Aspects of Physics Beyond the Standard Model in the Leptonic Sector

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After a great theoretical and experimental progress, it is now known that neutrinos have mass. However, there is no theoretical explanation for their almost vanishing mass and other issues. The Seesaw mechanism answers some of these and creates new phenomenology that can help answer several other open problems in Particle Physics, like the matter-antimatter asymmetry. In this thesis, a minimal extension to the Standard Model with three positive chirality neutrinos is devised, under the Seesaw Type I framework. Notation is fixed and a novel parametrization is exploited. This parametrization enables to control all deviations from unitarity through a single 3×3 matrix, which is denoted by X, that also connects the mixing of the light and heavy neutrinos in the context of type I seesaw. This parametrization is adequate for a general and exact treatment, independent of the scale of the right handed neutrino mass term. The models with controlled one-loop mass corrections are classified according to the heavy neutrino mass hierarchies they must possess - cases A, B and C. Cases B and C can have sizable deviations from unitarity. This means that if an almost sterile neutrino is discovered in the near future, heavy neutrinos mass hierarchies might be like the ones of case B - at least two almost degenerate neutrinos, or like the ones of case C - at least two eV or KeV neutrinos.

Keywords: Standard Model, Neutrinos, Seesaw, Deviations from Unitarity, One-Loop.

I. INTRODUCTION

The discovery of neutrino oscillations and at least two non-vanishing neutrino masses, provides clear evidence for Physics Beyond the Standard Model (SM). The simplest extension of the SM accommodating two non-vanishing neutrino masses involves the addition of at least two right-handed neutrinos. The most general gauge invariant Lagrangian includes a right-handed bare Majorana mass matrix M. As a result, the scale of M can be much larger than the electroweak scale, which leads to an elegant explanation for the smallness of neutrino masses, through the seesaw mechanism [1], [2], [3], [4], [5]. The seesaw mechanism necessarily implies violations from 3×3 unitarity of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, as well as Z-mediated lepton flavour violating couplings. The introduction of these heavy right-handed neutrinos can also have profound cosmological implications since they are a crucial component of the Leptogenesis mechanism to create the observed Baryon Asymmetry of the Universe (BAU) [6]. At this stage, it should be emphasized that within the seesaw type I framework, the observed pattern of neutrino masses and mixing does not require that all the heavy neutrino masses be much larger than the electroweak scale. In this paper, we carefully examine the question of whether it is possible, within the seesaw type I mechanism, to have experimentally detectable violations of 3×3 unitarity, taking into account the present experimental constraints. In particular, we address the following questions:

i) In the seesaw type I mechanism, is it possible to have significant deviations from 3×3 unitarity of the leptonic mixing matrix? By significant, we mean deviations which are sufficiently small to conform to all present stringent experimental constraints on these deviations, but are sufficiently large to be detectable in the next round of experiments. These experimental constraints arise from bounds on rare processes. ii) In the case the scenario described in (i) can indeed be realised within the framework of seesaw type I, what are the requirements on the pattern of heavy neutrino masses? For definiteness, we will work in a framework where three right-handed neutrinos are added to the spectrum of the SM. Our analysis starts with the introduction of the unitary 6×6 mixing matrix $\mathcal{V}$, characterising all the leptonic mixing. We write this 6×6 mixing matrix in terms of four blocks of 3×3 matrices. Using unitarity of $\mathcal{V}$, we show that the full matrix $\mathcal{V}$ can be expressed in terms of only three blocks of 3×3 matrices. Then we apply these results to the diagonalisation of the 6×6 neutrino mass matrix, including both Dirac and Majorana mass terms. Through the use of a specially convenient exact parametrisation of the 6×6 leptonic unitary mixing matrix, we evaluate deviations from 3×3 unitarity and derive the maximum value of the lightest heavy neutrino mass which is required in order to generate significant deviations from unitarity in the framework of the seesaw type I mechanism. The paper is organised as follows. In the next section, we review the seesaw mechanism, define our notation and introduce a specially convenient exact parametrisation of the 6×6 leptonic mixing matrix $\mathcal{V}$. We evaluate the size of the deviations of 3×3 unitarity in the present framework and derive a constraint on the magnitude of the mass of the heavy Majorana neutrinos, in order to have significant deviations of unitarity. Sections IV and V dwell on the importance of finite one-loops corrections for these kind of models [7]. Numerical examples are given in section VI. Finally we present our conclusions in the last section. The research work on which part of this thesis is based on can be found at [8].

II. DEVIATIONS FROM UNITARITY IN THE LEPTONIC SECTOR

A. Type I Seesaw mechanism

In the context of the Type I seesaw mechanism, with only three right-handed neutrinos added to the Lagrangian of the SM, the leptonic mass terms are given by:

$$\begin{aligned}\mathcal{L}_m &= -[\overline{\nu_L} M^\nu \nu_R + \frac{1}{2}\nu_R^T C\ M \nu_R + \overline{l_L^0} m_l l_R^0] + h.c. \\ &= -[\frac{1}{2}\chi_L^T C\ \mathcal{M}^* \chi_L + \overline{l_L} m_l l_R] + h.c.\ . \end{aligned} \quad (1)$$

There is no loss of generality in choosing a weak basis where m_l is already real and diagonal. The analysis that follows is performed in this basis. The neutrino mass matrix $\mathcal{M}$ is a 6×6 matrix and has the form:

$$\mathcal{M} = \begin{pmatrix} 0 & M^\nu \\ M^{\nu T} & M_R \end{pmatrix}\ . \quad (2)$$

This matrix is diagonalised by the unitary transformation

$$\mathcal{V}^T \mathcal{M}^* \mathcal{V} = \mathcal{D} \qquad \text{i.e.} \qquad \mathcal{V}^\dagger \mathcal{M} = \mathcal{D}\mathcal{V}^T\ , \quad (3)$$

where

$$\mathcal{D} = \begin{pmatrix} d & 0 \\ 0 & d_R \end{pmatrix}\ , \quad (4)$$

with $d = \text{diag.}(m_1, m_2, m_3)$ and $d_R = \text{diag.}(M_1, M_2, M_3)$ denoting respectively, the light and the heavy Majorana neutrino masses. The unitary 6×6 matrix $\mathcal{V}$ is often denoted in the literature as:

$$\mathcal{V} = \begin{pmatrix} K & R \\ S & Z \end{pmatrix}\ , \quad (5)$$

where K, R, S and Z are 3×3 matrices. For K and Z non singular, we may write

$$\mathcal{V} = \begin{pmatrix} K & 0 \\ 0 & Z \end{pmatrix} \begin{pmatrix} 1\!\!1 & Y \\ -X & 1\!\!1 \end{pmatrix}; \quad -X = Z^{-1}S; \quad Y = K^{-1}R\ . \quad (6)$$

From the unitary relation $\mathcal{V}\ \mathcal{V}^\dagger = 1\!\!1_{(6\times 6)}$, we promptly conclude that

$$Y = X^\dagger\ . \quad (7)$$

The matrix $\mathcal{V}$ can thus be written:

$$\mathcal{V} = \begin{pmatrix} K & KX^\dagger \\ -ZX & Z \end{pmatrix}\ . \quad (8)$$

