

Information and Communication Theory

2023

Problem Set 2

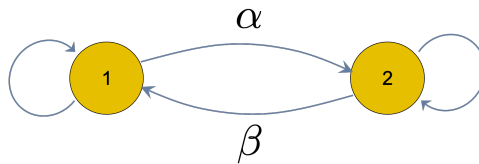
Department of Electrical and Computer Engineering,

Instituto Superior Técnico, Lisboa, Portugal

1. Consider a first-order Markov source with the probability transition matrix

$$\mathbf{P} = \begin{bmatrix} 0.4 & 0.6 & 0 \\ 0.5 & 0.3 & 0.2 \\ 0.2 & 0.7 & 0.1 \end{bmatrix}$$

- a) Compute $\mathbb{P}(X_4 = 3 | X_2 = 1) = f_{X_4 | X_2}(3, 1)$.
 - b) Assuming a uniform initial distribution, compute $\mathbb{P}(X_2 = 3)$.
2. Considering the same transition matrix in the previous question and a uniform initial distribution, compute $f_{X_1, X_2, X_3}(2, 3, 1)$, $f_{X_1, X_2, X_3}(2, 1, 3)$, and $f_{X_2, X_4}(1, 3)$
 3. Considering the same transition matrix in the previous question and \mathbf{f}_{X_1} uniform, compute \mathbf{f}_{X_2} and \mathbf{f}_{X_3}
 4. Is the source with the transition matrix given in Problem 1 irreducible and aperiodic? Justify.
 5. Find the stationary distribution of the following Markov process, where $\alpha, \beta \geq 0$:



Is it irreducible and aperiodic for any α, β ?

6. Determine the conditional entropy rate $H'(X)$ for the process defined in the previous question.
7. Consider a time-invariant Markov source producing symbols from the alphabet $\mathcal{X} = \{1, 2, 3\}$, with probability transition matrix:

$$\mathbf{P} = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}$$

- a) Find the stationary distribution of this source and its conditional entropy rate.
- b) Compare with the entropy of the stationary distribution and comment the result.

8. Repeat the previous problem, now with the following transition matrix:

$$\mathbf{P} = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

9. Consider a source with alphabet $\mathcal{X} = \{a, b, c\}$, restricted not to generate the same symbol in **two consecutive times**, but apart from that the symbols are equiprobable. Is this source memoryless? Write a model for the source, compute its stationary distribution $\boldsymbol{\mu}$ and conditional entropy rate $H'(X)$.
10. Repeat the previous problem, but now the restriction is that the source does not generate the same symbol in **three consecutive times**.
11. Compute the stationary distribution of a source with transition matrix

$$\mathbf{P} = \begin{bmatrix} 1/3 & 1/2 & 1/6 \\ 1/6 & 1/3 & 1/2 \\ 1/2 & 1/6 & 1/3 \end{bmatrix},$$

as well as its conditional entropy rate.