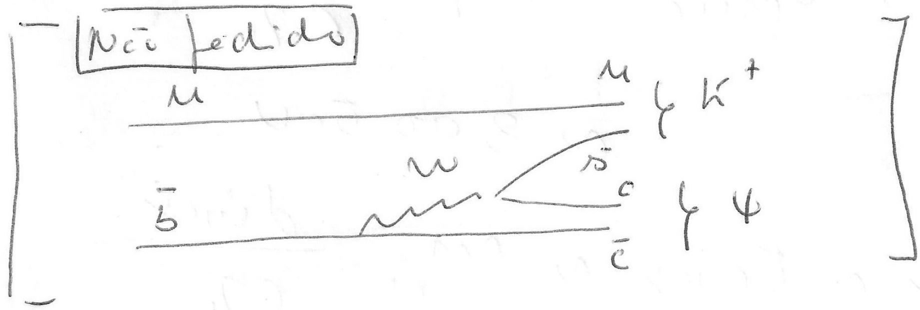
Resolução 2 data

1º teste

1 a) $B^+ \to \psi \, K^+$

$B^+ (u\bar{b})$, $\psi (c\bar{c})$, $K^+ (u\bar{s})$

Não pedido

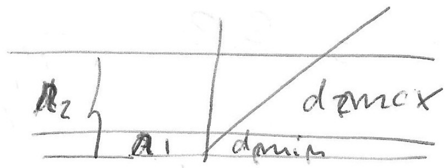


b) i) $B^+ \to e^+ \bar{\nu}_e$ $\boxed{\text{Não}}$ viola nº leptónico

ii) $B^+ \to p\, \pi^0$ $\boxed{\text{Não}}$ viola nº bariónico

iii) $B^+ \to p\, \bar{\Lambda}\, \pi^0$ $\boxed{\text{Sim}}$ embora a estranheza e a beleza ambas violadas isso é possível em decaimentos fracos!

c)



$d_{min} = \frac{r_1}{\sin 30} = 3 \text{ cm}$

$d_{max} = \frac{r_2}{\sin 30} = 274 \text{ cm}$

distancia percorrida em linha recta, no tempo medio de vida ②

$$d = \gamma \beta c \tau = \frac{E}{m} \frac{p}{E} c\tau = \frac{p}{m} c\tau$$

$c\tau \sim 492\ \mu m = 492\ 10^{-6}\ m$, $m_{B^+} \sim 5.3\ GeV$

donde

$$p_{min} \simeq E_{min} \sim m_{B^+} \frac{d_{min}}{c\tau}$$

$$\sim 300\ GeV$$

$$p_{max} \sim E_{max} \sim m_{B^+} \frac{d_{max}}{c\tau}$$

$$\sim 25\ TeV$$

- Nota:
Com este nivel de momento e deflexão no campo magnético é pequena donde pode ser desprezada

$$R = 3.3 \frac{m c^2/GeV \ \ \sim 1/c}{q/e \ \ B/tesla}$$

$$R \gg d_{max}$$

d) $\psi \to \mu^+ \mu^-$

$m_\psi = 3.1$ GeV

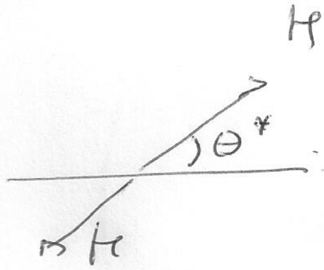
$m_\mu = 0.113$ GeV

$\boxed{\text{no c.m}}$

$$E^*_{\mu^+} = E^*_{\mu^-} = \frac{m_\psi}{2} = 1.55$$

$$p^*_\mu = \sqrt{E^2 - m^2} = 1.546$$

$$\beta_\mu = \frac{p}{E} = 0.9975$$



$\boxed{\text{no laboratory}}$

$$\gamma_\psi = \frac{E_\psi}{m_\psi} \sim 30$$

$E_\psi = 93$ GeV

$$\beta_\psi = \frac{p_\psi}{E_\psi}$$

$p_\psi = \sqrt{E^2 - m^2} = 92.948$

$\beta_\psi \sim 0.9995$

como

$\beta_\psi > \beta^*_\mu$ há inversão!

