Chapter 5

Modelling electrical systems

> By that time Mordor was deservedly being called "smithy of the nations," and it could trade its manufactured goods for any amounts of food from Khand and Umbar. Trading caravans went back and forth through the Ithilien Crossroads day and night, and more and more voices in Barad-dúr were saying that the country has had enough tinkering with agriculture, which was nothing but a net loss anyway, and the way to go was to develop what nobody else had — namely, metallurgy and chemistry. Indeed, the industrial revolution was well underway; steam engines toiled away in mines and factories, while the early aeronautic successes and experiments with electricity were the talk of the educated classes.

Kirill YESKOV (1956 — ...), *The Last Ringbearer*, I 3 (transl. Yisroel Markov)

This chapter addresses the modelling of electrical systems.

5.1 Passive components

The three simplest elements in an electrical circuit are:

1. A **resistor**. This component (see Figures 5.1 and 5.2) dissipates energy according to Ohm's law: *Resistor* *Ohm's law*

$$R(t) = \frac{U(t)}{I(t)} \tag{5.1}$$

   Here R is the resistance, U is the voltage (or tension, or electric potential difference), and I is the current.

2. A **capacitor**. This component stores energy and its most usual model is *Capacitor*

$$U(t) = \frac{1}{C}Q(t) \tag{5.2}$$

   where $Q(t)$ is the electrical charge stored, and C is the capacity. Since $I(t) = \frac{\mathrm{d}Q(t)}{\mathrm{d}t}$,

$$U(t) = \frac{1}{C}\int_0^t I(t)\,\mathrm{d}t \tag{5.3}$$

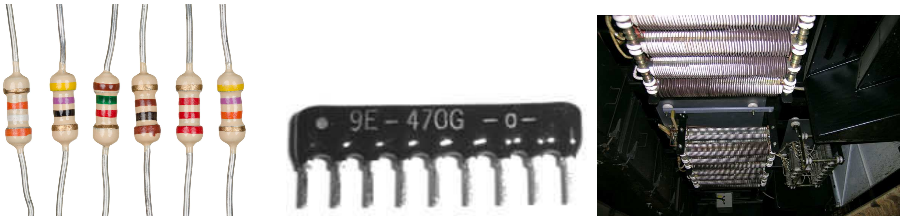

Figure 5.1: Different types of resistors. Left: individual resistors for use in electronics; centre: many resistors in one encasing; right: wirewound resistors for high tensions and currents in a train. (Source: Wikimedia.) There are still other types of resistors.

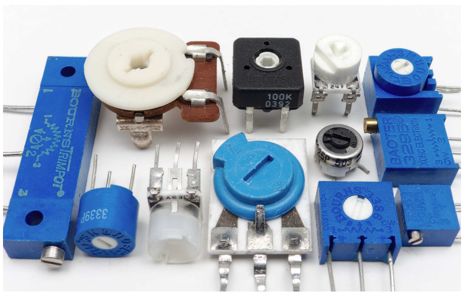

Figure 5.2: Potentiometers (or variable resistors, or rheostats) have a slider or a screw to move the position of a terminal, and thus the length of the resistor which is actually employed; resistance is proportional to this length, and can be varied in this manner (source: Wikimedia).

Differentiating, we get

$$I(t) = C\,\frac{\mathrm{d}U(t)}{\mathrm{d}t} \tag{5.4}$$

3. An **inductor**. This component also stores energy and its most usual model is *Inductor*

$$I(t) = \frac{1}{L}\lambda(t) \tag{5.5}$$

where $\lambda(t) = \int_0^t U(t)\,\mathrm{d}t$ is the flux linkage, and L is the inductance. Differentiating, we get

$$U(t) = L\frac{\mathrm{d}I(t)}{\mathrm{d}t} \tag{5.6}$$

The transfer functions of the resistor, the capacitor, and the inductor, corresponding to (5.1), (5.4), and (5.6), considering always tension $U(s)$ as the output and current $I(s)$ as the input, are

$$\frac{U(s)}{I(s)} = R \tag{5.7}$$

$$\frac{U(s)}{I(s)} = \frac{1}{Cs} \tag{5.8}$$

$$\frac{U(s)}{I(s)} = Ls \tag{5.9}$$

Remark 5.1. Notice that Ohm's law (5.1) or (5.7) corresponds to a static system. □

Remark 5.2. It cannot be overstated that relations (5.7)–(5.9) are not followed by many components:

- Many resistances do not follow a linear relation between U and I such as (5.1), and are thus called non-ohmic resistors. Still, Ohm's law can be a good approximation in a limited range of values (see Figure 4.3 again). *Non-ohmic resistors*

- Many capacitors have variable capacity C, depending on the voltage applied. Others follow differential equations of fractional order.

- Inductances always have some resistance, which is often not neglectable. So their transfer function would more accurately be $R + Ls$.

- Even when (5.7)–(5.9) are accurately followed, this only happens for a limited range of values. Increase U or I too much, and any electrical component will cease to function (burn, melt. . .). What is too much depends on the particular component: there are components that cannot stand 1 V while others work at 10^4 V and more.

Table 5.1: Effort, flow, accumulators and dissipators in electrical systems

	Electrical system	SI
effort e	voltage U	V
flow f	current I	A
effort accumulator	inductor with induction L	H
accumulated effort $e_a = \int e\,\mathrm{d}t$	flux linkage $\lambda = \int U\,\mathrm{d}t$	Wb
relation between accumulated effort and flow $e_a = \varphi(f)$	flux linkage $\lambda = LI$	
accumulated energy $E_e = \int e_a\,\mathrm{d}f$	inductive energy $E_e = \frac{1}{2}LI^2$	J
flow accumulator	capacitor with capacity C	F
accumulated flow $f_a = \int f\,\mathrm{d}t$	charge $Q = \int I\,\mathrm{d}t$	C
relation between accumulated flow and effort $f_a = \varphi(e)$	charge $Q = CU$	
accumulated energy $E_f = \int f_a\,\mathrm{d}e$	capacitative energy $E_f = \frac{1}{2}C\dot{V}^2$	J
dissipator	resistance R	Ω
relation between effort and flow $e = \varphi(f)$	$U = RI$	
dissipated energy $E_d = \int f\,\mathrm{d}e$	$E_d = \frac{1}{2}RI^2$	J

5.2 Energy, effort and flow

Because $\dot{E}(t) = U(t)I(t)$ and $E(t) = \int_0^t U(t)I(t)\,\mathrm{d}t$, effort and flow variables are U and I. While either can once more play each of the roles, by universal convention,

- U is the effort variable,
- I is the flow variable, and thus
- the inductor is the effort accumulator,
- the capacitor is the flux accumulator,
- the resistor is the dissipator.

Table 5.1 sums up the passing information and relations.

