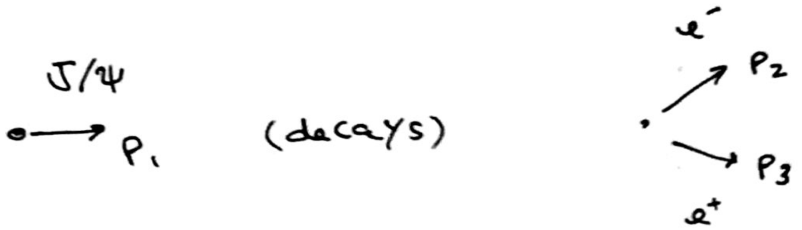
Exame 2 FPar (2017 - 2018)

1- a) LAB



$p_1^{\mu} = (E_1, \vec{p}_1)$ $\qquad$ $p_T^{\mu} = (E_2 + E_3 , \vec{p}_2 + \vec{p}_3)$

$$s = p_{1\mu} p_1^{\mu} = E_1^2 - \vec{p}_1^{\,2} \Rightarrow \sqrt{s} = m_{J/\psi}$$

$$s = p_{T\mu} p_T^{\mu} = (E_2 + E_3)^2 - (\vec{p}_2 + \vec{p}_3)^2$$

$E_i , \vec{p}_i$ - energy and momenta measured in LAB

$$\Rightarrow m_{J/\psi} = \sqrt{(E_2 + E_3)^2 - (\vec{p}_2 + \vec{p}_3)^2}$$

b) $E_e \equiv E_p = 1\,\text{GeV}$; $\Delta t \sim 1\,\text{ms}$

$$x = vt = \beta c t = \frac{p}{E} c t \Leftrightarrow t = \frac{E}{pc} x$$

$$t_p - t_e > 1\,\text{ms} = \frac{x}{c}\left[\frac{E_p}{p_p} - \frac{E_e}{p_e}\right] \Leftrightarrow \quad \text{since } p = \sqrt{E^2 - m^2}$$

$$x = \frac{\Delta t \, c}{\left[\frac{E_p}{p_p} - \frac{E_e}{p_e}\right]} \simeq 1\,\text{m}$$

c) $x = vt = \beta c t = \beta c \gamma \tau$

$$= \frac{p}{E} c \frac{E}{m} \tau = \frac{p}{m} c \tau \quad ; \quad (\Gamma \tau \sim \hbar)$$

$$\Rightarrow x = \frac{p}{m} c \frac{\hbar}{\Gamma} = 6{,}8 \times 10^{-13}\,\text{m}$$

where it was used

$p = 1\ GeV/c$

$m = 3,096\ GeV/c^2$

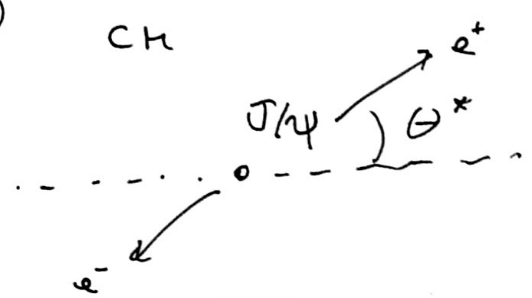
$c = 3 \times 10^8\ m/s$

$\Gamma = 92,9\ keV$

$\hbar = 6,58 \times 10^{-22}\ MeV\ s$

Γ or τ do not depend on the decay mode so the mean distance covered before it decay would be the same.

1- d) CM



$$E_e^* = \frac{M^2 + m_e^2 - m_e^2}{2M} \simeq \frac{M}{2}$$

with $M \equiv m_{J/\psi}$

$$p_e^* = \sqrt{E_e^{*2} - m_e^2} = 1,548\ GeV/c$$

$$\beta_{CM} = \frac{P}{E} = \frac{\sqrt{E^2 - M^2}}{E} \simeq 0,936 \quad (E = 10\ GeV)$$

$$\beta_e^* = \frac{P_i^*}{E_i^*} \simeq 1$$

since $\beta_e^* > \beta_{CM}$ the particle momentum cannot be inverted by the boost and therefore is easy to see that

$\theta_{min} = 0°$ (along) and $\theta_{max} = 180°$ (against) the boost (J/ψ LAB momentum)

1 - e)
$$\begin{pmatrix} E \\ P_\ell \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} E^* \\ P_\ell^* \end{pmatrix}$$

$$E_e = \gamma \left(E^* + \beta P^* \cos\theta^* \right)$$

$$\Rightarrow \quad E_e^{min} = \gamma \left[E^* - P^* \beta \right] = 1,53 \text{ GeV}$$

$$E_e^{max} = \gamma \left[E^* + \beta P^* \right] = 6,21 \text{ GeV}$$

where,

$$\gamma = \frac{\cancel{E} E}{M} \quad ; \quad \beta = \frac{P}{E} \quad \text{of } J/\psi \text{ in LAB}$$

