TÉCNICO LISBOA

1^{st} test

January 16^{th} 2018: 15h00

Duration of the test: 1h30

Duration of the exam: 3h00

Mestrado em Eng. Física Tecnológica (MEFT)
Particle Physics
1^{st} semester of 2018-19

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- The allowed elements for consult during the test are:
  - the PDG (Particle Data Book)
  - one single A4 page with formulas.
- Carefully justify all your answers.
- The exam has 5 questions (2 pages) plus a formulary.

1^{st} test

With the discovery of the Higgs boson, the search for physics beyond the Standard Model become increasingly important. One important test was performed by the LHCb collaboration[1] using the B_s^0 decay into $\mu^+\,\mu^-$, a channel with a high sensitivity to possible hidden sectors.

1. [5 val] The Large Hadron Collider beauty (LHCb) is a single-arm forward spectrometer designed to study particles containing b or c quarks. One of its main physics goals is to measure the branching ratio of $B_s^0 \to \mu^-\mu^+$.

a) Knowing that the search of this decay was done with a total integrated luminosity of $4.4\,\text{fb}^{-1}$ and that the production cross-section of particles with at least one b quark in pp collisions is $\sim 100\,\mu b$, compute the number of produced b-particles.

b) Compute the mean free path of a B_s^0 produced in the LAB with a momentum of $100\,\text{GeV/c}$.

c) Consider now the decay of the B_s^0 meson with energy $100\,\text{GeV}$ into muons ($B_s^0 \to \mu^+\mu^-$):

   i) What is the minimum and maximum energy that the muons can have in the B_s^0 reference frame?

   ii) Compute that minimum and maximum angle between the two muons in the LAB reference frame.

   iii) What is the minimum energy that the B_s^0 needs to have in the LAB to produce a muon of $8\,\text{GeV}$?

[1] the first report of this measurement was done LHCb and the CMS.

2. **[3 val]** Verify, from the point of view of quantum numbers, if the following reactions are possible and if not explain why.

a) $p\,p \to p\,e^+\gamma$

b) $p\,p \to p\,n\,\gamma$

c) $p\,p \to p\,n\,e^+\nu_e$

d) $p\,p \to \pi^+\pi^-K^+$

e) $p\,p \to p\,p\,K^0$

f) $p\,p \to p\,p\,K^+K^-$

3. **[2 val]** LHCb is a multi-detector experiment to be able to identify the different kind of particles that emerge from the decays of b-hadrons. In particular, it has two Ring-Imaging Cherenkov (RICH) detectors which are important in the discrimination of pions from Kaons. Assume that a π^+ and a K^+ each with 1 GeV/c:

a) What is the minimum (or maximum) medium refractive index to see Cherenkov photons from the pion but not from the Kaon?

b) Compute the produced Cherenkov light ratio between pions and Kaons. Assume that the particles travel the same distance and that the medium refractive index of the RHIC detector is $n = 1.2$.

2^{nd} *test*

4. **[7 val]** Consider the following interaction possible in proton-proton collisions at the LHC:

$$q(\hat{p}_1) + \bar{q}(\hat{p}_2) \to \mu^-(p_3) + \mu^+(p_4)$$

in the center of mass of the $q, \bar{q}$ pair, where $\hat{p}_i$ is the momenta carried by the each quark q. Neglect the masses of all fermions. For simplicity consider only up and down quarks while solving the following exercises.

a) Draw the corresponding first order Feynman diagram(s).

b) Considering the conditions of the problem write the amplitude $\mathcal{M}$ in its simplest form. Explain all the approximations.

Consider for the next questions that $\hat{s} = (\hat{p}_1 + \hat{p}_2)^2 = (M_Z)^2$. Present the results in terms of the general coupling constants.

c) In the above situation evaluate the spin averaged squared amplitude $\langle|\mathcal{M}|^2\rangle$.

d) For this process evaluate the differential cross section $d\sigma/d\Omega$ in the CM frame (of the elementary process) as a function of the square of the energy in the CM frame, $\hat{s} = (\hat{p}_1 + \hat{p}_2)^2$ and scattering angle, $\hat{\theta}$, for this elementary process.

e) Evaluate the total elementary cross-section for $\sqrt{\hat{s}} = M_Z$.

f) How could the proton-proton total cross-section for this process be computed? (no calculation is needed).

5. **[3 val]** Consider now the decay $B_s^0 \to \mu^-\mu^+$.

a) What is the quark content of the B_s^0 meson?