Electrical impedance

Remark 5.3. Transfer functions (5.7)–(5.9) have the flux as input and the effort and output. They consequently give the impedance of the corresponding components. □

Kirchoff's laws

To model electric systems with these components, (5.7)–(5.9) are combined with Kirchoff's laws:

Kirchoff's current law

- The current law states that the sum of the currents at a circuit's node is zero.

Kirchoff's voltage law

- The voltage law states that the sum of the voltages around a circuit's closed loop is zero.

Voltage divider

Example 5.1. Consider the system in Figure 5.3 known as voltage divider. The input is $V_i(t)$ and the output is $V_o(t)$. Applying the current law at point B, we see that the current flowing from A to B must be the same that flows

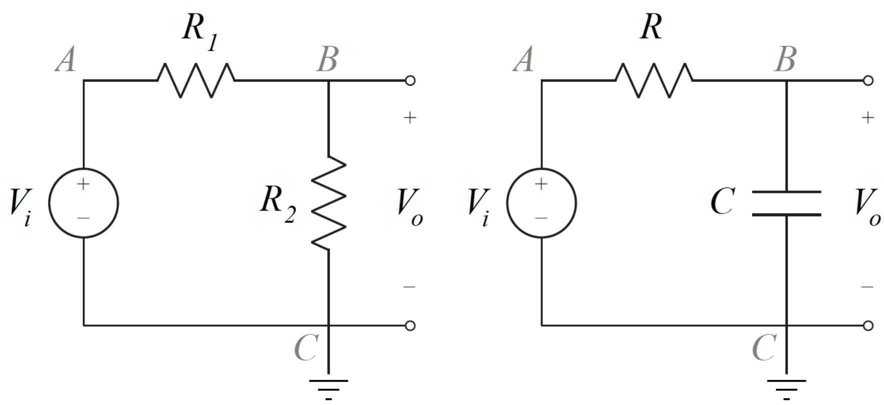

Figure 5.3: Left: voltage divider. Right: RC circuit.

from B to C. Applying Ohm's law (5.1) to the two resistances, we see that

$$R_1 = \frac{V_B(t) - V_A}{I(t)} = \frac{V_o(t) - V_i(t)}{I(t)} \Rightarrow I = \frac{V_o - V_i}{R_1} \tag{5.10}$$

$$R_2 = \frac{V_C(t) - V_B(t)}{I(t)} = \frac{0 - V_o(t)}{I(t)} \Rightarrow I = \frac{-V_o}{R_2} \tag{5.11}$$

In the last equalities above, we dropped the dependence on t to alleviate the notation. Consequently,

$$(V_o - V_i)R_2 = -V_o R_1 \Leftrightarrow V_o(R_1 + R_2) = V_i R_2 \Leftrightarrow \frac{V_o}{V_i} = \frac{R_2}{R_1 + R_2} \tag{5.12}$$

Notice that this system is static, and from $\frac{V_o(t)}{V_i(t)} = \frac{R_2}{R_1+R_2}$ we get $\frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_1+R_2}$. □

Remark 5.4. Remember that, similarly to what happens with the positive direction of displacements in mechanical systems, it is irrelevant if a higher tension is presumed to exist to the left or to the right of a component. Current is always assumed to flow from higher to lower tensions; as long as equations are coherently written, if in end current turns out to be negative, this only means that it will flow the other way round. □

Example 5.2. The transfer function of the system in Figure 5.3 known as RC circuit can be found in almost the same manner, thanks to impedances: *RC circuit*

$$R = \frac{V_B(s) - V_A(s)}{I(s)} = \frac{V_o(s) - V_i(s)}{I(s)} \Rightarrow I = \frac{V_o - V_i}{R} \tag{5.13}$$

$$\frac{1}{Cs} = \frac{V_C(s) - V_B(s)}{I(s)} = \frac{0 - V_o(s)}{I(s)} \Rightarrow I = -V_o Cs \tag{5.14}$$

In the last equalities above, we dropped the dependence on s to alleviate the notation. Consequently,

$$V_o - V_i = -V_o RCs \Leftrightarrow V_o(1 + RCs) = V_i \Leftrightarrow \frac{V_o}{V_i} = \frac{1}{1 + RCs} \tag{5.15}$$

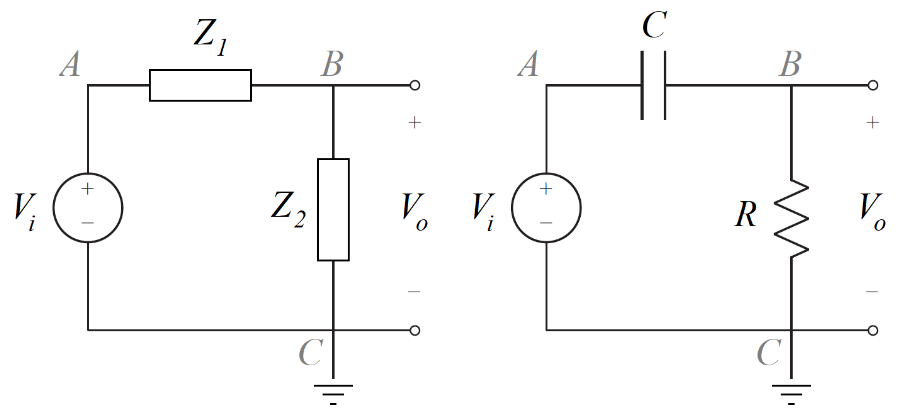

Figure 5.4: Left: generic electrical system with two impedances, of which the voltage divider (Figure 5.3), the RC circuit (Figure 5.3) and the CR circuit (to the right) are particular cases. Right: CR circuit.

Notice that this system is dynamic, and from $V_o(s)(1 + RCs) = V_i(s)$ we get $V_o(t) + RC\frac{\mathrm{d}V_o(t)}{\mathrm{d}t} = V_i(t)$. $\square$

Two generic impedances

Example 5.3. Both systems above are particular cases of the generic system in Figure 5.4 with two impedances:

$$Z_1(s) = \frac{V_B(s) - V_A(s)}{I(s)} = \frac{V_o(s) - V_i(s)}{I(s)} \Rightarrow I = \frac{V_o - V_i}{Z_1} \tag{5.16}$$

$$Z_2(s) = \frac{V_C(s) - V_B(s)}{I(s)} = \frac{0 - V_o(s)}{I(s)} \Rightarrow I = \frac{-V_o}{Z_2} \tag{5.17}$$

Consequently,

$$(V_o - V_i)Z_2 = -V_o Z_1 \Leftrightarrow \frac{V_o}{V_i} = \frac{Z_2}{Z_1 + Z_2} \tag{5.18}$$

Replacing $Z_1(s) = R_1$ and $Z_2(s) = R_2$ in (5.18), we obtain (5.12).

