Planning home health care services – a routing and scheduling problem

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Abstract

Key-words: Home Health Care, Routing and Scheduling, MPVRPTW , Mathematical Modelling, MILP

In the context of a growing aging population, the demand for Home Health Care (HHC) services has been rapidly increasing. Therefore, efficiency in planning human resources, namely in route design and scheduling, emerges as an area in which optimization can have a significant impact.
This thesis proposes a mathematical model that aims at helping a HHC provider to plan the routes associated to domiciliary services. The model is an extension of a MPVRPTW formulation. Beyond the typical VRP characteristics, the most relevant features of HHC services addressed are: 1) hard time windows, 2) work and break regulations and 3) continuity of care, both within a day and throughout the week. The daily continuity of care is modeled as a soft-constraint. The optimization can be performed focusing on two separate objective functions. The first permits the minimization of the travelling time, whereas the second aims at optimizing the workload balance amongst caregivers teams. In spite of introducing an exact model, a solution heuristic is also presented required to solve large instances in a reasonable amount of time.
The model was tested with the instances of a Portuguese institution of social solidarity, APOIO. Respecting all constraints, the minimization of travelling time yield a reduction, per week, of about 7%. In turn, when workload balance is concerned, there is a decrease of imbalances verified in the solution associated to the travelling time minimization. For the first objective function the maximum imbalance between teams is of 158 minutes, whereas for the second the same criteria assumes the value of 80 minutes. This is a reduction of about 50%. The results presented solidly validate the model.

Resumo

Palavras-Chave: Optimização, Serviços de Apoio Domiciliário, Desenho e planeamento de rotas, MPVRPTW, Modelo matemático, MILP

O aumento da população idosa, gera a necessidade de Serviços de Apoio ao Domicílio (SAD), tendência mais significativa em países desenvolvidos. Estando as atividades das instituições limitadas por orçamentos torna-se imperativo otimizar o planeamento de recursos humanos, emergindo a optimização do desenho e planeamento de rotas como uma área de relevo.

Esta dissertação visa introduzir um modelo matemático cuja finalidade é auxiliar prestadores de SAD no desenho e planeamento das suas rotas a domicílios. O modelo assenta na extensão da formulação de um *MPVRPTW*. Para além das características típicas de um *VRP* a formulação modela: 1) *hard time constraints,* 2) regulações laborais e 3) continuidade dos cuidados, durante o dia e a semana. A continuidade de cuidados diária é assegurada recorrendo a uma *soft-constraint*. A optimização foca duas FO diferentes: uma diz respeito à minimização do tempo de viagem, enquanto que a segunda optimiza a distribuição de carga laboral. Apesar de o modelo ser exato, é proposta uma heurística para obter uma solução dentro um intervalo de tempo razoável.

O modelo foi testado com dados reais de uma IPSS em Portugal. A diminuição do tempo de viagem verificada na solução da primeira função objetivo (FO) foi de 7%, em comparação com a solução atual. Focando solução da segunda FO, verifica-se que existe uma diminuição do desequilíbrio da carga laboral entre equipas. Para a primeira FO a desigualdade máxima é de 158 minutos, enquanto que para a segunda FO o valor é 80 minutos, uma redução de 50%, validando o modelo.

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Abbreviations

ADLs – Activities of daily living

APOIO – APOIO-instituição de solidariedade social

A-Teams – Autonomous patients' teams

B-Teams – Bedridden patients' teams

CPLEX - IBM ILOG CPLEX Optimization Studio

CVRP – Capacitated Vehicle Routing Problem

DVRP – Dynamic Vehicle Routing Problems

ENEPRI - European Network of Economic Policy Research Institutes

GAMS – Generic Algebraic Modelling System

HHC – Home Health Care

IADLs – instrumental activities of daily living

ILP – Integer Linear Programming

IPSS – *Instituição Privada de Solidariedade Social* (Social Solidarity Particular Institutions)

MDVRP – Multi-Depot Vehicle Routing Problem

MILP – Mixed Integer Linear Problem

MPVRPTW – Multi-Period Vehicle Routing Problem with Time Windows

OECD - Organization for Economic Cooperation and Development

OF – Objective Function

OVRP – Open Vehicle Routing Problem

PH – Personal Hygiene

PVRP – Periodic Vehicle Routing Problem

SCM – Supply Chain Management

SPO – Swarm Particle Optimization

ST – Service Time

SVRP – Stochastic Vehicle Routing Problems

TSP – Travelling Salesman Problem

TT- Travelling Time

TW – Time-Window

UN – United Nations

VRP – Vehicle Routing Problem

VRPTW – Vehicle Routing Problem with Time Windows

WT – Waiting Time

List of Symbols

Indexes

a, e – Teams of assistants

i, j – Patients and deposits

t, z – Days of the week

Parameters

D_{ij} – Distance between nodes *i* and *j*

w_{it} – Duration of visit to patient *i* on day *t*

NV_{it} – Number of visits to patient *i* on day *t*

P – Penalty value

K – Number of teams for lunch distribution

H – Working time limit

e_{it} – Time-Window earlier time of arrival to node *i* on day *t*

l_{it}- Time-Window later time of arrival to node *i* on day *t*

M_{ijt} – Big M value

lag_{ijt} – Schedule feasibility measure

Variables

x_{ijat} – Allocates teams *a* to arc (*i,j*) on day *t*

s_{ait} – Time at which the visit from team *a* to patient *i* on day *t* must be initiated

a_{ait}- First auxiliary variable for soft-constraint

b_{ait}- Second auxiliary variable for soft-constraint

1. Introduction

1.1 Motivation

Over the last decades, in developed countries, a generalized increase in life expectancy with an accentuated decrease in the fertility rates of the younger generations, has been propelling the aging of the population. The OCDE34 life expectancy at birth is of 80,5 years, increasing about 4 months every year without showing signs of slowing down. In Portugal, the average value stands at 80,8 years. In approximately two thirds of the OECD countries, the percentage of population aged over 65 years is expected to increase up to 25%, from the 15% verified in 2010 (OECD, 2015).

The increase in life expectancy, however, does not mean that the extra years gained are healthy. Regarding the way health is perceived by the Portuguese population over 65 years, it is possible to observe that they feel considerably limited. In the Figure 1.1, it is possible to note that the population's health status perception is the worst amongst other OECD countries. In terms of actual limitations in activities of daily living reported, Portugal is not the worst, but ranks high as one of the countries in which the elderly population feels more moderate or high limitations in those activities.

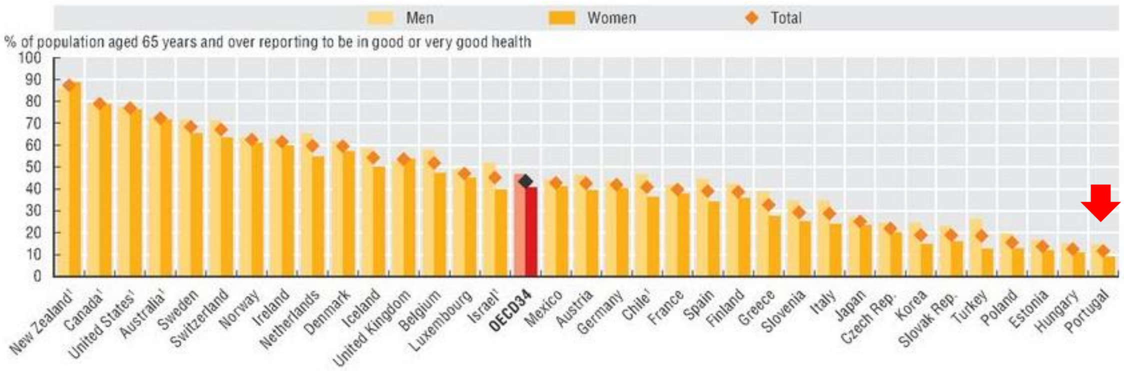


Figure 1.1: Reported good health as a percentage of population over 65 years. Source: OECD, European Observatory in Health Systems and Policies 2015

As people age, they tend to acquire disabilities, frequently resulting from non-communicable diseases. Dementia is a category of such diseases including, for example, Alzheimer's disease, that affects the memory of the elderly in an early stage and, as the disease progresses, it can cause disorientation, behavioral issues and disregard for self-care, amongst other issues. The inability to perform certain tasks leads to an independence loss and the need for some form of provision of care.

The care provided to the elderly and other bearers of non-communicable chronic diseases is known as long-term care and can be divided into formal and informal care. Institutions provide formal care, whereas informal care is related with family and friends. The ways in which long-term care provision varies amongst countries can be explored in Genet. et. al (2013), particularly, how each type of care is incentivized and its relevance for society. In Portugal, informal care has been declining, essentially due to the changes regarding family's organization: increase of woman's participation in the working market, families' atomization and a higher volatility of conjugal relationships (Pego et. al, 2013). The reduced capacity of the families to provide informal care places an additional pressure for the development of formal care.

The Social Security offers benefits in kind (personal care and home care) and cash in order to assure long-term care in Portugal. The institutions that provide the actual care are the IPSS – *Instituição Privada de Solidariedade Social* (Social Solidarity Particular Institutions), which are partially financed by the state (Genet et al, 2013). Currently, the services and facilities available to the elderly are various: day-care centers, home-based services and nursing homes (long-term and palliative care) for exceptionally dependent people (ENEPRI, 2010).

Despite possessing some infrastructures and organizations to provide long-term care, the government's expenditure on this type of care is remarkably limited when compared to other OECD countries. In Figure 1.2, it is possible to notice the difference on the expenditure on several functions of health care, namely, inpatient care, outpatient care, long-term care, medical goods and collective services. In comparison to other OECD countries, Portugal spends only 2% of its health budget in long-term care, while the OECD27 average is 12% and a majority of the most developed countries it stands above 20%. As a consequence, the lack of financial resources made available to IPSSs originates considerable financial strains, greatly restricting the patients eligible to receive care and limiting the number which can be enrolled in institutions.

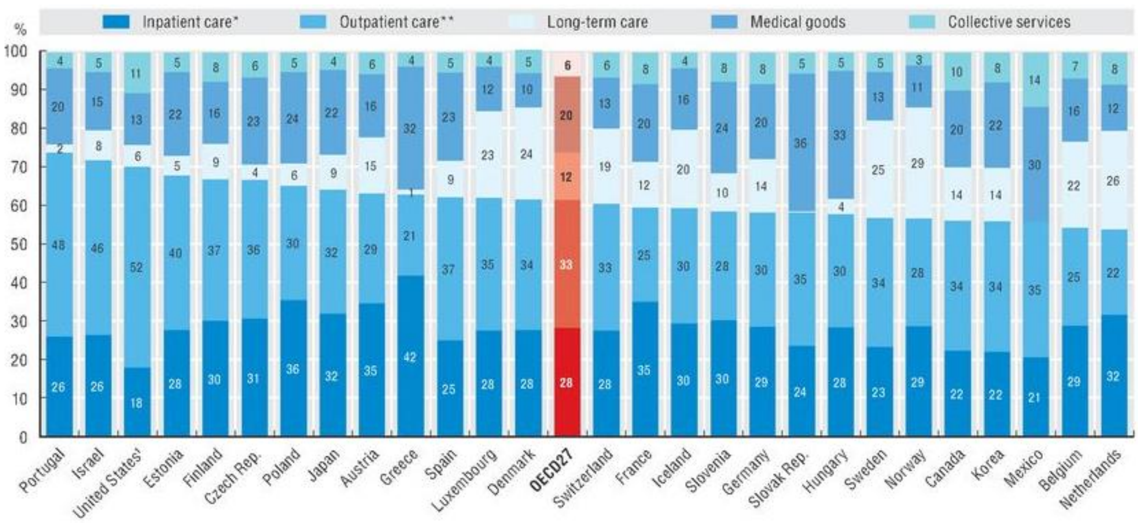


Figure 1.2: Current health expenditure by function of health care, 2013. Source: OECD, European Observatory in Health Systems and Policies 2015

The shortage of financial resources leads to the imperativeness of the implementing good management practices. Gutierrez and Vidal (2013) conduct a detailed review of the logistical problems in the field of Home Health Care (HHC), one of which is finding solutions for HHC scheduling and routing, the area in which this master thesis is focused. Solution procedures for this problem differ substantially between countries, due to the variations in terms of national and regulatory settings. In Fikar and Hirsch (2017), a review on HHC routing and scheduling models is performed, revealing that the most frequent approach to address this problem is through extensions of the Vehicle Routing Problem.

The main purpose of this master thesis is to propose a general mathematical model capable of aiding an HHC provider regarding the routing and scheduling of their domiciliary services. In order to accomplish that, a partnership with a solidarity institution was established, so that real-life instances can be obtained to test the model. The institution at stake is APOIO- *instituição de solidariedade social*, located in Oeiras, Portugal. In the context of VRPs, some particularities can be associated to the HHC, for example, the fact that the customers (patients in the case of HHC) may need to be visited not only several times in a week, but also more than once per day. Other characteristic example is that in some visits more than one person is required to provide the service. In addition, there is a preference to ensure the continuity of care, which means that within a day and throughout the week, the caregiver/nurse that provides the service to a patient should be maintained the same. All the details inherent to HHC routing and scheduling greatly increase the complexity of a mathematical formulation destined to solve it.

1.2 Objectives and Methodology

The objective of this master thesis is to propose and test a mathematical model and a solution method capable of generating solutions for HHC routing and scheduling problems. These solutions must be characterized by the sequence of the visits to the patients, as well as the time at which the visit should start, completely defining the route each team of assistants should perform, every day of the week. In addition, the solutions should respect the constraints originated by both the patient requests and the institutional operational preferences.

The problem is modelled as an extension of a MPVRPTW (Multi-Period Vehicle Routing Problem with Time Windows) and implemented in GAMS (Generic Algebraic Modelling System) as a MILP (Mixed Integer Linear Problem) program solved with CPLEX, an optimization software package.

The proposed MILP addresses real world constraints such as lunch breaks, patient visit frequency, and loyalty between caregiver and patient within a day and within a week. Two objective functions are considered separately: 1) minimizing travelling time, and 2) workload balance among teams. The institution’s objective is the minimization of the travelling time nonetheless, since the model is intended to serve as a decision support tool, it benefits from possessing other features to be optimized. With different optimization perspectives, a more informed and conscious decision can be assured.

Nonetheless, since the modeled problem is NP-Hard and the instances are quite large (9 assistants to be allocated to 36 patients in 7 different days), the computational capacity is not enough to run the general model. As such, a heuristic solution methods designed, so that the model can be solved in a reasonable time.

1.3 Thesis Outline

The present dissertation was structured into seven chapters, which are briefly described subsequently.

The first chapter contextualizes the problem by providing a motivation and a general description of the objectives to be attained in addition to the methodology used.

In the second chapter, the case study is presented, exploring the context and operations regulations of HHC services in APOIO. It contains information on the HHC services provided, the classification and description of its patients types, the characterization of the workforce and the definition of the operational procedures that the institution desires to be respected. The chapter is concluded by resuming the main features of the problem.

The third chapter is titled literature review and was developed aiming to comprehend how problems in a similar context are modeled and the methodologies' main components. The analysis of other works permitted the elaboration of suggestions on some other general factors that could be encompassed in the model, such as waiting times and overtimes, of which the impact in the outcome could be of interest for the partner institution.

The fourth and fifth chapters are strongly related. While the fourth presents the general MILP model namely, the constraints, objective functions, variables and parameters, the fifth describes the heuristic solution methodology used to solve it .

The sixth chapter comprises the presentation of the results for the real-case study, developing a comprehensive analysis encompassing the functionality of the constraints and the objective functions.

The last chapter incorporates the systematization of all the pros and cons of this model, in addition to the generation of suggestions for further work.

2. Case Study

2.1 APOIO: The partner institution

The development of this work and its validation is based on a real world case study. In an attempt to create a partnership, we contacted some institutions and one of which, APOIO, was the most receptive to the idea of participating in this project.

APOIO is a Private Institution of Social Solidarity (PISS), which means that it was constituted by private initiative, with a non-profitable nature, with the purpose of providing solidarity services to those in need. In this institution’s case, the services provided fall under four types of classification: elder day care center, home health care, social canteen and baby nursery/infant day care center. Our focus is the home health care services.

2.2 Home health care in APOIO

The institution’s headquarters are localized in Outurela, in Oeiras municipality, one of the areas in Portugal with an ever-increasing percentage of elderly population (Pimentel, 2013). The home health care services provided by APOIO encompass essentially three types of activities, the most relevant of these being the visits to patients’ homes for service provision, occupying most of the assistant’s working time. In addition to this service, lunch distribution also demands labor time from the HHC assistants on a daily basis, requiring them to distribute lunch to patients in their homes. The final and least frequent service provided by the HHC department of APOIO is patient transportation, from their home to the day care center and vice-versa, usually solicited by patients no longer capable of walking on their own. Assistants specifically allocated to the HHC department perform the services mentioned. They display homogeneous skills, allowing them to perform any HHC service.

The problem arises when there is a need to allocate teams of assistants to patients spread across a wide area and those patients have preferences regarding the time at which their visit should start. With the aim of understanding how to approach the problem, detailed information describing the nature of the services, the workforce, the patient typologies and the operational preferences of the institution is needed and, therefore, is explored in the following subsections.

2.2.1 Services in HHC

In APOIO, the range of services provided is remarkably wide. The nature of the services extends from personal hygiene (PH) and comfort to habitational hygiene maintenance, including clothes washing, alimentation, health care services and exterior accompaniment of patients as well. Within the nature of each service there are several sub-types of services that might be required by the patients, with some examples presented in Table 2.1. A certain degree of flexibility is associated to the features of the service type provided, with the possibility of negotiating with the administration of APOIO for their addition or adaptation.

Table 1.2: Examples of types of services available to the patients.

PH and comfort	Habitational hygiene maintenance	Clothes washing	Alimentation	Health care services	Exterior escort of patients
Complete PH PH Maintenance Diaper Maintenance Head washing Body hydration Dressing Transferring	Dusting Floor mopping Vacuuming Taking out the trash Making the bed Exchange of bed sheets	Collection Delivery	Distribution Accompaniment	Patient escort to the doctor's office Medicine administration	Shopping Payments Others

The frequency with which a service is required by a patient is yet another important aspect characterizing the services provided. A patient can require services of several different natures and of whichever type, moreover, these are required with the frequency that suits the patient's needs. In general, the frequency varies from once a week to four times per day and the duration of the same service provided to different patients may be distinct, due to the specific needs associated to each patient. A deeper characterization of the patients is subsequently developed.

2.2.2 The patients

Generally, the patients are elderly people with some degree of dependency. Among them, the care needs vary greatly. On one side, there are patients who need help only with washing their clothes, demanding one visit per week, and on the other side there are bedridden patients, who often cannot move without aid, and are sometimes entitled to as much as four visits per day. The patients visited more frequently also display more limitations, specifically presenting restrictions at the mobility level. After understanding both the limitations and service requisition patterns of the patient population, it is clear that they are classifiable into two types: autonomous and bedridden. Autonomous patients can perform most of the activities of daily living by themselves, requiring only sporadic care, whereas bedridden patients are completely dependent on assistance, necessitating help for activities as simple as bathing, dressing and feeding, and therefore demanding more visits. In Figure 2.1, the locations of the patients' homes are identified, discriminating between autonomous and bedridden patients. The image also exhibits the location of the day care center. In the moment of data collection, there were 36 patients requiring the HHC services, of which 10 were bedridden and 26 were autonomous.

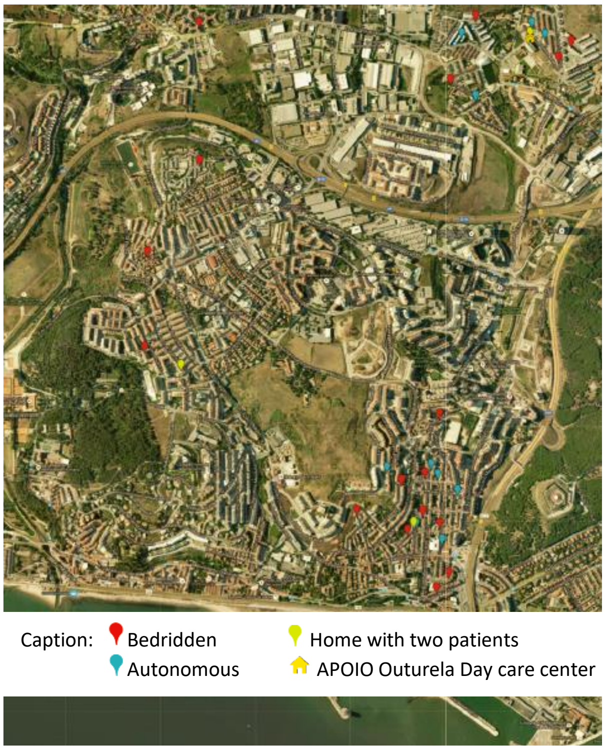


Figure 2.1- Geographical distribution of patient homes and location of APOIO day care center.

Throughout the week, elderly people value having a tendentiously fixed routine. Intending to stick to that routine as much as possible, the start of visit to the patients respects a fixed time interval, called a time window (TW). These time intervals may be larger or smaller depending both on the availability of the patient and on the characteristics of the service, for example, administration of medicine that must occur strictly at a specific time. The introduction of TWs considerably restrain the problem's feasible solutions and, in addition to the patient typology, influence the allocation of teams of assistants to patients. These assistant teams are further described in the following subsection dedicated to the constitution of the workforce.

2.2.3 Workforce

In APOIO there are assistants permanently allocated to certain departments. In this case, a homogeneous workforce constituted by nine assistants secures the delivery of HHC services. However, depending on the nature and type of the service performed, two assistants may be required instead of one. Such a fact leads to the allocation of assistants into two types of teams, differing in the number of assistants constituting them, which can be of either one or two. Usually, bedridden patients require the most demanding services, which is the main reason for arranging the assistants into teams of two. Despite APOIO's willingness to allocate teams of two assistants to bedridden patients (B-Teams) and teams of one assistant to autonomous ones (A-Teams), it is a preference the is still not currently enforced and assuring it is one of the institutions objectives.

Regarding the assistant's scheduling, there is also the question about how each of the assistants is assigned to the teams and how their rotation is processed amongst schedules. APOIO's schedules have a time horizon of one week, a period after which the assistants in a team change. Since the assistants' qualifications are homogeneous, there is no concern over which assistant is assigned to each team in that regard. However, in teams of two assistants, in order to improve the transmission of patient's health status knowledge between teams, it would be beneficial if one of the assistants was maintained in the same team from one week to the following. Assuring such conditions would significantly improve the continuity of care, thus enhancing the quality of the care provided. In addition to these operational preferences regarding the continuity of care, several other institutions' operational preferences are determinant for both the scheduling and routing of these teams. APOIO's operational procedures are consequently detailed in the next subsection.

