1st Exam 2018 / 2019

1- a)

$$N = \mathcal{L}_{int}\,\sigma = 4{,}4 \times 10^{11} \text{ b-particles produced}$$

b) $x = vt = \beta c t = \beta c \gamma \tau = \frac{p}{E}\frac{E}{m} c\tau \Leftrightarrow$

$$x = \frac{p}{m} c\tau = 8{,}4 \text{ mm}$$

$c\tau = 452{,}7\ \mu m$

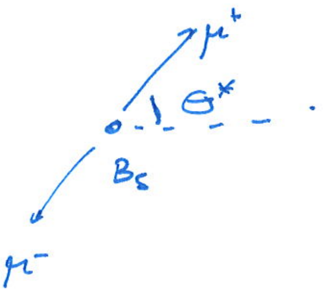
$m_{B_s^0} = 5{,}367$ GeV

c)

i) LAB

$B_s^0 \longrightarrow E = 100$ GeV

CM



In the CM-frame

$P_B = (m_B, \vec{0})$ since $P_B = P_{\mu^+} + P_{\mu^-}$

$P_{\mu^+} = (E^*_{\mu^+}, P^*_\mu)$ $m_B = 2E^*_\mu$

$P_{\mu^-} = (E^*_{\mu^-}, -P^*_\mu)$ $\Rightarrow E^*_\mu = \dfrac{m_B}{2} = 2{,}68$ GeV

ii) First one should start by checking if a muon can be inverted by the boost when it is emitted backwards,

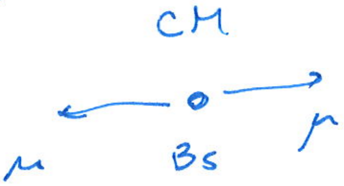
$$\beta_\mu^* = \frac{P_\mu^*}{E_\mu^*} = \frac{\sqrt{E_\mu^{*2} - m_\mu^2}}{E_\mu^*} = 0,9992$$

$$\beta_{CM} = \frac{P_B}{E_B} = \frac{\sqrt{E_B^2 - m_B^2}}{E_B} = 0,9986$$

$\Rightarrow \quad \beta_\mu^* > \beta_{CM}$ and the boost does not invert the direction of the muon

Hence,

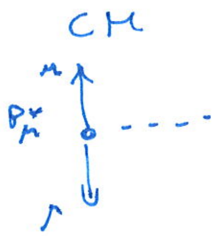
Θ_{MAX}:

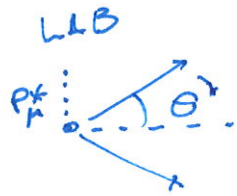


$$\begin{pmatrix} E \\ P_\ell \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} E^* \\ P_\ell^* \end{pmatrix}$$

$$\Theta_{max} = 180^\circ$$

Θ_{MIN}:





$$tg\,\theta' = \frac{P_\mu^*}{P_{\mu\ell}} = \frac{P_\mu^*}{\gamma\beta E_\mu^* + \underbrace{\gamma P_\mu^* \cos\theta^*}_{=0}} = \frac{P_\mu^*}{E_\mu^* \frac{E_B}{m_B} \frac{P_B}{E_B}} \quad (*)$$

$$\theta' = arctg\left(\frac{P_\mu^*}{E_\mu^*} \frac{m_B}{P_B}\right)$$

$$\Rightarrow \quad \theta = 2\theta' = 0,1 \text{ rad} = 6,15^\circ$$

1- c)

The minimum energy of B_s^0 will occur when the μ is emitted along the boost direction.