Replacing $Z_1(s) = R$ and $Z_2(s) = \frac{1}{Cs}$ in (5.18), we obtain (5.15).

We can also obtain the transfer function of the case where the resistor and the capacitor are switched as also shown in Figure 5.4: when $Z_1(s) = \frac{1}{Cs}$ and $Z_2(s) = R$, we have $\frac{V_o(s)}{V_i(s)} = \frac{R}{R+\frac{1}{Cs}} = \frac{RCs}{1+RCs}$. This is known as a CR circuit. $\square$

CR circuit

RLC circuit

Example 5.4. Consider the system in Figure 5.5 known as RLC circuit. The input is $V_i(t)$ and the output is $V_o(t)$. Applying the current law, we see that the current flowing from A to B must be the same that flows from B to C and the same that flows from C to D. Then

$$\begin{cases} R = \frac{V_B(t)-V_A}{I(t)} = \frac{V_B(t)-V_i(t)}{I(t)} \Rightarrow V_B = RI + V_i \\ Ls = \frac{V_C(t)-V_B(t)}{I(t)} = \frac{V_o(t)-V_B(s)}{I(t)} \Rightarrow ILs = V_o - V_B \\ \frac{1}{Cs} = \frac{V_D(s)-V_C(s)}{I(s)} = \frac{0-V_o(s)}{I(s)} \Rightarrow I = -V_o Cs \end{cases} \tag{5.19}$$

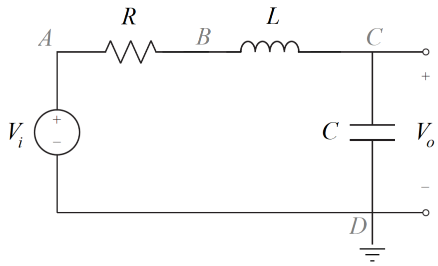

Figure 5.5: RLC circuit.

We now replace the first equation in the second, and use it together with the third to get

$$ILs = V_o - RI - V_i \Leftrightarrow I(R + Ls) = V_o - V_i \tag{5.20}$$

$$\Rightarrow \frac{V_o - V_i}{R + Ls} = -V_oCs \Leftrightarrow V_o + V_oCRs + V_oCLs^2 = V_i \Leftrightarrow \frac{V_o}{V_i} = \frac{1}{CLs^2 + CRs + 1} \tag{5.21}$$

From $V_o(s) + V_o(s)CRs + V_o(s)CLs^2 = V_i(s)$ we get

$$V_o(t) + CR\frac{\mathrm{d}V_o(t)}{\mathrm{d}t} + CL\frac{\mathrm{d}^2V_o(t)}{\mathrm{d}t^2} = V_i(t) \quad \square \tag{5.22}$$

Remark 5.5. We could have established (5.22) first, without using impedances:

$$\begin{cases} R = \frac{V_B(t)-V_A(t)}{I(t)} \Rightarrow I(t) = \frac{1}{R}V_B(t) - \frac{1}{R}V_i(t) \\ V_C(t) - V_B(t) = L\frac{\mathrm{d}I(t)}{\mathrm{d}t} \Rightarrow V_o(t) - V_B(t) = \frac{L}{R}\frac{\mathrm{d}V_B(t)}{\mathrm{d}t} - \frac{L}{R}\frac{\mathrm{d}V_i(t)}{\mathrm{d}t} \\ V_D(t) - V_C(t) = \frac{1}{C}\int I(t)\,\mathrm{d}t \Rightarrow -\frac{\mathrm{d}V_o(t)}{\mathrm{d}t} = frac1CI(t) \end{cases} \tag{5.23}$$

Replacing the first equation in the third, and then the result in the second,

$$-\frac{\mathrm{d}V_o(t)}{\mathrm{d}t} = \frac{1}{RC}V_B(t) - \frac{1}{RC}V_i(t) \Rightarrow V_B(t) = V_i(t) - RC\frac{\mathrm{d}V_o(t)}{\mathrm{d}t} \tag{5.24}$$

$$V_o(t) - V_i(t) + RC\frac{\mathrm{d}V_o(t)}{\mathrm{d}t} = \frac{L}{R}\left(\frac{\mathrm{d}V_i(t)}{\mathrm{d}t} - RC\frac{\mathrm{d}^2V_o(t)}{\mathrm{d}t^2}\right) - \frac{L}{R}\frac{\mathrm{d}V_i(t)}{\mathrm{d}t} \tag{5.25}$$

Rearranging terms in the last equality gives (5.22). Applying the Laplace transform, we then obtained transfer function (5.20). The results are of course the same. Notice that in both cases zero initial conditions were implicitly assumed (i.e. integrals were assumed to be zero at $t = 0$; in the case of the Laplace transform, this means that there is no $f(0)$ term in (2.41). We will address this further in Chapter 9. $\square$

Remark 5.6. Transfer function (5.20) is similar to transfer functions (4.9) and (4.10). As you know, this is one case of a so-called electrical equivalent of a mechanical system, or of a mechanical equivalent of an electrical system. The

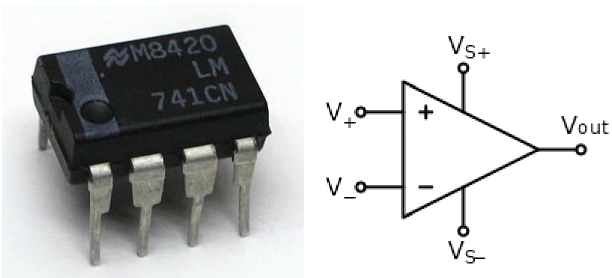

Figure 5.6: Left: an integrated circuit with a 741 OpAmp, one of the most usual types of OpAmps (source: Wikimedia). Other OpAmp types are manufactured in integrated circuits that have several OpAmps each, sharing the same power supply. Right: the symbol of the OpAmp (source: Wikimedia). Power supply tensions are often omitted in diagrams for simplicity, but never forget that an active component without power supply does not work.

notions of effort and flux make clear why this parallel between models exists: both consist of an effort accumulator, a flux accumulator, and a dissipator. But notice that the parallel is not complete: (4.9) has a flux as input and an accumulated effort as output; both the input and the output of (5.20) are efforts. □

5.3 The operational amplifier (OpAmp), an active component

Passive components

Active components

The resistor, the capacitor and the inductor are called passive components because they do not need a source of energy to function. Components that need a source of energy to function are called active components. Among them are diodes and transistors, together with sensors that we will study in Chapter 12. A component we will study right away because of its importance is the **operational amplifier**, or in short the OpAmp.