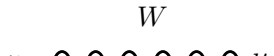
b) What is the lowest order diagram(s) for this decay? (note that the lowest order has one loop)

c) How could one distinguish the above B_s^0 decay from a Z^0 decaying into muons?

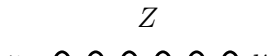
Formulary

Propagators

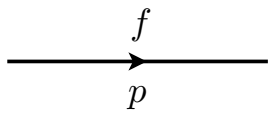
$$-i\frac{g_{\mu\nu}}{k^2} \tag{1}$$



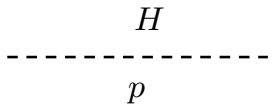
$$-i\frac{g_{\mu\nu}-\frac{k_\mu k_\nu}{M_W^2}}{k^2-M_W^2+i\,M_W\Gamma_W} \tag{2}$$



$$-i\frac{g_{\mu\nu}-\frac{k_\mu k_\nu}{M_Z^2}}{k^2-M_Z^2+i\,M_Z\Gamma_Z} \tag{3}$$



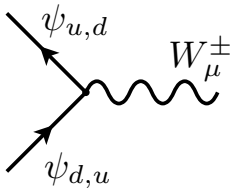
$$\frac{i(\not{p}+m_f)}{p^2-m_f^2} \tag{4}$$



$$\frac{i}{p^2-M_H^2+i\,M_H\Gamma_H} \tag{5}$$

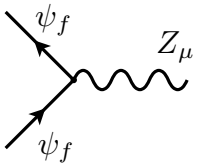
Vertices

Charged Current

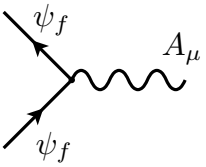


$$-i\frac{g}{\sqrt{2}}\gamma_\mu\frac{1-\gamma_5}{2} \tag{6}$$

Neutral Current



$$-i\frac{g}{\cos\theta_W}\gamma_\mu\left(g_V^f-g_A^f\gamma_5\right)$$

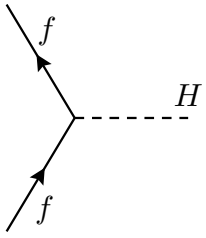


$$-ieQ_f\gamma_\mu$$

where

$$g_V^f=\frac{1}{2}T_f^3-Q_f\sin^2\theta_W,\quad g_A^f=\frac{1}{2}T_f^3\,. \tag{7}$$

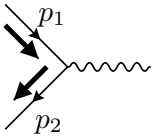
Higgs Interactions with fermions



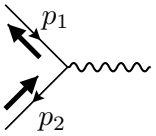
$$-i\,\frac{g}{2}\,\frac{m_f}{M_W}\equiv -i\,g_H^f \tag{8}$$

Results for the Helicity Vector Currents

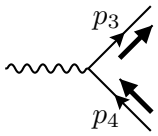
s-channel



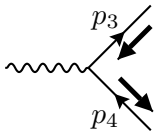
$$J_{u_1 v_2}(\uparrow,\downarrow) = \sqrt{s}\,(0,-1,-i,0) \tag{9}$$



$$J_{u_1 v_2}(\downarrow,\uparrow) = \sqrt{s}\,(0,-1,i,0) \tag{10}$$

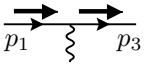


$$J_{u_3 v_4}(\uparrow,\downarrow) = \sqrt{s}\,(0,-\cos\theta,i,\sin\theta) \tag{11}$$

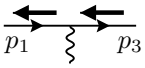


$$J_{u_3 v_4}(\downarrow,\uparrow) = \sqrt{s}\,(0,-\cos\theta,-i,\sin\theta) \tag{12}$$

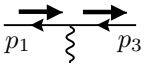
t-channel



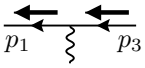
$$J_{u_1 u_3}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{13}$$



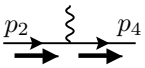
$$J_{u_1 u_3}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{14}$$



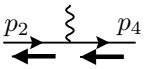
$$J_{v_1 v_3}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{15}$$



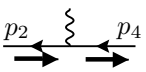
$$J_{v_1 v_3}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{16}$$



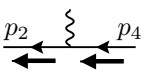
$$J_{u_2 u_4}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{17}$$



$$J_{u_2 u_4}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{18}$$



$$J_{v_2 v_4}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{19}$$



$$J_{v_2 v_4}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{20}$$