1 -

a) $x = vt = \beta c t = \beta c \gamma \tau$

knowing that $p = \gamma m \beta c \Leftrightarrow \frac{p}{m} = \gamma \beta c$

$\Rightarrow x = \frac{p}{m} \tau \, (c)$ and from PDG $c\tau_{\pi^+} = 7,8\,m$

↳ to put in S.I. units

$$x = \frac{145\,MeV}{140\,MeV} \, 7,8\,m = 8,08\,m$$

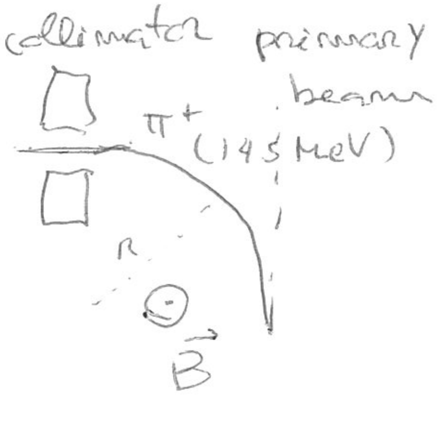
b) (possible answer)

→ Use magnetic fied to deflect particles

→ Deflection depends of particle mass, charge, velocity and of B

→ Use a collimator to stop close-by unwanted particle and create beam

→ Collimator has to be put before the π^+ decays $(x \geq \frac{2\pi r}{4})$



$r = \frac{p_\pi}{qB}$; $r \sim \frac{2x}{\pi} \simeq 5\,m$

x ≡ mean ~~free~~ travelled distance

$$\Rightarrow B = \frac{p_\pi}{2x\,|e|} \left(\frac{e}{c} \right) = 0,1\,T$$

↳ S.I.

1 -

c)

i) $\frac{dN}{dx} = - \underbrace{\frac{\sigma N_A \rho}{A}}_{\lambda} N \Rightarrow N = N_0 e^{-\lambda x}$

$$\ln\left(\frac{N}{N_0}\right) = -\sigma \underbrace{\left(\frac{N_A \rho x}{A}\right)}_{\text{projected number of protons} \equiv y} = -\sigma y$$

$$\sigma = \frac{\ln(N_0/N)}{y} = 1{,}29 \times 10^{-25}\ cm^2 = 129\ mb$$

where it was used

$$\frac{N_0}{N} = 1{,}116 \ ; \ y = 8{,}5 \times 10^{23}\ cm^{-2} \ ; \ 1b = 10^{-24}\ cm^2$$

ii) From the PDG

$$\frac{dE}{dX} \sim 5\ MeV/(g\ cm^{-2})$$

$$E_{loss} = \frac{dE}{dX}[MeV/g\,cm^2] \cdot X\,[g\,cm^{-2}]$$

Knowing that the molar mass of hydrogen is

$M_H = 1\ g/mol$ then,

~~\$~~ 1) c) (ii) (cont)

$1 g$ —— $6,022 \times 10^{23}$ atoms $\equiv$ mol

m —— $8,5 \times 10^{23}$ atoms

$\Rightarrow m = 1,41 g$ and therefore $X = 1,41 \text{ g cm}^{-2}$ (depth)

Hence,

$$E_{loss} = \frac{dE}{dX} \cdot X = 7,05 \text{ MeV}$$

~~From~~ For the dispersion from the PDG one has that the angle due to Coulomb multiple scattering is given ~~from~~ by,

$$\Theta_0 = \frac{13,6 \text{ MeV}}{\beta c p} z \sqrt{\frac{X}{X_0}} \left[1 + 0,038 \ln\left(\frac{X}{X_0}\right)\right]$$

$X_0(H_2) = 63,04 \text{ g cm}^{-2}$ (end of PDG)

$X = 1,41 \text{ g cm}^{-2}$; $c = 1$; $z = 1$

$\Rightarrow \Theta_0 = 0,017 \text{ rad} \simeq 0,96°$

1-d) The angle that the μ^+ can have with the beam depends of the relation between β^*_μ and $\beta_{CM} \equiv \beta_\pi$

