1 - a)

$$\cos\theta_c = \frac{1}{\beta n}$$ condition to have Cherenkov photons is that $\cos\theta_c \leq 1$

$\Rightarrow \beta \geq \frac{1}{n}$; knowing that $\gamma = \frac{1}{\sqrt{1-\beta^2}} \Leftrightarrow \beta = \sqrt{1-\frac{1}{\gamma^2}}$

moreover $\gamma = \frac{E}{m}$. Therefore,

$$\gamma = \frac{E}{m} \geq \sqrt{\frac{1}{1-\frac{1}{n^2}}} \Leftrightarrow E \geq m\sqrt{\frac{1}{1-\frac{1}{n^2}}}$$

Using $m_\mu = 105$ MeV and $n = 1,33$

$E_\mu \geq 159$ MeV

b) i) From the PDG (30.43)

$$\frac{d^2N}{dE\,dX} \simeq 370 \sin^2\theta_c \,[\text{ph eV}^{-1}\,\text{cm}^{-1}]$$

knowing that $E_\mu = 5$ GeV

$$\beta = \sqrt{1-\left(\frac{m_\mu}{E_\mu}\right)^2} = 0,9998$$ and

$$\theta_c = \arccos\left(\frac{1}{\beta n}\right) = 41,25^\circ$$

$$N_\gamma \simeq 370 \ \text{sen}^2 \theta_c \ \Delta E \ \Delta x \quad [ph]$$

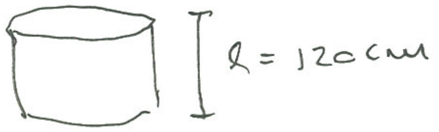
where it was used $n(E) = cte$. Thus

$$N_\gamma = 370 \ \text{sen}^2(41{,}25°)(1\,eV)(120\,cm) = 19362 \text{ photons}$$

1- b)

ii) Reading the plot dE/dX in the PDG (fig. 30.2)

one gets $\dfrac{dE}{dX} \sim 2{,}1 \ [MeV/(g\,cm^{-2})]$



The traversed matter for the μ is $X = l\,[cm] \times \rho_{water}\,[g/cm^3]$ -

$$\Rightarrow X = 120 \times 1 = 120 \ g\,cm^{-2}$$

Thus,

$$E_{loss}(\text{ionization}) = X \cdot \frac{dE}{dX} = 120 \times 2{,}1 = 252 \ MeV$$

For Cherenkov:

Assuming that in average each photon carried $\sim 3{,}5\,eV$

$$E_{loss}(\text{Cherenkov}) = \langle E \rangle_\gamma \cdot N_\gamma = 3{,}5\,eV \times 19302\,ph = 67557\,eV$$

$$= 67 \ KeV$$

$$\frac{E_{ion}}{E_{Ch}} = 3730{,}2$$

1 -

c) The rate of decays in an interval dt is

$$\frac{dN}{dt} = -\lambda N .$$ Integrating,

$N(t) = N_0 e^{-\lambda t}$ particles that survived after t

Therefore the number of particles that decayed in t is,

$$N_0 - N(t) = N_0 \left(1 - e^{-\lambda t}\right)$$

Thus, the probability of a particle to decay in an interval t is $P = 1 - e^{-\lambda t} = 1 - e^{-\frac{t}{t_0}}$ where t_0 is the mean lifetime of the particle in the LAB

~~Then~~ The relation with the particle lifetime in its proper frame is $\tau = \frac{t_0}{\gamma}$

Hence, $P = 1 - e^{-\frac{t}{\gamma\tau}} = 1 - e^{-\frac{t}{\tau}\sqrt{1-\beta^2}}$

From, $\beta c = \frac{x}{t} \Leftrightarrow t = \frac{x}{\beta c}$ one gets finally

$$P = 1 - \exp\left(-\frac{x}{\beta c}\frac{\sqrt{1-\beta^2}}{\tau_\mu}\right)$$

$P(\beta \to 1) = 0$ and $P(\beta \to 0) = 1$

1 - d) From the previous problem one can see that the muon velocity should be small (nearly stopped) in order to have a large probability of decaying inside the tank.

Therefore the signal should be something like

→ μ enters the tank ⇒ Cherenkov light

→ μ stops ⇒ no signal (light)

→ μ^- decays into $e^- \bar{\nu}_e \nu_\mu$ ⇒ e^- produces light (Cherenkov electromagnetic cascade)

In short:

~~Reg~~ → Signal → Δt with no signal → Signal

2.

a) $pp \to ppp\bar{p}$ (possible)

b) $p\bar{p} \to \gamma$ (not possible) violates energy-momentum conservation

c) $e^- \to \nu_e \gamma$ (not possible) violates charge conservation

d) $p \to n e^+ \nu_e$ (not possible) $m_p < m_n$ a particle cannot decay into another heavier

