Estimating Vibration, Acoustic and Vibro-Acoustic responses using Transmissibility functions

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Abstract

With this work, it is proposed to study ways to numerically estimate Vibration, Acoustic and Vibro-Acoustic (V-A) responses through Transmissibility functions. The author proposes to extend existing methodologies of dynamic displacement transmissibility and acoustic pressure transmissibility to the V-A case. So far, only experimental data for scalar V-A Transmissibility has been presented in available literature.

The methodology and results of its' implementation addresses initially the vibrational and acoustic Transmissibility Verification. Then a 3D Finite Element Method (FEM) implementation created with a fluid-structure interface, from which pressure and displacement response are calculated, and estimation method proposed with Single and Multiple degrees of freedom (SDOF and MDOF) Transmissibility functions obtained with Frequency Response Functions (FRFs) extracted from the coupled system. Primarily a scalar Transmissibility is proposed, followed by a matricial one which relates sets of displacements with pressures (structural-fluid). This is done for a range of frequencies, and assuming harmonic plane waves.

In conclusion, the concept of V-A Transmissibility was implemented and is still in development. The implementation is described and discussed. However, the process is still quite complex and the simulations for coupled Finite Elements (FE) are relatively heavy and time costly. The procedure and results presented are considered a contribution in the direction of a full answer to the challenge.
Keywords: Vibro-Acoustic Transmissiblity, Frequency domain, Coupled Systems, Finite Element Method, Fluid-Structure Interface

1. Introduction

Since the presence of V-A interface transmissibility is scarce, almost non-existent among the scientific community, and an indefinite answer has yet to be achieved towards actual response estimation, there is a certain interest towards transmissibility between variables of different natures. Indeed it would be of interest to further deepen the relation between pressure disturbances in an acoustic medium and displacements (and subsequent loads) existing in a certain nearby structure, if the interaction is properly established and both mediums perform an enclosure.

In previous works, there has only been proposed experimental V-A Transmissiblity analysis, where no model is actually suggested for prediction. Such is the case in [1], where the author only retrieves experimental data from FRFs (called transfer functions obtained along a transfer path) relating pressure with acceleration/force applied, in a car, as inputs and outputs (measured with microphones and accelerometers, respectively), or similarly for a wooden acoustic cavity in [2], where there is only measured, and no actual prior estimation verified and validated (from Transmissibility functions). So, this work aims to provide a 3D FEM model through which V-A Transmissibility is proposed and sets of dynamic displacements in structures can be estimated from pressure excitation inside an acoustic medium, and vice-versa. A "simple" model is proposed for this purpose, starting in a primary phase with Vibrational and Acoustic Transmissibility verification, followed by the proposed approach to the actual V-A problem.

The applicability of the developed method could be relevant in areas like acoustic and vibrational fatigue assessments, where authors (see [3]) have already delved into, to analyze how pressure excitations and dynamic loadings affect the "healthiness" of aerospace structures. This could be specially relevant in areas where human access is close to impossible, problem which as already been touched by authors like Guedes in [4], where an acoustic source localization (ASL) method is proposed, based on acoustic transmissibility, for 1D and 2D detection.

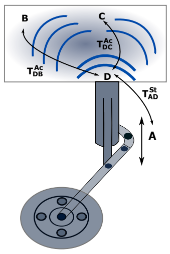

Figure 1: Vibro-acoustic Transmissiblity in a landing gear, with **B** and **C** located inside the fuselage (cavity)

In figure 1 an illustration is presented for V-A transmissibility from a landing gear to a fuselage. In this case, the interface in D models displacement-pressure interaction.

1.1. Brief State-of-the-Art regarding Vibration, Acoustic and Vibro-Acoustic Transmissibility

The concept of transmissibility is not new in Engineering, for there is already a good amount of work/literature in this specific area, being it in Vibrations or acoustics.

In [5], the authors proposed a relationship between FRFs for MDOFs systems with diagonal mass matrices and tri-diagonal stiffness ones, through transmissibility functions, for Vibrations.

In [6], it is discussed the prediction of motion transfer through single-point and multi-point FRFs, with a standard scalar approach and a more complex one through transformation matrices.

In Acoustics, in works like [7], an OAMA (operational acoustic modal analysis) is done through transmissibility measurements defined as a ratio between pressure values in specific points, originated by applying volume acceleration sources (or pressure excitations) in multiple places within the defined volume (along a centerline).

For V-A besides [1], in more recent years, with [2] (as already mentioned) the author conducted an acoustic and structural characterization of a wooden cavity, in terms of natural frequencies and mode shapes to allow for a proper characterization and consequent vibration transmissibility analysis in terms of V-As (if possible). The nature of this work was based around numerical simulation (numerical measuring) and experimental results, so there was still no progress with regards to estimating actual transmissibility.

Lastly, in [8], a method to estimate V-A transmissibility is announced, based on frequency response functions, but the model was not developed.

2. Background

In this chapter, the theoretical foundation for this work, regarding the fields of Vibrations and Acoustics will be established. As well as the notions of Transmissibility in both fields, and between them.

