2^{nd} test December 19th 2017: 18h00 Duration of the test: 1h30	Mestrado em Eng. Física Tecnológica (MEFT) **Particle Physics** 1^{st} semester of 2017-18 Prof. Jorge Romão Prof. Mário Pimenta Prof. Ruben Conceição

- The allowed elements for consult during the test are:
  - the PDG (Particle Data Book)
  - one single A4 page with formulas.
- Carefully justify all your answers.
- The test has 2 questions (2 pages) plus a formulary.

The SLAC national accelerator laboratory, originally named Stanford Linear Accelerator Center, hosts the longest linear accelerator ever built (3.2 km). The research done in it produced 3 Nobel prizes among them the discovery of the charm quark and the tau lepton. In the early to mid 1990s the Stanford Linear Collider (SLC) investigated the properties of the Z boson, using electron-positron beams which could be polarized.

1. **[10 val]** Consider the process $e^- + e^+ \to \mu^- + \mu^+$ at the CM energy $\sqrt{s} = M_Z$. Neglect the masses of the leptons.

a) What is the dominant diagram at these energies and why? Draw the corresponding Feynman diagram.

b) Considering the conditions of a) write the amplitude $\mathcal{M}$ in its simplest form. Explain all the approximations.

c) Evaluate the spin averaged squared amplitude $\langle|\mathcal{M}|^2\rangle$.

d) For this process evaluate the differential cross section $d\sigma/d\Omega$ in the CM frame as a function of the square of the energy in the CM frame, $s = (p_1 + p_2)^2$.

e) Determine the expression for total cross section $\sigma(e^- + e^+ \to \mu^- + \mu^+)$ at the energy $\sqrt{s} = M_Z$. Find its value in pb.

f) Find the value of the **total** cross section

$$\sigma((e^- + e^+ \to \text{All})$$

at $\sqrt{s} = M_Z$ in pb.

2. **[10 val]** One of the most important features of SLC was the use of polarization of the electron beam. A very important quantity measured was the asymmetry A_{LR} defined by

$$A_{LR} \equiv \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R},$$

where σ_L (σ_R) are the cross sections for the L and R polarized beams, respectively. Consider that $\sqrt{s} = M_Z$ and, as before, neglect the masses of the leptons.

a) Assuming 100% polarizations evaluate the spin averaged squared amplitudes $\langle |\mathcal{M}_L|^2 \rangle$ and $\langle |\mathcal{M}_R|^2 \rangle$ for the process $e^- + e^+ \to \mu^- + \mu^+$ for the cases when the electron beam is left-handed polarized (L) and righ-handed polarized (R), respectively. You can re-use the calculations of the helicity amplitudes of Problem 1.

b) Evaluate the total cross sections σ_L and σ_R for the two polarizations of the electron beam.

c) Show that A_{LR} can be written as,

$$A_{LR} = \frac{2\, g_V^e\, g_A^e}{(g_V^e)^2 + (g_A^e)^2} \,.$$

Explain why the result does not depend on g_V^μ and g_A^μ.

d) In practice the electron beam is not 100% polarized, but has some degree of polarization P_e. It is known that in this case

$$A_{LR} = \frac{1}{P_e} \frac{N_L - N_R}{N_L + N_R} \,,$$

where N_L (N_R) are the number of events for L (R) polarized beams. In the 1992 run of the experiment the following numbers were obtained (neglecting errors)

$$P_e = 0.224, \quad N_L = 5226, \quad N_R = 4998 \,.$$

Then determine A_{LR}.

