TÉCNICO LISBOA

2^{nd} test

December 20th 2016: 18h00

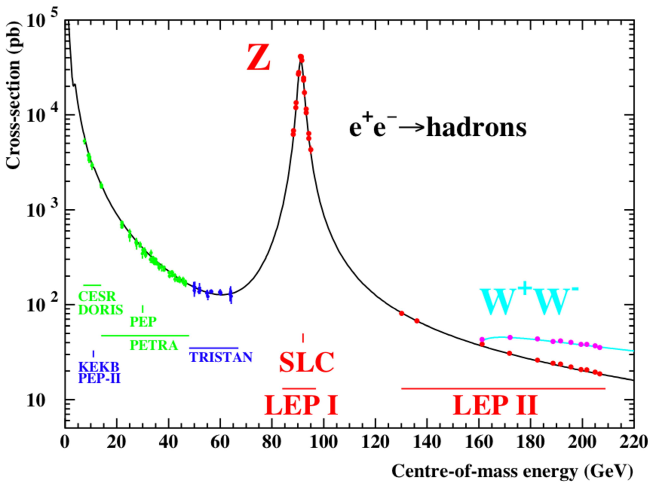
Duration of the test: 1h30

Mestrado em Eng. Física Tecnológica (MEFT)
Particle Physics
1^{st} semester of 2016-17

Prof. Jorge Romão
Prof. Mário Pimenta
Prof. Ruben Conceição

- The allowed elements for consult during the test are:
  - the PDG (Particle Data Book)
  - one single A4 page with formulas.
- Carefully justify all your answers.
- The test has 3 questions (2 pages).

The total cross-section of $e^+e^- \to hadrons$ is shown as a function of the centre of mass energy of the collision in following figure:



1. **[9 val]** Consider the process $e^- + e^+ \to \tau^- + \tau^+$ at the CM energy $\sqrt{s} = 20$ GeV. Neglect the masses of the leptons.

a) Taking in account the figure, what is the dominant diagram at these energies? Draw the corresponding Feynman diagram.

b) Considering the conditions of a) write the amplitude $\mathcal{M}$.

c) Evaluate the spin averaged squared amplitude $\langle |\mathcal{M}|^2 \rangle$.

d) For this process evaluate the differential cross section $d\sigma/d\Omega$ in the CM frame as a function of the square of the energy in the CM frame, $s = (p_1 + p_2)^2$.

e) Evaluate the total cross section $\sigma(e^- + e^+ \to \tau^{\cdot} + \tau^+)$ at this energy. Show that

$$\sigma(e^- + e^+ \to \text{hadrons}) = \beta\, \sigma(e^- + e^+ \to \tau^- + \tau^+)$$

Determine β and compare numerically with the figure (where the cross section is given in pb).

2. [9 val] Consider now the same process $e^- + e^+ \to \tau^- + \tau^+$ at the first phase of LEP, where the CM energy was tuned to be $\sqrt{s} = M_Z$. As before neglect the masses of the leptons.

a) Taking in account the figure, what is the dominant diagram at these energies? Draw the corresponding Feynman diagram.

b) Considering the conditions of a) write the amplitude $\mathcal{M}$. Discuss all the approximations you have done.

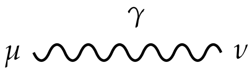
c) Evaluate the spin averaged squared amplitude $\langle|\mathcal{M}|^2\rangle$. It is simpler to leave the results in terms of $g_L^f = g_V^f + g_A^f$, and $g_R^f = g_V^f - g_A^f$, for $f = e, \tau$.

d) For this process evaluate the differential cross section $d\sigma/d\Omega$ in the CM frame as a function of the square of the energy in the CM frame, $s = (p_1 + p_2)^2$.

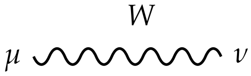
e) Evaluate the total cross section at these energies. Make a rough estimative of the increase of the cross section at $\sqrt{s} = M_Z$ in comparison with what it would be without this diagram and compare with the figure.

3. [2 val] Comment the following statement: *Experimentally the semi-leptonic decay of the τ (where one of the τ decays leptonically and the other hadronically) is one of the decay forms with less background.*

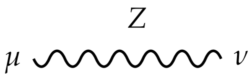
Propagators



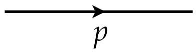
$$-i\frac{g_{\mu\nu}}{k^2} \tag{1}$$



$$-i\frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{M_W^2}}{k^2 - M_W^2 + i\, M_W \Gamma_W} \tag{2}$$



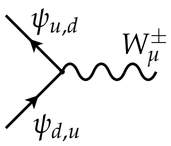
$$-i\frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{M_Z^2}}{k^2 - M_Z^2 + i\, M_Z \Gamma_Z} \tag{3}$$



$$\frac{i(\not{p} + m_f)}{p^2 - m_f^2} \tag{4}$$

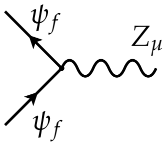
Vertices

Charged Current

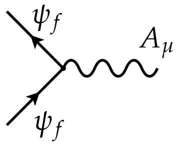


$$-i\frac{g}{\sqrt{2}}\gamma_\mu \frac{1-\gamma_5}{2} \tag{5}$$

Neutral Current



$$-i\frac{g}{\cos\theta_W}\gamma_\mu \left(g_V^f - g_A^f \gamma_5\right)$$



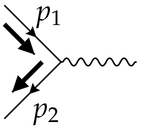
$$-ieQ_f\gamma_\mu \tag{6}$$

where

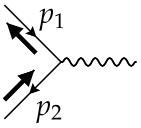
$$g_V^f = \frac{1}{2}T_f^3 - Q_f \sin^2\theta_W, \quad g_A^f = \frac{1}{2}T_f^3\,. \tag{7}$$

Results for the Helicity Currents

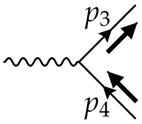
s-channel



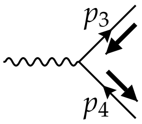
$$J_{u_1 v_2}(\uparrow,\downarrow) = \sqrt{s}\,(0,-1,-i,0) \tag{8}$$



$$J_{u_1 v_2}(\downarrow,\uparrow) = \sqrt{s}\,(0,-1,i,0) \tag{9}$$

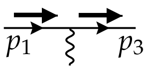


$$J_{u_3 v_4}(\uparrow,\downarrow) = \sqrt{s}\,(0,-\cos\theta,i,\sin\theta) \tag{10}$$

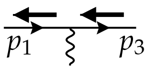


$$J_{u_3 v_4}(\downarrow,\uparrow) = \sqrt{s}\,(0,-\cos\theta,-i,\sin\theta) \tag{11}$$

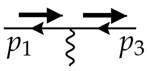
t-channel



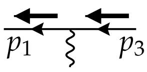
$$J_{u_1 u_3}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{12}$$



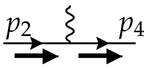
$$J_{u_1 u_3}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{13}$$



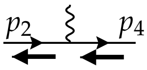
$$J_{v_1 v_3}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{14}$$



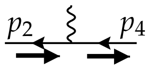
$$J_{v_1 v_3}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{15}$$



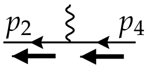
$$J_{u_2 u_4}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{16}$$



$$J_{u_2 u_4}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{17}$$



$$J_{v_2 v_4}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{18}$$



$$J_{v_2 v_4}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{19}$$