2^{nd} exam 2015-16

1- a) The number of Higgs produced in the process $H \to \gamma\gamma$ is given by

$N_H = \sigma_H \, \mathcal{L} \, \Delta t \, BR$ where $\mathcal{L}\,\Delta t$ is the integrated luminosity $= 20\, fb^{-1}$

Hence,

$$N = 21 \times (20 \times 10^{+3}) \times (2{,}28 \times 10^{-3}) = 957$$

b) When a photon enters a calorimeter it converts into a pair ($e^- e^+$). These particles will radiate (Bremsstrahlung) photons that, having enough energy, with convert into $e e^-$. This process repeats originating an electromagnetic cascade. The cascade stops when the produced particles reach a critical energy ($2 m_e$). After reaching these energies these particles will produced photons γ, by excitation or ionization, which can be detected.

The measurement of the number of these low energy photons enables an estimation of the primary photon energy.

From the PDG it can be seen that the energy resolution can be parameterized as

$$\frac{\sigma_E}{E} \propto \frac{a}{\sqrt{E}} \oplus b \oplus \frac{c}{E}$$

a - is related with the stochastic fluctuations of the e.m. cascade

b - related to detector non-uniformity, calibration uncertainty ...

c - electronic noise

c) Knowing that in the LAB

$E_1 = 123,3$ GeV ; $E_2 = 36,7$ GeV and $\theta = 136,3°$

$E_1 = |p_1|$ and $E_2 = |p_2|$ as $m_\gamma = 0$

Then
$$\vec{P}_H = \vec{P}_1 + \vec{P}_2$$

$$(\vec{P}_H)^2 = |\vec{P}_1|^2 + |\vec{P}_2|^2 + 2 P_1 P_2 \cos\theta \iff$$

$$|\vec{P}_H| \simeq 100 \text{ GeV}$$

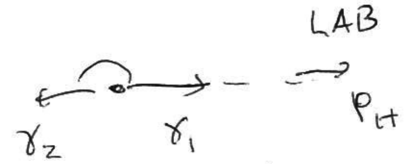
$$m_H = \sqrt{s} = (E_1 + E_2)^2 - \underbrace{(P_1 + P_2)^2}_{P_H^2} \simeq 124,9 \text{ GeV}$$

d) i) For $P_H = 0 \Rightarrow$ LAB is equivalent to CM and the angle between the photons has to be $\theta = 180°$ (back-to-back)

1- d) (i) (cont...)

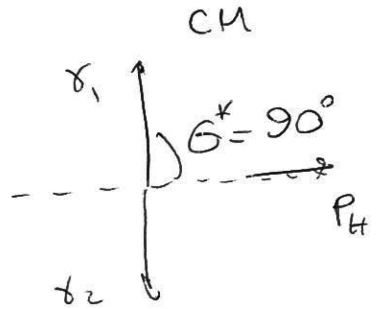
For $P_H = 100$ GeV

Maximum angle is $\theta_{max} = 180°$

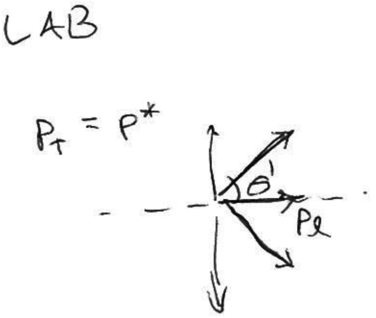


As the boost of the Higgs could never invert the momentum of a photon (mass equal to zero).

The minimum angle occurs when



$\Longrightarrow$



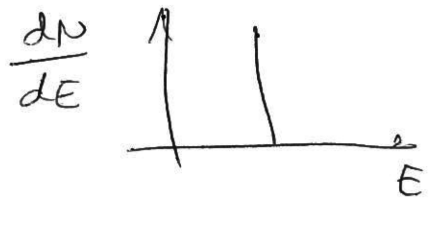
$$tg\theta' = \frac{P_T}{P_\ell} = \frac{m_H/2}{P_H/2}$$

$\rightarrow$ ~~since~~ the photons share the momentum of the Higgs boson in the LAB

$$tg\theta' = \frac{62,5}{50} \Leftrightarrow \theta_{min} = 2\theta' \simeq 102°$$

d) (ii) For $P_H = 0 \Rightarrow E_{max} = E_{min} = E^* = \frac{m_H}{2} = 62,5$ GeV

And so the energy spectrum is



1 - d)

(ii) (cont...)