An OpAmp is an electronic component that presents itself as an integrated circuit (see Figure 5.6) and amplifies the difference between its two inputs V^- and V^+:

$$V^{\text{out}} = K\left(V^+ - V^-\right) \tag{5.26}$$

The output V^{out} is limited to the power supply tensions:

$$V^{S+} \leq V^{\text{out}} \leq V^{S-} \tag{5.27}$$

No output if no power supply

As can be expected from the fact that the OpAmp is an active component, if no power is supplied, i.e. if the corresponding pins of the integrated circuit are disconnected and thus $V^{S+} = V^{S-} = 0$ V, then $V^{\text{out}} = 0$ V, i.e. there is no output. The gain of the OpAmp K should ideally be infinite; in practice it is very large, e.g. 10^5 or 10^6. See Figure 5.7.

The other important characteristic of the OpAmp is that the impedance between its two inputs V^- and V^+ is very large. Ideally it should be inifinite; in practice it is 2 MΩ or more.

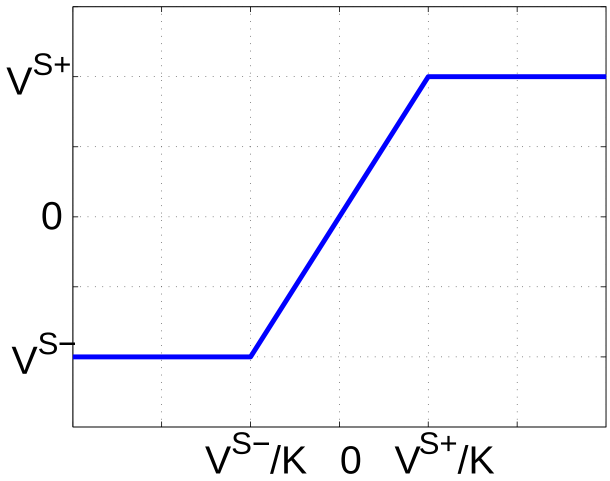

Figure 5.7: The output of an OpAmp.

Example 5.5. The OpAmp can be used to compare two tensions, connected to the two inputs V^- and V^+. Because K is very large, if $V^+ > V^-$, even if only by a very small margin, the output will saturate at tension V^{S+}. Likewise, if $V^+ < V^-$, even if only by a very small margin, the output will saturate at tension V^{S-}.

Comparator

Only if V^+ and V^- are equal, and equal to a great precision, will the output be 0 V. Consider the case of a 741 OpAmp, typically supplied with $V^{S\pm} = \pm 15$ V. Suppose that $K = 10^5$. Then the output V^{out} will not saturate at either $+15$ V or -15 V only if $|V^+ - V^-| < 15 \times 10^{-5}$ V. □

Example 5.6. OpAmps are very usually employed in the configuration shown in Figure 5.8, known as **inverting OpAmp** or **inverter**. In this case, because the OpAmp's input impedance is very large, the current I will flow from input V_i to output V_o, as shown in Figure 5.8. Consequently,

Inverting OpAmp or inverter

$$\begin{cases} V_o = K(V^+ - V^-) \Leftrightarrow V^- = -\frac{V_o}{K} \\ Z_2 = \frac{V_o - V^-}{I} \Leftrightarrow I = \frac{V_o - V^-}{Z_2} \\ Z_1 = \frac{V^- - V_i}{I} \Leftrightarrow I = \frac{V^- - V_i}{Z_1} \end{cases} \tag{5.28}$$

Eliminating I and V^-, we get

$$\frac{V_o + \frac{V_o}{K}}{Z_2} = \frac{-\frac{V_o}{K} - V_i}{Z_1} \Leftrightarrow V_o Z_1 + V_o \frac{Z_1}{K} + V_o \frac{Z_1}{K} = -V_i Z_2 \Leftrightarrow \frac{V_o}{V_i} = -\frac{Z_2 K}{Z_1 K + Z_1 + Z_2} \tag{5.29}$$

Because K is large, (5.29) reduces to

$$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1} \quad \square \tag{5.30}$$

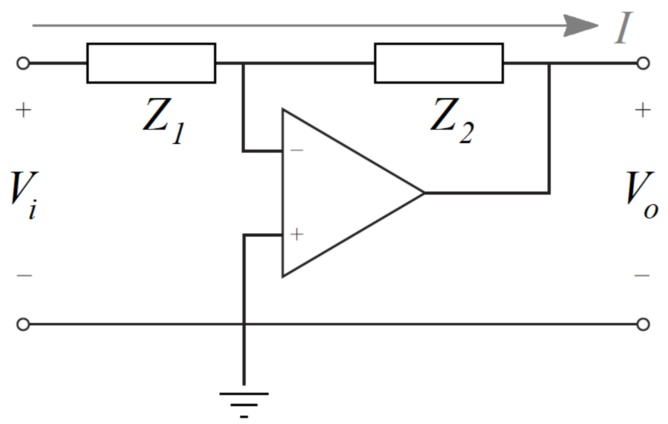

Figure 5.8: Inverting OpAmp with two generic impedances.

Remark 5.7. Notice how (5.30) shows that we can assume

$$V^+ = V^- \tag{5.31}$$

(and so in this case $V^- = 0$). This is because of the high input impedance. □

Inverting amplifier

Example 5.7. If in Figure 5.8 we make $Z_1 = R_1$ and $Z_2 = R_2$, we obtain the circuit in Figure 5.9, known as **inverting amplifier**, for which

$$\frac{V_o}{V_i} = -\frac{R_2}{R_1} \tag{5.32}$$

Notice that, because $R_1, R_2 > 0$ (there are no negative resistances!), in this circuit the signs of V_i and V_o are always opposite. In spite of the circuit's name, it can

Amplification

- amplify the input (i.e. $|V_o| > |V_i|$, if $R_2 > R_1$), or

Attenuation

- attenuate the input (i.e. $|V_o| < |V_i|$, if $R_1 > R_2$). □

Example 5.8. Consider the circuit in Figure 5.10, which is another variation of the negative feedback OpAmp. From (5.31) and the Kirchoff law of nodes, implicit in the currents shown in Figure 5.10, we get

$$\begin{cases} Z_1 = \frac{V_1 - 0}{I_1} \Leftrightarrow I_1 = \frac{V_1}{Z_1} \\ Z_2 = \frac{V_2 - 0}{I_2} \Leftrightarrow I_2 = \frac{V_2}{Z_2} \\ Z_3 = \frac{0 - V_o}{I_1 + I_2} \Leftrightarrow \frac{V_1}{Z_1} + \frac{V_2}{Z_2} = \frac{-V_o}{Z_3} \Leftrightarrow V_o = -\frac{Z_3}{Z_1}V_1 - \frac{Z_3}{Z_2}V_2 \end{cases} \tag{5.33}$$

Consider what happens when all the impedances are resistors:

Inverting summer or inverting summing circuit

- If $Z_1 = Z_2 = Z_3 = R$ this circuit is called **inverting summer** or **inverting summing circuit**. The output V_o is the sum of the two inputs V_1 and V_2, but with the sign inverted.

