2nd Exam (2016-2017)

1- a) $\mathcal{L} = \dfrac{N_1 N_2 N_b f}{A}$

$\mathcal{L} = 10^{31}\, cm^{-2} s^{-1}$
$A = 0{,}02\, mm^2$
$N_b = 1$
$f = 30\, Hz$

$N \equiv N_1 = N_2$

$\Rightarrow \quad N = \sqrt{\dfrac{\mathcal{L} A}{N_b f}} = 8{,}2 \times 10^{12}$ muons

b) $N_{decay} = \displaystyle\int_0^{60} \frac{dN}{dt}\, dt \; ; \quad \frac{dN}{dt} = -\lambda t$

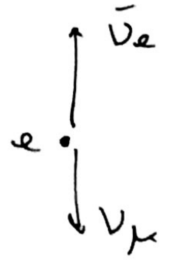
$\lambda = \dfrac{1}{t_{decay}} = \dfrac{1}{\gamma \tau_\mu} = \dfrac{c\, m_\mu}{E_\mu (c\tau_\mu)}$ s^{-1}

$\Rightarrow N_{decay} = N_0 (1 - e^{-\lambda t}) \simeq N_0$ so all μ decay and the collider has to be refilled with $N_0 = 8{,}2 \times 10^{12}$

c) $P_e = P_{\nu_e} + P_{\nu_\mu}$ and $E_e = \sqrt{P_e^2 + m_e^2}$ and $P_\nu = E_\nu$

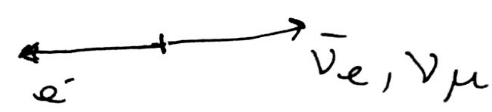
Then in the CM,

Min:



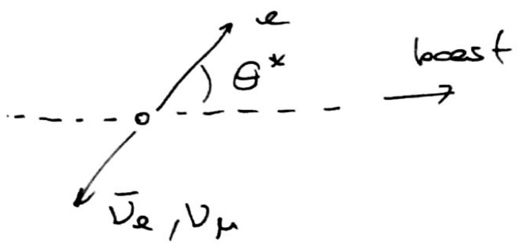
e^- is at rest and
$|P_{\nu_e}| = -|P_{\nu_\mu}|$

Max:



$|P_e| = -|P_{\bar{\nu}_e} + P_{\nu_\mu}|$

d) Take the max energy for the electron configuration for the electron in the CM,



$$P_\mu = P_e + P_{\nu_e} + P_{\nu_\mu}$$

$$(P_\mu - P_e)^2 = (P_{\nu_e} + P_{\nu_\mu})^2$$

As
$$\left.\begin{aligned} P_\mu &= (m_\mu, \vec{0}) \\ P_e &= (E_e^*, P_e^*) \\ P_\nu &= (E_\nu^*, E_\nu^*) \end{aligned}\right\} \Rightarrow \quad m_\mu^2 + m_e^2 - 2m_\mu E_e^* = 0$$

$$E_e^* = \frac{m_\mu^2 + m_e^2}{2m_\mu} = 52{,}8 \text{ MeV}$$

$$\gamma = \frac{P_\mu}{m_\mu} \; ; \quad \beta = \frac{P_\mu}{E_\mu} = \frac{\sqrt{E_\mu^2 - m_\mu^2}}{E_\mu}$$

$$\begin{pmatrix} E \\ P \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} E^* \\ P^* \end{pmatrix} \Rightarrow E_e^{lab} \equiv E_e = \gamma (E_e^* + \beta P_e^* \cos\theta^*)$$

$$E_e^{min} = \gamma (E_e^* - \beta P_e^*) = 1{,}5 \text{ MeV}$$

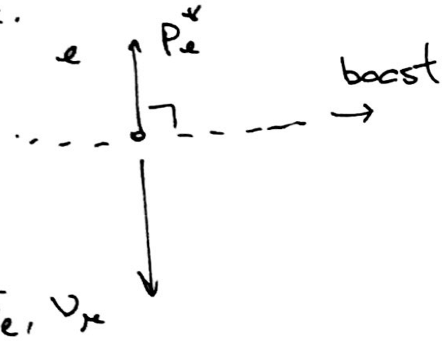
$$E_e^{max} = \gamma (E_e^* + \beta P_e^*) = 62{,}8 \text{ GeV}$$