We have thus made clear that the unitary 6×6 matrix $\mathcal{V}$ can be expressed in terms of three independent 3×3 matrices. From the unitarity of $\mathcal{V}$, we obtain:

$$\begin{aligned} K\ \left(1\!\!1 + X^\dagger X\right)\ K^\dagger &= 1\!\!1\ , \\ Z\ \left(1\!\!1 + X\ X^\dagger\right)\ Z^\dagger &= 1\!\!1\ . \end{aligned} \quad (9)$$

showing that the matrix X parametrizes the deviations from unitary of the matrices K and Z. More explicitly:

$$\begin{aligned} K\ K^\dagger &= 1\!\!1 - K\ X^\dagger X\ K^\dagger\ , \\ Z\ Z^\dagger &= 1\!\!1 - Z\ XX^\dagger\ Z^\dagger\ . \end{aligned} \quad (10)$$

From Eq. (3) we derive:

$$-X^\dagger Z^\dagger M^{\nu T} = dK^T\ , \quad (11)$$

$$K^\dagger M^\nu - X^\dagger Z^\dagger M_R = -dX^T Z^T\ , \quad (12)$$

$$Z^\dagger M^{\nu T} = d_R X^* K^T\ , \quad (13)$$

$$XK^\dagger M^\nu + Z^\dagger M_R = d_R Z^T\ , \quad (14)$$

replacing $Z^\dagger M^{\nu T}$ from Eq. (13) into Eq. (11) we get

$$d = -X^T\ d_R\ X\ , \quad (15)$$

which implies that:

$$X = \pm i \sqrt{d_R^{-1}} O_c \sqrt{d}\ , \quad (16)$$

where O_c is a complex orthogonal matrix, i.e., $O_c^T O_c = \text{I}$, or explicitly:

$$|X_{ij}| = \left| (O_c)_{ij} \sqrt{\frac{m_j}{M_i}} \right|\ . \quad (17)$$

It should be stressed that the parametrisation of the 6×6 unitary matrix $\mathcal{V}$ given by Eq. (8) has the especial property of allowing to connect in a straightforward and simple way the masses of the light and the heavy neutrinos through an orthogonal complex matrix O_c, as can be seen from Eqs. (15) and (16). This is an important new result which plays a crucial rôle in our analysis. Previously, in a variety of contexts, there have been various attempts at describing the possibility of having non-unitary lepton mixing [23]. The key point of our analysis is its generality. The parametrisation of Eq. (8) allows one to write the unitary 6×6 mixing matrix $\mathcal{V}$ in terms of two quasi-unitary matrices K and Z and a matrix X which controls the deviations from unitarity of both K and Z. This is explicitly shown in Eq. (10). The analysis we present here has the advantage of leading to a clearcut quantification of the phenomenological non-unitarity bounds in terms of the related 6×6 lepton mixing matrix parameters. Another important point is the fact that, unlike in the case of Ref. [25], we do not assume conservation of lepton number. In the sequel, we shall also present a clear and straightforward relation between the non-unitary deviations and the scale of the neutrino Dirac mass matrix.

Since O_c is an orthogonal complex matrix, not all of its elements need to be small; furthermore, not all the M_i need to be much larger than the electroweak scale, in order for the seesaw mechanism to lead to naturally suppressed neutrino masses. These observations about the size of the elements of X are specially relevant in view of the fact that some of the important physical implications of the seesaw model depend crucially on X. In particular, the deviations from 3×3 unitarity are controlled by X, as shown in Eq. (10).

Given the importance of the matrix X, one may ask whether it is possible to write the 6×6 unitary matrix $\mathcal{V}$ in terms of 3×3 blocks, where only 3×3 unitary matrices enter, together with the matrix X. It can be shown that the matrix $\mathcal{V}$ can be written:

$$\mathcal{V} = \begin{pmatrix} K & R \\ S & Z \end{pmatrix}$$
$$= \begin{pmatrix} \Omega\left(\sqrt{\mathbb{1}+X^\dagger X}\right)^{-1} & \Omega\left(\sqrt{\mathbb{1}+X^\dagger X}\right)^{-1} X^\dagger \\ -\Sigma\left(\sqrt{\mathbb{1}+X\, X^\dagger}\right)^{-1} X & \Sigma\left(\sqrt{\mathbb{1}+X\, X^\dagger}\right)^{-1} \end{pmatrix} \tag{18}$$

where Ω and Σ are 3×3 unitary matrices given by:

$$\Omega = U_K U^\dagger \qquad \Sigma = W_Z\, W^\dagger \tag{19}$$

and U, W are the unitary matrices that diagonalise respectively $X^\dagger X$ and $XX^\dagger$:

$$U^\dagger\; X^\dagger X\; U = d_X^2; \qquad W^\dagger X X^\dagger\; W = d_X^2 \tag{20}$$

Where U_K and W_Z defined by:

$$U_K \equiv K\; U\; \sqrt{(\mathbb{1}+d_X^2)} \qquad W_Z \equiv Z\; W\; \sqrt{(\mathbb{1}+d_X^2)} \tag{21}$$

are in fact unitary matrices.

As will be explained in the next section, this will allow us, in our analysis, to trade the matrix K by the combination $U_K U^\dagger$ which we identify as the best fit for U_{PMNS} derived under the assumption of unitarity, multiplied by the remaining factor that parametrises the deviations from unitarity.

B. On the Size of Deviations from Unitarity

In the framework of the type I seesaw, it is the block K of the matrix $\mathcal{V}$ that takes the rôle played by U_{PMNS} matrix at low energies in models with only Dirac-type neutrino masses. Clearly, in this framework, K is no longer a unitary matrix. However, present neutrino experiments are putting stringent constraints on the deviations from unitarity. In our search for significant deviations from unitarity of K, we must make appropriate choices for the matrix X in order to comply with the experimental bounds, while at the same time obtain deviations that are sizeable enough to be detected experimentally in the near future. Our aim is to show how one can achieve this result with at least one of the heavy neutrinos with a mass at most at the TeV scale, without requiring unnaturally small Yukawa couplings and still have light neutrino masses not exceeding one eV.

Deviations from unitarity [26], [27], [28],[29], [30], [32] of K have been parametrised as the product of an Hermitian matrix by a unitary matrix [30]:

$$K = (\mathbb{1}-\eta)V \tag{22}$$

where η is an Hermitian matrix with small entries. In order to identify the different components of our matrix K, given in Eq. (18), with the parametrisation of Eq. (22) we rewrite K as:

$$\begin{aligned} K &= U_K U^\dagger\left(\sqrt{\mathbb{1}+X^\dagger X}\right)^{-1} \\ &= \left[U_K U^\dagger\left(\sqrt{\mathbb{1}+X^\dagger X}\right)^{-1} U U_K^\dagger\right] U_K U^\dagger \end{aligned} \tag{23}$$

inside the square brackets we wrote the Hermitian matrix that we identify with $(\mathbb{1}-\eta)$, and which will parametrise the deviations from unitarity. The matrix $V \equiv U_K U^\dagger$ is a unitarity matrix which is identified with U_{PMNS} obtained from the standard parametrisation [33] for a unitary matrix. One can also write:

$$\begin{aligned} &\left[U_K U^\dagger\left(\sqrt{\mathbb{1}+X^\dagger X}\right)^{-1} U U_K^\dagger\right] \\ &\equiv \left[U_K\left(\sqrt{\mathbb{1}+d_X^2}\right)^{-1} U_K^\dagger\right] \end{aligned} \tag{24}$$

where d_X^2 is a 3×3 diagonal matrix, introduced in Eq. (20). Identifying the second expression of Eq. (24) to $(\mathbb{1}-\eta)$ we derive:

$$\eta = \mathbb{1} - U_K\left(\sqrt{\mathbb{1}+d_X^2}\right)^{-1} U_K^\dagger \approx \frac{1}{2} U_K\; d_X^2\; U_K^\dagger \tag{25}$$

for small d_X^2 . The matrix U_{PMNS} is then fixed making use of the present best fit values obtained from a global analysis based on the assumption of unitarity. As pointed out in [30], from the phenomenological point of view it is very useful to parametrise K with the unitary matrix on the right, due to the fact that experimentally it is not possible to determine which physical light neutrino is produced, and therefore, one must sum over the neutrino indices. As a result, most observables depend on $KK^\dagger$ which depends on the following combination:

$$\left(KK^\dagger\right)_{\alpha\beta} = \delta_{\alpha\beta} - 2\eta_{\alpha\beta} + \mathcal{O}(\eta_{\alpha\beta}^2) \tag{26}$$

