TÉCNICO LISBOA

2^{nd} exam
February 1^{st} 2018: 15h00
Duration of the exam: 3h00

Mestrado em Eng. Física Tecnológica (MEFT)
Particle Physics
1^{st} semester of 2018-19

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- The allowed elements for consult during the test are:
  - the PDG (Particle Data Book)
  - one single A4 page with formulas.
- Carefully justify all your answers.
- The exam has 4 questions (2 pages) plus a formulary.

1^{st} test

Leon Lederman, Melvin Schwartz and Jack Steinberger received the 1988 Physics Nobel prize for their discovery, in 1962, of the muon-neutrino. At the time, only the electron-neutrino was known and they were able to detected a new type of neutrino using the Brookhaven's Alternating Gradient Synchrotron.

1. **[6 val]** The analysis of the energy spectrum of the electron that arises from the muon decay indicated that there must exist two neutrinos.

Consider an atmospheric muon of 10 GeV produced at 5000 m of altitude. To solve the following questions consider also that:

- The atmosphere has a composition of 20% Oxygen and 80% Nitrogen.
- In the isothermal approximation, the depth X of the atmosphere at a height h (i.e., the amount of atmosphere above h) can be approximated as $X = X_0\, e^{-h/7\,\mathrm{km}}$, with $X_0 \approx 1037\,\mathrm{g/cm^2}$.

a) Can the muon reach the ground ($h = 0\,\mathrm{km}$)? To answer to this question:

   i. Compute the probability of the muon to decay.
   ii. Estimate the energy lost by the muon by ionization.
   iii. Can an electron produced with the same energy and at the same point reach the ground? Why?

b) Compute the minimum and maximum energy that the electron, arising from the muon decay, can carry in the muon reference frame.

c) Compute the minimum and maximum energy that the electron, arising from the muon decay, can carry in the LAB framework.

d) Why do the two neutrinos arising from the muon decay are supposed to be different?

2. **[4 val]** To prove that muon and electron neutrinos where two different particles it was conceived a test involving the scattering of a muon neutrino beam in matter. The produced pions and muons that do not decay strikes a 13.5 m thick iron shield wall at a distance of 21 m. For pions with energies $\sim 1 - 10\,\text{GeV}$ the attenuation in iron has been measured to be less than 0.24 m.

a) If 10^{17} pions are able to reach the iron wall, how many would be able to completely cross it?

b) Compute the cross-section pion-iron, $\sigma_{\pi-Fe}$. Express the result in milibarns (mb).

c) The interaction of muon neutrinos occurred in a spark chamber. To prove that ν_μ was a different neutrino they considered the following interactions:

i. $\nu_\mu\, n \to p\, e^-$

ii. $\overline{\nu}_\mu\, p \to n\, e^+$

iii. $\nu_\mu\, n \to p\, \mu^-$

iv. $\overline{\nu}_\mu\, p \to n\, \mu^+$

How could they prove that ν_μ was indeed a different neutrino? Justify.

d) Describe succinctly how to distinguish an 1 GeV electron from a muon with the same energy in a spark chamber. Justify your answer.

2^{nd} test

3. **[6 val]** Consider the following channel, where the neutrino has an energy of 1 GeV and the quark is at rest.

$$\nu_\mu(p_1) + q(p_2) \to \mu^-(p_3) + q'(p_4)$$

a) Identify the valence quarks q and q'.

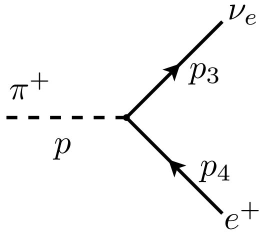
b) Draw the corresponding Feynman diagram(s) at tree level.

c) Considering the conditions of the problem write the amplitude $\mathcal{M}$ in its simplest form. Assume that all fermion masses can be neglected. Justify your answer.

d) Evaluate the spin averaged squared amplitude $\langle|\mathcal{M}|^2\rangle$.

e) For this process evaluate the differential cross section $d\sigma/d\Omega$ in the CM frame as a function of the square of the energy in the CM frame, $s = (p_1 + p_2)^2$ and the scattering angle θ.