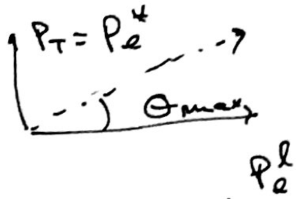
e)

CM

LAB

θ_{max}:



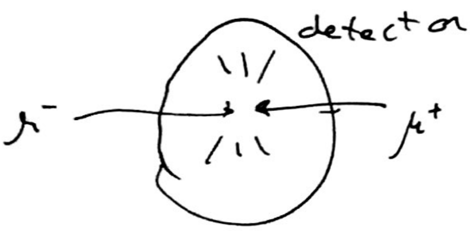


$$\theta_{max} = \operatorname{arctg}\left(\frac{p_e^*}{p_e^l}\right)$$

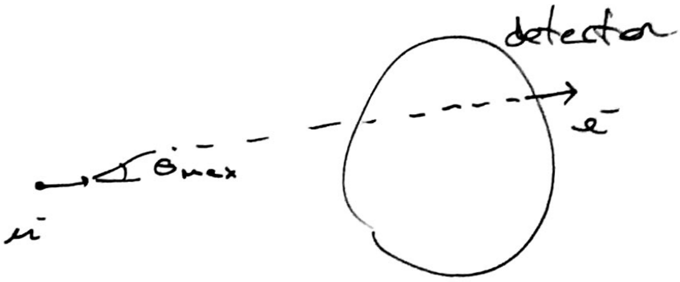
$$p_e^l = \gamma\beta E_e^* + \underbrace{\gamma\, p_e^* \cos\theta^*}_{=0} = \gamma\beta E_e^* = 31{,}4 \text{ GeV}/c$$

$\Rightarrow$ $\theta_{max} = 0{,}1°$

$\Rightarrow$ There should be a cut in the forward region of the detector



(normal interaction)



(muon decay

f) Let;

LN ≡ leptonic number
BN ≡ baryonic number
Ch ≡ charge

All (✓) ⇒ Allowed
One (×) ⇒ Forbidden

(i) $\mu^+ \rightarrow e^+ \nu_e \nu_\mu$

LN: $-1 \rightarrow +1$ ×
BN: $0 \rightarrow 0$ ✓
Ch: $+1 \rightarrow +1$ ✓

(ii) $\mu^+ \mu^- \rightarrow e^+ e^-$

LN: $0 \rightarrow 0$ ✓
BN: $0 \rightarrow 0$ ✓
Ch: $0 \rightarrow 0$ ✓

(iii) $p n \rightarrow n n \mu^+$

LN: $0 \rightarrow -1$ ×
BN: $2 \rightarrow 2$ ✓
ch: $+1 \rightarrow +1$ ✓

(iv) $\mu^+ \mu^- \rightarrow \nu_\mu \bar{\nu}_\mu$

LN: $0 \rightarrow 0$ ✓
BN: $0 \rightarrow 0$ ✓
Ch: $0 \rightarrow 0$ ✓

g) $\mathcal{L} = 10^{31}\ cm^{-2} s^{-1}$, $\sigma_H \sim 40 pb$; $\Delta t = 3600 \times 24 \times 365$

$N_{Higgs} = \mathcal{L} \sigma \Delta t$

$\sigma_H = 40 \times 10^{-12} \times 10^{-24} = 4 \times 10^{-35}\ cm^2$

$N_{Higgs} = 12615$ / year

h) The b quark is a very massive quark, so any particle containing a b-quark will decay very near the interaction point (IP).

Therefore, one strategy would be to have a detector sensitive to charge particles near the interaction point (micro-vertex detector) and looking for traces that emerge from the IP that decay into new traces rapidly.

2° EXAM 2/2/2017 (2nd test) 1

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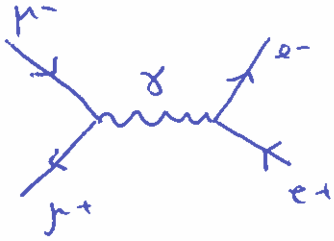
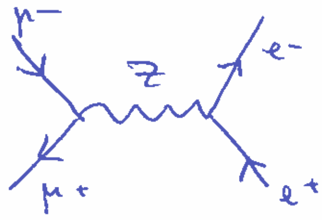
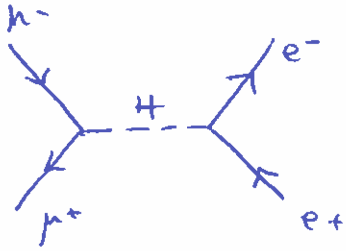
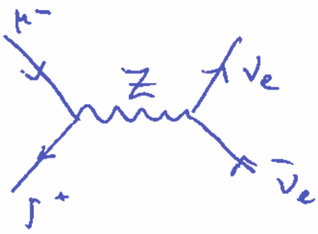
a) $\boxed{f = e^-}$