2.2.4 APOIO'S Operational procedures

The administration of the partner institution has decided to implement some operational rules in order to plan the activities it develops. Some of these rules have a direct effect on the routing and scheduling of the teams of assistants and, therefore, this subsection serves the purpose of describing them.

In terms of operating period during the weekdays, the day care opens at 8 a.m. and ceases its activities at 8 p.m., corresponding to a total operating period of 12 hours. On weekends and national holidays, the working period begins at 8 a.m. and ends at 1 p.m., a reduction in the operating period to 5 hours, assuring that the institution still operates in every day of the week. Each HHC assistant is required to work, at most, a mean of 37 hours per week over a period of 1,5 months and are entitled to two days off per week. The assistants should always have the lunch break at the day care center facilities, with a duration of one hour, which must begin within the time interval between 12 p.m. and 2 p.m., a flexibility useful when adjusting the schedule of a team to the patients' requests. All assistants must arrive to the Outurela center before starting their route and, after carrying out the assigned route, they should go back to the departing site.

Lunch distribution, as previously mentioned, is one of the services provided by APOIO. The distribution of meals should begin at 12 p.m. and has a duration of 90 minutes. It requires the presence at the day care center of 3 assistants. After performing this task, the assistants should return to the day care center.

One of the most demanding activities for the assistants is traveling. The most commonly used means of transportation is walking, however, when the walking distances surpass a 15 minutes' walk, there are motorized vehicles available to transport the teams to the patients' location. In total, APOIO possesses four motorized vehicles, one of which is customized for meals distribution and, as such, is allocated to the lunch distribution service. The vehicles can be allocated for activities such as teams' transportation to the patient's address, grocery shopping and patient transportation to the day care center.

2.3 The challenge

Judging by the APOIO's operational characteristics previously described, designing routes is an activity of transcendent complexity. Furthermore, this activity is presently developed by the social assistant in charge manually, resulting in the generation of routes which are inefficient and that do not respect all of the institution's preferences for operating procedures. For example, currently, the solution deployed does not respect the preference for assigning teams of two assistants to bedridden patients. Besides the disregard for some of the operating procedures, there is also a long waiting list of patients hoping to enroll and to start receiving home services, which continues to grow.

Both the inability to respect the preferred operational procedures and the excessive waiting list for enrolling in the institution's HHC services could be solved through the optimization of routes' design and scheduling. Therefore, the main objective of this work is to propose a mathematical model to minimize the weekly aggregated traveling time, while respecting all of the preferences regarding operational procedures. In a posterior stage, the model purpose is meant to be incorporated into a user friendly-user software, enabling solidarity institutions to freely use it as a tool to optimize their routes.

3. Literature Review

3.1 The changes in Long-Term Care: from informal care to formal care

A tendency towards population ageing is taking place in almost all countries. The demographic trend is defined as an increase in the share of older people in the society, and is caused by both the increase in life expectancy and, in developed countries, mainly by a decrease in the fertility rate. The number of elderly people is expected to more than double by 2050, varying from 841 million people in 2013 to approximately 2 billion. The increase in life expectancy, however, enhances the probability of the incidence and prevalence of morbidities, more frequently non-communicable diseases such as memory loss, urinary incontinence, depression and falls or immobility, also known as the *"four giants of geriatrics"* (UN, 2013). Most of these morbidities will accompany the elderly for the rest of their lives, deteriorating their capacity to perform both activities of daily living (ADLs), such as eating, dressing or bathing and instrumental activities of daily living (IADLs), for example, household chores, meal preparation, or shopping. (Genet et al 2012)The loss of independence brought by these disabilities generates a need for care services aimed at mitigating them, called long-term care.

The WHO (2000) defines long-term care as *"the system of activities undertaken by informal caregivers (family, friends and/or neighbors) and/or professionals (health and social services) to ensure that a person who is not fully capable of self-care can maintain the highest possible quality of life, according to his or her individual preferences, with the greatest possible degree of independence, autonomy, participation, personal fulfilment and human dignity".* Therefore, two types of systems to provide long-term are distinguished: the formal and the informal. In most cases, informal care is the most cost-effective solution in what concerns long-terms health, though it depends on the type of dependency. However, the accelerated life style, conditioning the capacity of the natural informal caregivers to provide care, enhancing the pressure on the formal system.

Bonsang (2008) studied the two types of formal care closer to informal care, probably the ones for which, in the future, demand will increase the most, namely, paid domestic help and nursing care. Paid domestic help is defined as the professional or paid home help for performing household chores such as doing cleaning the house or shopping for groceries. Nursing care consists of the medical or personal care provided by professional nurses. Three main conclusions are yield by the study. The first is a re-confirmation of the existence of a substitution relationship between informal and formal home care. Then, the previous re-confirmation is extended, finding relationships between the informal care and the two types of formal care. Informal care is found to decrease paid domestic help, demanding low-skilled personnel, while it complements nursing care, which requires a higher degree of skill level. The last conclusion shows that the substitution effect of the first conclusion disappears when elderly suffering from more severe disabilities are concerned. The two types of formal care previously mentioned fall under the classification of home health care services, since they are care services provided at the patient's home.

3.2 Home Health Care (HHC)

Integrating both the personal and medical care needs a patient may require, home health care has as main objective the suppression of a patient’s needs at home, increasing their quality of life. According to Bricon-Souf et al. (2005), five principal components constitute a home health care system, namely, (1) the patient, (2) the person responsible for engaging the services of the HHC institution, (3) the people involved in the logistic implementation (coordinator for the evaluation the patient’s needs) and (4) the health care team (nurses, practitioner), and (5) the team responsible for the patient’s well-being (caregivers, family).

When a patient enrolls in an institution, a wide variety of services becomes available, for example, cleaning, laundry assistance, preparing food, and even more personal needs such as getting out of bed, bathing, dressing, and dosing medicine. After being admitted, the patient is informed about the long-term plan of visits, which states approximately the time at which they should expect arrival of a caregiver professional. Then, after the agreement on the visit times, a route schedule has to be designed for the teams of caregivers, incorporating several of the patients (Ramussen et al 2012). This is defined in Bricon-Souf et al. (2002) as the logistic process, the first of the two processes he associates to home care, developed by the administrative staff and with the aim of efficiently allocating resources. For example, a nurse, a human resource with a high skill level, should not be assigned to a patient simply requiring household chores. Afterwards, the teams providing care are dispatched to perform the visits, initiating the health care process. This process regards the provision of care by actors to the patient at his home, in addition to the monitoring and control of the progression of his health status.

Through the previously described process of HHC, it is straightforward to understand these services as a delivery network with several actors. The logistic management of HHC networks presents complex situations in the context of decision making, which, according to Vidal et al (2013), are categorized into three dimensions. The first deals with the duration and influence of the planning decisions through the planning horizon. In turn, the second dimension disseminates the main logistics functions through four groups of management decisions: network design, transportation management, staff management and inventory management. Finally, the third dimension describes the services processes at the medical, patient and support services levels. The increase in the demand predicted for HHC services due to population ageing, in parallel with budget restrictions, will necessarily require the creation of optimization tools to support the decision-makers. The focus of this literature review is placed upon the second dimension, particularly in transportation management.

3.2.1 Optimizing HHC routing and scheduling: A variety of objectives and constraints

The characteristics of long-term care systems and, consequently, HHC services is remarkably distinct between countries, especially due to regulation and even to the nature of the system. In some countries, the provision of these services is a responsibility of the social security, in others it is accountable to the municipalities or even to the health care system, also being possible a mixture of the responsibilities of the institutions. As a result, the optimization of the HHC network will emphasize different objectives and constraints. Over the last few decades, various researchers have published mathematical models, which differ at several levels, explicitly in what concerns the objectives, the constraints and the solution methods used.

In Duque et. al. (2015) and Fikar and Hirsch (2017) some of the characteristics associated to routing and scheduling mathematical models are reviewed and their characteristics clustered. Regarding the optimization's objectives, the most widely considered are:

- **Travel time:** Minimizing the time spent by caregivers travelling between patient's homes;
- **Travel cost:** Minimizing the cost associated to the movements between patient's residences;
- **Traveling distance:** Minimizing the distance travelled by caregivers;
- **Waiting Times:** Minimizing waiting times, which corresponds to the time when a caregiver cannot perform the service due to arriving to the patient's residence before the agreed time of visit;
- **Overtime:** Minimization of the overtime a caregiver must work in order to perform all of the visits to her assigned;
- **Preferences:** Maximize the convenience for the patient in terms of, for example, caregiver preference or visit time. Preferences are always simultaneously optimized with other objective;
- **Number of caregivers:** Minimize the number of caregivers needed to perform all the visits required;
- **Constraint violations:** Aims at minimizing soft-constraint violations, and it is optimized together with other main objective;
- **Workload Balance:** Intents to balance the workload amongst caregivers**;**
- **Number of visits:** To maximize the total number of visits performed;
- **Continuity of care:** Assuring that the patient is assigned to the same caregiver.

The main focus of the studies on routing and scheduling optimization is usually placed on the travelling issue, however, instead of the traditional focus on minimizing the total distance, in an HHC network the caregivers' working times are often considered as the main cost factor, allowing the inclusion of both overtime and waiting times in the optimization (Fikar and Hirsch, 2017). The formulation is used to integrate some complexity inherent to this setting. Nonetheless, the terms accounted for in an optimization can vary considerably from studies that optimize a single term to models that consider up to thirteen terms. For example, the model presented by Akjiratikarl et al.

(2007), which uses a swarm particle optimization (SPO) metaheuristic to minimize the total distance traveled whereas Himermann et al. (2015) purposes a metaheuristic for solving a multimodal home health care scheduling problem in which the objective function is formed by 13 terms.

Summarizing all the objectives, it is possible to highlight two main HHC characteristics usually subjected to optimization and that are simultaneously considered in the models. The first characteristic is related with the total distance traveled or the routing costs of the nurses, in which are accounted, to some extent, the resources consumed on the travel. It can encompass terms for overtime costs, or caregivers/client preferences such as adding a penalty for deviations from preferred visit times or for assigning less suitable caregiver to a patient. The second characteristic contains the consideration of preferences of caregivers/patients or service consistency, i.e. attempting to keep the number of different caregivers per patient low. The former two characteristics associated to the optimization objectives are described in Braekers et al. (2016). Nonetheless, he also distinguishes two other HHC characteristics associated with constraints.

The factors that restrict a HHC routing problem are of many different natures. Bertels and Fahler (2006) classify those factors into six categories, namely, preferences of patients, preferences of nurses, the legal aspects, qualifications/experience, ergonomics and a last category named "others" for any other factor that is not accounted in the previous categories. To conveniently

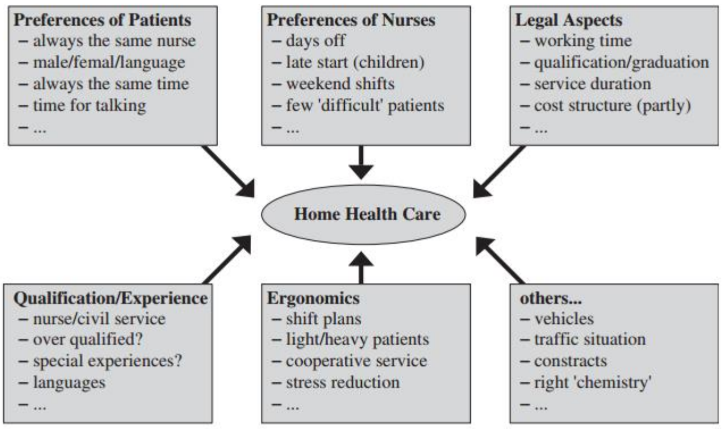


Figure 3.1: Categories of factors contributing to the restriction of HHC problems and examples of factors (Bertels and Fahle,2006)

illustrate the factors present in each category, in Figure 3.1 a scheme with some examples of factors depicting their category are displayed. According to Fikar and Hirsch (2017), the most common constraints modeled are the time windows to visit the patients, followed by the patients' skills requirements and the working time regulations. Then, less works consider caregivers' breaks and synchronization of resources and, lastly, fewer study precedence and uncertainty associated to HHC problems.

3.3 Vehicle Routing Problem

In the context of transportation, scheduling, distribution and logistics, particularly in Supply Chain Management (SCM), one of the most relevant challenges faced by managers concerns the definition of a strategy to optimize the delivery of products by suppliers to clients, while respecting some constraints (Surekha e Sumathi, 2011). The previously described problem is generally recognized as a Vehicle Routing Problem (VRP). Despite the easiness in describing it, due to the considerable variability amongst the conditions from one setting to the other, the objectives and constraints encountered in practice are remarkably diverse, generating a wide variety of constraints. Laporte (2007) states the diversity of the constraints as the reason for the unavailability a single universally accepted definition of the VRP.

Nowadays, several types of VRP have been introduced, originated from real-life needs. Nonetheless, its solution aims at designing optimal delivery or collection routes departing from a central depot (or more) to a set of geographically scattered clients, bounded by several restrictions, such as vehicle capacity, time windows, route length, precedence relations amongst clients, etc. A schematic representation can be found in Figure 3.2.

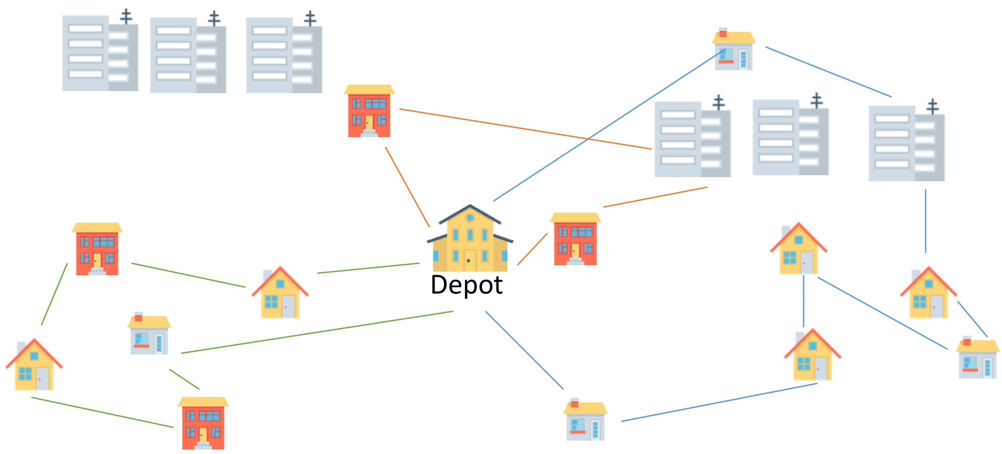


Figure 3.2: Schematic representation of the routes elaborated by solving a VRP.

Determining the optimal solution to VRP is NP-hard, conditioning its solvability, through the use of mathematical programming. Consequently, resorting to heuristics due to the size and frequency of real world VRPs becomes essential.

In early literature, VRPs were first introduced in 1954 by Dantzig et al., with the Travelling Salesman Problem (TSP). Since then, it has become one of the most well-known and studied optimization problems. Its main objective is to find the shortest route to visit all *n* locations. More generally, the model is given as input a symmetric *n x n* matrix with the distances between all nodes $D=(d_{IJ})$, then nodes must be arranged in a cyclical order, in such a way that the sum of all the distances between them is minimal. However, this formulation does not answer most of the real world requirements that nowadays industries are facing. As a consequence, over the last decades, new formulations were created, attending to the problem's characteristic being addressed, and diverging into different classes of VRP.

3.3.1 Types of VRP

The different operational contexts surrounding companies create a wide variety of optimization preferences that culminated in the diversification of the mathematical formulations. Throughout literature, it is possible to identify some of the VRP types most commonly applied in real-life. Five of the most relevant are:

- Capacitated Vehicle Routing Problem (CVRP);
- Vehicle Routing Problem with Time Windows (VRPTW);
- Multi-Depot Vehicle Routing Problem (MDVRP);
- Open Vehicle Routing Problem (OVRP);
- Periodic Vehicle Routing Problem (PVRP).

3.3.1.1 Capacitated Vehicle Routing Problem

In CVRP typical problems, each client has a known demand for a product, the fleet of vehicles to perform the delivery routes is homogeneous with a fixed capacity and each route must depart and arrive to the same depot. (Pillac et. al 2013). This is the equivalent to several TSP problems, as many as the number of vehicles in the fleet. An additional constraint must be added stating that, for each cyclic route, the sum of the demands must be lower or equal to the capacity of the vehicles. The objective is to minimize the total distance associated to all routes.

3.3.1.2 Vehicle Routing Problem with Time Windows

Xu et. al (2015) states that the VRPTW continues to be one of the most difficult problems in combinatorial optimization, still presenting a challenge. Similarly, to most VRPs, a homogeneous fleet is meant to deliver goods to clients demanding them. The particularity resides in the fact the deliveries are performed within a pre-specified time window.

The objective is to design routes that allow the available vehicles to serve all clients while minimizing the cost, respecting the constraints associated to the capacity, travel time of the vehicles and time windows of the clients. To date, there are no consistent exact algorithms for optimally solving VRPTW. Nonetheless, heuristic methods have been elaborated to solve these problems in a nearly optimal manner (Tan et al, 2001).

A central theme in VRPTW are the Time Window (TW) constraints. The strategies found in literature for modelling them vary between the implementation of hard or soft constraints. The most commonly employed are the hard constraints, which assures that the delivery is initiated within the time interval defined by the TW. Violations of the constraint are not permitted, which means that vehicles are allowed to wait with no cost if they arrive early and they are prohibited to serve if they arrive late. On the other hand, soft constraints can be violated at the cost of a penalty added to the objective function (Taş, et al., 2014).

Nonetheless, several studies involving soft constraints have been developed and once again due to the wide diversity of contexts, the definition of a soft constraint may vary slightly. For example, in gas stations the delivery of fuel may not respect both lower and upper bounds, with the delivery starting either before or after the time interval defined by the TW. The time difference between the upper and lower times of the TW are then converted into a penalization. When these TW are considered, the VRPTW type can be subdivided into VRPSTW (soft TW constraints) and VRPHTW (hard TW constraints). (Figliozzi, 2008)

3.3.1.3 Open Vehicle Routing Problem

In Open Vehicle Routing Problem, and similar to the CVRP, the demand and geographical location of each client are known and the vehicles possess a finite capacity, being only differentiated by not requiring the final node of the route to be the same as the departing one: as soon as the final client has been visited, the route is allowed to end. The main objective is to minimize both the number of vehicles and for the necessary vehicles, to minimize the total distance (or time) travelled. This indicates an additional vehicle should be associated to a cost larger than any savings that may be accomplished by the reduction of the total travel time (Fleszar et. al, 2009). The OVRP are commonly observed in situations in which subcontractors do deliveries resorting to their own vehicles to perform the task. Once all the delivery locations in the route have been visited they do not return to the depot.

3.3.1.4 Multi-Depot Vehicle Routing Problem

In some practical situations, single-depot VRP are not suitable for modeling the problem, hence creating the conditions for researchers to develop the multi-depot VRPs. As in all the previous types of VRPs, the client's geographic position and the demands are known beforehand, with homogeneous finite capacity vehicles, however there is more than one node serving as the depot. Each vehicle must start and end at the same depot. In accordance to Ho et. al (2008), these problems are usually solved through a process of hierarchical decision, composed of three steps. The first stage is grouping, in which clients are assigned to depots, followed by the routing of the customers, the stage responsible for allocating clients to vehicles. The final stage, concerning the scheduling, creates the sequence by which the clients will be visited by the vehicle (Surekha and Sumathi, 2011). Generally, the main objective of the MDVRP is to reduce the total distance of the routes, in addition to the number of routes/vehicles needed.

3.3.1.5 Periodic Vehicle Routing Problem

In the periodic vehicle routing problem, in addition to the typical client's characteristics, they are also associated with a set of dates in which the client may be served. The vehicles available leave the depot and are restricted by the shift time and vehicle capacity, at the end of which the driver must return to the depot. The main objective is the minimization of the distance travelled during the time horizon. The solution of the problem resides in the assignment of a visiting schedule to each of the and, for day of the time horizon, design the rotes including the customers scheduled to be visited that day (Angelelli and Speranza, 2002).

3.3.2 Stochastic and Dynamic VRP

The technological development, especially associated to information and communication, has dramatically increase the extend of data available. Telematics, such as positioning services and mobile communication facilitates the collection of real-life information and the exact monitoring of vehicles, creating the conditions for real-time decision support in vehicle routing (Ritzinger et al, 2012).

Dynamic Vehicle Routing Problems (DVRP) enables the inclusion of information in real-time, because it acknowledges that not all the information about the instances is known from the beginning. Some of the dynamic events that might occur is the arrival of new orders. Nonetheless, these events may also be related with service times and travel times. Pillac et. al (2013) state that in comparison with static problem, a DVRP possesses more complexity, due to the increase in the number of degrees of freedom, and hamper the consensual definition of an objective function.

In turn, Stochastic Vehicle Routing Problems approach uncertainty as a fundamental characteristic of real-life instances. Usually, there is information about events within historical data, which can be converted and used in mathematical models. The stochastic VRP (SVRP) can be seen as any VRP presenting one or more stochastic parameters, which means that future events are random variables characterized by a known probability distribution (Ritzinger et al, 2012).

3.4 Solving Routing Problems

VRPs are NP-hard problems, which become exponentially more complex to solve with the size of the problem. For this reason, essentially two types of solution methods appear in the literature: exact algorithms and heuristics. The two classes will be presented subsequently.

3.4.1 Exact Algorithms

Several families of exact algorithms have been proposed for the VRP, being based on integer linear programming (ILP), dynamic programming, and branch-and-bound. However, only three have been proven to be a workable methodology, namely, the three families of ILP based branch-and-cut algorithms. Despite yielding the most optimal results, they all require a significant mathematical programming machinery and their ability to solve real-life size instances is limited Laporte (2007).

3.4.2 Heuristics

In Laporte (2007) two major classes of heuristics are considered: the classical heuristics and the metaheuristics. The term "classical" is associated with methods that, in each step, always proceed from a current solution to a better one in its neighborhood, with the condition of stopping being the impossibility of further improving in a near solution. Classical heuristics can naturally be divided into constructive heuristics and improvement heuristics. The other class of heuristics is known as metaheuristics which, in contrast with the first class, permits the consideration of non-improving or even infeasible intermediate solutions. The importance of these latter heuristics has been growing for about two and a half decades.

3.4.2.1 Constructive Heuristics

A constructive heuristic can be seen as a solution method which begins with an empty solution and that iteratively extends the current solution until a feasible solution to the problem is obtained.