$$E_\mu = \gamma E_\mu^* + \gamma\beta P_\mu^* \underbrace{\cos\theta^*}_{=1}$$

E_μ^*, P_μ^* calculated in 1- c) i)

$$E_\mu = \frac{E_B}{m_B} E_\mu^* + \frac{E_B}{m_B}\frac{P_B}{E_B} P_\mu^* \Leftrightarrow$$

$$E_\mu m_B = E_B E_\mu^* + \sqrt{E_B^2 - m_B^2}\, P_\mu^*$$

$$\Rightarrow E_B = \frac{E_\mu^* E_\mu m_B \pm \sqrt{-E_\mu^{*2} m_B^2 P_\mu^{*2} + E_\mu^2 m_B^2 P_\mu^{*2} + m_B^2 P_\mu^{*4}}}{E_\mu^{*2} - P_\mu^{*2}}$$

$E_B = 8{,}9$ and 20615 GeV

$\Rightarrow$ The minimal energy is $E_B = 8{,}9$ GeV

2- a) Not possible, violates Baryon and lepton number

b) Not possible, violates Baryon number

c) Possible.

d) Not possible, violate Baryon number, charge and strangeness.

e) Not possible, violates strangeness

f) Possible.

3 -

a) $\cos\theta_c = \dfrac{1}{\beta n} \leq 1$ for Cherenkov emission

$$n_i \geq \frac{E}{p} = \frac{\sqrt{p^2+m_i^2}}{p}$$

$p = 1\ GeV/c$

$m_\pi = 0,140\ GeV$

$m_K = 0,494\ GeV$

$\Rightarrow n_\pi = 1,01$

$n_K = 1,12$ Hence, $n_\pi \leq n < n_K$

3 - b)

$$\frac{d^2N}{dE\,dx} \simeq 370\, Z^2 \operatorname{sen}^2\theta_c$$

for π, K $\quad Z^2 = 1$.

$\Rightarrow \quad N_\gamma \propto 370 \operatorname{sen}^2\theta_c$

$$\frac{N_\gamma^\pi}{N_\gamma^K} = \frac{\operatorname{sen}^2\theta_c^\pi}{\operatorname{sen}^2\theta_c^K} = \frac{1-\cos^2\theta_c^\pi}{1-\cos^2\theta_c^K}$$

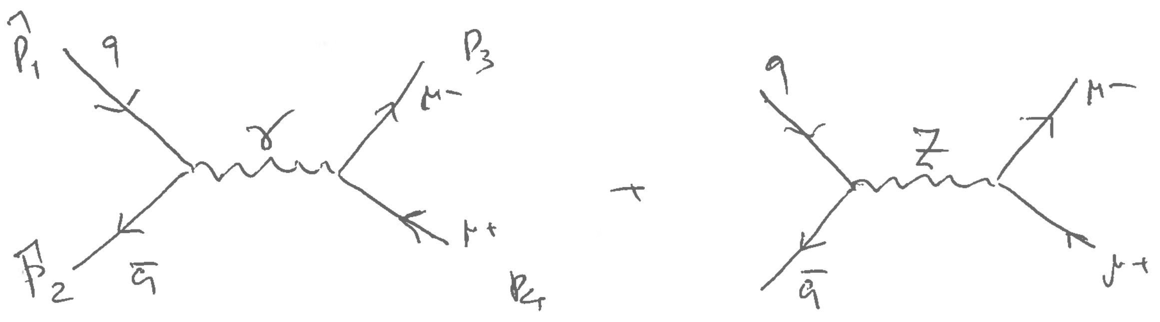
$$= \frac{1-\left(\frac{1}{\beta_\pi n}\right)^2}{1-\left(\frac{1}{\beta_K n}\right)^2} = 2,15$$

where $\beta_i = \dfrac{p}{\sqrt{p^2+m_i^2}}$

a) At the quark level the quark q can be,

$$q = u, d$$

As $Q[q + \bar{q}] = 0$ we have an s-channel neutral current. So the exchanged particle can be the γ or Z



1) 2)

b) we have

$$M = M_1 + M_2$$

where

$$i M_1 = (ie)^2 \, \bar{v}(\hat{p}_2)\gamma^\mu u(\hat{p}_1) \frac{-i g_{\mu\nu}}{(\hat{p}_1 + \hat{p}_2)^2} \bar{u}(p_3)\gamma^\nu v(p_4)$$

