Fisica & Particels   Test 2   18/12/2018

Problem 1

a) Neglecting the neutrino mass we have

$$P_\nu = (E_\nu, 0, 0, E_\nu)$$

for a $\bar{\nu}_e$ moving in the z direction. For the electron we have

$$P_e = (m, 0, 0, 0)$$

We should have

$$s = (P_\nu + P_e)^2 = M_W^2$$

But

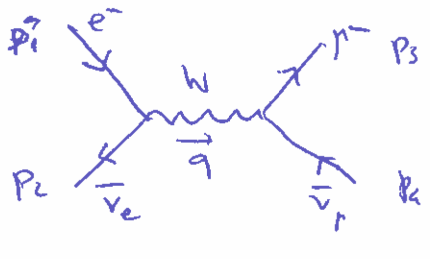
$$(P_\nu + P_e)^2 = P_\nu^2 + P_e^2 + 2 P_e \cdot P_\nu = m_e^2 + 2 E_\nu m_e$$

so

$$E_\nu = \frac{M_W^2 - m_e^2}{2 m_e} = 6.34 \times 10^{6}\ \text{GeV} = 6.34 \times 10^{15}\ \text{eV}$$

b) $e(p_1) + \bar{\nu}_e(p_2) \to \mu(p_3) + \bar{\nu}_\mu(p_4)$

The only diagram contributing is the charged current



$q = p_1 + p_2$

c)
$$iM = \left(-\frac{ig}{\sqrt{2}}\right)^2 \bar{v}(p_2)\gamma^\mu P_L u(p_1) \frac{-i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2}\right)}{q^2 - M_W^2 + i M_W \Gamma_W} \bar{u}(p_3)\gamma^\nu P_L v(p_4)$$

We can neglect the momentum in the numerator of the W propagator. In fact

$$\bar{v}(p_2)\not{q} P_L u(p_1) = \bar{v}(p_2)(\not{p}_1 + \not{p}_2) P_L u(p_1)$$
$$= \bar{v}(p_2)\not{p}_2 P_L u(p_1) + \bar{v}(p_2) P_R \not{p}_1 u(p_1)$$
$$= 0$$

because for massless fermions the Dirac equation gives

$$\bar{v}(p_2)\not{p}_2 = 0 \quad \text{and} \quad \not{p}_1 u(p_1) = 0$$

on the other hand we can write

$$\frac{g^2}{2} = \frac{4 G_F M_W^2}{\sqrt{2}}$$

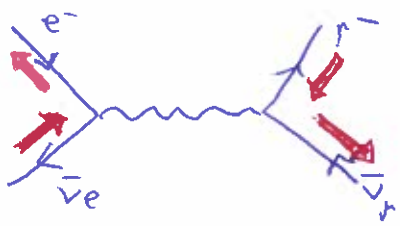
therefore

$$\boxed{M = \frac{4G_F}{\sqrt{2}} \frac{M_W^2}{s - M_W^2 + i M_W \Gamma_W} \bar{v}(p_2)\gamma^\mu P_L u(p_1) \; \bar{u}(p_3)\gamma_\mu P_L v(p_4)}$$

d) using

$$P_L u(p_1) = u(p_1, \downarrow)$$
$$P_L v(p_4) = v(p_4, \uparrow)$$

the only non-vanishing amplitude is



and

$$M(\downarrow\uparrow;\downarrow\uparrow) = \frac{4G_F}{\sqrt{2}} \frac{M_W^2}{s - M_W^2 + i M_W \Gamma_W} J_{u_1 v_2}(\downarrow\uparrow) \cdot J_{u_3 v_4}(\downarrow\uparrow)$$

$$= \frac{4G_F}{\sqrt{2}} \frac{M_W^2}{s - M_W^2 + i M_W \Gamma_W} s\,(0,-1,i,0)\cdot(0,-\cos\theta,-i,\sin\theta)$$

$$= -\frac{4G_F}{\sqrt{2}} \frac{M_W^2}{s - M_W^2 + i M_W \Gamma_W} s\,(1+\cos\theta)$$

e) Therefore

$$\langle |M|^2 \rangle = \frac{1}{2} |M(\downarrow\uparrow;\downarrow\uparrow)|^2 = 4 G_F^2 s^2 |F_W|^2 (1+\cos\theta)^2$$

where we have defined

$$F_W \equiv \frac{M_W}{s - M_W^2 + i M_W \Gamma_W}$$

e) $$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \langle |M|^2 \rangle$$ (for massless initial and final state fermions)

$$= \frac{G_F^2 s}{16\pi^2} |F_W|^2 (1+\cos\theta)^2$$

f) $$\sigma = \int d\Omega \frac{d\sigma}{d\Omega} = 2\pi \int_{-1}^{1} d\cos\theta \, \frac{G_F^2 s}{16\pi^2} |F_W|^2 (1+\cos\theta)^2$$

$$= \frac{G_F^2 s}{3\pi} |F_W|^2$$

$$= \frac{G_F^2 s}{3\pi} \frac{M_W^4}{(s - M_W^2)^2 + M_W^2 \Gamma_W^2}$$

At $\sqrt{s} = M_W$

$$\boxed{\sigma(\sqrt{s} = M_W) = \frac{G_F^2}{3\pi} \frac{M_W^4}{\Gamma_W^2} = 53.98 \text{ nb}}$$

g) As we are at the resonance $\sqrt{s} = M_W$ we should have

$$\sigma(e^- + \bar{\nu}_e \to \text{all}) = \sigma(e^- \bar{\nu}_e \to \mu^- + \bar{\nu}_\mu) \frac{\Gamma(W^- \to \text{all})}{\Gamma(W^- \to \mu^- \bar{\nu}_\mu)}$$