2.1. Vibrations in MDOF systems

The equation of motion for a steady-state dynamically excited structure is the following:

$$[\mathbf{K}]\{x(t)\} + [\mathbf{C}]\{\dot{x}(t)\} + [\mathbf{M}]\{\ddot{x}(t)\} = \{f(t)\} \quad (1)$$

With structural stiffness $[\mathbf{K}]$, mass $[\mathbf{M}]$, damping $[\mathbf{C}]$, displacement $x(t)$ and applied harmonic force $f(t)$.

From (1) one derives the dynamic stiffness matrix $\mathbf{Z}$, as well as the Frequency response function $\mathbf{H}$:

$$[\mathbf{Z}] = [\mathbf{K}] + i\omega[\mathbf{C}] - \omega^2[\mathbf{M}] = [\mathbf{H}]^{-1} \quad (2)$$

which essentially describe behavioural responses of systems, in the frequency domain, from an initially established movement equation [9].

2.2. Transmissibility in Solid Structures

SDOF Transmissibility can be found clarified in an array of textbooks, for instance [10], when it comes to SDOF systems, where this relation can be quite evident and linear and a displacement or load is transmitted immediately and predictably from point A to point B in the system, following a unidimensional scalar input/output mathematical relation.

Just recently, methods have been developed to predict how randomly applied dynamic loads (and again, forced dynamic displacements) are transmitted in MDOF (multiple degrees of freedom) to also randomly chosen points, et al [11, 12]. With MDOF systems the mentioned relation is not so simple anymore, therefore needing matricial representation, where random multiple inputs will have multiple responses in an array of outcomes.

2.2.1 Load Transmissibility

In accordance with [12], the definition of a set of generalized coordinates for a generic MDOF system needs to be done. For this to be achieved, some assumptions have to be made.

Firstly, it is established a set of coordinates, that will be called K, where the external (known) loads are to be applied.

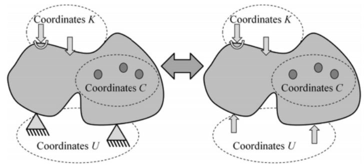

Figure 2: Set of Generalized Coordinates K, U and C (source Y.E. Lage et al [12])

Then, there is U, which defines another set where the unknown reaction forces will appear, and ultimately the C set which encompasses all the other coordinates, as described by fig. 2.

$$\begin{Bmatrix} \mathbf{Y}_K \\ \mathbf{Y}_U \end{Bmatrix} = \begin{bmatrix} \mathbf{H}_{KK} & \mathbf{H}_{KU} \\ \mathbf{H}_{UK} & \mathbf{H}_{UU} \end{bmatrix} \begin{Bmatrix} \mathbf{F}_K \\ \mathbf{F}_U \end{Bmatrix} \quad (3)$$

From the receptance frequency response matrix $[\mathbf{H}]$(where the receptance is defined by other submatrices), which relates, in steady-state conditions, the dynamic displacement amplitudes $[\mathbf{Y}]$ (at discretized nodes of the structure), with the force amplitudes $[\mathbf{F}]$,one describes a free body moving through space. This is all expressed in (3).

Taking into account that the supports at U constrain the displacements, one can consider $Y_U = 0$, which implies the following:

$$\mathbf{H}_{UK}F_K + \mathbf{H}_{UU}F_U = 0 \quad (4)$$

which is equivalent to:

$$\mathbf{F}_U = -(\mathbf{H}_{UU})^{-1}\mathbf{H}_{UK}\mathbf{F}_K \quad (5)$$

and ultimately comes to:

$$\mathbf{T}_{UK}^{(f)} = -(\mathbf{H}_{UU})^{-1}\mathbf{H}_{UK} \quad (6)$$

from which, comes the load transmissibility matrix:

$$(\mathbf{T}_{UK}^{(f)})^{+} = -(\mathbf{H}_{UK})^{+}\mathbf{H}_{UU} \quad (7)$$

Besides being obtainable from the Receptance Matrix, the Transmissibility, linked to MDOF, can also be obtained from the Dynamic Stiffness Matrix, just like in [11].

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As it is done for FRFs, also in steady-state conditions, one can relate dynamic displacements and external loading through the dynamic stiffness matrix $[\mathbf{Z}]$. In this case, as it was done Neves and Maia [11], and knowing that the coordinates K and C can be grouped into a new one, which will be named G (grouped), the relation between known loads and unknown ones can be written like (and keeping the same designations as for the receptance case):

$$\begin{Bmatrix} \mathbf{F}_G \\ \mathbf{F}_U \end{Bmatrix} = \begin{bmatrix} \mathbf{Z}_{GG} & \mathbf{Z}_{GU} \\ \mathbf{Z}_{UG} & \mathbf{Z}_{UU} \end{bmatrix} \begin{Bmatrix} \mathbf{Y}_G \\ \mathbf{Y}_U \end{Bmatrix} \quad (8)$$

If one assumes that $\mathbf{Y}_U$ becomes a null vector, this will result in:

$$\mathbf{F}_U = \mathbf{Z}_{UG}(\mathbf{Z}_{GG})^{-1}\mathbf{F}_G \quad (9)$$

which therefore will result in a load transmissibility matrix:

$$\mathbf{T}_{UG}^{(f)} = \mathbf{Z}_{UG}(\mathbf{Z}_{GG})^{-1} \quad (10)$$

The harmonic loads applied on C are considered null. This change will result in the disregarding of the column and row that will be present in the final transmissibility matrix, corresponding to this coordinate.