In Ref. [30] global constraints are derived on the matrix η through a fit of twenty eight observables including the W boson mass, the effective mixing weak angle θ_W, several ratios of Z fermionic decays, the invisible width of the Z, several ratios of weak decays constraining EW universality, weak decays constraining CKM unitarity and some radiative lepton flavour violating (LFV) processes. It can be shown that, using the properties of orthogonal complex matrices, only one of the d_{X_i}, which we take for definiteness as d_{X_3}, can have a significant value (e.g. $d_{X_3} \approx 10^{-2}$), while the other two are negligible. In fact from the eigenvalue equation of X we find:

$$\begin{aligned} &d_{X_1}^2 d_{X_2}^2 d_{X_3}^2 = \tfrac{m_1 m_2 m_3}{M_1 M_2 M_3} \\ &\tfrac{d_{X_1}^2 d_{X_2}^2 + d_{X_1}^2 d_{X_3}^2 + d_{X_2}^2 d_{X_3}^2}{d_{X_1}^2 + d_{X_2}^2 + d_{X_3}^2} \lessapprox O(\tfrac{m_3}{M_1}) \end{aligned} \tag{27}$$

choosing $d^2_{X_3}$ large enough to be experimentally relevant forces the other two eigenvalues to be extremely small.

Using this in Eq. (25), one obtains:

$$\eta = \frac{1}{2} {d_X}^2_3 \cdot \begin{pmatrix} |U_{K13}|^2 & U_{K13} \cdot U^*_{K23} & U_{K13} \cdot U^*_{K33} \\ U_{K23} \cdot U^*_{K13} & |U_{K23}|^2 & U_{K23} \cdot U^*_{K33} \\ U_{K33} \cdot U^*_{K13} & U_{K33} \cdot U^*_{K23} & |U_{K33}|^2 \end{pmatrix} . \tag{28}$$

With this one sees that when ${d_X}^2_3$ approaches 0, all entries of η will approach zero. Furthermore, if the entries of U_K are of same order of magnitude, such that every product of U_{Kij} yields ~ 1, η is a democratic matrix, dominated by ${d_X}^2_3$. The experimental bounds given in Ref. [30] constrain much more the entries that are proportional to U_{K23} than the rest. Looking at Eq. (26), one concludes that $KK^\dagger - I \sim -2\eta$, for a small η. Therefore:

$$|\eta| \leq \begin{pmatrix} 1.25 \times 10^{-3} & 1.20 \times 10^{-5} & 1.35 \times 10^{-3} \\ 1.20 \times 10^{-5} & 2.00 \times 10^{-4} & 6.00 \times 10^{-4} \\ 1.35 \times 10^{-3} & 6.00 \times 10^{-4} & 2.8 \times 10^{-3} \end{pmatrix} . \tag{29}$$

Thus, to achieve such non-democratic deviations from unitarity like in [eq. 29], one will need a non-democratic U_K matrix. This suggests that, if one wants a model that has deviations from unitarity matching the experimental bounds, one will need to find a U_K with the 23 entry small enough such that the entries proportional to it are controlled by it and the rest is controlled by ${d_X}^2_3$.

One identifies $U_K.U^\dagger$ with the best fit for U_{PMNS}, which contains the Majorana phases, α_i, without any constrain. Therefore, to achieve a small U_{K23} one needs to control the quantity:

$$\begin{aligned} &Line_2[U_{PMNS}] \times Column_3[U] = \\ &U^{21}_{PMNS} \cdot U^{13} + U^{22}_{PMNS} \cdot e^{i\alpha_1} \cdot U^{23} + U^{23}_{PMNS} \cdot e^{i\alpha_2} \cdot U^{33} . \end{aligned} \tag{30}$$

Thus, one can choose the α_i such that there is a cancellation and the above quantity is small.

For an O_c of the form:

$$O_c = \begin{pmatrix} 0 & \sqrt{x^2+1} & ix \\ 0 & ix & -\sqrt{x^2+1} \\ 1 & 0 & 0 \end{pmatrix} , \tag{31}$$

one gets $U^{13} = 0$. This puts too much strain on the process of controlling U_{K23}. Thus, the following O_c can be used:

$$O_c = O'_c.O = \begin{pmatrix} 0 & \sqrt{x^2+1} & ix \\ 0 & ix & -\sqrt{x^2+1} \\ 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \tag{32}$$

This angle θ will generate a controllable non-zero U^{13} without changing the eigenvalues of X. This procedure proves that that the Majorana phases may have a crucial role on the size of a given entry of η.

The main conclusion of this section is that, for $\frac{M_i}{m_j} >> 1$, the only eigenvalue of $X^\dagger X$ that contributes to the deviations from unitarity is ${d_X}^2_3$. For a fixed light and heavy mass scale, this variable depends on the parameter x, which is totally free. Thus, the conclusion seems to be that one can generate any size of deviations from unitarity, independently of the masses involved. However, there is a catch. In this general approach to the seesaw mechanism, the Dirac mass matrix, M^ν, is proportional to the matrix X, and, thus, also depends on the parameter x. In conclusion, the desirable size of the Yukawa couplings constrains the parameter space.

III. CONSTRAINING THE DEVIATIONS FROM UNITARITY USING THE ENTRIES OF THE DIRAC MASS MATRIX

In this section, it will become clear that the entries of the Dirac mass matrix constrain the possible deviations from unitarity for a given value of the lightest heavy neutrino mass. In other words, it is shown that there is a correlation among:

- The size of deviations from 3×3 unitarity of the leptonic mixing matrix K
- The mass of the lightest heavy neutrino, M_1.

From Eq. (13) one obtains:

$$M^\nu = K X^\dagger d_R \left(Z^*\right)^{-1} . \tag{33}$$

As said before, the experimental fact that K is almost unitary implies that Z is also almost unitary. Therefore the Dirac mass matrix M^ν is of the same order as X times d_R. Notice that the scale of d_R may be of the order of the top quark mass, so that indeed the Yukawa couplings need not be extremely small.