Defining $\hat{s} = (\hat{p}_1 + \hat{p}_2)^2$ we have

$$\boxed{M_1 = \frac{e^2}{\hat{s}} \bar{v}(\hat{p}_2)\gamma^\mu u(\hat{p}_1) \; \bar{u}(p_3)\gamma_\mu v(p_4)}$$

For M_2 we have $(q = \hat{p}_1 + \hat{p}_2)$ $(c_W = \cos\theta_W)$

$$i M_2 = \left(-i \frac{g}{c_W}\right)^2 \bar{v}(\hat{p}_2)\gamma^\mu (g_V^q - g_A^q \gamma_5) u(\hat{p}_1) \frac{-i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_Z^2}\right)}{\hat{s} - M_Z^2 + i M_Z \Gamma_Z} \bar{u}(p_3)\gamma^\nu (g_V^\mu - g_A^\mu \gamma_5) v(p_4)$$

approximations: Neglecting the fermion masses we can neglect the momenta in the Z propagator. In fact

$$\bar{v}(\hat{p}_2)\gamma^\mu(g_V^q - g_A^q\gamma_5)u(\hat{p}_1)q_\mu =$$

$$= \bar{v}(\hat{p}_2)\hat{\not{p}}_1(g_V^q - g_A^q\gamma_5)u(\hat{p}_1) + \bar{v}(\hat{p}_2)\hat{\not{p}}_2(g_V^q - g_A^q\gamma_5)u(\hat{p}_1)$$

$$= \bar{v}(\hat{p}_2)(g_V^q + g_A^q\gamma_5)\hat{\not{p}}_1 u(\hat{p}_1) + \bar{v}(\hat{p}_2)\hat{\not{p}}_2(g_V^q - g_A^q\gamma_5)u(\hat{p}_1)$$

$$= 0$$

where we used, for massless fermion the Dirac Equation

$$\hat{\not{p}}_1 u(\hat{p}_1) = 0 \quad , \quad \bar{v}(\hat{p}_2)\hat{\not{p}}_2 = 0$$

Simplifications

Defining
$$F_Z = \frac{M_Z^2}{\hat{s} - M_Z^2 + i M_Z\Gamma_Z}$$

and
$$\frac{g^2}{c_W^2 M_Z^2} = \frac{g^2}{M_W^2} = \frac{8G_F}{\sqrt{2}}$$

we can finally write

$$\boxed{M_2 = \frac{8G_F}{\sqrt{2}} F_Z\ \bar{v}(\hat{p}_2)\gamma^\mu(g_V^q - g_A^q\gamma_5)u(\hat{p}_1)\ \bar{u}(p_3)\gamma_\mu(g_V^\mu - g_A^\mu\gamma_5)v(p_4)}$$

In order to use the helicity amplitudes we can write M_2 in terms of P_L and P_R. In fact