2.2.2 Displacement Transmissibility

In this segment, just like in the load transmissibility one, there is a need to generate a set of coordinates in a structure (free elastic body). There is a small difference in this case though. Now there are no constraints applied by supports, so the structure presents no reaction forces, where there used to be ones. But just like for the previous case, there will firstly be an A set where the known external loads $\mathbf{F}$ are being applied, then a K set which has the known $\mathbf{Y}$ responses, followed by a U set, where the unknown $\mathbf{Y}$ responses exist, and finally a C set, where all the remaining coordinates of the structure are.

The FRF/receptance matrix $[\mathbf{H}]$ relates dynamic displacement and external dynamic loading according to the following equation:

$$\begin{Bmatrix} \mathbf{Y}_A \\ \mathbf{Y}_U \\ \mathbf{Y}_K \\ \mathbf{Y}_C \end{Bmatrix} = \begin{bmatrix} \mathbf{H}_{AA} \\ \mathbf{H}_{UA} \\ \mathbf{H}_{KA} \\ \mathbf{H}_{CA} \end{bmatrix} \{\mathbf{F}_A\} \quad (11)$$

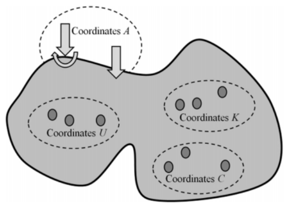

Figure 3: Free elastic body with four sets of coordinates A, U, K, C (source Y.E. Lage et al [12])

If the displacements in coordinates U and K (as in fig. 3) are caused by an harmonically applied load, and if equation (11) is added to the matter, one comes by:

$$\begin{Bmatrix} \mathbf{Y}_U \\ \mathbf{Y}_K \end{Bmatrix} = \begin{bmatrix} \mathbf{H}_{UA} \\ \mathbf{H}_{KA} \end{bmatrix} \{\mathbf{F}_A\} \quad (12)$$

which utimately, if the external loads are disregarded and considering that there are no restrictions to how the A set is constructed, comes to:

$$\mathbf{T}_{UK}^{(d)} = \mathbf{H}_{UU}(\mathbf{H}_{KU})^{+} \quad (13)$$

After some mathematical, from equations (7) and (13) one can relate the displacement transmissibility with the load one [12]:

$$\mathbf{T}_{UK}^{(d)} = ((\mathbf{T}_{UK}^{(f)})^{T})^{+} \Longrightarrow \mathbf{T}_{UK}^{(f)} = ((\mathbf{T}_{UK}^{(d)})^{+})^{T} \quad (14)$$

where T is for transposed and $+$ is for pseudo-inverse.

2.3. Acoustic Harmonic Plane Waves

The loss-less wave equation is defined by (with no acoustic source):

$$\vec{\nabla}^2 p' = \frac{1}{c^2}\frac{\partial^2 p'}{\partial t^2} \quad (15)$$

with $\vec{\nabla}.\vec{\nabla}=\vec{\nabla}^2$, c the sound speed and p' the pressure perturbation [13].

If an acoustic source q is considered and knowing that $k = \omega/c$, one obtains the final Helmholtz Equation, which serves as a basis for every acoustic FEM [13]:

$$k^2 p'(x,y,z) + \vec{\nabla}^2 . p'(x,y,z) = -i\omega\rho_0 q(x,y,z) \quad (16)$$

where $i = \sqrt{-1}$ and ρ_0 the equilibrium density of the medium.

Similarly to a dynamic structural system, which in steady-state regime can be modelled in the frequency domain by equation (1) (with damping or not), so can a dynamic acoustic one (where the damping would be the acoustic impedance Z), through a similar equation, as introduced in [7]:

$$\left\{([\mathbf{K}] - \omega^2[\mathbf{M}] + i\omega[\mathbf{C}])\right\}\{P(\omega)\} = \left\{\dot{Q}(\omega)\right\} \quad (17)$$

$$\{\mathbf{Z}(\omega)\}\{P(\omega)\} = \left\{\dot{Q}(\omega)\right\}$$

where $[\mathbf{K}]$ is the Acoustic Global Stiffness Matrix, $[\mathbf{M}]$ is the Acoustic Global Mass Matrix, $[\mathbf{C}]$ is the Acoustic Global Damping Matrix, $P(\omega)$ is the pressure vector (equivalent of the displacement vector $\mathbf{Y}$), $\dot{Q}(\omega)$ is the volume acceleration vector (equivalent of the mechanical load vector $\mathbf{F}$).