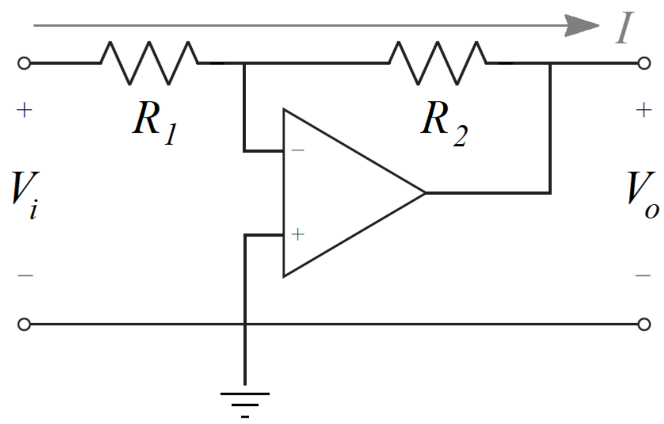

Figure 5.9: Inverter amplifier.

rting amplifying summer or inverting summing ifier

- If $Z_1 = Z_2 = R$ and $Z_3 = R_3$ this will be an **inverting amplifying summer** or **inverting summing amplifier**. The amplifying ratio is $-\frac{R_3}{R}$ (and can correspond to amplification or attenuation).

rting weighted sum-

- If all the resistances are different, we will have an **inverting weighted summer**. If $\frac{R_3}{R_1} + \frac{R_3}{R_2} = 1$ there is no amplification or attenuation; otherwise there is. □

Remark 5.8. Notice that the circuit in Figure 5.10 is a MISO system. □

Example 5.9. If in Figure 5.8 we have $Z_1 = R$ and Z_2 consists in a resistor R and a capacitor C in parallel, we obtain the circuit in Figure 5.11, with

$$Z_2 = \frac{1}{\frac{1}{R} + \frac{1}{\frac{1}{Cs}}} = \frac{R}{1 + RCs} \tag{5.34}$$

$$\frac{V_o}{V_i} = -\frac{1}{1 + RCs} \tag{5.35}$$

and similar to the RC circuit from Example 5.2 with transfer function (5.15).

If in Figure 5.8 we have $Z_2 = R$ and Z_1 consists in a resistor R and a capacitor C in series, we obtain the circuit in Figure 5.12, with

$$Z_1 = R + \frac{1}{Cs} = \frac{1 + RCs}{Cs} \tag{5.36}$$

$$\frac{V_o}{V_i} = -\frac{RCs}{1 + RCs} \tag{5.37}$$

and similar to the CR circuit from Example 5.3. □

Example 5.10. Other than the inverter configuration in Figure 5.8, the most usual configuration with which OpAmps are used is the on in Figure 5.13, known as the **non-inverting OpAmp** or **non-inverter**. Because of the very large

Non-inverting OpAmp or non-inverter

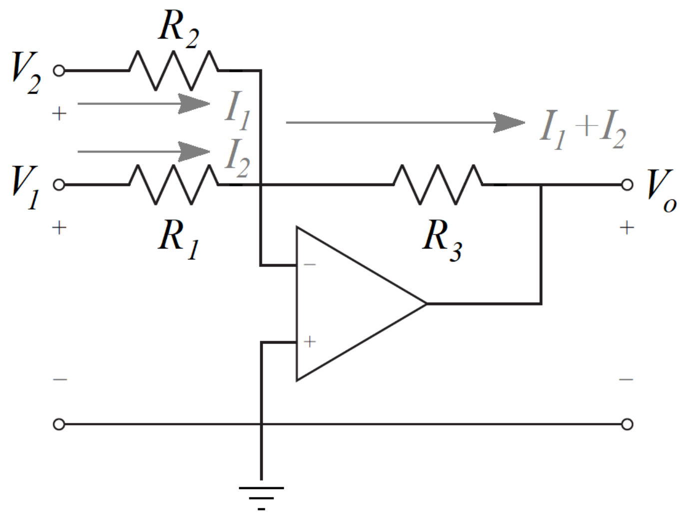

Figure 5.10: Inverting summer or summing circuit.

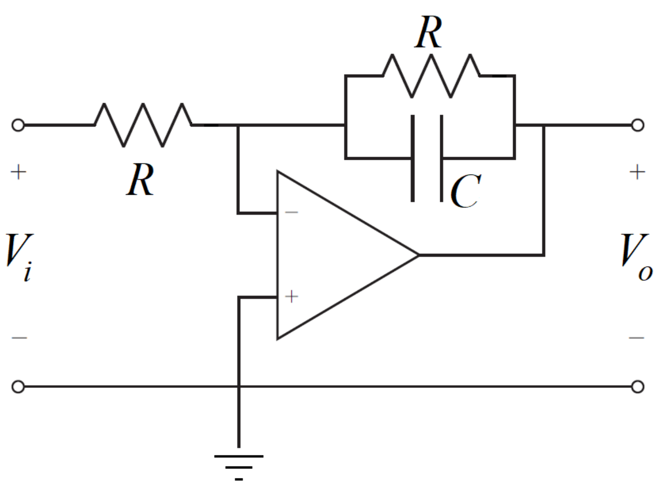

Figure 5.11: Inverting RC circuit with an OpAmp.

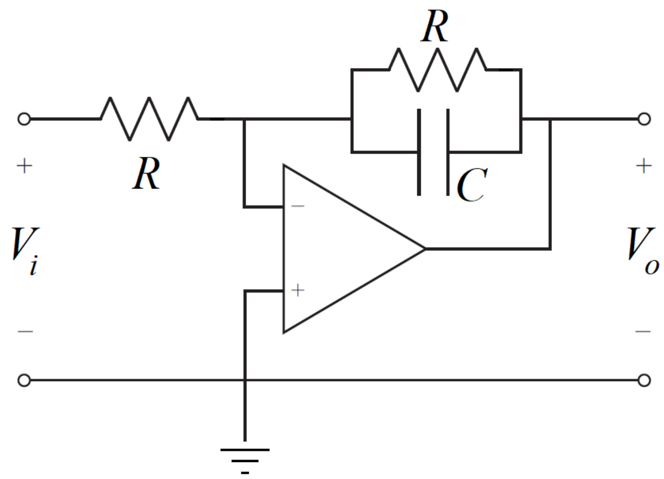

Figure 5.12: Inverting CR circuit with an OpAmp.

