Planning home health care services – a routing and scheduling problem.

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Thesis to obtain the Master of Science Degree in Biomedical Engineering

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June 2017

Abstract - *In the context of a growing aging population, it is associated loss of independency, the demand for Home Health Care (HHC) services has been rapidly increasing. The tendency is particularly relevant in developed countries which are struggling to provide the care required within restricted budgets. Therefore, efficiency in planning human resources, namely in route design and scheduling, emerges as an area in which optimization can have a significant impact.*

This thesis proposes a mathematical model that aims at helping a HHC provider to plan the routes associated to domiciliary services. The model is an extension of a multi-period vehicle routing problem with time windows (MPVRPTW) formulation. Beyond the typical VRP characteristics, the most relevant features of HHC services addressed by the model are: 1) hard time windows, 2) work and break regulations and 3) continuity of care, both within a day and throughout the week. The daily continuity of care is modeled as a soft-constraint. The optimization can be performed focusing on two separate objective functions. The first permits the minimization of the travelling time, whereas the second aims at optimizing the workload balance amongst caregivers teams. In spite of introducing an exact model, a solution heuristic is also presented required to solve large instances in a reasonable amount of time.

The model was tested with the instances of a Portuguese institution of social solidarity, APOIO. Due to the data size, there was a need to resort to the heuristic, implying that the solutions found can be sub-optimal. Nevertheless, the results are promising. Respecting all constraints, the minimization of travelling time yield a reduction, per week, of about 7%. In turn, when workload balance is concerned, there is a decrease of imbalances verified in the solution associated to the travelling time minimization. For the first objective function the maximum imbalance between teams is of 158 minutes, whereas for the second the same criteria assumes the value of 80 minutes. This is a reduction of about 50%. The results presented solidly validate the mode

Keywords — Home Health Care, Routing and Scheduling, MPVRPTW, Programming Model, MILP, GAMS

I. Introduction

Over the last decades, in developed countries, a generalized increase in life expectancy with an accentuated decrease in the fertility rates has been propelling the population aging. The OCDE34 life expectancy at birth is of 80,5 years, increasing about 4 months every year without showing signs of slowing down. In Portugal, the average value stands at 80,8 years. In approximately two thirds of the OECD countries, the percentage of population aged over 65 years is expected to increase up to 25%, very significant when compared with the 15% verified in 2010 (OECD, 2015).

As people age, they tend to acquire disabilities, frequently resulting from non-communicable diseases. Dementia is a category of such diseases including, for example, Alzheimer's disease, that affects the memory of the elderly in an early stage and, as the disease progresses, it can cause disorientation, behavioral issues and disregard for self-care, amongst others. The inability to perform certain tasks leads to an independence loss and the need for some form of provision of care.

The care provided to the elderly and other bearers of non-communicable chronic diseases is known as long-term care and can be divided into formal and informal care. Institutions provide formal care, whereas informal care is related with family and friends. The ways in which formal and informal care vary greatly amongst countries can be explored in Genet. et. al (2013). In Portugal, informal care has been declining, essentially due to the changes regarding family's organization: increase of woman's participation in the working market, families' atomization and a higher volatility of conjugal relationships (Pego et. al, 2013). The reduced capacity of the families to provide informal care places an additional pressure for the development of formal care.

Despite possessing some infrastructures and organizations to provide long-term care, the government's expenditure on this type of care is remarkably limited when compared to other OECD countries. Portugal spends only 2% of its health budget in long-term care, while the OECD27 average is 12% and a majority of the most developed countries it stands above 20%. (OECD, 2015) The shortage of financial resources leads to the imperativeness of the implementation of good management practices. Gutierrez and Vidal (2013) conduct a detailed review of the logistical problems in the field of Home Health Care (HHC), one of which is finding solutions for HHC scheduling and routing, the area in which this master thesis is focused. Solution procedures for this problem differ substantially between countries, due to the variations in terms of national and regulatory settings. In Fikar and Hirsch (2017), a review on HHC routing and scheduling models is performed, revealing that the most frequent approach to address this problem is through extensions of the Vehicle Routing Problems.

The objective of this master thesis is to propose and test a mathematical model and a solution method capable of generating solutions for HHC routing and scheduling problems. These solutions must be characterized by:

- the sequence of the visits to the patients;
- the time at which the visit should start.

In addition, the solutions should respect the constraints originated by both the patient requests and the institutional operational preferences. .The problem is modelled as an extension of a MPVRPTW (Multi-Period Vehicle Routing Problem with Time Windows) and implemented in GAMS (Generic Algebraic Modelling System) as a MILP (Mixed Integer Linear Problem) program solved with CPLEX, an optimization software package.

The proposed MILP addresses real world constraints such as lunch breaks, patient visit frequency, and loyalty between caregiver and patient within a day and throughout a week. Two objective functions are considered separately: 1) minimizing travelling time, and 2) workload balance among teams. The institution's objective is the minimization of the travelling time nonetheless, since the model is idealized to serve as a decision support tool, it benefits from possessing other features to be optimized. With different optimization perspectives, a more informed and conscious decision can be assured.

