1º Exam 2017/2018

1 -

a) $E_\nu = 10^{15} eV = 10^6 GeV$; $m_p = 0{,}938 GeV/c^2$

$\xrightarrow{E_\nu}$ $\cdot\, m_p$ $\quad P_\mu = (E_\nu + m_p, E_\nu)$

$s = (E_\nu + m_p)^2 - E_\nu^2 = 2E_\nu m_p + m_p$

$\sqrt{s} = \sqrt{2E_\nu m_p + m_p} = 1369{,}7 \text{ GeV} \simeq 1{,}4 \text{ TeV}$

b) $s = 1{,}876 \times 10^6$ GeV from a)

$\sigma(E = 1 \text{ PeV}) = 3{,}6\, s \sqrt{\dfrac{1{,}8 \times 10^4}{s + 1{,}8 \times 10^4}} = 6{,}58 \times 10^5 \text{ fb}$

$1 \text{ b} — 10^{-24} cm^2$
$6{,}58 \times 10^5 \times 10^{-15} — x \quad \Rightarrow \sigma(E = 1 \text{ PeV}) = 6{,}58 \times 10^{-34} cm^2$

$\dfrac{dN}{dx} = -\sigma n N \Rightarrow N = N_0 e^{-x/\lambda}$

where $\lambda = \dfrac{1}{\sigma n_{target}} = \dfrac{1}{\sigma N' \frac{\rho}{A} N_A} = \dfrac{1}{\sigma \rho N_A}$

with $\langle A \rangle = 30$ g/mol

$\langle N' \rangle = 30$

$N_A = 6{,}022 \times 10^{23}$

$\rho = 8 \text{ g cm}^{-3}$

$\Rightarrow \lambda = 3{,}15 \times 10^8$ cm

Prob. to survive: $\dfrac{N}{N_0} = e^{-x/\lambda}$

$\Rightarrow$ Prob to decay: $1 - e^{-x/\lambda}$

using for the diameter of the Earth

$d_E = 12742 \times 10^5$ cm

Prob. decay $= 1 - e^{-d_E/\lambda} = 0{,}98 = 98\%$

c) $\cos(\theta_c) = \frac{1}{\beta n} = \frac{1}{\frac{p_\mu}{E_\mu} n}$

$$\Rightarrow \theta_c = \arccos\left[\frac{1}{\frac{\sqrt{E_\mu^2 - m_\mu^2}}{E_\mu} n}\right] = 0{,}97 \text{ rad} = 55{,}8°$$

using $E_\mu = 10$ TeV $= 10^3$ GeV

$m_\mu = 105{,}6$ MeV $= 0{,}1056$ GeV

$n = 1{,}78$

d) From PDG 2016 (pag. 275)

$$\frac{d^2N}{dE\,dx} \simeq 370 \sin^2\theta_c \text{ eV}^{-1}\text{cm}^{-1}$$

$$\Rightarrow N_\gamma^{ch} = 370 \sin^2\theta_c \, \Delta x \, \underbrace{\Delta E}_{\langle E_\gamma \rangle}$$

$$= 370 \sin^2\theta_c (500 \times 10^2)(1 \text{ eV}) = 1{,}3 \times 10^7 \text{ photons}$$

2 - a) Not possible, violates charge

b) Not possible, violates charge

c) Not possible, violates lepton number

d) Possible

e) Not possible, violates lepton number

3 -

E_ν $\quad$ m_p $\qquad$ $P_\mu = (E_\nu + m_p, E_\nu)$

$$s = (E_\nu + m_p)^2 - E_\nu^2 = 2 E_\nu m_p + m_p$$

$$E_\nu = \frac{s - m_p^2}{2 m_p} \quad \text{if } s \gg m_p \Rightarrow E_\nu = \frac{s}{2 m_p}$$

$$\sigma = 3{,}6\, s \sqrt{\frac{1{,}8\, C}{s + 1{,}8\, C}} \; [fb] \qquad s\,[\mathrm{GeV}^2] \quad C = 10^4\,[\mathrm{GeV}^2]$$

$$\lambda = \frac{1}{\sigma\, n_{target}} = \frac{1}{\sigma N' \frac{\rho}{A} N_A} = \frac{1}{\sigma \rho N_A} \quad (1)$$

$$\frac{N}{N_0} = 10^{-3} = e^{-x_T/\lambda} \Leftrightarrow \lambda = \frac{x_T}{\ln(10^3)} \quad (2)$$