~~For~~ In the CM, the photons have

$$E^* = p^* = \frac{m_H}{2}$$

Using the Lorentz transformations, ~~tra~~

$$E_{1,2} = \gamma\left(E^* \pm \beta p^*\right) = \gamma E^* (1 \pm \beta)$$

~~Usi~~ $\gamma = \frac{E_H}{m_H}$; $\beta = \frac{p_H}{E_H}$; $E_H = \sqrt{p_H^2 + m_H^2}$

$$\Rightarrow \begin{cases} E_{min} = 30 \text{ GeV} \\ E_{max} = 130 \text{ GeV} \end{cases}$$

The energy spectrum of the photons in the LAB is given by,

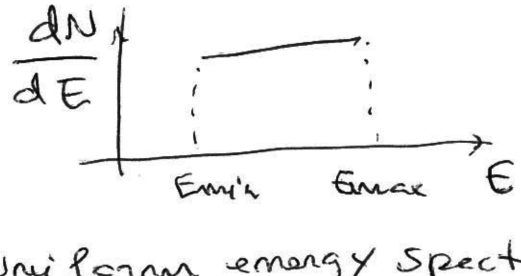
$$\frac{dN}{dE} = \underbrace{\frac{dN}{d\cos\theta^*}}\frac{d\cos\theta^*}{dE}$$

cte → as in the CM the photons are produced isotropically (Higgs boson is a scalar)

From the Lorentz transformations one has,

$$E = \gamma(E^* + \beta p^* \cos\theta^*)$$

Thus, $\frac{dE}{d\cos\theta^*} = \gamma\beta p^*$



Hence

$$\frac{dN}{dE} = \text{cte} \cdot \frac{1}{\gamma\beta p^*} \Rightarrow$$

Uniform energy spectrum

(1)

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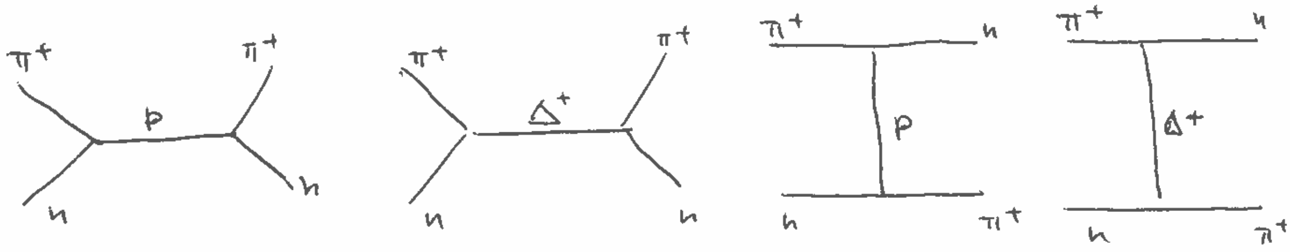
Problem (2)

a) With the vertices given and conserving charge we have



b) $P_1 = P_2 + P_3$. In the decaying frame of $\Delta^+(1232)$ we have

$$P_1 = (m_\Delta, \vec{0}) \quad ; \quad P_2 = (E_2, \vec{P}_{CM}) \quad ; \quad P_3 = (E_3, -\vec{P}_{CM})$$

we have

$$P_1 - P_2 = P_3 \Rightarrow (P_1 - P_2)^2 = P_3^2$$

which gives

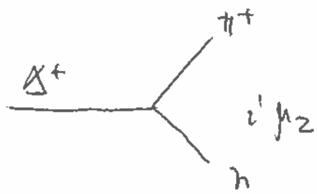
$$m^2_{\Delta^+} + m^2_{\pi^+} - 2\, P_1 \cdot P_2 = m_n^2$$

using $P_1 \cdot P_2 = E_2 m_{\Delta^+}$ we get finally

$$E_2 = \frac{m^2_{\Delta^+} + m^2_{\pi^+} - m_n^2}{2 m_\Delta} = 265.63 \text{ MeV}$$