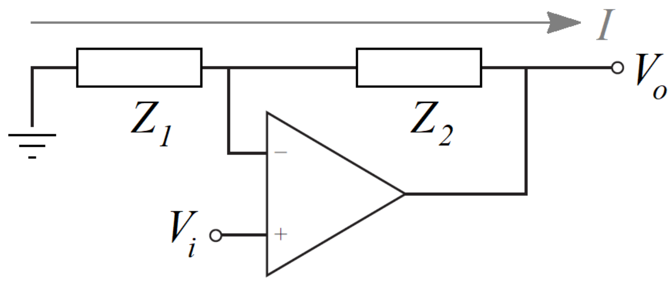

Figure 5.13: Non-inverting OpAmp with two generic impedances.

input impedance, current flows as shown, and

$$\begin{cases} V_o = K(V_i - V^-) \Leftrightarrow V^- = V_i - \frac{V_o}{K} \\ Z_2 = \frac{V_o - V^-}{I} \Leftrightarrow I = \frac{V_o - V^-}{Z_2} \\ Z_1 = \frac{V^- - 0}{I} \Leftrightarrow I = \frac{V^-}{Z_1} \end{cases} \tag{5.38}$$

Eliminating I and V^-, we get

$$\frac{V_o - V_i + \frac{V_o}{K}}{Z_2} = \frac{V_i - \frac{V_o}{K}}{Z_1} \Leftrightarrow V_o Z_1 - V_i Z_1 + V_o \frac{Z_1}{K} = V_i Z_2 - V_o \frac{Z_2}{K} \Leftrightarrow \frac{V_o}{V_i} = \frac{Z_1 + Z_2}{Z_1 + \frac{Z_1}{K} + \frac{Z_2}{K}} \tag{5.39}$$

Because K is large, (5.39) reduces to

$$\frac{V_o}{V_i} = \frac{Z_1 + Z_2}{Z_1} \quad \square \tag{5.40}$$

Remark 5.9. (5.40) shows once again that we can assume (5.31). We would have arrived sooner at the same result. □

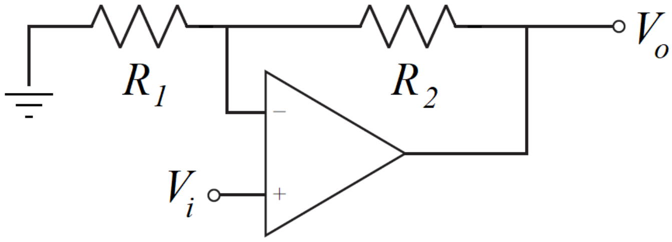

Figure 5.14: Non-inverting amplifier.

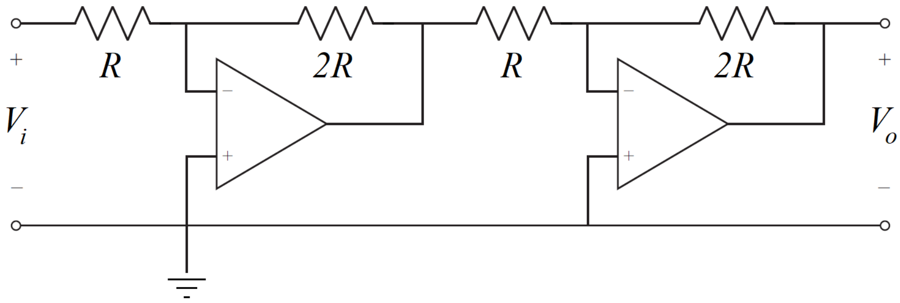

Figure 5.15: Two inverting amplifiers, that amplify the input 4 times.

Non-inverting amplifier

Example 5.11. If in Figure 5.13 we make $Z_1 = R_1$ and $Z_2 = R_2$, we obtain the circuit in Figure 5.14, known as **non-inverting amplifier**, for which

$$\frac{V_o}{V_i} = \frac{R_1 + R_2}{R_1} \tag{5.41}$$

Notice that, because $R_1, R_2 > 0$, not only in this circuit the signs of V_i and V_o are always the same, as the input is always amplified (i.e. $|V_o| > |V_i|$): it is impossible to attenuate the input. □

Example 5.12. Suppose that we want to amplify a tension 4 times. We can use the non-inverting amplifier of Figure 5.14 with $R_2 = 3R_1$. As an alternative, we can use two inverting amplifiers as in Figure 5.15. □

Remark 5.10. When we want to attenuate a tension without inverting its signal, a non-inverting amplifier cannot be used, since it must be always true that $\frac{V_o}{V_i} > 1$; two inverting amplifiers in series must be used instead, as in the previous example. □

Example 5.13. Consider the circuit in Figure 5.16. Because of (5.31), we have $V^+ = V^- = V_i$, and $V_o = V^-$; hence

$$V_o = V_i \tag{5.42}$$

While at first sight this may seem a good candidate for the prize of the most useless circuit, it is in reality a most useful one. We can be sure that $V_o = V_i$ and that whatever components are connected to V_o will not affect V_i, because

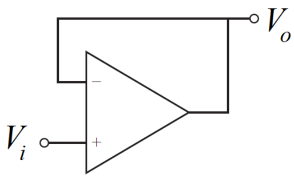

Figure 5.16: Voltage buffer.

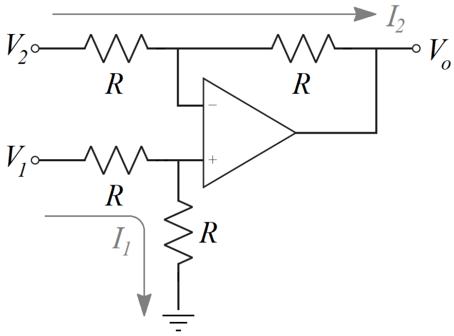

Figure 5.17: Subtractor.

there is no current flowing between V_i and V_o. (The source of energy is the OpAmp's power supply.) If it were not for the OpAmp, anything connected to V_o would modify the value of V_i. This circuit is known as **tension buffer**. □ *Tension buffer*

Example 5.14. The MISO system in Figure 5.17 is know as **subtractor**, because *Subtractor*

$$\begin{cases} R = \frac{V_2 - V^\pm}{I_2} \\ R = \frac{V^\pm - V_o}{I_2} \\ R = \frac{V_1 - V^\pm}{I_1} \\ R = \frac{V^\pm - 0}{I_1} \end{cases} \tag{5.43}$$