Two situations can be identified

$\beta^*_\mu > \beta_{CM}$

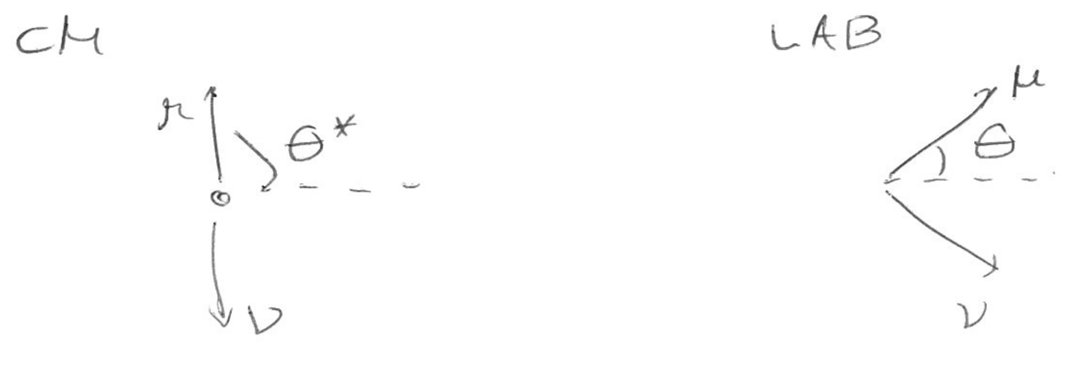
(LAB)

The μ is emitted backwards and the system boost is not big enough to reverse its velocity.

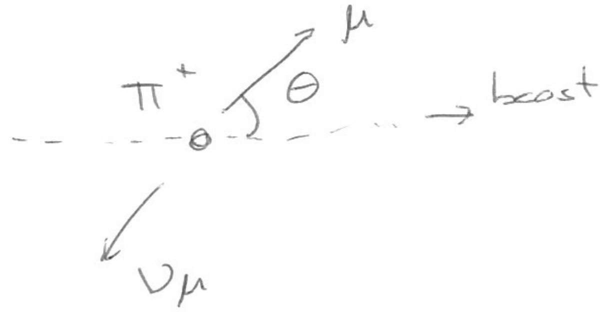
In this case the maximum possible angle with the beam is 180°

If the μ velocity (β^*_μ) in the CM is smaller than β_{CM} then the μ is always emitted in the forward direction.

In this case the μ maximum angle with the beam line occurs when the muon is emitted tranverselly to the boost (beam) direction. $\theta^* = \pi/2$



1 - d) In the CM



$\gamma = \frac{E_\pi}{m_\pi}$ | and $E_\pi = \sqrt{P_\pi^2 + m_\pi^2}$

$\beta = \frac{P_\pi}{E_\pi}$

First one needs to check if the muon can be emitted backwards in the LAB, i.e., β_μ^* is greater that $\beta_{CM} \equiv \beta_\pi$ ~~a~~

$$\beta_\mu = \frac{P_\mu^*}{E_\mu^*} \quad \text{and} \quad \beta_{CM} = \frac{P_\pi}{\sqrt{P_\pi^2 + m_\pi^2}} = 0{,}72$$

To get P_μ^* (in the CM) one uses,

$$P_\pi = P_\nu + P_\mu \Leftrightarrow \text{(4- vectors)}$$

$$(P_\pi - P_\mu)^2 = P_\nu^2 \Leftrightarrow P_\pi^2 + P_\mu^2 - 2P_\pi . P_\mu = P_\nu^2 \quad \text{(eq. 1)}$$

knowing that in CM

$P_\pi = (m_\pi, \vec{0})$

$P_\mu = (E_\mu^*, P_\mu^*)$

$P_\nu = (P_\mu^*, -P_\mu^*)$

because $P_\mu = -P_\nu$ and $E_\nu = P_\nu$

Using the 4-vector inner product

$$P_\mu P^\mu = E^2 - P^2 \quad \text{and} \quad E^2 = P^2 + m^2$$

1- d) (cont.)

one can transform eq. 1 into

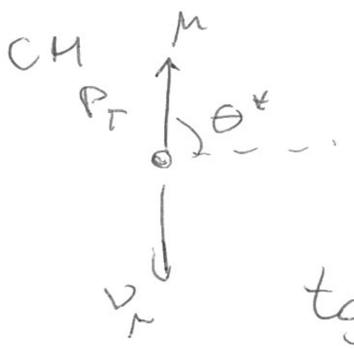
$$m_\pi^2 + m_\mu^2 - 2E_\mu^* m_\pi = 0 \Leftrightarrow E_\mu^* = \frac{m_\pi^2 + m_\mu^2}{2m_\pi}$$