c) For decays in the rest frame of a massive particle we have

$$\Gamma = \frac{1}{8\pi m_\Delta^2} |\vec{P}_2^{CM}| \, |M|^2$$



with

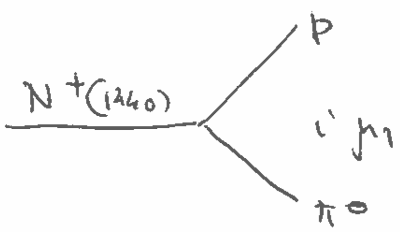
$$M = \mu_2$$

$$|\vec{p}_2^{\,CM}| = \frac{1}{2m_\Delta}\sqrt{m_\Delta^4 + m_{\pi^+}^4 + m_n^4 - 2m_\Delta^2 m_{\pi^+}^2 - 2m_\Delta^2 m_n^2 - 2m_{\pi^+}^2 m_n^2}$$

we get

$$\boxed{\Gamma = \frac{\mu_2^2}{16\pi m_\Delta^3}\sqrt{m_\Delta^4 + m_{\pi^+}^4 + m_n^4 - 2m_\Delta^2 m_{\pi^+}^2 - 2m_\Delta^2 m_n^2 - 2m_{\pi^+}^2 m_n^2}}$$

d) The diagram is



$$\boxed{M = \mu_1}$$

$$\Gamma = \frac{1}{8\pi m_{N^+}^2}|\vec{p}_2^{\,CM}|\,|M|^2$$

we finally get (is the same calculation as in c)

$$\boxed{\Gamma = \frac{\mu_1^2}{16\pi m_{N^+}^3}\sqrt{m_{N^+}^4 + m_p^4 + m_{\pi^0}^4 - 2m_{N^+}^2 m_p^2 - 2m_{N^+}^2 m_{\pi^0}^2 - 2m_{\pi^0}^2 m_p^2}}$$

e) From the PDG

$$\Gamma(\Delta^+) \simeq 117\,\mathrm{MeV}$$

$$\Gamma(N^+(1440)) = 350\,\mathrm{MeV}$$

$$BR(N^+(1440) \to p\,\pi^0) \simeq 65\% \quad (\text{average value})$$

$$BR(\Delta^+(1220) \to \pi^+ n) \simeq 100\%$$

therefore

$$\Gamma(\Delta^+(1220) \to \pi^+ n) = 117\,\mathrm{MeV}$$

$$\Gamma(N^+(1440) \to p\,\pi^0) = 0.65 \times 350\,\mathrm{MeV} = 227.5\,\mathrm{MeV}$$

we have then

$$\mu_2 = \left[\frac{16\pi\, m_{\Delta^+}^3\, \Gamma(\Delta^+(1220) \to \pi^+ n)}{\sqrt{m_{\Delta^+}^4 + m_{\pi^+}^4 + m_n^4 - 2m_{\Delta^+}^2 m_{\pi^+}^2 - 2m_{\Delta^+}^2 m_n^2 - 2m_{\pi^+}^2 m_n^2}} \right]^{1/2}$$

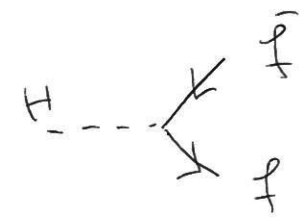
$$\mu_1 = \left[\frac{16\pi\, m_{N^+}^3\, \Gamma(N^+(1440) \to p\pi^0)}{\sqrt{m_{N^+}^4 + m_{\pi^0}^4 + m_p^4 - 2m_{N^+}^2 m_{\pi^0}^2 - 2m_{N^+}^2 m_p^2 - 2m_{\pi^0}^2 m_p^2}} \right]^{1/2}$$

Numerically:

$\mu_1 = 5455\,\mathrm{MeV}$ $\qquad$ $\mu_2 = 4444\,\mathrm{MeV}$

3-

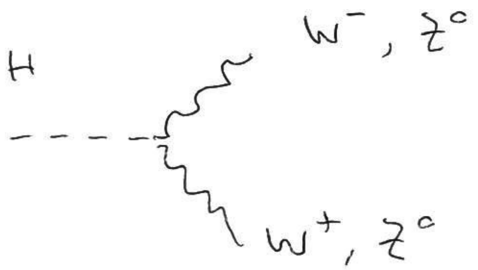
a) The Higgs boson can decay into:

- 

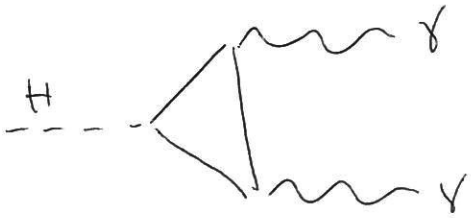
where f can be:

→ quark (u, d, s, c, b)

→ lepton (e, μ, τ)

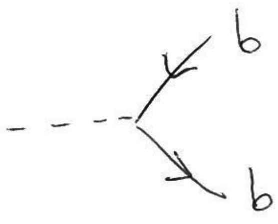
- 

Or into two photons via:

- 

The Higgs coupling is proportional to the masses of the decay products.

Since $m_H = 125$ GeV the "heaviest" pair into which the Higgs boson ~~will~~ decay more often is



$2 \times m_b \simeq 10$ GeV $< m_H$

$(m_H < 2 m_W < 2 m_Z)$

3-b) In LEP the initial state is an e^+e^-, whereas in the LHC the collisions are between the quarks (and gluons) inside of the protons.

This means that, while looking for a specific process, one has to deal not only with all the spectators, i.e., remaining quark and gluons that don't participate in the process of interest, but also the momenta of the quarks are not directly accessible, as these depend on the proton parton distribution functions (PDFs).

4-

a) $B^- - \bar{u}b$ $\quad B^+ - u\bar{b}$

$\bar{B}^0 - \bar{d}b$ $\quad B^0 - d\bar{b}$

$\bar{B}_s^0 - \bar{s}b$ $\quad B_s^0 - s\bar{b}$

$\bar{B}_c - \bar{c}b$ $\quad B_c^+ - c\bar{b}$

$\Upsilon - b\bar{b}$

b) (i) $B^0 \to e^+ \gamma$

Forbidden due to charge violation.

(ii) $B^0 \to \pi^- e^+ \nu_e$

Allowed

(iii) $B^0 \to p\bar{p}n$

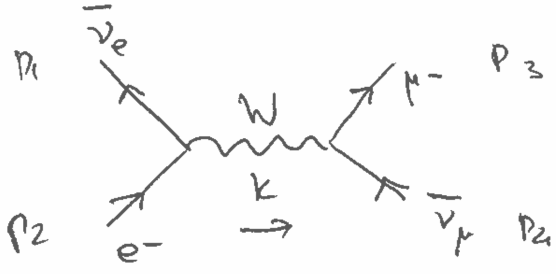
Forbidden. Violates baryonic number.

(iv) $B^0 \to e^+ e^- \bar{\nu}_e$

Forbidden. Violates leptonic number

Problem 5

a) The diagram is



$$k = p_1 + p_2 = p_3 + p_4$$

$$k^2 = s$$

with the amplitude

$$i\mathcal{M} = \left(-i\frac{g}{\sqrt{2}}\right)^2 \bar{v}(p_1)\gamma^\mu P_L u(p_2) \frac{(-i)\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{M_W^2}\right)}{k^2 - M_W^2 + iM_W\Gamma_W} \bar{u}(p_3)\gamma^\nu P_L v(p_4)$$

b) In these conditions we have

$$\frac{1}{k^2 - M_W^2 + iM_W\Gamma_W} \longrightarrow \frac{1}{-M_W^2}$$

and we can also neglect the terms in $k_\mu k_\nu$ in the numerator of the propagator. In fact take the electron line as an example. We have

$$\bar{v}(p_1)\gamma^\mu P_L u(p_2) k_\mu = \bar{v}(p_1)(\not{p}_1 + \not{p}_2) P_L u(p_2)$$

$$= \bar{v}(p_1)\not{p}_1 P_L u(p_2) + \bar{v}(p_1) P_R \not{p}_2 u(p_2) = 0$$

because $\bar{v}(p_1)\not{p}_1 = 0$; $\not{p}_2 u(p_2) = 0$

for massless fermions.