$\langle A \rangle = 30\ g/mol$; $\langle N' \rangle = 30$

$x_T = 12742 \times 10^5\ cm$; $\rho = 8\ g\,cm^{-3}$

$N_A = 6{,}022 \times 10^{23}$

Combining (1) and (2)

$$\frac{x_T}{\ln(10^3)} = \frac{1}{\sigma \rho N_A} \Leftrightarrow \sigma = \frac{\ln(10^3)}{x_T \rho N_A} = \alpha \; [cm^2]$$

$$\Rightarrow \quad 1\,b - 10^{-24}\,cm^2$$
$$10^{-15}\,b - y = 10^{-39}\,cm^2$$

$$\Rightarrow \sigma = 3,6 \times 10^{-39}\, s \sqrt{\frac{1,8\,C}{s + 1,8\,C}} \; [cm^2] = \alpha$$

solve to get s:

$$\beta s \sqrt{\frac{1,8\,C}{s+1,8\,C}} = \alpha \Leftrightarrow \frac{\beta}{\alpha} s = \sqrt{\frac{s+1,8\,C}{1,8\,C}} \Leftrightarrow$$

$$1,8\,C\left(\frac{\beta}{\alpha}\right)^2 s^2 = s + 1,8\,C \Leftrightarrow$$

$$1,8\,C\left(\frac{\beta}{\alpha}\right)^2 s^2 - s - 1,8\,C = 0$$

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1 + 4 \times 1,8 \left(\frac{\beta C}{\alpha}\right)^2}}{2C\left(\frac{\beta}{\alpha}\right)^2}$$

$$s \to \{-1,79 \times 10^4;\; 5,4 \times 10^6\} \; [GeV^2]$$

Taking the positive solution and noticing that $s \gg m_p$ ($m_p = 0,938$ GeV)

$$E_\nu = \frac{s}{2 m_p} = 2,9 \times 10^6\ GeV = 2,9\ PeV$$

4 - a)

- Interaction ν_τ, the mean free path is:

$$\lambda = \frac{1}{\sigma\, n_{target}} = \frac{1}{\sigma \rho N_A} = 3{,}15 \times 10^{8}\ \text{cm}$$

calculated in b)

- Decay of τ:

$$\langle x \rangle = vt = \beta c t = \beta c \tau \gamma = \frac{p}{E} c \tau \frac{E}{m} \quad (*)$$

$$\langle x \rangle = \frac{p}{m} c\tau = \frac{\sqrt{E^2 - m^2}}{m} c\tau$$

$m = m_\tau = 1{,}78\ \text{GeV}/c^2$ | $c\tau\,(\tau \text{ particle}) = 87{,}03\ \mu m$
$E = E_\tau = 10^6\ \text{GeV}$ | $= 87{,}03 \times 10^{-6}\ m$

$$\Rightarrow \langle x \rangle = 48{,}89\ m$$

Dividing by the diameter of Earth

$$d_t = 12742\ \text{Km}$$

$$\tau: \quad \frac{\langle x \rangle}{d_t} \simeq 3{,}8 \times 10^{-6}$$

$$\nu_\tau: \quad \frac{\lambda}{d_t} = 0{,}25$$

4-b) Let's consider the following decay

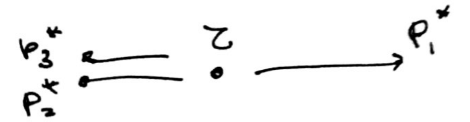
$$\tau \rightarrow \nu_\tau \quad e^- \quad \bar{\nu}_e$$

$$(P) \qquad (P_1) \quad (P_2) \quad (P_3)$$

In the LAB $P(E, P) = (0{,}9 \times 1\,\text{PeV}, P)$

To get the max and min energy of ν_τ let's start by evaluating the max and min in the centre-of-mass of the τ.

