1º Exame (2016/2017)

1 -

a)  W | $\rho_W = 19.3 \ g \ cm^{-3}$

$L = 1 \ m = 100 \ cm$

$J_{beam} = 10^{12}$ part s^{-1} | $\sigma_{pp} = 40$ mb

$N_{total \ interactions} = 10^{17}$

$$\sigma = \frac{J_{int}}{J_{beam} \times N_{target}}$$

$$N_{target} = \frac{L \times \rho_W N_A}{A} = 1{,}16 \times 10^{27} \ cm^{-2}$$

$$N_{tot \ int} = \int J_{int} \, dt \Rightarrow \Delta T = \frac{N_{tot}}{J_{int}}$$

$J_{int} = 4{,}64 \times 10^{13}$ int/s

$\Delta T = 2155$ s

b) $L_{decay} < L_{int}$

$$L_{decay} = \gamma \beta c \tau_\pi = \frac{E_\pi}{m_\pi} \frac{P_\pi}{E_\pi} c \tau_\pi = \frac{P_\pi}{m_\pi} c \tau_\pi$$

$$L_{int} = \frac{1}{\sigma_{\pi p} \rho_{target}}$$

$$\rho_{target} = \frac{\rho_W N_A}{A} = 1{,}16 \times 10^{25} \ cm^{-3}$$

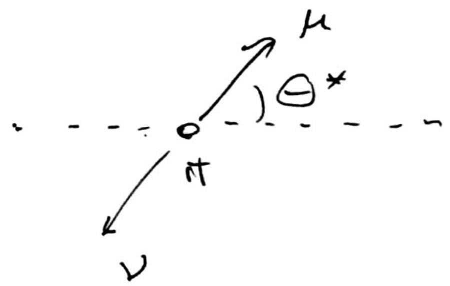
$L_{int} = 3{,}2$ cm

$$\Rightarrow P_\pi < \frac{m_\pi}{\frac{2}{3} \sigma_{pp} c \tau_\pi \rho_{target}} = 0{,}58 \ MeV/c$$

$$E_\pi = \sqrt{P_\pi^2 + m_\pi^2} \simeq 140 \ MeV$$

c) Let us consider the decay $\pi \to \mu \nu$ in the center-of-mass (CM) frame

CM



To get the energy E_μ^* and its momentum P_μ^* in the CM one uses

$$P_\pi = P_\nu + P_\mu \Leftrightarrow \quad (4\text{-vectors})$$

$$(P_\pi - P_\mu)^2 = P_\nu^2 \Leftrightarrow P_\pi^2 + P_\mu^2 - 2 P_\pi \cdot P_\mu = P_\nu^2$$

knowing that in the CM

$$P_\pi = (m_\pi, \vec{0})$$

$$P_\mu = (E_\mu^*, P_\mu^*)$$

$$P_\nu = (P_\mu^*, -P_\mu^*)$$

because $P_\mu = -P_\nu$ and $E_\nu = P_\nu$

Then,

$$m_\pi^2 + m_\mu^2 - 2 E_\mu^* m_\pi = 0 \Leftrightarrow E_\mu^* = \frac{m_\pi^2 + m_\mu^2}{2 m_\pi}$$

$$\Rightarrow \begin{cases} E_\mu^* = 110 \text{ MeV} \\ P_\mu^* = 30 \text{ MeV}/c \end{cases}$$

To get the maximum and minimum energy of the muon in the LAB one uses the Lorentz transformations

$$\begin{pmatrix} E_\mu \\ p_\mu^{l} \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} E^* \\ P_\mu^{l*} \end{pmatrix}$$

$$E_\mu = \gamma E_\mu^* + \gamma \beta P_\mu^* \cos\theta^* \quad \text{with}$$

$$\gamma = \frac{E_\pi}{m_\pi} \quad ; \quad \beta = \frac{P_\pi}{E_\pi}$$

$$E_\mu^{min}(\theta^* = 180°) = \gamma(E_\mu^* - \beta P_\mu^*) \simeq 5,7 \text{ GeV}$$

$$E_\mu^{max}(\theta^* = 0°) = \gamma(E_\mu^* + \beta P_\mu^*) \simeq 10 \text{ GeV}$$

