2^{nd} Exam
February 2^{nd} 2017: 8h00
Duration of the exam: 3h00

Mestrado em Eng. Física Tecnológica (MEFT)
Particle Physics
1^{st} semester of 2016-17

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- The allowed elements for consult during the test are:
  - the PDG (Particle Data Book)
  - one single A4 page with formulas.
- Carefully justify all your answers.
- The problem on the subject of the 1st test has 1 question with 8 items while the second has 1 question with 9 items

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1^{st} test

Muons couples directly to a standard-model Higgs particle in s-channel production. Thus, a muon collider could be an ideal device for precision measurements of the Higgs boson. Consider a muon collider ($\mu^+\mu^-$) with a beam energy of 63 GeV. The production of the muons that feed the accelerator come from pion decays that are created through proton-proton interactions.

1. **[10 val]**

a) The interaction of the single muon bunch per beam occurred with a repetition rate of 30 Hz. Knowing that the experiment is planned to achieve an instantaneous luminosity of 10^{31} cm^{-2}s^{-1}, and that in the interaction point the beam has a transverse area of 0.02 mm^2, compute the average number of muons in each beam.

b) Indicate the number of muons that have to be injected per minute in the collider to maintain the luminosity constant. Consider that the only factor for the disappearing of muons is its decay.

c) Consider the decay of one muon in its proper reference frame, into an electron and two neutrinos ($\overline{\nu}_e$ and ν_μ). Indicate the configuration, angles between particles, for which the electron has a minimum energy and the configuration for the maximum energy.

d) Determine the minimum and maximum energy that the electron can have in the LAB reference frame.

e) The products of the muon decays are, in these colliders, an important contamination for the selection of the Higgs event candidates. Discuss the angular region where this contamination is more important.

f) Explicitly show, using the quantum numbers conservation, which of the following reactions/decays involving muons are possible.

(i) $\mu^+ \to e^+\, \nu_e\, \nu_\mu$ (ii) $\mu^+\, \mu^- \to e^+\, e^-$ (iii) $p\, n \to n\, n\, \mu^+$ (iv) $\mu^+\, \mu^- \to \nu_\mu\, \overline{\nu}_\mu$

g) Estimate the number of Higgs bosons produced in one year using the instantaneous luminosity given in question a), and an Higgs production cross-section of $\sigma_H \sim 40\,\text{pb}$.

h) One of the most probable decay of the Higgs is into $b\bar{b}$. Discuss how experimentally one can discriminate this decay from other hadronic decays of the Higgs, indicating the type of detectors and its typical dimension and position with respect to the interaction point.

2^{nd} *test*

2. [10 val] In the muon collider, the $\mu^{\mp}$ beams have a contamination of electrons (and positrons) due the μ^- (μ^+) decays. In these context consider the following questions:

a) Draw all Feynman diagrams, in lowest order in the Standard Model (SM), for the processes:

$$\mu^- + \mu^+ \to f + \overline{f}$$

where f is one of the following: $f = e^-, \nu_e, \nu_\mu, b$. Consider the neutrinos to be massless, without interaction with the Higgs boson H.

b) Draw all Feynman diagrams, in lowest order in the SM, for the process

$$\mu^- + e^+ \to \mu^- + e^+$$

c) Consider now that $\sqrt{s} \simeq M_H$ and the process,

$$\mu^-(p_1) + \mu^+(p_2) \to b(p_3) + \overline{b}(p_4)$$

Taking only the dominant diagram for this energy, write down the amplitude $\mathcal{M}$ for this process. **Hint:** See in the formulary below a discussion of the Higgs scalar currents.

d) Neglecting the fermion masses everywhere, except in the Higgs couplings, write down the non-vanishing helicity amplitudes amplitudes for this process in the CM frame.

e) In these conditions evaluate $\left\langle |\mathcal{M}|^2 \right\rangle$, and $\dfrac{d\sigma}{d\Omega}$ in the CM frame.

f) In the same conditions evaluate the total cross-section for $\mu^- + \mu^+ \to b + \overline{b}$. For $\sqrt{s} = M_H$ express this cross section in *pb*.

g) Determine the expression for the ratio

$$R_1 = \frac{\sigma(\mu^- + \mu^+ \to b + \overline{b})}{\sigma(e^- + e^+ \to b + \overline{b})}$$

at $\sqrt{s} = M_H$ and give its numerical value.