Although the decay of the τ is a 3-body system it is easy to see that in the CM P_1^* maximum occurs when



(I)

while the minimum is $P_1^* = 0$.

(II)

As such, the maximum energy of $\nu_\tau (P_1)$ is dotain when P_1^* is maximum and the particle is emitted along the direction of the primary (τ-boost) such that from Lorentz transformation one gets,

$$E_\nu = \gamma\,(E_1^* + \beta P_1^* \cos(0^\circ))$$, and since $m_\nu \simeq 0$

$$\Rightarrow E^* = P^*$$

$$E_\nu = \gamma\,(P_1^* + \beta P_1^*) \quad \text{with} \quad \gamma = \frac{E_\tau}{m_\tau}$$

$$\beta = \frac{P_\tau}{E_\tau} = \frac{\sqrt{E_\tau^2 - m_\tau^2}}{E_\tau}$$

The calculation of P_i^* in configuration I can be done if one realizes that the masses of the τ decay sub-products are much smaller than the mass of the τ

$$m_\tau \gg m_e \gg m_{\nu_\tau} \sim m_{\bar{\nu}_e}$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ \sim 1{,}8 \text{ GeV} & 511 \text{ keV} & \sim \text{eV} \end{array}$$

In this case one can simply write

$$E_\tau = E_2^* + E_3^* \longleftarrow \bullet \longrightarrow E_1^*$$

and $E_1^* = P_1^* = \dfrac{m_\tau}{2}$

Alternatively one could also assume the system $e^-, \bar{\nu}_e$ as a single particle Q and use the 2-body decay expression:

$$E_1^* = \frac{M^2 - m_Q^2 + m_1^2}{2M} = \frac{M^2 - (m_2^2 + m_3^2) + m_1^2}{2M} \simeq \frac{M}{2} \sim \frac{m_\tau}{2}$$

Hence, the maximum energy that the particle ν_τ can get in the LAB is $E_1 = \gamma\left(\frac{m_\tau}{2} + \beta\frac{m_\tau}{2}\right) \Rightarrow$

$$E_1 = \frac{E_\tau}{2} + \frac{P_\tau}{2} \qquad (E_\tau \sim P_\tau)$$

$$E_{\nu_\tau} \simeq E_\tau = 0{,}9 \times 10^{15} \text{ eV}$$

From configuration II it can be seen that the minimum energy in CM of ν_Z is ~~...~~ when it is at rest,

If one applies the boost the ν_Z would acquire positive momentum in the. However, it can be easily seen that one can play with the momenta of the e^- and the $\bar{\nu}_e$, and choose a momentum such that it cancels the boost action

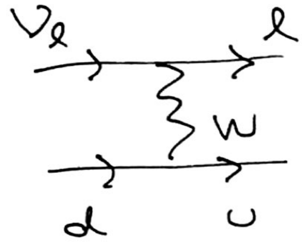
$$E_i = \gamma\left(E_i^* + \beta \cos(180^\circ)\, p_i^*\right)$$

$$p_i = \gamma\beta E^* + \gamma p_i^* \cos(180^\circ) = 0 \Leftrightarrow$$

$$\gamma\beta E_i^* = \gamma p_i^* \Leftrightarrow \beta_{CM} = \frac{p_i^*}{E_i^*} = \beta_i^*$$

Hence, the minimum energy of the ν_Z in the LAB is equal to its mass (m_{ν_Z}).

5 - To distinguish between different neutrino flavours one could exploit its charged current interaction, for instance,



where $l = e, \mu, \tau$

the leptons arising from this interaction would interact differently with the ice and as such neutrinos could be distinguished using the event topology, for instance:

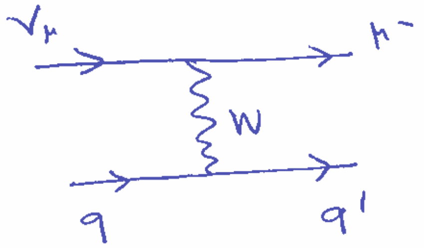
- μ: long-lived with nearly no interaction with matter $\Rightarrow$ produce a long trace of Cherenkov light 

- e: produce~~d~~ electromagnetic cascades $\Rightarrow$ shorter trace and broader 

- τ: would decay after some meters. ~~and into~~ ~~as~~ If it decays into quarks (electrons) it will produce a hadronic (e.m) shower.