4. **[4 val]** Although the π meson is a composite particle (quark-anti-quark) for many problems it is a good approximation to consider it as a point particle with an effective interaction. In this approximation the vertex responsible for the decay of the pion in the channel $\pi^+ \to e^+ + \nu_e$ is,



$$i\sqrt{2} G_F\, V_{ud}\, f_\pi\, m_e P_R$$

where $V_{ud} = 0.974$ is a constant (element of the V_{CKM} matrix), f_π the pion decay constant, m_e the electron mass and $P_R = \frac{1+\gamma_5}{2}$ is the right chiral projector.

a) Write the amplitude $\mathcal{M}$ for the process $\pi^+ \to e^+ + \nu_e$.

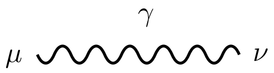
b) **Use the scalar currents in the formulary** to evaluate the non-vanishing helicity amplitude(s) in the limit where you neglect the fermion masses everywhere except in the coupling.

c) Use the results of b) to compute the width for this process, $\Gamma(\pi^+ \to e^+ + \nu_e)$.

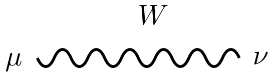
d) Using the fact that the Branching Ratio for this channel $\pi^+ \to e^+ + \nu_e$ is known, and knowing the lifetime of the pion, determine the constant f_π. Use the PDG to get the relevant numbers to solve this exercise. Express your result in MeV.

Formulary

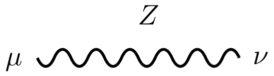
Propagators



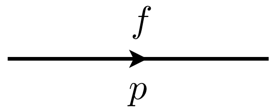
$$-i\frac{g_{\mu\nu}}{k^2} \tag{1}$$



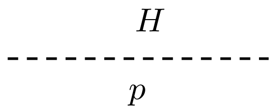
$$-i\frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{M_W^2}}{k^2 - M_W^2 + i\,M_W\Gamma_W} \tag{2}$$



$$-i\frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{M_Z^2}}{k^2 - M_Z^2 + i\,M_Z\Gamma_Z} \tag{3}$$



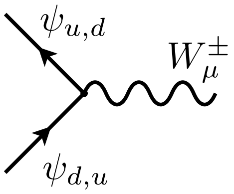
$$\frac{i(\not{p} + m_f)}{p^2 - m_f^2} \tag{4}$$



$$\frac{i}{p^2 - M_H^2 + i\,M_H\Gamma_H} \tag{5}$$

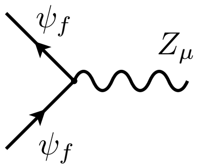
Vertices

Charged Current

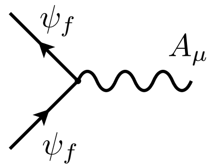


$$-i\frac{g}{\sqrt{2}}\gamma_\mu\frac{1-\gamma_5}{2} \tag{6}$$

Neutral Current



$$-i\frac{g}{\cos\theta_W}\gamma_\mu\left(g_V^f - g_A^f\gamma_5\right)$$

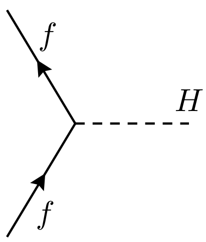


$$-ieQ_f\gamma_\mu$$

where

$$g_V^f = \frac{1}{2}T_f^3 - Q_f\sin^2\theta_W, \quad g_A^f = \frac{1}{2}T_f^3\,. \tag{7}$$

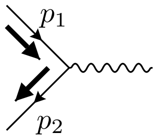
Higgs Interactions with fermions



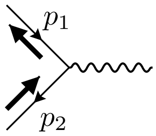
$$-i\,\frac{g}{2}\,\frac{m_f}{M_W} \equiv -i\,g_H^f \tag{8}$$

Results for the Helicity Vector Currents

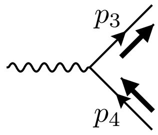
s-channel



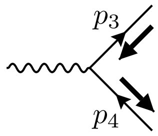
$$J_{u_1 v_2}(\uparrow,\downarrow) = \sqrt{s}\,(0,-1,-i,0) \tag{9}$$



$$J_{u_1 v_2}(\downarrow,\uparrow) = \sqrt{s}\,(0,-1,i,0) \tag{10}$$

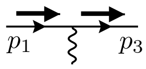


$$J_{u_3 v_4}(\uparrow,\downarrow) = \sqrt{s}\,(0,-\cos\theta,i,\sin\theta) \tag{11}$$