The most popular constructive heuristic was introduced by Clarke and Wright (1964) and is called Savings Algorithm. In Laporte (2007) the parallel methodis described. The heuristic is initiated with the construction of n routes which leave the depot, serve a client and come back to the depot $(0, i, 0)$ $(i = 1, . . . , n)$, which are all feasible. A general iteration of the algorithm comprises the merging of a route ending at i with another route starting at j, resulting in the feasible route $(0,i,j,0)$, providing a saving of sij = ci0 +c0j –cij (with cij the cost of travelling from node i to node j). The merge with the largest saving is selected at each iteration. The solution is found when no profitable and feasible merges are possible.

Other generally known heuristic is the Sweep Algorithm, popularized by Gillet and Miller (1974). It must be applied solely to planar instances of VRPs. Clusters of nodes are obtained by rotating a ray centered at the depot. A route is then designed for each set of nodes by solving a TSP (Laporte et al, 2000).

3.4.2.2 Improvement Heuristics

Improvement algorithms applied to obtain VRP solutions are essentially of two types. Intra-route methods optimize each solution's route separately by means of a TSP improvement heuristic, whereas Inter-route heuristics are based on moving vertices from one to another route. However, the performance of classical improvement heuristics is good but not excellent, being more frequently as building blocks within metaheuristics. (Laporte, 2007)

Thompson and Psaraftis (1993) describe a cyclic transfer algorithm in which a circular permutation of routes is considered, and a fixed number of vertices per route is shifted to the next route of the cyclic permutation.

3.4.2.3 Metaheuristics

Metaheuristics development in the field of VRPs has been particularly significant over the last years. These heuristics explore solutions beyond the immediate neighboring solutions to avoid converging to the first local optimum encountered. The procedures frequently incorporate some classical form of heuristics. Most of the schemes that have been put forward can be classified into three categories: 1) local search, 2) population search, and 3) learning mechanisms (Laporte, 2007).

In local search the heuristic begins with an initial solution s_0, not necessarily feasible, and moves at each iteration *t* from solution *s*t of value *f(st)* to another solution located in the neighborhood *N(st)* of *st* . The heuristic ends when the best known solution $s*$ has been identified, usually after a stopping criterion has been reached (for example, a predetermined number of iterations, or a number of consecutive iterations without changing the currently best solution). One example of these metaheuristics is the Tabu Search introduced by Glover (1986), being one of the most used metaheuristics.

The second category, named population search, essentially relies in genetic algorithms or derivatives. These algorithms generate optimal/sub-optimal solutions by resorting to natural selection-inspired operators such as mutation, crossover and selection. In the VRPs case, the crossover, for example, takes two routes (the parents) from the solution's population, combining them into one or two new routes (the offspring solutions). Many authors first apply Tabu Search algorithms and store the best solutions, which are then recombined through genetic algorithms in an attempt to produce the optimal solution (Laporte, 2007). A simple and flexible genetic algorithm is proposed by Prins (2004), which was successfully applied to standard benchmark instances ranging from 50 to 483 clients.

Finally, concerning learning mechanism, a limited number of heuristics have been proposed for the VRP. No neural networks based algorithms have been proved to be satisfactory, and the early ant colony based heuristics were far from performing as well as the best available approaches (Cordeau et al.,2007). Nevertheless, Reimann et al. (2004) have proposed a well-performing heuristics called D-ants (Decomposing-Ants), in which a population of artificial agents is continuously creating solutions to the problem resorting to a joint population memory and some heuristic information. After the elaboration of the following solution by each member of the population, the memory is updated with a preference being attributed to better solutions.

Gradually, the memory will enhance its influence on the solutions constructed by the agents and the solutions will evolve until optimality is reached. Recent work is being developed in this area. Zhang et. al (2014) propose a model integrating both an Ant Colony and a Tabu Search approach for solving time dependent VRP.

3.5 VRP in Home Health Care

The economic strains placed on HHC providers, for both private and public institutions, associated to the increasing elder population, have resulted in an enhancement of the interest in optimizing several aspects, in order to reduce expenditure while maintaining service level. One of the types of problems that have been given particular emphasis are routing and scheduling problems.

Generally, the problem consists in trying to assign a set of heterogeneous patients, who are spread across the geographical area of operations, to caregivers. They can place several service requirements with features that have an impact in way caregivers are assigned. The nature of those features may rely, for example, on TW preferences or specific nurses' skill. The assignment of caregivers is also considerably affected by the regulatory contexts which meaningfully vary amongst countries. For this reason, the formulations of VRPs applied to HHC vary considerably regarding constrains, objectives to optimized and the solution method used (Fikar and Hirch, 2017). Taking this into account, the literature will be divided into classes that state the most relevant feature being modeled.

Work and break regulations

In Trautsamwieser et. al (2011) a model formulation and a solution procedure are proposed. The objective is to minimize caregivers' travelling time and the dissatisfaction level of both clients and caregivers. The traveling times considered are the sum of the driving times and waiting times. Regarding constraints, the authors consider suitable allocations of caregivers to clients, working time regulations, hard TW and mandatory breaks. The model is shown to run for small instances, of 20 clients and 4 caregivers, but for real-life instances, a metaheuristic was designed based on the Variable Neighborhood Search algorithm. It allows multiple shifts, and breaks can only take place after a pre-established duration is exceeded.

Multi-Objective

A bi-objective model is presented in Braekers et al. (2016) in order to find a set of Pareto optimal solutions. The authors take into consideration two conflicting objectives: minimizing total costs and minimizing client inconvenience. The first consists of the sum of both travel costs and overtime costs, while the second depends on patient preferences in terms of caregiver skill level and visit schedules. To study large instances, a metaheuristic is elaborated based in a large neighborhood search algorithm in the multi-directional local search framework. The output can be multiple solutions and decision makers can investigate trade-offs.

Duque et al. (2014) defines a home care routing and scheduling problem, with two objectives: the service level and the total distance travelled by all caregivers. This differs from Braekers et al. (2016) since it is a multi-period VRP, which means that solution regards several days, instead of just one day. The optimization of the objectives is implemented in a hierarchical order prioritizing service level, which differentiates this work from other approaches in the literature. Normally, an aggregated function of the quantities to optimize is considered. The mathematical formulation proposed is based on the set partitioning problem. The major advantage of hierarchical procedure appears to be related with the easiness with which it can be implemented in organizations, due to being quite straightforward when trade-off analysis is concerned.

Stochastics

The works in the area of stochastics place special relevance on the fact that not all of the data is known in advance. According to Lanzarone and Matta (2014), a particularly feature ignored in the literature is the variability of patients' demand. Therefore, the authors propose an exact model that deals with uncertainty in demands and service times. The solution of the assignment problem considers continuity of care while minimizing overtimes incurred by caregivers. In common practice, new patients are assigned to the caregiver with the highest expected available capacity, determined as the difference between her capacity and the actual expected workload. Both the generally used policy for patient assignment and the one proposed were compared in real-life cases and, specifically for groups of patients with high variability of visits requirements, such as for non-palliative patients, the purposed policy has been shown to reduce overtime and increase workload balance.

Dynamics

Bowers et. al (2014) uses instances from home post-natal care. A particularity of this type of care is the irrefutable evidence regarding the benefits from continuity of care. However, shift patterns and part-time working disrupt continuity of care considerably. The authors propose a modified Clarke-Wright heuristic, embedded in a Monte-Carlo simulation framework, in which mothers are sampled according to specified geographical distributions. The method allows the assessment of the increase in travel time as a consequence of the continuity of care policies.

Temporal precedence and synchronization

In Rasmussen et al (2012) the problem is formulated as a set partitioning problem with side constraints and a branch-and-price solution algorithm is developed. The relevance of this work lies on the consideration of five different types of precedence, all of which are modelled as generalized precedence constraints, enforced through the branching. Precedence is related with, for example, the necessity that a patient be visited after a first visit was performed, such is the case of putting the patient's clothes in the washing machine and two hours later a caregiver needs to visit the same patient again to hang the clothes. Another example happens when a patient must be lifted out of bed, a task that must be performed by two caregivers, a type of precedence known as synchronization. As such, the objective of the model is rather unusual, which in this case is to maximize the number of served tasks performed respecting the precedence constraints.

Nonetheless, these models focused on the provision of simultaneous services or temporal precedence induces an increase in the complexity of the models, substantially increasing the computational time required to achieve high quality solutions (Fikar and Hirch, 2017).

Works similar to the case study

An interesting work is developed in Gomes and Ramos (2016). The authors aim at developing a toll capable of supporting a social assistant in the job of route design and scheduling, for domiciliary care associated with a social parochial center. They propose a mathematical model that is an extension of a VRPTW. One of the particularities of this model relies on the ability to elaborate different routes and schedules for each day of the week, respecting the time intervals at which the domiciliary services may be started. The previous statement grants the model features similar to the PVRP, in which there is an initial distribution of the patients to the days, in accordance with the frequency required and the allowable combinations of visits days. However, since the input in not the frequency of visits but rather a more restrained and aperiodic plan of visits, the model becomes a multi-period vehicle routing problem with time windows (MPVRPTW), as stated Archetti et al, 2015. Regarding constraints, the model considers many of the ones found in the case study, namely meals' distribution, considers lunch breaks and, most relevantly, addresses the continuity of care throughout the week. The optimization may focus on two objective functions separately, the first is to minimize the weekly distance travelled, one of the optimization objectives found more often, and the second is workload balance, which tries to distribute as evenly as possible the workload amongst teams of assistants.

This model has been applied to a relatively small real-case study, in which 17 patients and 15 residences were considered. In order for the model to run in a reasonable amount of time, the authors developed a heuristic that permits the independent run of the model for each day of the week. The solution yield for the first objective function was capable of reducing the weekly traveled time by 23%. Nonetheless, when the workload of this first solution was under analysis, it was verified that the workloads had become quite unbalanced. After running the model for the second objective function (with the heuristic starting on Monday – the heuristic may generate different solution depending on the day with which it is initiated) there was a reduction on the weekly traveled time of 6% and the workload was visibly much closer to being equally distributed.

3.6 Conclusions

The literature review permitted the deepening in the context of HHC and, more precisely, in typical transportation management problems. As it has been shown, the complexity inherent to each HHC provider and its environment generates a characteristic particularities that originate the wide diversity of formulations. Real-life instances are frequently data intensive, requiring the development of heuristics in order to solve the model in a reasonable period of time, resulting in sub-optimal solutions. Nonetheless, those sub-optimal solutions are regularly accompanied by a certain percentage of optimization, reflecting the utility of the area under study.

However, not all of the formulations presented can serve as a foundation to addressing the problems presented by APOIO. For example, the OVRP in which the last node in the route is a client cannot model the operational procedure affirming that the routes must start and end at the Day care center. Apparently, the most suitable approach to model the institutions' situation is through an extension of the model proposed by Gomes and Ramos (2016). The reasons for supporting this model are its capacity for handling multi-period contexts, a characteristic of APOIO's services, in which the patients vary from one day to the other, in addition to the TW consideration, relevant for the patients. The most important characteristic, however, is the continuity of care throughout the week, a characteristic that is not frequently modelled (especially due to the fact that most HHC models are single-period) despite being a common characteristic amongst HHC providers.

4. Model for the Routing and Scheduling of a HHC problem

As stated throughout this work, the main goal of this thesis is to help social solidarity institutions improve the planning of domiciliary care services by developing a model capable of elaborating the schedules of each caregiver/team, identifying the patients to visit in each day, as well as the visit sequence. This problem can be modelled as an extension of a MPVRPTW and, therefore, the mathematical model proposed will be an extension of the work of Gomes and Ramos (2016). In this chapter, the modelling approach is firstly described, followed by the presentation of the structural components of the model, namely, the sets, the parameters and the decision variables. Finally, the mathematical formulation of the problem is revealed, discriminating all the constraints and objective functions.

4.1 Problem modelling

The objective of this model is to obtain a schedule for a team of assistants to visit patients. In order to do so, the patients' visits were modeled as nodes in a graph. Each patient is then defined by a node and the service paid to that patient is characterized by the duration of the visit and a time-window, i.e., a time interval in which the visit must start. This modeling approach arises three obvious questions. The first is concern with the fact that if the visits do not discriminate the type of service provided, how are the differences accounted for in terms of service nature? Secondly, how to deal with patients that need more than one visit per day? And, finally, how to assure the operational preferences imposed by APOIO, such as the lunch break and the lunch distribution, are respected?

4.1.1 Accounting for distinct service natures

Regarding the first question, since all assistants are able to perform any type of service, there is no need to differentiate between types of service. Instead, the visit to a patient is characterized exclusively by the time required to perform the service and the time window in which that service can be initiated. The type of service will simply imply distinct levels of resource consumption, specifically, the working time of a team. Therefore, a service is described by associating a parameter, specifically, the duration of the visit, in which its nature is taken in account. For example, to a node representing a diaper change would be associated a duration of 10 minutes while for a house keeping activity would be associated a period of 20 minutes.

4.1.2 Variable frequency of visits

Commonly, in HHC, each patient requires more than one visit per day, a particularly frequent situation for bedridden patients. The model is able to include those extra visits through the creation of replicas, which are nodes with the same location, but with different time windows and durations according to the service required.

4.1.3 Respecting institutional operational regulations

Concerning the operative regulations of the institution, three operational regulations are modeled as hard-constraints, namely, the teams' working hours, the teams' lunch time and the lunch distribution service that needs to be done by a number of teams. In addition, one operational regulation is modeled as a soft-constraint - the preference for loyalty between a team and a patient within a day of the week. The following subsections explain the modeling approach chosen to address each regulation.

4.1.3.1 Teams' Lunch Time

The institution required the lunch break to be started at the day care center. Moreover, the break starting time must take place within a time interval, defined by an early lunch time and a later lunch time, with a predefined duration. To model this feature, a replica of the day care center was created, representing an activity characterized by a TW and the duration of, for example, one hour. The hard-constraint added to the model imposes that every team visits this replica.

4.1.3.2 Lunch distribution

The distribution of lunch to patients who are not at the day care center requires a fixed number of K teams of assistants that must arrive at the day care center at a pre-defined time to start distributing the meals. This activity is expected to last a pre-established time, for example, 90 minutes. Similar to the previous regulation, the distribution is modeled by generating a new replica of the day care center. This time, however, the TW associated to the node is very short to assure the assistance arrive on time. Each day, two teams, one of two assistants and another of one assistant, are be required to visit this replica.

4.1.3.3 Working time and operating period

In APOIO, the daily operating period far exceeds the daily working time of a team. To prevent the assignment of teams to routes that surpass their daily working time limit, a constraint was imposed assuring that the time interval between a team leaving and returning to the day care center must be below H minutes (e.g. 8 hours). Notice that the lunch break is part of the route's duration.

4.1.3.4 Daily Loyalty Preference

Continuity of care is an important factor not only regarding the psychological health of the patient but also in what concerns the monitoring of a patient's health status evolution. In order to allow for continuity of care, the same team should be assigned to all visits to a patient. However, as the operating hours exceed the working hours of a team, there are cases in which the last visit of the day to a patient must be performed after the working period of the team that performed the visits prior to the last. Consequently, a hard-constraint could not be a solution and, therefore, a soft-constraint was added to the model, penalizing the objective function in the event of more than one team being assigned to the same patient in one day. Hence, it is assured that maintaining the same team associated to a patient is preferred, unless it is impossible as a result of the incompatibility between the last replica time-window and the team's working time limit.

4.2 Sets

The model integrates two types of entities: the nodes (V) and the teams (U). The teams are defined by a set $a \in U = \{1, \dots, u\}$, representing the u existing teams. In turn, the nodes set has several subsets associated to it. For the sake of clarity, given its complexity, the group of subsets is detailed in the next subsection.

4.2.1 Nodes classification

The classes of nodes that constitute the network are available in Table 4.1:

Table 4.1: Nodes set partitioning

V	Set of all the nodes in the network
V_R	Subset of real day care center, $i \in V_R \subset V$;
V_F	Subset of day care center replicas for returning, $i \in V_F \subset V$;
V_L	Subset of day care center replicas for lunch break, $i \in V_L \subset V$;
V_D	Subset of day care center replicas for lunch distribution, $i \in V_D \subset V$;
V_P	Subset of patients and all the patients replicas, $i \in V_P \subset V$;
R_i	Subset composed subsets each containing exclusively patient i and his/her replicas $R_i \subset V_P$.

Further clarifications are demanded in order to define such set partitioning. Firstly, V_R is composed by the nodes that serve as teams' departing point. All routes start in this node. On the other hand, V_F is the subset of nodes serving as a landmark for all teams arrive. Afterwards, there are the subsets resulting from the operational constraints previously mentioned, V_L contains the day care replica representing the lunch time of all teams. It is composed by a single node which has to be visited by all teams. The second subset is V_D, comprising one node which, in contrast with the previous subset, must only be visited by one team (it can be viewed as a fictitious patient). Then, there is a subset associated to the patients and their corresponding replicas, V_p. Finally, subset R is the union of all subsets containing the patient and his/her replicas. This subset allows the modelling of the daily-loyalty preference constraints.

4.2.2 Time

Time is modeled as a set T defined by all of the periods comprising the time horizon (e.g. the days in a week).

4.2.3 Arcs classification

The network created by the model includes two elements, the nodes and the connections between them, commonly known as arcs. The set encompassing the totality of the connections between the nodes is A. Each arc depends on three dimensions: the departing node $i \in V$, the arrival node,

$j \in V$ and the day in which the connection is established, $t \in T$. The arc set is also partitioned into classes, which can be acknowledged by examining Table 4.2.

Table 4.2: Arcs set partitioning

A_{RT}	Valid arcs from V_R to V_P or V_F, $(i, j, t) \in A_{RT} \subset A$
A_{FT}	Valid arcs from V_R or V_P to V_F, $(i, j, t) \in A_{FT} \subset A$
A_{LT}	Valid arcs from V_L to V_P or vice versa, $(i, j, t) \in A_{LT} \subset A$
A_{DT}	Valid arcs from V_D to V_P or vice versa, $(i, j, t) \in A_{DT} \subset A$
A_{PT}	Valid arcs from V_P to V_P, $(i, j, t) \in A_{PT} \subset A$
A_{allT}	All valid arcs on day t, $(i, j, t) \in A_{allT} \subset A$

The inclusion of an arc into a set takes into consideration essentially two parameters, named *node visited* and the *schedule feasibility measure*. The first, node visited, is represented by NV_{it}, and states whether or not a node is visited on day t. The main reason supporting its consideration is that if a node in the arc isn't visited in that day, there is no need to include the arc in the set (remember that an arc is also defined by the day). The second parameter, the schedule feasibility measure, denoted by lag_{ijt}, has the function of associating a measure of "time feasibility" to the arc. If the arc is not feasible, given the time constrictions, it is excluded from the set. These parameters are further explored in the next subsection. The nature of the arc subsets inherent to the model are subsequently defined. The A_{RT} arcs start in the real day care center and arrive to a fictitious center or to a patient. Arcs departing from a real day care center or a patient and arriving to a fictitious center belong to the A_{FT} subset. The third subset, A_{LT}, encompasses arcs leaving the lunch replica and arriving at a patient node or vice versa. Similarly, A_{DT} contains arcs leaving the lunch distribution replica and arriving at a patient or vice versa. The A_{PT} subset includes only arcs leaving and arriving at patients. At last, the A_{allT} subset is formed by all the arcs previously considered valid $A_{allT} = A_{RT} + A_{FT} + A_{LT} + A_{DT} + A_{PT}$.

4.3 Parameters

Table 4.3: Parameter's definition

D_{ij}	Travelling time between node *i* and *j* (in minutes);
w_{it}	Visit duration in node *i* on day *t* (in minutes);
NV_{it}	Node Visited: = 1 if patient *i* is visited on day *t*;
P	Penalization value for the daily loyalty preference (in minutes);
K	Number of teams required for lunch distribution;
H	Team's maximum daily working time (in minutes);
e_{it}	Early time of TW for node *i* visited on day *t* (in minutes);
l_{it}	Later time of TW for node *i* visited on day *t* (in minutes);
M_{ijt}	Big M, a value large enough to assure constraint feasibility (in minutes);
lag_{ijt}	"Schedule feasibility" measure (in minutes);

4.4 Variables

Two different decisions should be taken. One concerns the sequence to which nodes are visited and establishes the time each visit should start. Therefore, two variables were defined:

- $\mathbf{x_{ijat}}$ a binary variable that equals 1 if team *a* is assigned to the arc (*i* , *j*) on day *t*; and zero otherwise**.** This variable will provide the nodes visiting sequence for team a on day t. .
- $\mathbf{s_{iat}}$ a continuous variable defining the starting time of the visit from team *a* to node *i* on day *t*,

Two binary auxiliary variables are needed to model the daily-loyalty preference constraint ($\mathbf{a_{iat}}$ and $\mathbf{b_{iat}}$)**.**

4.5 Model formulation

The essential feature of a linear programming model are the mathematical relationships that model the real world problems. In optimization, these assume two natures: they are either the objective function (OF) or a constrains. The first represents a measure of the advantage or disadvantage attributed to a feasible solution, whereas the constraints are the conditions that must be satisfied by feasible solutions. The constraints will dictate how the solution is conditioned by the characteristics of the real problem. Resorting to the previously introduced nomenclature, the model's mathematical formulation is presented subsequently.

4.5.1 Objective functions (OFs):

First Objective Function – Travelling Time

$$\min_{x} \sum_{i,j,a,t} (D_{ij} \times x_{ijat}) + P \sum_{i,a,t} b_{iat} \qquad (1)$$

The aim of the objective function is to minimize the time spent travelling (the first term). However, an additional term was essential, as a result of the introduction of a soft-constraint for modeling the daily-loyalty preference, accumulating a penalty value for each time the preference is not verified.

Second Objective Function – Workload balance

$$\min_{x} \max_{a,t} \sum_{i,j} \left(D_{ij} \times x_{ijat} + w_{it} \times x_{ijat} \right) + P \sum_{i} b_{iat} \quad (2)$$

The second objective function is different in nature. While the first objective function minimizes the total travelling time, in the second, per day, the maximum service time amongst the teams must be minimized, an objective that assures that a more balanced workload distribution.

Although this function is not linear, a linearization technique was used to allow it to be the second objective function used.

