2nd test / 1st exam

January 12$^{\text{th}}$ 2015: 8H00

Duration of the test: 1H30

Duration of the exam: 3H00

Mestrado em Eng. Física Tecnológica (MEFT)
Particle Physics
1st semester of 2014-15

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Prof. Jorge Romão
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- The allowed elements for consult during the exam are:
  - the PDG (Particle Data Book)
  - one single A4 page with formulas.
- Clearly identify all pages of the exam.
- The exam has 4 questions (3 pages).

1st test

1. **[5 val]** The luminosity at the Large Electron-Positron Collider (LEP) was determined by measuring the elastic e^+e^- scattering (Bhabha scattering) as its cross-section at low angles is well known from QED. In fact, assuming small polar angles, the Bhabha scattering cross-section integrated between a polar angle θ_{min} and θ_{max} is given at first order by,

$$\sigma \approx \frac{1040\,\text{nb}}{s\ [\text{GeV}^2]}\left(\frac{1}{\theta^2_{max} - \theta^2_{min}}\right)$$

a) Determine the luminosity of a run of LEP knowing that this run last 4 hours, the number of identified Bhabha scattering events was 1200 in the polar range of $\theta \in [29; 185]$ mrad. Take into account a detection efficiency of 95% and a background of 10% at $\sqrt{s} = m_Z$.

b) At LEP the electrons and positrons where detected in an electromagnetic calorimeter.

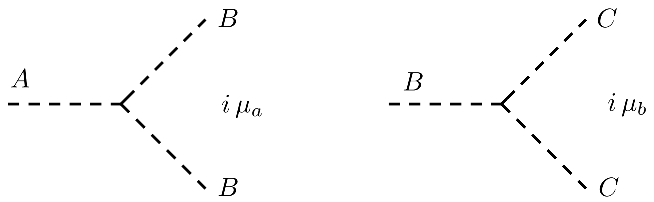
   i. Describe qualitatively the development of the electromagnetic shower in the calorimeter.

   ii. Discuss which type of material the calorimeter should be made in order to minimize the total length.

c) The effective energy of the interaction can be changed by the radiation of a photon by the particles of the beam (initial radiation), which is peaked at very small angles. Supposing that the measured e^+e^- has the following transverse momenta $p^t_1 = p^t_2 = 5\,\text{GeV}$ and the radiated photon is collinear with the beam and has an energy of 10 GeV, determine the effective energy of the interaction of the electron and positron in the centre of mass, $\sqrt{s_{e^+e^-}}$. Consider that the beam was tuned for a $\sqrt{s} = m_Z$.

d) Draw the QED Feynman diagrams at low order for the elastic e^+e^- scattering and discuss why the Bhabha scattering measurements at LEP are done at very small polar angle.

2.[5 val] Consider a theory with three neutral scalars A, B, C (spin 0). They interact in such way that the Feynman rules for the interactions are



where μ_a e μ_b have dimensions of mass ($\hbar = c = 1$). The masses are such that $m_A = m_B > 2m_C$.

a) Find the total width $\Gamma(B \to C + C)$ as a function of the masses of the particles.

b) For the process $A + B \to C + C$ draw the diagram(s) that contribute in lowest order and write down the amplitude(s).

c) Find the differential cross section $d\sigma/d\Omega$ in the CM frame as a function of the Madelstam variable $s = (p_1 + p_2)^2$ which represents the square of the energy in the CM frame.

d) In the limit $\sqrt{s} \gg m_A, m_B, m_C$ neglect the masses and evaluate the total cross section in the CM frame.

e) Verify that you could have obtained this result, up to a numerical factor, by dimensional analysis.

2^{nd} *test*

3.[7 val] At the Large Electron-Positron collider (LEP) one of the studied reactions was:

$$e^-(p_1) + e^+(p_2) \to \mu^-(p_3) + \mu^+(p_4)$$

a) Draw the Feynman diagrams that contribute in the Standard Model, in lowest order, to this process.

b) Consider now that $\sqrt{s} \simeq m_Z$. Write down the amplitude for the most important diagram. Neglect all fermion masses.

c) Show that if one neglects the fermions masses one can also neglect the terms proportional to the momenta in the numerator of the Feynman propagator for the massive gauge bosons. Write down the simplified amplitude.

d) Show that the electron current obey the relation

$$\overline{v}(p_2)\gamma^\mu(g_V^e - g_A^e\gamma_5)u(p_1) = g_L^e\,\overline{v}(p_2)\gamma^\mu P_L u(p_1) + g_R^e\,\overline{v}(p_2)\gamma^\mu P_R u(p_1)$$

where

$$P_L = \frac{1}{2}(1 - \gamma_5), \quad P_R = \frac{1}{2}(1 + \gamma_5), \quad g_L^e = g_V^e + g_A^e, \quad g_R^e = g_V^e - g_A^e$$

with similar expressions for the μ current.

e) Use the previous result to write down the non-vanishing helicity amplitudes for this process with the previous assumptions.

f) Calculate the spin averaged amplitude $\langle |\mathcal{M}|^2 \rangle$ and write the expression for the differential cross section $d\sigma/d\Omega$ in the CM frame.

