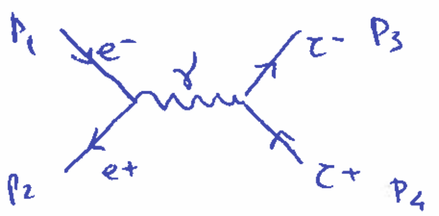
Particle Physics - Test 2 - 20/12/2016

1

a) At low energies ($\sqrt{s} \ll M_Z$) the dominant diagram is the photon exchange (QED). The Diagram is



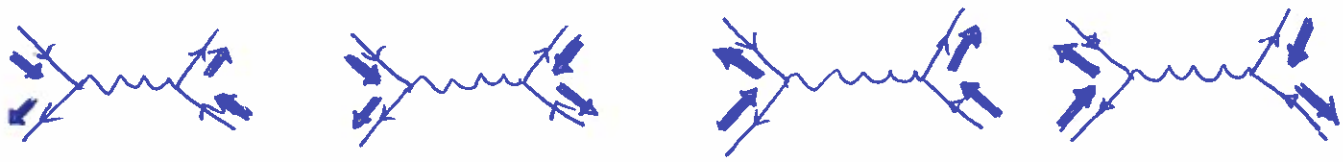
b) $$iM = \bar{v}(p_2)\gamma^\mu(ie)u(p_1)\,\frac{-ig_{\mu\nu}}{(p_1+p_2)^2}\,\bar{u}(p_3)\gamma^\nu(ie)v(p_4)$$

or

$$M = \frac{e^2}{s}\,\bar{v}(p_2)\gamma^\mu u(p_1)\,\bar{u}(p_3)\gamma_\mu v(p_4)$$

where $s = (p_1+p_2)^2$

c) For massless fermions there are 4 non-vanishing helicity amplitudes combinations



$M(\uparrow\downarrow;\uparrow\downarrow)$ $M(\uparrow\downarrow;\downarrow\uparrow)$ $M(\downarrow\uparrow;\uparrow\downarrow)$ $M(\downarrow\uparrow;\downarrow\uparrow)$

We get

$$M(\uparrow\downarrow;\uparrow\downarrow) = \frac{e^2}{s}\,J_{u_1v_2}(\uparrow\downarrow)\cdot J_{u_3v_4}(\uparrow\downarrow)$$

$$= \frac{e^2}{s}\,s\,(0,-1,-i,0)\cdot(0,-\cos\theta,i,\sin\theta)$$

$$= e^2\left[0-(\cos\theta+1)\right] = -e^2(1+\cos\theta)$$

c)

$$M(\uparrow\downarrow;\downarrow\uparrow) = \frac{e^2}{s}\, J_{u_1 v_2}(\uparrow\downarrow)\cdot J_{u_3 v_4}(\downarrow\uparrow)$$

$$= \frac{e^2}{s}\, s(0,-1,-i,0)\cdot(0,-\cos\theta,-i,\sin\theta)$$

$$= e^2\left[0-(\cos\theta-1)\right] = e^2(1-\cos\theta)$$

$$M(\downarrow\uparrow;\uparrow\downarrow) = \frac{e^2}{s}\, J_{u_1 v_2}(\downarrow\uparrow)\cdot J_{u_3 v_4}(\uparrow\downarrow)$$

$$= \frac{e^2}{s}\, s(0,-1,i,0)\cdot(0,-\cos\theta,i,\sin\theta)$$

$$= e^2(1-\cos\theta)$$

$$M(\downarrow\uparrow;\downarrow\uparrow) = \frac{e^2}{s}\, J_{u_1 v_2}(\downarrow\uparrow)\cdot J_{u_3 v_4}(\downarrow\uparrow)$$

$$= \frac{e^2}{s}\, s(0,-1,i,0)\cdot(0,-\cos\theta,-i,\sin\theta)$$

$$= -e^2(1+\cos\theta)$$

Therefore

$$\langle|M|^2\rangle = \frac{1}{4}\left[\,|M(\uparrow\downarrow;\uparrow\downarrow)|^2+|M(\uparrow\downarrow;\downarrow\uparrow)|^2+|M(\downarrow\uparrow;\uparrow\downarrow)|^2+|M(\downarrow\uparrow;\downarrow\uparrow)|^2\right]$$

$$= \frac{e^4}{4}\left[2(1+\cos\theta)^2+2(1-\cos\theta)^2\right]$$

$$= e^4\left[1+\cos^2\theta\right]$$

$$= (4\pi\alpha)^2\left[1+\cos^2\theta\right]$$

d) $$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \langle |M|^2 \rangle$$ for massless particles in the CM.

