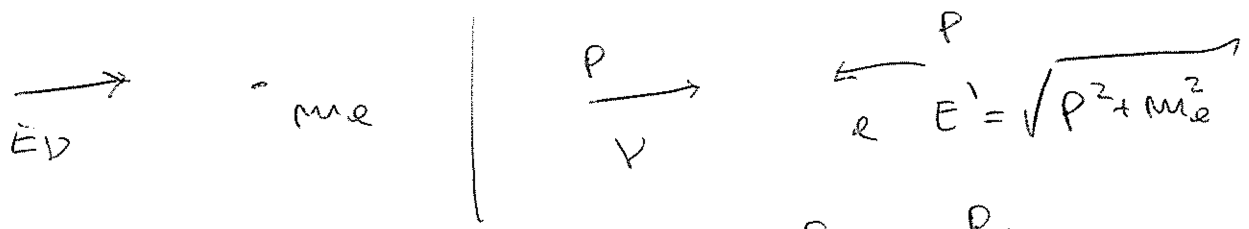
1^{st} Exam (2015 - 2016)

1.

a) LAB | CM



$$p \equiv p_\nu = -p_e$$

LAB: $P_\mu = (E_\nu + m_e, E_\nu)$ since $E_\nu = P_\nu$ as $m_\nu = 0$

$$P_\mu P^\mu = s = (E_\nu + m_e)^2 - E_\nu = \cancel{E_\nu^2} + m_e^2 + 2E_\nu m_e - \cancel{E_\nu^2}$$

CM:

$$P_\mu = (E' + p, 0)$$

$$s = (E' + p)^2 = E'^2 + p^2 + 2E'p \Leftrightarrow$$

$$s = 2p^2 + m_e^2 + 2E'p \Leftrightarrow s - 2p^2 - m_e^2 = 2E'p \Leftrightarrow$$

$$(s - 2p^2 - m_e^2)^2 = 4(p^2 + m_e^2)p^2 \Leftrightarrow$$

$$(s - m_e^2)^2 + 4\cancel{p^4} - 2(2p^2(s - m_e^2)) = \cancel{4p^4} + 4p^2 m_e^2 \Leftrightarrow$$

$$(s - m_e^2)^2 - 4p^2(s - \cancel{m_e^2}) = \cancel{4p^2 m_e^2} \Leftrightarrow$$

$$(s - m_e^2)^2 - 4p^2 s = 0 \Leftrightarrow$$

$$p = \frac{s - m_e^2}{2\sqrt{s}} = \frac{\cancel{m_e^2} + 2E_\nu m_e - \cancel{m_e^2}}{2\sqrt{m_e^2 + 2E_\nu m_e}}$$

$$P = \frac{E_\nu m_e}{\sqrt{m_e^2 + 2E_\nu m_e}} \simeq 1{,}6 \text{ MeV}$$

$$E_\nu^* = P_e^* = P_\nu^* = 1{,}6 \text{ MeV}$$

$$E_e^* = \sqrt{P^{*2} + m_e^2} = 1{,}7 \text{ MeV}$$

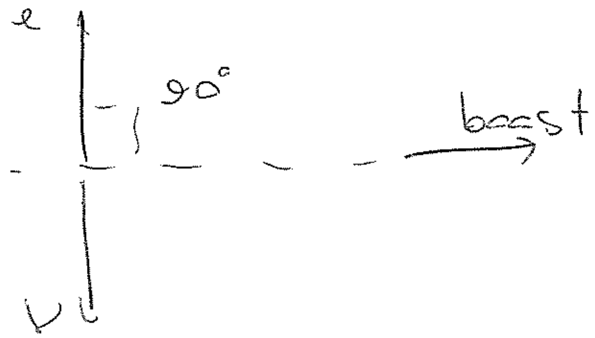
b)
$$\underbrace{\begin{pmatrix} E^* \\ P^* \end{pmatrix}}_{CM} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \underbrace{\begin{pmatrix} E \\ P \end{pmatrix}}_{LAB}$$

since in LAB and before collision $P_e = 0$

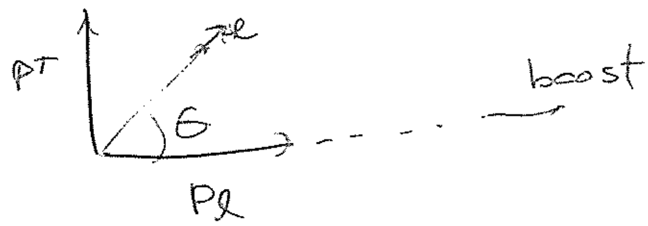
$$\begin{cases} E^* = \gamma(E - \beta P) \\ P^* = \gamma(-\beta E + P) \end{cases} \Leftrightarrow \begin{cases} E^* = \gamma E \\ P^* = -\gamma\beta E \end{cases}$$

$$\Rightarrow \frac{E^*}{P^*} = -\frac{1}{\beta} \Leftrightarrow \beta_{CM} = -\frac{P^*}{E^*} \simeq 0{,}95$$