g) In the LEP experiment an important quantity that was measured was the *Forward-Backward* asymmetry defined by

$$A_{FB} = \frac{\int_0^{\pi/2} d\Omega \, d\sigma/d\Omega - \int_{\pi/2}^{\pi} d\Omega \, d\sigma/d\Omega}{\int_0^{\pi/2} d\Omega \, d\sigma/d\Omega + \int_{\pi/2}^{\pi} d\Omega \, d\sigma/d\Omega}$$

Explain the name and why it might be useful. The following expressions are useful:

$$(g_L^e)^2(g_L^\mu)^2 + (g_R^e)^2(g_R^\mu)^2 + (g_L^e)^2(g_R^\mu)^2 + (g_R^e)^2(g_L^\mu)^2 = 4\left((g_V^e)^2 + (g_A^e)^2\right)\left((g_V^\mu)^2 + (g_A^\mu)^2\right)$$
$$(g_L^e)^2(g_L^\mu)^2 + (g_R^e)^2(g_R^\mu)^2 - (g_L^e)^2(g_R^\mu)^2 - (g_R^e)^2(g_L^\mu)^2 = 16\, g_V^e g_A^e g_V^\mu g_A^\mu$$

4.[3 val] At LEP final states with electromagnetic and strong interactions were detected. Taking the $\sqrt{s} = m_Z$:

a) Compute at the first order the ratio

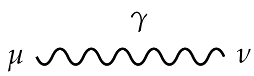
$$\frac{\sigma(e^-e^+ \to \nu\overline{\nu})}{\sigma(e^-e^+ \to \mu^+\mu^-)}$$

b) Compute at the first order the ratio

$$\frac{\sigma(e^-e^+ \to hadrons)}{\sigma(e^-e^+ \to \mu^+\mu^-)}$$

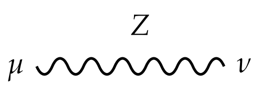
c) Discuss how was possible to determine in LEP that the number of light neutrino family is 2.9840 ± 0.0082.

Propagators

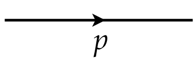


$$-i\frac{g_{\mu\nu}}{k^2} \quad (1)$$

$$-i\frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{M_W^2}}{k^2 - M_W^2} \quad (2)$$



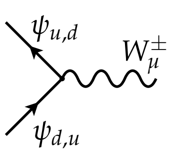
$$-i\frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{M_Z^2}}{k^2 - M_Z^2} \quad (3)$$



$$\frac{i(\not{p} + m_f)}{p^2 - m_f^2} \quad (4)$$

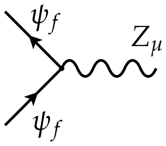
Vertices

Charged Current

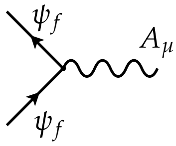


$$-i\frac{g}{\sqrt{2}}\gamma_\mu \frac{1-\gamma_5}{2} \quad (5)$$

Neutral Current



$$-i\frac{g}{\cos\theta_W}\gamma_\mu\left(g_V^f - g_A^f\gamma_5\right)$$



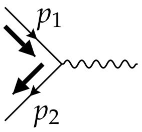
$$-ieQ_f\gamma_\mu \tag{6}$$

where

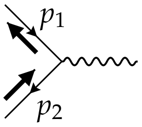
$$g_V^f = \frac{1}{2}T_f^3 - Q_f\sin^2\theta_W, \quad g_A^f = \frac{1}{2}T_f^3\,. \tag{7}$$

Results for the Helicity Currents

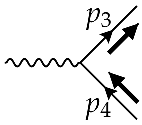
s-channel



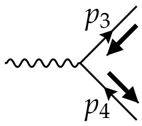
$$J_{u_1v_2}(\uparrow,\downarrow) = \sqrt{s}\,(0,-1,-i,0) \tag{8}$$



$$J_{u_1v_2}(\downarrow,\uparrow) = \sqrt{s}\,(0,-1,i,0) \tag{9}$$

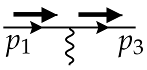


$$J_{u_3v_4}(\uparrow,\downarrow) = \sqrt{s}\,(0,-\cos\theta,i,\sin\theta) \tag{10}$$

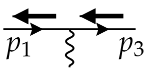


$$J_{u_3v_4}(\downarrow,\uparrow) = \sqrt{s}\,(0,-\cos\theta,-i,\sin\theta) \tag{11}$$

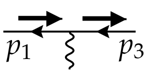
t-channel



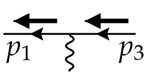
$$J_{u_1u_3}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{12}$$



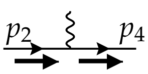
$$J_{u_1u_3}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{13}$$



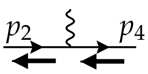
$$J_{v_1v_3}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{14}$$



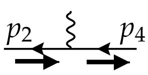
$$J_{v_1v_3}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{15}$$



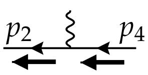
$$J_{u_2u_4}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{16}$$



$$J_{u_2u_4}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{17}$$



$$J_{v_2v_4}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{18}$$



$$J_{v_2v_4}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{19}$$