2^{nd} Exam

February 1st 2018: 8h00

Duration of the exam: 3h00

Mestrado em Eng. Física Tecnológica (MEFT)

Particle Physics

1^{st} semester of 2017-18

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- The allowed elements for consult during the exam are:
  - the PDG (Particle Data Book)
  - one single A4 page with formulas.
- Carefully justify all your answers.
- The exam has 6 questions (3 pages) plus a formulary.

The meson constituted by a charm and an anti-charm quarks, was discovered in 1974 independently by two research groups, one at the Brookhaven National Laboratory (BNL) and other at the Stanford Linear Accelerator Center (SLAC). For that reason the meson was called J/ψ, the combination of the name given by each group.

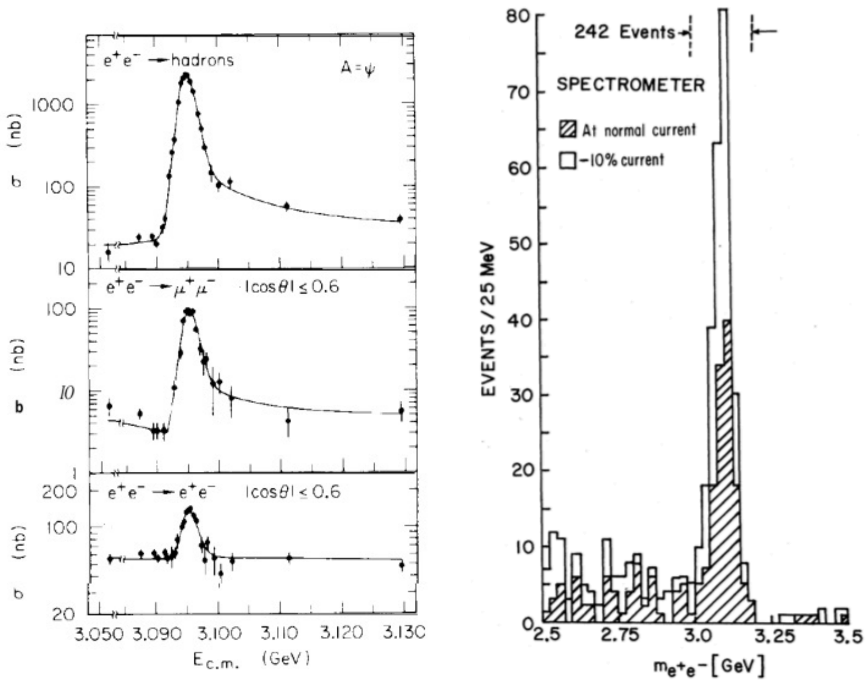


Figure 1: Original plots for the discovery of charm at SLAC (left) and at BNL (right).

1. **[5 val]** At BNL, the observation of J/ψ was achieved by colliding protons into a Beryllium target.

a) The identification of the J/ψ was done through the identification of a pair of electron-positron. Discuss how to evaluate the meson mass in the LAB framework.

b) One of the techniques used to distinguish electrons from protons is to measure the time of flight of the particle (ToF). Considering an electron and a proton, both with an energy of 1.5 GeV, and knowing that the detector time resolution is ~ 1 ns, determine the minimum distance necessary to distinguish the two types of particles.

c) Consider a J/ψ with a momentum of 10 GeV/c in the LAB. What is the mean distance that the meson can travel before it decays into an e^-e^+ pair? What would be the distance if the J/ψ decays instead into $\rho\pi$?

d) Taking into account the following decay, $J/\psi \to e^+e^-$, and knowing that the meson has an energy of 5 GeV, compute the minimum and maximum angle that the leptons can make with the J/ψ flight direction.

e) In the conditions of the previous question, what is now the minimum and maximum energy that the electron/positron can take in the LAB framework.

