2nd Exam (2018/2019)

1- a)

$$\frac{dN}{dt} = \lambda N \Rightarrow N = N_0 e^{-\lambda t} \Rightarrow \underbrace{Prob = e^{-\lambda t}}_{\text{Probability of survival}}$$

Then, the probability of decay is, $Prob = 1 - e^{-t/t_0} = 1 - \exp\left(-\frac{x}{\beta c \gamma \tau}\right)$

$$Prob = 1 - \exp\left(-\frac{x\, m_\mu}{P_\mu\, c\tau}\right) = 0{,}077 \simeq 7{,}7\,\%$$

Where, $E_\mu = 1\,GeV$; $x = 5000\,m$; $m_\mu = 105{,}7\,MeV/c^2$

$c\tau = 658{,}6\,m$; $P_\mu = \sqrt{E_\mu^2 - m_\mu^2}$

ii) $\Delta X = X(h=0) - X(h=5\,Km) = X_0\left(1 - e^{-h/7}\right)_{\infty}$

$\Delta X = 529{,}35\ g/cm^2$

$\delta \equiv \left\langle -\frac{dE}{dX}\right\rangle_{carbon} \cancel{\mu}(10\,GeV) \sim 2{,}2\ MeV/g\,cm^{-2}$

$E_{loss} = \Delta X \cdot \delta = 1{,}165\ GeV$

1- iii)

No. An electron with such high-energy would radiate a lot via bremsstrahlung.
In fact, such electron would give origin to an electromagnetic shower, similarly to what happens in a calorimeter, preventing the original particle to reach the ground.
Bremsstrahlung radiation is proportional to the inverse square of the particle mass and so the μ isn't affected by it

$$\left(\frac{E_{loss}(\text{Brem})\, e^-}{E_{loss}(\text{Brem})\, \mu^-}\right) \propto \left(\frac{\frac{1}{m_e^2}}{\frac{1}{m_\mu^2}}\right) \simeq 43000$$

the μ loses energy essentially by ionization.

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Combining the above questions is clear that a 10 GeV μ produced at h = 5000 m has a high probability of reaching the ground.

1 - b) In CM the e^- minimum energy occurs when:

$$-P \longleftarrow \underset{e^-}{\circ} \longrightarrow P \qquad \Rightarrow \quad E_e^* = \; m_e = 511 \text{ KeV}$$

$\nu_\mu \qquad e^- \qquad \nu_e$

The maximum energy:

$$\nu \begin{cases} \nu_\mu \longleftarrow \\ \nu_e \longleftarrow \end{cases} \underset{\mu}{\cdot} \longrightarrow e^-$$

$$P_\mu = (m_\mu, \vec{0})$$

$$P_\nu = (E_\nu^*, -P^*) \text{ , where } E_\nu^* = E_{\nu_\mu}^* + E_{\nu_e}^*$$

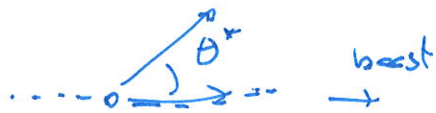
$$P_e = (E_e^*, P^*)$$

$$P_\mu = P_\nu + P_e \iff (P_\mu - P_e)^2 = P_\nu^2 \iff$$

$$P_\mu^2 + P_e^2 + 2 P_\mu P_e = 0 \iff m_\mu^2 + m_e^2 - 2 E_e m_\mu = 0$$

$$E_e^* = \frac{m_\mu^2 + m_e^2}{2 m_\mu} = 52{,}8 \text{ MeV}$$

c)
$$\begin{pmatrix} E \\ P_\ell \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} E^* \\ P_\ell^* \end{pmatrix}$$



$$P_\ell^* = P^* \cos\theta^* \qquad \text{min}: \theta^* = 180° \; ; \; \text{max}: \theta^* = 0°$$

$$E_e^{min} = \gamma E_e^* - \gamma\beta P_e^* \simeq \cancel{2_8}\, 513 \text{ KeV}$$

$$E_e^{max} = \gamma E_e^* + \gamma\beta P_e^* \simeq 9{,}99972 \simeq 10 \text{ GeV}$$

where $\gamma = \dfrac{E_\mu}{m_\mu}$; $\beta = \dfrac{P_\mu}{E_\mu}$; $P_\mu = \sqrt{E_\mu^2 + m_\mu^2}$

$$P_e^* = \sqrt{E_e^{*2} - m_e^2}$$

1 - d)

$$\mu^- \rightarrow e^- \; \bar{\nu}_e \; \nu_\mu$$

$$L_e \quad 0 \rightarrow 1 \quad -1 \quad 0$$

$$L_\mu \quad 1 \rightarrow 0 \quad 0 \quad 1$$

The neutrinos have to be different in order to conserve leptonic number.