2.4. Transmissibility in the field of Acoustics

The inputs and outputs of the transmissibility functions will now be imposed pressures values, as it is described in [7]. So whenever there is a pressure disturbance (or excitation source, volume acceleration) imposed in a certain point in an acoustic fluid, a transmissibility model will indicate how this disturbance is manifested in another point in the said field, by conveying a pressure response. This model can be described with the helping hand of the following equation:

$$T^k_{ir}(\omega) = \frac{p_i(\omega)}{p_r(\omega)} = \frac{H_{ik}(\omega)\dot{q}_k(\omega)}{H_{rk}(\omega)\dot{q}_k(\omega)} \quad (18)$$

Where for a single acoustic source present at a known DOF k, $T^k_{ir}(\omega)$ is the ratio between the fixed pressure measured at a reference-output DOF r and a measured one at DOF i. Since both the reference and the source points or locations are fixed, and the reference is well chosen.

2.4.1 Frequency Response Method

Following a similar method (as for structures) and considering $\mathbf{F}_C$=0.

$$\begin{Bmatrix} \mathbf{P}_K \\ \mathbf{P}_U \end{Bmatrix} = \begin{bmatrix} \mathbf{H}_{KK} & \mathbf{H}_{KU} \\ \mathbf{H}_{UK} & \mathbf{H}_{UU} \end{bmatrix} \begin{Bmatrix} \mathbf{F}_K \\ \mathbf{F}_U \end{Bmatrix} \quad (19)$$

$$\mathbf{P}_K = \mathbf{H}_{KK}\mathbf{F}_K + \mathbf{H}_{KU}\mathbf{F}_U \quad (20a)$$

$$\bar{P}_U = \mathbf{H}_{UK}\mathbf{F}_K + \mathbf{H}_{UU}\mathbf{F}_U \quad (20b)$$

Now, by defining $\mathbf{F}_U$ as a function of $\bar{P}_U$ and substitutes it in (20a) while considering $\mathbf{F}_K = 0$ the result is :

$$\mathbf{P}_K = \mathbf{H}_{KU}(\mathbf{H}_{UU})^{-1}\bar{P}_U \quad (21)$$

which will consequently give out the corresponding transmissibility matrix, between the sets:

$$T^a_{KU} = \mathbf{H}_{KU}(\mathbf{H}_{UU})^{-1} \quad (22)$$

2.5. V-A Transmissibility

An analogy with the acoustic case will be established for the V-A interaction, as shown in:

$$\left(\begin{bmatrix} K_s & K_c \\ 0 & K_a \end{bmatrix} + i\omega \begin{bmatrix} C_s & 0 \\ 0 & C_a \end{bmatrix} - \omega^2 \begin{bmatrix} M_s & 0 \\ -\rho_0 K_c^T & M_a \end{bmatrix}\right) \begin{Bmatrix} \mathrm{u}_i \\ \mathrm{p}_i \end{Bmatrix} = \quad (23)$$

$$= \begin{Bmatrix} \mathrm{F}_{si} \\ \mathrm{F}_{ai} \end{Bmatrix}$$

Above is presented the fluid-structure coupling equation. One can assume that, from this equation, like in pure structural mechanics and acoustics, the appearing matrices can be grouped into a general dynamic stiffness matrix [14]:

$$\mathbf{Z}\begin{Bmatrix} \mathbf{u}_i \\ \mathbf{P}_i \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_{si} \\ \mathbf{F}_{ai} \end{Bmatrix} \leftrightarrow \begin{Bmatrix} \mathbf{u}_i \\ \mathbf{P}_i \end{Bmatrix} = \mathbf{H}\begin{Bmatrix} \mathbf{F}_{si} \\ \mathbf{F}_{ai} \end{Bmatrix} \quad (24)$$

Now, adopting the same perspective as when (22) was generated, an analogy can established between the loads of acoustic and structure interaction ($\mathbf{F}_{ai}$ and $\mathbf{F}_{si}$), and the loads applied in regions U and K inside an enclosed volume where an additional fluid-structure interface is regarded (see also [8]). The same is done for the displacement and pressure in the first set, resulting in:

$$\begin{Bmatrix} \mathbf{u}_K \\ \mathbf{P}_U \end{Bmatrix} = \begin{bmatrix} \mathbf{H}_{KK} & \mathbf{H}_{KU} \\ \mathbf{H}_{UK} & \mathbf{H}_{UU} \end{bmatrix} \begin{Bmatrix} \mathbf{F}_K \\ \mathbf{F}_U \end{Bmatrix} \quad (25)$$

If the second equation of the system above is solved with respect to $\mathbf{F}_U$ and $\mathbf{F}_K$=0, the first one solved with respect to $\mathbf{u}_K$ yields:

$$\mathbf{u}_K = \mathbf{H}_{KU}\mathbf{H}_{UU}^{-1}\mathbf{P}_U \quad \text{where} \quad \mathbf{H}_{KU}\mathbf{H}_{UU}^{-1} = T^{FS}_{KU} \quad (26)$$

If an imposed dynamic displacement on the plate is considered instead of pressure in the fluid (now u_U and P_K), one obtains:

$$T^{SF}_{KU} = \mathbf{H}_{KU}\mathbf{H}_{UU}^{-1} \quad (27)$$

3. Methodologies

To try answer the question on how to estimate Vibro-Acoustic responses through transmissibility functions, this work follows a sequence.

