1st Test – Problem 1 – 12/1/2015

1- a)

$$\mathcal{L} = \frac{N_B - N_{background}}{\varepsilon\, \sigma_B\, \Delta t} = \frac{N_B - 0{,}1\, N_B}{\varepsilon\, \sigma_B\, \Delta t}$$

Knowing that

$\varepsilon = 0{,}95$ $\qquad \Delta t = 60 \times 60 \times 4 = 14400\ s$

$N_B = 1200$

$\sigma_B = 3{,}75\ mb$ where it was used $\theta_{min} = 29\ mrad$

$\theta_{max} = 185\ mrad$ and $s = (m_Z)^2 = 91{,}2^2\ GeV^2$

Hence,

$$\mathcal{L} = \frac{0{,}9}{0{,}95} \cdot \frac{1200}{3{,}75} \cdot \frac{1}{14400} = 0{,}021\ mb^{-1} s^{-1} = 2{,}1 \times 10^{31}\ cm^{2} s$$

where it was used $1b = 10^{-24}\ cm^2$

b) i) A high energy electron or positron in a calorimeter produces an electromagnetic (e.m.) shower. The cascade of particles increases basically due to two phenomena:

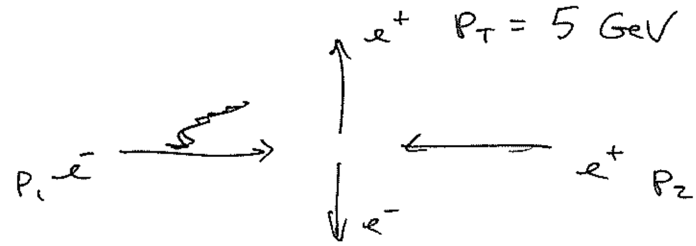
- e^+e^- pair creation by photons
- Emission of Bremsstrahlung radiation from e^-/e^+

The number of particles in the shower increases accordingly to the scheme until the produced photons don't have enough energy to convert into a e^+e^- pair. The development of the cascade depends on the radiation length in the calorimeter's material.

ii) $\sigma_{pair} \propto Z^2$; $\left(\frac{dE}{dx}\right)_{Bremsstr.} \propto Z^2$

Both processes that drive the e.m. shower development (e^+e^- pair creation; Bremsstrahlung) are proportional to the square of the number of protons in the calorimeter. Therefore, to minimize the total length of the e.m. shower the material from which the calorimeter is made should have a high atomic number (Z) and a high density (ρ) to increase the number of interactions.

1- c)



$P_1 = (E - E_\gamma, 0, 0, E - E_\gamma)$; $P_2 = (E, 0, 0, -E)$

where the masses of the leptons where neglected.

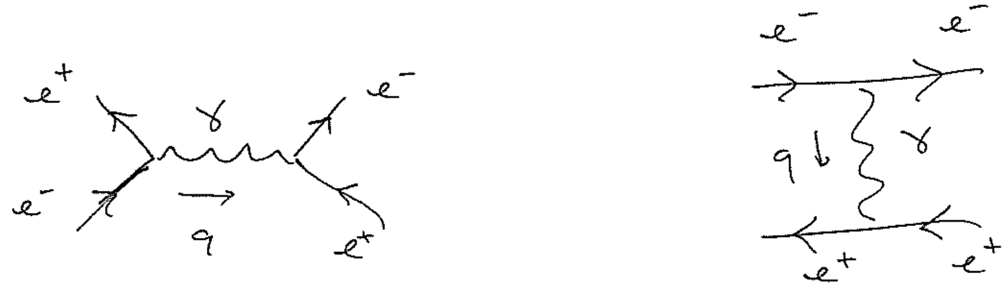
As the beam was tunned so that $\sqrt{s} = m_Z \Rightarrow E_{beam} = m_Z/2$

$$P_\mu = P_1 + P_2 = (2E - E_\gamma, -E_\gamma)$$

$$s'_{e^+e^-} = P_\mu P^\mu = 4E^2 + E_\gamma^2 - 4EE_\gamma - E_\gamma^2 = 4E^2\left(1 - \frac{E_\gamma}{E}\right)$$

$$\Rightarrow \sqrt{s'_{e^+e^-}} = 2E\sqrt{1 - \frac{E}{E_\gamma}} = 80{,}6 \text{ GeV}$$