Knowing that in CM the muon angular production distribution is isotropic

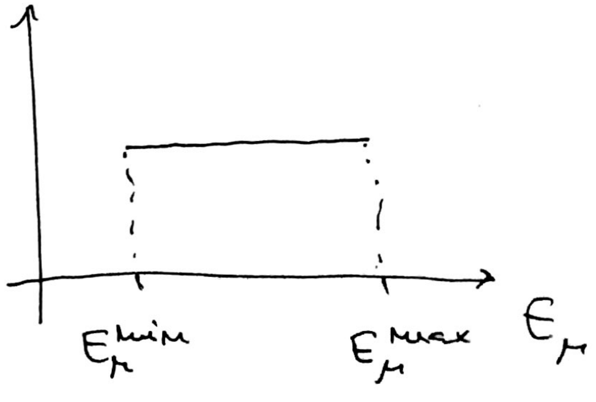
$$\Rightarrow \frac{dN}{d\cos\theta^*} = cte \text{ (constant)}$$

$$\underbrace{\frac{dN}{dE}}_{\text{energy spectrum in the LAB}} = \frac{dN}{d\cos\theta^*} \frac{d\cos\theta^*}{dE}$$

Since, $\cos\theta^* = \dfrac{E_\mu - \gamma E_\mu^*}{\gamma\beta P_\mu^*}$

$$\frac{dN}{dE} = cte \times \frac{1}{\gamma\beta P_\mu^*} = cte \Rightarrow \text{uniform}$$

Hence, $\dfrac{dN}{dE}$



d) $E_\mu = 5\ GeV \Rightarrow P_\mu = \sqrt{E_\mu^2 - m_\mu^2} \simeq 5\ GeV/c$

$L = 1\ m = 100\ cm$

$\left\langle \frac{dE}{dx} \right\rangle$ from the PDG using the Lead

$$\left\langle \frac{dE}{dx} \right\rangle \simeq 1{,}5\ MeV / g\ cm^{-2}$$

$$\langle E_\mu^{loss} \rangle = \left\langle \frac{dE}{dx} \right\rangle \cdot \Delta X$$

$$\Delta X = \rho_w [g\ cm^{-3}] \times L\ [cm] = 19{,}3 \times 100 = 1930\ g\ cm^{-2}$$

$$\langle E_\mu^{loss} \rangle = 1{,}5 \times 1930 = 2895\ MeV \simeq 2{,}9\ GeV$$

(ρ_{Pb} was also accepted)

e)

$$N_{D_s^+} = N_{\text{total interactions}} \frac{\sigma_{D_s^+}}{\sigma_{pp}} = 7{,}5 \times 10^{11}$$

f)

$c\bar{c} \rightarrow J/\psi$

$c\bar{d} \rightarrow D^+$

$c\bar{s} \rightarrow D_s^+$

$c\bar{b} \rightarrow B_c^+$

$c\bar{u} \rightarrow D^0$

g) D_s^+ is a $c\bar{s}$ mesons, and weak decays allows for the violation of strangeness and charm number. Hence, this quantity will not be checked for conservation in the following decays.

(i) $D_s^+ \rightarrow \mu^+ \bar{\nu}_\mu$

BN	: Baryonic number	$0 \rightarrow 0$	✓
LN	: leptonic number	$0 \rightarrow -2$	×
Ch	: Charge conservation	$+1 \rightarrow +1$	✓

Not allowed as violates lepton number

(ii) $D_s^+ \rightarrow \pi^+ \pi^+ \pi^-$

BN:	$0 \rightarrow 0$	✓	Allowed
LN:	$0 \rightarrow 0$	✓	
Ch:	$+1 \rightarrow +1$	✓	

(iii) $D_s^+ \rightarrow p\, K^+\, K^-$

BN : $0 \longrightarrow 1$ ✗ | Not allowed as violates baryonic number

LN : $0 \rightarrow 0$ ✓

Ch : $+1 \rightarrow +1$ ✓

(iv) $D_s^+ \longrightarrow K^+ K^- \pi^+$

BN : $0 \longrightarrow 0$ ✓ | Allowed

LN : $0 \rightarrow 0$ ✓

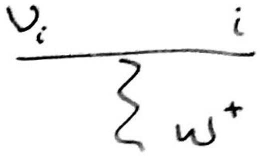
Ch : $+1 \rightarrow +1$ ✓

h) $D_s^+ \rightarrow \tau^+ \nu_\tau$ BR: $5{,}55 \pm 0{,}24\ \%$

$N_{D_s^+}$ from question e)

$$\Rightarrow N_{\nu_\tau} = BR \times N_{D_s^+} = 4{,}125 \times 10^{10}$$

i) The interaction of neutrinos with matter gives as charged particles



$\nu_e \to e^- \, W^+$

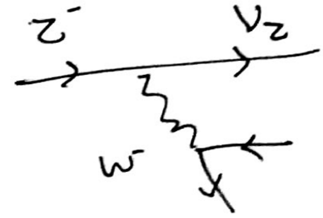
$\nu_\mu \to \mu^- \, W^+$

$\nu_\tau \to \tau^- \, W^+$

So the lepton will be the first charged trace.