4.5.2 Constraints

1) **To visit all patients**

All patient nodes must be visited only once by the teams.

$$\sum_{a} \sum_{j} x_{ijat} = 1, \quad \forall\, i \in V_p, \forall\, t : \; NV_{it} = 1 \quad (3)$$

2) **Teams' lunch break**

All teams must have a lunch break. This was modelled imposing all teams to visit the day care center replica corresponding to the lunch break.

$$\sum_{i} x_{ijat} = 1, \qquad \forall\, a \in A, \forall\, t \in T, \forall\, j \in V_L \quad (4)$$

3) **Weekly loyalty constraint**

Each patient node cannot be visited by two different teams in different days of the week

$$\sum_{j} x_{ijat} + \sum_{j} x_{ijez} \leq 1, \quad \forall a, e \in A: a \neq e, \forall\, t, z \in T: t \neq z, \forall\, i \in V_P \quad (5)$$

4) **Daily loyalty preference**

For each patient with more than one visit per can be visited by more than one team

$$\sum_j x_{ijat} + a_{iat} = \sum_j x_{(i+1)jat} + b_{(i+1)at} \quad \forall i \in R_i, \forall a \in A, \forall t \in T \tag{6}$$

with $(i+1)$ representing the node flowing node *i* in set R_i.

Notice that if one team cannot perform all the day visits, variable b will be one and will introduce a penalization in the objective function**.**

5) All teams must depart from the day care center.

$$\sum_{j \in V_P \cup V_L} x_{ijat} = 1, \qquad \forall a \in U, \forall\, t \in T, \forall\, i \in V_R \tag{7}$$

6) All teams must arrive to the day care center.

$$\sum_{i \in V_P \cup V_L} x_{ijat} = 1, \qquad \forall a \in U, \forall\, t \in T, \forall\, j \in V_F \tag{8}$$

7) Route continuity

Equation (9) states that the team that enters a node must also leave from it.

$$\sum_i x_{ijat} - \sum_i x_{jiat} = 0, \quad \forall a \in U, \forall\, t \in T, \forall j \in V_p \cup V_L \tag{9}$$

8) Lunch distribution

Equation (10) assures that exactly *K* teams should be available for lunch distribution.

$$\sum_a \sum_i x_{ijat} = K, \qquad \forall\, t \in T, \forall j\ \in V_D \tag{10}$$

9) Visit starting time

Constraint (11) assures that if the same team visits nodes *i* and *j*, the corresponding starting times allow the travelling between nodes (D_{ij}) and the working time at node i. Notice that if team a does not visits these two nodes, M_{ijt} is a value large enough to make the constraint redundant.

$$s_{iat} + D_{ij} + w_{it} + M_{ijt}\left(1 - x_{ijat}\right) \le s_{jat}, \quad \forall a \in U, \forall\, t \in T, \forall i, j \in V \tag{11}$$

10) Teams daily work time

Constraint (12) assures to each team the daily working time of H minutes is respected.

$$s_{iat} - s_{jat} \le H \ \ \forall a \in U, \forall\, t \in T, \forall\, i \in\ V_F, j \in\ V_R \tag{12}$$

11) Non-negativity

$$\boldsymbol{s_{iat}} \geq \mathbf{0}, \quad \forall \boldsymbol{a} \in \boldsymbol{U}, \forall\ \boldsymbol{t} \in \boldsymbol{T}, \forall \boldsymbol{i} \in \boldsymbol{V} \tag{13}$$

12) Binary Variables

$$\boldsymbol{x_{ijat}} \in \{\mathbf{0}, \mathbf{1}\}, \qquad \forall \boldsymbol{a} \in \boldsymbol{U}, \forall\ \boldsymbol{t} \in \boldsymbol{T}, \forall\ \boldsymbol{i}, \boldsymbol{j} \in \boldsymbol{V} \tag{14}$$

$$\boldsymbol{a_{iat}} \in \{\mathbf{0}, \mathbf{1}\}, \qquad \forall \boldsymbol{a} \in \boldsymbol{U}, \forall\ \boldsymbol{t} \in \boldsymbol{T}, \forall\ \boldsymbol{i} \in \boldsymbol{V} \tag{15}$$

$$\boldsymbol{b_{iat}} \in \{\mathbf{0}, \mathbf{1}\}, \qquad \forall \boldsymbol{a} \in \boldsymbol{U}, \forall\ \boldsymbol{t} \in \boldsymbol{T}, \forall\ \boldsymbol{i} \in \boldsymbol{V} \tag{16}$$

5. Solution approach

VRP are NP-hard problems, what implies that achieving an optimal solution is computationally intractable for large-scale instances. The instances provided by APOIO are of medium size, with the routes planned for 6 teams visiting daily approximately 57 nodes, for every day of the week (a lessened number of nodes on weekends). Consequently, in order to overcome the experienced computational difficulties, a solution approached based on the mathematical formulation was designed. The procedures constituting the solution methodology are discussed in the following subsection.

Solution methodology

The first strategy was to decompose the model in two dimensions: patient typology and days of the week. The division into dimensions results from analyzing the case study characteristics. By dividing into days of the week, the solved instance becomes much smaller since data provide to the model concerns one single day (at most 57 nodes to be visited). In regards to the dimension

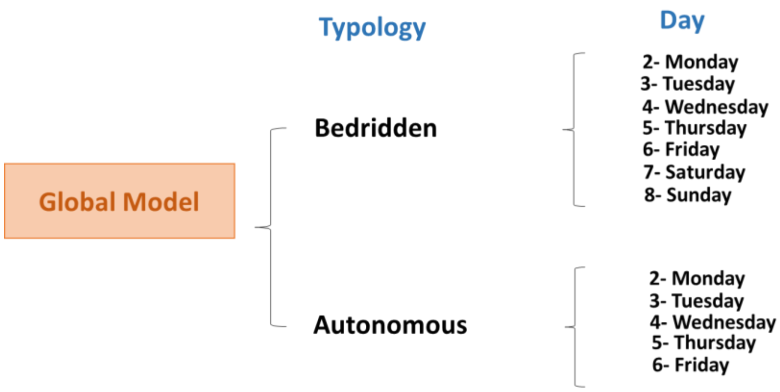


Figure 5.1: Schematic of the dimensions considered the model.

of patient typology, the division allowed the differentiation of the teams used. When the model is run for the bedridden typology, the teams have to be composed of two caregivers while for autonomous only one assistant performs the visits. A schematic representation of the decomposition strategy is depicted in Figure 5.1. Moreover, it is noteworthy that the dimensional division previously mentioned automatically assures one of the requests made by APOIO, namely, assigning teams of two assistants to bedridden patients and those teams of one to semi-dependent patients.

The model was run in accordance with the dimensional division, for each of the objective functions. Nonetheless, there was one aspect about the operational preferences of the institution that was would not become assured with this solution: the weekly loyalty preference, which states that every visit to a patient within the same week should be assigned to one single team when possible. This separation of the weekdays implies that equation (5) cannot be implemented by the model (it does not take into account the multi-period characteristic of the model). In order to account for that preference, the following procedure was designed.

Weekly loyalty procedure

After solving each day independently, optimized routes for each day of the week are obtained. However, a patient served by team one on day 1 may be assigned to team 3 on day 2. This aspect is not relevant for bedridden patients, since the services to be performed are the same on weekdays, varying only on weekends. The weekly loyalty was assured by fixing the patients to the teams of the weekday, and solving the problem only for weekends.

In contrast, the services paid to autonomous patients are considerably variable each day. To assure the week loyalty, a procedure was established and is defined as follows. Firstly, the days of the weekdays are sorted in a decreasing manner, with respect to the number of patients to visit. Then, in the second step, the MILP model is resolved for the first day (t_1), establishing the first partial patients assignment to the teams. Afterwards, in the third step, the model is resolved for the second day (t_2) with the partial patients' assignment fixed in beforehand. The preceding step guarantees that the assignments attained in d_1 are maintained in t_2 and that the patients that are not served in day d_1 are one of the model solution. Finally, the procedure described in the third step is repeated sequentially for the other days, assigning the totality of the patients to the teams. A diagram of the procedure is depicted in Figure 5.2.

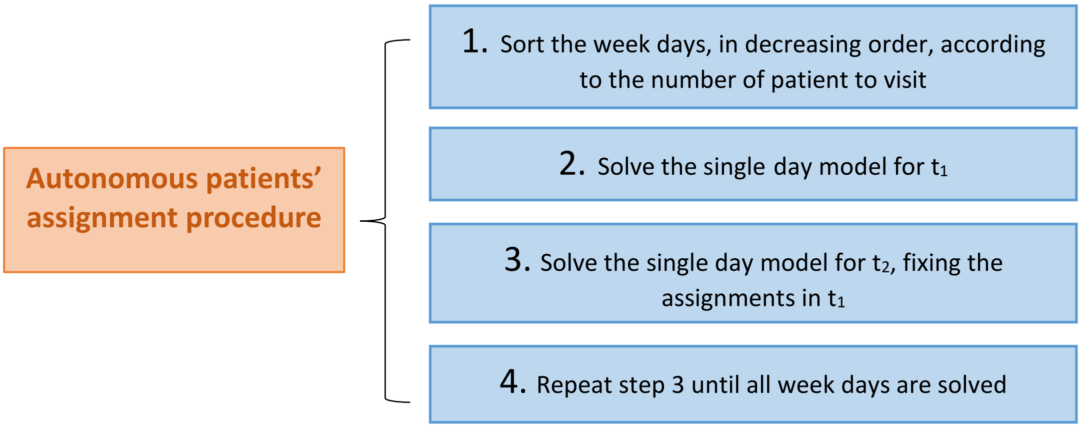


Figure 5.2: Diagram of the steps constituting Autonomous patients' assignment procedure to the teams.

For the bedridden patients the t_1 was Monday, for it was the day with the highest value of OF. Since all the days of the week have the same number of patients, all days were fixed based on Monday. For the autonomous patients, the day chosen as t_1 was Friday, with 20 patients, followed by Monday - 18 patients, Tuesday – 13 patients, Wednesday – 13 patients and, finally, Thursday with 12 patients.

6. Case study results and discussion

Considering the two objective functions and the solution methodology, it was considered that the analysis of the results should be carried out in four scenarios. These are characterized by two components: the objective function that is under scrutiny and the application of the weekly loyalty constraint. For each scenario, a solution is obtained and analyzed then, they are compared among each other so that the functionalities of the model can be more accurately understood and validated. A schematic of the scenario's organization can be found in Figure 6.1.

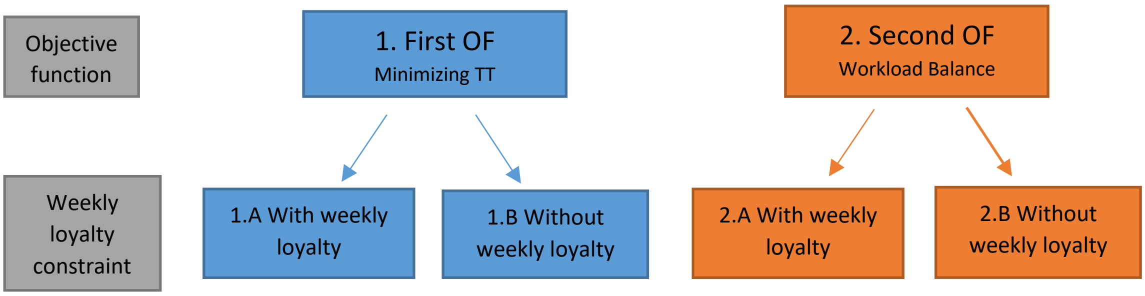


Figure 6.1: Scenarios analyzed.

6.1 Indicators

The analysis of the scenarios demands the creation of some indicators allowing the comparison between them. Therefore, the travelling time, the service time, the waiting time, the route balance and the route duration balance are the indicators selected to study the quality of solutions.

The travelling time indicator is defined as the sum of the times each team spends traveling between patients, being defined as:

$$TT_{at} = \sum_{i} D_{ij} * x_{ijat} \tag{17}$$

It is a relevant indicator, since the main objective of the first OF is the minimization of the sum of the travelling time values of all teams within a week.

The second indicator is the service time, and it considers both the duration of the visits and the traveling time amongst nodes, being determined as:

$$ST_{at} = \sum_{i} w_{it} + \sum_{i} D_{it}. \tag{18}$$

This indicator is particularly relevant for the second objective function, which aims at balancing the workload among teams.

In contrast with the former two indicators, the third, named waiting time, is not significant in what concerns the designing of the routes; however, its analysis is believed to be relevant especially since, it has been subject of optimization in the literature. The waiting time indicator is defined as:

$$WT_{at} = s_{DFit} - s_{DRit} - \sum_i w_{it} - \sum_i D_{it} \tag{19}$$

or, more simply,

$$WT_{at} = s_{aDFt} - s_{aDRt} - ST_{at} , \tag{20}$$

with *DF* is the arriving day care center node and the *DR* is the departing day care center node.

The other two indicators differ from the previous, as they do not characterize a route, but rather a day of the week. The creation of the route balance indicator is directly related with the nature of the second OF and it impacts on the variation of the teams' service times. The most appropriate method for assessing its correct functioning would be through the analysis of the variation in the difference between the team with the maximum and the team with the minimum value of service time, within the same team type and for the same day. Consequently, the indicator of route balance is defined as follows:

$$iRB_t = Max(ST_{at}) - Min(ST_{et}) \tag{21}$$

in which $Max(ST_{at})$ stands for the maximum value amongst all the $a \in U$ teams of the same type, on day t and while $Min(ST_{at})$ is the minimum value in the same set.

Finally, a route duration balance indicator was introduced to explore how the total duration of the routes varies as different objectives are optimized. The route duration balance indicator is defined as:

$$iRDB_t = Max(s_{aDFt} - s_{aDRt}) - Min(s_{eDFt} - s_{eDRt}). \tag{22}$$

It can be seen as the difference between the maximum and minimum duration of all the routes in day *t*.

6.2 Case study data

Selecting and assessing the input data is a crucial stage when testing a model, as it bears significant influence over the solutions achieved. The present section aims at introducing and justifying the decisions made in what concerns data input. One of the first aspects regards the nomenclature associated to the nodes of patients and its replicas that, despite being modeled as different nodes, must be identifiable as the same patient. All of the other relevant issues result from either selecting the data source for a parameter or defining how to calculate its value.

6.2.1 Node nomenclature

The distinction between patients and replicas, while simultaneously identifying them as the same patient, is assured by adding multiples of 100 to the identification number of the patient. For example, if a patient number is 2 and he is visited four times during a day, three replicas are needed. Therefore, the first replica is 102 (that represents the second visit of the day), the second replica is 202 (that represents the third visit of the day) and the third replica is 302 (that represents the fourth visit of the day). In Table 6.1 are presented the bedridden patients with associated replicas, on Monday. Currently, the bedridden patients with replicas, as well as the number of replicas, are constant for every weekday, and are equal to the ones in Table 6.1.

Table 6.1: Sets of Monday bedridden patients' nodes, with their associated replicas

Sets of patient's nodes
{2,102,202,302}
{3,103,203}
{5,105,205}
{6,106}
{57,157,257}
{58,158}
{59,159}

Beyond the patients and replicas, the day care center also has replicas for lunch break and lunch distribution. In Table 6.2 the nomenclature used is provided along with their definition.

Table 6.2: Label and function of the Day Care Centre replicas

Identity Number	Node function
0	Departing node to all routes
3001	Lunch break node
3002	Lunch distribution node
3000	Arrival node

It can be observed that, to the day care center replicas were attributed relatively high identification numbers, permitting not only that they are easily spotted in a route, but also permitts the existence of a considerable number of replicas of a patient. For example, according to the replicas' labeling method, for the patient labeled 1 to need a replica identified as 3001 would require 30 visits in the same day, a situation which is highly unlikely.

6.2.2 Parameters determination

The approach used for determining the parameters can significantly influence the implementation of the results obtained and, therefore, a further description of each parameter and process used for their determination is consequently addressed.

Visit duration w_{it}, TW early time e_{it} and TW later time l_{it}

These three parameters have their values determined through the analysis of the operational datasheet of the teams of one week, thus resulting in parameters with values originated in services that have actually been performed.

The visit duration w_{it} is defined as the quantity of minutes required to provide a visit to a patient located at node *i*, occurring on day *t*. Another important nodes' characteristic is the time window, inputted into the model through the parameters e_{it} and l_{it} which, respectively, stipulate the earlier and later times at which the provision of service might be initiated. In this way it is guaranteed not only that the duration used as input is the real time necessary for providing that service, specifically to that patient, but also that the service is initiated within a time interval compatible with what is requested by the patient. Some of the nodes are characterized by particular values for these parameters. Node 3001, representing the lunch break, has a visit duration of 60 minutes with a TW starting at 12 p.m. and 14 p.m., a time interval important for the route design flexibility. Regarding node 3002, the TW parameters are both fixed at 12 p.m , so that the teams initiate the distribution of meals exactly at that time. Its duration is 90 minutes. Finally, nodes 0 and 3000 are characterized by a duration of 0 minutes, serving simply as landmarks for initiating and finishing the routes.

Ideally, the data retrieved from the datasheets should correspond to several weeks and the values used should be the average, since there might be unpredicted situations at one specific day that increase the value of the visit duration, for example. However, the institution's teams only maintain a paper record of this type of data, preventing an easy computational access to it, which has to be retrieved manually. Hence, the data collected respected just one week, so that the study could be finished within the deadline.

Time travel between nodes D_{ij}

The HHC assistants often travel by foot or by van. Parameter D_{ij} is actually a measure of the time the team spent on traveling from node *i* to node *j*. The data used for this parameter were obtained from google maps, resorting to an API - application programming interface- which gives the real distances between the coordinates associated to the patient's addresses. This information is then converted into walking or driving time. Despite the time differences between walking and driving, the main objective of the model is to reduce the total time spent on traveling and, therefore, minimizing it as a walking time or as a driving time will generate the same result, as they are assumed to be proportionally related.

Number of visits, *NV*$_{it}$

The binary parameter NV_{it} contains information regarding whether or not a patient node *i* is visited on day *t*. It has the value 1 if it is visited on day *t* and 0 otherwise. The relevance of this parameter is related with the set partitioning associated with the arcs, permitting subsets associated to day *t* to be constituted solely by arcs between nodes visited on day *t*, decreasing the amount of time needed to retrieve a solution. The information is retrieve from the analysis of the team's operational datasheets.

Big M parameter

This parameter derives from the need to linearize one of the model's constraints, namely the Visit Starting Time constraint, equation (11). Originally, the equation (11) is written as:

$$(s_{jat} - s_{iat} - D_{ij} - w_{it}) * x_{ijat} \leq 0 \tag{23}$$

which through the addition of the parameter M_{ijt}, sufficiently large, the previous equation is linearized and is rewritten as:

$$s_{iat} + D_{ij} + w_{it} + M_{ijt}(1 - x_{ijat}) \leq s_{jat} \quad . \tag{24}$$

The introduction of the M_{ijt} big enough allows that, when $x_{ijat} = 0$, the restriction becomes redundant. The parameter must be large enough to prevent the s values from being conditioned when $x_{ijat} = 0$. The previous situation is verified when M_{ijt} as:

$$M_{ijt} = l_{it} - e_{jt} + w_{it} + D_{ij}. \tag{25}$$

Penalty value (P)

The soft-constraint used to implement the daily loyalty preference requires the introduction of a value that penalizes the objective function when the constraint is violated. The determination of a general value for this parameter is not straightforward and, therefore, a study on its behavior becomes pertinent. It was decided that the values for P would be based on a sensitive analysis. For the B-Teams, the value is derived from a sensitive analysis performed for Monday, whereas for the A-Teams the value is set by a similar analysis but for Friday. The reasoning behind the different weekdays chosen is related with the solution methodology, with the value for P being determined for the day that is assumed as t_1 in the solution method (Friday for the autonomous and Monday for the bedridden patients). The analysis for the two cases is now presented.

Sensitive analysis B-Teams, Monday

Firstly, the impact of the variation of the value for P is analyzed considering two criteria: the number of penalties in the solution and the value of the solution's iRB (indicator of route balance), with the analysis being performed for both objective functions. As it can be observed, in Figures 6.2 and 6.3, for the first OF, from a value for P of 12 minutes, both the number of penalties and the indicator iRB maintain their values constants. Nevertheless, it must be remembered that the model is meant to be general and, as a result, it was decided that the value used to obtain the

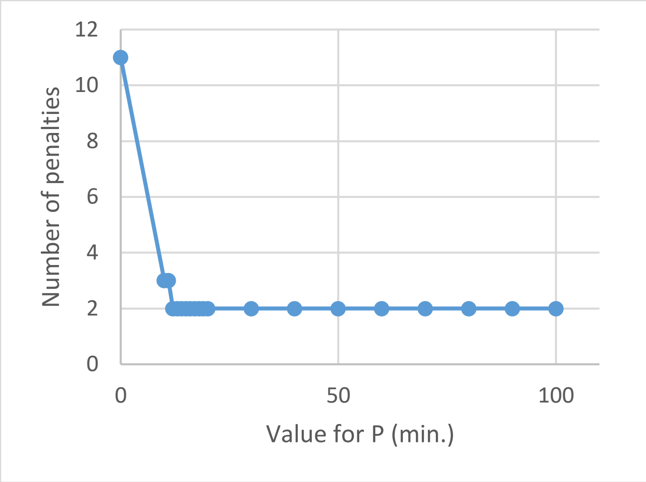


Figure 6.2: Variation of the number of penalties as a function of the value of the parameter P for the first OF, Monday, B-Teams, with/without WLC.

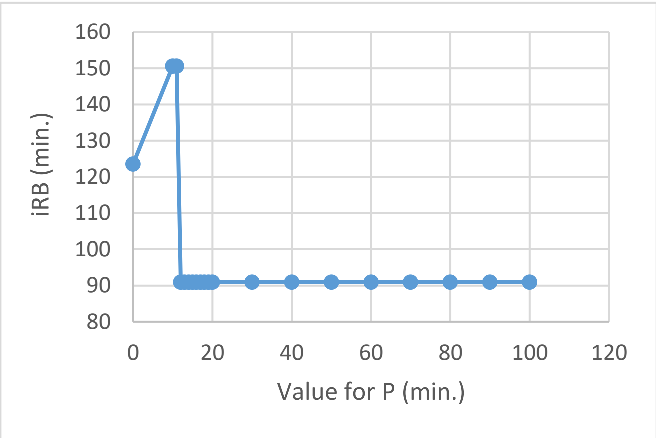


Figure 6.3: Variation of the value of the indicator of route balance as a function of the value of the parameter P for the first OF, Monday, B-Teams, with/without WLC.

solution would be 100 minutes. Setting the value for P at such a great value will ensure that in other instances, the probability of the responsibility of the Value of P for having more penalties than the strictly required by problem characteristics is significantly reduced.