1 - d) The lowest order QED Feynman diagrams for the e^+e^- scattering (Bhabha) are

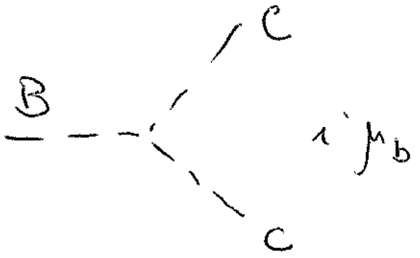


While for the s-channel there is a minimum energy that the propagator must have ($2m_e$) for the t-channel diagram the propagator energy can be very small ($E_\gamma \to 0$). Therefore, the latter diagram (t-channel) can happen without having to exchange a significant amount of energy making it more likely to occur.

In fact, the less energy transfered (q) the more probable it is, but also the less deflected are the initial particles.

Hence, for $\theta \to 0$ the e^+e^- scattering cross-section $\sigma \to \infty$ (similarly to what happens in the Rutherford experiment) and so the only relevant diagram for this process is the t-channel.

a) As $m_B > 2m_C$ the decay



is possible. The amplitude is

$$\mathcal{M} = i\,(i\mu_b) = -\mu_b$$

The formula for the differential decay width in the rest frame of the decaying particle is

$$\frac{d\Gamma}{d\Omega} = \frac{S}{32\pi^2 m_B^2}\,|\vec{p}_c|\,|\mathcal{M}|^2$$

with $S = 1/2$. As nothing depends on the angles we get

$$\Gamma = \frac{\mu_b^2}{16\pi m_B^2}\,|\vec{p}_c|$$

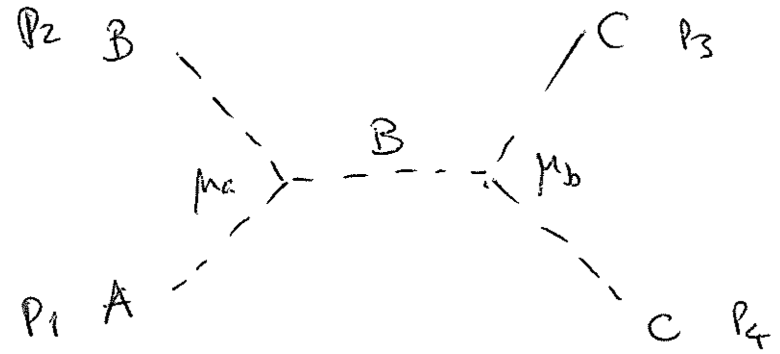
To calculate $|\vec{p}_c|$ we know that in the rest frame of B

$$E_c = \frac{m_B}{2} \Rightarrow |\vec{p}_c| = \sqrt{\frac{m_B^2}{4} - m_c^2} = \frac{m_B}{2}\sqrt{1-\frac{4m_c^2}{m_B^2}}$$

and therefore

$$\boxed{\Gamma_B = \frac{\mu_b^2}{32\pi\, m_B}\sqrt{1-\frac{4m_c^2}{m_B^2}}}$$

b) The only Diagram, in lowest order, is



the amplitude is

$$\mathcal{M} = i\,(i\mu_b)(i\mu_a)\frac{i}{s-m_B^2+i m_B\Gamma_B} = \frac{\mu_a\mu_b}{s-m_B^2+i m_B\Gamma_B}$$

c) $$\frac{d\sigma}{d\Omega} = \frac{S}{64\pi^2 s}\frac{|\vec{p}_3|}{|\vec{p}_1|}|\mathcal{M}|^2 \quad \text{with } S=\frac{1}{2!}$$

$$= \frac{1}{128\pi^2 s}\frac{|\vec{p}_3|}{|\vec{p}_1|^2}\frac{\mu_b^2\mu_a^2}{(s-m_B^2)^2+m_B^2\Gamma_B^2}$$

$$|\vec{p}_1| = \sqrt{E_A^2-m_A^2} \quad \text{with} \quad E_A = \frac{s+m_A^2-m_B^2}{2\sqrt{s}}$$

