1^{st} Exam
January 17^{th} 2018: 11h30
Duration of the test: 1h30
Duration of the exam: 3h00

Mestrado em Eng. Física Tecnológica (MEFT)
Particle Physics
1^{st} semester of 2017-18

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- The allowed elements for consult during the test are:
  - the PDG (Particle Data Book)
  - one single A4 page with formulas.
- Carefully justify all your answers.
- The exam has 8 questions (3 pages) plus a formulary.

1^{st} test

IceCube is a neutrino observatory build at the South Pole, which uses thousands of detectors distributed over a cubic kilometre of volume under the Antarctic ice. The neutrino is detected when interacts with the ice atoms producing relativistic charged particles. These particles will emit Čerenkov light which can be detected by the surrounding photomultipliers. Although the experiment is sensitive to different flavours of neutrinos one of the most sensitive channels is the muon neutrino.

The neutrino-nucleon charged current cross-section can be given, in first approximation, by,

$$\sigma(\nu N)_{CC} \approx 3.6\, s \sqrt{\frac{1.8\, C}{s + 1.8\, C}}\ [\text{fb}] \tag{1}$$

where s is the square of center-of-mass energy in GeV^2 and $C = 10^4\,\text{GeV}^2$.
Properties of the planet Earth, necessary to solve the following problems:

- Earth's average density $\approx 8\, g\, cm^{-3}$;
- Earth's average molar mass $\sim 30\,\text{g}\,\text{mol}^{-1}$;
- Earth's average number of nucleons per atom ~ 30;
- Earth's average diameter $\approx 12742\,\text{km}$;
- Antarctic deep ice refraction index ≈ 1.78.

1. [6 val]

a) Consider a neutrino with an energy of 1 PeV (10^{15} eV) interacting with a proton at rest, and determine the center-of-mass energy of the collision.

b) Compute the probability of a 1 PeV (10^{15} eV) muon neutrino to interact inside the Earth knowing that it crosses it through the center.

c) The interaction of a neutrino with a nucleus can produce a muon. Calculate the Čerenkov critical angle for a muon with an energy of 10 TeV.

d) How many Čerenkov photons are produced if the muon crosses half the detector (500 m)? Assume that the mean energy of the produced Čerenkov photons is ≈ 1 eV.

2. **[5 val]** Verify, from the point of view of quantum numbers, if the following reactions are possible and if not explain why.

a) $\nu_\mu\ p \to n\ \nu_\mu$

b) $\nu_\mu\ p \to n\ e^-\ \overline{\nu_e}\ \nu_\mu$

c) $\nu_\mu\ n \to n\ e^+\ e^-$

d) $\nu_\mu\ p \to \Delta^+\ \nu_\mu$

e) $\nu_\tau\ n \to n\ \pi^-\ \tau^+$

3. **[3 val]** As the neutrino energy increases the Earth becomes gradually opaque, i.e. the neutrino interacts with its atoms. Assuming that the neutrino crosses through the center of the Earth determine the ν_μ energy for which its survival probability is less than 0.1%.

4. **[4 val]** Although the tau neutrino has, similarly to the muon neutrino, an energy for which the Earth is opaque, it can be regenerated via the decay of the tau lepton. Consider for the next questions the following reactions/decays occurring inside Earth: $\nu_\tau\ p \to \tau^-\ X\ \ \to \nu_\tau\ e^-\ \overline{\nu_e}\ X$, where X are hadronic particles.

a) What is the interaction mean free path of ν_τ and the decay mean free path of the τ lepton, if both of them have an energy of 1 PeV (10^{15} eV)? Compare it with the diameter of Earth.

b) Consider a ν_τ with an energy of 1 PeV (10^{15} eV). Knowing that the τ takes 90% of the neutrino energy, compute the maximum and minimum energy that the ν_τ, emerging from the τ decay, can get.

5. **[2 val]** Discuss how could the IceCube experiment distinguish between different neutrino flavours.

2^{nd} *test*

The events studied at IceCube correspond to charged current interactions of the type

$$\nu_\mu(p_1) + N(p_2) \to \mu^-(p_3) + X(p_4)$$

where N denotes the nucleons in the ice and X the final state excluding the muon.