The τ^- can be distinguished from the other leptons as it decays very fast (near the neutrino interaction point) as it has a $c\tau = 87{,}03 \, \mu m$.

A τ^- can decay into:



$\tau^- \to \nu_\tau \, \bar{\nu}_e \, e^-$

$\tau^- \to \nu_\tau \, \bar{\nu}_\mu \, \mu^-$

$\tau^- \to \nu_\tau$ quarks (ex: $\bar{u} d$)

Choosing the leptonic decay which as only a charged particle in the final state. The energy of the lepton has to be shared with the remaining two neutrinos which means that it is unprobable that the lepton would follow the direction of the τ^-.

This means that the signature would ~~are~~ be for the ν_τ: one charged track (the τ^-) followed by another track with different direction (kink) which is the lepton (e^- or μ^-) coming from the τ^- decay.

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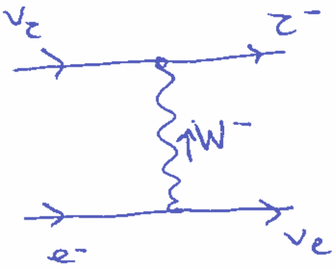
a)

for $\nu_e e^- \to e^- \nu_e$



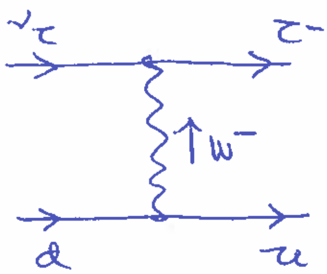
for $\nu_\tau e^- \to \tau^- \nu_e$

b) We must have

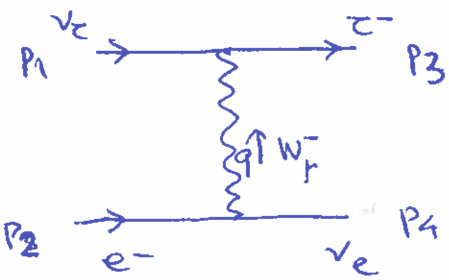
$$Q(\nu_\tau) + Q(q) = Q(\tau^-) + Q(q')$$

$$Q(q) = -1 + Q(q') \Rightarrow \boxed{Q(q') - Q(q) = 1}$$

therefore $q' \equiv u$ and $q = d$ $\left(\frac{2}{3} - \left(-\frac{1}{3}\right) = 1\right)$. The diagram is then



c)



$$q = p_3 - p_1$$

$$i\mathcal{M} = \left(-i\frac{g}{\sqrt{2}}\right)^2 \bar{u}(p_3)\gamma^\mu P_L u(p_1) \frac{(-i)\left[g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2}\right]}{q^2 - M_W^2} \bar{u}(p_4)\gamma^\nu P_L u(p_2)$$

d) Neglecting the lepton mass we can neglect the momenta (2)

in the numerator of the W propagator. In fact, consider the electron line $(q = p_2 - p_4)$

$$\bar{u}(p_4) \not{q} P_L u(p_2) = \bar{u}(p_4) \not{p}_2 P_L u(p_2) - \bar{u}(p_4) \not{p}_4 P_L u(p_2)$$

$$= \bar{u}(p_4) P_R \not{p}_2 u(p_2) - \bar{u}(p_4) \not{p}_4 P_L u(p_2)$$

$$= 0$$

because for massless fermions the Dirac equation gives

$$\not{p} u(p) = 0 \quad \text{and} \quad \bar{u}(p) \not{p} = 0$$

On the other hand, if $\sqrt{s} \ll M_W^2$ we can neglect q^2 in the denominator and obtain for M $(q^2 = t \simeq -\frac{s}{2}(1-\cos\theta))$