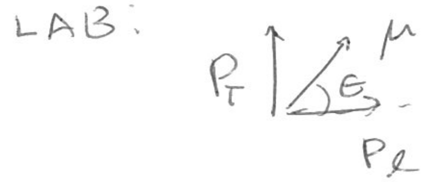
Hence, $E_\mu^* = 110$ MeV and

$$\beta_\mu^* = \frac{P_\mu^*}{E_\mu^*} = 0{,}298$$

Therefore $\beta_\mu^* < \beta_{CM}$ and the μ is always emitted in the direction of the boost.

Thus, the maximum angle occurs for $\theta^* = 90°$





$$tg\,\theta = \frac{P_T}{P_\ell}$$

$P_\mu^* = P_T^* = P_T$ (no boost in the transverse direction)

$$\begin{pmatrix} E \\ P_\ell \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} E^* \\ P_\ell^* \end{pmatrix}$$

$$\Rightarrow P_\ell = \gamma\beta E_\mu^* + \gamma \underbrace{P_\mu^* \cos\theta^*}_{0} = \gamma\beta E_\mu^*$$

$$tg\,\theta = \frac{P_\mu^*}{\gamma\beta E_\mu^*} = \frac{\sqrt{E_\mu^{*2} - m_\mu^2}}{\frac{E_\pi}{m_\pi}\frac{P_\pi}{E_\pi} E_\mu^*} \Rightarrow \theta \simeq 16°$$

1 - d)

LAB energy

$$E_\mu = \gamma E_\mu^* + \gamma \beta \cos\theta^*$$

Max E: $\theta^* = 0 \Rightarrow E_\mu^{max} = \gamma E_\mu^* + \gamma\beta P_\mu^* = 190{,}3$ MeV

Min E: $\theta^* = 180° \Rightarrow E_\mu^{min} = \gamma E_\mu^* - \gamma\beta P_\mu^* = 126{,}5$ MeV

knowing that in CM the muon angular production distribution is isotropic

$$\Rightarrow \frac{dN}{d\cos\theta^*} = cte$$

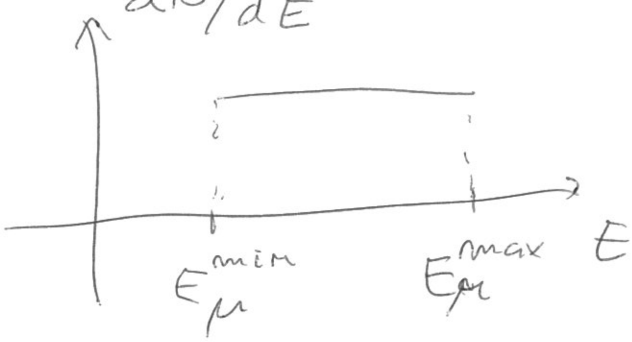
$$\frac{dN}{dE} = \frac{dN}{d\cos\theta^*} \frac{d\cos\theta^*}{dE}$$

↳ energy spectrum in the LAB

Since, $\cos\theta^* = \dfrac{E_\mu - \gamma E_\mu^*}{\gamma\beta P_\mu^*}$

$$\frac{dN}{dE} = cte \times \frac{1}{\gamma\beta P_\mu^*} = cte \Rightarrow \text{uniform}$$

Hence



Particle Physics – TEST 1 – 24/10/2015

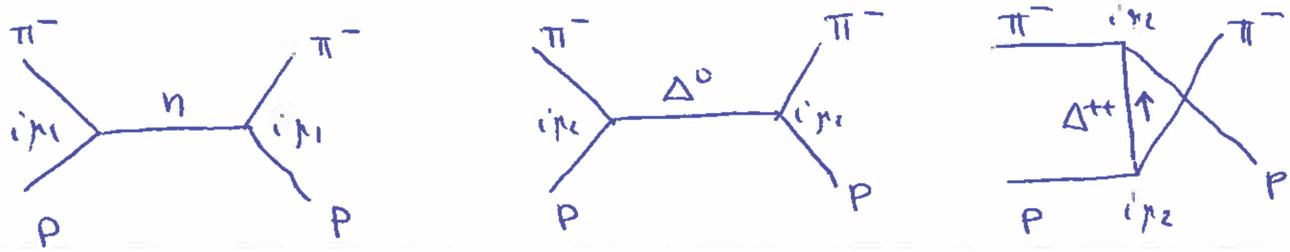
Problem 2

a) To be able to draw all diagrams we need to know the possible charges for the particles. The n is neutral and the p is positively charged. As for the pion it is a triplet of isospin 1 with π^+, π^- and π^0 as you can look in the PDG.