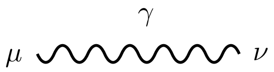
e) Derive an expression for $s_W^2 \equiv \sin^2\theta_W$, where θ_W is the weak mixing angle, in terms of A_{LR}.

f) Use the previous results to determine the value of s_W^2. Knowing that the experimental error on A_{LR} in the above experiment was ± 0.044, discuss if the determination is reasonable considering the present measured value

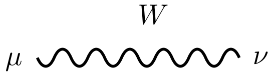
$$s_W^2 = 0.23129 \pm 0.00005 \,.$$

Formulary

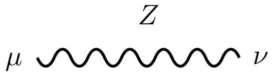
Propagators



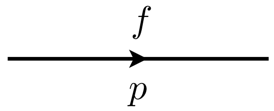
$$-i\frac{g_{\mu\nu}}{k^2} \tag{1}$$



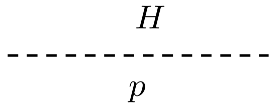
$$-i\frac{g_{\mu\nu}-\frac{k_\mu k_\nu}{M_W^2}}{k^2-M_W^2+i\,M_W\Gamma_W} \tag{2}$$



$$-i\frac{g_{\mu\nu}-\frac{k_\mu k_\nu}{M_Z^2}}{k^2-M_Z^2+i\,M_Z\Gamma_Z} \tag{3}$$



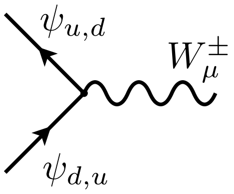
$$\frac{i(\not{p}+m_f)}{p^2-m_f^2} \tag{4}$$



$$\frac{i}{p^2-M_H^2+i\,M_H\Gamma_H} \tag{5}$$

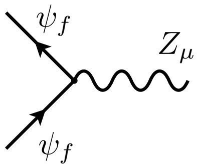
Vertices

Charged Current

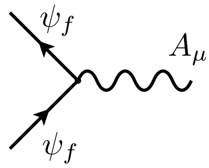


$$-i\frac{g}{\sqrt{2}}\gamma_\mu\frac{1-\gamma_5}{2} \tag{6}$$

Neutral Current



$$-i\frac{g}{\cos\theta_W}\gamma_\mu\left(g_V^f-g_A^f\gamma_5\right)$$

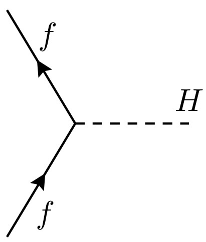


$$-ieQ_f\gamma_\mu$$

where

$$g_V^f=\frac{1}{2}T_f^3-Q_f\sin^2\theta_W, \quad g_A^f=\frac{1}{2}T_f^3\,. \tag{7}$$

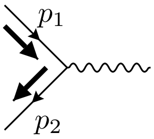
Higgs Interactions with fermions



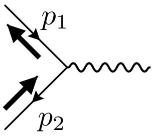
$$-i\,\frac{g}{2}\,\frac{m_f}{M_W}\equiv -i\,g_H^f \tag{8}$$

Results for the Helicity Vector Currents

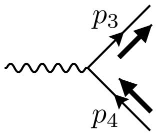
s-channel



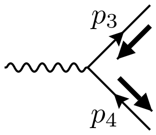
$$J_{u_1v_2}(\uparrow,\downarrow) = \sqrt{s}\,(0,-1,-i,0) \tag{9}$$



$$J_{u_1v_2}(\downarrow,\uparrow) = \sqrt{s}\,(0,-1,i,0) \tag{10}$$

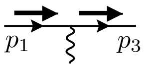


$$J_{u_3v_4}(\uparrow,\downarrow) = \sqrt{s}\,(0,-\cos\theta,i,\sin\theta) \tag{11}$$

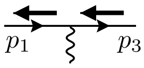


$$J_{u_3v_4}(\downarrow,\uparrow) = \sqrt{s}\,(0,-\cos\theta,-i,\sin\theta) \tag{12}$$

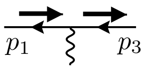
t-channel



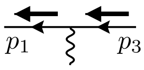
$$J_{u_1u_3}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{13}$$



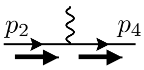
$$J_{u_1u_3}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{14}$$



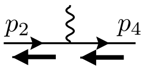
$$J_{v_1v_3}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{15}$$



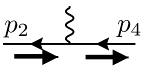
$$J_{v_1v_3}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{16}$$



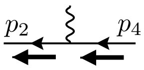
$$J_{u_2u_4}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{17}$$



$$J_{u_2u_4}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{18}$$



$$J_{v_2v_4}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{19}$$



$$J_{v_2v_4}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{20}$$