Influence of spectra model on the ship response

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ABSTRACT: The prediction of the ship responses in ocean-waves excitement is of marked importance on the design, as well as in lifetime operations. The structures have to withstand the wave induced loads as to guarantee motions and accelerations to be contained within acceptable limits, so that the comfort and safety of people and cargo onboard are not overlooked. Wave energy spectra parametric models can be used to describe the distribution of wave energy into both the frequency and directional domains. The ship responses can be estimated from them, regarding the correspondent Response Amplitude Operators (RAOs). This work aims at investigating the influence of the spectra models on the ship responses, studying the significance of the wave climate on the differences between the models, so that suited spectral models can be recommended according to the seaways the vessels are designed to operate. Cases in which the traditional uni-directional single-peaked approach provides enough accurate responses are to be highlighted, as also those in which a better description of the wave field energy in terms of wave components and directionality is needed. Single and double-peaked models were found to present agreeing results in dominated wave fields. Uni- and multi-directional models did not show to provide expectable agreement, although higher similarity in more severe wave climate was verified in heave, suggesting that simpler models can be used. A more marked variability on the differences in roll was observed, meaning that a better description of the wave field energy is suggested.

Keywords: Spectra Models, Ship Responses, Wave Climate

LIST OF SYMBOLS

T_m	Average Wave Period
S_ζ	Heave Response Spectrum(a)
$\Phi_{\zeta_w\zeta}$	Heave Transfer Function
θ_H	Relative wave direction
MWD	Mean Wave Direction
ζ_s	Significant Heave Amplitude
φ_s	Significant Roll Amplitude
H_s	Significant Wave Height
S_{ζ_w}	Wave Energy Spectrum(a)

1. INTRODUCTION

A major design concern about man-made marine structures is the capability of withstanding wave induced loads that shall act upon them during the operating life. Besides, motions, such as accelerations, are, in most cases, design priority, as they are intended to not be violent, which express the concerning about the safety of people and cargo on-board.

The description of the wave field energy is crucial part of the ship response computations. The wave energy spectrum is largely used to describe the wave energy distribution into both the frequency and directional domains. Full representation of the energy spectrum is in most of the cases impractical, thus parametric models are usually adopted to fit the energy distribution to classical distributions proposed in the literature. Integral parameters required to describe the wave field main characteristics are, for instance: H_s, T_m and MWD. The wave energy spectrum can be expressed by different parametric models, such as uni- or multi-directional, single or double-peaked ones, reflecting the presence of multiple wave systems. Depending on the sea-state characteristics, the variability on the ship response estimations when changing the parametric model can be more or less significant, in such way that it shows the importance of selecting a reliable model in order to avoid eventual under-estimations on the calculation of loads and motions.

The influence of selecting either a parametric spectra formulation or the full spectrum on both the ship responses and route planning was investigated [1]. One shows that different decisions on route planning can eventually be derived whether using the parametric formulation over the full spectra. Detailed and reliable estimates on the ship responses such as vertical bending moment, shear stresses and accelerations could be derived thanks to wave spectra partitioning into wave components, swell and wind-sea [2]. Aspects such as operational life extension could be approached for a FPSO operating at the North Sea, which heading was found to be dominated by wind-sea waves. Such information would be lost whether the combined wave system was considered, showing therefore the relevance of the wave directionality on the response computations. Aside the mentioned studies, the significance of spectra models on the response estimates is such a subject that has not yet been explored deeply by the scientific community, though.

The present work aims at verifying the influence of the spectra model over the responses and deriving the relevance of the wave climate on the difference between the models. The particular cases when the usual approach of describing the energy spectrum by a simple single-peaked uni-directional parametric model can be reliable for the ship response estimation are intended to be highlighted such as those when a more complete description of the wave field energy distribution is suggested. The selection of a suitable model shows to be crucial for the reliable response estimations, being important for both the design and operational purposes.

2. SHIP SEAKEEPING

2.1 Seakeeping problem and strip theory

To compute the ship responses, both the wave energy spectrum and the ship transfer functions are needed. The response spectrum, $S_r(\omega,\theta)$, respectively to an arbitrary response r, in terms of both the wave frequency and direction domains can be found as formulated in Equation 1, as described in [3]:

$$S_r(\omega,\theta) = \left|\Phi_{\zeta_w r}(\omega,\theta)\right|^2 \times S_{\zeta_w}(\omega,\theta) \qquad (1)$$

The terms are such that $\left|\Phi_{\zeta_w r}(\omega,\theta)\right|$ is the module of the transfer function, meaning the Response Amplitude Operator (RAO) and $S_{\zeta_w}(\omega,\theta)$, the wave energy spectrum, which, for instance, can be expressed in terms of integral parameters (H_s, T_m and MWD) such as by the parametric formulation of the JONSWAP spectrum.

RAOs are usually obtained either experimentally from tests in model basins or can be determined computationally, by solving the seakeeping problem with numerical methods such as Strip Theory. This latter methodology is the most commonly used in ship response analysis, and the one applied in this work.

The definition of the fluid loads acting upon a floating structure performing harmonic oscillatory motions and the motion amplitudes, transfer functions and other measurements of interest, define the seakeeping problem. The fluid is considered to be ideal, meaning incompressible, inviscid, irrotational and without surface tension. Incident harmonic waves are present and the water depth is finite. The linearized Bernoulli equation is applied to compute the fluid pressures on the body surface. Such linearization is possible due to the assumption of small amplitudes and motion velocities. The Bernoulli equation depends on the solution of the velocity potential of the fluid.