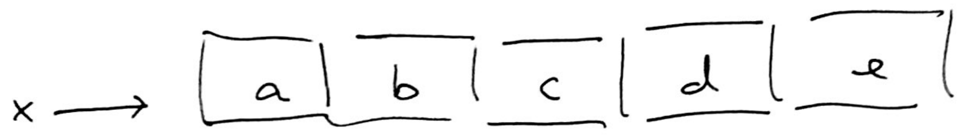
2 - a) LAB

$$P_\mu = \left(E_{e^+} + m_e ,\ P_{e^+} \right)$$

$$S = \left(E_{e^+} + m_e \right)^2 - P_{e^+}^2 = m_{J/\psi}^2 \Leftrightarrow$$

$$E_{e^+} = \frac{m_{J/\psi}^2 - 2 m_e^2}{2 m_e} \simeq 9377 \text{ GeV} \simeq 10 \text{ TeV}$$

2-b) Example of accepted answer



particle path

a - inner detector sensitive to charged particles

b- electromagnetic calorimeter

c - hadronic calorimeter

d - iron block

e - detector sensitive to charged particles

label : (x) seen ; (o) not seen

Part id	a	b	c	d	e
γ	o	x		–	o
e	x	x	o	–	o
p	x	(x)	x	–	o
μ	x	x	x	–	x

The particle energy can be reconstructed from the calorimeter (e, p) or using the curvature due to a magnetic field (μ)

3 -

a) Not possible, violates charge

b) Not possible, violates baryon number

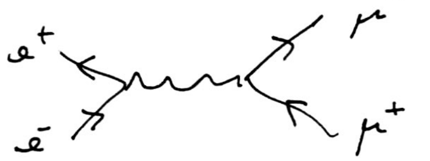
c) Not possible, violates strangeness

d) Possible

e) Not possible, violates lepton number

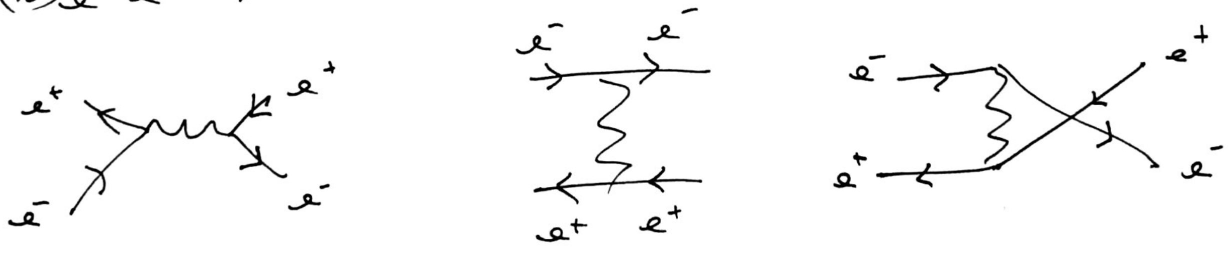
4 - Diagrams that contribute for:

(a) $e^+ e^- \to \mu^+ \mu^-$



(only channel - s)

(b) $e^+ e^- \to e^+ e^-$

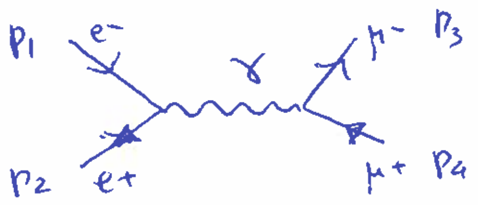


(channel s, t and u)

Hence, for (a) the result would be essentially the same but (b) would have an enormous contribution from the forward region that would hide the J/ψ ressonance.

(5) This process occurs in the Standard Model through

a) The diagrams



1)

+

2)

at the energies $\sqrt{s}$ $[2, 8]$ GeV we are in the condition $\sqrt{s} \ll M_Z$ and the dominant diagram is the photon exchange diagram 1)

b) we have for the amplitude of diagram 1)

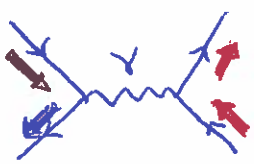
$$i\mathcal{M} = (ie)^2\, \bar{v}(p_2)\gamma^\mu u(p_1)\, \frac{-i g_{\mu\nu}}{s}\, \bar{u}(p_3)\gamma^\nu v(p_4)$$

therefore

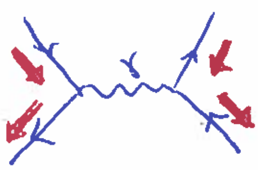
$$\boxed{\mathcal{M} = +\frac{e^2}{s}\, \bar{v}(p_2)\gamma^\mu u(p_1)\, \bar{u}(p_3)\gamma_\mu v(p_4)}$$

where $s = (p_1 + p_2)^2$ is the square of the center of mass energy.