$\Rightarrow$ signature would appear as a double-bump structure

(6) a) This a charged current process



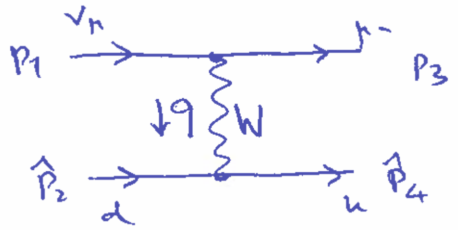
To conserve charge

$$Q(q') - Q(q) = 1$$

As q, q' are valence quarks (u, d) the solution is

$$q = d, \quad q' = u$$

b)



$$q = p_1 - p_3$$

c) $$i\mathcal{M} = \left(-i\frac{g}{\sqrt{2}}\right)^2 \bar{u}(p_3)\gamma^\mu P_L u(p_1)(-i)\frac{g_{\mu\nu} - q_\mu q_\nu/M_W^2}{q^2 - M_W^2}\bar{u}(\hat{p}_4)\gamma^\nu P_L u(\hat{p}_2)$$

Now we can neglect the terms proportional to q_μ in the numerator for massless fermions. In fact, for instance

$$\bar{u}(p_3)\gamma^\mu P_L u(p_1) q_\mu = \bar{u}(p_3)\not{p}_1 P_L u(p_1) - \bar{u}(p_3)\not{p}_3 P_L u(p_1)$$

$$= \bar{u}(p_3) P_R \not{p}_1 u(p_1) - \bar{u}(p_3)\not{p}_3 P_L u(p_1)$$

$$= 0$$

where we have used Dirac equation

$$\not{p}\, u(p) = 0 \quad \text{and} \quad \bar{u}(p)\not{p} = 0$$

therefore we can write

$$\mathcal{M} = \frac{g^2}{2} \frac{1}{q^2 - M_W^2} \bar{u}(p_3)\gamma^\mu P_L u(p_1)\, \bar{u}(\hat{p}_4)\gamma_\mu P_L(\hat{p}_2)$$

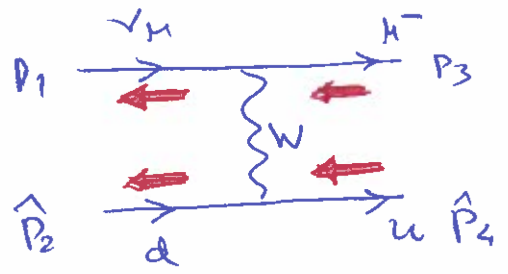
Now use $\frac{g^2}{2} = \frac{G_F}{\sqrt{2}} 4 M_W^2$; $Q^2 = -q^2$ to set

$$\boxed{\mathcal{M} = -\frac{4 G_F}{\sqrt{2}} \frac{M_W^2}{Q^2 + M_W^2} \bar{u}(p_3)\gamma^\mu P_L u(p_1)\, \bar{u}(\hat{p}_4)\gamma_\mu P_L u(\hat{p}_2)}$$

d) As the charged current is left handed we have just one helicity combination. Using

$$P_L u(p_1) = u(p_1, \downarrow) \quad ; \quad P_L u(\hat{p}_2) = u(\hat{p}_2, \downarrow)$$

we must have



and

$$\mathcal{M}(\downarrow\downarrow;\downarrow\downarrow) = -\frac{4 G_F}{\sqrt{2}} \frac{M_W^2}{Q^2 + M_W^2} J_{u_1 u_3}(\downarrow\downarrow) \cdot J_{u_2 u_4}(\downarrow\downarrow)$$

$$= -\frac{4 G_F}{\sqrt{2}} \frac{M_W^2}{Q^2 + M_W^2} \hat{s} \left(\cos\frac{\hat{\theta}}{2}, \sin\frac{\hat{\theta}}{2}, -i\sin\frac{\hat{\theta}}{2}, \cos\frac{\hat{\theta}}{2}\right) \cdot \left(\cos\frac{\hat{\theta}}{2}, -\sin\frac{\hat{\theta}}{2}, -i\sin\frac{\hat{\theta}}{2}, -\cos\frac{\hat{\theta}}{2}\right)$$