The velocity potential can then be expressed in terms of three components, as by Equation 2:

$$\phi(x,y,z,t) = \phi_r + \phi_w + \phi_d \qquad (2)$$

They are:

- ϕ_r, the so-called radiation potential, which represents the disturbances in the fluid due to the oscillatory motion of the body.
- ϕ_w, the velocity potential associated to the incident harmonic waves, called undisturbed wave potential.
- ϕ_d, the diffraction potential, which represents the velocity potential of the fluid associated to the diffracted waves by the body, in case of fixed condition.

The solution of the total velocity potential, expressed in the left hand of Equation 2, must satisfy a set of boundary conditions. The definition of the velocity potential and the imposition of the boundary conditions define the so-called Boundary Value Problem. The linearized Bernoulli equation has to be applied to compute the pressure field upon the hull, from which the exciting forces and moments can be obtained. In the end, these exciting loads are equated to the inertial ones, these latter associated to the acceleration of the body mass, as described in [4], in order to derive the general equations of the motions, from which the transfer functions are obtained.

The ship transfer functions are to be obtained numerically from an in-house seakeeping code developed in CENTEC, in which strip theory is implemented to solve the seakeeping problem regarding the potential theory. Good estimations on the RAOs, heave and pitch, specifically, were found to be derived from the code, where a comparative investigation between three codes (the in-house developed one, PDStrip and MaxSurf) and experimental data from two fast displacement hulls in head waves derived from model testing was performed [5].

2.2 Ship description

The ship selected is the S-175 container ship due to the considerable number of works already published in which this hull is object of study as for seakeeping analysis such as those carried out during the 15th International Towing Tank Conference (ITTC) in which a comparative numerical study is performed on motion prediction, [6], or for loads assessment such as described in [7], when nonlinear responses are compared to published experimental data.

The container ship main particulars are presented in Table 1 and the bodylines, in Figure 1.

Table 1. S175 Container Ship main particulars

Length between perpendiculars	L_{pp} [m]	175
Beam	B [m]	25.4
Depth	D [m]	15.4
Draft	T [m]	9.5
Displacement	Δ [ton]	24742
Longitudinal position of CG	LCG [m]	-2.43

The program input file is prepared, in which the submerged hull is described. The ship forward speed is hereafter considered to be such that $F_n = 0.2$, meaning, approximately 16 knots. Ship courses in a range with step of 10 degrees from 0° to 180° are implemented to obtain the RAOs.

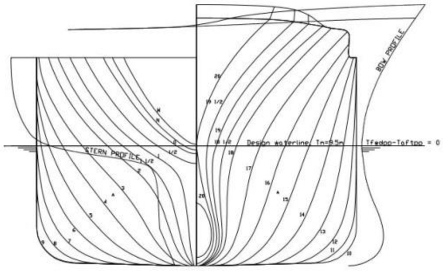

Figure 1. S175 Container Ship bodylines

3. WAVE SPECTRA MODELS AND FORMULATIONS

3.1 The JONSWAP spectrum

In 1964, Pierson and Moskowitz [8] presented for a fully developed sea-state the uni-directional wave energy spectrum (P-M Spectrum). Developing seas, however, were found to have a more peaked shape, which came to be better described by the JONSWAP Spectrum, where the dependency on the wind speed and fetch was introduced, as proposed by the formulation shown in [9]. This latter is considered to be a generalization of the P-M Spectrum through the introduction of the mentioned parameters, and particularly when the so-called peak enhancement factor equals 1, it simplifies into the P-M formulation, as shown in.

In 1976, the parametrization of the former JONSWAP spectrum was proposed in terms of H_s and T_m [10]. This latter formulation is used in this work to describe the wave energy distribution into the frequency domain (and further distribution into the directional domain is to be performed over this model in order to obtain the directional ones), with which the ship responses are to be computed. In Equation 3 the JONSWAP parametric frequency spectrum is presented.

$$S_{\zeta_w}(f) = 0.11 H_v^2 T_v (T_v f)^{-5} \exp[-0.44 (T_v f)^{-4}] F_1^{-1} \gamma^{\exp\left[-\frac{1}{2\sigma^2}(1.296 f T_v - 1)^2\right]} \quad (3)$$

The terms are:

- $H_v = \frac{H_s}{\sqrt{F_1}}$, where H_s is the significant wave height in m;
- $T_v = \frac{T_z}{F_2}$, where T_z is the average period, according to [10] and [11], based on the first moment, meaning that $T_z = \left(\frac{1}{2\pi}\right)\left(\frac{m_0}{m_1}\right)$, where m_0 and m_1 are the spectral moments of order 0 and 1, respectively;
- F_1 and F_2 are correction factors for area and peak period of a P-M frequency spectrum, respectively;
- γ is the JONSWAP peak enhancement factor, and for the "average JONSWAP spectrum" is considered to be such that $\gamma = 3.3$, moreover, whether the factor equals 1, the spectrum turns to be the same as the P-M one;
- σ is a parameter associated to the peak width, which is defined as follows:

In this work, the values of F_1 and F_2 are assumed to be those in which the *average* JONSWAP spectrum is considered ($\gamma = 3.3$), hence $F_1 = 1.52$ and $F_2 = 0.92$.

3.2 Double-peaked parametric model

Assuming the wave energy spectrum to be single-peaked can eventually be inadequate to suitably represent the wave field energy, as it describes the energy distribution of a resulting average wave system, in such way that the separate contribution of each component (swell and wind-sea) is not considered. Depending on the wave climate, both swell and wind-sea can be equally relevant about the total wave field energy in such way that this information as well as the correspondent peak frequencies of each wave component are lost whether considering single-peaked formulation. Even worse, the peak of the spectrum assessed with the single-peak model, can be located relatively far from the ranges of frequencies where there is the highest concentration of energy. This can significantly distort the results that are sensitive to the vicinity of the wave energy to the natural frequency of the motions.