therefore

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4 s} (1 + \cos^2\theta)$$

e) $$\sigma = \int d\Omega \, \frac{d\sigma}{d\Omega} =$$

$$= \frac{\alpha^2}{4 s} \times 2\pi \times \underbrace{\int_{-1}^{1} d\cos\theta \, (1 + \cos^2\theta)}_{8/3}$$

and then

$$\boxed{\sigma = \frac{4\pi\alpha^2}{3 s}}$$

Let us call

$$\sigma_0 = \sigma(e^+ e^- \to \tau^- \tau^+)$$

Then

$$\sigma(e^+e^- \to \text{hadrons}) = \sum_i \sigma(e^+e^- \to \bar{q}_i q_i) \quad \text{with} \quad i = u, d, c, s, b$$

$$= \Big[\underset{\substack{\uparrow \\ d,s,b}}{3} \times \underset{\substack{\uparrow \\ \text{color}}}{3} \times \underset{\substack{\uparrow \\ \text{charge}}}{\left(-\tfrac{1}{3}\right)^2} + \underset{\substack{\uparrow \\ u,c}}{2} \times \underset{\substack{\uparrow \\ \text{color}}}{3} \times \underset{\substack{\uparrow \\ \text{charge}}}{\left(\tfrac{2}{3}\right)^2} \Big] \sigma_0$$

$$= \frac{11}{3} \sigma_0 \quad \Rightarrow \quad \beta = \frac{11}{3}$$

Numerically at $\sqrt{s}=20$

$$\sigma_0 = 217.25 \text{ pb}$$

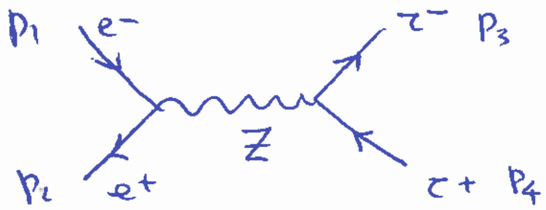
and then

$$\sigma(e^+e^- \to \text{hadrons}) = \frac{11}{3}\sigma_0 \simeq 796.6 \text{ pb}$$

in agreement with the figure.

2

a) At $\sqrt{s} = M_Z$ the dominant diagram is the Z exchange diagram. The feynman diagram is



$$q = p_1 + p_2 = p_3 + p_4$$

b) The amplitude is

$$iM = \left(-i\frac{g}{\cos\theta_W}\right)^2 \bar{v}(p_2)\gamma^\mu(g_V^e - g_A^e\gamma_5)u(p_1)\frac{-i\left[g_{\mu\nu} - \frac{q_\mu q_\nu}{M_Z^2}\right]}{q^2 - M_Z^2 + iM_Z\Gamma_Z}\bar{u}(p_3)\gamma^\nu(g_V^\tau - g_A^\tau\gamma_5)v(p_4)$$

<u>Approximations</u>: neglecting the fermions mass we can neglect the momentum terms in the numerator of the propagator of the Z. In fact

$$\bar{v}(p_2)\not{q}(g_V^e - g_A^e\gamma_5)u(p_1)$$
$$= \bar{v}(p_2)\not{p}_2(g_V^e - g_A^e\gamma_5)u(p_1) + \bar{v}(p_2)\not{p}_1(g_V^e - g_A^e\gamma_5)u(p_1)$$
$$= \bar{v}(p_2)\not{p}_2(g_V^e - g_A^e\gamma_5)u(p_1) + \bar{v}(p_2)(g_V^e + g_A^e\gamma_5)\not{p}_1 u(p_1) = 0$$

Because For massless fermions, the Dirac equation gives

$$\not{p}\,u(p) = 0 \quad \text{and} \quad \bar{v}(p)\not{p} = 0$$

then

$$\mathcal{M} = \left(\frac{g}{M_Z \cos\theta_W}\right)^2 F_Z \, \bar{v}(p_2)\gamma^\mu(g_V^e - g_A^e\gamma_5)u(p_1)\, \bar{u}(p_3)\gamma_\mu(g_V^\tau - g_A^\tau\gamma_5)v(p_4)$$

$$= \frac{8G_F}{\sqrt{2}} F_Z \, \bar{v}(p_2)\gamma^\mu(g_V^e - g_A^e\gamma_5)u(p_1)\, \bar{u}(p_3)\gamma_\mu(g_V^\tau - g_A^\tau\gamma_5)v(p_4)$$

where

$$F_Z \equiv \frac{M_Z^2}{s - M_Z^2 + iM_Z\Gamma_Z} \quad ; \quad \frac{g^2}{8M_W} = \frac{G_F}{\sqrt{2}} \; ; \; M_W = M_Z\cos\theta_W$$