$$= -\frac{4 G_F}{\sqrt{2}} \frac{M_W^2}{Q^2 + M_W^2} \hat{s} \left[\cos^2\frac{\hat{\theta}}{2} - \left(-\sin^2\frac{\hat{\theta}}{2} - \sin^2\frac{\hat{\theta}}{2} - \cos^2\frac{\hat{\theta}}{2}\right)\right]$$

$$= -\frac{8 G_F}{\sqrt{2}} \frac{M_W^2}{Q^2 + M_W^2} \hat{s}$$

Therefore

$$\langle |M|^2 \rangle = \frac{1}{2} \left| M(\downarrow\downarrow;\downarrow\downarrow) \right|^2$$

$$= \frac{1}{2} \, 32 G_F^2 \hat{s}^2 \left[\frac{M_W^2}{Q^2 + M_W^2} \right]^2 = 16 G_F^2 \hat{s}^2 \left[\frac{M_W^2}{Q^2+M_W^2} \right]^2$$

e) $$\frac{d\hat{\sigma}}{d\Omega} = \frac{1}{64\pi^2 \hat{s}} \langle |M|^2 \rangle$$

then

$$\boxed{\frac{d\hat{\sigma}}{d\Omega} = \frac{G_F^2 \hat{s}}{4\pi^2} \left[\frac{M_W^2}{Q^2+M_W^2} \right]^2}$$

7 a) Now

$$Q(\bar{q}') - Q(\bar{q}) = 1$$

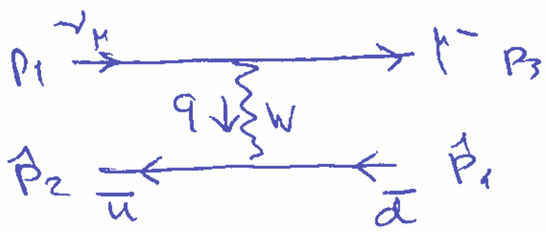
this means $Q(\bar{q}) = -\frac{2}{3}$ $Q(\bar{q}') = \frac{1}{3}$. The possibilities are

$$(\bar{q}, \bar{q}') = (\bar{u}, \bar{d}), \ (\bar{c}, \bar{s}), \ (\bar{t}, \bar{b})$$

The light quarks will have an higher probability, so we choose

$$\bar{q} = \bar{u}, \quad \bar{q}' = \bar{d}$$

b) The Diagram is now



c) The amplitude is

$$iM = \left(-i \frac{g}{\sqrt{2}}\right)^2 \bar{u}(p_3) \gamma^\mu P_L u(p_1) (-i) \frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2} \bar{v}(\vec{p}_2) \gamma^\nu P_L v(\vec{p}_4)$$

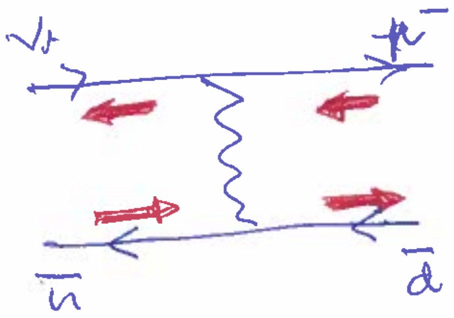
with the same approximation of ⑥ we get

$$\boxed{\mathcal{M} = -\frac{4G_F}{\sqrt{2}}\frac{M_W^2}{Q^2+M_W^2}\,\bar{u}(p_3)\gamma^\mu P_L u(p_1)\,\bar{v}(\vec{p}_2)\gamma_\mu P_L v(\vec{p}_4)}$$

d) There is again just one diagram. Using

$$P_L\, v(\vec{p}_4) = v(\vec{p}_4 \uparrow)$$

we have just the helicity combinations



and

$$M(\downarrow\uparrow;\downarrow\uparrow) = -\frac{4G_F}{\sqrt{2}}\frac{M_W^2}{Q^2+M_W^2}\,J_{u_1u_3}(\downarrow\downarrow)\cdot J_{v_2v_4}(\uparrow\uparrow)$$