In that way, the double-peaked spectral modeling can be expressed by the sum of two JONSWAP spectra models. Considering swell (S_S) and wind-sea waves (S_W) as the possible wave components of the wave field system, the total spectrum can be expressed by:

$$S_{\zeta_w}(\omega) = S_{\zeta_w,S}(\omega) + S_{\zeta_w,W}(\omega) \qquad (4)$$

Both the swell and wind-sea energy spectra are, in this work, suitably represented by the JONSWAP formulation, on regards of their correspondent H_s, T_m and MWD.

3.3 Directional wave distribution

The frequency (uni-directional) wave spectrum $S(\omega)$ can be spread into the direction domain, yielding the directional spectrum. The directional distribution is performed by weighting it with a probability distribution function, as described in [12].

$$S_{\zeta_w}(\omega,\theta) = D(\theta|s,\theta_W).S_{\zeta_w}(\omega) \qquad (5)$$

The term $D(\theta|s,\theta_W)$ is defined as *direction dispersion function*. It represents the wave energy distribution at a given frequency into the direction domain around the main propagation direction, θ_W. It is formulated as follows:

$$D(\theta|s,\theta_W) = \frac{2^{2s-1}\Gamma(s+1)\Gamma(s)}{\pi\Gamma(2s)}\cos^{2s}(\theta - \theta_W) \qquad (6)$$

Where:

- s is the factor of the dispersion function, in this work, considered to be 1;
- θ_W is the dominant wave propagation direction;
- $\Gamma(s)$ is the Gamma Function.

The factor of the dispersion function, s, may assume different values for swell and wind-sea, as these two components present quite different directionality, inherently associated to their generation mechanisms. Wind-sea is more commonly to be found propagating in different directions while swell waves usually propagate towards a fairly defined one. The energy directionality of both components at a single point can present, therefore, different distributions. The ship responses are to be, however, computed over a grid of points on the North Atlantic, in such way that the definition of the suitable dispersion factor for each point comes to be impractical, then one assumes the same value for both wave components. Further investigations on the influence of the dispersion factor on the responses may be, nevertheless, requited.

3.4 Ocean wave data

The representation of the wave energy spectrum is made through the models so far presented. The H_s, T_m and MWDs of the combined wave system and components, swell and wind-sea, are, therefore, the main ocean-wave data required as prior information. These data are retrieved from the ERA-Interim database, which is an ocean atmospheric reanalysis provided by the European Centre of Medium-Range Weather Forecasts (ECMWF) [13]. Environmental data estimates are performed by combination of data assimilated majorly from satellite observations and prior information from a forecast model. Ocean surface analysis in terms of wave height are performed using observations from space-born radar altimeters and background estimates from such model, which describes the evolution of two-dimensional wave spectrum at the sea surface, considering the contribution of swell and wind-sea waves. The analyzed wave heights are used to continuously adjust the model-predicted wave spectra. A 6-hourly global ocean-wave data from 2017 is retrieved and used in this work.

The forecast model is composed by a set of equations that describe some of the physical aspects associated to climatology in general.

The spectra representation of the atmospheric dynamic variables constitutes the main role of the atmospheric model on the estimates. The impact of ocean waves on the airflow via transfer of energy and momentum is the main focus of the ocean waves forecast model, on the other hand. Atmospheric variables that influence the wave growth are introduced to this latter. The consequential impact of the resulting sea-state on the ocean surface roughness is the output of the process.

The wave model is based on the WAM approach. This latter was introduce to the Integrated Forecast System (IFS) and was the first one on solving the energy balance equation, in which nonlinear wave interactions were included.

4. RESPONSES TO DIFFERENT SPECTRA MODELS

In this section, some differences between the outputs of the parametric models on heave motion are highlighted. The objective is to verify what they are related to, regarding the influence of the wave climate on the relation between the energy spectrum and the RAOs.

The relevance of wind-sea waves over the total amount of energy is measured according to the parameter shown in Equation 7. Hereafter, a wave component is considered *negligible* if the ratio between the energy carried by that component is lower than 10% of the total spectral energy, such that, with regards to wind-sea:

$$\left(\frac{H_S^{WW}}{H_S}\right)^2 < 0.1 \tag{7}$$

The wave data, which the wave spectra models are related to, are from a point near to the Azores archipelago (GPS coordinates: 40º N, 26º W). The ship is sailing at $F_n = 0.2$, service speed at, approximately, 16 knots and course of 45 degrees, North-East direction.

In this particular case, the models produced different results. Wind-sea waves are considered to be relevant, as $\left(\frac{H_S^{WW}}{H_S}\right)^2 = 0.15$.

The heave RAO and the wave energy spectrum about the single and double-peaked frequency models are displayed in Figure 2. The combined wave system main characteristics are shown in the textbox.

The T_m of wind-sea waves is less than half of the swell period, as $T_m^{WW} = 4.06$ [s] and $T_m^S = 8.76$ [s]. As shown either in Figure 2 or in Figure 4.B, the energy peak associated to the wind-sea is not represented, as the peak frequency of that component overcomes the frequency threshold until which waves can excite the ship on heave motion.