(emissão no centro de massa $\theta^* = 90$)

donde

$\boxed{\theta_{min} = 0}$ $\boxed{\tan\theta_{max} = \frac{p_\perp}{p_\psi/2} = \frac{2 \times 1.546}{92.948}}$

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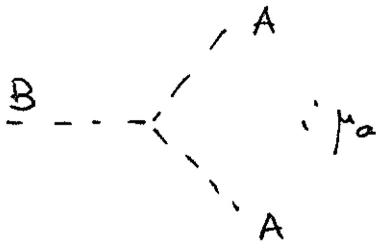
e) Basta adicionarmos os quadrivetores dos dois feixes e quadrar

$$m_\psi = \sqrt{s} = \left[p_{\mu^+} + p_{\mu^-} \right]^2$$

$$= \sqrt{(E_1 + E_2)^2 - (\vec{p}_{\mu^+} + \vec{p}_{\mu^-})^2}$$

2nd Exam – 1st TEST – Problem 2 – 27/1/2013

a) As $m_B > 2m_A$ the decay $B \to AA$ is possible



Therefore the corresponding amplitude is

$$\mathcal{M} = i\,(i\mu_a) = -\mu_a$$

For the total width we have

$$\Gamma = \frac{S}{8\pi m_B^2}\,|\vec{p}_A|\,|\mathcal{M}|^2$$

with $S = \frac{1}{2!} = \frac{1}{2}$ for identical particles in the final state

also

$$|\vec{p}_A| = \sqrt{E_A^2 - m_A^2} \quad \text{with} \quad E_A = \frac{1}{2} m_B$$

so

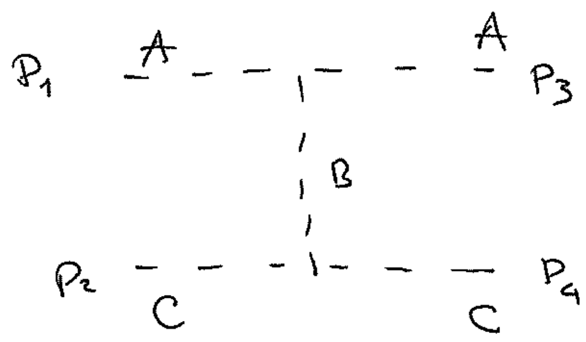
$$|\vec{p}_A| = \sqrt{\frac{1}{4} m_B^2 - m_A^2} = \frac{1}{2} m_B \sqrt{1 - \frac{4m_A^2}{m_B^2}}$$

so we get the final result

$$\boxed{\Gamma = \frac{\mu_a^2}{32\pi\, m_B}\sqrt{1 - \frac{4m_A^2}{m_B^2}}}$$

b) $A + C \to A + C$

the only diagram is the t-channel



the Amplitude is

$$\mathcal{M} = i\,(i\mu_a)(i\mu_b)\,\frac{i}{(p_1-p_3)^2 - m_B^2}$$

$$\boxed{\mathcal{M} = \frac{\mu_a \mu_b}{t - m_B^2}}$$ where $t \equiv (p_1-p_3)^2$

c) In the CM frame

$$\frac{d\sigma}{d\Omega} = \frac{S}{64\pi^2 s}\,\frac{|\vec{p}_3|}{|\vec{p}_1|}\,|\mathcal{M}|^2$$ with $S = 1$ (no identical particles in final state)

$$= \frac{1}{64\pi^2 s}\,\frac{|\vec{p}_3|}{|\vec{p}_1|}\,\frac{\mu_a^2 \mu_b^2}{(t - m_B^2)^2}$$

Now $E_1 = \dfrac{s + m_A^2 - m_C^2}{2\sqrt{s}}$; $E_3 = \dfrac{s + m_A^2 - m_C^2}{2\sqrt{s}}$

As particles 1 and 3 have the same mass and the same energy they have the same absolute value for the momenta in the CM

$$|\vec{p}_3| = |\vec{p}_1|$$

therefore $d\sigma/d\Omega$ simplifies to

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\,\frac{M_a^2\, M_b^2}{(t - m_B^2)^2}}$$

Finally using the kinematics $(|\vec{p}_1| = \beta E_1)$

$$p_1 = (E_1, 0, 0, \beta E_1) \quad ; \quad p_3 = (E_1, \beta E_1 \sin\theta, 0, \beta E_1 \cos\theta)$$

$$(p_1 - p_3)^2 = -2\beta^2 E_1^2 (1-\cos\theta) = -2|\vec{p}_1|^2 (1-\cos\theta)$$

$$|\vec{p}_1|^* = \sqrt{\left(\frac{s + m_A^2 - m_C^2}{2\sqrt{s}}\right)^2 - m_A^2}$$

$$= \frac{1}{2\sqrt{s}} \sqrt{s^2 + m_A^4 + m_C^4 - 2m_A^2 s - 2m_C^2 s - 2m_A^2 m_C^2}$$