6. **[8.5 val]**

Consider first the interaction of the ν_μ with the valence quarks of the nucleon N, that is,

$$\nu_\mu(p_1) + q(\hat{p}_2) \to \mu^-(p_3) + q'(\hat{p}_4)$$

in the center of mass of the ν_μ, q pair, where $\hat{p}_2$ is the momenta carried by the quark q. Neglect the masses of all fermions.

a) Identify the valence quarks q and q'.

b) Draw the corresponding Feynman diagram(s).

c) Considering the conditions of the problem write the amplitude $\mathcal{M}$ in its simplest form. Explain all the approximations.

d) Evaluate the spin averaged squared amplitude $\langle|\mathcal{M}|^2\rangle$.

e) For this process evaluate the differential cross section $d\sigma/d\Omega$ in the CM frame (of the elementary process) as a function of the square of the energy in the CM frame, $\hat{s} = (p_1 + \hat{p}_2)^2$ and scattering angle, $\hat{\theta}$, for this elementary process.

7. [8.5 val]

The neutrinos detected at IceCube have an energy sufficiently large to interact with the anti-quarks inside the nucleons with high probability. Consider then the elementary interaction of the ν_μ with the anti-quarks inside the nucleon N, that is,

$$\nu_\mu(p_1) + \overline{q}(\hat{p}_2) \to \mu^-(p_3) + \overline{q}'(\hat{p}_4)$$

in the center of mass of the $\nu_\mu, \overline{q}$ pair, where $\hat{p}_2$ is the momenta carried by the anti-quark $\overline{q}$. Neglect the masses of all fermions.

a) Identify the anti-quarks $\overline{q}$ and $\overline{q}'$.

b) Draw the corresponding Feynman diagram(s).

c) Considering the conditions of the problem write the amplitude $\mathcal{M}$ in its simplest form. Explain all the approximations.

d) Evaluate the spin averaged squared amplitude $\langle |\mathcal{M}|^2 \rangle$.

e) For this process evaluate the differential cross section $d\sigma/d\Omega$ in the CM frame as a function of the square of the energy in the CM frame (of the elementary process), $\hat{s} = (p_1 + \hat{p}_2)^2$ and scattering angle, $\hat{\theta}$, for this elementary process.

8. [3 val]

The expressions for the differential cross sections obtained in the previous problems were given in terms of $\hat{s}$ and $\hat{\theta}$, respectively the energy and scattering angle in the CM frame of the elementary process.

a) Explain why these are not good variables to compare with the experiment.

b) To solve the previous problem, one introduces Lorentz invariant (frame independent) variables. In this case we use the deep inelastic invariant variable

$$y \equiv \frac{p_2 \cdot (p_1 - p_3)}{p_2 \cdot p_1} = \frac{\hat{p}_2 \cdot (p_1 - p_3)}{\hat{p}_2 \cdot p_1}$$

Explain the second equality in the above equation and show that

$$y = \frac{1}{2}(1 - \cos\hat{\theta})$$

c) The final expression for the cross section of the process (ν_μ on proton)

$$\nu_\mu(p_1) + p(p_2) \to \mu^-(p_3) + X(p_4)$$

is given by

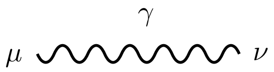
$$\sigma_{\nu p \to \mu X}(s) = \int_0^1 dx \int_0^1 dy \, \frac{G_F^2 x s}{\pi} \, \left[d(x) + (1-y)^2 \overline{u}(x)\right] \, \left[\frac{M_W^2}{Q^2 + M_W^2}\right]^2$$

where $Q^2 = -(p_1 - p_3)^2$. You should be able to derive this expression using the results of the previous problems. But here do not worry about that and just answer the following intermediate necessary steps:

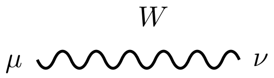
1. Why do you sum the cross sections of problems 1 and 2 and not the amplitudes?
2. What is the meaning of the variable x? What is the relation between s and $\hat{s}$?
3. Find a relation between $d\sigma/d\cos\hat{\theta}$ and $d\sigma/dy$.
4. Explain the integration limits in the above expression.
5. Explain the meaning of the functions $d(x)$ and $\overline{u}(x)$.