$$g_V^f - g_A^f \gamma_5 = (g_V^f - g_A^f \gamma_5)(P_L + P_R)$$

$$= (g_V^f + g_A^f) P_L + (g_V^f - g_A^f) P_R$$

$$\equiv g_L^f P_L + g_R^f P_R$$

with

$$g_L^f = g_V^f + g_A^f \quad ; \quad g_R^f = g_V^f - g_A^f$$

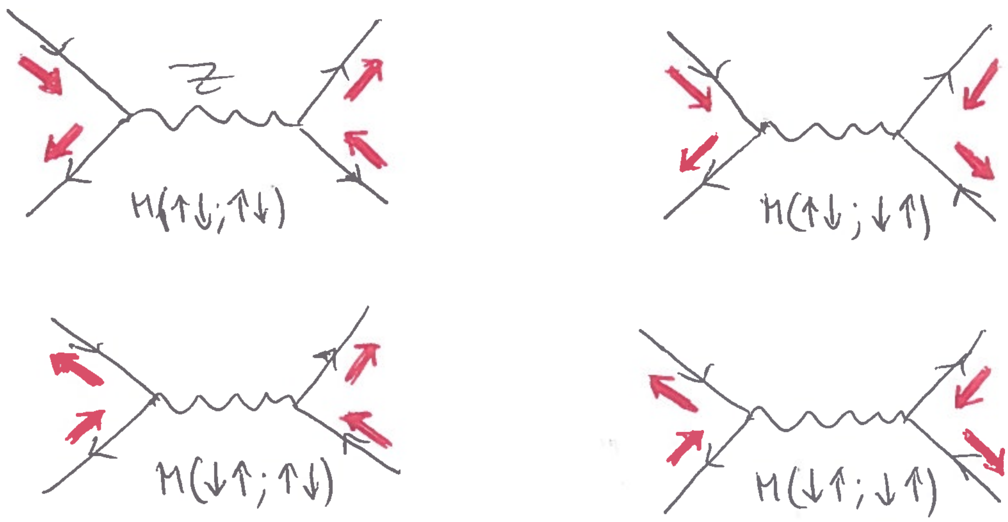
Therefore we get

$$\boxed{M_Z = \frac{8 G_F}{\sqrt{2}} F_Z \, \bar{v}(\hat{p}_2) \gamma^\mu (g_L^q P_L + g_R^q P_R) u(\hat{p}_1) \, \bar{u}(p_3) \gamma_\mu (g_L^\mu P_L + g_R^\mu P_R) v(p_4)}$$

c) for $\hat{s} = M_Z^2$ the dominant diagram is the Z exchange, that is,

$$M = \frac{8 G_F}{\sqrt{2}} F_Z \, \bar{v}(\hat{p}_2) \gamma^\mu (g_L^q P_L + g_R^q P_R) u(\hat{p}_1) \, \bar{u}(p_3) \gamma_\mu (g_L^\mu P_L + g_R^\mu P_R) v(p_4)$$

we have 4 helicity combinations $M_i(h_1 h_2 ; h_3 h_4)$



using

$$P_L u(\hat{p}_1) = u(\hat{p}_1, \downarrow) \quad ; \quad P_R u(\hat{p}_1) = u(\hat{p}_1 \uparrow)$$

$$P_L v(p_4) = v(p_4, \uparrow) \quad ; \quad P_R v(p_4) = v(p_4, \downarrow)$$

we set

$$M(\uparrow\downarrow;\uparrow\downarrow) = \frac{8G_F}{\sqrt{2}} F_Z g^q_R g^\mu_R \; J_{u_1 v_2}(\uparrow\downarrow) \cdot J_{u_3 v_4}(\uparrow\downarrow)$$

$$= \frac{8G_F}{\sqrt{2}} F_Z g^q_R g^\mu_R \, s \; (0,-1,-i,0)\cdot(0,-\cos\theta, i, \sin\theta)$$

$$= -\frac{8G_F}{\sqrt{2}} F_Z g^q_R g^\mu_R \, s\,(1+\cos\theta)$$

$$M(\uparrow\downarrow;\downarrow\uparrow) = \frac{8G_F}{\sqrt{2}} F_Z g^q_R g^\mu_L \; J_{u_1 v_2}(\uparrow\downarrow) \cdot J_{u_3 v_4}(\downarrow\uparrow)$$

$$= \frac{8G_F}{\sqrt{2}} F_Z g^q_R g^\mu_L \, s \; (0,-1,-i,0)\cdot(0,-\cos\theta,-i,\sin\theta)$$