II. CASE STUDY

The validation of this work is based on a real world case study. The instances used belong to APOIO, a Private Institution of Social Solidarity (PISS), with a non-profitable nature, with the purpose of providing solidarity services to those in need, with a day care centre located in Oeiras, Portugal. Despite providing several types of services, our focus is the home health care services. The problem arises when there is a need to allocate teams of assistants to patients spread across a wide area and those patients have preferences regarding the time at which their visit should start. With the aim of understanding how to approach the problem, detailed information describing the nature of the services, the workforce, the patient typologies and the operational preferences of the institution is needed and, therefore, displayed below..

A. Services in HHC

The institution offers a wide range of services from Personal hygiene and comfort, Habitational hygiene maintenance, Alimentation, Health care services to Exterior escort of patients. Within each of the nature services previously mentioned, there are different types of services. There can even be an added diversity through the negotiation of some service characteristics with the patients. The services are, thus, remarkably diverse.

B. The patients

The patients are classified into two types: autonomous and bedridden. Autonomous patients can perform most of the activities of daily living by themselves, requiring only sporadic care, whereas bedridden patients are completely dependent on assistance, necessitating help for activities as simple as bathing, dressing and feeding, therefore demanding more visits. In the institution there are 36 patients requiring HHC services, of which 10 are bedridden and 26 are autonomous

C. Workforce

The workforce allocated to HHC services in APOIO is constituted by 9 assistants of homogeneous skill levels, implying that every assistant can perform any service required. Nonetheless, there are services that require teams of two assistants, particularly those associated to bedridden patients. As such, the institution would like to allocate teams of two assistants to bedridden patients (B-Teams) and teams composed by a single assistant to autonomous patients (A-Teams).

D. APOIO's Operational Procedures

The administration of the partner institution has decided to implement some operational rules:

- **Operating Time**: During the weekdays opens at 8 a.m. and ceases its activities at 8 p.m. On weekends and national holidays, the working period begins at 8 a.m. and ends at 1 p.m. ;

- **Labour Time**: Each HHC assistant is required to work a mean of 37 hours per week;
- **Lunch Break**: Takes place at the day care centre, with a duration of one hour, which must begin within the time interval between 12 p.m. and 2 p.m;
- **Lunch distribution**: Begin at 12 p.m. and has a duration of 90 minutes. It requires the presence at the day care centre of 3 assistants.

Designing routes is an activity of transcendent complexity, currently done manually resulting in the generation of routes which are inefficient and that do not respect all of the institution's preferences for operating procedures.

III. Literature Review

A. Vehicle Routing Problem

In the context of transportation, scheduling, distribution and logistics, one of the most relevant challenges faced by managers concerns the definition of a strategy to optimize the delivery of products by suppliers to clients, while respecting some constraints (Surekha e Sumathi, 2011). The previously described problem is generally recognized as a Vehicle Routing Problem (VRP).

In early literature, VRPs were first introduced in 1954 by Dantzig et al., with the Travelling Salesman Problem (TSP). Over the last decades, new formulations were created, attending to the problem's characteristic being addressed, and diverging into different classes of VRP. Five of the most relevant are for the case study are:

•Capacitated Vehicle Routing Problem (CVRP);
•Vehicle Routing Problem with Time Windows (VRPTW);
•Periodic Vehicle Routing Problem (PVRP).

Xu et. al (2015) states that the VRPTW similarly, to most VRPs, a homogeneous fleet is meant to deliver goods to clients demanding them, with the particularity residing in delivering it within a pre-specified time window. A central theme in VRPTW are the Time Window (TW) constraints, which are implemented through hard or soft constraints. The most commonly employed are the hard constraints, which assures that the delivery is initiated within the time interval. On the other hand, soft constraints can be violated at the cost of a penalty added to the objective function (Taş, et al., 2014).

B. VRP in Home Health Care

Generally, the problem consists in trying to assign a set of heterogeneous patients, who are spread across the geographical area of operations, to caregivers. They can place several service requirements with features that have an impact in way caregivers are assigned. The nature of those features may rely, for example, on TW preferences or specific nurses' skill. The assignment of caregivers is also considerably affected by the regulatory contexts which meaningfully vary amongst countries. For this reason, the formulations of VRPs applied to HHC vary considerably regarding constrains, objectives to optimized and the solution method used (Fikar and Hirch, 2017). Taking this into account, the literature will be divided into classes that state the most relevant feature being modelled. .