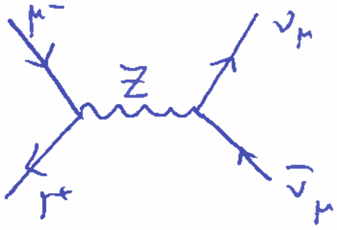


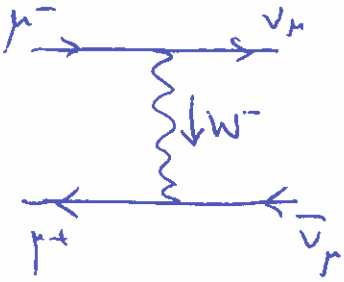


$\boxed{f = \nu_e}$

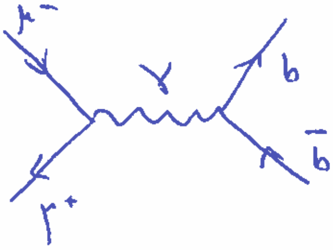


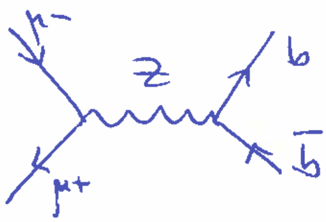
$\boxed{f = \nu_\mu}$

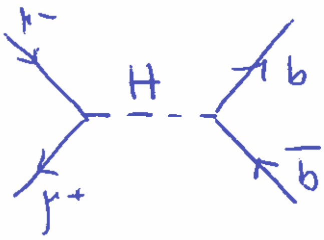




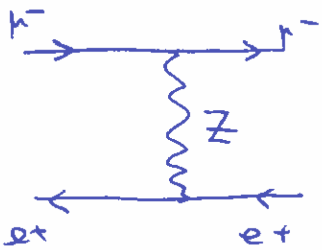
$\boxed{f = b}$

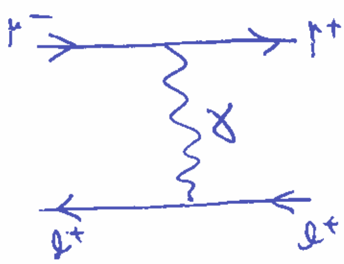


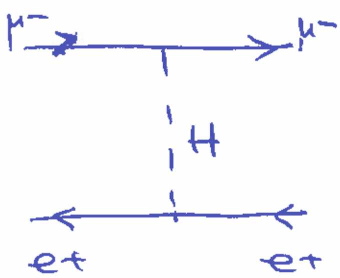




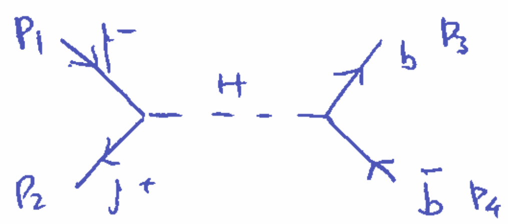
b) $\mu^- + e^+ \to \mu^- + e^+$







c) For $\sqrt{s} \simeq M_H$ only the diagram with the s-channel Higgs exchange is relevant



the amplitude is

$$i\mathcal{M} = (-i g_H^\mu)(-i g_H^b)\, \bar{v}(p_2) u(p_1) \frac{i}{s - M_H^2 + i M_H \Gamma_H} \bar{u}(p_3) v(p_4)$$

Escrevendo

$$\bar{v}(p_2,h_2) u(p_1,h_1) \equiv J_{u_1 v_2}(h_1,h_2)$$

$$\bar{u}(p_3,h_3) v(p_4,h_4) \equiv J_{u_3 v_4}(h_3,h_4)$$

Obtemos

$$\boxed{\mathcal{M}(h_1 h_2; h_3 h_4) = -g_H^\mu g_H^b \frac{1}{s - M_H^2 + i M_H \Gamma_H} J_{u_1 v_2}(h_1,h_2)\, J_{u_3 v_4}(h_3,h_4)}$$