Regarding the second OF, the same analysis is presented in Figures 6.4 and 6.5. Differently from what was observed in the first OF, there is a degree of compromise between the number of penalties and the iRB, modulated through the Value fo P: the greater the value of P, the less penalties are allowed in the solution, however, it will also increase the iRB, decreasing the solutions' quality. Analyzing the two previously mentioned graphics, it is possible to conclude that the best value for P is 20 minutes, particularly because it is the first value for which the number of penalties assumes the value 2 (Figure 6.4) and, for values for P greater than 20, the iRB always deteriorates.

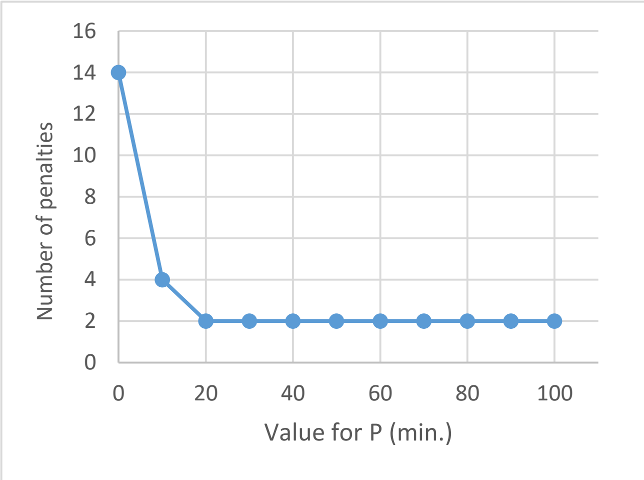


Figure 6.4: Variation of the number of penalties as a function of the value of the parameter P for the second OF, Monday, B-Teams, with/without WLC.

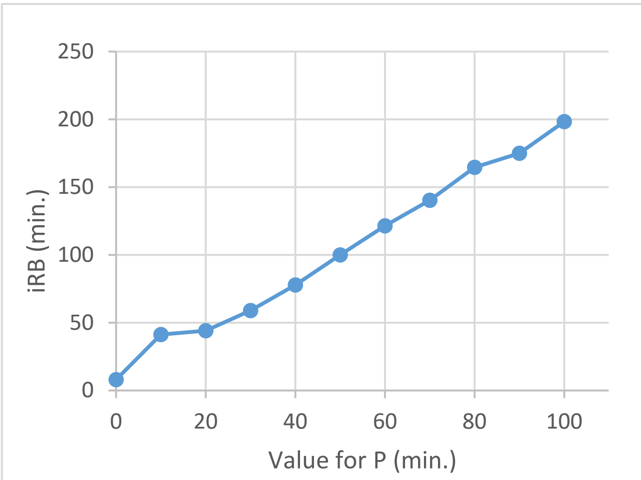


Figure 6.5: Variation of the value of the indicator of route balance as a function of the value of the parameter P for the second OF, Monday, B-Teams, with/without WLC.

Sensitive analysis A-Teams, Friday

The study of the sensitive analysis for Friday A-Teams has a similar structure to the analysis for Monday B-Teams. For the first OF, however, there is no need to present the graphics of the variations of the number of penalties nor of the iRB as a function of the value for P, because they are both constant. The number of penalties is constantly equal to 0, even when the value for P is also 0. This is justified by the low number of replicas, in addition to a lower number of patients per team when compared with the B-Teams, a conjecture that allows the only replica to be optimally served by the same team as the main node. Similarly to Monday, when the number of penalties hits a limit, beyond which it no longer varies, the iRB is also kept constant. The variation limit had already been found from the beginning, since the number of penalties is 0 from the strat, the iRB never changes. As result, any value would be suitable for running the model in this day. For a matter of simplicity, the value used to obtain the solution is 100 minutes, the same as in the B-Teams.

For the second OF, the results of the sensitive analysis are less monotonous. In Figure 6.6, it is possible to observe that, at the beginning, the number of penalties is 1 and when the parameter P assumes the value 7, the number of penalties decreases to zero, remaining zero for greater values of P. It can also be observed, in Figure 6.7, that for the value of P equal to 7, the iRB attains its lowest value, maintaining it for values of the paramater greater than 7. Also for the sake of simplicity, it was decided that the value used for the parameter P would be 20 minutes, the same as the value used for the B-Teams.

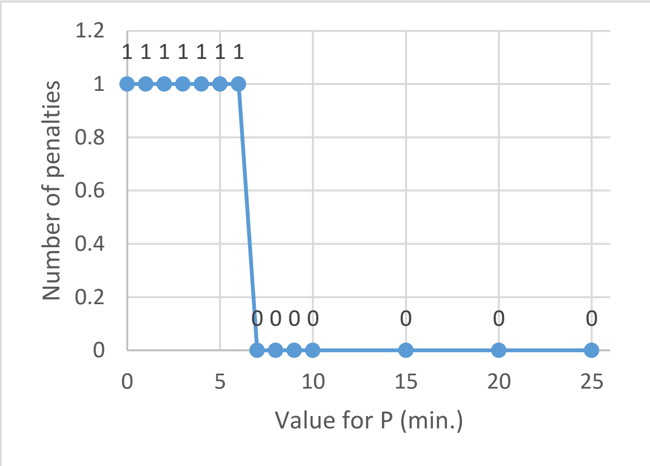


Figure 6.6: Variation of the number of penalties as a function of the value of the parameter P for the second OF, Friday, A-Teams, with/without WLC.

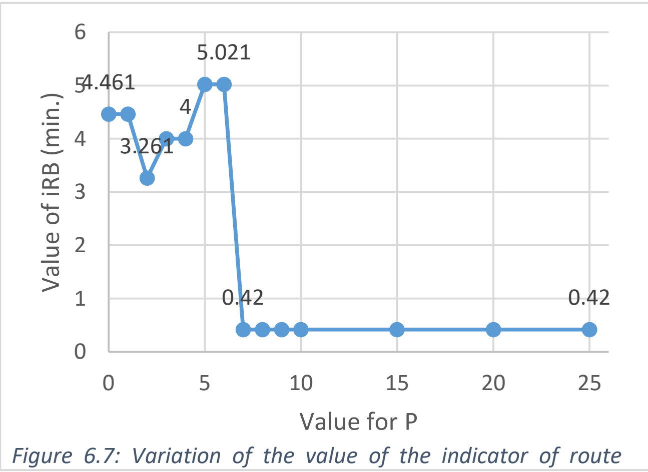


Figure 6.7: Variation of the value of the indicator of route balance as a function of the value of the parameter P for the first OF, Friday, A-Teams, with/without WLC.

For A-Teams on Friday and for the B-Teams on Monday, the solutions are the same with or without the WLC. However, the same is not verified for all of the other days of the week. In order to understand the impact of the introduction of the WLC into the model through the solution method, it was decided that the analysis should be extended also to a day in which the variations of the two quantities analyzed differ when the WLC is enforced or disregarded. The day selected for the study was Wednesday, due to it never being considered in the first step of the solution methodology for both team types. The analysis is available in the annex C.1. (Sensibility analysis for Wednesday).

6.3 Current Solution

One of the most relevant pieces of information is the current solution, for which only the visit sequence is known. The traveling time was computed given the matrix *Dij* in order to be possible to compare the current solution with the solutions proposed. One example of the routes performed on Tuesday are displayed in Table 6.3.

Table 6.3: Current solution operated by APOIO on Tuesday.

Current solution - Tuesday			
Team	Route	TT (min)	TOTAL
1	0-9-3000	15,2	166,3
5+6	0-56-57-58-59-3000	5,7	
6	0-2-3000	14,4	
4	0-23-24-13-3001-23-3000	19,4	
2	0-10-15-14-62-3000	20,7	
1+2	0-2-4-5-3-1-58-59-57-3000	26,3	
3+5	0-2-3-5-3000	18,6	
3	0-55-7-3-5-2-3000	24,2	
2+5	0-6-2-3000	8,4	
5	0-57-6-3000	13,5	

It is possible to understand that the routes are quite confusing. For example, in Tuesday, the assistant number 5 has to perform three small routes. Also, the bedridden patients are not always assigned to teams of two assistants, as is the case for patients 57 and 6 for the last route. The sequence for all the routes currently performed can be found on the Annex B.1. (Current Solution). For comparison purposes, the most relevant data is presented in Table 6.4, the daily TT.

Table 6.4: Traveling Times for APOIO's current solution

Current Solution Traveling Times	
Week days	**TT (min.)**
Monday	151,8
Tuesday	166,3
Wednesday	157,64
Thursday	136,7
Friday	169,7
Saturday	33,4
Sunday	38,4
Total	853,2

6.4 Scenario 1- Minimizing Travelling Time

With Weekly Loyalty Constraint (Scenario 1.A)

In accordance with the solution methodology, the solution is divided into two parts, the bedridden patients and the autonomous patients, the Wednesday's routes are depicted in Table 6.5. The complete solution is available in the annex C1 (Scenario 1.A Solution).

Table 6.5: Wednesday's solution for Scenario 1.A (First OF solution, respecting WLC). TT- Travelling Time; ST – Service Time; WT – Waiting Time

Week Day	Route nb.	Route	TT (min.)	ST (min.)	WT (min.)
Wednesday	1B	0-3-102-1-3001-202-103-105-205-203-302-3000	37	274	206
	2B	0-56-58-3002-3001-158-3000	4	246	216
	3B	0-57-2-6-4-5-59-3001-157-257-106-159-3000	36	337	137
	1A	0-9-12-15-22-3001-115-3000	33	246	29
	2A	0-3002-3001-55-107-3000	18	209	117
	3A	0-23-24-62-11-13-3001-46-3000	20	283	140

The solution obtained for the scenario 1. A. respects all of the constraints imposed by the problem, and a tendency for respecting the preference associated to the soft-constraint can be verified. For the sake of clarity, place the focus on Wednesday. In the B-Teams routes, only patients number 2 and 5 have replicas visited by more than one team. Only the first visit of the day is performed by a different team and, for both patients, the first visits are performed by the same team. Team 1B starts working later than the other two, suggesting precisely that the preference is not respected due schedule imcompatibility: if the teams can only work 480 minutes, the teams performing the first visit can never perform the last. Patients 3, 6, 57, 58 and 59 and their replicas are visited by the same team. The same reasoning is applicable to all of the other days and to the routes of the A-Teams (even though it is never verified for the latter team type). The complete routes for scenario 1.A are present in annex C1.

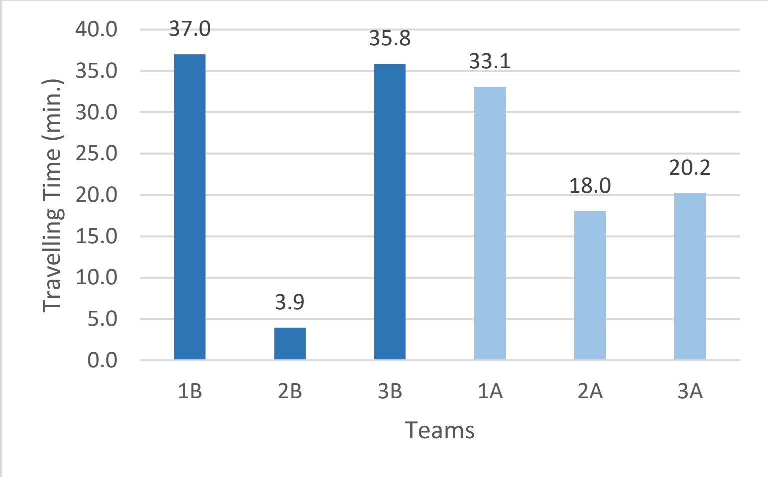


Figure 6.8: Wednesday Travelling Time for the Solution of Scenario 1.A

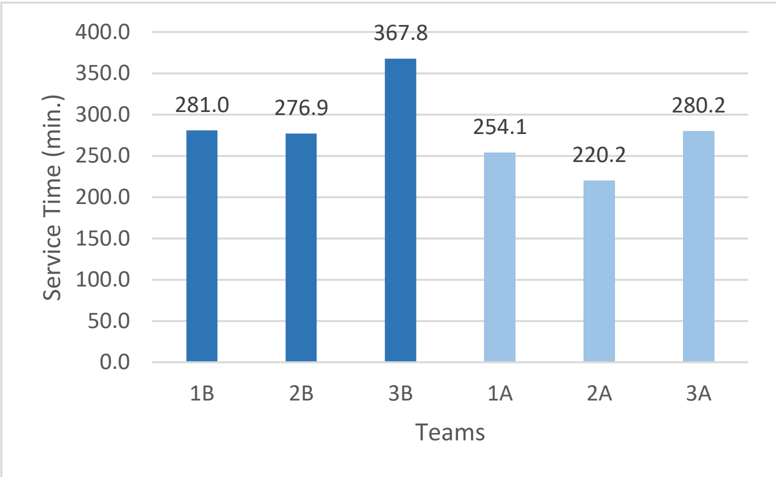


Figure 6.9: Wednesday Service Time for the Solution of Scenario 1.A

Regarding the routes indicators, in Figure 6.8 it is possible to observe how imbalanced are the routes concerning TT. In terms of workload distribution, it is noticeable that there is a wide difference among teams of a same day. For the B-Teams on Wednesday, Figure 6.9, team 2B has a workload of 276,9 minutes (only service time), whereas team 3B is assigned a work schedule encompassing 367,8 minutes of service, presenting also high variability concerning waiting time (Table 6.5). The former fact demonstrates that the workload is unbalanced among teams and, since the solution has a direct impact on the well-being of the assistants, it is believed that the additional solution in which the workload distribution is balanced will proportionate a relevant tool for decision-making. As such, the aforementioned solution is presented in the following subsection.

Another perceptible situation affecting route balance is the variation between the maximum value and the lowest value of TT within a set of teams of the same type. For example, on Wednesday, the maximum value of TT for a team is of 37 minutes whereas the minimum is 4 minutes. This behavior is frequently seen in optimization models for minimizing distances. The most advantageous way to allocate a patient to a route is to assign it to a team that is already on the field, a tendency that leaves some routes with many more nodes, but which is also the most optimized.

Without Weekly Loyalty Constraint (Scenario 1.B)

The complete second solution is displayed in the annex C.2. For the moment, the attention is centered, once again, on Wednesday's solution (Table 6.6). Every constraint is respected, except for the WLC. The preference for the daily loyalty is tendentiously respected, similarly to what was observed in Scenario 1.A. In Table 6.6 it is possible to notice that without the WLC there is only one patient with replicas visited by more than one team, namely, patient number 2. Patient number 5 is no longer visited by more than one team contrarily to what happened in Scenario 1.A. This was expected since this scenario looses one restriction.

Table 6.6: Wednesday's solution for Scenario 1.B (First OF solution, without respecting WLC). TT- Travelling Time; ST – Service Time; WT – Waiting Time

Week day	Route nb.	Route	TT	ST	WT
Wednesday	1B	0-57-58-3002-157-3001-158-257-3000	4	274	168
	2B	0-56-2-6-4-59-3001-102-202-106-159-3000	36	306	174
	3B	0-5-3-1-3001-103-105-205-203-302-3000	38	278	202
	1A	0-23-3001-55-3000	19	141	321
	2A	0-24-11-9-12-15-13-22-115-3001-46-3000	33	306	77
	3A	0-62-3002-3001-107-3000	18	278	261

Regarding the model's indicators for Wednesday, Figures 6.10 and 6.11, amongst teams of the same type there is an explicit imbalance both in terms of TT and ST. Comparing the service times for the B-Teams in 1.A with the ones present in Figure 6.11 it is verified that the workload is more balanced after the introduction of the WLC. The most significant variations in terms of service

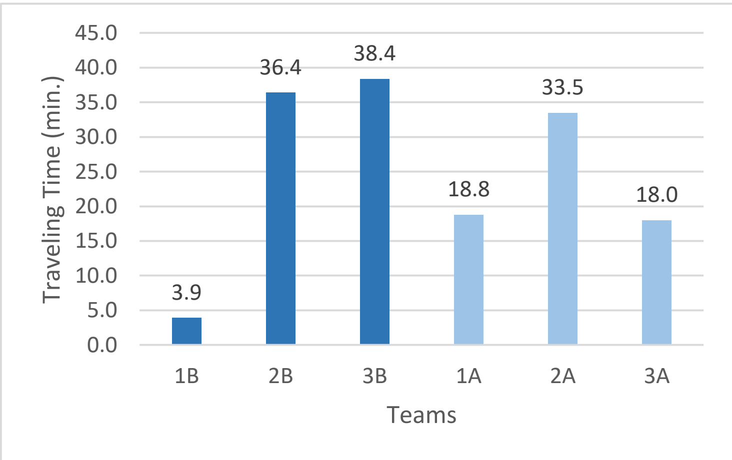


Figure 6.10: Wednesday Travelling Time for the Solution of Scenario 1.B

time are verified in Wednesday, for A-Teams, in which the team with the minimum ST value is kept at 141 minutes, while the maximum reached 306 minutes, which more than doubles the

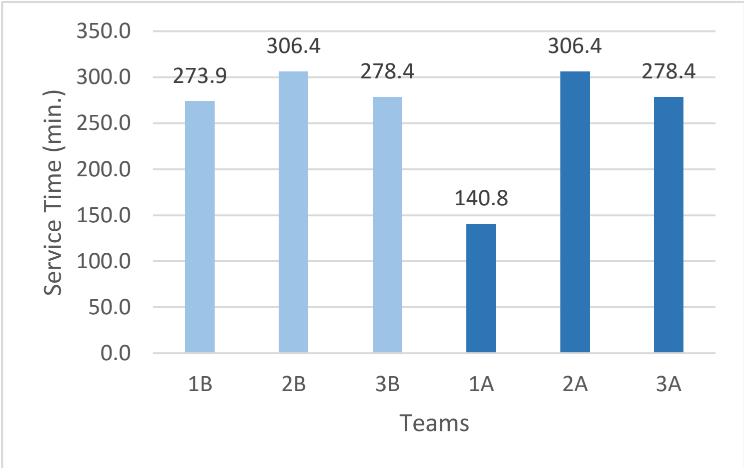


Figure 6.11: Wednesday Service Time for the Solution of Scenario 1.B

minimum value (Figure 6.11). The routes' TT imbalance typical in TT optimization is clear on Thursday, A-Teams, with one team spending 46 minutes traveling while there is one that spends virtually none.

In order to compare the two solutions presented, a systematic and holistic approach can be adopted through the overview of the values of the objective function.

Minimizing Travelling Time: values and interpretation

The objective function includes the time traveled plus the penalty. In Table 6.7, the values of the OF for each day and team type are shown, also discriminating the number of penalties associated to each run associated to scenarios 1.A and 1.B.

Table 6.7: First Objective Function's values associated to both the solutions with and without respecting the WLC. #P = Number of penalties; n.d. = non-defined.

Travel Time Minimization - 1st OF								
Week Day	Respecting weekly loyalty (1.A)				Without weekly loyalty (1.B)			
	Bedridden		Autonomous		Bedridden		Autonomous	
	OF value	#P	OF value	#P	OF value	#P	OF value	#P
Monday	276,8	2	79,4	0	276,8	2	76,9	0
Tuesday	276,8	2	66,0	0	178,7	1	62,6	0
Wednesday	276,8	2	71,3	0	178,7	1	70,3	0
Thursday	277,3	2	59,0	0	179,5	1	54,0	0
Friday	276,8	2	77,8	0	178,7	1	77,8	0
Saturday	28,8	0	n.d.	n.d	28,8	0	n.d	n.d.
Sunday	29,4	0	n.d.	n.d.	29,4	0	n.d	n.d

As verified in Table 6.7, the values of the objective function, when it respects the weekly loyalty constraint, are greater or equal to the values of the non-respecting solution. The maintenance of the TT values for Saturdays and Sundays is due to the services required being performed by a single B-Team of the bedridden. As a result, the value of the OF will be the same regardless of the constraints influencing the optimization. For all weekdays the value of the OF is greater when the weekly loyalty is respected, assuring that the model is functioning as it was expected to. In terms of penalties, for the B-Teams, there is a clear increase in the number of penalties (#P column) when the problem is more constrained, whereas for the A-Teams the number of penalties remains zero.

The main reasons supporting the discrepancy at the penalties level are both the nature of the daily loyalty constraint and, to a certain degree, the heuristic used to solve the WLC constraint. The introduction of the daily loyalty constraint has a more significant influence on a problem with more replicas of the same patient. In APOIO, the bedridden patients' sub-problem has seven sets of up to four replicas, every weekday, whereas in the autonomous patient's sub-problem has, at most, two sets of only two replicas, enhancing the natural complexity inherent to the satisfaction of the highlighted constraint for the bedridden sub-problem.

The causality attributed to the heuristic of the solution methodology is related to the fact that the replicas were modelled as separate patients. Actually, in the day with a greater number of patients, if the first visit to a patient is assigned to a team and the second visit has to be assigned to another team, for a matter of scheduling incompatibilities, that assignment will be maintained for all the days of that week. When possible, the daily loyalty constraint will assure that the patients and its replicas are assigned to the same team within a day. However, if in a previous step of the heuristic a patient and a replica were assigned to different teams, they will remain in those teams, even if the incompatibility that demanded they were separated is no longer verified. The former reasoning is supported by the lower number of penalties obtained when the weekly loyalty constraint is removed, which can be verified in right side of Table 6.7.

Nevertheless, after obtaining the solutions respecting and ignoring the WLC, it is possible to conclude that despite the addition of the weekly loyalty constrain the value of the traveling time is slightly decreased for the B-Teams (notice that in Table 6.7, by removing the value 100(#p) the daily travel time is obtained, for example, for Monday B-Teams, there are 2 penalties. By removing subtracting the value 200 to the OF, the TT of 76,8 minutes is achived). This fact is due to one of the other constraint being violated: the daily loyalty preference. The previous situation will enlarge value of the objective function and, since its violation is irrevocable (the imposition by the WLC is dominant), the model can take advantage of the situation and decrease the traveling time of the team. This was a phenomenon that was verified and, yet, unexpected. In contrast, for the A-Teams, the addition of the WLC results in a generalized slight increase of the team's daily TT.

6.5 Minimizing the maximum workload

With Weekly Loyalty Constraint (Scenario 2.A)

In order to smooth the workload among teams, a second objective function is considered. This objective function minimizes the maximum of the service time values of the routes of the routes of one type, within a day. In Table 6.8 are represented the paths forming each of the solution's routes.