$$= \frac{8G_F}{\sqrt{2}} F_Z g^q_R g^\mu_L \, s\,(1-\cos\theta)$$

$$M(\downarrow\uparrow;\uparrow\downarrow) = \frac{8G_F}{\sqrt{2}} F_Z g^q_L g^\mu_R \; J_{u_1 v_2}(\downarrow\uparrow) \cdot J_{u_3 v_4}(\uparrow\downarrow)$$

$$= \frac{8G_F}{\sqrt{2}} F_Z g^q_L g^\mu_R \, s \; (0,-1,i,0)\cdot(0,-\cos\theta, i, \sin\theta)$$

$$= \frac{8G_F}{\sqrt{2}} F_Z g^q_L g^\mu_R \, s\,(1-\cos\theta)$$

$$M(\downarrow\uparrow;\downarrow\uparrow) = \frac{8G_F}{\sqrt{2}} F_Z g^q_L g^\mu_L \; J_{u_1 v_1}(\downarrow\uparrow) \cdot J_{u_3 v_4}(\downarrow\uparrow)$$

$$= \frac{8G_F}{\sqrt{2}} F_Z g^q_L g^\mu_L \, s \; (0,-1,i,0)\cdot(0,-\cos\theta,-i,\sin\theta)$$

$$= -\frac{8G_F}{\sqrt{2}} F_Z g^q_L g^\mu_L \, s\,(1+\cos\theta)$$

Therefore

$$\langle |M|^2 \rangle = \frac{1}{4}\, 32\, G_F^2\, |F_Z|^2\, s^2 \Big[(1+\cos\theta)^2 \big[(g_R^q g_R^\mu)^2 + (g_L^q g_L^\mu)^2 \big] + (1-\cos\theta)^2 \big[(g_R^q g_L^\mu)^2 + (g_L^q g_R^\mu)^2 \big] \Big]$$

where

$$|F_Z|^2 = \frac{M_Z^4}{(\hat{s} - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \underset{\hat{s}=M_Z^2}{=} \frac{M_Z^2}{\Gamma_Z^2}$$

d) for massless particles

$$\frac{d\sigma_{q\bar{q}}}{d\Omega} = \frac{1}{64\pi^2 \hat{s}} \langle |M|^2 \rangle$$

$$= \frac{1}{8\pi^2} G_F^2 |F_Z|^2 \hat{s} \Big[(1+\cos\theta)^2 \big[(g_R^q g_R^\mu)^2 + (g_L^q g_L^\mu)^2 \big] + (1-\cos\theta)^2 \big[(g_R^q g_L^\mu)^2 + (g_L^q g_R^\mu)^2 \big] \Big]$$

e)

$$\sigma_{q\bar{q}}(\hat{s}) = \int d\Omega \frac{d\sigma}{d\Omega}$$

$$= 2\pi \int_{-1}^{1} d\cos\theta \frac{d\sigma}{d\Omega}$$

using

$$\int_{-1}^{1} d\cos\theta\, (1+\cos\theta)^2 = \int_{-1}^{1} dx\, (1+2x+x^2) = \frac{8}{3}$$

$$\int_{-1}^{1} d\cos\theta\, (1-\cos\theta)^2 = \int_{-1}^{1} dx\, (1-2x+x^2) = \frac{8}{3}$$

we get

$$\boxed{\sigma_{q\bar{q}}(\hat{s}) = \frac{2G_F^2|F_Z|^2\hat{s}}{3\pi}\left[(g_R^q g_R^\mu)^2 + (g_L^q g_L^\mu)^2 + (g_R^q g_L^\mu)^2 + (g_L^q g_R^\mu)^2\right]}$$

where

$$q = u, d.$$

at $\hat{s} = M_Z^2$ we get

$$\boxed{\sigma_{q\bar{q}}(M_Z^2) = \frac{2G_F^2 M_Z^4}{3\pi\,\Gamma_Z^2}\left[\ldots\ldots\right]}$$

