1^{st} exam

January 13^{th} 2016: 11h30

Duration of the test: 1h30

Duration of the exam: 3h00

Mestrado em Eng. Física Tecnológica (MEFT)
Particle Physics
1^{st} semester of 2015-16

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- The allowed elements for consult during the test are:
  - the PDG (Particle Data Book)
  - one single A4 page with formulas.
- Clearly identify all pages of the test.
- The exam has 3 questions (2 pages).

1^{st} test

1. **[5 val]** The Sudbury Neutrino Observatory (SNO) was able to observe neutrino flavour oscillations by measuring the different possible interactions of the solar neutrinos with heavy water. One of these interactions is the elastic scattering of the neutrinos with the electrons present in the water,

$$\nu_X + e^- \to \nu_X + e^-$$

where ν_X can be a neutrino of an electron, muon or tau. Assuming an energy for the neutrino of 10 MeV, neglecting its mass and knowing the following properties of the heavy water: $n = 1.33$ and $\rho_{D_2O} = 1.11\,\text{g}\,\text{cm}^{-3}$,

a) Determine the energy of the neutrino and the electron in the centre-of-mass (CM) reference frame, before and after the reaction.

b) Compute the velocity, β of the centre-of-mass (CM) reference frame.

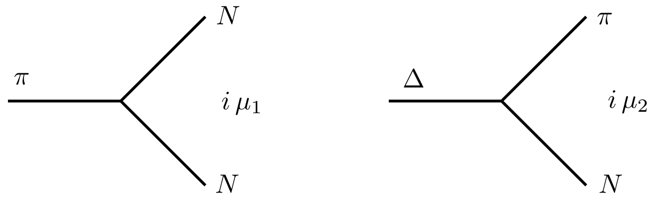
c) Calculate the maximum angle that the electron can have with respect to the direction of the incoming neutrino.

d) Apart from the neutrino elastic scattering with the electrons, the SNO experiment also uses the interaction of the neutrinos with the deuterium ($D = p\,n$), present in the heavy water to detect them. Knowing that the area of detection is composed by 1000 tons of heavy water (D_2O), surrounded by PMTs, and that the neutrino interaction cross-section with nucleons is $\sigma_{\nu N} \sim 10^{-42}\,\text{cm}^2$, estimate the expected number of neutrinos to be detected per day. Consider a neutrino solar flux of $\sim 5 \times 10^6\,\text{cm}^{-2}\,\text{s}^{-1}$ and an experimental detection efficiency of $\sim 50\%$.

e) Assuming that the scattered electron travels 5 cm before stopping, estimate the number of photons emitted by Cherenkov radiation.

2. **[5 val]**

A very simple model for NN and πN scattering (N=nucleon=neutron or proton), can be used when we neglect spin. The model is described by the interactions



where the constants μ_1 e μ_2 have dimension of a mass in our natural system of units ($\hbar = c = 1$). The charges of the particles are such that the charge is conserved at each vertex.

a) Consider the elastic scattering $\pi^0 + n \to \pi^0 + n$. Draw the Feynman Diagram(s) for the process.

b) Consider now the process $\pi^-(p_1) + p(p_2) \to \pi^-(p_3) + \Delta^+(p_4)$ in the Lab frame where the proton is at rest. What is the minimum energy of the pion beam for the process to take place.

c) For process b) draw the Feynman diagram(s) and obtain the correspondent amplitude.

d) For process b) evaluate the differential cross section $d\sigma/d\Omega$ in the CM frame as a function of the square of the energy in the CM frame, $s = (p_1 + p_2)^2$ and the masses of the particles.

e) In the limit that $\sqrt{s} \gg m_\pi, m_N, m_\Delta$ neglect the masses and evaluate the total cross section in the CM frame. Show that it has the correct dimensions for a cross section.

2^{nd} *test*

3. **[10 val]** The solar neutrinos interact with the nucleus of the heavy water (D_2O with $D = pn$) through the following channels: charged current (CC), neutral current (NC). Additionally, it can undergo elastic scattering (ES) with the electrons present in the heavy water.

a) Draw all the possible diagrams, at the tree level, for the following channels:

   i. Charged current (exchange of a W boson)

   ii. Neutral current (exchange of a Z boson)

   iii. Elastic scattering

b) The charged current interaction is described, at quark level, by the process,

$$\nu_e(p_1) + d(p_2) \to e^-(p_3) + u(p_4)\,.$$

c) Write down the amplitude $\mathcal{M}$ for this process.

d) Assuming that the energy is such that you can neglect the lepton and quark masses, but not the gauge boson masses, simplify the expression for the amplitude. Justify the various steps.

e) In these conditions evaluate $\left\langle |\mathcal{M}|^2 \right\rangle$,

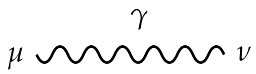
f) In the same conditions evaluate the total cross-section.

g) Write the cross-section of the process $\nu_e + D \to p + p + e^-$ in terms of the parton cross-sections. Consider only valence quarks and leave the result in its integral form.