So

$$\boxed{t = (p_1 - p_3)^2 = -\frac{1}{2s}(s^2 + m_A^4 + m_C^4 - 2m_A^2 s - 2m_C^2 s - 2m_A^2 m_C^2)(1-\cos\theta)}$$

d) Limit $m_B \gg \sqrt{s}, m_A, m_C$

$$t - m_B^2 \rightarrow -m_B^2$$

therefore

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{\mu_a^2 \mu_b^2}{64\pi^2 s\, m_B^4}}$$

and for the total cross section

$$\boxed{\sigma = \frac{\mu_a^2 \mu_b^2}{16\pi\, s m_B^4}}$$

e) For $m_B \gg \sqrt{s}, m_A, m_C$

$$|M|^2 \Rightarrow \frac{\mu_a^2 \mu_b^2}{m_B^4}$$

and

$$\sigma \propto s^{\alpha} |M|^2 = \frac{\mu_a^2 \mu_b^2}{s^{\alpha} m_B^4}$$

As $[\sigma] = M^{-2} = [s]^{-\alpha} = [M]^{-2\alpha} \Rightarrow \alpha = 1$

So

$$\sigma \propto \frac{\mu_a^2 \mu_b^2}{s\, m_B^4}$$

Note: $[\mu_a] = [\mu_b] = [m_B]$ $= M$

a) One must have

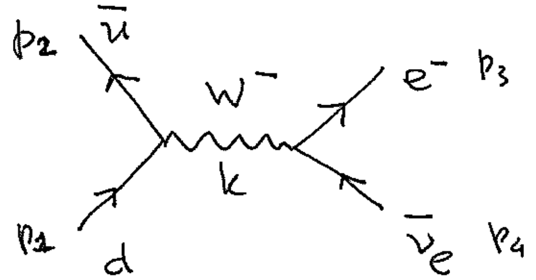
$$Q[q] - Q[q'] = -1$$

As the possible values for the charges are $-\frac{1}{3}$ and $\frac{2}{3}$ the solution is

$$Q[q] = -\frac{1}{3} \Rightarrow q = d \quad \text{(valence quark)}$$

$$Q[q'] = \frac{2}{3} \Rightarrow q' = u \quad \text{(valence quark)}$$

b) The process corresponds to the Feynman Diagram



$$k = p_1 + p_2 = p_3 + p_4$$

c) The amplitude is

$$\mathcal{M} = i\left(-i\frac{g}{\sqrt{2}}\right)^2 \bar{v}(p_2)\gamma^\mu P_L u(p_1) \frac{-i\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{M_W^2}\right)}{k^2 - M_W^2 + i M_W \Gamma_W}$$

$$\bar{u}(p_3)\gamma^\nu P_L v(p_4)$$

with $P_L = \frac{1-\gamma_5}{2}$.

d) Let us look, for instance, at the electron-neutrino line. We have one term

$$A = \bar{u}(p_3)\gamma^\nu P_L v(p_4) k_\nu$$

$$= \bar{u}(p_3)\gamma^\mu P_L v(p_4)(p_3+p_4)_\nu$$

$$= \bar{u}(p_3)\not{p}_3 P_L v(p_4) + \bar{u}(p_3)\not{p}_4 P_L v(p_4)$$

$$= \bar{u}(p_3)\not{p}_3 P_L v(p_4) + \bar{u}(p_3) P_R \not{p}_4 v(p_4)$$

where we used

$$\not{p} P_L = P_R \not{p}.$$

Now for massless fermions

$$\not{p}_4 v(p_4) = 0$$

$$\bar{u}(p_3)\not{p}_3 = 0$$

so

$$A = 0$$

and the term proportional to $\frac{k_\mu k_\nu}{M_W^2}$ vanishes. The simplified amplitude is then

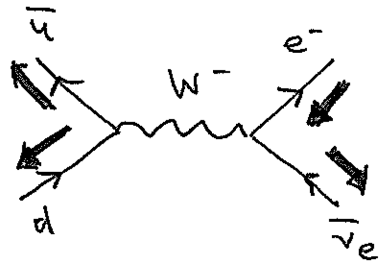
$$\boxed{\mathcal{M} = -\frac{g^2}{2}\frac{1}{k^2-M_W^2+iM_W\Gamma_W}\bar{v}(p_2)\gamma^\mu P_L u(p_1)\,\bar{u}(p_3)\gamma_\mu P_L v(p_4)}$$

e) As the W only couples to left-handed currents we have only one non-zero helicity amplitude. Using the notation

$$\mathcal{M}(h_d\, h_{\bar{u}}\,;\, h_{e^-}\, h_{\bar{\nu}_e})$$

we have

$$\mathcal{M}(\downarrow\uparrow\,;\,\downarrow\uparrow) \equiv$$



$$= -\frac{g^2}{2}\,\frac{1}{\hat{s}-M_W^2+iM_W\Gamma_W}\,J_{u_1 v_2}(\downarrow\uparrow)\cdot J_{u_3 v_4}(\downarrow\uparrow)$$