To continue it is better to write the couplings in terms of the L and R parts. We use

$$g_V^f - g_A^f\gamma_5 = (g_V^f - g_A^f\gamma_5)(P_L + P_R)$$

$$= (g_V^f + g_A^f)P_L + (g_V^f - g_A^f)P_R$$

$$\equiv g_L^f P_L + g_R^f P_R$$

Then the final amplitude is

$$\boxed{\mathcal{M} = \frac{8G_F}{\sqrt{2}} F_Z \, \bar{v}(p_2)\gamma^\mu(g_L^e P_L + g_R^e P_R)u(p_1)\, \bar{u}(p_3)\gamma_\mu(g_L^\tau P_L + g_R^\tau P_R)v(p_4)}$$

c) As in problem 1) we only have 4 non-zero helicity amplitudes. To find them we use

$$P_L u(p) = u(p\downarrow) \; ; \; P_R u(p) = u(p,\uparrow) \; ; \; P_L v(p) = v(p,\uparrow) \; ; \; P_R v(p) = v(p\downarrow)$$

then

$$M(\uparrow\downarrow;\uparrow\downarrow) = \frac{8G_F F_Z}{\sqrt{2}} g_R^e g_R^\tau \quad J_{u_1v_2}(\uparrow\downarrow)\cdot J_{u_3v_4}(\uparrow\downarrow)$$

$$= -\frac{8G_F F_Z s\, g_R^e g_R^\tau(1+\cos\theta)}{\sqrt{2}}$$

where we have used the results of Problem 1. In the same way

$$M(\uparrow\downarrow;\downarrow\uparrow) = \frac{8G_F F_Z}{\sqrt{2}} g_R^e g_L^\tau \quad J_{u_1v_2}(\uparrow\downarrow)\cdot J_{u_3v_4}(\downarrow\uparrow)$$

$$= \frac{8G_F F_Z s\, g_R^e g_L^\tau(1-\cos\theta)}{\sqrt{2}}$$

$$M(\downarrow\uparrow;\uparrow\downarrow) = \frac{8G_F F_Z}{\sqrt{2}} g_L^e g_R^\tau \quad J_{u_1v_1}(\downarrow\uparrow)\cdot J_{u_3v_4}(\uparrow\downarrow)$$

$$= \frac{8G_F F_Z s}{\sqrt{2}} g_L^e g_R^\tau\,(1-\cos\theta)$$

$$M(\downarrow\uparrow;\downarrow\uparrow) = \frac{8G_F F_Z}{\sqrt{2}} g_L^e g_L^\tau \quad J_{u_1v_2}(\downarrow\uparrow)\cdot J_{u_3v_4}(\downarrow\uparrow)$$

$$= -\frac{8G_F F_Z}{\sqrt{2}} s\, g_L^e g_L^\tau\,(1+\cos\theta)$$

Therefore

$$\langle|M|^2\rangle = \frac{1}{4}\, 32 G_F^2 |F_Z|^2 s^2 \Big[(1+\cos\theta)^2 \left((g_R^e g_R^\tau)^2 + (g_L^e g_L^\tau)^2\right)$$
$$+ (1-\cos\theta)^2\left((g_L^e g_R^\tau)^2 + (g_R^e g_L^\tau)^2\right)\Big]$$

where

$$|F_Z|^2 = \frac{M_Z^4}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2}$$

d) $$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \langle |M|^2 \rangle$$

$$= \frac{G_F^2 |F_Z|^2 s}{8\pi^2} \Big[(1+\cos\theta)^2 \left[g_R^{e\,2} g_R^{\tau\,2} + g_L^{e\,2} g_L^{\tau\,2} \right] + (1-\cos\theta)^2 \left[g_R^{e\,2} g_L^{\tau\,2} + g_L^{e\,2} g_R^{\tau\,2} \right] \Big]$$

e) $$\sigma = \int d\Omega \, \frac{d\sigma}{d\Omega}$$

$$= \frac{G_F^2 |F_Z|^2 s}{8\pi^2} \times 2\pi \times \int_{-1}^{1} d\cos\theta \, [\quad]$$

using

$$\int_{-1}^{1} d\cos\theta \, (1 \pm \cos\theta)^2 = \frac{8}{3}$$

we get

$$\sigma = \frac{2 G_F^2 |F_Z|^2 s}{3\pi} \left[g_R^{e\,2} g_R^{\tau\,2} + g_L^{e\,2} g_L^{\tau\,2} + g_L^{e\,2} g_R^{\tau\,2} + g_R^{e\,2} g_L^{\tau\,2} \right]$$

the increase at $\sqrt{s} = M_Z$ comes from the resonance. An estimate is given by

$$|F_Z(\sqrt{s} = M_Z)|^2 = \frac{M_Z^2}{\Gamma_Z^2} \simeq 1300$$

that is why LEP was constructed at this energy.

(3) The τ^- decays into a ν_τ and (via a virtual W) into a:

- $q\bar{q}$ pair, or;
- $e^- \bar{\nu}_e$ pair, or;
- $\mu^- \bar{\nu}_\mu$ pair.

[τ^+ has similar decays]

- If the two τs decay leptonically, the dominant background will be the decays of the Z^0 into e^+e^- or $\mu^+\mu^-$.
- If the two τs decay hadronically the dominant background will be the hadronic decays of the Z^0.
- In the semi-leptonic decay we have a mixed signature and therefore it is easier to identify from the other possible backgrounds.