2 -

a)

$$N = N_0 \, e^{-13,5/0,24} \quad \text{with} \quad N_0 = 10^{17}$$

$N = 3,7 \times 10^{-8}$ which means that in average no pion would be able to cross the wall.

b) $$\lambda = \frac{1}{\sigma n} = \frac{1}{\sigma \frac{\rho N_A}{A}} \Leftrightarrow$$

$$\sigma = \frac{A}{\lambda \rho N_A} = 4,92 \times 10^{-25} \text{ cm}^2 \simeq 0,49 \text{ b}$$

with $\lambda = 24$ cm ; $\rho = 7,874 \text{ g cm}^{-3}$

$A = 56$; $N_A = 6,022 \times 10^{23}$

2 -

c) Because only interactions (iii) and (iv) would be possible, due to lepton number conservation, and therefore seem.

d) The muon only loses energy by ionization and therefore would appear as a single track.

The electron, on the other hand, would produce and electromagnetic cascade (γ, e^+, e^-, generated via bremsstrahlung radiation and pair creation) producing a broader (more thick) track.

2nd Exam "Particle Physics"

2nd Test

3) a)

$$Q[\nu] + Q[q] = Q[\mu^-] + Q[q']$$

So using $Q[\nu]=0$, $Q[\mu^-]=-1$ we get

$$Q[q'] - Q[q] = 1$$

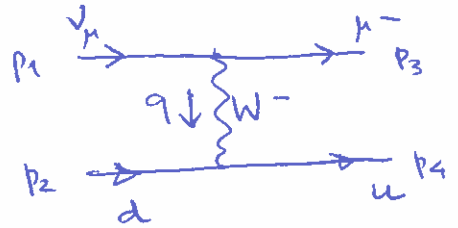
and therefore

$$Q[q'] = \frac{2}{3} \quad ; \quad Q[q] = -\frac{1}{3}$$

for valence quarks we have

$$q' = u, \quad q = d$$

b)



$$q = p_1 - p_3$$

c) $$i\mathcal{M} = \left(-i\frac{g}{\sqrt{2}}\right)^2 \bar{u}(p_3)\gamma^\mu P_L u(p_1)\, \frac{-i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2}\right)}{t - M_W^2}\, \bar{u}(p_4)\gamma^\nu P_L u(p_2)$$

where $q = p_1 - p_3$ and $t = q^2 = (p_1 - p_3)^2$. No need for the $iM_W\Gamma_W$ term as $t<0$ always.

Neglecting all fermion masses we can also neglect the momenta in the W propagator. In fact

$$\bar{u}(p_3)\not{q} P_L u(p_1) = \bar{u}(p_3)\not{p}_1 P_L u(p_1) - \bar{u}(p_3)\not{p}_3 P_L u(p_1)$$

$$= \bar{u}(p_3) P_R \not{p}_1 u(p_1) - \bar{u}(p_3)\not{p}_3 P_L u(p_1)$$

$$= 0 \qquad \text{using} \qquad \not{p}_1 u(p_1) = 0 \quad \text{and} \quad \bar{u}(p_3)\not{p}_3$$

which results from the Dirac equation for massless fermions.

On the other hand $E_\nu = 1$ GeV $<< M_W$, so we can neglect t in the denominator as we shall have always

$$t << M_W^2$$

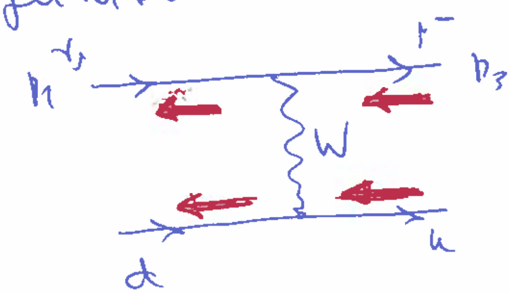
In the end

$$M = -\frac{g^2}{2M_W^2}\, \bar{u}(p_3)\gamma^\mu P_L u(p_1)\, \bar{u}(p_4)\gamma_\mu P_L u(p_2)$$

using $\frac{g^2}{8M_W^2} = \frac{G_F}{\sqrt{2}}$, we get finally

$$\boxed{M = -\frac{4G_F}{\sqrt{2}}\, \bar{u}(p_3)\gamma^\mu P_L u(p_1)\, \bar{u}(p_4)\gamma_\mu P_L u(p_2)}$$

b) As the W only couples to left-handed currents we have only one non-vanishing helicity amplitude