Because the couplings of the W are all equal independent of the fermions it couples to. Now the W can have the decays ④

$$W^- \to (e^- \bar{\nu}_e)\ ;\ (\mu^- \bar{\nu}_\mu);\ (\tau^- \bar{\nu}_\tau);\ (d\,\bar{u})\ ;\ (s\,\bar{c})$$

because there is no decay in (bt) (kinematically forbidden)

So counting we have

$$\Gamma(W^- \to all) = \Gamma(W^- \to \mu^- \bar{\nu}_\mu)\Big[\underset{\substack{\uparrow \\ \text{leptons}}}{3} + \underset{\substack{\uparrow \\ \text{color}}}{3} \times \underset{(d\bar{u})+(s\bar{c})}{2}\Big]$$

$$= 9\,\Gamma(W^- \to \mu^- \bar{\nu}_\mu)$$

and finally (at $\sqrt{s} = M_W$)

$$\sigma(e^- + \bar{\nu}_e \to all) = 9\,\sigma(e^- + \bar{\nu}_e \to \mu^- + \bar{\nu}_\mu)$$

$$= 9 \times 53.98\,nb$$

$$= 485.6\,nb$$

Problem 2

a) $\nu_e + q \to e^- + q'$

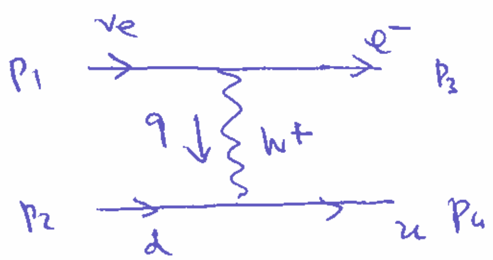
We should have

$$Q[\nu_e] + Q[q] = Q[e^-] + Q[q']$$

so

$$Q[q'] - Q[q] = 1$$

therefore $q' = u$ and $q = d$



$$q^2 = t = (p_1 - p_3)^2$$

$$iM = \left(-i\frac{g}{\sqrt{2}}\right)^2 \bar{u}(p_3)\gamma^\mu P_L u(p_1) \frac{-i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2}\right)}{t - M_W^2 + i M_W \Gamma_W} \bar{u}(p_4)\gamma^\nu P_L u(p_2)$$

The terms in the numerator of the W propagator proportional to the momentum can be neglect for massless fermions (see 1c)). Also we can neglect the Γ_W because $t<0$

Using $\frac{g^2}{2} = \frac{4 G_F M_W^2}{\sqrt{2}}$

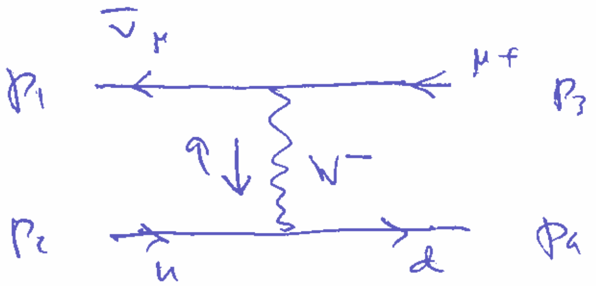
we finally write

$$\boxed{M = \frac{4 G_F}{\sqrt{2}} \frac{M_W^2}{t - M_W^2} \bar{u}(p_3)\gamma^\mu P_L u(p_1)\, \bar{u}(p_4)\gamma_\mu P_L u(p_2)}$$

b) $\bar{\nu}_\mu + q \to \mu^+ + q'$

$$Q[q'] - Q[q] = -1 \quad \Rightarrow \quad q' = d \; ; \; q = u$$

so



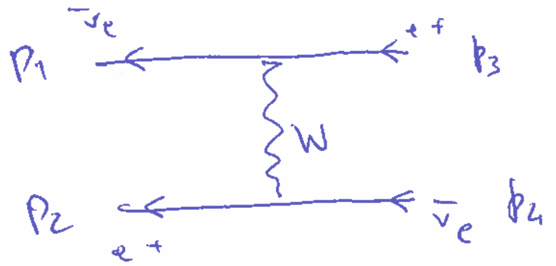
$$q^2 = t$$

$$i\mathcal{M} = \left(-i\frac{g}{\sqrt{2}}\right)^2 \bar{v}(p_1)\gamma^\mu P_L v(p_3) \frac{-i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2}\right)}{t - M_W^2 + iM_W\Gamma_W} \bar{u}(p_4)\gamma^\nu P_L u(p_2)$$