$$= -\frac{4G_F}{\sqrt{2}}\frac{M_W^2}{Q^2+M_W^2}\,\hat{s}\left(\cos\frac{\hat\theta}{2}, \sin\frac{\hat\theta}{2}, -i\sin\frac{\hat\theta}{2}, \cos\frac{\hat\theta}{2}\right)\cdot\left(\cos\frac{\hat\theta}{2}, -\sin\frac{\hat\theta}{2}, i\sin\frac{\hat\theta}{2}, -\cos\frac{\hat\theta}{2}\right)$$

$$= -\frac{4G_F}{\sqrt{2}}\frac{M_W^2}{Q^2+M_W^2}\,\hat{s}\left[\cos^2\frac{\hat\theta}{2} - \left(-\sin^2\frac{\hat\theta}{2} + \sin^2\frac{\hat\theta}{2} - \cos^2\frac{\hat\theta}{2}\right)\right]$$

$$= -\frac{4G_F}{\sqrt{2}}\frac{M_W^2}{Q^2+M_W^2}\,\hat{s}\;2\cos^2\frac{\hat\theta}{2}$$

$$= -\frac{4G_F}{\sqrt{2}}\frac{M_W^2}{Q^2+M_W^2}\,\hat{s}\,(1+\cos\hat\theta)$$

Therefore

$$\langle |M|^2 \rangle = \frac{1}{2} \left| M(\downarrow\uparrow;\downarrow\uparrow) \right|^2$$

$$= \frac{1}{2} 8 G_F^2 \hat{s}^2 \left[\frac{M_W^2}{Q^2 + M_W^2} \right]^2 (1+\cos\hat{\theta})^2$$

$$= 4 G_F^2 \hat{s} \left[\frac{M_W^2}{Q^2 + M_W^2} \right]^2 (1+\cos\hat{\theta})^2$$

e)

$$\frac{d\hat{\sigma}}{d\Omega} = \frac{1}{64\pi^2 \hat{s}} \langle |M|^2 \rangle$$

then

$$\boxed{\frac{d\hat{\sigma}}{d\Omega} = \frac{G_F^2 \hat{s}}{16\pi^2} \left[\frac{M_W^2}{Q^2 + M_W^2} \right]^2 (1+\cos\hat{\theta})^2}$$

8 a) They are not good variables because the CM of the elementary process is not the CM of the overall process and the variables are not available.

b) If x is the fraction of the proton (nucleon) momentum carried by the quark we must have

$$\hat{P}_2 = x P_2$$

and therefore

$$y = \frac{P_2 \cdot (p_1 - p_2)}{P_2 \cdot p_1} = \frac{\hat{P}_2 \cdot (p_1 - p_3)}{\hat{P}_2 \cdot p_1}$$

Now this quantity is a Lorentz invariant and can be calculated in the CM of the elementary process (in fact in any frame, but to relate y to $\hat{\theta}$, this the one that matters).

In this frame, neglecting fermion masses, we have

$$p_1 = (E, 0, 0, E) \qquad \hat{p}_2 = (E, 0, 0, -E)$$

$$p_3 = (E, E\sin\hat{\theta}, 0, E\cos\hat{\theta}) \qquad \hat{p}_4 = (E, -E\sin\hat{\theta}, 0, -E\cos\hat{\theta})$$

where the energy $E = \frac{\sqrt{\hat{s}}}{2}$. We have then

$$p_1 - p_3 = (0, -E\sin\hat{\theta}, 0, E - E\cos\hat{\theta})$$

and

$$\hat{p}_2 \cdot (p_1 - p_3) = -(-E)\,E(1-\cos\hat{\theta}) = E^2(1-\cos\hat{\theta})$$

$$\hat{p}_2 \cdot p_1 = 2E^2$$

then

$$y = \frac{\hat{p}_2 \cdot (p_1 - p_3)}{\hat{p}_2 \cdot p_1} = \frac{E^2(1-\cos\hat{\theta})}{2E^2}$$

or

$$\boxed{y = \frac{1}{2}(1-\cos\hat{\theta})}$$

c) 1) they are distinct processes so sum the probabilities of each, that is the cross sections.