$$|\vec{p}_3| = \sqrt{E_3^2-m_C^2} \quad \text{with} \quad E_3 = \frac{\sqrt{s}}{2}$$

therefore

$$|\vec{p}_3| = \frac{\sqrt{s}}{2}\sqrt{1 - \frac{4m_c^2}{s}}$$

$$|\vec{p}_1| = \frac{1}{2\sqrt{s}}\sqrt{(s + m_A^2 - m_B^2)^2 - 4 s m_A^2}$$

$$= \frac{1}{2\sqrt{s}}\sqrt{s^2 + m_A^4 + m_B^4 - 2 s m_A^2 - 2 s m_B^2 - 2 m_A^2 m_B^2}$$

and finally

$$\frac{d\sigma}{d\Omega} = \frac{1}{128\pi^2 s} \frac{s\sqrt{1-\frac{4m_c^2}{s}}}{\sqrt{s^2 + m_A^4 + m_B^4 - 2 s m_A^2 - 2 s m_B^2 - 2 m_A^2 m_B^2}} |M|^2$$

d) We take $\sqrt{s} >> m_A, m_B, m_C$. Then

$$|M|^2 \rightarrow \frac{\mu_a^2 \mu_b^2}{s^2}$$

$$\frac{|\vec{p}_3|}{|\vec{p}_1|} \rightarrow 1$$

$$\frac{d\sigma}{d\Omega} \simeq \frac{1}{128\pi^2 s} \frac{\mu_a^2 \mu_b^2}{s^2}$$

and

$$\boxed{\sigma = \frac{\mu_a^2 \mu_b^2}{32\pi\, s^3}}$$

e) The cross section has dimension (4)

$$[\sigma] = \text{Energy}^{-2}$$

So

$$\sigma = \lambda \frac{\mu_a^2 \mu_b^2}{s^\alpha} \Rightarrow [\sigma] = \text{Energy}^{2+2-2\alpha}$$

where μ_a^2, μ_b^2 come from the cm. phys. We have then (Remember $[s] = \text{Energy}^2$)

$$4 - 2\alpha = -2$$

or $\boxed{\alpha = 3}$

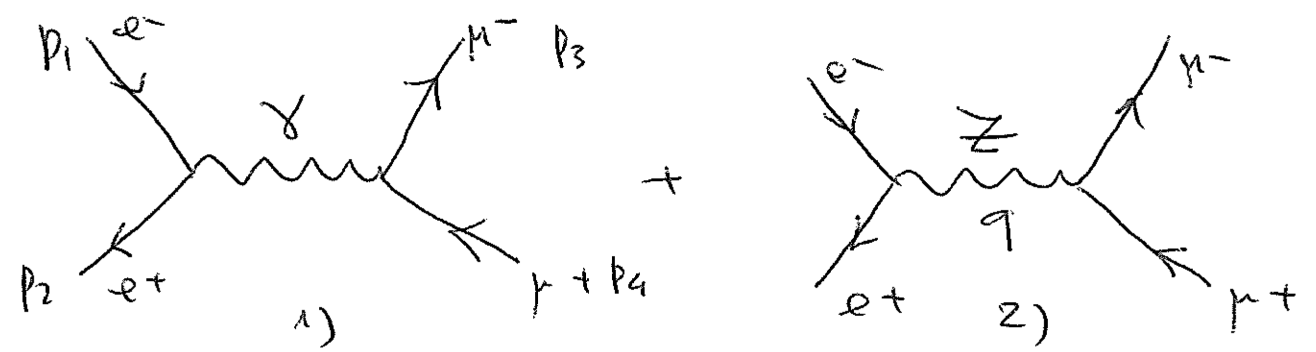
So final result

$$\sigma = \lambda \frac{\mu_a^2 \mu_b^2}{s^3}$$

Dimensionless constant

2^{nd} Test - Problem 3 12/1/2015

a)



b) If $\sqrt{s} \simeq M_Z$ the second diagram is the largest because of the Z denominator

$$\frac{1}{s - M_Z^2 + i M_Z \Gamma_Z} \simeq \frac{1}{i M_Z \Gamma_Z} \quad ; \quad \Gamma_Z << M_Z$$