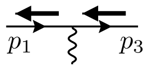


$$J_{u_3 v_4}(\downarrow,\uparrow) = \sqrt{s}\,(0,-\cos\theta,-i,\sin\theta) \tag{12}$$

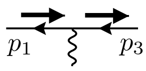
t-channel



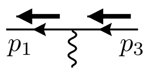
$$J_{u_1 u_3}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{13}$$



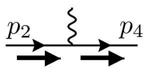
$$J_{u_1 u_3}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{14}$$



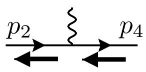
$$J_{v_1 v_3}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{15}$$



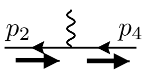
$$J_{v_1 v_3}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{16}$$



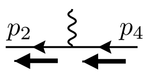
$$J_{u_2 u_4}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{17}$$



$$J_{u_2 u_4}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{18}$$



$$J_{v_2 v_4}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{19}$$



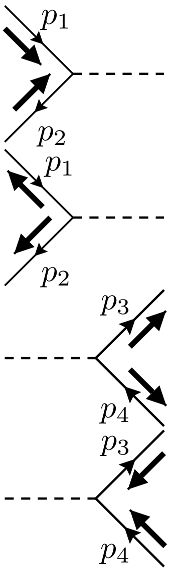
$$J_{v_2 v_4}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{20}$$

Results for Scalar Helicity Currents

The interactions of fermions with the scalar bosons are **scalar** interactions and therefore the scalar currents are numbers, **not four-vectors**! Neglecting the fermion masses they are given below, for the s and t channels.

Important Note:
Notice that the rule for the spin arrows for scalar currents is opposite to that of the vector helicity currents. This means that for s-schannel the only non-vanishing helicity amplitudes are $\uparrow,\uparrow$ *and* $\downarrow,\downarrow$*, while for the t-channel amplitudes they are* $\uparrow,\downarrow$ *and* $\downarrow,\uparrow$*.*

s-channel



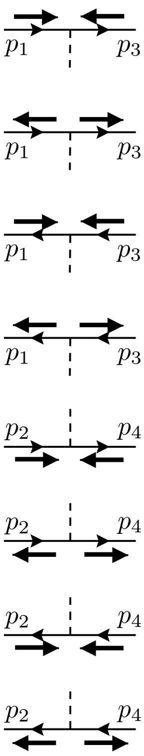
$$J_{u_1v_2}(\uparrow,\uparrow) = \sqrt{s} \tag{21}$$

$$J_{u_1v_2}(\downarrow,\downarrow) = -\sqrt{s} \tag{22}$$

$$J_{u_3v_4}(\uparrow,\uparrow) = \sqrt{s} \tag{23}$$

$$J_{u_3v_4}(\downarrow,\downarrow) = -\sqrt{s} \tag{24}$$

t-channel



$$J_{u_1u_3}(\uparrow,\downarrow) = -\sqrt{s}\sin\frac{\theta}{2} \tag{25}$$

$$J_{u_1u_3}(\downarrow,\uparrow) = \sqrt{s}\sin\frac{\theta}{2} \tag{26}$$

$$J_{v_1v_3}(\uparrow,\downarrow) = -\sqrt{s}\sin\frac{\theta}{2} \tag{27}$$

$$J_{v_1v_3}(\downarrow,\uparrow) = \sqrt{s}\sin\frac{\theta}{2} \tag{28}$$

$$J_{u_2u_4}(\uparrow,\downarrow) = \sqrt{s}\sin\frac{\theta}{2} \tag{29}$$

$$J_{u_2u_4}(\downarrow,\uparrow) = -\sqrt{s}\sin\frac{\theta}{2} \tag{30}$$

$$J_{v_2v_4}(\uparrow,\downarrow) = \sqrt{s}\sin\frac{\theta}{2} \tag{31}$$

$$J_{v_2v_4}(\downarrow,\uparrow) = -\sqrt{s}\sin\frac{\theta}{2} \tag{32}$$