Firstly, it starts with an initial verification of results for Vibrations and Acoustics already present in the literature. The methodologies presented are shown and justified accordingly, following the theoretical background introduced in section 2.

After, both the acoustic and vibrational transmissibility are verified, the implementation of the V-A model for V-A point by point and matricial transmissiblity estimation is presented.

3.1. Dynamic Force and Displacement Transmissibility Verification

As a way of verifying results present in Neves and Maia [11], for force transmissibility matrices, the following methodology is presented. This was done firstly for a specific mass/spring system, and then for a simply supported beam.

3.1.1 Mass/Spring System and simply supported Beam

So, a MATLAB® (see manual [15]) algorithm is made in order to calculate the assembled global matrices of a considered mass/spring system, followed by transmissibility matrices, using the dynamic stiffness as well as the Receptance/Frequency response matrices.

Initially, the mass and stiffness constants are defined for mass and spring elements, as well as the already assembled, global mass and stiffness matrices are defined as inputs to be manipulated. The load vector to be applied in the system is also declared in this step. Then, a cycle will be created, where in each iteration, a number of procedures occur, namely obtaining $\mathbf{Z}(\omega)$, definition of submatrices of $\mathbf{Z}(\omega)$, which relate the sets of coordinates, between which, the transmissibility matrix will be computed, according to equation (10), then $\mathbf{Z}(\omega)$ is inverted, originating $\mathbf{H}(\omega)$, subsequent submatrices will also be declared. The Transmissibility will then be computed, following equation (7).

For the beam part, the algorithm is pretty similar, but with small differences. In this case the implementation was based in Lage et al [12]. The assemblage of the beam was done through 2D Euler-Bernoulli beam elements (4 DOFs, see [16]), instead of springs and masses, and the force and displacement transmissibilities were obtained from the frequency response matrix.

3.2. Beam Modelling in ANSYS APDL

Besides aiming for a verification of already produced models, one also desires to give an introduction to how one can work around the ANSYS APDL software in order to implement a Vibro-Acoustic interaction model and extract data from it, with the purpose of verifying V-A transmissibility further ahead in this work.

The beam element that will be used in this new model will be the BEAM3 element.

In PREP7, the APDL routine used for this section is as follows:

- Firstly, the type of beam element to be used is chosen, along with the properties of the material;
- Then, the mesh is assembled;
- Vertical loads are imposed to allow remesh.

In the solution phase:

- A Modal analysis type is done and the global matrices are written into a file;
- After assembled and written onto a file (dumped in Harwell-Boeing (HB) format), the file is read through a MATLAB® routine which converts HB format sparse matrices into MATLAB® ones, using [17].
- The Transmissibility matrices are assembled accordingly to the DOFs desired and, again, following the principles in [12].
- Finally, the rest of the processing and post-processing is done in MATLAB® environment.

3.3. Transmissibility - Aspects of Computational Implementation

In this section are briefly presented the implementation aspects followed to obtain pressure transmissibility inside a 1D tube, as well as for a 3D one (for SDOF and MDOF), with either purely reflective or anechoic top extremities (only shown reflective results). This is done following two different paths. Primarily, for a single imposed pressure in a certain point and compared with the literature, as in [7], both for the 1D and 3D cases. Since there is only one point from which there is either a source or an imposed pressure, the transmissibility in this case will be solely scalar, as dictated by equation (18).

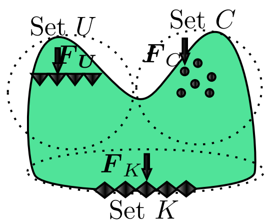

Figure 4: Sets of Coordinates U, K and C for an Acoustic Enclosed Domain

Then, sets of coordinates (U,K and C) will be considered (fig. 4) and MDOF Transmissiblity will be computed through matrices.

3.3.1 Transmissibility using a code in ANSYS

Initially, with the purpose of validating by superposition, results already obtained in 1D through the MATLAB® Software, the same method for scalar transmissibility will be used to calculate/estimate this same transmissibility with the help of equation (18) which, as already mentioned, is based upon [7]. But, in this particular situation, this will be done through a FE commercial software, ANSYS APDL, followed by further processing in MATLAB®, just like before:

- First, the properties of the 3D acoustic fluid element fluid30, will be established;
- Then, the model, a tube, is to be developed. Based upon the number of elements per wavelength, the model will have it's nodes orderly generated from a single one, which will be copied along the x and $y-axis$ (and $z-axis$, just once for now), while aiming to produce a square base with square elements.
  Ultimately, this pattern will be reproduced along the $z-axis$ and the mesh assembled.
  The position of chosen nodes (to calculate T.) will be represented in equation (18), with z_{ref} being the position of the reference nodes, about which $T^k_{ir}(\omega)$ will be calculated, and i being the point where it is actually computed;
- The values obtained are then compared with the equivalent 1D case.

When the objective is a transmissibility calculated in regions of the fluid (from 2 points onward), matrices that specify grouping of DOFs inside a region have to be used, much like it already was in Dynamic Structures.