The PDG also tells you that the Δ has $J = \frac{3}{2}$ and therefore $2 \times \frac{3}{2} + 1 = 4$ states. The PDG tells us that we have

$$\Delta^{++}, \Delta^{+}, \Delta^{0}, \Delta^{-}$$

With this knowledge we have the diagrams



where the indicated charges flow from left to right except the Δ^{++} that flows as indicated.

b) For the process to occur we should have in the CM frame

$$\sqrt{s} \geq m_p + m_{\Delta^0}$$

On the other hand the 4-momenta of the initial particles in the Lab are

$$p_1 \equiv (E_1, 0, 0, \sqrt{E_1^2 - m_n^2})$$

$$p_2 = (M_p, 0, 0, 0)$$

the Mandelstam variable $s = (p_1 + p_2)^2$ is an invariant and has the meaning of the square of the CM energy. Therefore

$$s = (p_1 + p_2)^2 = m_p^2 + m_n^2 + 2E_1 m_p \geq (m_p + m_{\Delta^0})^2$$

or

$$2E_1 m_p = m_{\Delta^0}^2 + 2m_p m_{\Delta^0} - m_n^2$$

$$\boxed{E_1 = \frac{m_{\Delta^0}^2 + 2m_p m_{\Delta^0} - m_n^2}{2m_p}}$$

Numerically (PDG)

$m_p = 938.27$ MeV

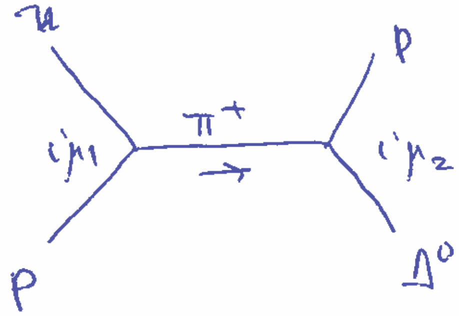
$m_n = 939.56$ MeV

$m_{\Delta^0} = 1232$ MeV

and we get

$$E_1 = 1570.4 \text{ MeV}$$

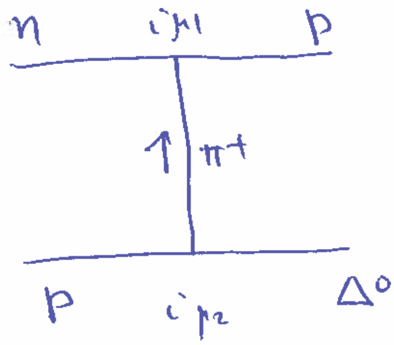
c)

1) 

$$\Rightarrow \quad -iM_1 = (i\mu_1)(i\mu_2)\frac{i}{(p_1+p_2)^2 - m_\pi^2}$$

or

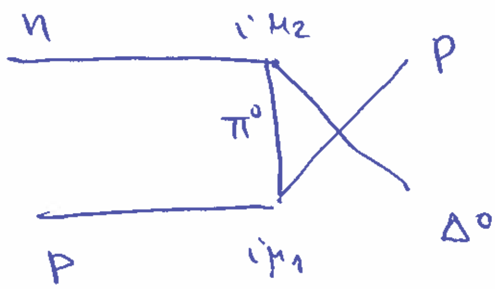
$$M_1 = \frac{\mu_1\mu_2}{s - m_\pi^2} \qquad s = (p_1+p_2)^2$$

