1 -

a) $E_e = 26{,}7\ \text{GeV}$ $\quad \mathcal{L}_{int} = 2{,}1\ \text{nb}^{-1}$

$E_p = 820\ \text{GeV}$

$$P_\mu = (E_e + E_p,\ p_e + p_p)$$

$$s = P_\mu P^\mu = (E_e + E_p)^2 - (p_e - p_p)^2$$

$$= (E_e + E_p)^2 - \left(\sqrt{E_e^2 - m_e^2} - \sqrt{E_p^2 - m_p^2}\right)^2$$

$$\Rightarrow \sqrt{s} = 295{,}93\ \text{GeV}$$

b) $$P_\mu = (E_e' + m_p,\ p_e')$$

$$s = (E_e' + m_p^2) - p_e'^2 = E_e'^2 + m_p^2 + 2E_e' m_p - p_e'^2 \Leftrightarrow$$

$$s = m_e^2 + m_p^2 + 2E_e' m_p \Leftrightarrow E_e' = \frac{s - m_e^2 - m_p^2}{2m_p}$$

$$E_e' = 46{,}68\ \text{TeV}$$

c) $\sigma = 4{,}8 \times 10^{-32}\ \text{cm}^2$; $\quad 1\ \text{barn} = 10^{-24}\ \text{cm}^2$

$\varepsilon = 0{,}8$

$N = \mathcal{L}\sigma\varepsilon$

$1\text{b} - 10^{-24}\ \text{cm}^2$

$x - 4{,}8 \times 10^{-32}\ \text{cm}^2$

$$\sigma = \frac{4{,}8 \times 10^{-32}}{10^{-24}} = 4{,}8 \times 10^{8}\ \text{b}$$

$$\Rightarrow N = 2{,}1 \times 48 \times 0{,}8 \simeq 81\ \text{evts}$$

d) In the framework where the proton is at rest

$$\nu = E - E' \; ; \quad x = \frac{Q^2}{2M\nu} \qquad M \equiv m_p \quad E \equiv E_e'$$

$$\Rightarrow E - E' = \frac{Q^2}{2Mx} \Leftrightarrow E' = E - \frac{Q^2}{2Mx} = 35{,}63 \text{ TeV}$$

$$Q^2 \simeq 4EE' \operatorname{sen}^2(\theta/2) \Leftrightarrow$$

$$\theta = 2 \operatorname{arcsen}\left(\sqrt{\frac{Q^2}{4EE'}}\right) = 0{,}004° = 7{,}1 \times 10^{-5} \text{ rad}$$

2 -

a) Not possible. Violates Barion number

b) Not possible. Violates strangeness

c) Possible.

d) Not possible. Violates lepton number

e) Possible.

3 - a) $\Delta x_U = 3{,}3$ mm

PDG (pag. 338)

Rad Length $X_0 = 6\ g\,cm^{-2}$ | $\rho_U = 18{,}950\ g\,cm^{-3}$

$$N_0 \text{ rad length} = \frac{\Delta x_U \times \rho_U}{X_0} = \frac{0{,}33\ cm \times 18{,}95\ g\,cm^{-3}}{6\ g\,cm^{-2}} =$$

$$= 1{,}04 \simeq 1$$

b) For high-energy particles two processes dominate having nearly the same cross-section:

→ Pair creation ($e^+ e^-$)

→ Bremsstrahlung

This produces a shower of e^-, e^+, γ until a critical energy is reached.

↳ that increases in number of particles

Uranium : "forces" the particles to interact and the shower to develop.

⊕ absorber for low energy photons

Scintillator : low energy particles excite the material atoms producing light that can be channeled and detected by PMTs.

3 - c)

$E_k = 20\ \text{GeV}$; $m_k = 493{,}68\ \text{MeV}$

$$\beta\gamma = \frac{p}{E}\frac{E}{m} = \frac{p}{m} = \frac{\sqrt{E^2 - m^2}}{m} = 40{,}5$$

$$\left\langle \frac{dE}{dx} \right\rangle \Big|_{Pb} = 1{,}5\ \text{MeV g}^{-1}\text{cm}^{-2}$$

$$\langle E \rangle_{loss} = \left\langle \frac{dE}{dx} \right\rangle \Delta X = \left\langle \frac{dE}{dx} \right\rangle \Delta x_o \rho_o =$$

$$= 1{,}5 \times 0{,}33\ \text{cm} \times 18{,}95\ \text{g cm}^{-3} = 9{,}4\ \text{MeV}$$

4- a)

$x = vt = \beta c t = \beta c \gamma \tau$

$\tau \Gamma = \hbar \Rightarrow x = \beta c \gamma \dfrac{\hbar}{\Gamma}$

$c = 3 \times 10^{8}\ m/s$

$\hbar = 6{,}5 \times 10^{-22}\ MeV\,s$

$\Gamma = 117\ MeV$

$P = 4\ GeV/c$

$m_\Delta = 1{,}232\ GeV/c^2$

$\beta = \dfrac{P}{E} = \dfrac{P}{\sqrt{P^2 + m_\Delta^2}} = 0{,}9557$

$\gamma = \dfrac{E}{m_\Delta} = 3{,}397$

$\Rightarrow x = 5{,}4 \times 10^{-15}\ m$

b)

$\Delta^+ \to p \pi^0 \ ; \ \Delta^+ \to n \pi^+$

$|3/2\ 1/2\rangle = a\, |1/2\ 1/2\rangle |1 0\rangle + b\, |1/2\ {-1/2}\rangle |1 1\rangle$

C. G. coef. $\Rightarrow a = \sqrt{2/3}$

$b = \sqrt{1/3}$

$\Rightarrow \dfrac{\Gamma(\Delta^+ \to p \pi^0)}{\Gamma(\Delta^+ \to n \pi^+)} = \dfrac{a^2}{b^2} = \dfrac{\frac{2}{3}}{\frac{1}{3}} = 2$

4- c)

$$x_{\Delta^+} = 5{,}4 \times 10^{15} \, m$$

$$x_{\pi^0} \propto c\tau = 25{,}5 \times 10^{-9} \, m$$

$\Rightarrow$ After 1 m (nearly) all particle have decayed

Since $\Delta^+ \to p\pi^0 \to p\gamma\gamma$ ($2/3 \, N_0$)

$$\Rightarrow N_\gamma = 2 \times \frac{2}{3} \times N_0 = \frac{4}{3} N_0$$