Formulary

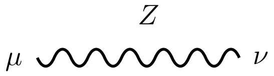
Propagators



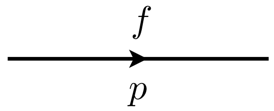
$$-i\frac{g_{\mu\nu}}{k^2} \tag{2}$$



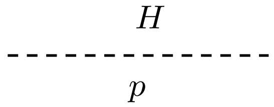
$$-i\frac{g_{\mu\nu}-\frac{k_\mu k_\nu}{M_W^2}}{k^2-M_W^2+i\,M_W\Gamma_W} \tag{3}$$



$$-i\frac{g_{\mu\nu}-\frac{k_\mu k_\nu}{M_Z^2}}{k^2-M_Z^2+i\,M_Z\Gamma_Z} \tag{4}$$



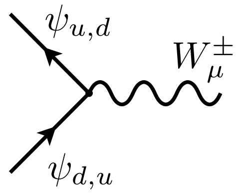
$$\frac{i(\not{p}+m_f)}{p^2-m_f^2} \tag{5}$$



$$\frac{i}{p^2-M_H^2+i\,M_H\Gamma_H} \tag{6}$$

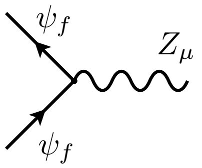
Vertices

Charged Current

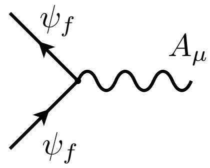


$$-i\frac{g}{\sqrt{2}}\gamma_\mu\frac{1-\gamma_5}{2} \tag{7}$$

Neutral Current



$$-i\frac{g}{\cos\theta_W}\gamma_\mu\left(g_V^f-g_A^f\gamma_5\right)$$

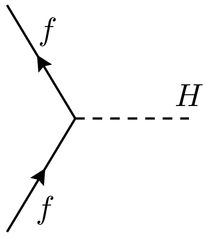


$$-ieQ_f\gamma_\mu$$

where

$$g_V^f=\frac{1}{2}T_f^3-Q_f\sin^2\theta_W,\quad g_A^f=\frac{1}{2}T_f^3\,. \tag{8}$$

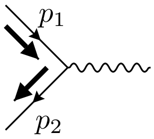
Higgs Interactions with fermions



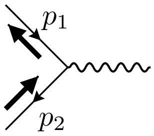
$$-i\,\frac{g}{2}\,\frac{m_f}{M_W}\equiv -i\,g_H^f \tag{9}$$

Results for the Helicity Vector Currents

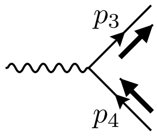
s-channel



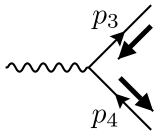
$$J_{u_1 v_2}(\uparrow,\downarrow) = \sqrt{s}\,(0,-1,-i,0) \tag{10}$$



$$J_{u_1 v_2}(\downarrow,\uparrow) = \sqrt{s}\,(0,-1,i,0) \tag{11}$$

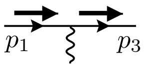


$$J_{u_3 v_4}(\uparrow,\downarrow) = \sqrt{s}\,(0,-\cos\theta,i,\sin\theta) \tag{12}$$

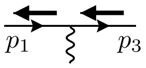


$$J_{u_3 v_4}(\downarrow,\uparrow) = \sqrt{s}\,(0,-\cos\theta,-i,\sin\theta) \tag{13}$$

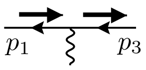
t-channel



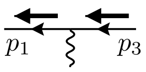
$$J_{u_1 u_3}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{14}$$



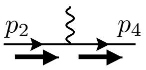
$$J_{u_1 u_3}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{15}$$



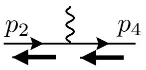
$$J_{v_1 v_3}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{16}$$



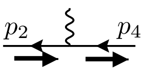
$$J_{v_1 v_3}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},\cos\frac{\theta}{2}\right) \tag{17}$$



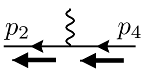
$$J_{u_2 u_4}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{18}$$



$$J_{u_2 u_4}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{19}$$



$$J_{v_2 v_4}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{20}$$



$$J_{v_2 v_4}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2},-\sin\frac{\theta}{2},-i\sin\frac{\theta}{2},-\cos\frac{\theta}{2}\right) \tag{21}$$