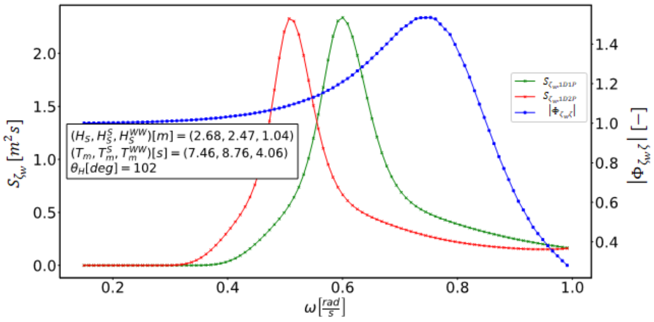

Figure 2. Wave spectra and Heave RAO, 1D models comparison

The wind-sea recorded has, in other words, relatively short wave length compared with the ship size, meaning that the excitations produced are irrelevant. Nevertheless, the combined wave system is still affected by the contributions of both the swell and wind-sea components, meaning that the resulting energy peak locates in between the peaks of the components. This effect can be seen in Figure 2, as the combined wave system peak (1D1P) is slightly shifted to higher frequencies. The heave RAO shows to amplify the excitations, as the relative wave direction ($\theta_H = 102$ [deg]) is a high excitation direction, in this case. The magnification performed over the 1D1P model ends up to be higher, since the resulting energy peak is shifted towards the increasing RAO amplification. A more energetic response is obtained compared to the correspondent frequency double-peaked model ($SHA_{1D1P} = 1.56$ [m] and $SHA_{1D2P} = 1.39$ [m]), as seen in Figure 3, in which this latter response is approximately 11% lower than the first one.

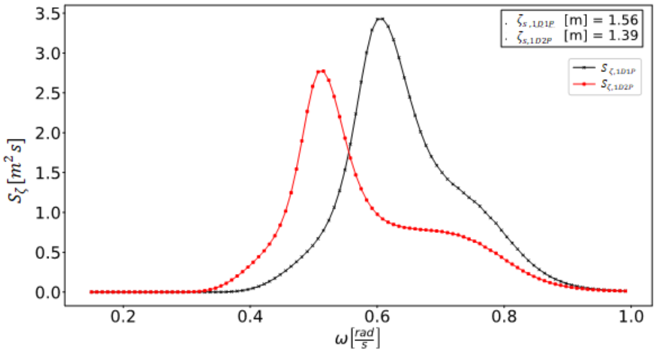

Figure 3. Heave response spectra, 1D models comparison

It shows, therefore, that waves which eventually do not even excite ship motions, but are energetically relevant for the wave field, are inherently sensed on the general measurement of the combined wave system and that can also have an impact on the responses. Such disagreement can eventually be relevant on the design or for operational purposes.

The energy concentration at θ_H, considered as a high excitation direction, makes the frequency model to produce higher responses compared to the directional ones. The significant heave amplitude of the single-peaked directional model is $SHA_{2D1P} = 1.30$ [m], which is 17% lower than the correspondent frequency one. The same explains the difference between the double-peaked frequency and directional models ($SHA_{2D2P} = 1.14$ [m], which is 18% less than the 1D2P rmodel and 27% less than the 1D1P one).

The directional wave spectra are displayed in Figure 4. The difference between the single and double-peaked directional models is pronounced, as $SHA_{2D1P} = 1.30$ [m] and $SHA_{2D2P} = 1.14$ [m], as shown in Figure 6, panels A and B, respectively. Not only the effect of the wind-sea T_m over the combined wave system one is verified but also the MWD of the wind-sea component contributed for the wave field represented by the 2D1P model to be slightly shifted towards higher excitation directions.

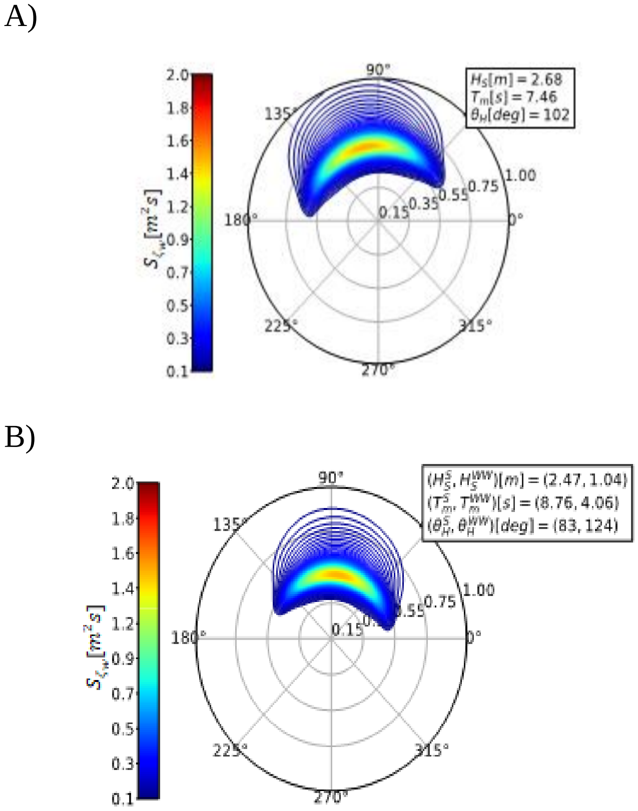

Figure 4. Directional wave spectra. A) is the 2D1P model and B) is the 2D2P one

The single-peaked directional spectrum shown in Figure 4.A appears to have the energy peak located at the second quadrant while about the double-peaked model, see Figure 4.B, such energetic response peak is less marked. The resulting heave response spectrum about the 2D1P model showed, therefore, to present a pronounced energy peak at the second quadrant, as the responses are magnified about that directions by the heave RAO. The heave spectra are presented in Figure 6.