The elements of the neutrino Dirac mass matrix, M^ν, are connected to the deviations from unitarity of the leptonic mixing matrix, K, in the following way:

$$M^\nu = U_K \left(\sqrt{(\mathbb{1} + d_X^2)}\right)^{-1} d_X \, W^\dagger \, d_R \, W^* \left(\sqrt{(\mathbb{1} + d_X^2)}\right) W_Z^T , \tag{34}$$

where Eqs. (20), (21), (23), (24) were used. An interesting quantity that gives an insight on the order of the entries of M^ν is:

$$\begin{aligned} &Tr\left[M^\nu {M^\nu}^\dagger\right] \\ &= Tr\left[\left(\sqrt{(\mathbb{1} + d_X^2)}\right)^{-1} d_X \; W^\dagger \; d_R \; W^* \; \left(\mathbb{1} + d_X^2\right) \; W^T \; d_R \; W \; d_X \; \left(\sqrt{(\mathbb{1} + d_X^2)}\right)^{-1}\right] \end{aligned} \tag{35}$$

As previously emphasized, deviations from 3×3 unitarity in the leptonic mixing matrix, K, are controlled by the matrix X. For $X = 0$, there are no deviations from unitarity. Small deviations from unitarity correspond to d_X small and, in that case, one has, in a very good approximation:

$$Tr\left[M^\nu {M^\nu}^\dagger\right] \approx Tr\left[d_X \; W^\dagger d_R^2 W \; d_X\right] = Tr\left[d_X^2 \; W^\dagger d_R^2 W\right] , \tag{36}$$

where the terms with powers higher than 2 of d_X were neglected. This can be written as:

$$Tr\left[M^\nu M^{\nu\dagger}\right] = d_{X_1}^2\left(M_1^2\left|W_{11}\right|^2 + M_2^2\left|W_{21}\right|^2 + M_3^2\left|W_{31}\right|^2\right) + d_{X_2}^2\left(M_1^2\left|W_{12}\right|^2 + M_2^2\left|W_{22}\right|^2 + M_3^2\left|W_{32}\right|^2\right) + d_{X_3}^2\left(M_1^2\left|W_{13}\right|^2 + M_2^2\left|W_{23}\right|^2 + M_3^2\left|W_{33}\right|^2\right) . \tag{37}$$

Thus, one finds in good approximation:

$$Tr\left[M^\nu M^{\nu\dagger}\right] \approx d_{X_3}^2\left(M_1^2\left|W_{13}\right|^2 + M_2^2\left|W_{23}\right|^2 + M_3^2\left|W_{33}\right|^2\right) . \tag{38}$$

Using the unitary of W:

$$Tr\left[M^\nu M^{\nu\dagger}\right] \approx d_{X_3}^2 M_1^2\left(1 + \left(\frac{M_2^2}{M_1^2} - 1\right)\left|W_{23}\right|^2 + \left(\frac{M_3^2}{M_1^2} - 1\right)\left|W_{33}\right|^2\right) , \tag{39}$$

which, with the choice $M_3 \geq M_2 \geq M_1$, leads to:

$$d_{X_3}^2 M_1^2 \leq Tr\left[M^\nu M^{\nu\dagger}\right] = \sum_{i,j}\left|M_{ij}^\nu\right|^2 . \tag{40}$$

From Eq. (40), it is clear that for significant values of d_{X_3}, M_1 cannot be too large in order to avoid a too large value of $Tr\left[M^\nu M^{\nu\dagger}\right]$, which in turn would imply that at least one of the $\left|M_{ij}^\nu\right|^2$ is too large. This can be seen in Fig. (1), where the plot of $\frac{1}{2}d_{X_3}^2$ versus M_1 is presented. This is done for a large x^2, the parameter of the matrix O_c. For the case when $x^2 << 1$,the deviations from unitarity are totally controlled by the heavy mass scale. Both cases yield similar plots. Significant values of $d_{X_3}^2$ can only be obtained for $M_1 \leq 1\ TeV$, in the large x region. Of course that for a very small M_1, to obtain deviations from unitarity of this order ($\sim 10^{-3}$), $Tr\left[M^\nu M^{\nu\dagger}\right]$ would yield a very small result and this is also not wanted.

Thus, the quantity $Tr\left[M^\nu M^{\nu\dagger}\right]$ constrains the lightest heavy neutrino mass by giving a lower and an upper bound, for a given quantity of $d_{X_3}^2$. In the following plot, it is required that $Tr\left[M^\nu M^{\nu\dagger}\right] \leq m_t^2$. To create them, the case of normal ordering was considered, and the values of light neutrinos masses m_i, were varied up to $m_3 = 0.5$ eV. Concerning the heavy Majorana masses M_i, M_3 was allowed to reach values of the order of $10^4 m_t$ and the O_c were randomly generated with a large x. In Fig. (1), the condition $|\eta_{12}| \leq 2\times 10^{-5}$ is imposed.

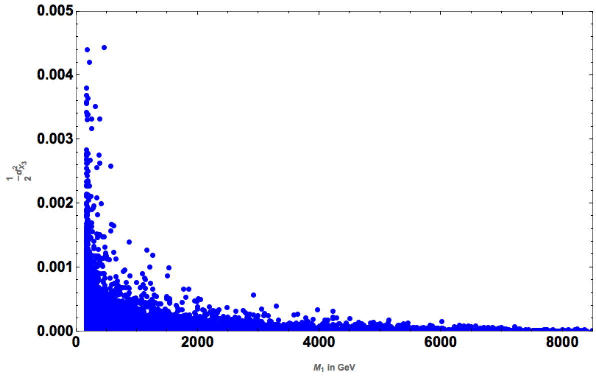

FIG. 1: Maximum deviations from unitarity as a function of M_1, generated under the condition that $\mathrm{Tr}(M^\nu M^{\nu\dagger}) \leq m_t^2$ and $|\eta_{12}| \leq 2\times 10^{-5}$.

IV. THE IMPORTANCE OF LOOP CORRECTIONS

Loop corrections can be of two kinds: renormalizable and intrinsically finite. The renormalizable pieces consist of corrections to the tree level parameters already present in the Lagrangian. In the case of corrections to the masses, these are suppressed with respect to the tree level ones by the loop factor $\frac{1}{16\pi^2}$ and by being proportional to leptonic Yukawa couplings [31]. The intrinsically finite corrections are terms which need to be finite since there are no counterterms that could be used to absorb possible divergences arising from them. They are only suppressed by the loop factor, and, thus, can be potentially large. At one-loop level, the generalized mass matrix, $\mathcal{M}$ from Eq. (2) turns into:

$$\mathcal{M} = M^{tree} + M^{loop} , \tag{41}$$

where

$$M^{tree} = \begin{pmatrix} 0 & M^\nu \\ M^{\nu T} & M_R \end{pmatrix} , \quad M^{loop} = \begin{pmatrix} \delta M_L & \delta M^\nu \\ (\delta M^\nu)^T & \delta M_R \end{pmatrix} . \tag{42}$$

Using the previous equation one can conclude that δM_L will be the potentially dangerous correction, since it is the one without a tree level counterpart. The renormalizable and suppressed corrections are given by δM^ν. Discussing δM_R is cumbersome since M_R and M_i are free parameters of the theory.

The corrections stem from the two point function known as neutrino self energy, $\Sigma(p)$, where p is the neutrino momentum. This is calculated in the mass basis, then Eq. (3) is used to transform it to the interaction basis:

$$M^{loop} = V\Sigma(p)V^T . \tag{43}$$

The diagrams one should consider in order to calculate $\Sigma(p)$ at one-loop are given in Fig. 2. Fig. 2 should be seen in the following way: if $A = Z, H, \phi_Z$ then $B = \chi_K$ or else if $A = \phi^\pm, W^\pm$, then $B = l^\mp$ and

$$\chi = \begin{pmatrix} n \\ N \end{pmatrix} , \ \nu_L^0 = (K\ R)P_L\chi , \ {\nu'_L}^0 = (S\ Z)P_L\chi , \tag{44}$$

as in Eq. (1) and where n are the light neutrino states and N the heavy neutrino states.