$$M = -\frac{g^2}{2M_W^2} \bar{u}(p_3) \gamma^\mu P_L u(p_1) \bar{u}(p_4) \gamma_\mu P_L u(p_2)$$

$$= -\frac{4G_F}{\sqrt{2}} \bar{u}(p_3) \gamma^\mu P_L u(p_1) \bar{u}(p_4) \gamma_\mu P_L u(p_2)$$

e) Using $P_L u(p) = u(p\downarrow)$ we have only one non-vanishing amplitude

$$M(\downarrow\downarrow;\downarrow\downarrow) = -\frac{4G_F}{\sqrt{2}} \bar{u}(p_3\downarrow) \gamma^\mu u(p_1\downarrow) \bar{u}(p_4\downarrow) \gamma_\mu u(p_2\downarrow)$$

$$= -\frac{4G_F}{\sqrt{2}} J_{u_1 u_3}(\downarrow\downarrow) \cdot J_{u_2 u_4}(\downarrow\downarrow)$$

$$= -\frac{4G_F}{\sqrt{2}} s\left(\cos\frac{\theta}{2}, \sin\frac{\theta}{2}, -i\sin\frac{\theta}{2}, \cos\frac{\theta}{2}\right) \cdot \left(\cos\frac{\theta}{2}, -\sin\frac{\theta}{2}, -i\sin\frac{\theta}{2}, -\cos\frac{\theta}{2}\right)$$

then

$$\mathcal{M}(\downarrow\downarrow;\downarrow\downarrow) = -\frac{4G_F s}{\sqrt{2}}\left[\cos^2\frac{\theta}{2} - \left(-\sin^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} - \cos^2\frac{\theta}{2}\right)\right]$$

$$= -\frac{8G_F s}{\sqrt{2}}$$

and

$$\langle|\mathcal{M}|^2\rangle = \frac{1}{2}\,|\mathcal{M}(\downarrow\downarrow;\downarrow\downarrow)|^2 = 16\,G_F^2 s^2$$

f) As we have seen, in the CM, for massless particles

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\langle|\mathcal{M}|^2\rangle$$

and therefore

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{G_F^2 s}{4\pi^2}}$$

g) the total cross section is

$$\sigma = \int d\Omega\,\frac{d\sigma}{d\Omega} = 4\pi \times \frac{d\sigma}{d\Omega} = \frac{G_F^2 s}{\pi}$$

as $\frac{d\sigma}{d\Omega}$ does not depend on the CM angle θ. We have then $\boxed{\sigma = \frac{G_F^2 s}{\pi}}$ $\qquad [\sigma] = \frac{1}{M^2}$ ✓

From PDG:

$$G_F = 1.166\times10^{-5}\ \text{GeV}^{-2}$$

$$1\ \text{GeV}^{-2} = 0.3894\ \text{mb} = 0.3894\times10^{12}\ \text{fb}$$

then for $\sqrt{s} = 1\,\text{GeV}$

$$\sigma = \frac{G_F^2 s}{\pi} = 4.327\times10^{-11}\ \text{GeV}^{-2}$$

$$= 4.327\times10^{-11}\times0.3894\times10^{12}\ \text{fb}$$

or finally

$$\boxed{\sigma = 16.85\ \text{fb}}$$

h) As $\frac{d\sigma}{d\Omega}$ in the CM does not depend on the CM angle θ, it is as likely to be emitted forward or backwards

$$\int_0^{\pi/2} d\theta \sin\theta = \left[-\cos\theta\right]_0^{\pi/2} = 1$$

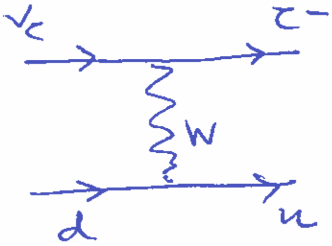
$$\int_{\pi/2}^{\pi} d\theta \sin\theta = \left[-\cos\theta\right]_{\pi/2}^{\pi} = 1$$

i) Neutrinos do not leave tracks on emulsion. So the neutrino beam comes from left and produces 3 tracks. one of this tracks must be the τ^- because

1) Travels some distance. From PDG $c\tau = 87.03\,\mu\text{m}$ for τ^- and 658 m for μ^-.

2) decays into charged lepton + neutrino

explaining the kink. So the Feynman diagram should be



with subsequent decay of τ^-. Cannot be $\nu_\tau + e^- \to \tau^- \nu_e$ because the track of the ν_e would not be seen in the emulsion.