then

$$\mathcal{M} \simeq M_2 = i\left(-\frac{ig}{\cos\theta_W}\right)^2 \bar{v}(p_2)\gamma^\mu(g_V^e - g_A^e\gamma_5)\, u(p_1)$$

$$-i\, \frac{\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_Z^2}\right)}{s - M_Z^2 + i M_Z \Gamma_Z} \cdot \bar{u}(p_3)\gamma^\nu(g_V^\mu - g_A^\mu\gamma_5)\, v(p_4)$$

c) One such term in the electron line would be proportional to

$$X = \bar{v}(p_2)\gamma^\mu(g_V^e - g_A^e\gamma_5)\, u(p_1)\, q_\mu$$

with $q = p_1 + p_2$.

therefore

$$X = \bar{v}(p_2)\, \not{p}_1 (g_V^e - g_A^e \gamma_5) u(p_1)$$

$$+ \bar{v}(p_2)\, \not{p}_2 (g_V^e - g_A^e \gamma_5) u(p_1)$$

$$= \bar{v}(p_2) (g_V^e + g_A^e \gamma_5) \not{p}_1 u(p_1)$$

$$+ \bar{v}(p_2)\, \not{p}_2 (g_V^e - g_A^e \gamma_5) u(p_1)$$

$$= 0$$

because for massless fermions we have

$$\not{p}_1 u(p_1) = 0$$

$$\bar{v}(p_2)\, \not{p}_2 = 0$$

therefore

$$\mathcal{M} = -\left(\frac{g}{\cos\theta_W}\right)^2 \frac{i}{i M_Z \Gamma_Z}\, \bar{v}(p_2)\gamma^\mu (g_V^e - g_A^e \gamma_5) u(p_1)\, \bar{u}(p_3)\gamma_\mu (g_V^\mu - g_A^\mu \gamma_5) v(p_4)$$

d) For the electron we have $(1 = P_L + P_R)$

$$\bar{v}(p_2)\gamma^\mu (g_V^e - g_A^e \gamma_5) u(p_1) = \bar{v}(p_2)\gamma^\mu (g_V^e - g_A^e \gamma_5)(P_L + P_R) u(p_1)$$

Now $\gamma_5 P_L = -P_L \;;\; \gamma_5 P_R = P_R$

So

$$\bar{v}(p_2)\gamma^\mu(g_V^e - g_A^e\gamma_5)u(p_1)$$

$$= \bar{v}(p_2)\gamma^\mu\left[(g_V^e + g_A^e)P_L + (g_V^e - g_A^e)P_R\right]u(p_1)$$

$$= g_L^e\,\bar{v}(p_2)\gamma^\mu P_L u(p_1) + g_R^e\,\bar{v}(p_2)\gamma^\mu P_R u(p_1)$$

with

$$g_L^e = g_V^e + g_A^e \quad ; \quad g_R^e = g_V^e - g_A^e$$

e) Using the result of d) we can write

$$\mathcal{M} = C_Z\left[g_L^e\,\bar{v}(p_2)\gamma^\mu P_L u(p_1) + g_R^e\,\bar{v}(p_2)\gamma^\mu P_R u(p_1)\right]$$

$$\left[g_L^\mu\,\bar{u}(p_3)\gamma_\mu P_L v(p_4) + g_R^\mu\,\bar{u}(p_3)\gamma_\mu P_R v(p_4)\right]$$

with $C_Z = -\left(\frac{g}{\cos\theta_W}\right)^2 \frac{1}{iM_Z\Gamma_Z}$

We therefore have

$$\mathcal{M} = C_Z\, g_L^e g_L^\mu\,\bar{v}(p_2)\gamma^\mu P_L u(p_1)\,\bar{u}(p_3)\gamma_\mu P_L v(p_4)$$

$$+ C_Z\, g_L^e g_R^\mu\, \bar{v}(p_2)\gamma^\mu P_L u(p_1)\, \bar{u}(p_3)\gamma_\mu P_R v(p_4)$$

$$+ C_Z\, g_R^e g_L^\mu\, \bar{v}(p_2)\gamma^\mu P_R u(p_1)\, \bar{u}(p_3)\gamma_\mu P_L v(p_4)$$

$$+ C_Z\, g_R^e g_R^\mu\, \bar{v}(p_2)\gamma^\mu P_R u(p_1)\, \bar{u}(p_3)\gamma_\mu P_R v(p_4)$$

$$= \mathcal{M}(\downarrow\uparrow;\downarrow\uparrow) + \mathcal{M}(\downarrow\uparrow;\uparrow\downarrow)$$