2. **[1.5 val]** At SLAC, the J/ψ meson was observed by colliding electrons with positrons.

a) What would be the energy of the positron necessary to produce a J/ψ if the electron is at rest?

b) As stated previously, the time of flight of a particle can be used to identify its mass. Describe another possible experimental apparatus to distinguish different types of highly relativistic particles, namely: electrons, protons and muons.

3. **[2.5 val]** Verify, from the point of view of quantum numbers, if the following reactions involving the J/ψ meson are possible and if not explain why.

a) $e^+ e^- \to J/\psi\, \mu^- \bar{\nu}_\mu$

b) $e^+ e^- \to J/\psi\, n$

c) $e^+ e^- \to J/\psi\, K^0$

d) $e^+ e^- \to J/\psi\, \pi^0$

e) $e^+ e^- \to J/\psi\, \nu_e$

4. **[1 val]** Figure 1 (left) was produced accepting events whose collision sub-products are within $|\cos\theta| \leq 0.6$, being θ the angle done with the beam direction. How would the plot for $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to e^+e^-$ change if all events were accepted independently of the angle θ?

5. **[5 val]** At SLAC/SPEAR e^- and e^+ beams were collided with a center of mass energy in the interval [2, 8] GeV. Consider first the process,

$$e^-(p_1) + e^+(p_2) \to \mu^-(p_3) + \mu^+(p_4)$$

in the center of mass, for energies in that range. Neglect the masses of all fermions.

a) Draw the corresponding Feynman diagram(s). Identify the most important process at these energies.

b) Considering the conditions of the problem write the amplitude $\mathcal{M}$ in its simplest form for the dominant diagram. Explain all the approximations.

c) Evaluate the spin averaged squared amplitude $\langle|\mathcal{M}|^2\rangle$.

d) For this process, evaluate the differential cross section $d\sigma/d\Omega$ in the CM frame as a function of the square of the energy in the CM frame, s, and scattering angle, θ, defined as the angle between the outgoing μ^- and the incoming e^-.

e) Find the expression for the total cross section. Evaluate it at $\sqrt{s} = 3.13$ GeV in nb.

f) To compare with the experimental result in Fig. 1 (middle panel of the left hand side) you have to make a cut in the scattering angle θ, as it was done in the experiment. Redo your calculation of the cross section for $|\cos\theta| \leq 0.6$ and compare with the results of the figure. To make this comparison consider the point that is well away from the resonance with $\sqrt{s} = 3.13$ GeV. To extract the values from the plots consider the curve that fits the points and not the data points. Comment on your results.

6. [5 val] In the experiment at SLAC, Burton Richter and collaborators, were scanning the center of mass energy $\sqrt{s} \in [2, 8]$ GeV in intervals of 200 MeV. The purpose was to measure the so-called ratio R defined by

$$R = \frac{\sigma(e^- + e^+ \to \text{hadrons})}{\sigma(e^- + e^+ \to \mu^- + \mu^+)}$$

They found that this ratio *jumped* around $\sqrt{s} \sim 3$ GeV. After this they did a finer scan as shown in Fig. 1.

a) Determine this ratio for $\sqrt{s} = 2$ GeV.

b) Consider now that $\sqrt{s} \sim 3.5$ GeV. Determine the ratio at this energy.

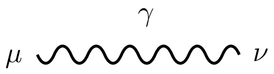
c) Calculate now R at $\sqrt{s} = 20$ GeV.

d) Make an approximate plot of the ratio R in the interval $\sqrt{s} \in [2, 20]$ GeV.

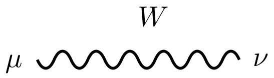
e) Use Fig. 1 to extract the value of R below and above the J/ψ resonance. Do not forget that you have to scale up the values for $\sigma(e^- + e^+ \to \mu^- + \mu^+)$ in the middle panel of Fig. 1 (left) to obtain the cross section for all angles. To extract the values from the plots consider the curve that fits the points and not the data points. Compare with your results.