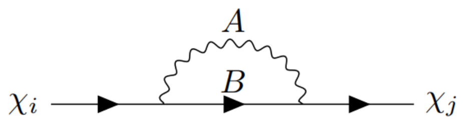

FIG. 2: Loop Diagrams used to calculate the neutrino self energy.

V. HOW TO CONTROL LIGHT NEUTRINO LOOP MASS CORRECTIONS

In very good approximation the loop generated mass term is given by:

$$\delta M_L \approx \frac{g^2}{64\pi^2 m_W^2} R \left[3L^M(m_Z) + L^M(m_H)\right] R^T \ , \quad (45)$$

where

$$L^M(m_B) = \begin{pmatrix} M_1^3 \frac{\log(M_1^2/m_B^2)}{M_1^2/m_B^2-1} & 0 & 0 \\ 0 & M_2^3 \frac{\log(M_2^2/m_B^2)}{M_2^2/m_B^2-1} & 0 \\ 0 & 0 & M_3^3 \frac{\log(M_3^2/m_B^2)}{M_3^2/m_B^2-1} \end{pmatrix} . \quad (46)$$

Controlling light neutrino loop mass corrections, reduces to control the quantity given in Eq. (45). Since $L^M(m_B)$ are, in general, large there are only three possibilities in order to generate a small δM_L:

A Having a very small R, such that $R\left(L^M(m_B)\right)R^T$ is suppressed.

B Having a R with entries of arbitrary order of magnitude but with a given structure such that combined with a proper choice of $L^M(m_B)$ it yields a small $R\left(L^M(m_B)\right)R^T$ due to cancellations.

C Having two small heavy neutrino masses (of the order of the eV or KeV, for example), such that two of the columns of $L^M(m_B)$ are small while the remaining heavy neutrino has a large mass. Along with the choice of a special type of O_c such that one of the rows of X has small entries, leading to a column with small entries in R. This column should match the one column of $L^M(m_B)$ that is not small, i.e., the one that corresponds to the heavy neutrino with large mass. This way, $R\left(L^M(m_B)\right)R^T$ is suppressed. [1]

[1] A scenario with 3 light heavy neutrino would also work but it is disfavoured due to always leading to unnaturally small neutrino Yukawa couplings, independently of the chosen deviations from unitarity.

VI. NUMERICAL EXAMPLES AND THE EFFECT OF DEVIATIONS FROM UNITARITY ON LOOP CORRECTIONS

This section is organized as follows. First, examples for the three models in which light neutrino loop mass corrections are controlled are presented: For case A- small R, and, thus, small deviations from unitarity, type II - sizable deviations from unitarity with quasi-degenerate heavy neutrinos and type III - sizable deviations from unitarity with two light heavy neutrinos.

The case A example is given for normal ordering and includes an analysis of the effect of the deviations from unitarity on the variation of the heaviest light neutrino mass after loop corrections.

The case B examples are given for normal and inverted ordering, each for a different pattern of deviations from unitarity. For the normal ordering scenario, an analysis of the effect of the deviations from unitarity on the variation of the heaviest light neutrino mass after loop corrections is also given.

A final case C example is given for normal ordering with M_1 of the order of the eV, M_2 of the order of the KeV and a large M_3, with sizable deviations from unitarity.

The numerical examples are given in the following tables, where the deviations from unitarity are expressed by the hermitian matrix η, defined in Eq. (25). The first row contains quantities that are the same at tree and loop level - heavy neutrino masses, Dirac mass matrix, M^ν and Heavy neutrino mass matrix, M_R. The second row contains relevant quantities - the light neutrino masses, the matrix X and the mixing matrix that connects light and heavy neutrinos through electroweak processes, R - at tree level. The third row contains the same quantities as the second row, but at one-loop level. The mixing matrix K has entries in the U_{PMNS} 1σ allowed range, both for tree and one loop level. The differences of the squared light neutrino masses, Δm^2_{ij}, at one loop level are in the 1σ range of the values given in [34]. All quantities with units of mass, except the light neutrino masses which are in eV, are expressed in terms of the top quark mass m_t. The matrix W_Z, defined in Eq. (21), was chosen to be

$$W_Z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \ , \quad (47)$$

since there are no experimental bounds for the Z matrix.

A. Case A: Small Deviations from Unitarity

For the example given in Table I the used O_c was of the type given in Eq. (32) with $\theta = \frac{\pi}{3}$ and $x = 4.8$. The Majorana phases were taken to be $\alpha_1 = \alpha_2 = 0$. The loop corrections become controlled near the minimum possible value for the deviations from unitarity $d^2_{X3} \sim \frac{m}{M}$, as

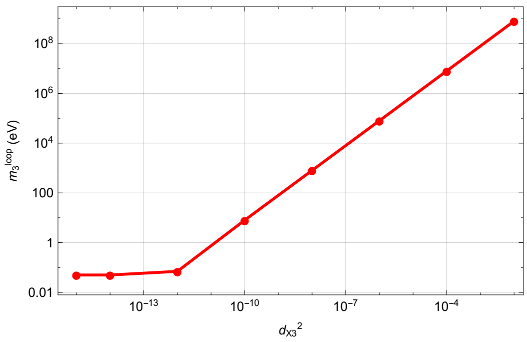

FIG. 3: m_3 after loop corrections as a function of d_{X3}^2, generated in the example given in Tab. I, varying the value of the parameter x, while $Tr[M^\nu M^{\nu\dagger}] \leq m_t^2$ and with everything else kept constant.

for small x one has $X \approx -i\sqrt{\frac{d}{d_R}}$. Higher level loop corrections on the example given in Tab. I are not expected to be very big due to the smallness of the entries of the R matrix.

B. Case B: Sizable Deviations from Unitarity with two Quasi-degenerate Heavy Neutrinos

For the example given in Table II the used O_c was of the type given in [eq. 32] with $\theta = \frac{\pi}{3}$ and $x = 2.36 \times 10^5$. The used Majorana phases were $\alpha_1 = \frac{53}{58}\pi$, $\alpha_2 = \frac{19}{34}\pi$. The loop corrections are essentially constant, independently of the size of the deviations from unitarity d_{X3}^2. This happens due to the cancelling structure of R and because of the quasi-degeneracy of M_1 and M_2. Higher level loop corrections on the example given in Tab. II are not expected to be very big due to the persistence of structure of R after loop corrections.

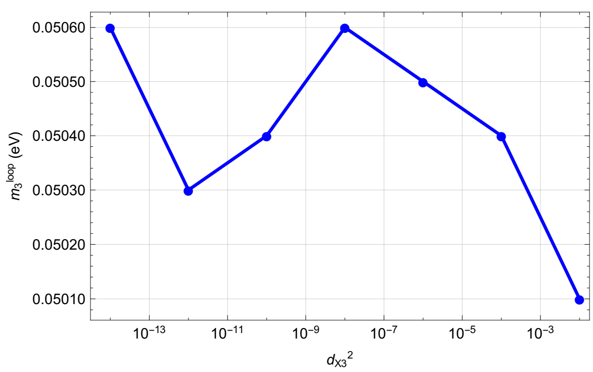

FIG. 4: m_3 after loop corrections as a function of d_{X3}^2, generated in the example given in Tab. II, varying the value of the parameter x, while $Tr[M^\nu M^{\nu\dagger}] \leq m_t^2$ and with everything else kept constant.