2) 

$$-iM_2 = (i\mu_1)(i\mu_2)\frac{i}{(p_1-p_3)^2 - m_\pi^2}$$

or

$$M_2 = \frac{\mu_1\mu_2}{t - m_\pi^2} \qquad t = (p_1-p_3)^2$$

3) 

$$-iM = (i\mu_1)(i\mu_2)\frac{i}{(p_1-p_4)^2 - m_{\pi^0}^2}$$

or

$$M_3 = \frac{\mu_1\mu_2}{u - m_{\pi^0}^2} \qquad u = (p_1-p_4)^2$$

d) We have

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{|\vec{p}_3^{\,CM}|}{|\vec{p}_1^{\,CM}|} |M|^2$$

we have

$$E_1^{CM} = \frac{s + m_n^2 - m_p^2}{2\sqrt{s}} \qquad E_3^{CM} = \frac{s + m_p^2 - m_{\Delta^0}^2}{2\sqrt{s}}$$

Therefore

$$|\vec{p}_1^{\,CM}| = \sqrt{(E_1^{CM})^2 - m_n^2} = \frac{1}{2\sqrt{s}}\sqrt{s^2 + m_n^4 + m_p^4 - 2s m_n^2 - 2s m_p^2 - 2m_p^2 m_n^2}$$

$$|\vec{p}_3^{\,CM}| = \sqrt{(E_3^{CM})^2 - m_p^2} = \frac{1}{2\sqrt{s}}\sqrt{s^2 + m_{\Delta^0}^4 + m_p^4 - 2s m_p^2 - 2s m_{\Delta^0}^2 - 2m_p^2 m_{\Delta^0}^2}$$

and

$$M = \mu_1\mu_2 \left[\frac{1}{s - m_{\pi^+}^2} + \frac{1}{t - m_{\pi^+}^2} + \frac{1}{u - m_{\pi^0}^2} \right]$$

we get finally

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{(\mu_1\mu_2)^2}{64\pi^2 s} \frac{\sqrt{s^2 + m_p^4 + m_{\Delta^0}^4 - 2s m_p^2 - 2s m_{\Delta^0}^2 - 2m_p^2 m_{\Delta^0}^2}}{\sqrt{s + m_n^4 + m_p^4 - 2s m_p^2 - 2s m_n^2 - 2m_n^2 m_p^2}} \left[\frac{1}{s - m_{\pi^+}^2} + \frac{1}{t - m_{\pi^+}^2} + \frac{1}{u - m_{\pi^0}^2} \right]}$$

e) Limit $\sqrt{s} >$ all mass. this means

$$\sqrt{s} \gg m_{\pi}, m_{p}, m_{\Delta}$$

but also

$$t \gg m_{\pi^+}^2 \quad ; \quad u \gg m_{\pi^0}^2$$

we get

$$\frac{d\sigma}{d\Omega} = \frac{(\mu_1 \mu_2)^2}{64\pi^2 s} \left[\frac{1}{s} + \frac{1}{t} + \frac{1}{u} \right]^2$$

Now, t and u have angles. Neglecting masses

$$t = (p_1 - p_3)^2 = -2 p_1 \cdot p_3 = -2\left(\frac{\sqrt{s}}{2}\right)^2 (1 - \cos\theta)$$

$$u = (p_1 - p_4)^2 = -2 p_1 \cdot p_4 = -2\left(\frac{\sqrt{s}}{2}\right)^2 (1 + \cos\theta)$$

where we have used for $\sqrt{s} \gg$ mass

$$p_1^{CM} = \frac{\sqrt{s}}{2}(1, 0, 0, 1)$$

$$p_3^{CM} = \frac{\sqrt{s}}{2}(1, 0, 0, \cos\theta)$$

$$p_4^{CM} = \frac{\sqrt{s}}{2}(1, 0, 0, -\cos\theta)$$

Now the integrations on the angles will diverge without the mass. This is not a problem in real experiments because θ can not be 0, or π (these angles are inside the pipe). But in order to simplify the problem was asked only to evaluate the contribution from s-channel there are no angles and the integration in $d\Omega$ gives 4π. We get therefore for that case

$$\sigma = \frac{(\mu_1\mu_2)^2}{64\pi^2 s} \frac{1}{s^2} \times 4\pi$$

$$\boxed{\sigma = \frac{(\mu_1\mu_2)^2}{16\pi\, s^3}}$$

As $[s] = \text{Mass}^2$ and $[\mu_1] = [\mu_2] = \text{Mass}$ we have

$$[\sigma] = \frac{(\text{Mass})^4}{(\text{Mass}^2)^3} = \frac{1}{\text{Mass}^2}$$ as required.