$$= +\frac{g^2}{2}\,\frac{\hat{s}}{\hat{s}-M_W^2+iM_W\Gamma_W}\,(1+\cos\theta)$$

f) $$\frac{d\hat{\sigma}}{d\Omega} = \frac{1}{64\pi^2\hat{s}}\,\frac{|\vec{p}_3|}{|\vec{p}_1|}\,\langle|\mathcal{M}|^2\rangle$$

for massless particles $|\vec{p}_3| = |\vec{p}_1|$ and therefore

$$\frac{d\hat{\sigma}}{d\Omega} = \frac{1}{64\pi^2\hat{s}}\,\langle|\mathcal{M}|^2\rangle$$

where

$$\langle |M|^2 \rangle = \frac{1}{4} \left| M(\downarrow\uparrow ; \downarrow\uparrow) \right|^2$$

$$= \frac{g^4}{8} \left| \frac{\hat{s}}{\hat{s} - M_W^2 + i M_W \Gamma_W} \right|^2 (1 + \cos\theta)^2$$

g) using

$$\int d\Omega \, (1+\cos\theta)^2 = 2\pi \int_{-1}^{1} dx \, (1+x)^2$$

$$= 2\pi \int_{-1}^{1} dx \left[1 + 2x + x^2 \right] = 2\pi \left(2 + \frac{2}{3} \right)$$

$$= \frac{16\pi}{3}$$

we get

$$\hat{\sigma}(\hat{s}) = \frac{1}{64\pi^2 \hat{s}} \, \frac{16\pi}{3} \, \frac{g^4}{8} \left| \frac{\hat{s}}{\hat{s} - M_W^2 + i M_W \Gamma_W} \right|^2$$

or

$$\boxed{\hat{\sigma}(\hat{s}) = \frac{g^4}{96\pi \hat{s}} \left| \frac{\hat{s}}{\hat{s} - M_W^2 + i M_W \Gamma_W} \right|^2}$$

h) Writing $p + \bar{p} \to W^- \to e^- + \bar{\nu}_e$

with momenta p_1 for proton e p_2 for anti-proton.

For the elementary process $d + \bar{u} \to W^- \ldots$

we consider d with momenta $\hat{p}_1$ and $\bar{u}$ with momentum $\hat{p}_2$ given by

$$\hat{p}_1 = x_1 \, p_1$$

$$\hat{p}_2 = x_2 \, p_2$$

where x_1 and x_2 are the fractions of the proton and anti-proton carried by the quarks. then, neglecting the mass

$$\hat{s} = (\hat{p}_1 + \hat{p}_2)^2 = 2\, \hat{p}_1 \cdot \hat{p}_2 = x_1 x_2 \; 2\, p_1 \cdot p_2 = x_1 x_2 \, s$$

that is

$$\boxed{\hat{s} = x_1 x_2 \, s}$$

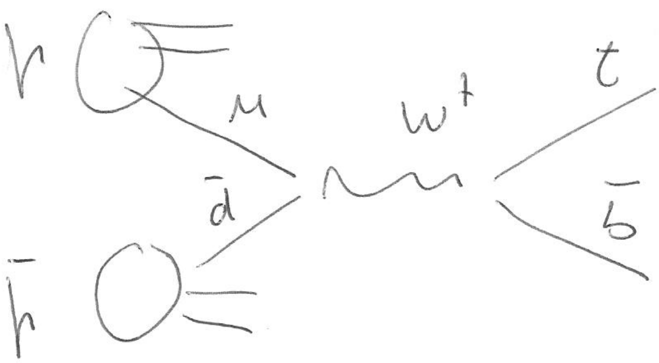
therefore

$$\boxed{\sigma_{p\bar{p} \to W^-}(s) = \int_0^1 dx_1 \int_0^1 dx_2 \; f_d(x_1)\, f_{\bar{u}}(x_2)\, \hat{\sigma}(x_1 x_2 s)}$$

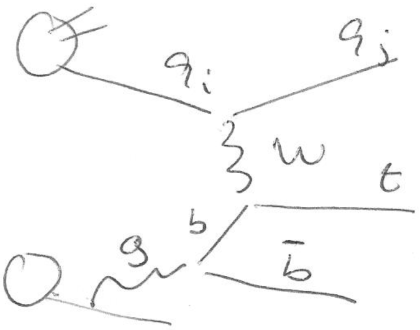
Resolução 2 data

2º teste

4 (a) first order s channel



— Não pedido [cond t]



(b) decaimentos top

$t \to W^+ b$ e o $W^+ \to q_i \bar{q}_j$

$\to l \bar{\nu}$

Os modos mais visíveis são os que apresentam leptões isolados. Os exclusivamente hadrónicos têm que "lutar" com um background considerável.