TÉCNICO LISBOA

2^{nd} test
December 18^{th} 2018: 18h00
Duration of the test: 1h30

Mestrado em Eng. Física Tecnológica (MEFT)
Particle Physics
1^{st} semester of 2018-19

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- The allowed elements for consult during the test are:
  - the PDG (Particle Data Book)
  - one single A4 page with formulas.
- Carefully justify all your answers.
- The test has 3 questions (2 pages).

IceCube is a neutrino observatory built at the South Pole, which uses thousands of detectors distributed over a cubic kilometre of volume under the Antarctic ice. The neutrino is detected when it interacts with the ice atoms producing relativistic charged particles. This experiment is sensitive to neutrinos and anti-neutrinos of all flavours.

1. **[12 val]** One interesting channel which was recently reported is the interaction between the anti-neutrino of the electron, $\overline{\nu}_e$, with an electron in the ice, as it is a resonant process via the W boson. This process is known as the Glashow resonance.

a) Consider the interaction between the electron and the anti-neutrino. Knowing that the electron is at rest, what is the energy of the anti-neutrino necessary to produce a W boson?

b) Consider now the process $e^-(p_1) + \overline{\nu}_e(p_2) \to \mu^-(p_3) + \overline{\nu}_\mu(p_4)$. Draw the corresponding Feynman diagram(s).

c) Write the amplitude $\mathcal{M}$ in its simplest form. Assuming that $m_{\text{fermions}} \ll \sqrt{s} \leq M_Z, M_W$ simplify as much as possible the amplitudes. Justify your answer.

d) Evaluate the spin averaged squared amplitude $\langle |\mathcal{M}|^2 \rangle$.

e) For this process evaluate the differential cross section $d\sigma/d\Omega$ in the CM frame as a function of the square of the energy in the CM frame, $s = (p_1 + p_2)^2$ and the scattering angle θ.

f) Determine the expression for total cross section $\sigma(e^- + \overline{\nu}_e \to \mu^- + \overline{\nu}_\mu)$ at the energy $\sqrt{s} = M_W$. Find its value in nb.

g) Find the value of the **total** cross section

$$\sigma(e^- + \overline{\nu}_e \to \text{All})$$

at $\sqrt{s} = M_W$ in nb.

2. **[6 val]** Consider the following processes with neutrinos. For each process draw the Feynman diagram(s) and write the amplitude. Assuming that $m_{\text{fermions}} \ll \sqrt{s} \leq M_Z, M_W$ simplify as much as possible the amplitudes. Justify your answer. Do not calculate anything.

a) $\nu_e + q \rightarrow e^- + q'$
Identify the possible quarks q and q' assuming that they are valence quarks of nucleons in matter not anti-quarks.

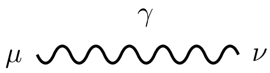
b) $\overline{\nu}_\mu + q \rightarrow \mu^+ + q'$
Identify the possible quarks q and q' assuming that they are valence quarks of nucleon in matter, not anti-quarks.

c) $\overline{\nu}_e + e^+ \rightarrow \overline{\nu}_e + e^+$
Indicate what are the non vanishing helicity amplitudes, by drawing in the diagrams the spin arrows.

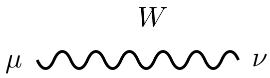
3. **[2 val]** The electron anti-neutrinos measured at IceCube should have an astrophysical origin. Given their extremely high energy, these $\overline{\nu}_e$ are most likely produced in SuperNova Remnants (SNR). Describe the most probable mechanism which can originate such flux of electron anti-neutrinos from the primary matter of a star, namely protons and electrons. (Remember that in SNR high-energy particles are essentially expected to be accelerated in the electromagnetic shock waves).

