compatibility angle consideration.
3.3. Design Objective

The focus of the work is to find the optimal fiber angles distribution that maximizes the relative deflections of the two stable states of a bi-stable composite.

The objective function may be explicitly defined in terms of out-of-plane displacements in stable state-1 (OPD1) and stable state-2 (OPD2), equation (4) The value used as the out-of-plane displacement value for a stable state is computed from the average value of the out-of-plane displacements of nodes C and mid-C, please refer to figure 2.

\[
\text{ObjectiveFunction} = \begin{cases} 
|\text{OPD1}| + |\text{OPD2}|, & \text{if } \text{OPD1} \leq 0 \text{ and } \text{OPD2} \geq 0 \\
0, & \text{otherwise}
\end{cases}
\] (4)

3.4. Design Framework Implementation

In order to develop a design framework it is necessary to be able to: one, model the stiffness variation; two, analyze the structural response associated with the stiffness variation and three, optimize the design to meet the imposed requirements.

This sub-section describes the implementation of the design framework used for construct the optimization problem and outlines the modeling tools used to execute each analysis.

This study considers the use of ModelCenter coupled with Abaqus FEA and Matlab to iteratively converge upon optimal designs in an automated fashion.

The design inputs are the fiber orientation angles and the objective is maximization of a function that quantifies the relative deflections of the two stable states of the structure.

Since the study involved changing the material properties of the above model, a fully parameterized Python script was written to automate the task in Abaqus. This script create the geometry, assembled the parts, applied the loads and boundary conditions, ran the analysis and exported relevant analysis information to a text file called Results.txt, all without any further user intervention.

The Python script was then wrapped using PHX ModelCenter’s QuickWrap tool. Quickwrap automatically exposed the input variables in the Python script. It was also necessary to define the output variables and the file in which they are located, Results.txt. The command to run Abaqus then simply took the Python script as an argument and performed the steps in the script.

Then, it was needed to define constrain variables in order to enforce compatibility of lamination sequences in adjacent cells and manufacturing limitations, as described in the previous section 4.2.
To achieve that a Matlab QuickWrap component that receive the design variables as input variables and generates constraint variables as output was created.

The Matlab script begins by creating two variables called sub-constraints given by equations (5a,b). Finally a global constraint is created. This constraint corresponds to the higher value of the two sub-constraints created earlier, equation (5c). Thus, each design variable must respect only one constraint (GlobalConstraint) in order to respect the inequality constraint expressed by equation (2).

\[
\text{SubConstraint}_1 = | \theta_{(R,C)} - \min(\theta_{(R-1,C-1)}, \\
\theta_{(R-1,C-1)}, \theta_{(R-1,C+1)}, \theta_{(R-1,C)}, \theta_{(R+1,C)}, \\
\theta_{(R-1,C+1)}, \theta_{(R,C+1)}, \theta_{(R+1,C+1)}) |
\]

(5a)

\[
\text{SubConstraint}_2 = | \theta_{(R,C)} - \max(\theta_{(R-1,C-1)}, \\
\theta_{(R-1,C-1)}, \theta_{(R-1,C+1)}, \theta_{(R-1,C)}, \theta_{(R+1,C)}, \\
\theta_{(R-1,C+1)}, \theta_{(R,C+1)}, \theta_{(R+1,C+1)}) |
\]

(5b)

\[
\text{GlobalConstraint} = \max(\text{SubConstraint}_1, \text{SubConstraint}_2)
\]

4. Results Blended Laminate Design

This chapter presents a series of numerical optimisation results using the problem formulation of section 3. All examples use T300/914 material properties, Table 1.