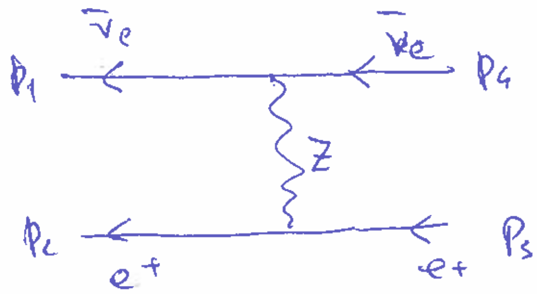
with the same aproximations

$$\boxed{\mathcal{M} = \frac{4G_F}{\sqrt{2}} \frac{M_W^2}{t - M_W^2} \bar{v}(p_1)\gamma^\mu P_L v(p_3)\bar{u}(p_4)\gamma_\mu P_L u(p_2)}$$

e) $\bar{\nu}_e + e^+ \to \bar{\nu}_e + e^+$



(1)



(2)

$$\mathcal{M}_1 = \frac{4G_F}{\sqrt{2}} \frac{M}{t - M_W^2} \bar{v}(p_1)\gamma^\mu P_L v(p_3)\, \bar{v}(p_2)\gamma_\mu P_L v(p_4)$$

for the second diagram we have

$$i\mathcal{M}_2 = \left(-i\frac{g}{\cos\theta_W}\right) \bar{v}(p_1)\gamma^\mu(g_V^\nu - g_A^\nu\gamma_5)v(p_4) \frac{-i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2}\right)}{t - M_W^2 + iM_W\Gamma_W}$$

$$\cdot\, \bar{v}(p_3)\gamma^\nu(g_V^e - g_A^e\gamma_5)v(p_4)$$

with the same approximations and using

$$g_V^\nu - g_A^\nu \gamma_5 = \frac{1}{2} P_L$$

$$g_V^e - g_A^e \gamma_5 = g_L^e P_L + g_R^e P_R$$

where

$$g_L^e = g_V^e + g_A^e$$
$$g_R^e = g_V^e - g_A^e$$

we have $(M_W = M_Z \cos\theta_W)$

$$\boxed{M_2 = \frac{8 G_F}{\sqrt{2}} \frac{M_Z^2}{t - M_Z^2} \frac{1}{2} \bar{u}(p_1)\gamma^\mu P_L u(p_4) \cdot \bar{v}(p_2)\gamma^\mu (g_L^e P_L + g_R^e P_R) v(p_3)}$$

the non-vanishing helicity are:

Diagram 1

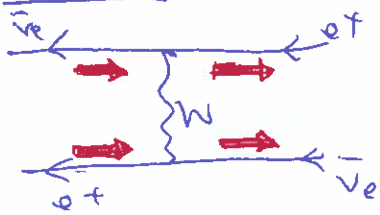
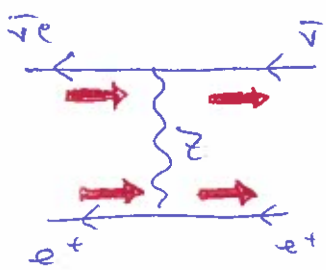
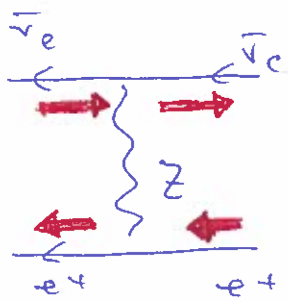


Diagram 2





3-

Neutrinos are neutral so they have to be the interaction/decay products of charge particles which are accelerated in the SNR electromagnetic (e.m.) fields.

High-energy (HE) electrons produces e.m. cascades where they can produce electrons, positrons and gammas.
Anti-neutrinos of the electron ($\bar{\nu}_e$) could be produced, for instance, via the following interaction: $e^+ + n \rightarrow \bar{\nu}_e + p$. It should be noted however, that this is not the main production mechanism of these particles since electrons cannot be effectively accelerated as they tend to lose energy through bremsstrahlung emissions.

HE protons, on the other hand, are less sensitive to these energy losse processes and can be accelerated to very high energies.
A HE proton will interact with other medium protons giving origin to hadronic particles

(mostly pions).

Charged pions decay into:

$$\pi^+ \rightarrow \mu^+ \nu_\mu$$

$$\pi^- \rightarrow \mu^- \bar{\nu}_\mu$$

The negative muon will ~~deca~~ then decay giving origin to ν_μ an e^- and the "desired" $\bar{\nu}_e$.

An alternative to the pion channel would be the creation of high-energy neutrons, resulting from high-energy proton collisions. Here, the channel to produce $\bar{\nu}_e$ would be $n \rightarrow p\, e^-\, \bar{\nu}_e$.

Note that a PeV neutron would travel in average 30 light-years before decaying which means that there is a high probability of re-interaction and consequente destruction of the neutron.