Social Robotics

PDEEC PhD course on Social Robotics

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Quick revisit to social (robot) dynamics

- Huge literature corpus on generic social dynamics of systems (not that much about social robot systems)

- Models often defined in terms of aggregated state variables (as opposed to state variables of each of the entities, e.g. the Lotka-Volterra models – simpler to deal with)

- From the arguments in the literature (see for instance [Singer, Spilerman, 1976], [Gintis, 2013], [Ntigwa, Ogutu, 2018]), one can conclude that memory is not that important to explain the evolution of a social entity/system

  An intuition is that if, at each state, the local evolution is known then it is possible to predict future evolution (at least within some bounded horizon)

Markov processes in social robotics

- Often models involve a countable set of states and some state transition map
- Evolution at instant k does not depend on the path up to k – no memory
- States or transitions are identifiable with robot actions – suitable to diagram representation
- Asynchronous events control the transition between states
- Useful both for single agent and social group dynamics modeling
- Can arbitrary behavioral processes resulting from biological processes be described by a MP ?

  Apparently not, see [Moore et al, 2018] for a example on how Bach is not describable by a MP

Types of Markov processes

- Discrete-time discrete-state, e.g., random walk
- Continuous-time discrete-state, e.g., Poisson process
- Continuous-time continuous-state, e.g., Brownian motion
- Discrete-time continuous state, e.g., auto-regressive processes
- Markov chains – MP with countable time (this may be relaxed) and/or countable space

  MC are easily represented by state diagrams – each state can be identified, for example, with actions involved in robot interactions

Basis concepts on Markov chains I

- Transition matrix, containing the probabilities of moving between arbitrary states
- Absorbing states – states that can not be left from
- Transient state – states visited a finite number of times, with probability 1
- Leaky set – set that can be left from to other states not in the set
- Irreducible set – non-leaky set of states – e.g., a bounded set of states a system stays in forever after some time
- Period – constant time between visits to a state
- Aperiodic state – the state is not visited periodically

Basis concepts on Markov chains II

- Long-run behavior
  - Number of visits to a specific state
  - Irreducible set of states after a certain time (eventually the whole set)
- Ergodicity – a single long-enough run distribution is enough to estimate the equilibrium distribution distribution (as it could be done from multiple long-runs) – sample mean equals temporal averaging
- Aggregation theorem, [Gintis, 2013]

Suppose a finite Markov process with state space S has a set of states $A \subset S$ with all $j \in A$ identically situated in the sense that $p_{ji} = p_{ki}$ for all states $i \in S$. Then there is a Markov process with the same transition probabilities as $\mathcal{M}$, except the states in A are replaced by a single state.

Sequences of states can be morphed/combined into a single state if they are indistinguishable from the point of view of the transition probabilities

Basis concepts on Markov chains III

- *Finite History Theorem*, [Gintis, 2013]

  Consider a stochastic process $\mathcal{S}$ with finite state space S and transition probability function P with k-period memory, where k>1. Then $\mathcal{S}$ is isomorphic to a MP $\mathcal{M}$, with state space S', where $i' \in S'$ is a k-dimensional vector $(i_1, \ldots, i_k) \in S^k$. The function ϕ is the identity on S, and ψ is the canonical isomorphism from $H^k_{\mathcal{S}}$ to S^k.

  Sequences of states that do not influence the transition probabilities can be morphed to yield smaller complexity models , i.e., as if the memory implicit in those sequences is not relevant

- Social systems tend to have a large number of states, hence it may be difficult to assess the behavior of the global system – necessary to look for aggregate properties of the irreducible processes (as discussed in the 1st slide of the session)

Markov chains and social dynamics models I

- Let a set of nodes A each representing an *actor* – an entity able to execute actions in a set
- Consider a directed graph (digraph) where an edge from node a_i to node a_j represents an interaction of some kind from a_i to a_j – for example an action that a_i performs on a_j
- These interactions are stochastic processes that form the dynamics of the actor in the social environment it interacts with
- Let X be a set of stochastic digraphs defined on A, i.e., a set of stochastic matrices representing digraphs
- What is the element of X that better fits a set of observations ?