From the last two equations, we get $2V^\pm = V_1$. From the first two equations, and replacing this last result,

$$V_2 = 2V^\pm - V_o \Leftrightarrow V_o = V_1 - V_2 \quad \square \tag{5.44}$$

5.4 Other components

Among the several other components that may be found in mechanical systems, we will study the model of the **transformer**, shown in Figure 5.18: *Transformer*

$$\frac{V_P}{V_S} = \frac{N_P}{N_S} \tag{5.45}$$

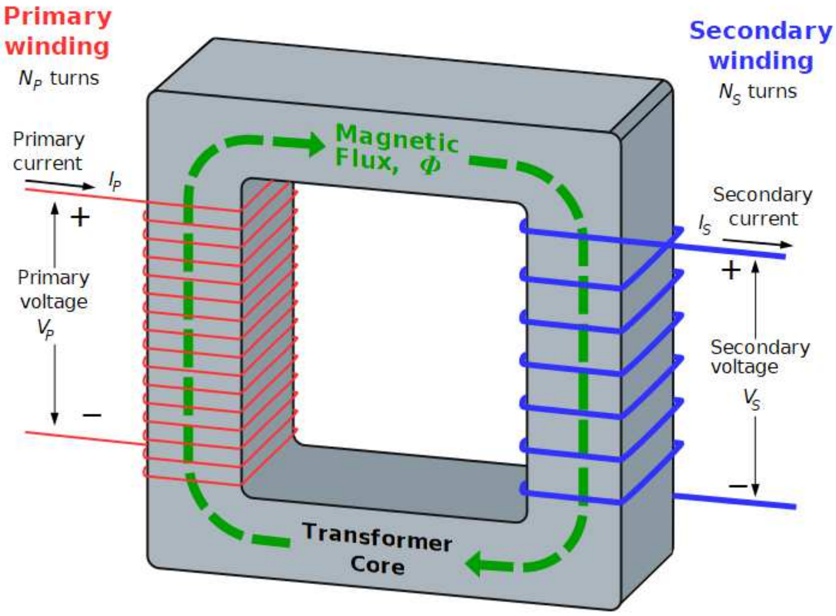

Figure 5.18: Transformer (source: Wikimedia commons).

Here V_P and V_S are the tensions in the two windings, and N_P and N_S are the corresponding numbers of turns in each winding. This is an ideal model; in practice, there are losses, but we will not need to use a more accurate expression.

Glossary

> Et le professeur Lidenbrock devait bien s'y connaître, car il passait pour être un véritable polyglotte. Non pas qu'il parlât couramment les deux mille langues et les quatre mille idiomes employés à la surface du globe, mais enfin il en savait sa bonne part.
>
> Jules Verne (1828 — †1905), *Voyage au centre de la Terre*, 2

active component componente ativo
amplification amplificação
attenuation atenuação
capacitance capacidade elétrica, capacitância (bras.)
capacitor condensador, capacitor (bras.)
current corrente, intensidade (de corrente elétrica)
electric potential difference voltagem, tensão, diferença de potencial elétrico
flux linkage fluxo magnético total
impedance impedância
inductance indutância

inductor bobina, indutor
inverting amplifier amplificador inversor
inverter inversor
inverting amplifying summer somador amplificador inversor
inverting OpAmp AmpOp inversor
inverting summer somador inversor
inverting summing amplifier amplificador somador inversor
inverting summing circuit circuito somador inversor
inverting weighted summer somador inversor ponderado
non-inverter não-inversor
non-inverting OpAmp AmpOp não-inversor
OpAmp AmpOp
operational amplifier amplificador operacional
passive component componente passivo
potentiometer potenciómetro, resistência variável, reóstato
resistance resistência
resistor resistência, resistor (bras.)
rheostat potenciómetro, resistência variável, reóstato
subtractor subtrator
tension voltagem, tensão, diferença de potencial elétrico
tension buffer AmpOp seguidor de tensão, *buffer* de tensão
variable resistor potenciómetro, resistência variável, reóstato
voltage voltagem, tensão, diferença de potencial elétrico

Exercises

1. Find the equations describing the dynamics of the systems in Figure 5.19, and apply the Laplace transform to the equations to find the corresponding transfer function.

2. Again for the systems in Figure 5.19, find the transfer function directly from the impedances of the components, and apply the inverse Laplace transform to the transfer functions to find the corresponding equations.

3. Show that the differential equations modelling the circuit in Figure 5.20 are similar to those of the mechanical system of Exercise 1 in Chapter 4. Explain why, using the concepts of effort and flow.

4. Find the mechanical systems equivalent to the circuits in Figure 5.21.

5. Find the transfer function of the circuit in Figure 5.8 from the impedance of the components for the following cases:

   (a) Impedance Z_1 is a resistor, impedance Z_2 is a capacitor.

   (b) Impedance Z_1 is a capacitor, impedance Z_2 is a resistor.

   (c) Impedance Z_1 is a resistor, impedance Z_2 is an inductor.

   (d) Impedance Z_1 is an inductor, impedance Z_2 is a resistor.

   (e) Both impedances Z_1 and Z_2 are capacitors.

   (f) Both impedances Z_1 and Z_2 are inductors.