Formulary

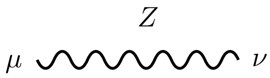
Propagators



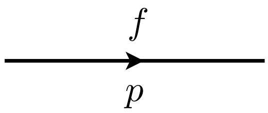
$$-i\frac{g_{\mu\nu}}{k^2} \tag{1}$$



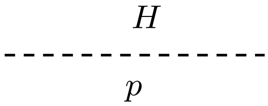
$$-i\frac{g_{\mu\nu}-\frac{k_\mu k_\nu}{M_W^2}}{k^2-M_W^2+i\,M_W\Gamma_W} \tag{2}$$



$$-i\frac{g_{\mu\nu}-\frac{k_\mu k_\nu}{M_Z^2}}{k^2-M_Z^2+i\,M_Z\Gamma_Z} \tag{3}$$



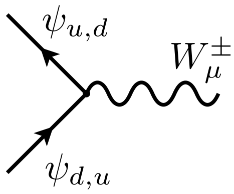
$$\frac{i(\not{p}+m_f)}{p^2-m_f^2} \tag{4}$$



$$\frac{i}{p^2-M_H^2+i\,M_H\Gamma_H} \tag{5}$$

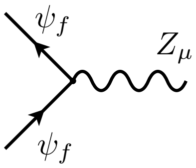
Vertices

Charged Current

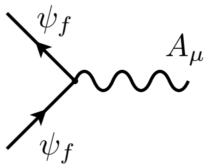


$$-i\frac{g}{\sqrt{2}}\gamma_\mu\frac{1-\gamma_5}{2} \tag{6}$$

Neutral Current



$$-i\frac{g}{\cos\theta_W}\gamma_\mu\left(g_V^f-g_A^f\gamma_5\right)$$

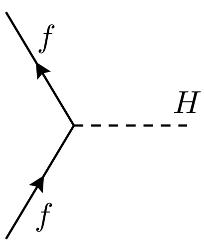


$$-ieQ_f\gamma_\mu$$

where

$$g_V^f=\frac{1}{2}T_f^3-Q_f\sin^2\theta_W,\quad g_A^f=\frac{1}{2}T_f^3\,. \tag{7}$$

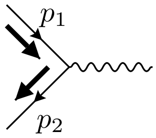
Higgs Interactions with fermions



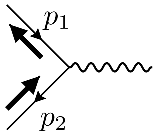
$$-i\,\frac{g}{2}\,\frac{m_f}{M_W}\equiv -i\,g_H^f \tag{8}$$

Results for the Helicity Vector Currents

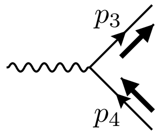
s-channel



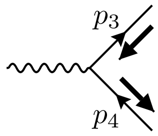
$$J_{u_1 v_2}(\uparrow,\downarrow) = \sqrt{s}\,(0,-1,-i,0) \tag{9}$$



$$J_{u_1 v_2}(\downarrow,\uparrow) = \sqrt{s}\,(0,-1,i,0) \tag{10}$$

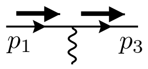


$$J_{u_3 v_4}(\uparrow,\downarrow) = \sqrt{s}\,(0,-\cos\theta,i,\sin\theta) \tag{11}$$

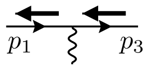


$$J_{u_3 v_4}(\downarrow,\uparrow) = \sqrt{s}\,(0,-\cos\theta,-i,\sin\theta) \tag{12}$$

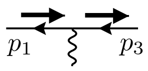
t-channel



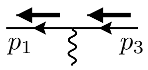
$$J_{u_1 u_3}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{13}$$



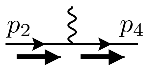
$$J_{u_1 u_3}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{14}$$



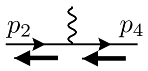
$$J_{v_1 v_3}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{15}$$



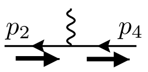
$$J_{v_1 v_3}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{16}$$



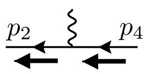
$$J_{u_2 u_4}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{17}$$



$$J_{u_2 u_4}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{18}$$



$$J_{v_2 v_4}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{19}$$



$$J_{v_2 v_4}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{20}$$