  Observations correspond to the interactions between actor and environment

Markov chains and social dynamics models II

- The goal is to maximize the likelihood that a set of measurements is generated by some chain $x \in X$, i.e., a transition matrix, i.e., a game that defines a sequence of decisions

- Assume that each measurement in the set is independent of the others (this eliminates any temporal dependence between measurements)

$$\text{Prob}(\theta|o_1, \ldots, o_n) = \text{Prob}(\theta|o_1)\ldots\text{Prob}(\theta|o_n)$$

  where θ is a vector of parameters, e.g., the elements of a matrix, and the o_i are the observations

- (How to) select a model for $\text{Prob}(\theta|o_i)$

  For instance, using an histogram with the relative frequency of the o_i, to compute $\text{Prob}(o_i)$, and some a priori model – knowledge – of the process, $\text{Prob}(o_i|\theta)$, and using Bayes rule

$$\text{Prob}(\theta|o_i) = \text{Prob}(o_i|\theta)\,\frac{\text{Prob}(\theta)}{\text{Prob}(o_i)}$$

Markov chains and social dynamics models III

- Often using

$$\lambda(\theta) = \log(\mathrm{Prob}(\theta|o_1, \ldots, o_n)) = \log(\mathrm{Prob}(\theta|o_1)) + \ldots + \log(\mathrm{Prob}(\theta|o_n))$$

  simplifies the calculations

- Computing $\partial\lambda/\partial\theta = 0$ yields the ML estimate

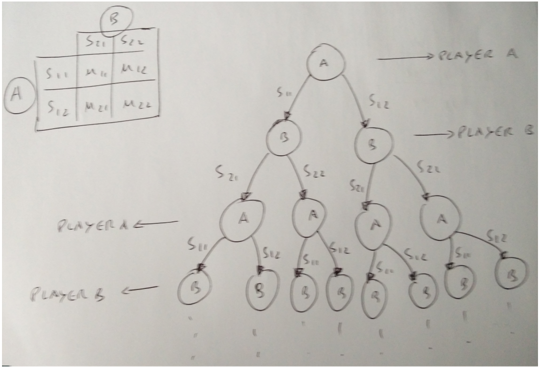
- If $\lambda(\theta)$ is augmented with additional data (to facilitate estimation in cases where there is missing data that is replaced by interpolated data) compute instead $E[\partial\lambda/\partial\theta] = 0$

  (see [Snijders et al, 2010] for additional details)

Markov chains and games I

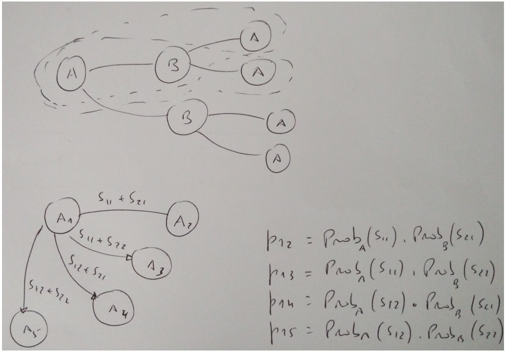
- Tabular games are memoryless – they simply encode instantaneous decisions – and therefore they are representable by Markov chains

- A sequence of decisions from a game table can be expanded into a tree diagram (with each level identified with a player)



Markov chains and games II

- Game tables can be identified with Markov chains



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Markov chains and games III

- Game payoffs/utilities can be mapped into probabilities using, for example, a monotone function, $f(\cdot)$, of the payoffs/utilities u_k involved (possibly blind to external information))

$$p_{ij} = f(u_1, u_2, \ldots), \quad \text{subject to } \forall i, \ \sum_j p_{ij} = 1$$

- Game tree diagrams are mapped directly into a Markov chain – with the aforementioned assumptions on the conversion between payoffs/utilities and probabilities
- There is an identification between Markov chains and (transition) matrices – problems relating to sequences of decisions using game theory can be framed in a space of stochastic matrices

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