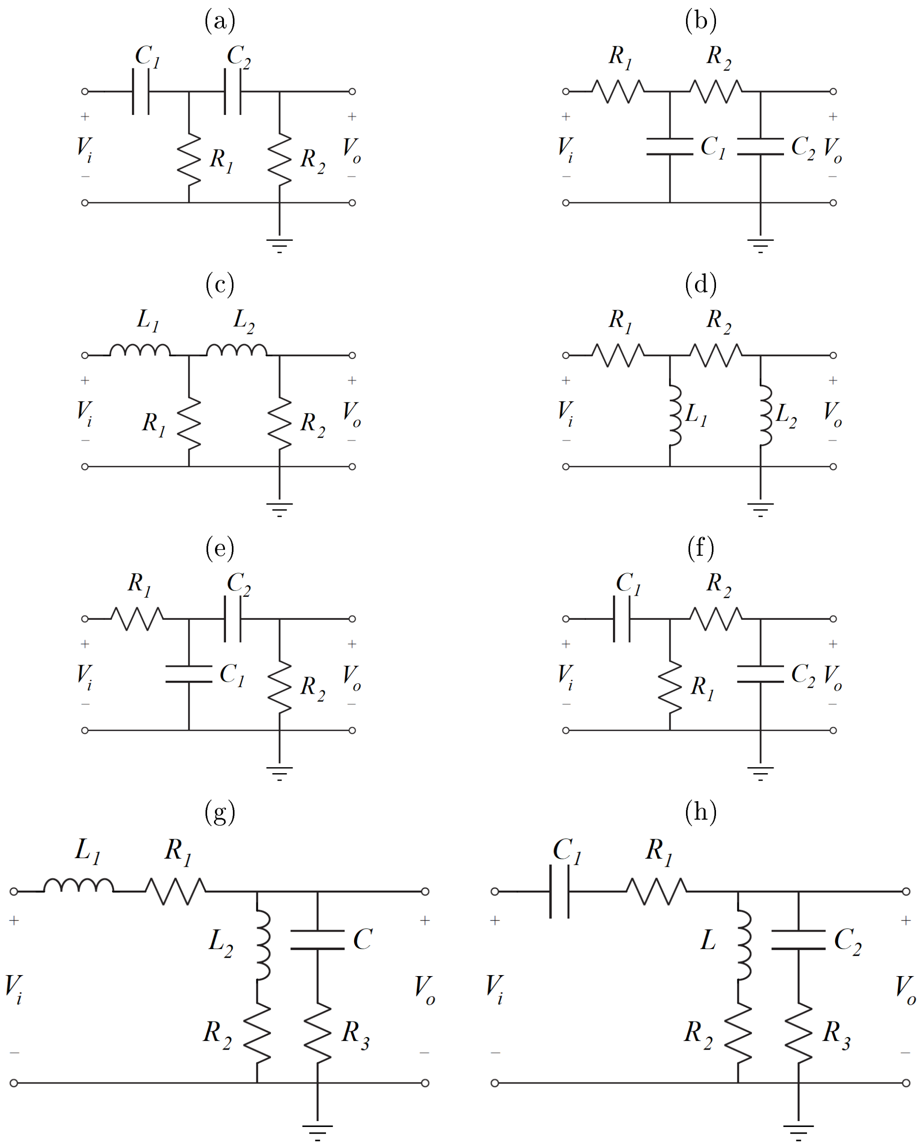

Figure 5.19: Systems of Exercises 1 and 2.

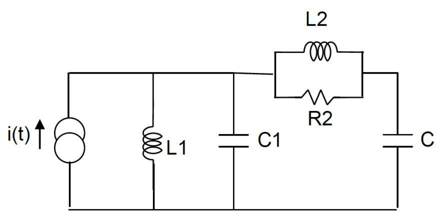

Figure 5.20: Circuit of exercise 3.

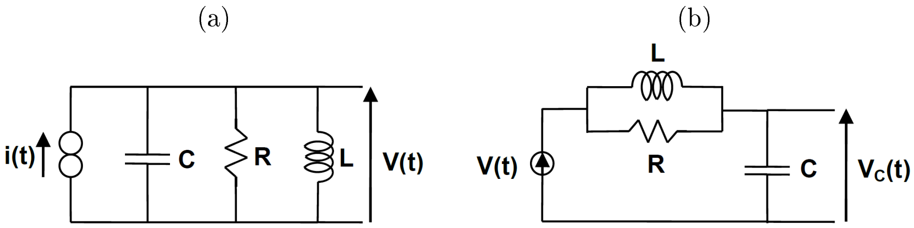

Figure 5.21: Systems of Exercise 4.

(g) Impedance Z_1 consists in a resistor and a capacitor in series, impedance Z_2 is a resistor.

(h) Impedance Z_1 consists in a resistor and a capacitor in parallel, impedance Z_2 is a resistor.

(i) Impedance Z_1 is a resistor, impedance Z_2 consists in a resistor and a capacitor in series.

(j) Impedance Z_1 is a resistor, impedance Z_2 consists in a resistor and a capacitor in parallel.

(k) Both impedances Z_1 and Z_2 consist in a resistor and a capacitor in series.

(l) Both impedances Z_1 and Z_2 consist in a resistor and a capacitor in parallel.

(m) Impedance Z_1 consists in a resistor and a capacitor in series, impedance Z_2 consists in a resistor and a capacitor in parallel.

(n) Impedance Z_1 consists in a resistor and a capacitor in parallel, impedance Z_2 consists in a resistor and a capacitor in series.

(o) Impedance Z_1 consists in a resistor and an inductor in series, impedance Z_2 is a resistor.

(p) Impedance Z_1 consists in a resistor and an inductor in parallel, impedance Z_2 is a resistor.

(q) Impedance Z_1 is a resistor, impedance Z_2 consists in a resistor and an inductor in series.

(r) Impedance Z_1 is a resistor, impedance Z_2 consists in a resistor and an inductor in parallel.

(s) Both impedances Z_1 and Z_2 consist in a resistor and an inductor in series.

(t) Both impedances Z_1 and Z_2 consist in a resistor and an inductor in parallel.

(u) Impedance Z_1 consists in a resistor and an inductor in series, impedance Z_2 consists in a resistor and an inductor in parallel.

(v) Impedance Z_1 consists in a resistor and an inductor in parallel, impedance Z_2 consists in a resistor and an inductor in series.

6. Find the transfer function of the circuit in Figure 5.22. Assume that all resistors are equal.

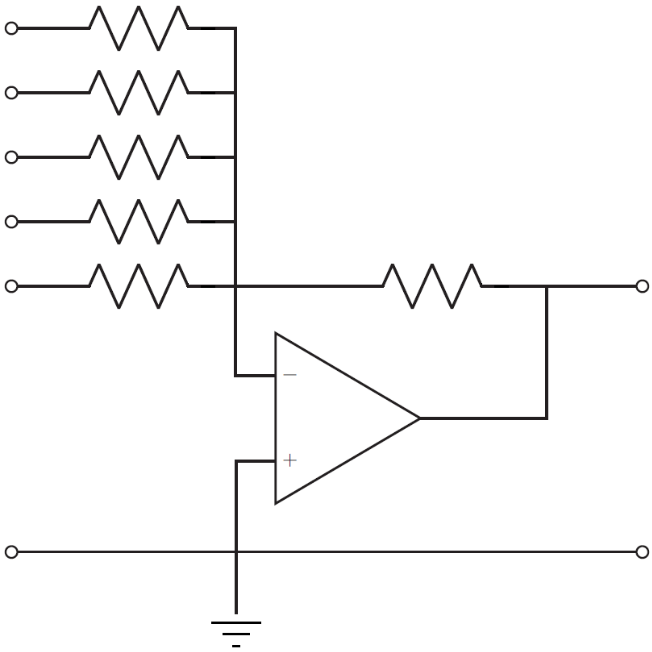

Figure 5.22: Circuit of exercise 6.

7. How could you perform operation $V_o = V_1 - V_2$ using two OpAmps and without resorting to the subtractor in Figure 5.17?

8. Design a circuit to perform the operation $V_o = V_1 + V_2 - V_3 + 2V_4 - 3V_5$. Use only one OpAmp.

9. Design a circuit to perform the operation $V_o = 10(V_1 + V_2 + \frac{1}{2}V_3)$. Use two OpAmps.

10. Modify the subtractor in Figure 5.17 so as to give:
    (a) $V_o = V_1 - \frac{1}{3}V_2$
    (b) $V_o = 5(V_1 - V_2)$

11. Suppose you are using an OpAmp with power supply $V^{S\pm} = \pm 20$ V as comparator. Use MATLAB to plot the expected output V_o for 0 s$\leq t \leq$ 10 s and the following inputs:
    (a) $V^+ = \sin(t\pi)$ V and $V^- = 5$ V
    (b) $V^+ = 5$ V and $V^- = \sin(t\pi)$ V
    (c) $V^+ = 10\sin(t\pi)$ V and $V^- = 5$ V
    (d) $V^+ = 5$ V and $V^- = 10\sin(t\pi)$ V