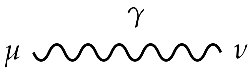
h) Without making any calculations make an estimative of the value of the ratio

$$R_2 = \frac{\sigma(\mu^- + \mu^+ \to b + \overline{b})}{\sigma(e^- + e^+ \to b + \overline{b})}$$

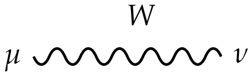
at $\sqrt{s} = M_Z$ and give its numerical value.

i) Comment the following sentence: *A muon collider is a much better Higgs factory than an e^-e^+ collider*. Justify carefully your answer.

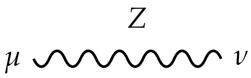
Propagators



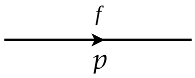
$$-i\frac{g_{\mu\nu}}{k^2} \tag{1}$$



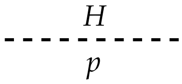
$$-i\frac{g_{\mu\nu}-\frac{k_\mu k_\nu}{M_W^2}}{k^2-M_W^2+i\,M_W\Gamma_W} \tag{2}$$



$$-i\frac{g_{\mu\nu}-\frac{k_\mu k_\nu}{M_Z^2}}{k^2-M_Z^2+i\,M_Z\Gamma_Z} \tag{3}$$



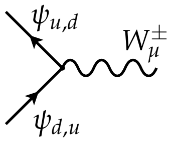
$$\frac{i(\not{p}+m_f)}{p^2-m_f^2} \tag{4}$$



$$\frac{i}{p^2-M_H^2+i\,M_H\Gamma_H} \tag{5}$$

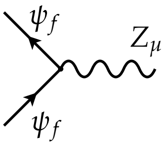
Vertices

Charged Current

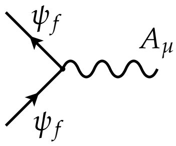


$$-i\frac{g}{\sqrt{2}}\gamma_\mu\frac{1-\gamma_5}{2} \tag{6}$$

Neutral Current



$$-i\frac{g}{\cos\theta_W}\gamma_\mu\left(g_V^f-g_A^f\gamma_5\right)$$

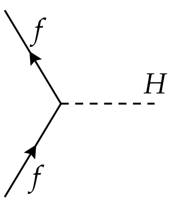


$$-ieQ_f\gamma_\mu \tag{7}$$

where

$$g_V^f=\frac{1}{2}T_f^3-Q_f\sin^2\theta_W, \quad g_A^f=\frac{1}{2}T_f^3\,. \tag{8}$$

Higgs Interactions with fermions



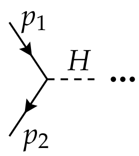
$$-i\,\frac{g}{2}\,\frac{m_f}{M_W}\equiv -i\,g_H^f \tag{9}$$

Higgs Scalar interactions (currents)

While the γ, W and Z gauge bosons are spin 1 particles (vectors) that interact with fermions via a vector current, the Higgs boson, H, is a scalar (spin 0) and therefore interacts with fermions through a scalar interaction (or current with an abuse of language). These are simpler that the vector currents.

s-channel

For instance, consider that we have an s-channel mediated H boson,



Then the amplitude is

$$
\begin{aligned}
\mathcal{M} &= -\, i\, g_H^f\, \overline{v}(p_2, h_2) u(p_1, h_1) \times \cdots \\
&\equiv -\, i\, g_H^f\, J_{u_1 v_2}(h_1, h_2) \times \cdots
\end{aligned}
$$

where

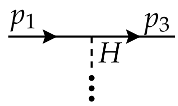
- $g_H^f = \frac{1}{2} g\, \frac{m_f}{M_W}$ is the Higgs coupling to fermions
- The dots $(\cdots)$ refer to the rest of the diagram (including the Higgs propagator not written)
- We have defined the Higgs scalar currents

  $$J_{u_1 v_2}(h_1, h_2) \equiv \overline{v}(p_2, h_2) u(p_1, h_1)$$

  where h_i are, as usually, the helicities of the fermions.

t-channel

In the same way for a t-channel diagram we would have



Then, with the same conventions, the amplitude is

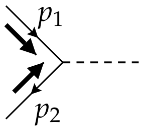
$$
\begin{aligned}
\mathcal{M} &= -\, i\, g_H^f\, \overline{u}(p_3, h_3) u(p_1, h_1) \times \cdots \\
&\equiv -\, i\, g_H^f\, J_{u_1 u_3}(h_1, h_3) \times \cdots
\end{aligned}
$$

Below you can find the non-vanishing scalar currents $J_{u_1 v_2}(h_1, h_2)$ for s-channel and $J_{u_1 u_3}(h_1, h_3)$ t-channel diagrams, in all possible non-vanishing combinations.