Com $\quad g_H^\mu = \frac{1}{2} g \frac{m_\mu}{M_W} \quad ; \quad g_H^b = \frac{1}{2} g \frac{m_b}{M_W}$

d) Neglecting the fermion masses we only two non-vanishing amplitudes for the initial and final states. There are therefore 4 non-vanishing helicity amplitudes

$$\mathcal{M}(\uparrow\uparrow;\uparrow\uparrow) = \mathcal{M}(\downarrow\downarrow;\downarrow\downarrow) = -\, g_H^\mu g_H^b \frac{s}{s - M_H^2 + i M_H \Gamma_H}$$

$$\mathcal{M}(\uparrow\uparrow;\downarrow\downarrow) = \mathcal{M}(\downarrow\downarrow;\uparrow\uparrow) = +\, g_H^\mu g_H^b \frac{s}{s - M_H^2 + i M_H \Gamma_H}$$

e) using d) we write

$$\langle|\mathcal{M}|^2\rangle = \frac{1}{4}\Big[|\mathcal{M}(\uparrow\uparrow;\uparrow\uparrow)|^2 + |\mathcal{M}(\downarrow\downarrow;\downarrow\downarrow)|^2 + |\mathcal{M}(\uparrow\uparrow;\downarrow\downarrow)|^2 + |\mathcal{M}(\downarrow\downarrow;\uparrow\uparrow)|^2 \Big]$$

$$\boxed{\langle|\mathcal{M}|^2\rangle = (g_H^\mu)^2 (g_H^b)^2 \frac{s^2}{(s-M_H^2)^2 + M_H^2 \Gamma_H^2}}$$

in the CM, neglecting masses we have

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \langle|\mathcal{M}|^2\rangle$$

$$= \frac{(g_H^\mu)^2 (g_H^b)^2}{64\pi^2} \frac{s}{(s-M_H^2)^2 + M_H^2 \Gamma_H^2}$$

f) As nothing depends on the scattering angle

$$\sigma = 4\pi \times \frac{d\sigma}{d\Omega} = \frac{(g_H^\mu)^2 (g_H^b)^2}{16\pi} \frac{s}{(s-M_H^2)^2 + M_H^2 \Gamma_t^2}$$

for $\sqrt{s} = M_H$ we get

$$\boxed{\sigma = \frac{(g_H^{\mu})^2 (g_H^b)^2}{16\pi} \frac{1}{\Gamma_H^2}}$$

numerically

$$g_H^{\mu} = \frac{1}{2} g \frac{m_\mu}{M_W} = 4.15 \times 10^{-4} \qquad m_\mu = 105.6\ \text{MeV}; M_W = 80.385\ \text{GeV}$$

$$g_H^{b} = \frac{1}{2} g \frac{m_b}{M_W} = 0.0163 \qquad (m_b = 4.15\ \text{GeV})$$

$$\Gamma_H = 4\ \text{MeV}$$

therefore

$$\sigma = 22.14\ \text{pb}$$

g) For the electron everything would be similar, therefore

$$\sigma(e^+e^- \to b\bar{b}) = \frac{(g_H^e)^2 (g_H^b)^2}{16\pi} \frac{1}{\Gamma_H^2}$$

and at $\sqrt{s} = M_H$

$$R_1 = \frac{(g_H^{\mu})^2}{(g_H^e)^2} = \frac{m_\mu^2}{m_e^2} = 4.275 \times 10^4$$

h) At $\sqrt{s} = M_Z$, the dominant diagram is the s-channel Z exchange. As the Z couples in the same way to μ^- and to e^- we will have

$R_2 = 1$

i) A "Higgs factory" means that we want to produce as many H as possible. This is achieved through the Higgs s-channel resonance at $\sqrt{s} = M_H$. As we have seen the cross section is inversely proportional to $\frac{1}{\Gamma_H^2}$ making it very large. However the Higgs couples proportional to mass and therefore the coupling to μ^- is $\left(\frac{m_\mu}{m_e}\right)$ times larger that to e^-. As we we have seen in g)

$R_1 = 4.275 \times 10^4$

making the μ^- collider much better to produce the Higgs boson.