Table 6.8: Wednesday's solution for Scenario 2.A (Second OF solution, respecting WLC). TT- Travelling Time; ST – Service Time; WT – Waiting Time

Week Day	Route nb.	Route	TT (min.)	ST (min.)	WT (min.)
Wednesday	1B	0-56-57-58-3-157-3001-257-158-3000	18	278	115
	2B	0-6-2-4-59-3002-3001-106-159-3000	34	321	271
	3B	0-5-102-1-3001-103-105-202-203-205-302-3000	45	278	129
	1A	0-9-12-13-3001-46-3000	20	227	159
	2A	0-22-3002-3001-55-107-3000	32	273	117
	3A	0-62-24-23-11-15-3001-115-3000	40	259	72

Remembering the solution obtained for the first OF and generally comparing it with the one above disclosed, the improvements regarding the workload balance become evident. For example, on Friday, for the A-Teams, the three teams present the same exact value for service time (annex C.3). This value is fixed at 273 minutes, while in the first OF's solution the values vary between 247 and 327, a deviation of 80 minutes. The result implies that, in this case, team 1A would work less about 25% in terms of service time, when compared with team 3A.

Nonetheless, for the remaining days, the workload balance is not as good as the found for Friday. The situation was expected, specifically because the solution here presented is the one respecting the weekly loyalty constraint. It is interesting to notice that the day chosen as t_1 in the solution methodology is leveled exactly. This implies that the routes are the same as the one not respecting WLC. Even though Friday is the day with the most autonomous patients to visit, it is also the day in which the workload is more evenly distributed. Considering other days, such as Wednesday, which is never a t_1, it is observed that the workload balance becomes more balanced (compare Figures 6.13 and 6.9). However, it does not necessarily mean that the TT will be balanced as well (Figure 6.12).

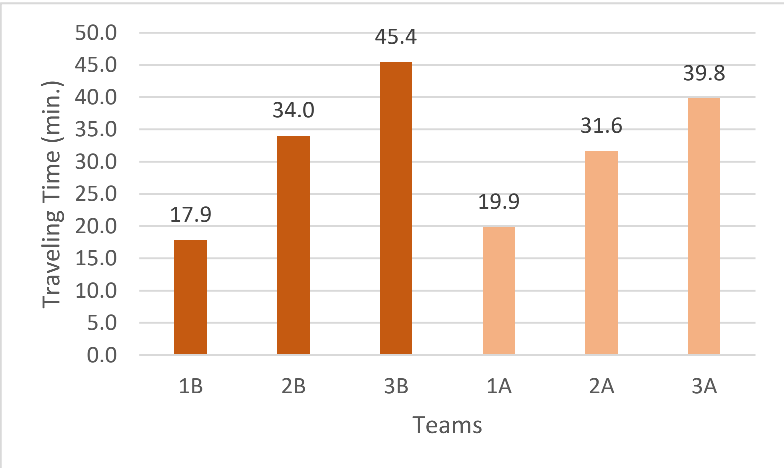


Figure 6.12: Wednesday Travelling Time for the Solution of Scenario 2.A

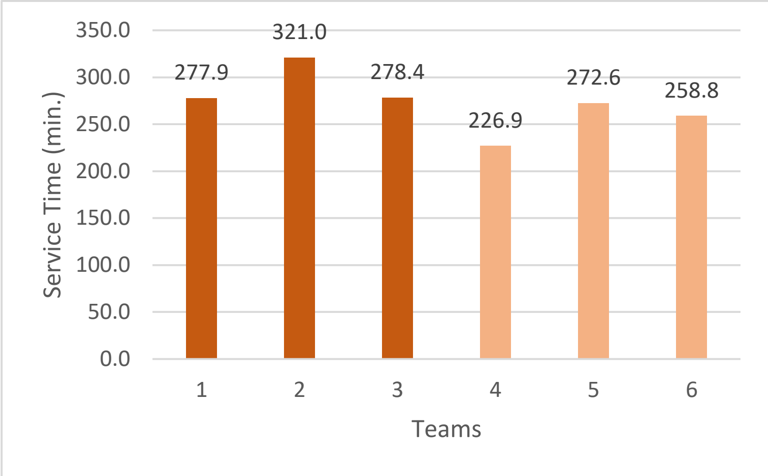


Figure 6.13: Wednesday Service Time for the Solution of Scenario 2.A

Without Weekly Loyalty Constraint (Scenario 2.B)

Finally, the last solution is associated with the scenario 2.B and the solution for Wednesday is exhibited in Table 6.9.

The immediate improvement detected, when this solution is compared with the one from scenario 2.A is related with the decrease in terms of imbalance associated to the different routes verified in scenario 2.B. This decrease can be assessed by comparing Figures 6.13 and 6.15. The improvement was already expected since the removal of a significant constraint permits an optimization of greater quality.

Table 6.9: Wednesday's solution for Scenario 2.A (Second OF solution, respecting WLC). TT- Travelling Time; ST – Service Time; WT – Waiting Time

Week Day	Route nb.	Route	TT (min.)	ST (min.)	WT (min.)
Wednesday	1B	0-6-2-58-59-3001-102-202-106-159-158-3000	38	297	173
	2B	0-56-57-4-3002-157-3001-257-3000	15	296	164
	3B	0-5-3-1-3001-103-105-205-203-302-3000	38	278	202
	1A	0-62-12-3002-3001-107-3000	32	251	229
	2A	0-23-9-15-115-3001-46-3000	18	251	178
	3A	0-24-11-13-22-3001-55-3000	37	252	228

The maximum service time difference amongst teams of the same type, in a day *t*, is verified on Thursday, with a variation of about 46 minutes, representing a reduction on this value of approximately 43% (annex C.4).The removal of the WLC is proved, once again, to considerably decrease the quality of the solution. It is also interesting to notice that comparing scenarios 2.B

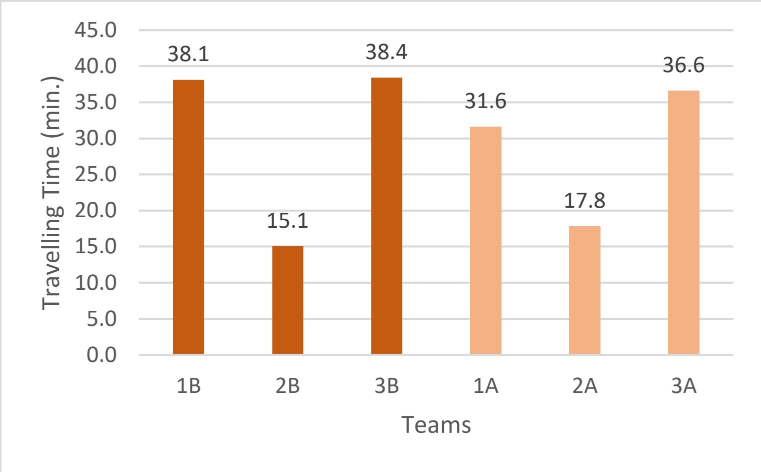


Figure 6.14: Wednesday Travelling Time for the Solution of Scenario 2.B

and 2.A (though Figures 6.14 and 6.12) the traveling time, a criteria encompassed in the workload optimization, is slightly more balanced. The same tendency is verified for every other day of the week.

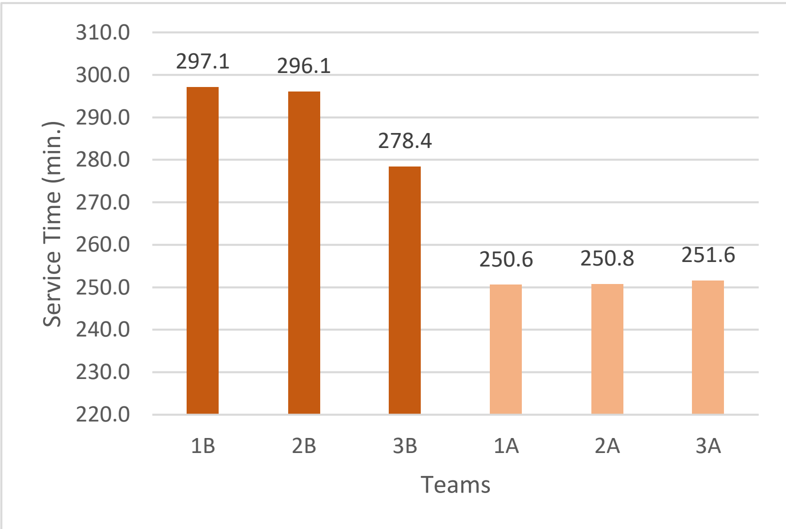


Figure 6.15: Wednesday Service Time for the Solution of Scenario 2.B

Second objective function values and interpretation

Table 6.10: Second Objective Function's values associated to both the solutions with and without respecting the WLC. #P = Number of penalties; n.d. = non-defined..

Work balance Optimization – 2nd OF								
Week Day	Respecting weekly loyalty				Without weekly loyalty			
	Bedridden		Autonomous		Bedridden		Autonomous	
	OF value (min.)	#P	OF value (min.)	#P	OF value (min.)	#P	OF value (min.)	#P
Monday	328,7	2	275,0	0	328,7	2	253,8	0
Tuesday	341,9	2	287,2	1	302,4	1	268	0
Wednesday	321	2	272,6	0	298,4	1	251,6	0
Thursday	337	2	259,4	1	311,7	2	241,2	0
Friday	337	2	273,0	0	308,9	1	273,0	0
Saturday	346,8	0	n.d.	n.d	346,8	0	n.d	n.d.
Sunday	348,4	0	n.d.	n.d.	348,4	0	n.d	n.d

Table 6.10 presents the values of the second objective function, which aims at balancing the workload amongst the teams. The workload is considered as the sum of the traveling time and the service time. The output values of this objective function, present in the table, are the maximum value of workload among the three teams of the respective day. Similarly to the Table 6.7, a comparison between the model with and without the heuristic is performed. This time, it is clear that the addition of the weekly loyalty constraint leads to an increase in the value of the objective function, meaning that the maximum value of workload is increased, what was expected. Respecting the WLC generates asymmetries in the distribution of the workload, resulting in the increase of the maximum value of workload attributed to one of the three teams. The values of the penalties either increase or do not change, in what concerns the integration of the weekly loyalty constraint. Once again, this situation is caused by the heuristic chosen, with the same reasoning used for the explanation relative to the first OF.

Despite the conclusions retrieved from Table 6.10, it is not enough to assure the correct functioning of the second OF. Its benefit is appropriately evaluated through the analysis of an indicator named route balance, which will be disclosed in the subsections ahead.

6.6 Discussion

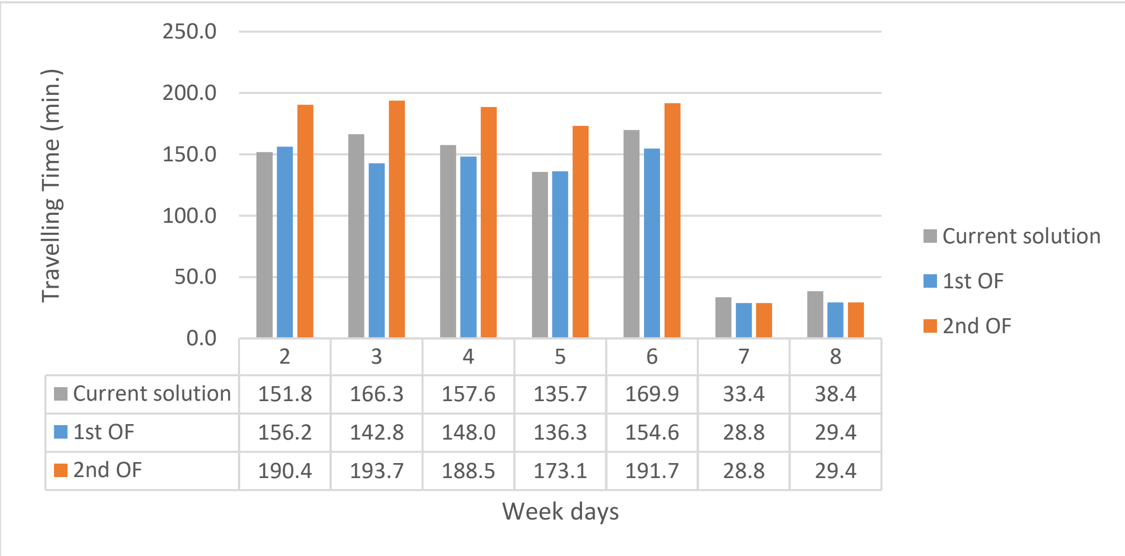
A more detailed analysis of the scenarios can be executed through the introduction of a calculated indicator. The indicators which will be approached are the travelling time, the indicator of route balance and the waiting time.

6.6.1 Travelling Time

Assessing the solution benefits for the institution demanded a weekly analysis of the daily traveling times, due to different natures of the routes of the current solution and the ones newly obtained. In the current solution, there are more routes than available teams, suggesting that the routes are segmented and, from one week to the other, those segments might not be performed by the same team, as one single route. A smaller route, for example route y, is not consistently performed by the same team that performed it in the previous week, being realized by either teams from routes x, y or z.

In addition, within a day, a team member of a team of two assistants may be included into two different teams of two assistants. As a result of such discrepancies, the only possible comparison amongst the current and the obtained solutions is the daily traveling time. In Figure 6.16, the values needed for the previous comparison are displayed.

Regarding the first OF, there is a clear decrease in the daily traveling time with respect to the

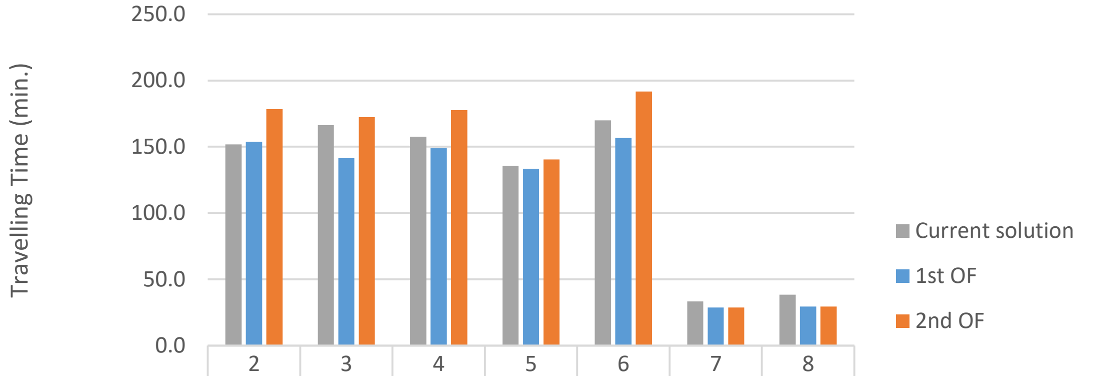


	2	3	4	5	6	7	8
Current solution	151.8	166.3	157.6	135.7	169.9	33.4	38.4
1st OF	156.2	142.8	148.0	136.3	154.6	28.8	29.4
2nd OF	190.4	193.7	188.5	173.1	191.7	28.8	29.4

Figure 6.16: Daily traveling time of the solutions obtained for both OF respecting the WLC, in comparison with the current solution. Week days are represented by their values of t index, in which 2 is Monday and, continuing orderly, 8 is Sunday.

current solution. In the week studied, for 5 days out of 7 the TT is reduced, verifying the adequacy of the first objective function and of the model developed. Focusing now on the second OF, the results in terms of TT are worse when compared to the current solution, resulting in an increase of TT for every day of the week. Nevertheless, the objective of the function is to balance the workload, an aspect addressed in the following sections.

As a matter of academic interest, the model was also run without respecting the WLC. Expectations were that under constraining the problem would result in a significant decrease of the daily TT, and it was verified. The effect is more relevant for the second OF, as can be observed by comparing the tables in Figures 6.16 and 6.17.



	2	3	4	5	6	7	8
Current solution	151.8	166.3	157.6	135.7	169.9	33.4	38.4
1st OF	153.7	141.3	149.0	133.5	156.6	28.8	29.4
2nd OF	178.4	172.3	177.6	140.5	191.7	28.8	29.4

Week days

Figure 6.17: Daily traveling time of the solutions obtained for both OFs, in comparison with the current solution, without respecting weekly loyalty between team and patient. Week days are represented by their model values for the t index, in which 2 is Monday and, continuing orderly, 8 is Sunday.

Weekly analysis of the scenarios

The analysis of the weekly travelling time grants a holistic view of the overall benefit of the solutions found, taking into account the greatest worry of the institution: reducing the TT. In Figure 6.18, two solutions are shown to have improved weekly TT, when compared with the current solution, namely the two originated from scenarios 1.A and 1.B. They both represent a decrease of 7%. Considering that not only the time is reduced, but all of the constraints proposed by the institution are accounted for, these are promising results. Notice that in the current solution bedridden patients may be visited by teams for the autonomous patients. On the other hand, however, the solutions found for scenarios 2.A and 2.B are worse than the common practices at the moment. For scenario 2.A there is an increase of 8% of the weekly TT, and for the last scenario the raise in quantity of total traveled time is of about 17%.

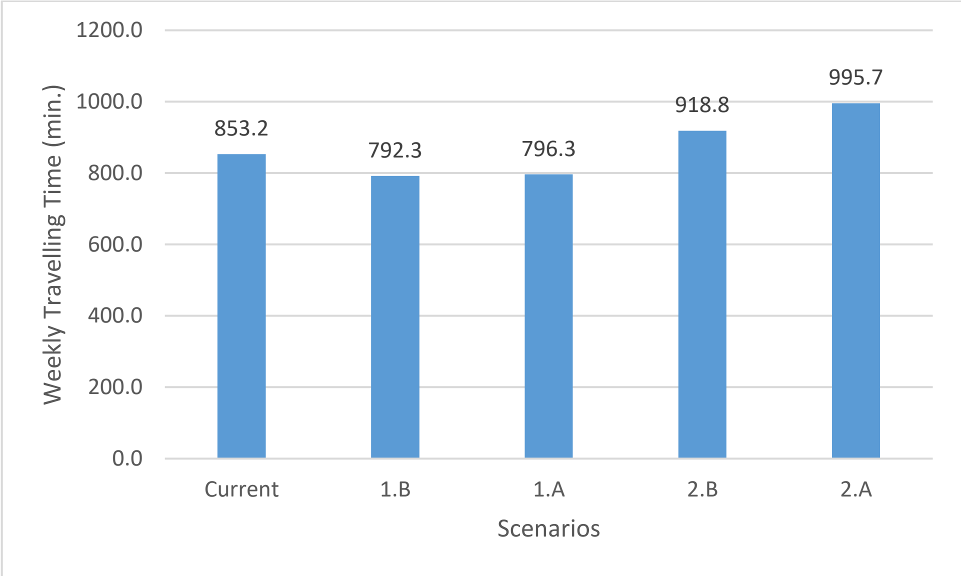


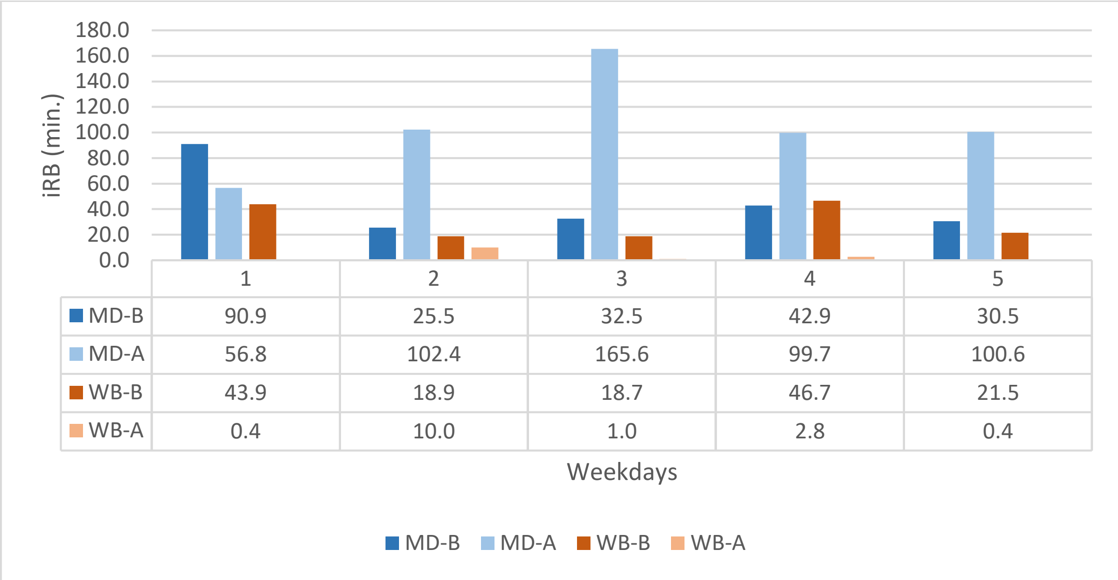
Figure 6.18: Weekly Travelling Time, for each of the four scenarios

6.6.2 Workload Balance

The nature of the second OF impacts on the variation of the teams' service times, in an attempt to balance them. As such, the most appropriate method for assessing its adequacy is through the analysis of the variation in the difference between the team with the maximum and the team with the minimum value of service time, within the same type of team and for the same day of the week. Consequently, the indicator of route balance (equation (21)) is presented in Figure 6.19.

The indicator of route balance confirms the correct functioning of the second OF. When the indicators from both the first solution and the second without WLC are compared, there is always a decrease in the value of the indicator, except for the B-Teams on Thursday. (Figure 6.19)

The lowest value of the indicator is verified for the A-Teams on Friday, an expected consequence of scenario characteristics (smaller number of patients with replicas and the smaller number of replicas associated to patients) and, once again, of the solution methodology. The daily loyalty constraint, a soft-constraint modelled by both objective functions, places a heavier influence on the B-Teams. While for the autonomous patients the maximum number of patients that need more that one visit is 2 (at most two replicas), the bedridden patients present 6 patients with a visit frequency needs varying between two and four times per day. This scenario discrepancy between the two patient types justifies the lower values verified for autonomous patients instead of the ones found for bedridden ones.



	1	2	3	4	5
MD-B	90.9	25.5	32.5	42.9	30.5
MD-A	56.8	102.4	165.6	99.7	100.6
WB-B	43.9	18.9	18.7	46.7	21.5
WB-A	0.4	10.0	1.0	2.8	0.4

Figure 6.19: Indicator of Route Balance for the two objective functions without the WLC. MD-B – Value of iRB for the solution Minimizing the Distance for the Bedridden patients; MD-A – Value of iRB for the solution Minimizing the distance for Autonomous patients; WB-B - Value of iRB for the solution optimizing Workload Balance for the Bedridden patients; WB-A - Value of iRB for the solution optimizing Workload Balance for the Autonomous patients

Respecting the weekly loyalty constraint, introduced by the solution methodology, also increases the value of the indicator under analysis, similarly to what was found for the TT indicator. When the third step of the solution methodology is performed for the second time, some of the patients and replicas have already been assigned to a team, restricting the problem. The latter situation significantly diminishes the ability of the model to balance the workload of the three teams. For the autonomous patients, the day with which the solution methodology is initiated is Friday. Consequently, this week day presents the lowest value for the indicator. In order to enhance the support to this reasoning, Figure 6.20 is presented, in which the values for the same indicator are displayed, except that, in turn, this are the values obtained when the weekly loyalty constraint is disregarded. In it, the maximum value for the iRB is 158 minutes.