Formulary

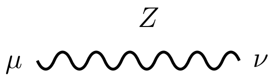
Propagators



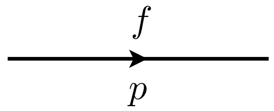
$$-i\frac{g_{\mu\nu}}{k^2} \tag{1}$$



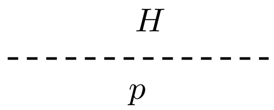
$$-i\frac{g_{\mu\nu}-\frac{k_\mu k_\nu}{M_W^2}}{k^2-M_W^2+i\,M_W\Gamma_W} \tag{2}$$



$$-i\frac{g_{\mu\nu}-\frac{k_\mu k_\nu}{M_Z^2}}{k^2-M_Z^2+i\,M_Z\Gamma_Z} \tag{3}$$



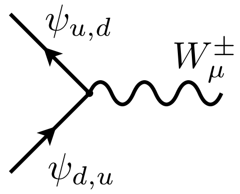
$$\frac{i(\not{p}+m_f)}{p^2-m_f^2} \tag{4}$$



$$\frac{i}{p^2-M_H^2+i\,M_H\Gamma_H} \tag{5}$$

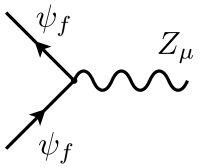
Vertices

Charged Current

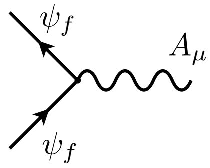


$$-i\frac{g}{\sqrt{2}}\gamma_\mu\frac{1-\gamma_5}{2} \tag{6}$$

Neutral Current



$$-i\frac{g}{\cos\theta_W}\gamma_\mu\left(g_V^f-g_A^f\gamma_5\right)$$

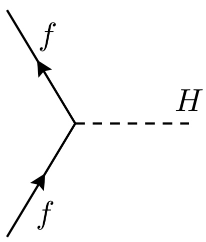


$$-ieQ_f\gamma_\mu$$

where

$$g_V^f=\frac{1}{2}T_f^3-Q_f\sin^2\theta_W,\quad g_A^f=\frac{1}{2}T_f^3\,. \tag{7}$$

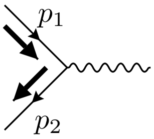
Higgs Interactions with fermions



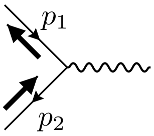
$$-i\,\frac{g}{2}\,\frac{m_f}{M_W}\equiv -i\,g_H^f \tag{8}$$

Results for the Helicity Vector Currents

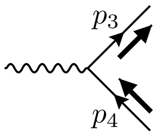
s-channel



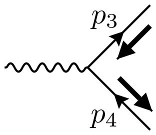
$$J_{u_1 v_2}(\uparrow,\downarrow) = \sqrt{s}\,(0,-1,-i,0) \tag{9}$$



$$J_{u_1 v_2}(\downarrow,\uparrow) = \sqrt{s}\,(0,-1,i,0) \tag{10}$$

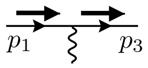


$$J_{u_3 v_4}(\uparrow,\downarrow) = \sqrt{s}\,(0,-\cos\theta,i,\sin\theta) \tag{11}$$

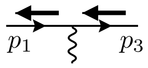


$$J_{u_3 v_4}(\downarrow,\uparrow) = \sqrt{s}\,(0,-\cos\theta,-i,\sin\theta) \tag{12}$$

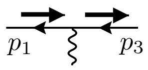
t-channel



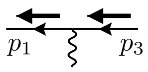
$$J_{u_1 u_3}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{13}$$



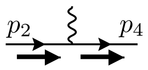
$$J_{u_1 u_3}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{14}$$



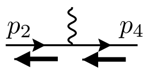
$$J_{v_1 v_3}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{15}$$



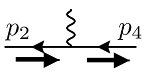
$$J_{v_1 v_3}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{16}$$



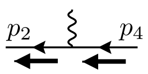
$$J_{u_2 u_4}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{17}$$



$$J_{u_2 u_4}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{18}$$



$$J_{v_2 v_4}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{19}$$



$$J_{v_2 v_4}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{20}$$