<table>
<thead>
<tr>
<th>Material</th>
<th>T300/914</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{11}(GPa)$</td>
<td>130</td>
</tr>
<tr>
<td>$E_{22}(GPa)$</td>
<td>10</td>
</tr>
<tr>
<td>$G_{12}(GPa)$</td>
<td>4.4</td>
</tr>
<tr>
<td>$v$</td>
<td>0.3</td>
</tr>
<tr>
<td>$\alpha_1(1/^\circ C)$</td>
<td>-1.8e-8</td>
</tr>
<tr>
<td>$\alpha_2(1/^\circ C)$</td>
<td>30e-6</td>
</tr>
<tr>
<td>$t(mm)$</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Table 1: Material properties of carbon fiber laminates used throughout the analysis

4.1. Rectangular Laminate

The steps used in the FE analysis are described below and used a value of $5 \times 10^{-10}$ for the damping factor (unless stated otherwise):

1. In the first step residual stresses are introduced by cooling it to the room temperature. The cool-down is simulated by applying an initial temperature of 140$^\circ$ and a final temperature of 0$^\circ$ to all nodes of the model.

2. Actuation loads are applied on the corners C and C’ to make the laminate deform in one of the stable shape of the laminate. ($c = 5 \times 10^{-7}$)

3. The loads are removed to acquire the first stable shape.

4. Actuation loads are now applied in the opposite direction to make the structure to snap-through and deform towards the possible second stable shape.

5. These loads are removed to observe a possible second stable shape.

The rectangular laminates have the dimensions reported in figure 4.

Figure 4: Rectangular laminate dimension and boundary conditions

Convergence studies were carried out to investigate the influence of both mesh density and the damping factor. Very fine meshes were computationally too demanding and extremely small damping factors resulted in models failing to converge. A good compromise between the model convergence and the computational effort was found for the reported values.

A mesh of 1974 doubly-curved, 4 node shell elements, S4R, is used to perform the numerical analysis of the rectangular laminate in Abaqus.

4.1.1 Unconstrained Parametric Study (Unidirectional Fibers)

A parametric study considering only unidirectional fibers was done. This study considers the set of fiber angles \{15$^\circ$, 30$^\circ$, 45$^\circ$, 60$^\circ$, 75$^\circ$, 90$^\circ$\} for the top two plies of the bi-stable region, which is the one considered in the optimization process. The best result of this analysis is reported in Table 2.
4.1.2 Preliminary Study

Problem description

In this section the optimization is performed to find the optimal distributions of fibers of a simplify model of the rectangular blended bi-stable laminate. This preliminary study has been conducted to test the developed design framework and also allows finding a possible initial fiber angle distribution to the final blended rectangular laminate.

In this preliminary study the bi-stable region is divided into 4 regions, as shown in figure 6. This results in a model with only four design variables.

The results of the preliminary study are reported in the following sub-section.

Generated Results

The Darwin algorithm was set to a population size of 21 with a multiple elitist selection scheme with a number of preserved designs \(N_p = 6\). The Use Memory feature of the algorithm was used.

The algorithm was run until convergence, i.e. if the change in objective function value, equation 4, isn’t better than the top design in the previous generation for 20 consecutive iterations, or until a maximum number of iterations was reached. The number of maximum generations specified was 1000. A crossover probability of 1.00 and a mutation probability of 0.05 was used. The percentage penalty was set to 0.5.

Results were obtained using the out-of-plane displacements of the stable states of the plate according to equation 4 as a design objective.

The convergence study obtained for the maximum deflection between states is plotted in figure 5.

Figure 5: Design history of the preliminary study of the rectangular laminate

Table 3, reports the result OPD of each state as well as the result objective function,

<table>
<thead>
<tr>
<th>OPD1 (mm)</th>
<th>OPD2 (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>405.086</td>
<td>755.992</td>
</tr>
<tr>
<td>Objective Funtion(mm)</td>
<td>1161.078</td>
</tr>
</tbody>
</table>

4.1.3 Final Blended Rectangular Laminate

Problem description

The rectangular laminates have the dimensions reported in figure 4. Each cell of the figure represents a fiber orientation angle, as explained in section 4.1.

The numerical predictions of the snap-through and snap-back are obtained by applying the boundary described in figure 7. A clamped boundary condition is imposed on the edge A-A'.

Generated results

The fiber angles \(\theta_{(R,C)}\) assigned to the cells in the top 5×10 grid are limited to possible values from the set \(\{0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ, 90^\circ\}\). Note that the thickness of the laminate was kept constant.

The Darwin algorithm was set to a population size of 50 with a multiple elitist selection scheme with a number of preserved designs \(N_p = 15\). The Memory feature of the algorithm was used.