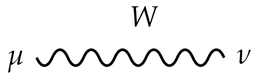
h) Consider the statement: *The total interaction cross-section of neutrinos with matter is proportional to the square of the CM energy*. Discuss this statement indicating its domain of validity.

i) The results obtained in the SNO experiment led to the Nobel Prize in 2015. Discuss briefly how the neutrino oscillations could be observed using the measurements of the channels described above. What is the importance of this result?

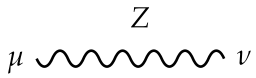
Propagators



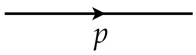
$$-i\frac{g_{\mu\nu}}{k^2} \tag{1}$$



$$-i\frac{g_{\mu\nu}-\frac{k_\mu k_\nu}{M_W^2}}{k^2-M_W^2+i\,M_W\Gamma_W} \tag{2}$$



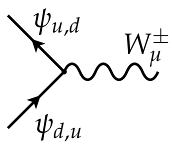
$$-i\frac{g_{\mu\nu}-\frac{k_\mu k_\nu}{M_Z^2}}{k^2-M_Z^2+i\,M_Z\Gamma_Z} \tag{3}$$



$$\frac{i(\not p+m_f)}{p^2-m_f^2} \tag{4}$$

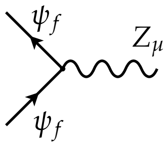
Vertices

Charged Current

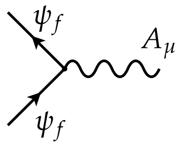


$$-i\frac{g}{\sqrt{2}}\gamma_\mu\frac{1-\gamma_5}{2} \tag{5}$$

Neutral Current



$$-i\frac{g}{\cos\theta_W}\gamma_\mu\left(g_V^f-g_A^f\gamma_5\right)$$



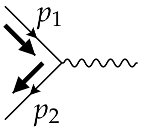
$$-ieQ_f\gamma_\mu \tag{6}$$

where

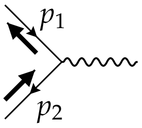
$$g_V^f=\frac{1}{2}T_f^3-Q_f\sin^2\theta_W,\quad g_A^f=\frac{1}{2}T_f^3\,. \tag{7}$$

Results for the Helicity Currents

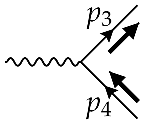
s-channel



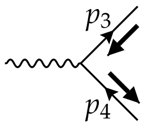
$$J_{u_1 v_2}(\uparrow,\downarrow) = \sqrt{s}\,(0,-1,-i,0) \tag{8}$$



$$J_{u_1 v_2}(\downarrow,\uparrow) = \sqrt{s}\,(0,-1,i,0) \tag{9}$$

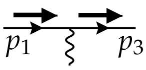


$$J_{u_3 v_4}(\uparrow,\downarrow) = \sqrt{s}\,(0,-\cos\theta,i,\sin\theta) \tag{10}$$

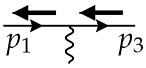


$$J_{u_3 v_4}(\downarrow,\uparrow) = \sqrt{s}\,(0,-\cos\theta,-i,\sin\theta) \tag{11}$$

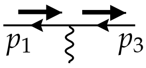
t-channel



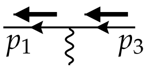
$$J_{u_1 u_3}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{12}$$



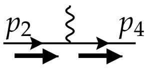
$$J_{u_1 u_3}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{13}$$



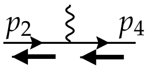
$$J_{v_1 v_3}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{14}$$



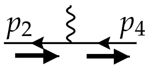
$$J_{v_1 v_3}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{15}$$



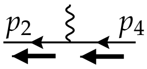
$$J_{u_2 u_4}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{16}$$



$$J_{u_2 u_4}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{17}$$



$$J_{v_2 v_4}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{18}$$



$$J_{v_2 v_4}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{19}$$