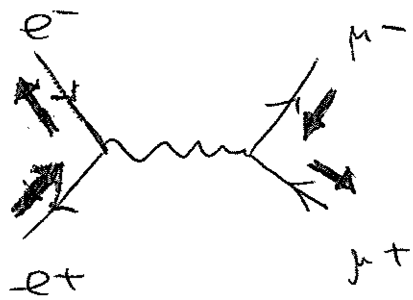
$$+ \mathcal{M}(\uparrow\downarrow;\downarrow\uparrow) + \mathcal{M}(\uparrow\downarrow;\uparrow\downarrow)$$

with the notation

$$\mathcal{M}(h_{e-}\, h_{e+};\, h_{\mu-}\, h_{\mu+})$$

we obtain therefore

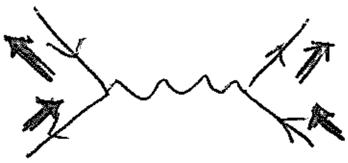
$$\mathcal{M}(\downarrow\uparrow;\downarrow\uparrow) \equiv$$



$$= C_Z\, g_L^e g_L^\mu\, J_{u_1 v_2}(\downarrow\uparrow)\cdot J_{u_3 v_4}(\downarrow\uparrow)$$

$$= C_Z\, g_L^e g_L^e\, s(0,-1,i,0)\cdot(0,-\cos\theta,-i,\sin\theta)$$

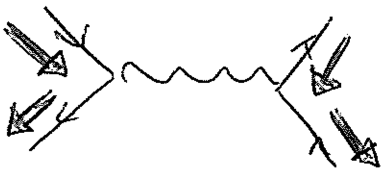
$$= -C_Z\, g_L^e g_L^e\,(1+\cos\theta)$$

$\mathcal{M}(\downarrow\uparrow;\uparrow\downarrow) \equiv$ 

$= C_Z\, g_L^e g_R^\mu \quad J_{u_1 v_2}(\downarrow\uparrow)\cdot J_{u_3 v_4}(\uparrow\downarrow)$

$= C_Z\, g_L^e g_R^\mu \; s\,(0,-1,i,0)\cdot(0,-\cos\theta,\, i,\sin\theta)$

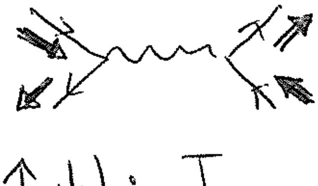
$= -C_Z\, g_L^e g_R^\mu\, s\,(1-\cos\theta)$

$\mathcal{M}(\uparrow\downarrow;\downarrow\uparrow) \equiv$ 

$= C_Z\, g_R^e g_L^\mu \quad J_{u_1 v_2}(\uparrow\downarrow)\cdot J_{u_3 v_4}(\downarrow\uparrow)$

$= C_Z\, g_R^e g_L^\mu \; s\,(0,-1,-i,0)\cdot(0,-\cos\theta,-i,\sin\theta)$

$= -C_Z\, g_R^e g_L^\mu \; s\,(1-\cos\theta)$

$\mathcal{M}(\uparrow\downarrow;\uparrow\downarrow) \equiv$ 

$= C_Z\, g_R^e g_R^\mu \quad J_{u_1 v_2}(\uparrow\downarrow)\cdot J_{u_3 v_4}(\uparrow\downarrow)$

