**A. Probabilidade condicionada** [Alternativa a 2.53. Adaptado de Moss (2013, pp. 29–31).[1]]

*For any one building the electricity needed can be either 5 or 10 units ($E_5$ or $E_{10}$), and the water needed can be either 1 or 2 units ($W_1$ or $W_2$). The owner ascribes the following probabilities (expert consensus) based on previous experience with similar type of buildings:* $P(E_5 \cap W_1) = 0.1$; $P(E_{10} \cap W_1) = 0.1$; $P(E_5 \cap W_2) = 0.2$; $P(E_{10} \cap W_2) = 0.6$.

(a) *What is the probability that a building needs 10 units of electricity ?*

(b) *What is the probability that a building require 2 units of water, given that it needs 10 units of electricity ?*

- **Quadro de acontecimentos e probabilidades**

| **Acontecimento** | **Probabilidade** |
|---|---|
| $E_i$ = building needs $i$ units of electricity ($i = 5, 10$) | $P(E_i) = ?$ |
| $W_j$ = building needs of $j$ units of water ($j = 1, 2$) | $P(W_j) = ?$ |
| | $P(E_5 \cap W_1) = 0.1$ |
| | $P(E_{10} \cap W_1) =$ |
| | $P(E_5 \cap W_2) =$ |
| | $P(E_{10} \cap W_2) =$ |

- **Probabilidades pedidas**

$$
\begin{aligned}
P(E_{10}) &= \\
&= \\
&= 0.7 \\
P(W_2 \mid E_{10}) &= \\
&= \\
&= \frac{6}{7} \\
&\simeq 0.8571.
\end{aligned}
$$

**B. Lei da probabilidade composta** [Adaptado de Moss (2013, pp. 33).]

*The foundation of a tall building may fail due to inadequate bearing capacity (B) or excessive settlement (S). The following probabilities are known for this type of building and failure situation:* $P(B) = 0.001$; $P(S) = 0.008$; $P(B \mid S) = 0.10$.

*What is the probability the building will experience excessive settlement but not bearing capacity failure ?*

- **Quadro de eventos e probabilidades**

| **Evento** | **Probabilidade** |
|---|---|
| $B$ = building with inadequate bearing capacity | $P(B) = 0.001$ |
| $S$ = building with excessive settlement | $P(S) =$ |
| | $P(B \mid S) =$ |

[1] Moss, R.E.S. (2013). *Applied Civil Engineering Risk Analysis*. Shedwick Publishing.

- **Probabilidade pedida**

$$\begin{aligned} P(S \cap \bar{B}) &\overset{lei\ prob.\ composta}{=} P(\bar{B} \mid S) \times P(S) \\ &= \\ &= \\ &= 0.0072. \end{aligned}$$

**C. Lei da probabilidade total** [Alternativa a 2.66. Adaptado de Moss (2013, p. 37).]

*Hurricanes are categorized by increasing wind speed, Category 1 through Category 5. Based on historical hurricane data along the Louisiana Coast the annualized probability of each category is:* $P(C_1) = 0.55$, $P(C_2) = 0.25$, $P(C_3) = 0.14$, $P(C_4) = 0.05$, $P(C_5) = 0.01$.[2]

*In this problem we are interested in the probability of structural damage due to hurricane winds. Reconnaissance of previous hurricane disasters has shown that structural damage can be approximated by the following conditional probabilities:* $P(D \mid C_1) = 0.05$, $P(D \mid C_2) = 0.10$, $P(D \mid C_3) = 0.25$, $P(D \mid C_4) = 0.60$, $P(D \mid C_5) = 1.00$.

*What is the annual probability of structural damage?*

- **Quadro de eventos e probabilidades**

| **Evento** | **Probabilidade** |
|---|---|
| $C_1$ = hurricane of category 1 | $P(C_1) = 0.55$ |
| $C_2$ = hurricane of category 2 | $P(C_2) =$ |
| $C_3$ = hurricane of category 3 | $P(C_3) =$ |
| $C_4$ = hurricane of category 4 | $P(C_4) =$ |
| $C_5$ = hurricane of category 5 | $P(C_1) =$ |
| $D$ = structural damage due to hurricane wind | $P(D) = ?$ |
| | $P(D \mid C_1) = 0.05$ |
| | $P(D \mid C_2) =$ |
| | $P(D \mid C_3) =$ |
| | $P(D \mid C_4) =$ |
| | $P(D \mid C_5) =$ |

- **Prob. pedida**

$$\begin{aligned} P(D) &\overset{lei\ prob.\ total}{=} \\ &= \\ &= 0.1275. \end{aligned}$$

**D. Lei da probabilidade total e teorema de Bayes** [Alternativa a 2.74. Adaptado de Moss (2013, p. 38).]

*There is aggregate being delivered to a construction site from two sources, $A$ and $B$. Trucks from sources $A$ deliver 600 loads a day of which* 3% *is bad (meaning it does not meet the project specifications). Trucks from source $B$ deliver 400 loads a day of which* 1% *is bad.*

(a) *What is the probability of bad aggregate ?*

[2] The probabilities in Moss (2013, p. 37) do not sum one.

- **Quadro de eventos e probabilidades**

| Evento | Probabilidade |
|---|---|
| $A$ = aggregate from source $A$ | $P(A) = \frac{600}{600+400} = 0.6$ |
| $B$ = aggregate from source $B$ | $P(B) =$ |
| $Bad$ = bad aggregate | $P(Bad) = ?$ |
| | $P(Bad \mid A) = 0.03$ |
| | $P(Bad \mid B) =$ |

- **Prob. pedida**

$$P(Bad) \overset{lei\ prob.\ total}{=} \quad = \quad = 0.022.$$

(b) Given that the aggregate is bad, what is the probability it was from source A ?

- **Prob. pedida**

$$P(A \mid Bad) \overset{teo.\ Bayes}{=} \quad = \quad \simeq 0.8182.$$

**E. Eventos independentes** [Alternativa a 2.79.]

*Resume Problem A. Are the events $E_{10}$ and $W_1$ independent?*

- **Averiguação de independência**

$$P(E_{10} \cap W_1) \overset{A.}{=}$$

$$P(E_{10}) \times P(W_1) \overset{A.}{=} P(E_{10}) \times [P(E_5 \cap W_1) + P(E_{10} \cap W_1)] \overset{A.}{=} \quad \overset{A.}{=} 0.14.$$

Logo, podemos afirmar que $E_{10}$ and $W_1$ são eventos .