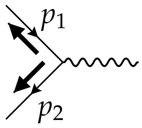
Results for Scalar Helicity Currents

The interactions of fermions with the Higgs boson are **scalar** interactions and therefore the scalar currents are numbers, **not four-vectors**! Neglecting the fermion masses they are given below, for the s and t channels.
Notice that the rule for the spin arrows is opposite to that of the vector helicity currents.

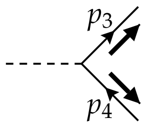
s-channel



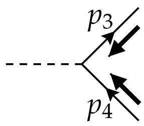
$$J_{u_1 v_2}(\uparrow,\uparrow) = \sqrt{s} \tag{10}$$



$$J_{u_1 v_2}(\downarrow,\downarrow) = -\sqrt{s} \tag{11}$$

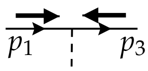


$$J_{u_3 v_4}(\uparrow,\uparrow) = \sqrt{s} \tag{12}$$

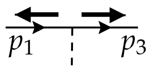


$$J_{u_3 v_4}(\downarrow,\downarrow) = -\sqrt{s} \tag{13}$$

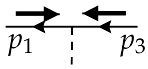
t-channel



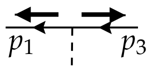
$$J_{u_1 u_3}(\uparrow,\downarrow) = -\sqrt{s}\sin\frac{\theta}{2} \tag{14}$$



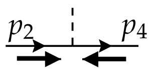
$$J_{u_1 u_3}(\downarrow,\uparrow) = \sqrt{s}\sin\frac{\theta}{2} \tag{15}$$



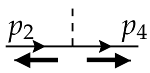
$$J_{v_1 v_3}(\uparrow,\downarrow) = -\sqrt{s}\sin\frac{\theta}{2} \tag{16}$$



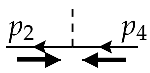
$$J_{v_1 v_3}(\downarrow,\uparrow) = \sqrt{s}\sin\frac{\theta}{2} \tag{17}$$



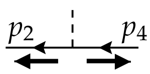
$$J_{u_2 u_4}(\uparrow,\downarrow) = \sqrt{s}\sin\frac{\theta}{2} \tag{18}$$



$$J_{u_2 u_4}(\downarrow,\uparrow) = -\sqrt{s}\sin\frac{\theta}{2} \tag{19}$$



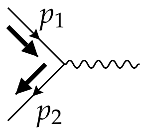
$$J_{v_2 v_4}(\uparrow,\downarrow) = \sqrt{s}\sin\frac{\theta}{2} \tag{20}$$



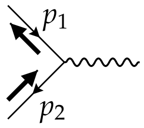
$$J_{v_2 v_4}(\downarrow,\uparrow) = -\sqrt{s}\sin\frac{\theta}{2} \tag{21}$$

Results for the Helicity Vector Currents

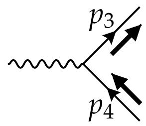
s-channel



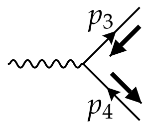
$$J_{u_1 v_2}(\uparrow,\downarrow) = \sqrt{s}\,(0,-1,-i,0) \tag{22}$$



$$J_{u_1 v_2}(\downarrow,\uparrow) = \sqrt{s}\,(0,-1,i,0) \tag{23}$$

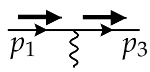


$$J_{u_3 v_4}(\uparrow,\downarrow) = \sqrt{s}\,(0,-\cos\theta,i,\sin\theta) \tag{24}$$

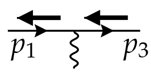


$$J_{u_3 v_4}(\downarrow,\uparrow) = \sqrt{s}\,(0,-\cos\theta,-i,\sin\theta) \tag{25}$$

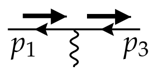
t-channel



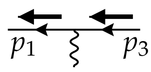
$$J_{u_1 u_3}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{26}$$



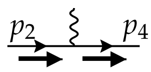
$$J_{u_1 u_3}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{27}$$



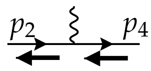
$$J_{v_1 v_3}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{28}$$



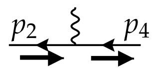
$$J_{v_1 v_3}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{29}$$



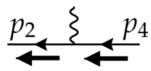
$$J_{u_2 u_4}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{30}$$



$$J_{u_2 u_4}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{31}$$



$$J_{v_2 v_4}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{32}$$



$$J_{v_2 v_4}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{33}$$