The algorithm was run until convergence, i.e. if the change in objective function value, equation 4, isn’t better than the top design in the previous generation for 90 consecutive iterations, or until a maximum number of iterations was reached. The number of maximum generations specified was 1000. A
crossover probability of 1.00 and a mutation probability of 0.10 was used. The percentage penalty was set to 0.5.

The initial fiber angle designs, or fiber angle seeds, used were the obtained in the previous preliminary study.

The convergence study obtained for the maximum deflection between states is plotted in figure 7.

Figure 7: Design History Rectangular Laminate

The best design was obtained for the fiber angles presented in figure 8. Note that the angles adjacent to the axis of anti-symmetry, i.e in the transition region are just the representative of the value of $\theta_{top}$.

Figure 8: Optimum fiber angle distribution rectangular laminate

**Numerical results**

The numerical prediction of the optimum bi-stable laminate is provided in this section.

In figure 9 it is possible to see the stable states of the obtained optimum design.

Table 4 reports the result OPD of each state as well as the result objective function.

Table 4: Results optimum design

<table>
<thead>
<tr>
<th></th>
<th>OPD1</th>
<th>OPD2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacements (mm)</td>
<td>438.941</td>
<td>748.525</td>
</tr>
<tr>
<td>Objective Function (mm)</td>
<td>1187.465</td>
<td></td>
</tr>
</tbody>
</table>

Figure 9: Stable states optimum rectangular laminate

The load-displacement plots allow us to compute the maximum out-of-plane load that the optimum laminates can withstand before changing configuration and are reported in figure 10. For convenience, it was defined that the first and second stable shapes as having negative and, respectively, positive values for the out-of-plane displacements. Thus, the actuation loads are acting in a positive direction to make the structure snap-through from the first to the second stable shape and in negative direction for opposite actuation.

Figure 10: Load-displacement relationship

4.2. Trapezoidal Laminate

The steps used in the pseudo-dynamic FE analysis are the same described in section 4.1.

A good compromise between the model convergence and the computational effort was found for a mesh of 1698 doubly-curved, 4 node shell elements, S4R.

The trapezoidal laminates have the dimensions reported in figure 11.

4.2.1 Unconstrained Parametric Study (Unidirectional Fibers)

As with the rectangular laminate a parametric study considering only unidirectional fibers was done. The study considers the set of fiber angles \{15°, 30°, 45°, 60°, 75°, 90°\} for the top two plies of the bi-stable region, which is the set considered in the optimization process. The best result of this analysis is reported in table 5.

4.2.2 Preliminary Study

Problem description
Figure 11: Trapezoidal laminate dimension and boundary conditions

Table 5: Parametric study trapezoidal laminate

<table>
<thead>
<tr>
<th>Fiber Orientation</th>
<th>OPD1 [mm]</th>
<th>OPD2 [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>75°</td>
<td>190.4721</td>
<td>746.2759</td>
</tr>
</tbody>
</table>

A preliminary analysis of a simple model of the bi-stable variable stiffness laminate has been conducted. The regions considered in this preliminary study are shown in figure 13.

**Generated Results**

The Darwin algorithm was set to a population size of 21 with a multiple elitist selection scheme with a number of preserved designs \( N_p = 6 \). The Memory feature of the algorithm was used. The algorithm was run until convergence, i.e. if the change in objective function isn’t better than the top design in the previous generation for 20 consecutive iterations, or until a maximum number of iterations was reached. The number of maximum generations specified was 1000. A crossover probability of 1.00 and a mutation probability of 0.05 was used. The percentage penalty was set to 0.5.

The convergence history obtained for the maximum deflection between states is plotted in figure 12. The best design was obtained for the fiber angles presented in figure 13.

Figure 13: Optimum design of the preliminary study of the trapezoidal laminate

Table 6: Results optimum preliminary design

<table>
<thead>
<tr>
<th>OPD1</th>
<th>OPD2</th>
</tr>
</thead>
<tbody>
<tr>
<td>256.4093</td>
<td>737.7249</td>
</tr>
<tr>
<td>Objective Function [mm]</td>
<td>994.1342</td>
</tr>
</tbody>
</table>

4.2.3 Final Blended Trapezoidal Laminate

**Problem description**

The regions considered in the final study of the trapezoidal laminate are reported in figure 11. The colored area represents the transition region.

