1st test (2018/2019)

1- a) $$r_g/m = 3.3 \frac{(E/\text{GeV})\ (v_\perp/c)}{(|q|/e)\ (B/T)}$$

$E = 63 \times 10^{-3}$ GeV

$m_{e^+} = 511 \times 10^{-6}$ GeV

$B = 1{,}7$ T

$$(v_\perp/c) = \beta = \frac{p}{E} = \frac{\sqrt{E^2 - m_{e^+}^2}}{E}$$

$\Rightarrow r_g = 0{,}12$ m $= 12$ cm

b) electron: if it was an electron in the opposite direction it would look like the e^- gained energy (the curvature radius of the track above is smaller than the one below), when crossing the lead plate which is not possible.

proton: $$p[\text{GeV}] \sim 0{,}3\, q\, B\, r \text{, with } q\,[C],\ B\,[T],\ r\,[m]$$

Hence, the momentum of the proton for the same $r = 0{,}12$ and $B = 1{,}7$ T would be $p = 0{,}06$ GeV/c.

The loss by ionization (see PDG 2016, fig 33.2) would be huge which means that the proton would stop inside the lead plate.

1- c)

$$\Delta E_{loss} = E_2 - E_1 = 40\ MeV$$

$E_2 = 63\ MeV$

$E_1 = 23\ MeV$

since $E_{loss} = \left(\frac{\Delta E}{\Delta x}\right)\Delta x$

$$\Rightarrow \Delta x = E_{loss}/(\Delta E/\Delta x)$$

From fig 33.2 (PDG 2016), taking into account that the target is Pb (lead) and the particle is a positron

$$\beta\gamma = \frac{p}{E}\frac{E}{m} = \frac{p}{m} = \frac{\sqrt{E^2 - m^2}}{m}$$

$$\left.\begin{array}{l} \text{For } E_1 \longrightarrow \beta\gamma \simeq 45 \\ \quad E_2 \longrightarrow \beta\gamma \simeq 123 \end{array}\right\} \Rightarrow \left(\frac{dE}{dx}\right) \sim 1{,}6\ MeV/g\,cm^{-2}$$

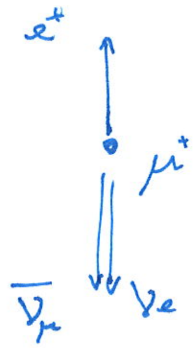
$$\Rightarrow \Delta x = 25\ g\,cm^{-2}$$

$$x = \frac{\Delta x\ [g/cm^2]}{\rho_{Pb}\ [g/cm^3]} = 2{,}2\ cm$$

with $\rho_{Pb} = 11{,}35\ g\,cm^{-3}$ (taken from pag. 338 of PDG 2016)

1 - d)

The most favorable scenario for a e^+ to be produced upwards is if the μ^+ is at rest, i.e. no boost.



We want the e^+ to take the maximum energy, so the best configuration of the decay is when ~~the~~ both the neutrinos travel in the opposite direction of the e^+.

$$P_e = \underbrace{P_{\bar{\nu}_\mu} + P_{\nu_e}}_{\hookrightarrow P_{\nu s}} \equiv P_{\nu s}$$

$$P_\mu = (m_\mu, \vec{0})$$
$$P_e = (E_e, \vec{P})$$
$$P_{\nu s} = (E_{\nu s}, -\vec{P})$$

$$P_\mu = P_e + P_{\nu s} \Leftrightarrow$$
$$(P_\mu - P_{\nu s})^2 = P_e^2$$

$$P_\mu^2 + P_{\nu s}^2 - 2P_\mu P_{\nu s} = P_e^2 \Leftrightarrow m_\mu^2 + 0 - 2m_\mu E_{\nu s} = m_e^2$$

$$E_{\nu s} = \frac{m_\mu^2 - m_e^2}{2m_\mu}$$ and since $E_{\nu s} = P_{\nu s}$

$$E_e = \sqrt{E_{\nu s}^2 + m_e^2} = 52,8 \text{ MeV}$$

Therefore the max energy that the positron can get is 52,8 MeV which is less than the measured 63 MeV.

1 - e) To know if it is possible to distinguish a e^+ from a p using this setup one has to compute the Cherenkov light production threshold.