For the example given in Table III the used O_c was of the type:

$$O_c = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} \sqrt{x^2+1} & 0 & ix \\ ix & 0 & -\sqrt{x^2+1} \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{48}$$

with $\theta = \frac{\pi}{10}$ and $x = 2.44 \times 10^5$. The used Majorana phases were $\alpha_1 = \frac{3}{2}\pi$, $\alpha_2 = \frac{47}{80}\pi$.

C. Case C: Sizable Deviations from Unitarity with two Light Heavy Neutrinos

For the example given in Table IV the used O_c was of the type given in Eq. (32) with $\theta = \frac{\pi}{3}$ and $x = 0.78$. The used Majorana phases were $\alpha_1 = \frac{52}{125}\pi$, $\alpha_2 = \frac{389}{200}\pi$.

This situation is possible because of the interplay of three things. The order of magnitude of the masses M_1 and M_2, the big deviations from unitarity and the O_c chosen to be like in Eq. (32). As the deviations from unitarity are sizable, and X is of this type, its third row is very small, thus cancelling the effect of a very large M_3 on eq. (45). Furthermore, because of the smallness of M_1 and M_2, the first two entries of Eq. (46) are small, thus controlling the loop generated mass matrix δM_L.

Studying the effect of the variation of the parameter x in these type of models, as done in Figs. (3, 4), is cumbersome, since x doesn't control the deviations from unitarity for these type of models. This happens because these models achieve sizable deviations from unitarity for $x \sim 1$. The only way to decrease the deviations from unitarity is increasing the heavy neutrino masses, and this, of course, has a big effect on the loop corrections. Summarizing, it's not possible to isolate the effect of the deviations of unitarity on the loop corrections for these type of models.

TABLE I: Example for case A, with Normal Ordering of light neutrino masses. This example gives the following phenomenological important quantities: $|m_{\beta\beta}| = 5.97 \times 10^{-3}\ eV$, $m_\beta = 9.67 \times 10^{-3}\ eV$, and $N_\nu = 2.999$.

Heavy Neutrino Masses (m_t)	$\|M^\nu\|\ (m_t)$	$Tr\left[M^\nu M^{\nu\dagger}\right](m_t^2)$	$\|M_R\|\ (m_t)$
$\begin{bmatrix} M_1 = 30 \\ M_2 = 60 \\ M_3 = 100 \end{bmatrix}$	$\begin{pmatrix} 1.02\times10^{-5} & 6.08\times10^{-8} & 1.10\times10^{-6} \\ 1.19\times10^{-5} & 3.52\times10^{-7} & 3.41\times10^{-6} \\ 7.97\times10^{-6} & 3.72\times10^{-7} & 4.32\times10^{-6} \end{pmatrix}$	3.41×10^{-10}	$\begin{pmatrix} 3.03\times10^{1} & 4.20\times10^{1} & 6.77 \\ 4.20\times10^{1} & 7.64\times10^{-1} & 9.38 \\ 6.77 & 9.38 & 9.90\times10^{1} \end{pmatrix}$
Tree Level Light Neutrino Masses (eV)	$\|\eta\|^{tree}$	X^{tree}	R^{tree}
$\begin{bmatrix} m_1 = 0.0062 \\ m_2 = 0.00902 \\ m_3 = 0.0542 \end{bmatrix}$	$\begin{pmatrix} 2.89\times10^{-14} & 3.40\times10^{-14} & 2.29\times10^{-14} \\ 3.40\times10^{-14} & 4.06\times10^{-14} & 2.72\times10^{-14} \\ 2.29\times10^{-14} & 2.72\times10^{-14} & 1.91\times10^{-14} \end{pmatrix}$	$\begin{pmatrix} -1.46\times10^{-7} & \left(-2.04\times10^{-7}\right)i & 2.30\times10^{-7} \\ \left(-1.06\times10^{-7}\right)i & 1.41\times10^{-7} & \left(1.66\times10^{-7}\right)i \\ \left(-8.85\times10^{-9}\right)i & 0 & \left(-4.94\times10^{-8}\right)i \end{pmatrix}$	$\begin{pmatrix} 1.3\times10^{-7}-\left(1.47\times10^{-7}\right)i & 1.02\times10^{-7}+\left(9.37\times10^{-8}\right)i & 6.9\times10^{-9}-\left(4.97\times10^{-9}\right)i \\ -1.97\times10^{-7}-\left(1.19\times10^{-7}\right)i & 8.24\times10^{-8}-\left(1.43\times10^{-7}\right)i & 6.68\times10^{-10}-\left(2.85\times10^{-8}\right)i \\ -1.01\times10^{-7}+\left(1.19\times10^{-7}\right)i & -8.29\times10^{-8}-\left(7.32\times10^{-8}\right)i & 7.69\times10^{-10}-\left(4.04\times10^{-8}\right)i \end{pmatrix}$
One Loop Light Neutrino Masses (eV)	$\|\eta\|^{loop}$	X^{loop}	R^{loop}
$\begin{bmatrix} m_1 = 0.00543 \\ m_2 = 0.0102 \\ m_3 = 0.0505 \end{bmatrix}$	$\begin{pmatrix} 2.92\times10^{-14} & 3.41\times10^{-14} & 2.27\times10^{-14} \\ 3.41\times10^{-14} & 4.09\times10^{-14} & 2.72\times10^{-14} \\ 2.27\times10^{-14} & 2.72\times10^{-14} & 1.88\times10^{-14} \end{pmatrix}$	$\begin{pmatrix} 1.39\times10^{-7}+\left(6.56\times10^{-9}\right)i & 1.98\times10^{-9}-\left(2.16\times10^{-7}\right)i & -2.24\times10^{-7}+\left(4.54\times10^{-10}\right)i \\ 4.60\times10^{-9}-\left(1.00\times10^{-7}\right)i & -1.50\times10^{-7}-\left(1.44\times10^{-9}\right)i & 2.90\times10^{-10}+\left(1.62\times10^{-7}\right)i \\ 2.90\times10^{-10}+\left(8.15\times10^{-9}\right)i & -4.07\times10^{-10}+\left(4.82\times10^{-10}\right)i & 5.04\times10^{-11}+\left(4.95\times10^{-8}\right)i \end{pmatrix}$	$\begin{pmatrix} 1.3\times10^{-7}-\left(1.47\times10^{-7}\right)i & 1.02\times10^{-7}+\left(9.37\times10^{-8}\right)i & 6.9\times10^{-9}-\left(4.97\times10^{-9}\right)i \\ -1.97\times10^{-7}-\left(1.19\times10^{-7}\right)i & 8.24\times10^{-8}-\left(1.43\times10^{-7}\right)i & 6.68\times10^{-10}-\left(2.85\times10^{-8}\right)i \\ -1.01\times10^{-7}+\left(1.19\times10^{-7}\right)i & -8.29\times10^{-8}-\left(7.32\times10^{-8}\right)i & 7.69\times10^{-10}-\left(4.04\times10^{-8}\right)i \end{pmatrix}$

TABLE II: Example for case B, with Normal Ordering of light neutrino masses. This example gives the following phenomenological important quantities: $|m_{\beta\beta}| = 6.58 \times 10^{-3}\ eV$, $m_\beta = 1.01 \times 10^{-2}\ eV$ and $N_\nu = 2.989$.