Comparing Figures 6.19 and 6.20 confirms the logic used to justify the connotation of the weekly loyalty constraint as a factor with major impact in the indicator of route balance, diminishing the model's capacity for producing routes with a more equitable distribution of workload. The IRB has a higher value for the first OF, practically every day of the week and for both team types. There is only one situation in which it is not verified, namely on Thursday, for the bedridden patients teams. After a thorough analysis comparing the two solutions, it was perceived that one of the visible differences were the number of penalties, within the bedridden patients of the second solution . For most of the B-Teams in the second solution there is a decrease in the number of penalties, when the WLC is removed (Table 6.10), except for Monday and Thursday. Monday had already been observed to had two penalties in the solution of the first OF without WLC, but on Thursday the number of penalties is 1. The immediate line of though was that altering that difference in penalty numbers could possibly lead to an approximation of the indicator values. As such, the model was run for the second OF altering the value of the P. The first value for which

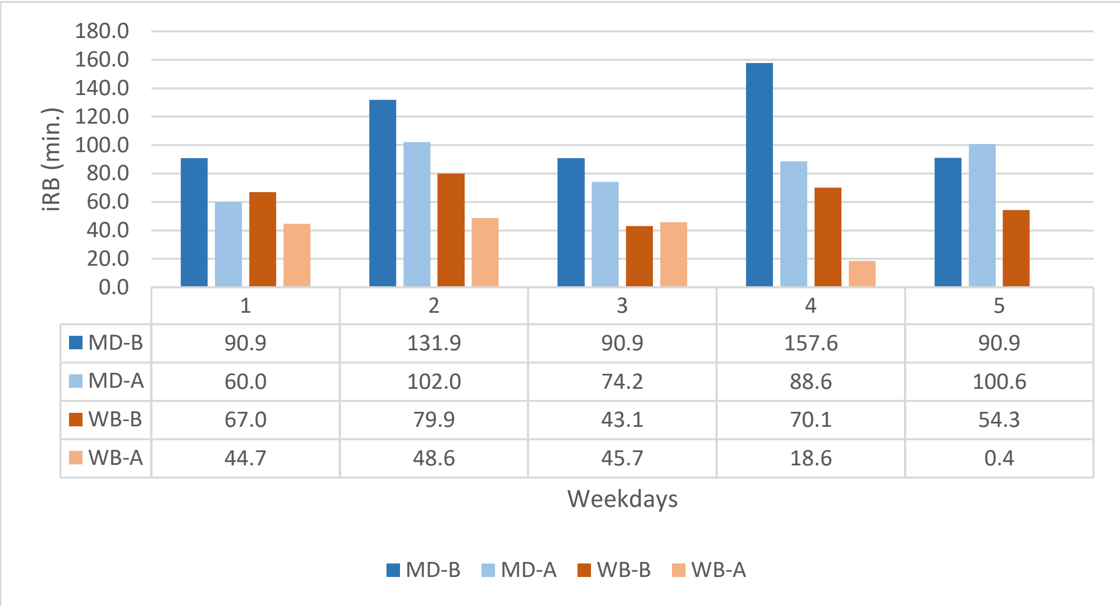


Figure 6.20: Indicator of Route Balance for the two objective functions with the WLC. MD-B – Value of iRB for the solution Minimizing the Distance for the Bedridden patients; MD-A – Value of iRB for the solution Minimizing the distance for Autonomous patients; WB-B - Value of iRB for the solution optimizing Workload Balance for the Bedridden patients; WB-A - Value of iRB for the solution optimizing Workload Balance for the Autonomous patients

the penalty number attained the value ones was 25. The number of penalties decreased to one and the value of the indicator 40 minutes, closer to that of the value found for the first solution B-Teams without WLC.

The work schedule

In the course of the results analysis, much emphasis has been placed on the routes' indicators, but their duration is yet to be analyzed. The most relevant aspect to confirm is the respect for the labor time constraint, which states that the routes must no surpass 480 minutes (the maximum working time). The compliance of this institutional rule can be visualized in Table 6.11 It incorporates both the route's total time and an indicator of variance similar to the indicator of workload balance. However, instead of evaluating the difference in terms of the total service time, does it for the route's duration with regard to the total working time, called indicator of route duration balance (iRDB), defined in equation (22).

Generally, there appears to be a tendency for the iRDB value to decrease from the first solution to the second. However, this indicator might be misleading, especially when remembering that the schedules will be put into practice in an organization. Comparing between the difference in schedules is common amongst coworkers, and despite not being considered to assess the quality of the routes obtained, its analysis possesses relevance.

Two opposite situations that reveal how it can be deceiving are the cases of the Monday B-Teams, for the traveling time OF, and the Friday A-Teams, regarding the workload OF. In the first situation, the value of the indicator of route duration balance is 0 minutes, leading the workers to believe that the work is quite well distributed, however, revisiting the table present in Figure 6.20 it is possible to verify that the difference in service time is of 90 minutes, representing one extra hour of service time. On the other hand, for the second objective function, Friday presents a dreadful value for the indicator (138 minutes), one of the worst found for either the first or the second objective functions. Nevertheless, in Figure 6.20 is possible to observe that the values of service time for each of the teams is exactly the same, fixed at 273 minutes (Annex C.3), within the proposed routes regarding workload.

Table 6.11: Route duration and Indicator of Route Duration Balance. n.d.- non-defined

Week days	Route Duration and iRDB				
	Team type	1st OF (min.)		2nd OF (min.)	
		Duration	IRDB	Duration	IRDB
Monday	1B	480	15,2	480	0
	2B	465		480	
	3B	480		480	
	1A	401	124,6	400	82,4
	2A	320		320	
	3A	445		403	
Tuesday	1B	480	17,2	408	72,26
	2B	463		433	
	3B	466		480	
	1A	374	145	362	40,2
	2A	321		321	
	3A	466		331	
Wednesday	1B	480	18,2	473	21,4
	2B	462		459	
	3B	478		480	
	1A	275	148,2	387	59
	2A	326		390	
	3A	423		331	
Thursday	1B	443	20,8	473	6,96
	2B	464		480	
	3B	459		480	
	1A	438	172	429	163,2
	2A	266		266	
	3A	426		331	
Friday	1B	480	16,2	430	52
	2B	464		428	
	3B	478		480	
	1A	388	64	463	138,8
	2A	324		324	

	3A	370		378	
Saturday	1B	460	n.d	460	n.d
Sunday	1B	460	n.d	460	n.d

In APOIO teams are meant to rotate amongst schedules, this will reduce the impact of the inequality in terms of total route duration. However, when schedules are allocated to a real team, some human resources conflicts might arise. In the studied problem, the difference in terms of working schedule, within the same type of teams and considering a single day of the week, varies between 0 hours and up to 2 hours and 52 minutes. Three hours have a very significant impact on the life of any worker therefore, from a worker's perspective, it would be important to decrease this discrepancy.

The origin of the divergence on the routes durations heavily relies on the third type of time associated to the routes, the waiting times. Although this time is not modeled in any of the objective functions, it is analyzed as an indicator in the following subsection.

6.6.3 Waiting Time

Route's waiting times are frequently not considered in optimization problems. The main reason is that employees are paid on a per month basis and minimization of waiting times or its equal distribution amongst the fleet workforce is not as significant as reducing the costs of traveling or maximizing the service delivery capacity. However, since the waiting times are a component of the routes resulting from the need to respect the patient's time windows availabilities, it greatly influences the duration of the route, as approached in the previous subsection. A general notion of the time spent on waiting by the teams can be obtained by analyzing the Figure 6.21, derived from the first OF and Figure 6.22 elaborated with the results obtained from the second OF.

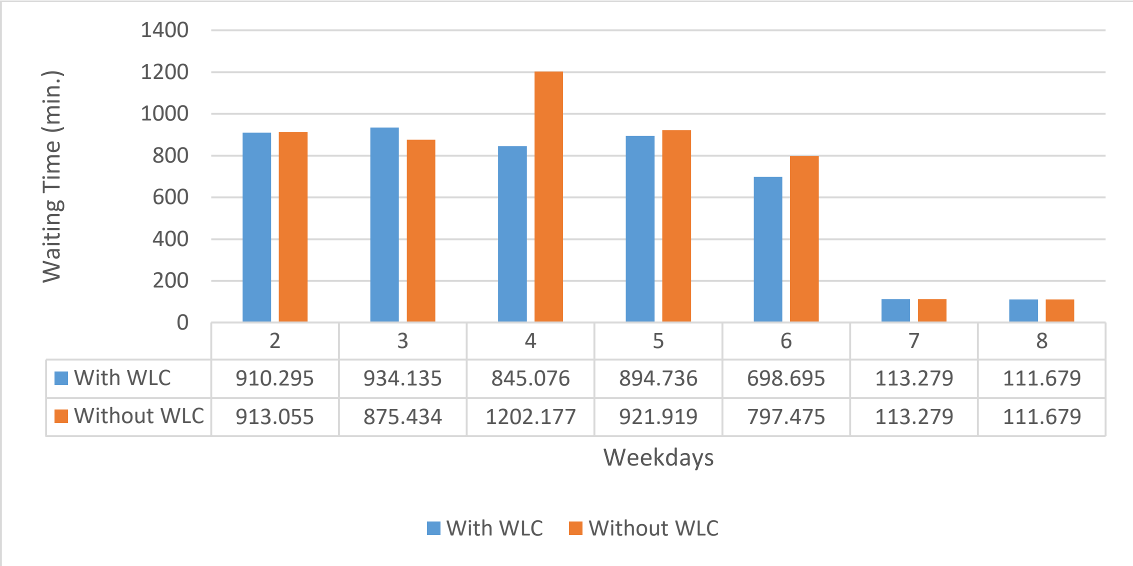


Figure 6.21: Daily Waiting Times for the First Objective function.

For the solutions attained from the first OF, the one respecting the weekly loyalty constraint is constituted by routes which, aggregately, amount to a total waiting time of 4596 minutes, representing for about 30% of the total time available to the teams. In turn, the solution that ignores that same constraint has the value of 4935 minutes, corresponding to approximately 32% of the 15360 minutes available for the teams to develop their activities.

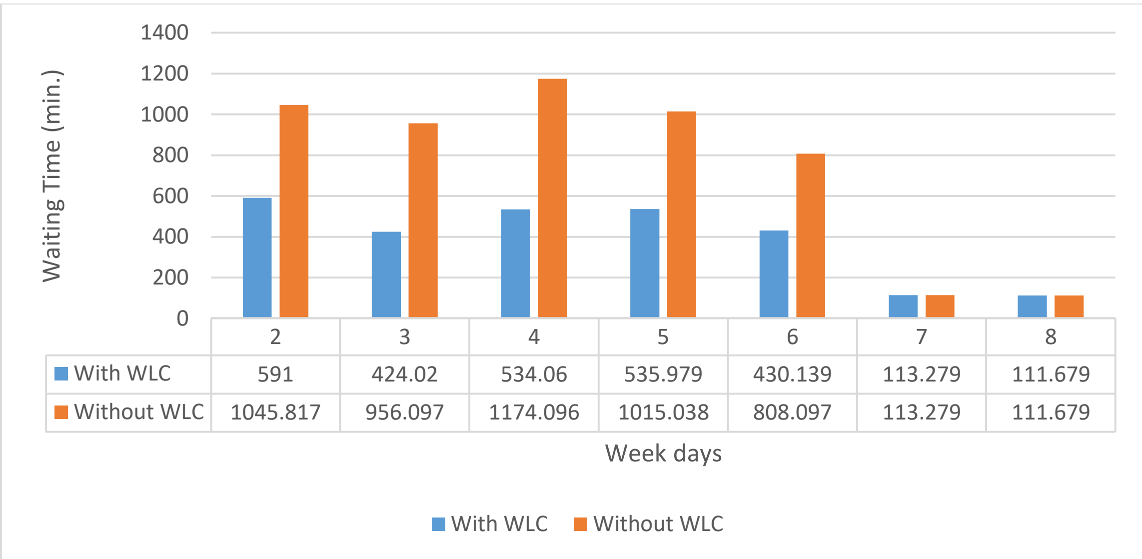


Figure 6.22: Daily Waiting Time for the Second Objective function.

Regarding the solutions associated to the second OF, an increase in waiting time is obvious when the model does not respect the WLC. The main reason is the fact that without the restriction the model is free to balance the workload resulting in the addition of waiting time. As a consequence, the duration of the routes is enlarged so that a more balanced route is achived. In turn, the solution respecting the WLC, some of the patients to visit have already been preassigned and the freedom of the model to add waiting times is, therefore, limited.

Reconsidering the differences in duration and the indicator of route duration in Table 6.11, for the A-Teams on Friday, the difference of 138,8 is due, in a considerable part, to waiting times. In the Annex C.3 it is possible to verify that the difference between the waiting times of the team with the highest and lowest route durations is of 138 minutes. This result shows that the total difference in duration is associated to waiting times, rendering this indicator as very relevant when designing routes with balanced durations.

6.6.4 Computational Results

The results obtained for all of the four scenarios were implemented resorting to the software GAMS, installed in a computer with a processor Intel(R) Core(TM) i7-3537U @ 2GHz with 8GB of RAM memory. In Table 6.12 it is possible to analyze some of the characteristics of model's runs, namely, for example, the number of variables and constraints, as well as the execution time, which is the time interval necessary for the model to generate an output., with the results associated to scenario 1.B.

Table 6.12: Computational results for scenario 1.B

Patient Type	Day	Nb. Variables	Nb. binary variables	Nb. Of constraints	Iterations	Execution time (s)	GAP (%)	Optimal solution
Auton.	2	939	861	1,002	14184	1,281	0	76,833
	3	582	483	589	479	0,281	0	62,604
	4	555	489	595	1185	0,609	0	70,261
	5	546	450	549	901	0,5	0	54,002
	6	1086	999	1154	1874	0,359	0	77,843
	7	-	-	-	-	-	-	-
	8	-	-	-	-	-	-	-
Bedr.	2	1569	1155	1345	206366	47,344	0	276,782
	3	1581	1179	1348	192488	23,722	0	178,722
	4	1578	1182	1351	84930	22,937	0	178,722
	5	1581	1185	1354	120952	22,343	0	179,521
	6	1569	1173	1342	47688	9,5	0	178,722
	7	95	73	85	37	0,125	0	28,841
	8	80	61	70	28	0,125	0	29,441

Since the value found for GAP for all the runs is 0, every solution presented throughout the work was an optimal one. In addition to being optimal, the solutions are obtained in a reasonable amount of time, as observed in column Execution time. Due to the complexity inherent to the sub-problem of the bedridden patients, they present an execution time generally larger than that verified for the autonomous patients. An exception is the weekend, a situation in which the problems complexity is greatly reduced by the low number of patients requiring services and for the fact that only one teams works on the weekend.

7. **Conclusions and Future Work**

7.1 Conclusions

One of the most significant global megatrends responsible for shaping nowadays society is population aging. Its origins lie on two major factors, the first being the increasing life expectancy, showing no signs of slowing down, and the second associated with the decreasing fertility rates. Accompanying aging is the increase in the prevalence of non-communicable chronic diseases that present consequences at the level of diminishing autonomy. One of those diseases that is most immediately associated with old age is dementia. Dementia causes, for example, memory loss and disorientation, two symptoms that considerably interfere in the patients' ability to perform the activities of daily living autonomously. The solution to this problem frequently resides in the requisition of services to institutions that provide HHC. In Portugal, the provision of long-term care and, in specific, formal care is of the responsibility of IPSS, which are non-profitable institutions mostly funded by the social security and a negligible percentage of co-payments. The non-profit nature of the institutions in addition to a cumulative waiting list of patients hoping to enroll generates a need to optimized management decisions. In some institutions, one of the dimensions in which the decision process is inefficient is the routing and scheduling of assistant teams. Generally, institution's administration staff design the routes manually. The aim of these study is thus to propose and validate a mathematical model to be used as a tool to support route design.

The validation of the mathematical model proposed is attained by resorting to the instances provided by APOIO, the partner institution. The model provides as outputs both the sequence of the visits that should be performed to patients and the times at which each visit should start. The model respects a series of institutional operational constraints, namely:

- One hour lunch breaks which can begin anywhere between 12 p.m. and 2p.m.;
- Lunch distribution, occupying 90 minutes and starting precisely at 12 p.m.;
- Modeling a labor time inferior to the functioning time of the institution;
- Preference of the assignment of a single team to a patient, whenever possible (daily-loyalty).

The MP-VRPTW model is extended to accommodate the aforementioned constraints. In order to do so, it is characterized by hard time windows, which obliges the visit to nodes to be initiated within the time interval it defines. Actually, all constraints are hard constraints, except for the one modeling the daily-loyalty. The reason for being modelled as a soft-constraint resides in the fact that there might be cases in which the difference in time between the first and last visit time windows may be large that the working time of a team is not sufficient to perform both visits. Such situations result in the attribution of two teams to one patient.

The MILP was implemented in GAMS and solved resorting to CPLEX. Due to the heavy computational memory requirement, it was imperative to use a heuristic to solve the model. The first part of the heuristic was to run the model separately for the two typologies of teams: the bedridden patients’ teams and the autonomous patients’ teams. Then, within each team type, the model was run for every day of the week. In that sense, it was assured that the bedridden patients were always assigned to teams of two patients, as asked by the institution. Interestingly, since the patients’ typologies distinguish them to the point of being either on the bedridden side or on the autonomous side, it was almost as if the model had been run for two problems with different characteristics: in the bedridden side, there are less patients, but with more replicas, when compared with the autonomous side. For that reason, the effects of the broken daily loyalty constraint are more relevant in the bedridden side. The second part of the heuristic involved the assignment of patients to teams. The assignment consisted of four steps process in which the model was run for the days, which had previously been ordered according to the number of patients to visit, and the patients are fixed to the teams. Whenever a patient is fixed to a team, the allocation transits to all the other days ranked below. This second stage of the heuristic is actually the function that ought to be realized by weekly loyalty constraint, the fourth equation in the model’s constraints. However, due to the previous stage, the only constraint that has to receive inputs from different days of the week cannot be used.

Despite the possibility of the obtaining sub-optimal results, the solutions obtained are quite satisfying. For the scenarios associated with the first objective function, which aims ate reducing weekly traveled time, both with and without the WLC, there is a reduction on the weekly traveled time. The scenario with WLC presents a reduction of 7% of the TT, equal to that of the scenario without WLC. The latter was an interesting result, since it was expected that the addition of a constraint would worsen the value of the reduction. Actually it does, the soft-constraint which penalizes the objective function worsens the solution’s value but, in turn, permits the amount of weekly TT to be slightly increased (A-Teams) or even reduced (B-teams). The optimization of the TT was the objective of the institution, and it was optimized when compared with the current solution.

The results regarding the second objective function are of a more difficult assessment. It was not possible to have access to the current solution’s service time. As such, the two were compared in terms of weekly TT, for which the results verified and increase of 8% without WLC and an increase of 17% when the same constraint is enforced. Nonetheless, the objective function aims at balancing the workload of the teams, thus making the comparison unsuitable to assess the correct functioning and utility of the function. It was then decided that a new indicator should be created, discriminating the difference regarding ST. The results clearly show that there is an improvement in the values of the indicator from the first to the second objective function. In addition, there is also an improvement in the indicator when the WLC is enforced.

7.2 Future work

The first suggestion of work regards the model presented. A study of a form to implement the second function based on the indicator of route balance being defined as the sum of the differences among teams’ service times. Of course, it would involve modulus, because the difference between the teams' service time can be either positive of negative, and that quantity should be minimized. However, I am familiar with the fact that it is not easy to model the idealized objective function in GAMS, what would probably require some time to investigate strategies to transform the equation into something that the GAMS can work with.

The second suggestion arises from the knowledge that APOIO is currently implementing a software that enables the insertion of arrivals and departures to patients' homes directly into a database, enhancing the accessibility of the data, which will no longer be stored in paper records. This also permits the monitoring of patients and of the uncertainty to them associated. The newly available type of details is exactly what is needed to develop a Stochastic HHC routing and scheduling problem. In spite of being common practice in other areas of problems formulated as VRPs, in HHC there is very little work done in the area as is stated by Fikar and Hirsch (2017).

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Annexes

A. Sensibility analysis for Wednesday

Minimizing Travelling Time

The first data to be analyzed was the variation of the number of penalties as a function of the value of the penalty parameter associated to the first OF. Some of the acquired information is available in Figures A.1 and A.2. Focusing in Figure A.1, when penalty value is of 8 minutes the number of penalties of the solution hits its lowest value, one penalty, having been verified that the solution remains also unchanged for the greater values of P. The value from which the solutions start to present a constant number of penalties is called variation limit. On Figure 6.3, the same situation is observed, with a variation limit of 12 minutes. In the Figure 6.2 associated to the B-Teams the number of penalties never assumes the value zero because one of the patients has replicas that need to be visited within an interval so extended that the working schedule of a team is not sufficient for the same team to perform both visits. It can be argued that the unsuppressed penalty is a result of excessive workload that can only be served by allowing one penalty. However, even when the model is run with 4 B-Teams, there is still one penalty, refuting this reasoning.

The results obtained were expected. The Value fo P does not affect the routes directly, having an

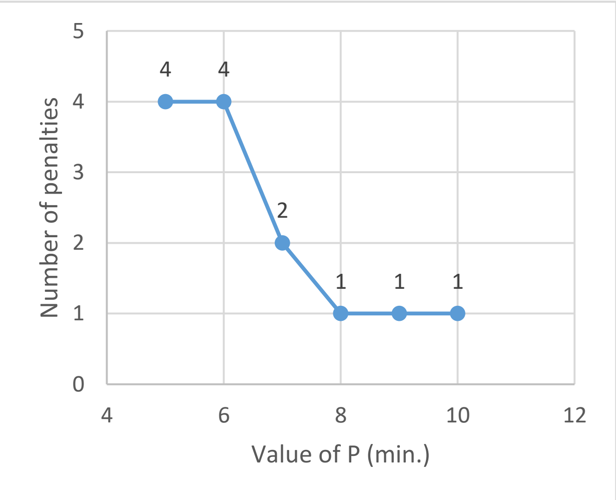

Figure A.1: Variation of the number of penalties as a function of the Penalty value associated to the first OF, B-Teams, without WLC, Wednesday.