1- c) Max angle with incoming neutrino (boost direction) occurs when, in the centre of mass CM



Hence in the LAB the electron,



Then,
$$tg\theta = \frac{P_T}{P_l} = \frac{P_e^*}{\gamma\beta E^*}$$
since,

$$P_l = \gamma\beta E^* + \gamma P^* \underbrace{\cos\theta^*}_{\phi} = \gamma\beta E^*$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = 3{,}25$$

So,
$$\theta = \operatorname{arctg}\left(\frac{1{,}6}{3{,}25 \times 0{,}95 \times 1{,}7}\right) \simeq 17°$$

1- d) First let's start by calculating the number of targets

Taking $m_p \sim m_n$ then,

$D_2O \rightarrow 4 + 16 = 20$ nucleons from which 4 are in the deuterium. Hence, only 1/5 of the total D_2O volume will interact with ν. Moreover,

$1M = 20g$ — N_A (Avogrado number)

$10^3 . 10^3 . 10^3 = 10^9 g$ — N_{mol}.

$$N_{target} = \frac{1}{5} 20 . N_{mol} = \frac{4 \times 10^9}{20} N_A = 1{,}2 \times 10^{32}$$

The number of interactions is,

$$N_{int} = \sigma_{\nu N} \cdot \phi \cdot N_t \cdot \varepsilon$$

$$= 10^{-42} \times 5 \times 10^6 \times 1{,}2 \times 10^{32} \times 0{,}5 = 3 \times 10^{-4} s^{-1}$$

And per day,

$$N_{int} = 15 \times 10^{-4} (60 \times 60 \times 24) \simeq 2\cancel{8}5{,}92 / day$$

1 - e) From the PDG

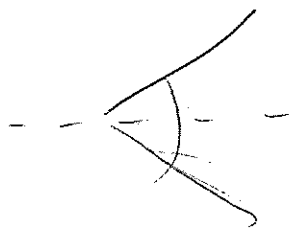
$$\frac{d^2N}{dE\,dx} \simeq 370 \sin^2\theta_c \ [\text{ph eV}^{-1}\text{cm}^{-1}]$$

$$\Rightarrow \frac{dN}{dE} \simeq 370 \sin^2\theta_c \,\Delta x \ [\text{ph eV}^{-1}]$$

↳ number of photons as a function of its energy

with $\Delta x = 5$ cm

knowing that the max angle with the incomming neutrino is $17°$ one can take the mean angle as zero.



In this case,

$$\beta = \sqrt{1-\left(\frac{m_e}{E_e}\right)^2} \simeq 0,9998$$

with $E_e = \gamma\left(E_e^* + \beta P_e^* \cos\theta^*\right) = 10,465$ MeV
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad 0°$

Then, $\theta_c = \arccos\left(\frac{1}{\beta n}\right) \simeq 41,23°$

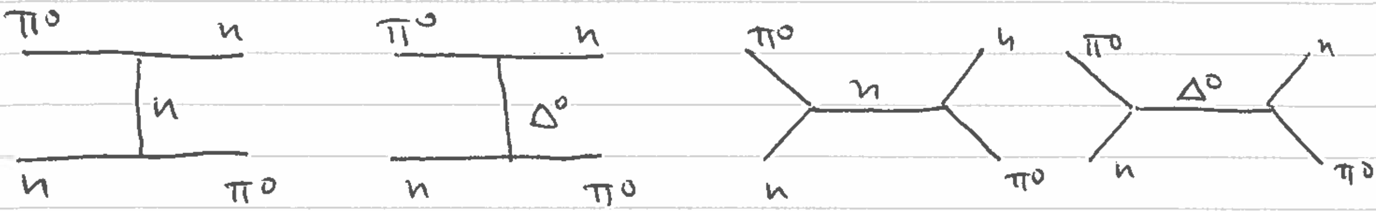
where $n = 1,33$

Hence, $\frac{dN}{dE} \simeq 804 \ [\text{ph eV}^{-1}]$

1º Teste 2015-2016 (13/11/2016)