3-

a) $\begin{cases} \pi^+ p \to \Delta^{++} \\ \pi^+ p \to \pi^+ p \end{cases}$ $\begin{cases} \pi^- p \to \pi^- p \\ \pi^- p \to \Delta^0 \\ \pi^- p \to n \pi^0 \end{cases}$

3-b) Using the isospin symmetries $|I \; I_3\rangle$ one can write the interactions as

$$p\pi^+ : |1/2 \; 1/2\rangle |1 \; 1\rangle = |3/2 \; 3/2\rangle$$

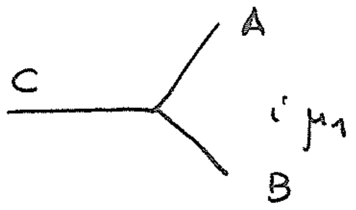
$$p\pi^- : |1/2 \; 1/2\rangle |1 \; -1\rangle = \sqrt{1/3}\,|3/2 \; -1/2\rangle - \sqrt{2/3}\,|1/2 \; -1/2\rangle$$

where it was used the sum rules for spin and the coefficients from the Clebsh-Gordon table (PDG)

As the energy of the interactions is ~~1323~~ 1232 MeV (Δ ressonance) only the state of isospin $I = 3/2$ are relevant for the computation of the cross-section. Hence,

$$\frac{\sigma(p\pi^+)}{\sigma(p\pi^-)} = \frac{\left(|3/2 \; 3/2\rangle\right)^2}{\left(\sqrt{\frac{1}{3}}\,|3/2 \; -1/2\rangle\right)^2} = 3$$

4) a) the decay is possible because $m_C > m_A + m_B$. The Feynman diagram is



$$\mathcal{M} = i\,(i\mu_1) = -\mu_1$$

Using

$$\Gamma = \frac{1}{8\pi m_C^2}\,|\vec{p}_A|\,|\mathcal{M}|^2$$

we get

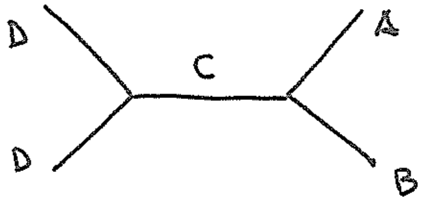
$$\Gamma = \frac{\mu_1^2}{8\pi m_C^2}\,|\vec{p}_A|$$

$$|\vec{p}_A| = \sqrt{E_A^2 - m_A^2} \quad ; \quad E_A = \frac{m_C^2 + m_A^2 - m_B^2}{2m_C}$$

$$= \sqrt{\left(\frac{m_C^2 + m_A^2 - m_B^2}{2m_C}\right)^2 - m_A^2}$$

$$= \frac{1}{2m_C}\sqrt{m_C^4 + m_A^4 + m_B^2 - 2m_A^2 m_B^2 - 2m_A^2 m_C^2 - 2m_B^2 m_C^2}$$

b)



$$\overset{D}{p_1} + \overset{D}{p_2} = \overset{A}{p_3} + \overset{B}{p_4}$$

$$\mathcal{M} = i\,(i\mu_1)(i\mu_2)\,\frac{i}{(p_1+p_2)^2 - m_C^2}$$

$$= \frac{\mu_1\mu_2}{s - m_C^2} \qquad s = (p_1+p_2)^2$$

c)

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\,\frac{|\vec{p}_3|}{|\vec{p}_1|}\,|\mathcal{M}|^2$$

$$|\vec{p}_1| = \sqrt{E_1^2 - m_1^2} \qquad E_1 = \frac{s + m_D^2 - m_D^2}{2\sqrt{s}} = \frac{1}{2}\sqrt{s}$$

$$= \sqrt{\frac{1}{4}s - m_D^2} = \frac{1}{2}\sqrt{s - 4m_D^2}$$

$$|\vec{p}_3| = \sqrt{E_A^2 - m_A^2} = \sqrt{\left(\frac{s + m_A^2 - m_B^2}{2\sqrt{s}}\right)^2 - m_A^2}$$

$$= \frac{1}{2\sqrt{s}}\sqrt{s^2 + m_A^2 + m_B^2 - 2\cdot s\,m_A^2 - 2\,s\,m_B^2 - 2\,m_A^2 m_B^2}$$

d) Neglecting all mass:

$$\mathcal{M} \simeq \frac{\mu_1 \mu_2}{s}$$

$$|\vec{p}_1| = \frac{1}{2}\sqrt{s}$$

$$|\vec{p}_3| = \frac{1}{2}\sqrt{s}$$

therefore

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{(\mu_1 \mu_2)^2}{s^2} = \frac{\mu_1^2 \mu_2^2}{64\pi^2 s^3}$$

As there are no angles the integration in $d\Omega$ gives 4π.

So

$$\boxed{\sigma = \frac{\mu_1^2 \mu_2^2}{16\pi s^3}}$$

$$[\sigma] = \frac{M^4}{M^6} = M^{-2}$$ (Mass = energy in our system)

$$= \text{GeV}^{-2}$$