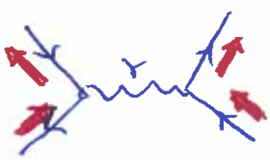
c) we have 4 non-vanishing contributions for the possible helicities for massless fermions



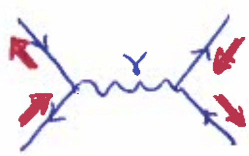
$\mathcal{M}(\uparrow\downarrow;\uparrow\downarrow)$



$\mathcal{M}(\uparrow\downarrow;\downarrow\uparrow)$



$\mathcal{M}(\downarrow\uparrow;\uparrow\downarrow)$



$\mathcal{M}(\downarrow\uparrow;\downarrow\uparrow)$

then

$$M(h_1 h_2; h_3 h_4) = + \frac{e^2}{s} \bar{v}(p_2,h_2)\gamma^\mu u(p_1,h_1)\ \bar{u}(p_3,h_3)\gamma_\mu v(p_4,h_4)$$

$$= + \frac{e^2}{s} J_{u_1 v_2}(h_1 h_2) \cdot J_{u_3 v_4}(h_3 h_4)$$

using the formulary

$$M(\uparrow\downarrow;\uparrow\downarrow) = + \frac{e^2}{s} s\,(0,-1,-i,0)\cdot(0,-\cos\theta, i, \sin\theta)$$

$$= + e^2 \left[0 - (\cos\theta + 1) \right] = - e^2 (1+\cos\theta)$$

$$= - (4\pi\alpha)(1+\cos\theta)$$

$$M(\uparrow\downarrow;\downarrow\uparrow) = \frac{e^2}{s} s\,(0,-1,-i,0)\cdot(0,-\cos\theta,-i,\sin\theta)$$

$$= e^2 \left[0 - (\cos\theta - 1) \right]$$

$$= (4\pi\alpha)(1-\cos\theta)$$

$$M(\downarrow\uparrow;\uparrow\downarrow) = \frac{e^2}{s} s\,(0,-1,i,0)\cdot(0,-\cos\theta, i, \sin\theta)$$

$$= e^2 \left[0 - (\cos\theta - 1) \right]$$

$$= (4\pi\alpha)(1-\cos\theta)$$

$$M(\downarrow\uparrow;\downarrow\uparrow) = \frac{e^2}{s} s\,(0,-1,i,0)\cdot(0,-\cos\theta,-i,\sin\theta)$$

$$= e^2 \left[0 - (\cos\theta + 1) \right]$$

$$= -(4\pi\alpha)(1+\cos\theta)$$

then

$$\langle |M|^2 \rangle = \frac{1}{4} \left[|M(\uparrow\downarrow;\uparrow\downarrow)|^2 + |M(\uparrow\downarrow;\downarrow\uparrow)|^2 \right.$$

$$\left. + |M(\downarrow\uparrow;\uparrow\downarrow)|^2 + M(\downarrow\uparrow;\downarrow\uparrow)|^2 \right]$$

$$= \frac{1}{4} (4\pi\alpha)^2 \left[2(1+\cos\theta)^2 + 2(1-\cos\theta)^2 \right]$$

$$= (4\pi\alpha)^2 \left[1 + \cos^2\theta \right]$$

d) $$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{|P_3|}{|P_1|} \langle |M|^2 \rangle$$

for massless fermions $|P_3| = P_1|$ and therefore

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} (4\pi\alpha)^2 (1+\cos^2\theta)$$

$$= \frac{\alpha^2}{4s} (1 + \cos^2\theta)$$

e) $$\sigma = \int d\Omega \frac{d\sigma}{d\Omega}$$

As nothing depends on φ we have

$$\sigma = 2\pi \times \int_{-1}^{1} d\cos\theta \, \frac{\alpha^2}{4s} (1+\cos^2\theta)$$

$$= \frac{2\pi\alpha^2}{4s} \int_{-1}^{1} dx (1+x^2) = \frac{2\pi\alpha^2}{4s} \left(2 + \frac{2}{3} \right)$$

finally

$$\boxed{\sigma = \frac{4\pi\alpha^2}{3\,s}}$$

using $1\ \text{GeV}^{-2} = 0.38938 \times 10^6\ \text{nb}$

$$\sigma = 4\times\pi\times\left(\frac{1}{137.036}\right)^2 \frac{1}{3}\ \frac{1}{(3.13)^2} \times 0.38938\times10^6\ \text{nb}$$

$$= 8.9\,\text{nb}$$

f) with the cut the cross section would be $(X_0 = |\cos\theta_L| = 0.6)$

$$\sigma = \frac{\pi\alpha^2}{2s}\int_{-X_0}^{X_0} dx\,(1+x^2) = \frac{\pi\alpha^2}{2s}\left(2X_0 + \frac{2X_0^3}{3}\right)$$