Problem 2

a) Using the interactions given and conserving charge we can have the following diagrams



b) $\pi^-(p_1) + p(p_2) \to \pi^-(p_3) + \Delta^+(p_4)$

The minimum energy of pion beam occurs for

$$\sqrt{s_{min}} = m_{\pi^-} + m_{\Delta^+}$$

The CM energy is

$$s = (p_1+p_2)^2 = p_1^2 + p_2^2 + 2p_1 \cdot p_2 = m_{\pi^-}^2 + m_p^2 + 2E_\pi m_p$$

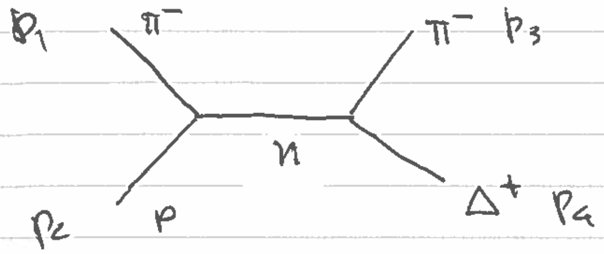
therefore

$$m_{\pi^-}^2 + m_p^2 + 2E_\pi^{min} m_p = (m_{\pi^-} + m_{\Delta^+})^2$$

$$2E_\pi^{min} m_p = m_{\Delta^+}^2 + 2m_{\pi^-} m_{\Delta^+} - m_p^2$$

$$E_\pi^{min} = \frac{m_{\Delta^+}^2 + 2m_\pi m_{\Delta^+} - m_p^2}{2m_p} = 522.97 \text{ MeV}$$

c) $\pi^- + p \to \pi^- + \Delta^+$



$$i\mathcal{M} = (i\mu_1)(i\mu_2)\frac{i}{s-m_n^2}$$

Then

$$\mathcal{M} = -\frac{\mu_1\mu_2}{s-m_n^2}$$

d)
$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\frac{|\vec{p}_3^*|}{|\vec{p}_1^*|}|\mathcal{M}|^2$$

where $\vec{p}_1^*$ and $\vec{p}_3^*$ are the momenta of particles 1 and 3 in the CM Frame. We have

$$E_1^* = \frac{s+m_\pi^2-m_p^2}{2\sqrt{s}} \quad ; \quad E_3^* = \frac{s+m_\pi^2-m_\Delta^2}{2\sqrt{s}}$$

$$|\vec{p}_1^*| = \sqrt{E_1^{*2}-m_\pi^2} = \frac{1}{2\sqrt{s}}\sqrt{s^2+m_\pi^4+m_p^4-2sm_\pi^2-2sm_p^2-2m_\pi^2m_p^2}$$

$$|\vec{p}_3^*| = \sqrt{E_3^{*2}-m_\pi^2} = \frac{1}{2\sqrt{s}}\sqrt{s^2+m_\pi^4+m_\Delta^4-2sm_\pi^2-2sm_\Delta^2-2m_\pi^2m_\Delta^2}$$

therefore

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\frac{\sqrt{s^2+m_\pi^4+m_\Delta^4-2sm_\pi^2-2sm_\Delta^2-2m_\pi^2m_\Delta^2}}{\sqrt{s^2+m_\pi^4+m_p^4-2sm_\pi^2-2sm_p^2-2m_\pi^2m_p^2}}\frac{(\mu_1\mu_2)^2}{(s-m_n^2)^2}$$

e) $\sqrt{s} \gg m_\pi, m_\Delta, m_p$

Then we have

$$\frac{d\sigma}{d\Omega} \simeq \frac{1}{64\pi^2 s} \frac{(\mu_1\mu_2)^2}{s^2} = \frac{(\mu_1\mu_2)^2}{64\pi^2 s^3}$$

and

$$\boxed{\sigma = 4\pi \frac{(\mu_1\mu_2)^2}{64\pi^2 s^3}}$$

As $[\mu_1\mu_2] = M^2$ ~~and~~ and $[s] = M^2$ we have

$$[\sigma] = \frac{(M^2)^2}{(M^2)^3} = \frac{1}{M^2} = L^2$$ in our system as

it should be (dimension of an area).