Heavy Neutrino Masses (m_t)	$\|M^\nu\|\ (m_t)$	$Tr\left[M^\nu M^{\nu\dagger}\right](m_t^2)$	$\|M_R\|\ (m_t)$
$\begin{bmatrix} M_1 = 3 \\ M_2 = 3+1\times10^{-10} \\ M_3 = 50 \end{bmatrix}$	$\begin{pmatrix} 0.140 & 4.12\times10^{-13} & 6.49\times10^{-7} \\ 0.000876 & 2.06\times10^{-12} & 2.32\times10^{-6} \\ 0.171 & 1.84\times10^{-12} & 3.17\times10^{-6} \end{pmatrix}$	0.0488	$\begin{pmatrix} 7.15\times10^{-10} & 2.99 & 1.76\times10^{-4} \\ 2.99 & 2.14\times10^{-11} & 3.85\times10^{-5} \\ 1.76\times10^{-4} & 3.85\times10^{-5} & 5.00\times10^{1} \end{pmatrix}$
Tree Level Light Neutrino Masses (eV)	$\|\eta\|^{tree}$	X^{tree}	R^{tree}
$\begin{bmatrix} m_1 = 0.00507 \\ m_2 = 0.0100 \\ m_3 = 0.0522 \end{bmatrix}$	$\begin{pmatrix} 1.09\times10^{-3} & 6.82\times10^{-6} & 1.33\times10^{-3} \\ 6.82\times10^{-6} & 4.27\times10^{-8} & 8.34\times10^{-6} \\ 1.33\times10^{-3} & 8.34\times10^{-6} & 1.63\times10^{-3} \end{pmatrix}$	$\begin{pmatrix} -0.0206 & -0.0328i & 0.0351 \\ -0.0206i & 0.0328 & 0.0351i \\ \left(-1.13\times10^{-8}\right)i & 0 & \left(-6.85\times10^{-8}\right)i \end{pmatrix}$	$\begin{pmatrix} -0.0262-0.0201i & -0.0201+0.0262i & 4.78\times10^{-9}+\left(4.53\times10^{-10}\right)i \\ 0.000137+0.000154i & 0.000154-0.000137i & -4.46\times10^{-8}-\left(1.28\times10^{-8}\right)i \\ -0.0066+0.0398i & 0.0398+0.0066i & -5.12\times10^{-8}-\left(4.92\times10^{-9}\right)i \end{pmatrix}$
One Loop Light Neutrino Masses (eV)	$\|\eta\|^{loop}$	X^{loop}	R^{loop}
$\begin{bmatrix} m_1 = 0.00491 \\ m_2 = 0.0100 \\ m_3 = 0.0504 \end{bmatrix}$	$\begin{pmatrix} 1.09\times10^{-3} & 6.82\times10^{-6} & 1.33\times10^{-3} \\ 6.82\times10^{-6} & 4.27\times10^{-8} & 8.33\times10^{-6} \\ 1.33\times10^{-3} & 8.33\times10^{-6} & 1.63\times10^{-3} \end{pmatrix}$	$\begin{pmatrix} -0.0203+\left(9.27\times10^{-6}\right)i & 6.29\times10^{-6}+0.0330i & -0.0350+\left(5.33\times10^{-7}\right)i \\ -9.27\times10^{-6}-0.0203i & -0.0330+\left(6.29\times10^{-6}\right)i & -5.33\times10^{-7}-0.0350i \\ 2.77\times10^{-14}-\left(1.14\times10^{-8}\right)i & 9.27\times10^{-11}+\left(2.11\times10^{-12}\right)i & 2.68\times10^{-15}+\left(6.85\times10^{-8}\right)i \end{pmatrix}$	$\begin{pmatrix} -0.0262-0.0201i & -0.0201+0.0262i & 4.78\times10^{-9}+\left(4.53\times10^{-10}\right)i \\ 0.000137+0.000154i & 0.000154-0.000137i & -4.46\times10^{-8}-\left(1.28\times10^{-8}\right)i \\ -0.0066+0.0398i & 0.0398+0.0066i & -5.12\times10^{-8}-\left(4.92\times10^{-9}\right)i \end{pmatrix}$

TABLE III: Example for case B, with Inverted Ordering of light neutrino masses. This example gives the following phenomenological important quantities: $|m_{\beta\beta}| = 1.76 \times 10^{-2}\ eV$, $m_\beta = 4.97 \times 10^{-2}\ eV$ and $N_\nu = 2.991$.

Heavy Neutrino Masses (m_t)	$\|M^\nu\|\ (m_t)$	$Tr\left[M^\nu M^{\nu\dagger}\right](m_t^2)$	$\|M_R\|\ (m_t)$
$\begin{bmatrix} M_1 = 3 \\ M_2 = 9 \\ M_3 = 9+1\times10^{-10} \end{bmatrix}$	$\begin{pmatrix} 0.425 & 3.23\times10^{-12} & 5.56\times10^{-7} \\ 0.00434 & 1.49\times10^{-12} & 5.31\times10^{-7} \\ 0.432 & 1.93\times10^{-12} & 5.47\times10^{-7} \end{pmatrix}$	0.367	$\begin{pmatrix} 1.54\times10^{-10} & 8.98 & 1.48\times10^{-7} \\ 8.98 & 5.35\times10^{-11} & 5.61\times10^{-8} \\ 1.48\times10^{-7} & 5.61\times10^{-8} & 3.00 \end{pmatrix}$
Tree Level Light Neutrino Masses (eV)	$\|\eta\|^{tree}$	X^{tree}	R^{tree}
$\begin{bmatrix} m_1 = 0.0509 \\ m_2 = 0.0516 \\ m_3 = 0.00852 \end{bmatrix}$	$\begin{pmatrix} 1.12\times10^{-3} & 1.14\times10^{-5} & 1.13\times10^{-3} \\ 1.14\times10^{-5} & 1.16\times10^{-7} & 1.16\times10^{-5} \\ 1.13\times10^{-3} & 1.16\times10^{-5} & 1.15\times10^{-3} \end{pmatrix}$	$\begin{pmatrix} \left(9.66\times10^{-8}\right)i & \left(-2.99\times10^{-7}\right)i & 0 \\ -0.0420i & -0.0137i & 0.0181 \\ 0.0420 & 0.0137 & 0.0181i \end{pmatrix}$	$\begin{pmatrix} 1.68\times10^{-7}-\left(7.89\times10^{-8}\right)i & 0.00532+0.0330i & 0.0330-0.00532i \\ 1.69\times10^{-7}+\left(5.18\times10^{-8}\right)i & -0.000225-0.000256i & -0.000256+0.000225i \\ -1.80\times10^{-7}-\left(2.80\times10^{-8}\right)i & -0.0148+0.0305i & 0.0305+0.0148i \end{pmatrix}$
One Loop Light Neutrino Masses (eV)	$\|\eta\|^{loop}$	X^{loop}	R^{loop}
$\begin{bmatrix} m_1 = 0.0501 \\ m_2 = 0.0508 \\ m_3 = 0.00828 \end{bmatrix}$	$\begin{pmatrix} 1.11\times10^{-3} & 1.14\times10^{-5} & 1.13\times10^{-3} \\ 1.14\times10^{-5} & 1.16\times10^{-7} & 1.16\times10^{-5} \\ 1.13\times10^{-3} & 1.16\times10^{-5} & 1.15\times10^{-3} \end{pmatrix}$	$\begin{pmatrix} 1.10\times10^{-9}+\left(7.73\times10^{-8}\right)i & 2.78\times10^{-10}+\left(3.05\times10^{-7}\right)i & 1.94\times10^{-11}-\left(1.74\times10^{-11}\right)i \\ -0.0000374+0.0428i & 0.000154-0.0110i & 0.0180-\left(5.06\times10^{-6}\right)i \\ 0.0428+0.0000374i & -0.0110-0.000154i & -5.06\times10^{-6}-0.0180i \end{pmatrix}$	$\begin{pmatrix} 1.68\times10^{-7}-\left(7.89\times10^{-8}\right)i & -0.00532-0.033i & 0.033-0.00532i \\ 1.69\times10^{-7}+\left(5.18\times10^{-8}\right)i & 0.000225+0.000256i & -0.000256+0.000225i \\ -1.8\times10^{-7}-\left(2.8\times10^{-8}\right)i & 0.0148-0.0305i & 0.0305+0.0148i \end{pmatrix}$