$E = E_e = E_p = 63\ MeV$

$\cos(\theta_c) = \frac{1}{\beta n} \leq 1$ $\qquad n = 1,33$ (PDG2016, pag. 339)

$$\beta \geq \frac{1}{n} \Leftrightarrow \frac{p}{E} \geq \frac{1}{n} \Leftrightarrow \frac{\sqrt{E^2 - m^2}}{E} \geq \frac{1}{n}$$

$$E^2 - m^2 \geq \frac{E^2}{n^2} \Leftrightarrow E^2 - \frac{E^2}{n^2} \geq m^2 \Leftrightarrow E^2\left(1 - \frac{1}{n^2}\right) \geq m^2$$

$$E \geq \frac{m}{\sqrt{1 - \frac{1}{n^2}}} \Rightarrow E_{e^+}^{crit} = 0,7\ MeV \Rightarrow \exists\ \check{C}\ \text{light}$$

$$E_p^{crit} = 1,4\ GeV \Rightarrow \nexists\ \check{C}\ \text{light}$$

Therefore, the answer is yes.

2. $\pi^+ \longrightarrow \mu^+ \nu_\mu$; $E_\pi = 10\,GeV$

a) $c\tau = 7,8\,m$

$x = vt = \beta c t = \beta c \gamma \tau = \beta\gamma\, c\tau$

$$x = \frac{P}{E}\frac{E}{m}\, c\tau = \frac{P}{m}\, c\tau = \frac{\sqrt{E^2 - m^2}}{m}\, c\tau = 557\,m$$

b)

$\rho \sim 1,1\,kg\,m^{-3} = 1100\,g\,m^{-3}$

$\sigma \sim 200\,mb$

$1b - 10^{-28}\,m^2$

$200 \times 10^{-3} b - \sigma = 2 \times 10^{-29}\,m^2$

Atomasphere 80% N and 20% O them,

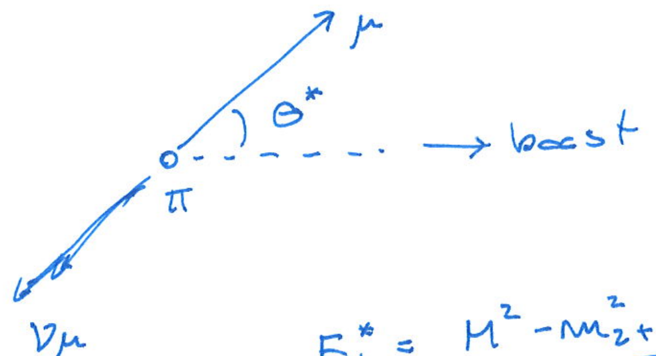
$A = 0,8 \times 14,007 + 0,2 \times 15,999 = 14,4\,g\,mol^{-1}$

$$\lambda = \frac{1}{\sigma n} = \frac{A}{\sigma \rho N_A} = 1087\,m$$

Hence, at this energy the pion is expected to decay before it interacts.

2-c)

CM



Two body decay
(similar to 1-d))

PDG 2016 - eq. 47.15

$$E_1^* = \frac{M^2 - m_2^2 + m_1^2}{2M} \quad \text{with} \quad \begin{aligned} M &= m_\pi \\ m_2 &= m_\nu = 0 \\ m_1 &= m_\mu \end{aligned}$$

$\Rightarrow E_\mu^* \simeq 110$ MeV

d) CM $\Rightarrow$ LAB
$\hookrightarrow$ apply boost (vel. of the pion)

$$\beta_{CM} = \frac{p_\pi}{E_\pi} \; ; \; \gamma_{CM} = \frac{E_\pi}{m_\pi} \; ; \; \beta_{CM} = \frac{\sqrt{E_\pi^2 - m_\pi^2}}{E_\pi}$$

$$\begin{pmatrix} E \\ p_l \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} E^* \\ p_l^* \end{pmatrix}$$

$$\Rightarrow E_\mu = \gamma E_\mu^* + \gamma\beta p_\mu^* \cos\theta^*$$

max: $\theta^* = 0°$; min: $\theta^* = 180°$

substituting:

$E_{min,\mu} \simeq 5{,}8$ GeV

$E_{max,\mu} \simeq 10$ GeV

3 - a) Not possible, violates leptonic number

b) Not possible, violates baryonic number

c) Possible.

d) Not possible, violates strangeness