The basis for this particular part of the methodology is explained in section 2.4.1. The Frequency response sub-matrices H_{KU} will be n_Kxn_U, H_{UU} will be n_Uxn_U and T_{KU}, also n_Kxn_U.

The acoustic analysis (coupled as well) is done, based on [18] and [19].

3.4. Vibro-Acoustic Transmissiblity

In this section is presented the methodology proposed to estimate Vibro-Acoustic Transmissibility.

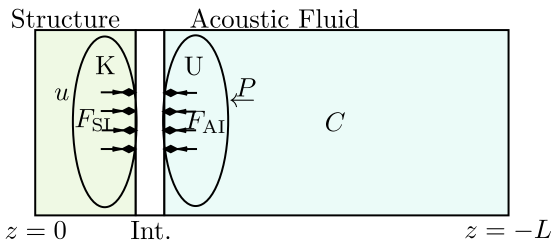

Figure 5: Vibro-Acoustic Interaction (Interface) depicted in Sets of Coordinates U, K and others C, with imposed pressure

The algorithm that is going to be employed to calculate the FRFs and estimate the resulting Transmissibility matrix that relates displacements in a structure with a pressure imposed in a fluid within the vicinity is essentially the same as the one used for the acoustic transmissibility. There are some extra steps though. These differences come from coupling. When the plate is inserted, in interface needs to be created between it and the acoustic fluid. This is done through an FSI flag (in APDL). Another factor is that, when a harmonic analysis is done, the all the DOFs need to be constrained, except for the ones at the interface. Nodes around the plate are fixed.

4. Results

Primarily, following the methodology described the obtained results are tested against ones already present in the literature. In a primary instance this will be done for verification of solutions regarding the field of Vibrations, namely the topic of Vibration Transmissibility (either Force or Displacement).

After the Transmissibility is verified for this section, the results for MDOF presented in [7] and FRF originated pressure transmissibility will be presented, for acoustics.

Finally, following the same approach as before, the results for numeric V-A Transmissiblity are presented.

4.1. Vibration Transmissibility

In a primary instance the verification of solutions regarding Vibration Transmissibility (either Force or Displacement), for a spring/mass system and then for a simply supported beam.

4.1.1 Spring/Mass System

Figure 6 shows the model chosen for verification in this section.

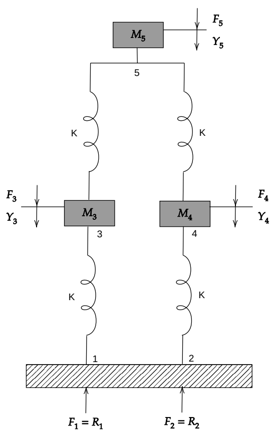

Figure 6: Mass/Spring System

Since a two node discretized spring element was used, the connectivity table, with the rigidity weights from every spring in every node was constructed and through it, the global stiffness matrix (5x5 since there are 5 nodes) was assembled. Each spring was assigned with a rigidity of $k = 10^3$ [N/m] and mass element with mass $m = 5$ [kg].

A set of constant amplitude known loads $\{\mathbf{F}_K\} = \{10, 10\}^T$ N were applied in nodes 4 and 5 of the system, while the reaction loads would be present in nodes 1 and 2.

Following the methodology described in section 3, the transmissiblity matrices were obtained using the global matrices in the system described in fig.6. The transmissibility between nodes 1 and 4 is presented in fig.7.

(a)

(b)

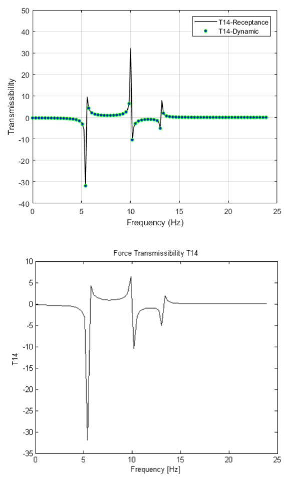

Figure 7: Comparison of T_{14} (from receptance), obtained in this work (a) with the one in [11] (b)

4.1.2 Simply Supported Beam

The beam assembled in the script has the properties indicated in [12], which are numbered in table 1.

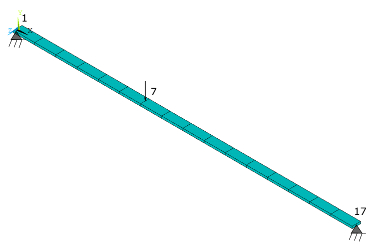

Figure 8: 16 Element Beam, generated in APDL

The initial purpose, as in [12] was to apply a dynamic load $\mathbf{F}_K$=100 N, in node number 7 ($x = 0.3$ m), and study how this load would be transmitted to the nodes where the structure was supported, so nodes 1 ($x = 0$) and 17 ($x = L$), which would, as already mentioned would have their transverse DOF fixed. Afterwards, in order to further verify results, the pseudo inverted transposed displacement transmissibility matrix was obtained (which is the same as just inverting, since the matrix is a square one). For this to be done, an additional dynamic load was considered in the load vector (of known applied loads) applied in node 9 (whose value was taken as null later), because sets U and K have to at least have the same number of coordinates (two each for inversion, so 2x2 matrices).