(7)

2) x is the fraction of momenta carried by the quark with respect to the momenta of the proton, that is

$$\boxed{\hat{P}_2 = x P_2}$$

Now

$$\hat{S} = (P_1 + \hat{P}_2)^2 = P_1^2 + \hat{P}_2^2 + 2P_1 \cdot \hat{P}_2 = 2 P_1 \cdot \hat{P}_2$$

where we neglect the fermion mass.

on the other hand

$$S = (P_1 + P_2)^2 = P_1^2 + P_2^2 + 2P_1 \cdot P_2 \simeq 2 P_1 \cdot P_2$$

where we neglect the neutrino but also the proton mass because the energy of the ν_μ is very large

using now $\hat{P}_2 = x P_2$ we get

$$\hat{S} = 2 P_1 \cdot \hat{P}_2 = x\, 2 P_1 \cdot P_2 = x S$$

$$\boxed{\hat{S} = x S}$$

3) From $y = \frac{1}{2}(1-\cos\hat{\theta})$

we get $(1+\cos\hat{\theta}) = 2(1-y)$

and

$$\frac{d\sigma}{dy} = \left|\frac{d\cos\hat{\theta}}{dy}\right| \frac{d\sigma}{d\cos\hat{\theta}} = 2\frac{d\sigma}{d\cos\hat{\theta})}$$

4) From the above relation between $\hat{\theta}$ and $\hat{y}$ we have

$$\theta = -\pi \Rightarrow y=1$$
$$\theta = \pi \Rightarrow y=0$$

so $y \in [0,1]$

x being the fraction of the momentum of the proton is obviously $x \in [0,1]$

5) For $\hat{\sigma}_{\nu, d\to \mu^- u}(xs)$

$$\frac{d\hat{\sigma}_{\nu d\to\mu^- u}}{d\cos\hat{\theta}^*} = 2\pi \frac{d\hat{\sigma}}{d\Omega} = 2\pi \frac{G_F^2 \hat{s}}{4\pi^2}\left[\frac{M_W^2}{Q^2+M_W^2}\right]^2$$

we get for a particular value of x

$$\boxed{\frac{d\hat{\sigma}_{\nu d\to\mu^- u}(xs)}{dy} = \frac{G_F^2\, xs}{\pi}\left[\frac{M_W^2}{Q^2+M_W^2}\right]^2}$$

for $\hat{\sigma}_{\nu,\bar{u}\to\bar{d}\mu^-}$, using $(1+\cos\hat{\theta}) = 2(1-y)$

$$\frac{d\hat{\sigma}_{\nu\bar{u}\to\mu^-\bar{d}}}{d\cos\theta^*} = 2\pi\frac{d\hat{\sigma}}{d\Omega} = \frac{2\pi\, G_F^2\hat{s}}{16\pi^2}\left[\frac{M_W^2}{Q^2+M_W^2}\right]^2 4(1-y)^2$$

$$= \frac{G_F^2\, xs}{2\pi}\left[\frac{M_W^2}{Q^2+M_W^2}\right]^2(1-y)^2$$

and we get for a particular $\underline{x}$

$$\frac{d\hat{\sigma}_{\nu\bar{u}\to\mu^-\bar{d}}(xs)}{dy} = \frac{G_F^2\, xs}{\pi}\left[\frac{M_W^2}{Q^2+M_W^2}\right]^2(1-y)^2$$

Now if $\underline{d(x)}$ is the probability of finding a quark $\underline{d}$ inside the proton with fraction of momentum x and $\bar{u}(x)$ the probability of finding an anti quark $\bar{u}$ with the same fraction of momentum x, The total cross section is clearly

$$\sigma_{\nu p\to\mu X} = \int_0^1 dx\int_0^1 dy\,\frac{G_F^2\, xs}{\pi}\left[d(x) + (1-y)^2\bar{u}(x)\right]\left[\frac{M_W^2}{Q^2+M_W^2}\right]^2$$

Note: Q^2 depends on x and y.