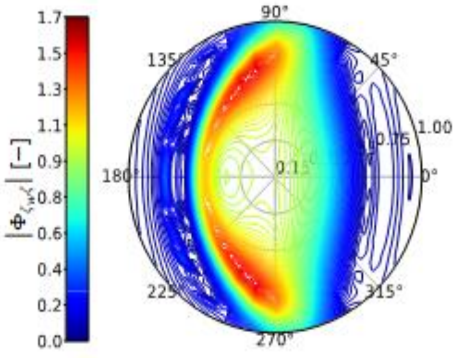

Figure 5. Heave RAO

Some mechanisms associated to the differences between the models could be highlighted and the relation between them and the wave climate was outlined, from this example.

Clearly, single and double-peaked models can differ from each other, for instance, when both wave components are energetically relevant about the total wave field energy. In this case, even though wind-sea showed to not excite the ship on heave motions, its influence on the combined wave system was effective in such way that single and double-peaked models came to provide different results.

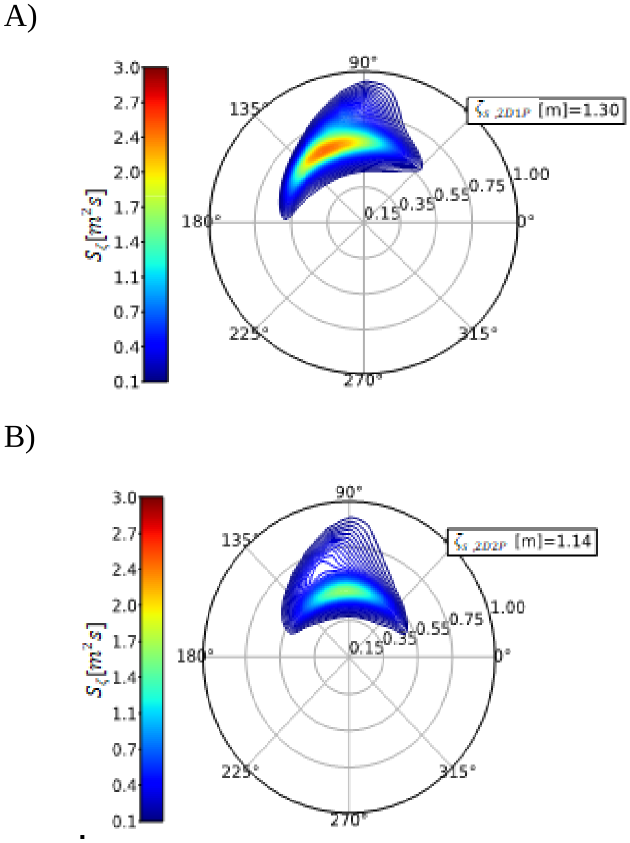

Figure 6. Heave response spectra, 2D models comparison. Same disposition as in Figure 4

The difference between the frequency and directional models strongly depends on θ_H and how the RAOs relate with the wave energy spectrum. The energy concentration at a single direction, considered by frequency models, plays a relevant role about the responses in that direction. Over or under-estimated responses can be produced, depending if θ_H is a high excitation direction or not. The agreement between 1D and 2D models is hard to be predicted, due to the stochastic nature of the wave climate on regards of the MWDs of the different wave components, therefore.

In other examples discussed in the main text it was found the single and double-peaked models tend to provide agreeing responses as long as either swell or wind-sea is energetically irrelevant about the total energy, in other words, when dominated wave fields are verified. The agreement between these latter models can, therefore, be predicted whether known if dominated wave fields are likely to occur on the seaways the vessels are meant to operate at.

5. SHIP RESPONSES ON THE NORTH ATLANTIC

Heave and roll responses computed over a grid about the North Atlantic are shown. Visual evidences of the differences between the models over the grid are shown and assessed in sight of the general wave climate expectable on the North Atlantic.

In order to take into account all possible courses that the ships can assume in each location, a discrete probability distribution function has been calculated for each considered grid point by analyzing the reports provided within the Voluntary Observing Ships (VOS) scheme [14] by vessels navigating in these areas. These ships are recruited by National Meteorological Services to sense and transmit these meteorological data, including visually observed wave heights, periods and directions of swell and wind-sea waves. That data are associated to the coordinates where the voluntary ships sail through. Furthermore, the measurements of the average H_s are in general over-estimated in comparison to those generated by remote sensing and numerical methods, in areas with small amplitudes, as shown in [15].

The dependency of ship responses on the course is then eliminated by weighting the responses at each direction by the correspondent course probability of occurrence, providing a map of the generic expectable behavior of the ship. In Figure 7, are presented the weighted averages of each model on the significant heave amplitude. Different ship headings (courses) are considered, from 0° to 315° with step of 45°, and for each, the models are applied in order to compute the ship responses at every single point of the grid, in which the North Atlantic is represented.