Table 1: Beam Properties

Young's Modulus - E	208 GPa
Density - ρ	7840 kg/m^3
Length - L	0.8 m
Section Width - b	20x10^{-3} m
Section Height - h	5x10^{-3} m
Second Moment of Area - I_{zz}	2.0833x10^{-10} m^4
Element Length - l_e	0.05 m

This beam was simply supported at both ends (nodes 1 and 17) and dynamic loads were applied in nodes 7 and 9. Then the force and displacement transmissibility were computed and plotted in a range of frequencies, from the frequency response matrix. The transmissibility between nodes 1 and 7, as well as the receptance between these nodes is plotted in fig. 9, from both softwares.

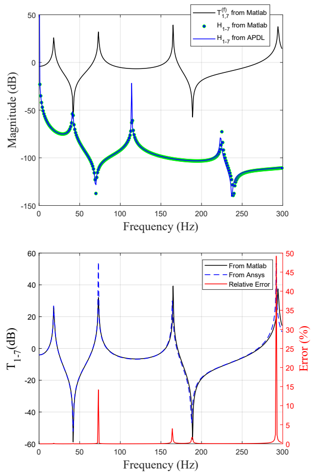

Figure 9: FRF and Transmissibility plots from APDL and MATLAB

This ends the verification of Vibration Transmissibility.

4.2. Acoustic Transmissibility

A tube of length L is considered with the data from table 2 and geometry in fig.10.

Table 2: Tube Properties (Fluid 30)

Sound Speed - c	344 m/s
Density - ρ	1.21 kg/m^3
Reference Pressure - P_{ref}	20x10^{-6} Pa
Length - L	4 m
Heigth and Width - b	0.1 m
f_{ref} for N	200 Hz

4.2.1 Scalar Transmissibility

In fig. 11, Transmissibility results are presented for a tube discretized with exactly 513 DOFs. On the other hand, as comes from before, the 1D tube does not need as many elements (DOFs) for it's discretization, having only 57 nodes/DOFs for the same N.

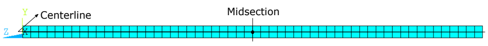

Figure 10: Side View of the 3D tube model ($z0y$ plane) with $N = 24$ elements per λ, 56 elements along the length and a reflective end at $z = -L$

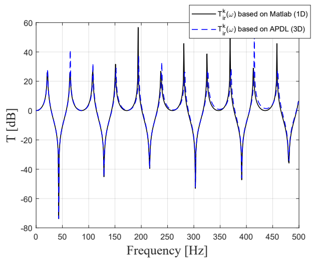

Figure 11: Results obtained from [4, 7] compared against the 3D model developed in APDL, for 24 elements per wavelength, and a reflective end. Pressure ratio in black, for 1D, and FRF ratio in blue for 3D

In either cases, the FRF matrices were used to compare scalar pressure transmissibility between the midpoint/midsection of the tube with the imposed pressure on one of the ends, in this case, the one upstream from the fluid.

4.2.2 MDOF Pressure Transmissibility from FRFs

The results that follow were obtained for specific regions K and U inside the volume enclosed by the tube, where the pressure values were respectively known and unknown. The methodology to achieve this is clarified in section 3.

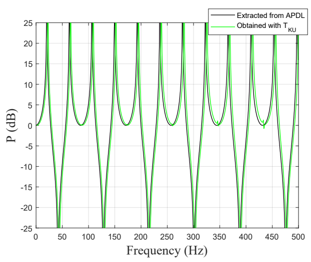

Figure 12: Pressure measured at the centre of the tube (black) and calculated with T^a_{KU} (green) with an imposed pressure of 1 Pa at z=0, and a reflective top, for $N = 36$

The model for MDOF Transmissibility. does not have a center-line defined by nodes, having only one element transversely. This ends the verification for Acoustic Transmissibility.

4.3. Vibro-Acoustic Transmissibility

The proposed model is an acoustic tube connected (at top $z = -L$) to a plate by means of a fluid-structure interface. The plate has the properties in table 3 and fig.13.

Table 3: Plate Properties (SHELL181)

Young's Modulus - E	210 GPa
Density - ρ	7800 kg/m^3
Section Width and Height - b	0.1 m
Thickness - t	0.001 m
Poisson's Ratio - ν	0.3

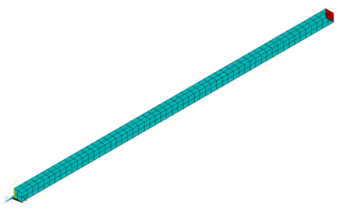

Figure 13: Model for the coupled System, with the plate at the end, in red at $z = -L$

In Fig. 14 the results were plotted for an imposed load at the center coordinates of the plate, and a "measured" pressure at the midsection of the tube.