$= C_Z\, g_R^e g_R^\mu \; s\,(0,-1,-i,0)\cdot(0,-\cos\theta,\, i,\, \sin\theta)$

$= -C_Z\, g_R^e g_R^\mu \; s\,(1+\cos\theta)$

f) $$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\langle|\mathcal{M}|^2\rangle$$ where

$$\langle|\mathcal{M}|^2\rangle = \frac{1}{4}\Big[|\mathcal{M}(\downarrow\uparrow;\downarrow\uparrow)|^2 + |\mathcal{M}(\downarrow\uparrow;\uparrow\downarrow)|^2 + |\mathcal{M}(\uparrow\downarrow;\downarrow\uparrow)|^2 + |\mathcal{M}(\uparrow\downarrow;\uparrow\downarrow)|^2\Big]$$

$$= \frac{1}{4}C_Z^2 s^2\Big[(1+\cos\theta)^2\left(g_L^{e2}g_L^{\mu 2} + g_R^{e2}g_R^{\mu 2}\right) + (1-\cos\theta)^2\left(g_L^{e2}g_R^{\mu 2} + g_R^{e2}g_L^{\mu 2}\right)\Big]$$

$$= \frac{1}{4}C_Z^2 s^2\Big[(1+\cos^2\theta)\left(g_L^{e2}g_L^{\mu 2} + g_R^{e2}g_R^{\mu 2} + g_L^{e2}g_R^{2} + g_R^{e2}g_L^{\mu 2}\right) + 2\cos\theta\left(g_L^{e2}g_L^{\mu 2} + g_R^{e2}g_R^{\mu 2} - g_L^{e2}g_R^{\mu 2} - g_R^{e2}g_L^{e2}\right)\Big]$$

g) $\int_0^{\pi/2} d\Omega$ selects events forward $(\theta < \pi/2)$

$\int_{\pi/2}^{\pi} d\Omega$ selects events backward $(\theta > \pi/2)$

therefore the name.

As $\int_0^{\pi/2} d\Omega\,(1+\cos^2\theta) = \int_{\pi/2}^{\pi} d\Omega\,(1+\cos^2\theta)$

and $\int_0^{\pi/2} d\Omega\,\cos\theta = -\int_{\pi/2}^{\pi} d\Omega\,\cos\theta$

A_{FB} is proportional to the terms in $\cos\theta$. These are proportional to

$$A_{FB} \propto \left(g_L^{e^2} g_L^{\mu^2} + g_R^{e^2} g_R^{\mu^2} - g_L^{e^2} g_R^{\mu^2} - g_R^{e^2} g_L^{\mu^2}\right)$$

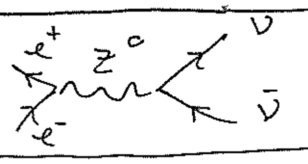
$$\propto g_V^e g_A^e g_V^\mu g_A^\mu$$

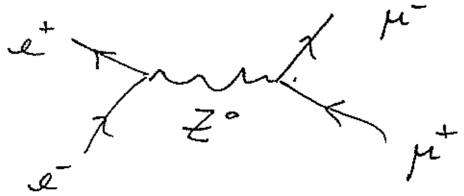
It is therefore sensitive to the sign of the SM. couplings, while the total cross section is not. This gives important information in testing the couplings of the Standard Model.

2nd Test Problem 4 12/1/2015

4. -

a) $\dfrac{\sigma(e^+e^- \to \bar{\nu}\nu)}{\sigma(e^+e^- \to \mu^+\mu^-)}$ =





Let's start by noting that at the Z^0 ressonance $\sqrt{s} = m_Z$ the relevant diagram are the ones above. To compute this ratio it is usefull to note that the only quantity that is different is the Z^0 vertice of the final state products.

Let us the consider the Z^0 vertice,

$$\frac{-ig}{\cos\theta_W}\gamma^\mu \underbrace{\left(g_V^f - g_A^f \gamma_5\right)}_{\frac{1}{2}\left(f_L(1-\gamma_5) + f_R(1+\gamma_5)\right)}$$

with

$$f_L = \frac{1}{2}(g_V + g_A) \; ; \; f_R = \frac{1}{2}(g_V - g_A)$$

Therefore the current of $\mathcal{M}$ can be split into

$$\bar{\psi}_L \gamma^\mu (g_V + g_A)\psi_L + \bar{\psi}_R \gamma^\mu (g_V - g_A)\psi_R$$