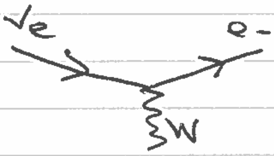
2º TEST 2015-2016 (13/11/2016)

Problem 3

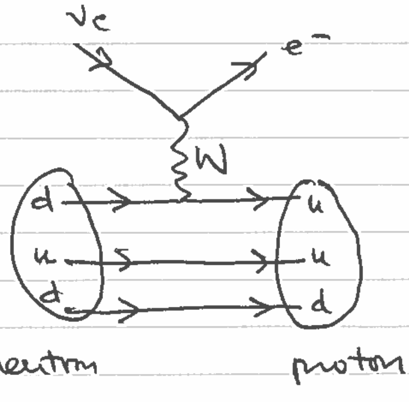
a) The solar neutrinos interact in SNO with the D in the heavy water molecule.

i) Charged Current

As the solar neutrinos are particles, through the charged current they can only give negatively charged leptons. In the experiment, for the energies of the neutrinos only electrons can be produced. So we should have the charged current vertex

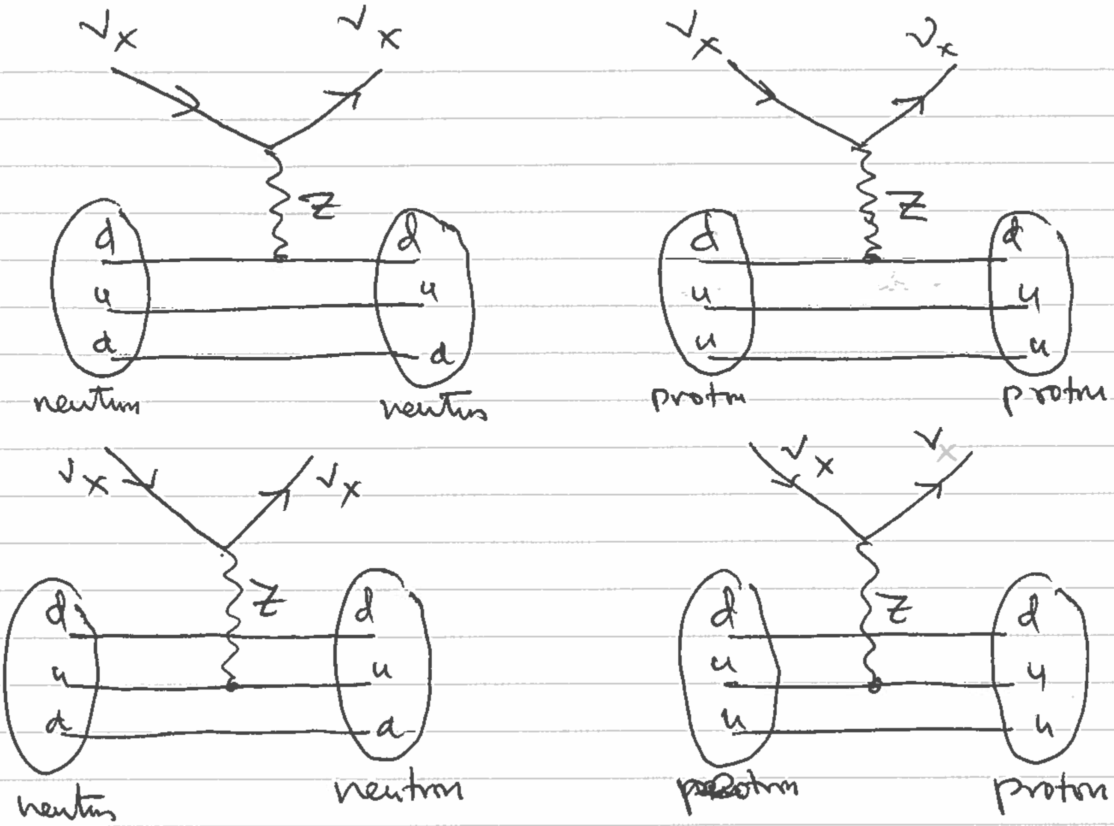


To conserve charge the W has to connect to the d quark and transform it into a u quark. The only possibility is



ii) Neutral Current

The neutral current is mediated by the Z^0. Now there is no change of charge and we can have



Notes: 1) The Z can interact with both u and d quarks inside the neutron and proton