f) First we have to evaluate

$$\sigma(pp \to \mu^- + \mu^+)$$

at $\hat{s} = M_Z^2$. If $f_q(x)$ and $f_{\bar{q}}(x)$ denote the probability of finding a quark or antiquark in the proton with momentum $\hat{p}_i = x p_i$ where p_i is the proton momentum, we have

$$\hat{s} = (\hat{p}_1 + \hat{p}_2)^2 = (x_1 p_1 + x_2 p_2)^2 =$$

$$= x_1^2 p_1^2 + x_2^2 p_2^2 + 2x_1 x_2\, p_1 \cdot p_2$$

$$= 2x_1 x_2\, p_1 \cdot p_2$$

where we neglect the mass. But neglecting the masses also

$$s = (p_1 + p_2) = 2\, p_1 \cdot p_2$$

therefore

$$\hat{s} = x_1 x_2 s$$

Then the total cross section $\sigma(pp \to \mu^+\mu^-)$ would be

$$\sigma_{pp\to\mu^-\mu^+}(s) = \int_0^1 dx_1 \int_0^1 dx_2 \Big[f_u(x_1)\, f_{\bar{u}}(x_2)\, \sigma_{u\bar{u}}(x_1 x_2 s)$$

$$+ f_d(x_1)\, f_{\bar{d}}(x_2)\, \sigma_{d\bar{d}}(x_1 x_2 s) \Big]$$

There is an important point here. In the experiment we control s (for instance $\sqrt{s} = 13\,\mathrm{TeV}$ at LHC) but not $\hat{s} = x_1 x_2 s$. So we can not use the simplified $\sigma_{q\bar{q}\to\mu^-\mu^-}(\hat{s} = M_Z^2)$ but the full form

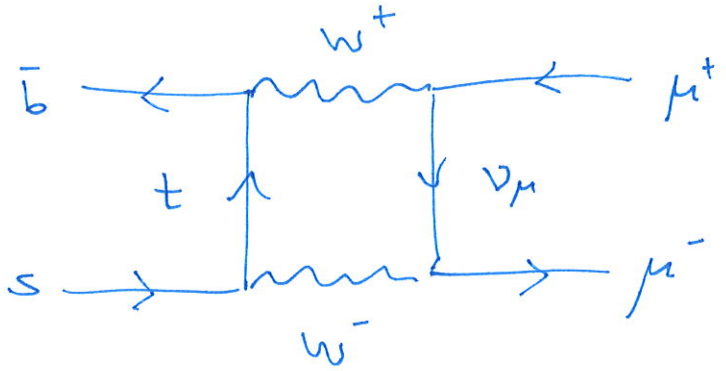
$$\sigma_{q\bar{q}}(\hat{s}) = \frac{2 G_F^2}{3\pi} \frac{\hat{s}\, M_Z^2}{(\hat{s} - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \Big[\cdots \Big]$$

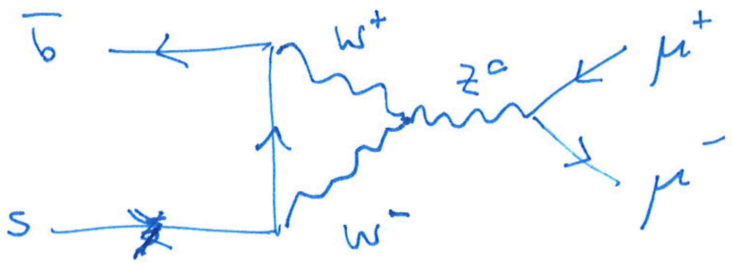
where $\hat{s} = x_1 x_2 s$.

5 -

a) $\bar{b}s$

b)





c) By computing the invariant mass which is a Lorentz invariant quantity

$$s = E^2 - P^2 = m^2_{B^0_s} \neq m^2_{Z^0}$$

From the μs:

$$s = (E_{\mu^+} + E_{\mu^-})^2 - (P_{\mu^+} + P_{\mu^-})^2 = m^2_{B^0_s}$$