$M(\downarrow\downarrow;\downarrow\downarrow)$

we have

$$M(\downarrow\downarrow;\downarrow\downarrow) = -\frac{4G_F}{\sqrt{2}} J_{u_1u_3}(\downarrow\downarrow)\cdot J_{u_2u_4}(\downarrow\downarrow)$$

$$= -4s\frac{G_F}{\sqrt{2}}\left[\cos\tfrac{\theta}{2}, \sin\tfrac{\theta}{2}, -i\sin\tfrac{\theta}{2}, \cos\tfrac{\theta}{2}\right]\cdot\left[\cos\tfrac{\theta}{2}, -\sin\tfrac{\theta}{2}, -i\sin\tfrac{\theta}{2}, -\cos\tfrac{\theta}{2}\right]$$

$$= -4\frac{G_F}{\sqrt{2}}s\left[\cos^2\frac{\theta}{2} - \left(-\sin^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} - \cos^2\frac{\theta}{2}\right)\right]$$

$$= -\frac{8G_F}{\sqrt{2}}s$$

therefore

$$\boxed{M(\downarrow\downarrow;\downarrow\downarrow) = -\frac{8G_F s}{\sqrt{2}}}$$

and

$$\boxed{\langle|M|^2\rangle = \frac{1}{2}\left|M(\downarrow\downarrow;\downarrow\downarrow)\right|^2 = 16G_F^2 s}$$

e) $$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\frac{|\vec{p}_3|}{|\vec{p}_1|}\langle|M|^2\rangle = \frac{1}{64\pi^2 s}\langle|M|^2\rangle$$

because when we neglect the mass we have $|\vec{p}_1| = |\vec{p}_3|$.

therefore

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}16G_F^2 s^2$$

or

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{G_F^2 s}{4\pi^2}}$$

In fact we could also obtain the total cross section trivially

$$\boxed{\sigma = \frac{G_F^2 s}{\pi}}$$

4) a)

The amplitude is

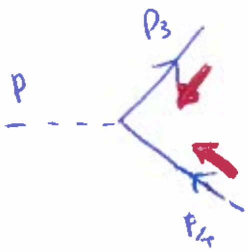
$$i\mathcal{M} = i\sqrt{2}\, G_F V_{ud} f_\pi m_e \quad \bar{u}(p_3) P_R v(p_4)$$

or

$$\boxed{\mathcal{M} = \sqrt{2}\, G_F V_{ud} f_\pi m_e \quad \bar{u}(p_3) P_R v(p_4)}$$

b) Because of the projector P_R the only non-vanishing amplitude is

$\mathcal{M}(\downarrow\downarrow)$



and therefore

$$\boxed{\begin{aligned}\mathcal{M}(\downarrow\downarrow) &= \sqrt{2}\, G_F V_{ud} f_\pi m_e \quad J_{u_3 v_4}(\downarrow\downarrow) \\ &= -\sqrt{2}\, G_F V_{ud} f_\pi m_e \sqrt{s}\end{aligned}}$$

$\sqrt{s} = m_\pi$

c) $$\frac{d\Gamma}{d\Omega} = \frac{1}{32\pi^2} \frac{1}{m_\pi^2} |\vec{p}_3| \langle |\mathcal{M}|^2 \rangle = \frac{1}{64\pi^2} \frac{1}{m_\pi} \langle |\mathcal{M}|^2 \rangle \qquad \boxed{|\vec{p}_3| = \frac{m_\pi}{2}}$$

As the π^+ has spin 0 we have

$$\langle |\mathcal{M}|^2 \rangle = |\mathcal{M}|^2 = 2 m_\pi^2 G_F^2 V_{ud}^2 f_\pi^2 m_e^2$$

$$\Gamma = \frac{1}{16\pi} \frac{1}{m_\pi} 2 m_\pi^2 G_F^2 V_{ud}^2 f_\pi^2 m_e^2$$

or

$$\boxed{\Gamma = \frac{1}{8\pi} m_\pi G_F^2 V_{ud}^2 f_\pi^2 m_e^2}$$

d) we get

$$f_\pi = \sqrt{\frac{8\pi \Gamma(\pi^+ \to \nu_e e^+)}{m_\pi m_e^2 V_{ud}^2 G_F^2}}$$

from $\boxed{\text{PDG}}$

$$\Gamma(\pi^+ \to \nu_e e^+) = \Gamma(\pi^+ \to \text{all}) \times BR(\pi^+ \to \nu_e e^+)$$

$$\Gamma(\pi^+ \to \text{all}) = \frac{1}{\tau} = \frac{1}{2.603 \times 10^{-8}} \ s^{-1}$$

$$= \frac{1}{2.603 \times 10^{-18}} \times 6.578 \times 10^{-25} \text{ GeV}$$

$$= 2.527 \times 10^{-17} \text{ GeV}$$

$$BR(\pi^+ \to e^+ \nu_e) = 1.23 \times 10^{-4}$$

therefore

$$\Gamma(\pi^+ \to \nu_e \pi^+) = 2.527 \times 10^{-17} \times 1.23 \times 10^{-4} \text{ GeV}$$

$$= 3.108 \times 10^{-21} \text{ GeV}$$

and

$$f_\pi = 0.1288 \text{ GeV} \Rightarrow \boxed{f_\pi = 128.8 \text{ MeV}}$$