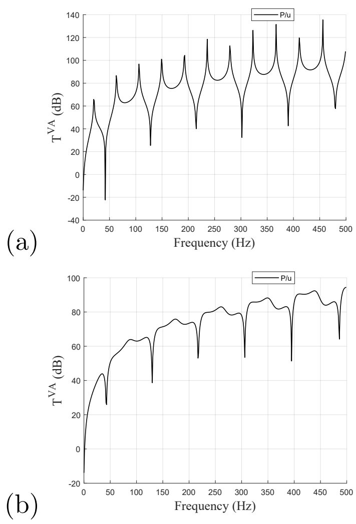

Figure 14: P/u Ratio results from the model in fig.13. Model with 567 DOFs total. Reflective (a) and anechoic end (b) in $z = 0$.

4.4. Vibro-acoustic Transmissibility Estimation through FRFs

Now that the scalar "measured" P/u is already computed and plotted, what follows is somewhat of an uncharted area, so there is still some uncertainties toward the approach chosen, namely for data verification.

The results obtained were based on the methodology described in [7], in a way that **H** sub-matrices were used to establish Fluid-Structure Transmissiblity, for the applied force set U (corresponding to a dynamic displacement in the centre of the plate, along the Oz axis) and the known K pressure set, as in equation (27). and considering $x, y = b/2$. Sets U and K are respectively $z = -L$ and $z = -L/2$. Inside the projected model, these coordinates were picked as DOFs inside the global FRF matrix, as in, the DOFs mapped to the corresponding node. So the model is ready and prepared to extract results, but is still a work in progress.

Initially, it would be expected that the computed FRF ratios would at least be close to the P/u ratios (as it was for pressure ratios in acoustic), but that was not the case, being verified a clear deviation in terms of results. So, verification is yet to be achieved, hence, the results obtained won't be showed in this work. Some reasons for this not to work are being rechecked:

- Unknown mapping/assigning of DOFs when the plate is inserted into the model;
- Little information about the inner workings of APDL (internal processes/routines);
- Low Refinement for a coupled model.

4.4.1 MDOF V-A Transmissibility

A routine was also developed to try to estimate V-A Transmissibility (similar to the acoustic one) between a pressure imposition $\mathbf{P}_U$ in a section of the tube U and a "known" displacement set (K) in the plate.

Commencing with obtaining **H** just like previously, the sub-matrices are then chosen accordingly to the DOFs to be related, so to speak, pressure somewhere inside the tube and nodal displacement along Oz. In this case, K consisted of a set with the DOF number corresponding to the nodal u_z displacement inside the vector of DOFs (pressure, rotations and displacements), and U to the pressure DOFs inside the same vector.

5. Conclusions

In the work presented along this document, a methodology is studied with the purpose of estimating transmissibility between points of a structure, inside an acoustic medium (cavity) or an acoustic fluid-structure medium, in the frequency domain.

By primarily verifying, based on the available literature on SDOF/MDOF transmissibility in dynamically excited structures (steady-state) as well as acoustic fluids, a V-A methodology was then proposed with the same goal. Provided that the available literature was scarce regarding V-A transmissibility (close to just some experimental results in [1]), the method developed was based on Vibration and Acoustic Transmissibility computation, and some theoretical projections in [14] for coupling and [8].

So, before actually suggesting a method for V-A Transmissibility computation in 3D, other models were developed to predict load and displacement transmissibility in structures (spring/mass system and beam), by means of a FEM. However, in Acoustics, the level of refining for a model to be proper requires a rather high number of elements per wavelength, which means that simulation times (hours) would have to get higher in order to avoid pollution [20, 21], and try to guarantee proper meshing. In some cases, this was not entirely possible, given the relatively high number of DOFs in the mesh and the low sparsity of the frequency response matrix used. Indeed, by using FEs one limits the analysis, sometimes to a certain maximum mesh refinement.

One last methodology was proposed to predict V-A Transmissibility based on frequency response matrices (sub-matrices), used to relate sets of pressure in acoustic coordinates with displacements in structure coordinates, and vice-versa. The frequency response matrices were extracted from a commercial FEM software (using a modal analysis) and the procedure for transmissibility computation was proposed (scalar u/P and P/u ratio and from the FRF the methodology was proposed and awaits verification), with some deviation. This deviation reflects the difficulty of interpreting how the mapping of the mixed DOFs (in Fluid-Structure Interaction (FSI) models) is done within the software and loaded into the matrices, after the structure is inserted into the model (Displacements, Rotations and Pressure),topic regarding which there is not much clear and concise information.

The model proposed in this work to estimate Vibration, Acoustic and V-A response through Transmissibility functions used a specific method based on the extraction of globally assembled matrices (**K**, **M** and **C**) from a commercial software and computation of the final **T** matrices, with Matlab environment. There are some considerations regarding future work on the topic that were not developed and tested due to lack of time. These are:

- Clarify the indexing of entries of matrices from the files in HB format for V-A models. Afterwards, it is expected that the methodology can be quickly tested and verified;
- Try and apply (after verification) the devel-

oped model inside an array of other acoustic cavities also with FSI;

- Instead of using the receptance method, one could try to obtain **T** from the Dynamic stiffness matrix of the system;
- Study V-A Transmissibility in the domain of time instead of frequency.

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