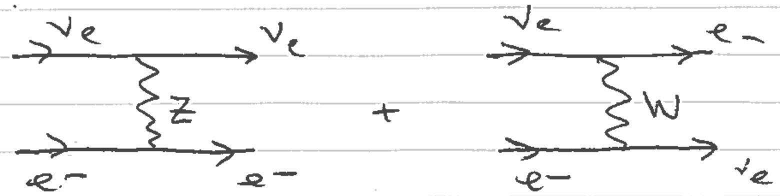
2) ν_x means $x = e^-, \mu^-, \tau^-$, that is all types of neutrino

iii) <u>Elastic Scattering</u>

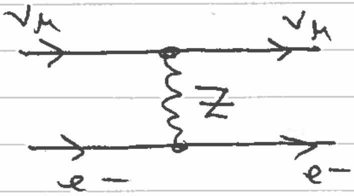
the elastic scattering is

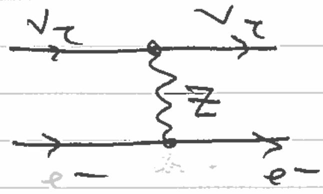
$$\nu_x + e^- \rightarrow \nu_x + e^- \qquad \text{for } (\nu_e, \nu_\mu, \nu_\tau)$$

For ν_e:



For ν_μ and ν_τ

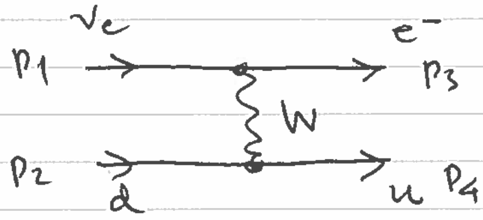




b) The diagram for process

$$\nu_e(p_1) + d(p_2) \to e^-(p_3) + u(p_4)$$

is



c) the amplitude is

$$iM = \left(-i\frac{g}{\sqrt{2}}\right)^2 (-i)\; \bar{u}(p_3)\gamma^\mu P_L u(p_1)$$
$$\frac{g_{\mu\nu} - k_\mu k_\nu/M_W^2}{t - M_W^2}\; \bar{u}(p_4)\gamma^\nu P_L u(p_2)$$

where $t = (p_1-p_3)^2$; $k = p_1 - p_3 = p_4 - p_2$

d) Neglecting the quark and lepton mass we can discard the terms in the numerator of the W propagator proportional to the momentum. To show this we take, for instance, the neutrino electron line. Then

$$k_\mu \bar{u}(p_3)\gamma^\mu P_L u(p_1) = \bar{u}(p_3)(\not{p}_1 - \not{p}_3) P_L u(p_1)$$
$$= -\bar{u}(p_3)\not{p}_3 P_L u(p_1) + \bar{u}(p_3) P_R \not{p}_1 u(p_1) = 0$$

For massless particles the Dirac equation gives

$$\not{p}_1 u(p_1) = 0 \quad \text{and} \quad \bar{u}(p_3)\not{p}_3 = 0$$

So the term is zero.

Also if we assume that the energies are such that $|t| \ll M_W^2$ we can colapse the propagator of the W into

$$\frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{M_W^2}}{t - M_W^2} \rightarrow -\frac{1}{M_W^2} g_{\mu\nu}$$

In this approximation we have

$$M = -\frac{g^2}{2M_W^2}\, \bar{u}(p_3)\gamma^\mu P_L u(p_1)\, \bar{u}(p_4)\gamma_\mu P_L u(p_2)$$

using

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

we get

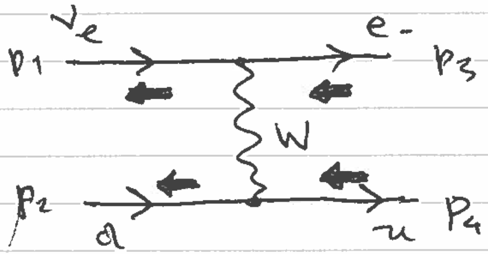
$$\boxed{M = -\frac{4G_F}{\sqrt{2}}\, \bar{u}(p_3)\gamma^\mu P_L u(p_1)\, \bar{u}(p_4)\gamma_\mu P_L u(p_2)}$$

e) Because the charged current is left handed there is only one helicity combination not vanishing. This can be seen because for massless particles we have