TABLE IV: Example for case C, with Normal Ordering of light neutrino masses. This example gives the following phenomenological important quantities: $|m_{\beta\beta}| = 1.13 \times 10^{-3}\ eV$, $m_\beta = 1.03 \times 10^{-2}\ eV$ and $N_\nu = 2.999$.

Heavy Neutrino Masses (m_t)	$\|M^\nu\|\ (m_t)$	$Tr\left[M^\nu M^{\nu\dagger}\right](m_t^2)$	$\|M_R\|\ (m_t)$
$\begin{bmatrix} M_1 = 2.88\times10^{-11} \\ M_2 = 1.44\times10^{-8} \\ M_3 = 5.76\times10^{12} \end{bmatrix}$	$\begin{pmatrix} 0.226 & 3.62\times10^{-13} & 3.24\times10^{-11} \\ 0.749 & 2.25\times10^{-12} & 8.69\times10^{-11} \\ 1.02 & 1.73\times10^{-12} & 1.22\times10^{-10} \end{pmatrix}$	1.66	$\begin{pmatrix} 5.76\times10^{12} & 9.01 & 4.93\times10^{2} \\ 9.01 & 1.54\times10^{-11} & 1.43\times10^{-9} \\ 4.93\times10^{2} & 1.43\times10^{-9} & 5.66\times10^{-8} \end{pmatrix}$
Tree Level Light Neutrino Masses (eV)	$\|\eta\|^{tree}$	X^{tree}	R^{tree}
$\begin{bmatrix} m_1 = 0.00500 \\ m_2 = 0.00987 \\ m_3 = 0.0627 \end{bmatrix}$	$\begin{pmatrix} 1.30\times10^{-3} & 1.86\times10^{-5} & 1.32\times10^{-3} \\ 1.86\times10^{-5} & 1.58\times10^{-6} & 1.88\times10^{-5} \\ 1.32\times10^{-3} & 1.88\times10^{-5} & 1.35\times10^{-3} \end{pmatrix}$	$\begin{pmatrix} -0.0217 & -0.0562i & 0.0407 \\ -0.00158i & 0.00154 & 0.00297i \\ \left(-3.31\times10^{-14}\right)i & 0 & \left(-2.21\times10^{-13}\right)i \end{pmatrix}$	$\begin{pmatrix} -0.0488+0.0141i & 0.000655+0.00218i & -3.21\times10^{-14}+\left(2.26\times10^{-14}\right)i \\ -0.000699+0.000381i & -0.000271-0.00156i & 2.22\times10^{-14}+\left(1.28\times10^{-13}\right)i \\ 0.0486-0.0179i & -0.000815-0.00230i & 2.55\times10^{-14}+\left(1.76\times10^{-13}\right)i \end{pmatrix}$
One Loop Light Neutrino Masses (eV)	$\|\eta\|^{loop}$	X^{loop}	R^{loop}
$\begin{bmatrix} m_1 = 0.00467 \\ m_2 = 0.00986 \\ m_3 = 0.0504 \end{bmatrix}$	$\begin{pmatrix} 1.23\times10^{-3} & 6.91\times10^{-6} & 1.32\times10^{-3} \\ 6.91\times10^{-6} & 1.35\times10^{-6} & 6.92\times10^{-6} \\ 1.32\times10^{-3} & 6.92\times10^{-6} & 1.34\times10^{-3} \end{pmatrix}$	$\begin{pmatrix} 0.0202-0.0000320i & 0.0000754+0.0559i & -0.0418-0.000156i \\ 9.71\times10^{-7}+0.00147i & -0.00153+\left(2.86\times10^{-6}\right)i & 5.67\times10^{-6}-0.00305i \\ 1.75\times10^{-16}-\left(4.22\times10^{-14}\right)i & 5.20\times10^{-18}-\left(3.55\times10^{-16}\right)i & -3.82\times10^{-17}-\left(2.20\times10^{-13}\right)i \end{pmatrix}$	$\begin{pmatrix} 0.0488-0.0141i & -0.000655-0.00218i & -3.21\times10^{-14}+\left(2.26\times10^{-14}\right)i \\ 0.000305-0.00015i & 0.000282+0.00158i & 2.21\times10^{-14}+\left(1.28\times10^{-13}\right)i \\ -0.0485+0.0179i & 0.000815+0.0023i & 2.55\times10^{-14}+\left(1.76\times10^{-13}\right)i \end{pmatrix}$

VII. CONCLUSIONS

In this work, a novel parametrization, adequate for the exact treatment of Seesaw Type I models independent of the scale of M_R was exploited. This revealed a matrix, X, defined in Eq. (16), responsible for the deviations from unitarity of the leptonic mixing matrix K. This parametrization clarifies the relation between heavy neutrino masses and deviations from unitarity which is explained in section III and can be summarized in Eq. (40), which means that to achieve natural values for the Yukawa couplings one needs to take both the size of the deviations from unitarity and the scale of the heavy neutrino masses into account. The possibly dangerously large one-loop corrections were studied, and from that, three types of models with controlled loop corrections were suggested.

Case A models, with small deviations from unitarity, without constraints on the heavy neutrinos masses and with unnaturally small Yukawa couplings. These are very complicated to falsify experimentally.

Case B models, with two quasi-degenerate heavy neutrino masses of the order of the top mass, sizable deviations from unitarity and without unnaturally small Yukawa couplings. These are appealing because they can be observed in the next round of experiments at the LHC. Moreover, it would be interesting to study if the existence of at least two quasi-degenerate heavy neutrinos enables the possibility of resonant Leptogenesis, providing an explanation to the observed matter-anti matter asymmetry [38, 39].

Case C models, with two light heavy neutrinos, sizable deviations from unitarity and without unnaturally small Yukawa couplings. These are appealing because KATRIN will be able to explore the existence of at least one heavy (mostly sterile) neutrino in the mass range of $1 - 18.5\ KeV$, with a mixing to the active neutrino ν_e as $|R_{11}|^2 \geq 10^{-6}$ [40]. Furthermore, they can explain the anomalies from short baseline oscillation experiments [41] and can give explanations to other Physics puzzles like dark matter (when M_2 has a mass on the KeV scale like in the example given in [Tab. IV]) as pointed out in [41].

The question of the possibility of Thermal Leptogenesis for case A and case C is highly relevant, and requires further study. All models explain the smallness of light neutrino masses and case C models have a dark matter candidate. Experimental input from KATRIN, the LHC and neutrino oscillation experiments will be fundamental to discern which, if any, of these models might match with Nature.

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