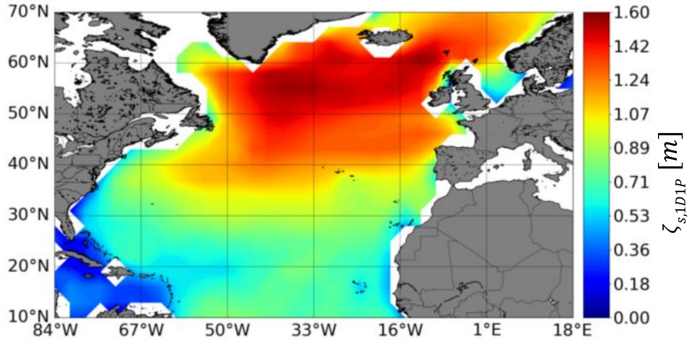

Figure 7. Weighted average mappings for the North Atlantic on the Heave Response, 1D1P model

In Figure 8, are displayed the relative differences between the frequency single and double-peaked models. These plots allow verifying that the differences, in these cases, tend to decrease towards the extratropical area. Furthermore, it was verified that the probability of dominated wave fields is high on the North Atlantic in the area within 20ºN and 40ºN, 10ºW and 60ºW (that will be better discussed in the next section). This can then be related to the decreasing differences verified at these locations. As swell dominated wave field is expected in that area, the differences between the single and double-peaked models are also expected to be lower. Therefore, in areas where wave system domination is verified, single-peaked models could preferably be used over double-peaked ones, which decision can be time saving from the computational point of view, such that, nevertheless, fairly good results are provided compared to the more complete and detailed models.

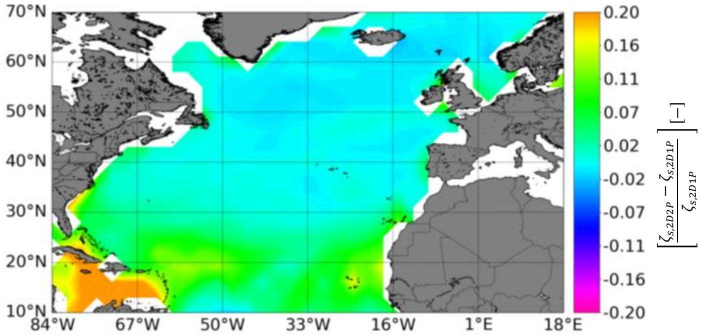

Figure 8. Relative difference mappings for the North Atlantic on the Heave Response, 1D2PX1D1P

Single and double-peaked models tend to be strongly correlated, as seen in Figure 9. The Pierson Correlation Coefficient is used to provide a quantitative measurement of how strongly are the models related to each other.

These models tend to produce agreeing responses under the occurrence of dominated wave fields, which is not necessarily true when comparing frequency and directional models. For that reason, more often than not, single and double-peaked models yielded similar results, resulting in stronger correlations.

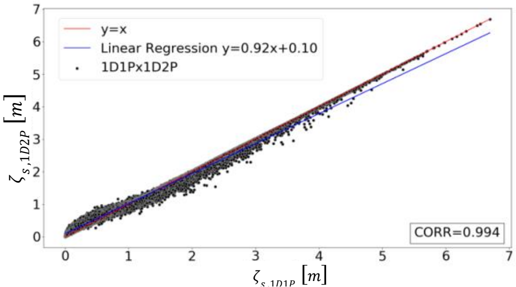

Figure 9. Correlation between the models, 1D1PX1D2P

Such discussion can be assisted by the results shown in Figure 10, where are displayed the algebraic difference between the models and its relation with the part from the total wave field energy carried by the wind-sea component.

The algebraic difference between both the single and double-peaked models tend to increase as wind-sea becomes relevant, $\left(\frac{H_S^{WW}}{H_S}\right)^2 > 0.1$, and tend to decrease as swell becomes negligible, $\left(\frac{H_S^{WW}}{H_S}\right)^2 > 0.9$. Since the difference decreases as long as one wave component becomes negligible, inherently the 1P and 2P models present stronger global correlation, as increased is the number of events when they provide agreeing responses.

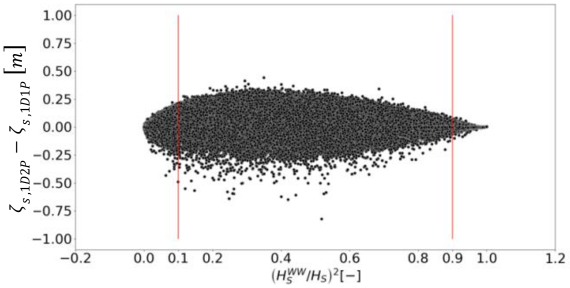

Figure 10. Relation between the significant heave amplitude absolute difference and the part of wind-sea energy from the total one, 1D1PX1D2P

It was found that, in general, for heave responses higher than 5 meters, the model comparison method adheres the $y = x$ line quite strongly, even from the comparison between frequency and directional models, although in this case the adherence was found to be less marked. Extreme stormy weather is, in general, scenario in which these responses are more likely to occur. The low differences in these events, mostly those verified in Figure 9, suggests that the wave-field was dominated by one wave component, as the agreement between the single and double-peaked models is significant. According to studies on sea-state classification shown in Table 1 from [16], the wave-field in Azores can be considered to be wind-sea dominated when stormy events are verified, where the H_s usually exceeds 8 [m]. In sight of that, wind-waves generated by the local wind in extreme wave-climate dominate the wave field in such way that ocean-going waves as swell tend to play a smaller role. Therefore, one can attribute the strong correlation between single and double-peaked models when extreme heave responses are verified due to wind-sea dominated wave-fields in stormy conditions.

As general rule, the higher the responses, the stronger the correlation between the models.