$$P_L u(p_1) = u(p_1 \downarrow)$$

$$P_L u(p_2) = u(p_2 \downarrow)$$

and we get



we have then

$$M(\downarrow\downarrow;\downarrow\downarrow) = -\frac{4G_F}{\sqrt{2}} J_{u_1u_3}(\downarrow\downarrow)\cdot J_{u_2u_4}(\downarrow\downarrow)$$

$$= -\frac{4G_F}{\sqrt{2}} s\left(\cos\frac{\theta}{2}, \sin\frac{\theta}{2}, -i\sin\frac{\theta}{2}, \cos\frac{\theta}{2}\right)\cdot$$

$$\cdot\left(\cos\frac{\theta}{2}, -\sin\frac{\theta}{2}, -i\sin\frac{\theta}{2}, -\cos\frac{\theta}{2}\right)$$

$$= -\frac{4G_F}{\sqrt{2}} s\left[\cos^2\frac{\theta}{2} - \left(-\sin^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} - \cos^2\frac{\theta}{2}\right)\right]$$

$$= -\frac{8G_F}{\sqrt{2}} s$$

$$\langle|M|^2\rangle = \frac{1}{2}|M(\downarrow\downarrow;\downarrow\downarrow)|^2 = \frac{32G_F^2 s^2}{2} = 16 G_F^2 s^2$$

f) We have

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \langle |M|^2 \rangle = \frac{1}{64\pi^2 s} 16 G_F^2 s^2 = \frac{G_F^2 s}{4\pi^2}$$

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = 4\pi \frac{G_F^2 s}{4\pi^2} = \frac{G_F^2 s}{\pi}$$

g) Denoting the momentum of the d quark by $\hat{p}_2$

we define

$$\hat{\sigma}(\hat{s}) = \frac{G_F^2 \hat{s}}{\pi}$$

where

$$\hat{s} = (p_1 + \hat{p}_2)^2$$

Now $\hat{p}_2 = x\, p_2$ where we now consider the p_2 the momentum of the neutron. We have for the system ν + neutron

$$s = (p_1 + p_2)^2 \simeq 2 p_1 \cdot p_2$$

therefore

$$\hat{s} = (p_1 + x p_2)^2 \simeq 2x\, p_1 \cdot p_2 = x\, s$$

And

$$\sigma_{\nu D}(s) = \int_0^1 dx\, f_d^n(x)\, \hat{\sigma}_{\nu d}(x s)$$

Where $f_d^n(x)$ is the probability of finding a quark d with fraction of momentum x inside the neutron.

Note: the photon mode for D is spectator in this process.

h) We have seen in f) that

$$\sigma_{\nu q} = \frac{G_F s}{\pi}$$

Therefore it goes like the square of the CM energy. But this is only true for the conditions of item f) that is

$$m_d, m_\nu, m_e \ll \sqrt{s} \ll M_W$$

If $\sqrt{s} \sim M_W$ or $\sqrt{s} \gg M_W$ this is not true anymore. If we go back we can make the replacement

$$M(\downarrow\downarrow;\downarrow\downarrow) = \frac{8 G_F s}{\sqrt{2}} \frac{M_W^2}{t - M_W^2}$$

and we would get for the differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2 s}{4\pi^2} \frac{M_W^4}{(t - M_W^2)^2}$$

Now $t = (p_1 - p_3)^2 = -2 p_1 \cdot p_3$

in the CM

$$p_1 = \frac{\sqrt{s}}{2}(1,0,0,1) \qquad p_3 = \frac{\sqrt{s}}{2}(1, \sin\theta, 0, \cos\theta)$$

So

$$t = -\frac{s}{2}(1 - \cos\theta)$$

Therefore t grows with s and cures the problem for large $\sqrt{s}$

i) The charged current is only sensitive to ν_e, while the neutral current and elastic scattering are sensitive to all neutrino species. The elastic scattering is smaller, then the other two are dominant

In the experiment they measured the ratio of charged to neutral currents

$$\frac{\Phi_{CC}}{\Phi_{NC}} \simeq 1/3$$

where Φ_{CC} and Φ_{NC} are the fluxes of the neutrino in the charged current (ν_e) and Φ_{NC} in the neutral current ($\nu_e + \nu_\mu + \nu_\tau$).

this result explains why we get less ν_e than those produced in the Sun, as the total number is ok. They oscillate into ν_μ and ν_τ. The elastic scattering plays the role of cross checking the result.

This is very important because to oscillate the neutrinos must have mass, therefore this experiment shows indirectly that the neutrinos are not massless.