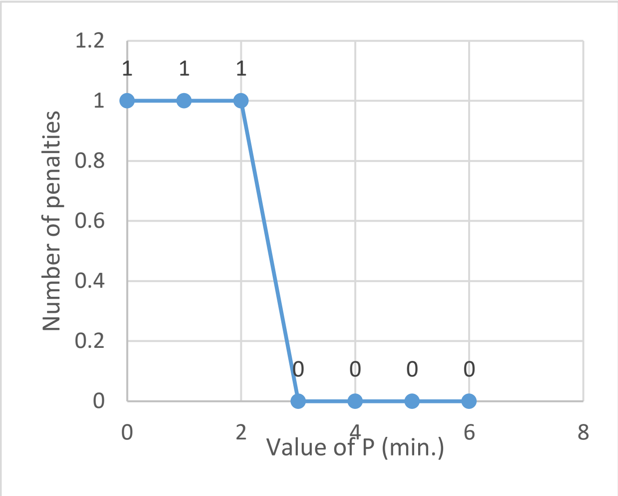

Figure A.2: Variation of the number of penalties as a function of the Penalty value associated to the first OF, for A-Teams, without WLC for Wednesday.

impact on the objective function as a whole. Sets of routes presenting a more favorable result in terms of TT but with too many penalties are discarded, assuming that the Value fo P is above the variation limit. The value for the penalties added to the discarded solution must surpass the gap in TT of the solution with no, or less, penalties. The value of the variation limit for the penalties will therefore vary with the characteristics of the problem. However, if the Value fo P is given a value great enough, for example 100 minutes, the value will most likely have surpassed all of the limits associated to each day and to the different team types. Notice that most days total TT never reaches 100 minutes, which means that not serving a replica will result in the addition of the equivalent to another route, placing an heavy preference on assigning the same team do all the replicas. That value even facilitates the process of determining the number of penalties associated to that day.

When the analysis moves to the solutions that respect the WLC, what is observed is the absence of variation. For the B-Teams the number of penalties is 2 for values of P extending from zero to one hundred, whereas for the A-Teams, for the same extension of values for P, the number of penalties is fixed at zero. This stability is due to the primary assignment of patients, which leaves a reduced margin for the model to either increase the number of penalties by decreasing the value of P or to decrease the number of penalties by increasing the value of the parameter.

In terms of the variation of iRB, similarly to Monday, for the B-Teams, after the variation limit of 8 minutes (Figure A.1) is attained, the iRB value is fixed at about 32 minutes. When the A-Teams are concerned, the value of iRB remains constant at 247 minutes, after the variance limit of 3 minutes (Figure A.2) is attained. Changing the focus towards the variation of iRB for scenario with the WLC, for both the B-Teams and the A-Teams the iRB indicator is kept constant, from a value for P equal to zero, at approximately 43 and 74 minutes, respectively. The former results enforce that, for the first OF, after a variation limit for the value for P, both the number of penalties and the iRB stabilize, reaching its lowest values and thus, any value greater than the variation limit can be used to obtain optimal results.

Workload Balance

A similar sensitivity analysis is performed for the two quantities, number of penalties and iRB, but, in this subsection, associated to the second OF. Focusing on the B-Teams respecting WLC, Figures A.3 and A.4 were elaborated.

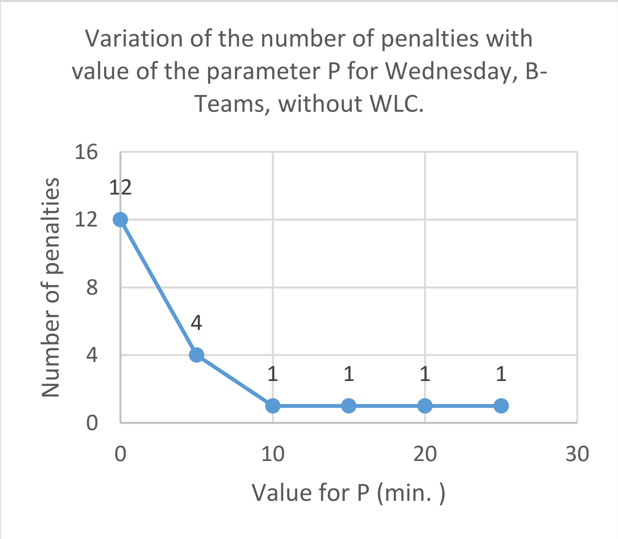

Figure A.3: Variation of the number of penalties as a function of the Penalty parameter value associated to the second OF, for B-Teams, without WLC, Wednesday.

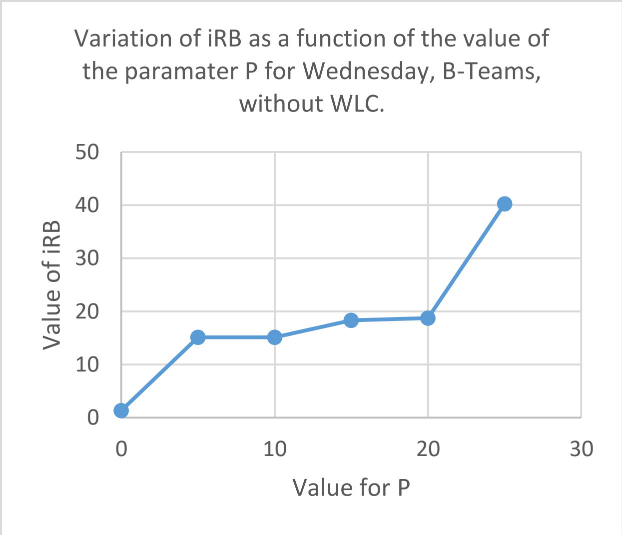

Figure A.4: Variation of the IRB as a function of the Penalty parameter value associated to the second OF, B-Teams, without WLC. Wednesday.

In Figure A.3, as the value for P increases, there is a tendency for the number of penalties to decrease. However, in contrast with what was observed for the first OF's analysis, the solution quality deteriorates as the parameter value increases which means that, even though the number of penalties is stabilized, the solutions are not constant. The reason for the detected behavior is that, in the second OF, the objective is to minimize the maximum value of the service time of verified within the three routes being designed. The penalty will directly influence the routes, attributing them more service time than what they are actually performing. The attribution of patients to the routes is conducted as though those minutes of penalty were service time, manipulating the distribution of the workload by attributing more service time to teams with no penalties, thus deteriorating the solution. When the WLC is respected, both the number of penalties and the iRB are constant from a value for P equal to zero, at 2 and 43 minutes, respectively. Similarly to the reason for the same stability in the first OF, the pre-assignment of the patients to the teams strongly conditions the influence of the value for P on the solutions.

For the A-Teams, with and without WLC, the values of iRB are always constant at 46 and 1 minutes, respectively, restating the decrease in the quality of the solution with the introduction of the WLC. In terms of penalties' number, when the WLC is respected, the number is kept at zero and when it is ignored, the variation limit for the parameter is 1, corresponding to zero penalties. This is essentially due to the reduced number of replicas, as is explained with more detail in chapter 6.5.

Nonetheless, there are also situations in which the variations are not so consistently behaved as what is presented in Figure A.3 and A.4, for example, on Monday B-Teams, for the value of P of 9 and 10 the number of penalties is of 3 and 4, opposing the generally decreasing tendency (Figure A.5). For the same day, the IRB for a value of P of 3 is 21 minutes, whereas for a P of 4 the iRB is 12,6 minutes, not verifying the increasing tendency (Figure A.6). This illustrates the

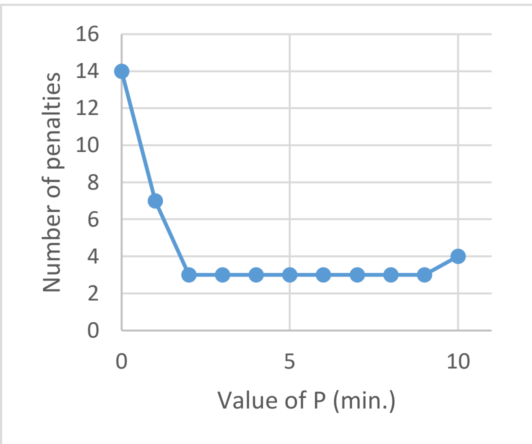

Figure A.5: Variation of the number of penalties as a function of Value fo P for the second OF, Monday, B-Teams, with/without WLC, in detail for P∈[0,10]

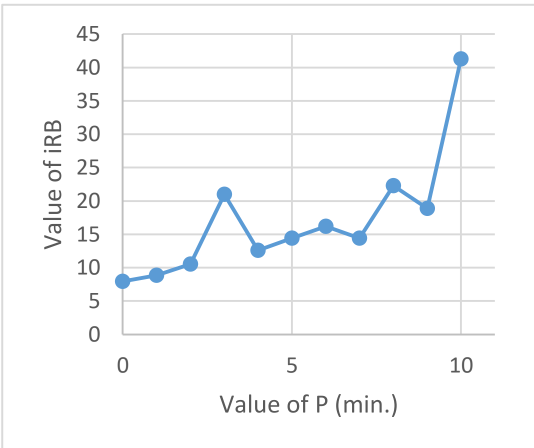

Figure A.6: Variation of iRB as a function of the Value fo P for the second OF, Monday, B-Teams, with/without WLC, in detail for P∈[0,10]

difficulty of establishing a general value for P for the second OF; for a constant number of penalties, the iRB can vary, a situation not verified for the first OF.

B. APOIO's Current Solution

Table B.1: Current solution operated at APOIO

Current Solution				
Week days	Team	Route	TT (min)	TOTAL
Monday	1+2	0-2-4-5-3-1-58-59-57-3000	21,2	151,84
	4	0-23-60-24-62-61-13-2-3000	24,84	
	5+6	0-56-57-58-2-3-5-3000	21,1	
	7	0-9-11-12-3000	28,8	
	6	0-57-55-15-3000	2,02	
	1+2	0-6-3000	13,6	
	8	0-15-3000	14	
Tuesday	1	0-9-3000	15,2	166,34
	5+6	0-56-57-58-59-3000	5,66	
	6	0-2-3000	14,4	
	4	0-23-24-13-3001-23-3000	19,4	
	2	0-10-15-14-62-3000	20,68	
	1+2	0-2-4-5-3-1-58-59-57-3000	26,28	
	3+5	0-2-3-5-3000	18,6	
	3	0-55-7-3-5-2-3000	24,2	
	2+5	0-6-2-3000	8,4	
	5	0-57-6-3000	13,52	
Wednesday	1+2	0-9-2-4-5-3-1-3001-58-59-57-3000	28,68	157,64
	5	0-57-62-61-15-55-3000	5,7	
	9	0-1-3000	14,4	
	3	0-46-7-3-5-2-3000	21,2	
	2+5	0-6-3000	13,6	
	9	0-23-24-11-13-12-3000	22,4	
	8	0-15-3000	14	
	6	0-6-3000	13,6	
	5+6	0-56-57-58-59-3001-2-5-3000	24,06	
Thursday	1+2	0-2-4-5-1-58-59-3000	21,76	135,66
	6	0-57-15-6-62-3000	1,84	
	3	0-50-3001-55-5-2-3000	20,2	
	4	0-23-24-13-7-2-3001-24-3000	24,4	
	2	0-1-3001-6-3000	28	
	5+6	0-56-57-58-59-3001-2-3-5-3000	24,26	
	1	0-9-3000	15,2	

Table B.2: Current solution operated at APOIO- continuation

Week days	Team	Route	TT(min)	Total
Friday	1+2	0-2-4-5-3001-58-59-6-3000	40,96	169,86
	5+6	0-56-57-58-59-3001-3-2-3000	20,86	
	9	0-23-24-11-12-3000	23,4	
	6	0-57-3-15-14-13-6-62-3000	3,84	
	2	0-9-3000	15,2	
	3	0-51-7-6-3-3000	16,8	
	1	0-10-3001-2-3000	32,4	
	10	0-55-3000	16,4	
Saturday	3+8	0-57-55-2-3-59-57-3000	19,04	33,44
	3	0-2-57-3000	14,4	
Sunday	1+3	0-57-2-3-59-57-3000	17,64	38,44
	3	0-55-2-3000	20,8	
TOTAL				853,22

Model Solutions

C.1 Scenario 1.A Solution

Week Day	Route nb.	Route	TT (min.)	ST (min.)	WT (min.)
Monday	1B	0-3-102-1-3001-202-103-105-205-203-302-3000	37	281	199
	2B	0-56-58-3002-3001-158-3000	4	277	188
	3B	0-57-2-6-4-5-59-3001-157-257-106-159-3000	36	368	112
	1A	0-60-61-9-12-15-115-3001-17-16-3000	39	254	147
	2A	0-3002-3001-19-18-55-107-3000	20	220	100
	3A	0-23-24-62-11-13-3001-48-3000	20	280	165
Tuesday	1B	0-3-102-1-3001-202-103-105-205-203-302-3000	37	280	200
	2B	0-56-58-3002-3001-158-3000	4	227	236
	3B	0-57-2-6-4-5-59-3001-157-257-106-159-3000	36	359	107
	1A	0-10-9-15-115-3001-50-3000	27	258	116
	2A	0-3002-3001-55-107-3000	18	205	116
	3A	0-23-24-62-13-14-3001-123-3000	21	307	159
Wednesday	1B	0-3-102-1-3001-202-103-105-205-203-302-3000	37	274	206
	2B	0-56-58-3002-3001-158-3000	4	246	216
	3B	0-57-2-6-4-5-59-3001-157-257-106-159-3000	36	337	137
	1A	0-9-12-15-22-3001-115-3000	33	246	29
	2A	0-3002-3001-55-107-3000	18	209	117
	3A	0-23-24-62-11-13-3001-46-3000	20	283	140
Thursday	1B	0-3-1-102-3001-202-103-105-205-203-302-3000	37	261	182
	2B	0-56-58-3002-3001-158-3000	4	232	232
	3B	0-57-2-6-4-5-59-157-3001-159-106-257-3000	37	390	69
	1A	0-61-7-9-15-115-3001-47-3000	23	262	177
	2A	0-3002-3001-55-3000	16	184	82
	3A	0-23-24-62-13-3001-124-3000	20	273	153
Friday	1B	0-3-102-1-3001-202-103-105-205-203-302-3000	37	277	203
	2B	0-56-58-3002-3001-158-3000	4	368	96
	3B	0-57-2-6-4-5-59-3001-157-257-106-159-3000	36	361	117
	1A	0-10-9-12-15-115-3001-17-16-3000	36	247	142
	2A	0-3001-19-21-20-18-55-107-3000	21	226	98
	3A	0-23-24-62-11-13-14-3001-51-3000	21	327	43
Saturday	1B	0-57-2-155-59-3-102-157-3001-3002-3000	29	347	113
Sunday	1B	0-57-2-3-59-102-157-3001-3002-3000	29	348	112
Total			796	9013	4509

C.2 Scenario 1.B Solution

Week day	Route nb.	Route	TT	ST	WT
Monday	1B	0-3-102-1-3001-202-103-105-205-203-302-3000	37	281	199
	2B	0-57-2-6-4-5-59-3001-157-257-106-159-3000	36	368	112
	3B	0-56-58-3002-3001-158-3000	4	277	188
	1A	0-3002-3001-19-18-55-107-3000	20	220	100
	2A	0-24-62-9-12-15-3001-115-16-17-3000	35	254	147
	3A	0-60-23-61-11-13-3001-48-3000	21	277	168
Tuesday	1B	0-56-6-2-4-59-3001-102-202-106-159-3000	36	300	170
	2B	0-5-3-1-3001-103-105-205-203-302-3000	38	295	185
	3B	0-57-58-3002-157-3001-158-257-3000	4	275	170
	1A	0-3002-3001-55-107-3000	18	205	116
	2A	0-23-9-15-115-3001-123-3000	20	295	180
	3A	0-10-24-62-13-14-3001-50-3000	24	307	54
Wednesday	1B	0-57-58-3002-157-3001-158-257-3000	4	274	168
	2B	0-56-2-6-4-59-3001-102-202-106-159-3000	36	306	174
	3B	0-5-3-1-3001-103-105-205-203-302-3000	38	278	202
	1A	0-23-3001-55-3000	19	141	321
	2A	0-24-11-9-12-15-13-22-115-3001-46-3000	33	306	77
	3A	0-62-3002-3001-107-3000	18	278	261
Thursday	1B	0-5-102-1-3001-202-105-205-302-3000	40	269	211
	2B	0-2-6-3-4-59-3001-159-103-106-203-3000	35	304	174
	3B	0-57-56-58-3002-157-3001-257-158-3000	5	312	132
	1A	0-61-9-7-15-13-3001-115-55-3000	46	284	196
	2A	0-3002-3001-47-3000	0	212	0
	3A	0-23-24-62-3001-124-3000	8	312	208
Friday	1B	0-56-2-6-4-59-3001-102-202-106-159-3000	36	316	163
	2B	0-5-3-1-3001-103-105-205-203-302-3000	38	289	191
	3B	0-57-58-3002-157-3001-257-158-3000	4	286	161
	1A	0-10-9-12-15-115-3001-17-16-3000	36	247	142
	2A	0-3002-3001-19-21-20-18-55-107-3000	21	226	98
	3A	0-23-24-62-11-13-14-3001-51-3000	21	327	43
Saturday	1B	0-57-2-155-59-3-102-157-3001-3002-3000	29	347	113
Sunday	1B	0-57-2-3-59-102-157-3001-3002-3000	29	348	112
Total			792	9019	4935

C.3 Scenario 2.A Solution

Week Day	Route nb.	Route	TT (min.)	ST (min.)	WT (min.)
Monday	1B	0-57-2-3-59-102-157-3001-3002-3000	18	311	190
	2B	0-3002-3001-48-18-55-107-3000	34	294	52
	3B	0-23-24-62-11-15-115-3001-17-16-3000	41	244	105
	1A	0-10-9-12-13-14-3001-19-20-21-3000	41	230	170
	2A	0-3002-3001-51-18-55-107-3000	18	268	53
	3A	0-11-15-16-17-23-24-62-115-3000-3001	39	275	128
Tuesday	1B	0-57-56-58-3-3002-157-3001-158-257-3000	18	342	66
	2B	0-2-6-59-4-3001-106-159-3000	45	262	171
	3B	0-5-102-1-3001-202-103-105-203-205-302-3000	45	293	187
	1A	0-10-9-13-14-3001-50-3000	33	287	74
	2A	0-3002-3001-123-55-107-3000	19	264	57
	3A	0-23-24-62-15-3001-115-3000	34	239	92
Wednesday	1B	0-56-57-58-3-157-3001-257-158-3000	18	278	115
	2B	0-6-2-4-59-3002-3001-106-159-3000	34	321	271
	3B	0-5-102-1-3001-103-105-202-203-205-302-3000	45	278	129
	1A	0-9-12-13-3001-46-3000	20	227	159
	2A	0-22-3002-3001-55-107-3000	32	273	117
	3A	0-62-24-23-11-15-3001-115-3000	40	259	72
Thursday	1B	0-56-57-58-3-157-3001-158-257-3000	18	267	98
	2B	0-2-6-4-59-3002-3001-106-159-3000	34	337	292
	3B	0-5-102-1-3001-202-103-105-205-203-302-3000	40	293	121
	1A	0-7-9-61-13-3001-47-3000	31	258	171
	2A	0-3002-3001-124-55-3000	16	239	27
	3A	0-23-24-62-15-3001-115-3000	34	244	87
Friday	1B	0-57-56-58-3-3001-157-257-158-3000	18	283	148
	2B	0-2-6-4-59-3002-3001-159-106-3000	34	337	91
	3B	0-5-102-1-3001-105-103-202-203-302-205-3000	44	289	191
	1A	0-10-9-12-13-14-3001-21-20-19-3000	40	273	190
	2A	0-3002-3001-51-18-55-107-3000	18	273	52
	3A	0-23-24-62-11-15-115-3001-17-16-3000	39	273	105
Saturday	1B	0-57-2-155-59-3-102-157-3001-3002-3000	29	347	113
Sunday	1B	0-57-2-3-59-102-157-3001-3002-3000	29	348	112
Total			996	8779	4297

C.4 Scenario 2.B Solution

Week day	Route nb.	Route	TT (min.)	ST (min.)	WT (min.)
Monday	1B	0-56-57-58-3-3001-157-257-158-3000	18	329	151
	2B	0-2-6-4-59-3002-3001-106-159-3000	34	328	152
	3B	0-5-102-1-3001-103-202-105-205-203-302-3000	41	285	195
	1A	0-62-3002-3001-19-17-16-18-55-107-3000	30	253	149
	2A	0-60-9-12-15-13-3001-115-3000	38	254	226
	3A	0-23-24-11-61-3001-48-3000	18	254	172
Tuesday	1B	0-57-6-2-3002-157-3001-106-257-3000	29	301	178
	2B	0-5-3-102-3001-202-103-105-205-203-302-3000	39	282	198
	3B	0-56-58-59-4-1-3001-159-158-3000	23	297	183
	1A	0-10-9-15-115-3001-50-3000	27	258	116
	2A	0-13-3002-3001-55-107-3000	33	258	116
	3A	0-23-24-62-14-3001-123-3000	20	268	166
Wednesday	1B	0-6-2-58-59-3001-102-202-106-159-158-3000	38	297	173
	2B	0-56-57-4-3002-157-3001-257-3000	15	296	164
	3B	0-5-3-1-3001-103-105-205-203-302-3000	38	278	202
	1A	0-62-12-3002-3001-107-3000	32	251	229
	2A	0-23-9-15-115-3001-46-3000	18	251	178
	3A	0-24-11-13-22-3001-55-3000	37	252	228
Thursday	1B	0-57-56-58-3002-157-3001-158-257-3000	5	312	131
	2B	0-6-2-4-5-59-3001-159-106-3000	36	310	119
	3B	0-3-102-1-3001-105-103-202-205-203-302-3000	41	265	215
	1A	0-62-9-7-15-115-3001-55-3000	36	238	242
	2A	0-24-3002-3001-124-3000	4	239	149
	3A	0-23-61-13-3001-47-3000	18	241	159
Friday	1B	0-57-2-6-4-1-157-3001-106-257-3000	32	298	182
	2B	0-56-58-59-3002-3001-159-158-3000	10	309	86
	3B	0-5-3-102-3001-202-103-105-205-203-302-3000	39	287	193
	1A	0-10-9-12-13-14-3001-19-20-21-3000	40	273	190
	2A	0-3002-3001-51-18-55-107-3000	18	273	52
	3A	0-23-24-62-11-15-115-3001-17-16-3000	39	273	105
Saturday	1B	0-57-2-155-59-3-102-157-3001-3002-3000	29	347	113
Sunday	1B	0-57-2-3-59-102-157-3001-3002-3000	29	348	112
Total			905	9006	5224