These findings allows to draw some important conclusions about the usage of different models for design or operational purposes. At the design stage regarding seakeeping under extreme weather conditions, the selection of simpler models is not expected to lead to substantial disagreements compared to results from more complete ones. Thus the traditional and simpler 1D1P model can be recommended. The general agreement in less severe sea-states, $SHA \leq 5.0$ [m], is not conclusive, though, as the pattern shows an increased disagreement about the ideal line. In sight of that, when considering the optimization of the operability in average climate, the parametric model to be used should be accurately selected. In this case, it is suggested to gather more information about the sea-ways the ship is supposed to sail through, as to determine if they are likely to be dominated by one wave component, or rather composed by mode wave systems, possibly from different directions. This allows, eventually, the selection of the most suited model.

6. SEA STATE CLASSIFICATIONS

It becomes vital to foreknow if the wave field is expected to be dominated, as in this case single-peaked models are sufficiently accurate to compute the desired responses. The knowledge of the relevance of the wave components on the wave climate can be achieved if the wave field is classified according to pre-defined parameters and if statistical analysis are performed over a certain time record in order to determine the probability of occurrence of each class.

The classification methodology used in this work follows a straightforward parametric classification methodology.

The classification is based on the wave field main characteristics. Three parameters are used to distinguish the further presented classes. They are:

$$\tilde{H} = \left(\frac{H_S^{WW}}{H_S}\right)^2 \tag{8}$$

$$\tilde{T} = abs(T_m^S - T_m^{WW}) \tag{9}$$

$$\tilde{\theta} = abs(\theta_W^S - \theta_W^{WW}) \tag{10}$$

The classes whose nomenclature is displayed below are:

- One-peaked swell dominated – OPS;
- One-peaked wind sea dominated – OPWS;
- Two-peaked, no crossing-seas occurrence – TPNCS;
- Two-peaked, with crossing-seas occurrence – TPCS;
- Undefined Mixed Condition – UMC.

The classes are mathematically defined as shown in Table 2.

Table 2. Mathematical description of the classes

Class	Description
OPS	$\tilde{H} \leq 0.1$
OPWS	$\tilde{H} \geq 0.9$
TPNCS	$0.1 < \tilde{H} < 0.9$ $\tilde{T} > 4.0\ s$ $\tilde{\theta} \leq 30°$
TPCS	$0.1 < \tilde{H} < 0.9$ $\tilde{\theta} > 30°$
UMC	$0.1 < \tilde{H} < 0.9$ $\tilde{T} \leq 4.0\ s$ $\tilde{\theta} \leq 30°$

Single-peaked events probability of occurrence can be computed over the North Atlantic grid, as shown in Figure 11. It represents the sum of both the swell (OPS) and wind-sea (OPWS) probability of occurrences, meaning when the wave field is likely to be dominated by a single component.

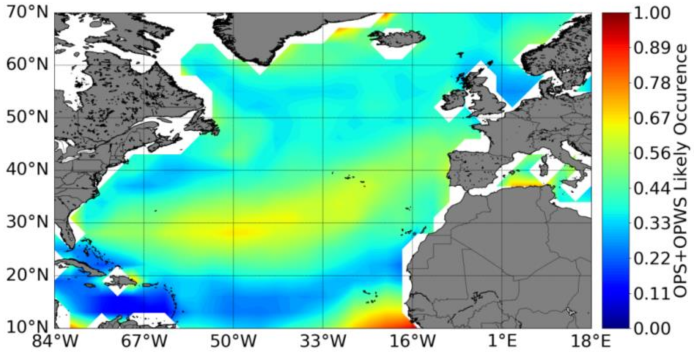

Figure 11. Occurrence probability of single-peaked events

High probability of single-peaked spectra is verified within 20°N and 50°N, therefore. The decreased differences observed in Figure 8 in that area can, therefore, be in part attributed to the higher occurrence of single-peaked spectrum events as well as to the fact that the amplitudes tend to increase from the inter towards the extratropical zone, as models tend to provide similar results as higher the amplitudes.

The direct influence of the classified sea states upon the differences between the models can be verified conditioning the statistical analysis of the wave field to the classifications proposed. It is expectable, indeed, that OPS and OPWS classes will be fairly represented by 1D1P or 2D1P models, TPNCS class will require at least 1D2P model, whereas TPCS must include both double peak and directionality in the spectral model as to catch the real physics of the wave field.

In Figure 12 are shown the relative differences conditioned to single-peaked wave field.

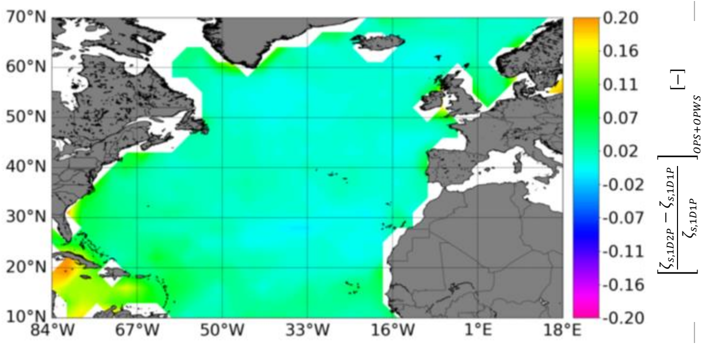

Figure 12. Same as Figure 8, but conditioned to single-peaked wave field only

The differences between single and double-peaked models are negligible over the whole area, excepting in marginal seas such as the Caribbean one. The differences on the extratropical area, around the parallel 50°N, are negligible, as these are the locations where high responses are verified. That figure provide visual evidence that in fact single and double-peaked models output agreeing responses as long as the wave field is dominated by a specific component as well as show the particular tendency of the differences to be lower in higher responses. The same agreement in dominated seas on the single and double-peaked models comparison was verified about roll responses, although the 1D1P model slightly over-estimated the responses about 50°N.