②

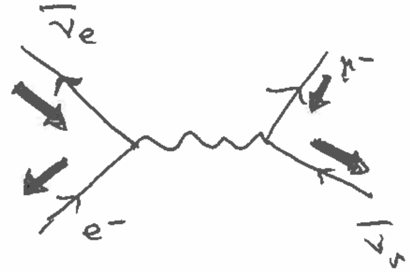
The simplified expression for $\mathcal{M}$ is therefore

$$\mathcal{M} = -\frac{g^2}{2M_W^2}\,\bar{v}(p_1)\gamma^\mu P_L u(p_2)\,\bar{u}(p_3)\gamma_\mu P_L v(p_4)$$

or

$$\boxed{\mathcal{M} = -\frac{4G_F}{\sqrt{2}}\,\bar{v}(p_1)\gamma^\mu P_L u(p_2)\,\bar{u}(p_3)\gamma_\mu P_L v(p_4)}$$

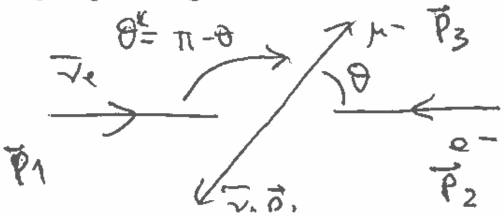
c) Because the anti-neutrinos have positive helicity, the only non-zero helicity combination is



or, with the notation $\mathcal{M}(h_{\bar{\nu}_e}, h_{e^-}; h_{\mu^-} h_{\bar{\nu}_\mu})$,

$$\mathcal{M}(\uparrow\downarrow;\downarrow\uparrow) = -\frac{4G_F}{\sqrt{2}}\,J_{\bar{v}_1 u_2}(\uparrow\downarrow)\cdot J_{u_3 v_4}(\downarrow\uparrow)$$

As $J_{\bar{v}_1 u_2}$ was not in the Formulary we can do the following. Call $p_1 \to p_2$ $p_2 \to p_1$. Then in the end we have to make $\theta^* \to \pi - \theta$



NOTE: By definition the angle θ is the angle of $\vec{p}_3$ with $\vec{p}_1$. θ^* is the angle of $\vec{p}_3$ with $\vec{p}_2$ (the electron) we set

$$\mathcal{M}(\underset{\bar\nu_e}{\uparrow}\underset{e^-}{\downarrow};\downarrow\uparrow) = -4\frac{G_F}{\sqrt{2}}\; J_{u_1 v_2}(\underset{e^-}{\downarrow}\underset{\bar\nu_e}{\uparrow}) \cdot J_{u_3 v_4}(\downarrow\uparrow)$$

$$= -4\frac{G_F}{\sqrt{2}} s\,(0,-1,i,0)\cdot(0,-\cos\theta^*, -i, \sin\theta^*)$$

$$= -4\frac{G_F}{\sqrt{2}} s\left[0-(\cos\theta^*+1)\right]$$

$$= \frac{4G_F}{\sqrt{2}} s\,(1+\cos\theta^*)$$

$$= \frac{4G_F}{\sqrt{2}} s\,(1-\cos\theta)$$

then

$$\langle|M|^2\rangle = \frac{1}{2}\,|M(\uparrow\downarrow;\downarrow\uparrow)|^2 = 4\,G_F^2 s^2 (1-\cos\theta)^2$$

d) we have

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\langle|M|^2\rangle = \frac{G_F^2 s}{16\pi^2}(1-\cos\theta)^2$$

e) $$\sigma = \int d\Omega\,\frac{d\sigma}{d\Omega} = 2\pi\,\frac{G_F^2 s}{16\pi^2}\underbrace{\int_{-1}^{1} dx\,(1-x)^2}_{\frac{8}{3}} = \frac{G_F^2 s}{3\pi}$$

If $\sqrt{s} \geq M_W$ we have to substitute in the propagator

$$\frac{1}{-M_W^2} \rightarrow \frac{1}{s - M_W^2 + i M_W \Gamma_W}$$

therefore we should correct with a factor of

$$\left| \frac{-M_W^2}{s - M_W^2 + i M_W \Gamma_W} \right|^2 = \frac{M_W^4}{(s-M_W^2)^2 + M_W^2 \Gamma_W^2}$$

As it is a s-chanel there are no angle θ ...

$$\sigma = \frac{G_F^2 s}{3\pi} \frac{M_W^4}{(s - M_W^2)^2 + M_W^2 \Gamma_W^2} \xrightarrow[\sqrt{s} \gg M_W]{} \frac{G_F^2 M_W^4}{3\pi s}$$

and decreases with the energy for large values of $\sqrt{s}$