Again noting that the dirac structure for the two process is the same, and considering the ratio between them

$$\sigma(Z^0 \to f\bar{f}) \propto |\mathcal{M}^2| \propto (g_V + g_A)^2 + (g_V - g_A)^2$$

$$\sigma(Z^0 \to f\bar{f}) \;\propto\; {g_V^f}^2 + {g_A^f}^2$$

Using,

$$g_V^f = \frac{1}{2} T_f^3 - Q_f \operatorname{sen}^2\theta_W \; ; \; g_A^f = \frac{1}{2} T_f^3 \; ; \; \operatorname{sen}^2\theta_W \simeq 0{,}231$$

One can build the following table for the Z^0 coupling with different fermions

f	T_f^3	Q	g_A	g_V	$g_V^2 + g_A^2$
ν_e, ν_μ, ν_τ	1/2	0	+0,25	0,25	0,125
e, μ, τ	-1/2	-1	-0,25	-0,02	0,06
u, c, t	1/2	2/3	+0,25	0,096	0,07
d, s, b	-1/2	-1/3	-0,25	-0,17	0,09

We can ~~R~~now evaluate

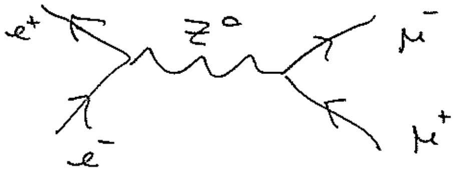
$$\frac{\sigma(e^+e^- \to \nu\bar{\nu})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{\left[(g_V^\nu)^2 + (g_A^\nu)^2\right] \times \text{number neutrinos}}{\left[(g_V^\mu)^2 + (g_A^\mu)^2\right]}$$

$$= \frac{3 \times 0{,}125}{0{,}06} = 6{,}25 \,//$$

4-

b)

$$\frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{\text{diagram 1}}{\text{diagram 2}} =$$



using the same strategy as in a)

$$= \frac{\underset{\text{color}}{3} \times \underset{\text{quarks}(d,s,b)}{3} \times 0{,}09 + \underset{\text{color}}{3} \times \underset{\text{quarks}(u,c)}{2} \times 0{,}07}{0{,}06} = 20{,}5$$

Note that ~~a~~ Z^0 ~~produced~~ mass is smaller than the top mass.

4- c) Due to the neutrino very low cross-section, neutrinos cannot be detected at the LEP detectors and thus the process $Z^0 \to \nu\bar{\nu}$ cannot be observed directly.

However, the total width of Z^0 boson is,

$$\Gamma_Z = 3\,\Gamma_{\ell\bar{\ell}} + \Gamma_{\text{hadrons}} + N_\nu \Gamma_{\nu\bar{\nu}}$$

where it was used the lepton universality of the weak interaction.

Hence, N_ν (number of neutrino families) can be measured using,

$$N_\nu = \frac{\Gamma_Z - 3\Gamma_{\ell\bar{\ell}} - \Gamma_{hadrons}}{\Gamma_{\nu\bar{\nu}}^{SM}} = \frac{\Gamma_{invisible}}{\Gamma_{\nu\bar{\nu}}^{SM}}$$

where,

$\Gamma_Z \rightarrow$ is the total width of Z° and is proportional to its lifetime and can be measured from any decay mode $\Gamma(Z^\circ \rightarrow f\bar{f})$

$\Gamma_{\ell\bar{\ell}} \rightarrow$ is the partial width of $Z^\circ \rightarrow \cancel{f} \ell\bar{\ell}$ (lepton states)

$\Gamma_{hadrons} \rightarrow$ is the partial width of Z° decaying into $q\bar{q}$ (final states of hadronic nature)

The partial widths can be measured through the measurement of the process cross-section,

$\sigma(e^+e^- \rightarrow Z^\circ \rightarrow f\bar{f}) \propto \Gamma(Z^\circ \rightarrow f\bar{f})$ i.e., $\Gamma_{f\bar{f}}$

$\Gamma_{\nu\bar{\nu}}^{SM} \rightarrow$ is the partial width for the process $Z^\circ \rightarrow \nu_i \bar{\nu}_i$ as predicted by the Standard Model