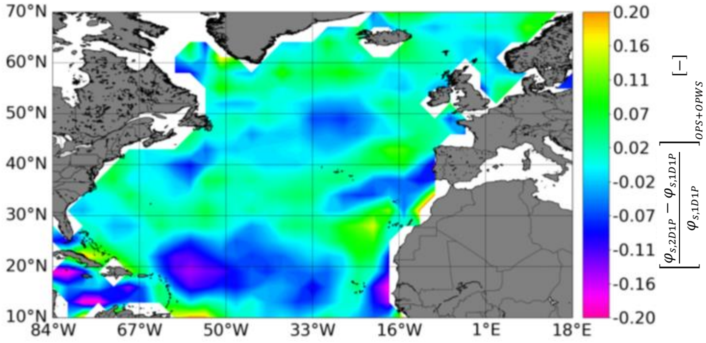

Figure 13. Relative difference mappings for the North Atlantic on the Roll Response, 2D1PX1D1P

The differences between frequency and directional models showed strong variability in roll motion, on the other hand, as seen in Figure 13. One notices that the frequency model either under or over-estimate the responses, in such way that similar results are quite uncommon to be seen. Differently from the heave motion where the differences tended to decrease towards the extratropical area, locations where agreeing responses are observed cannot be highlighted as pattern since in roll motion the differences between frequency and directional models seem to have a random nature.

7. CONCLUSIONS

In this work, the influence of the spectra model on the ship response is analyzed. The differences on the ship responses regarding the application of each parametric model and how they can be related to the wave climate in general is the main focus of this work. First, an understanding on the mechanisms that lead to the differences between the models is outlined and finally a relation between them and the wave climate is studied, as the agreement between the models vary according to the environmental characteristics recorded at each event. In sight of the sea-ways climate that the vessels are designed to work at, simpler parametric models such as the single-peaked one can be eventually suggested as long as they provide similar results compared to the more complete one. Avoiding the usage of more complete model in fact can be preferable for computational time saving and modeling simplicity.

Regarding relation between different models for the same sea-state condition, it was found that, in general, when both swell and wind-sea are energetically relevant about the wave field energy, the models tend to provide different results. The differences between single and double-peaked models regarding the same energy distribution tend to decrease as long as the relevance of one of the components shows to be lower compared to the total energy. In these cases, the resultant spectrum of the double-peaked model mostly represents the energy distribution of the most energetic component, often being swell. The differences between frequency and directional models, even considering dominated wave field, vary according to the relative wave direction. Frequency models can provide similar, and either significantly higher or lower results compared to the directions ones, depending whether the relative wave direction is a high excitation direction or not.

The differences between the models over the grid on the North Atlantic provided a visual evidence of their variability according to the locally dependent wave climate.

The sea state classification showed its importance, as the differences between the models could be analyzed regarding similar wave fields recorded in each classification. Regarding the heave conditioned to single-peaked events, both the single and double-peaked models showed to provide similar results all around the open North Atlantic, as seen in Figure 12. A higher variability is presented when comparing frequency and directional models, as expected, since the agreement between them strongly depends on the MHA and how the wave energy spectrum relates with the HTF. The differences in this latter case tended to decrease towards the extratropical zone, where higher responses are expected. About both the TPNCS and TPCS classifications, it was found that single-peaked models tend to under-estimate the responses compared to those from the double-peaked ones, although towards the extratropical zone, higher agreement is observed,

On regards of roll responses, on the other hand, aside the agreement verified between single and double-peaked models in dominated wave field, a general higher variability on the differences between the models was observed. When one component is not dominant, more often than not frequency models misestimated the responses compared to those from the double-peaked ones. The differences between frequency and directional models showed quite high variability in all conditioned seas. More complete and detailed models such as the directional double-peaked one showed to be more appropriate than the simpler ones, as the contribution of both the swell and wind-sea are separately taken into consideration.

It is clear that different parametric models can provide different responses depending on the wave climate and on the location. The prior knowledge of the climate of the sea-ways the structures are designed to operate at can be fundamental for the selection of a suitable model, as in some cases, simpler models can present fairly reliable results compared to those from more complete ones. In general, in dominated sea-states, the single-peaked directional model can be used rather than the double-peaked one. Besides, for the usage of frequency models in dominated seas, one should take into consideration whether extreme responses are expected, as the differences between them and the directional models tend to decrease under severe weather conditions. In locations where both the swell and wind-sea components are relevant, more complete models should be taken into consideration.

The findings of the present work allow to provide some important recommendation for the selection of the most appropriate spectral model:

- for design purposes, when limit state values are to be evaluated, the traditional approach to single-peak model can be accepted, however taking into account directional spreading is recommendable, especially when roll motion is considered.

- In case of design parameter influenced by frequent events of lower intensity, such as fatigue, more complex models may be needed, however further research on structural loads should be carried out to better investigate these aspects;
- for operational purposes, the expected sea-state should be carefully categorized and guided towards the model that better reflects the physics of such a wave field.

Finally, the performed analysis opens the path to further investigations in this directions. First of all, since the responses have been shown to be sensitive to the directional distribution of the wave energy, more effort should be put on the identification of the most appropriate parameters for the directional spreading function, depending on the wave component and, possibly, on the location. The complexity of the wave climate on closed seas and coastal areas suggests to study in more details the ship responses in these regions, especially considering that these are often the busiest operational areas. Moreover, a similar analysis, but focused on structural stresses can also lead to important conclusions for the design of marine structures.

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