$$= \frac{\pi\alpha^2}{3s}\left(3X_0 + X_0^3\right)$$

therefore

$$\frac{\sigma(X_0=0.6)}{\sigma(X_0=1)} = \frac{3\times0.6+0.6^3}{4} = 0.504$$

we would get

$$\sigma(X_0=0.6) = 0.504 \times 8.9\ \text{nb} \simeq 4.5\,\text{nb}$$

In the figure (middle plot of left fig 1) we have $\sigma \sim 5\,\text{nb}$ which a good agreement. We can see that errors are $\sim 1\,\text{nb}$.

6 a) In this range, the dominant diagram is the photon exchange. Photon couples in the same way to all charged particles with a coupling proportional to (eQ_f). Therefore for a fermion f we must have 5

$$\sigma(e^- e^+ \to f\bar{f}) = Q_f^2 N_c \, \sigma(e^- e^+ \to \mu^- \mu^+)$$

where Q_f is the charge of the fermion (in units of e)

and
$$N_c = \begin{cases} 1 & \text{for leptons} \\ 3 & \text{for quarks because they come in 3 colors} \end{cases}$$

Now for $\sqrt{s} \leq 2$ GeV the only quarks that contribute are the u, d and s. So

$$R = 3 \times \sum_{f=u,d,s} Q_f^2 = 3\left[\frac{4}{9} + 2 \times \frac{1}{9}\right] = \frac{18}{9} = 2$$

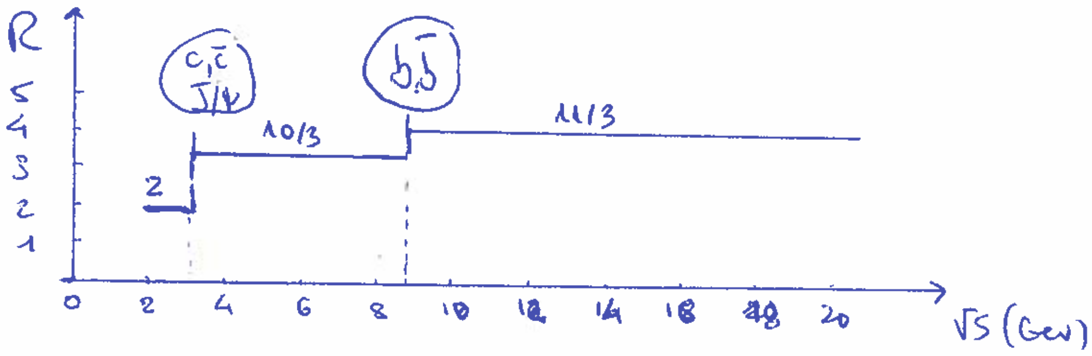
b) Now $\sqrt{s} = 3.5$ GeV, so we are above $c\bar{c}$ threshold (J/ψ). Then

$$R = 3 \times \sum_{f=u,d,s,c} Q_f^2 = 3\left[2 \times \frac{4}{9} + 2 \times \frac{1}{9}\right] = \frac{30}{9} = \frac{10}{3}$$

c) At $\sqrt{s} = 20$ GeV we are above $b\bar{b}$ thershold ($m_b \simeq 4.8$ GeV). So

$$R = 3 \times \sum_{f=u,d,s,c,b} Q_f^2 = 3\left[2 \times \frac{4}{9} + 3 \times \frac{1}{9}\right] = \frac{33}{9} = \frac{11}{3}$$

d)



R jumps at each new threshold.

e) <u>Below</u> J/ψ $\sqrt{s} \lesssim 3$ GeV

$\sigma_{hadrons} \simeq 20 nb$

$\sigma_{\mu^-\mu^+}(x_0=0.6) \simeq 5 nb \Rightarrow \sigma_{\mu^-\mu^-}(x_0=1) = \frac{5}{0.504} nb \simeq 9.9 nb$

$$\boxed{R(\sqrt{s} < 3 \text{ GeV}) \simeq \frac{20}{9.9} \simeq 2}$$

<u>Above</u> J/ψ $\sqrt{s} \gtrsim 3.1$ GeV

$\sigma_{hadrons} \simeq 35 nb$

$\sigma_{\mu^-\mu^+}(x_0=0.6) \simeq 5 nb \Rightarrow \sigma_{\mu^-\mu^+}(x_0=1) \simeq 9.9 nb$

$$\boxed{R(\sqrt{s} > 3.1 \text{ GeV}) \simeq \frac{35}{9.9} = 3.5}$$

this is to be compared with $R=2$ (below) and $R = \frac{10}{3} = 3.3$ (above). the agreement is very good taken in account the errors.