**Generated results**

The fiber angles \( \theta_{(R,C)} \) assigned to the cells in the top 5 × 10 grid are limited to possible values from the set \( \{ 0°, 15°, 30°, 45°, 60°, 75°, 90° \} \).

The Darwin algorithm was set to a population size of 50 with a multiple elitist selection scheme with a number of preserved designs \( N_p = 15 \). The Memory feature of the algorithm was used.

The algorithm was run until convergence, i.e. if the change in objective function value, equation 4, isn’t better than the top design in the previous generation for 90 consecutive iterations, or until a maximum number of iterations was reached. The number of maximum generations specified was 1000. A crossover probability of 1.00 and a mutation probability of 0.10 was used. The percentage penalty was set to 0.5.

The initial fiber angle designs, or fiber angle seeds, used were the ones obtained in the preliminary study. The convergence study obtained for the maximum deflection between states is plotted in figure 13.
The best design was obtained for the fiber angles presented in figure 15.

Figure 15: Optimum fiber angle distribution trapezoidal laminate

**Numerical results**

In figure 16 it is possible to see the stable states of the obtained optimum design.

Figure 16: Stable states optimum trapezoidal laminate

Table 7 reports the result OPD of each state as well as the result objective function.

<table>
<thead>
<tr>
<th>OPD1</th>
<th>OPD2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacements(mm)</td>
<td>367.7187</td>
</tr>
<tr>
<td>Objective Function(mm)</td>
<td>1084.687</td>
</tr>
</tbody>
</table>

The load-displacement plots are reported in figure 17.

Figure 17: Load-displacement relationship

5. Conclusions

This work presents a design framework created and used to design morphing bi-stable laminates, where the laminates were assumed to be a variable stiffness composite structure. The framework take the advantages offered by-tow-steering techniques and composite materials while ensures fiber continuity within the wider structure.

This was done by using laminates with two regions with different stacking sequences. In doing so, it is possible to simulate the interaction between a moving component and a stiff structure which is appropriate when considering morphing structures.

The extended modelling was applied to an optimization study where the objective function maximizes the performance of the bi-stable laminates considered. To achieve that this thesis considers the use of ModelCenter coupled with Abaqus FEA and Matlab to iteratively converge upon optimal designs in an automated fashion.

Different configurations of laminates geometries (rectangular and trapezoidal) and modeling approaches have been analyzed which include predicting their cured shapes and their stability characteristics due to a concentrate force leading to establishing the requirements to trigger snap-through behaviour.

It is important to note that the simplified preliminary works provided a close approximation to the more complex cases. Quantitatively, i.e, in terms of the optimum solutions objective function value, the global study of figure 8 provided a 2% higher value than that of the preliminary solution of figure 6. The global solution of the trapezoidal laminate provided a 9% higher value than that of its preliminary study (figure 15 and 13, respectively).

For the parametric studies that were made using unidirectional fibers and again, taking into account the value of the objective function of each solution, were obtained enhancements of 10% in the case of the optimum rectangular laminate and 20% for the optimum trapezoidal laminate. This analysis allow us to make a measure of how the increased design freedom that results from using a variable stiffness composites improves the performance of the laminates considered. Moreover, the use of laminates with curved fibers mitigates the effect of stress concentrations that is expected to occur when straight
fibers are used (due to the lay-up mismatch).

The Load-displacement graphics of the optimum rectangular and trapezoidal laminates were reported to compute the maximum out-of plane load that the laminates can withstand before changing configuration (figure 10 and 17). The snap-through loads can be defined as the loads for which large displacements occur. In the figures, these large displacements appear as the regions where the lines are horizontal.

The design and optimisation of a bi-stable composite was shown to be possible using the design framework presented in this work and the results reported herein are encouraging of applying the technique to several other structural cases. The overall conclusion is that a flexible computational tool set has been developed which can be used to produce an optimum variable stiffness bi-stable composite given in terms of an optimal fiber angles distribution.

References