Duque et al. (2015) defines a home care routing and scheduling problem, with two objectives: the service level and the total distance travelled by all caregivers. This differs from Braekers et al. (2016) since it is a multi-period VRP, which means that solution regards several days, instead of just one day. The optimization of the objectives is implemented in a hierarchical order prioritizing service level, which differentiates this work from other approaches in the literature. Normally, an aggregated function of the quantities to optimize is considered. The mathematical formulation proposed is based on the set partitioning problem. The major advantage of hierarchical procedure appears to be related with the easiness with which it can be implemented in organizations, due to being quite straightforward when trade-off analysis is concerned.

An interesting work is developed in Gomes and Ramos (2016). The authors aim at developing a toll capable of supporting a social assistant in the job of route design and scheduling, for domiciliary care associated with a social parochial center. They propose a mathematical model that is an extension of a VRPTW. One of the particularities of this model relies on the ability to elaborate different routes and schedules for each day of the week, respecting the time intervals at which the domiciliary services may be started. The previous statement grants the model features similar to the PVRP, in which there is an initial distribution of the patients to the days, in accordance with the frequency required and the allowable combinations of visits days. However, since the input in not the frequency of visits but rather a more restrained and aperiodic plan of visits, the model becomes a multi-period vehicle routing problem with time windows (MPVRPTW), as stated Archetti et al, 2015. Regarding constraints, the model considers many of the ones found in the case study, namely meals' distribution, considers lunch breaks and, most relevantly, addresses the continuity of care throughout the week. The optimization may focus on two objective functions separately, the first is to minimize the weekly distance travelled, one of the optimization objectives found more often, and the second is workload balance, which tries to distribute as evenly as possible the workload amongst teams of assistants.

This model has been applied to a relatively small real-case study, in which 17 patients and 15 residences were considered. In order for the model to run in a reasonable amount of time, the authors developed a heuristic that permits the independent run of the model for each day of the week. The solution yield for the first objective function was capable of reducing the weekly traveled time by 23%. Nonetheless, when the workload of this first solution was under analysis, it was verified that the workloads had become quite unbalanced. After running the model for the second objective function (with the heuristic starting on Monday – the heuristic may generate different solution depending on the day with which it is initiated) there was a reduction on the weekly traveled time of 6% and the workload was visibly much closer to being equally distributed.

C. MODEL FOR THE ROUTING AND SCHEDULING OF A HHC PROBEM

The proposed formulation models the problem displayed in section II.

A. Sets

Table 1: Sets of nodes

V	Set of all the nodes in the network
V_R	Subset of real day care center, $i \in V_R \subset V$;
V_F	Subset of day care center replicas for returning, $i \in V_F \subset V$;
V_L	Subset of day care center replicas for lunch break, $i \in V_L \subset V$;
V_D	Subset of day care center replicas for lunch distribution, $i \in V_D \subset V$;
V_P	Subset of patients and all the patients replicas, $i \in V_P \subset V$;
R_i	Subset composed subsets each containing exclusively patient *i* and his/her replicas $R_i \subset V_P$.

B. Parameters

D_{ij}	Travelling time between node i and j;
w_{it}	Visit duration in node i on day t;
NV_{it}	Node Visited: = 1 if patient i is visited on day t;
P	Penalization value for the daily loyalty preference
K	Number of teams required for lunch distribution
H	Team's maximum daily working time (in minutes)
e_{it}	Early time of TW for node i visited on day t;
l_{it}	Later time of TW for node i visited on day t;
M_{ijt}	Big M, a value large enough to assure constraint feasibility
lag_{ijt}	"Schedule feasibility" measure;

C. Variables

- x_{aijt} - Binary variable that equals 1 if team *a* is assigned to the arc (*i,j*) on day *t*; and zero otherwise. This variable will provide the nodes visiting sequence for team a on day *t*. .
- s_{iat} - Continuous variable defining the starting time of the visit from team a to node i on day t,

Two binary auxiliary variables are needed to model the daily-loyalty preference constraint (a_{iat} and b_{iat}).

D. Model Formulation

I. Objective functions

First Objective Function – Travelling Time

$$\min \sum_{i,j,a,t}\left(D_{ij} \times x_{ijat}\right) + P\sum_{i,a,t} b_{iat} \quad (1)$$

The aim of the objective function is to minimize the time spent travelling (the first term). However, an additional term was essential, as a result of the introduction of a soft-constraint for modeling the daily-loyalty preference, accumulating a penalty value for each time the preference is not verified.

Second Objective Function – Workload balance

$$\min \max_{a,t} \sum_{i,j}\left(D_{ij} \times x_{ijat} + w_{it} \times x_{ijat}\right) + P\sum_{i} b_{iat} \quad (2)$$

The second objective function minimizes, per day, the maximum service time amongst the teams,an objective that assures that a more balanced workload distribution. Although this function is not linear, a linearization technique was used to allow it to be the second objective function used.

II. Constraints

To visit all patients

All patient nodes must be visited only once by the teams.

$$\sum_a \sum_j x_{ijat} = 1, \quad \forall\, i \in V_p, \forall\, t: \; NV_{it} = 1 \quad (3)$$

Teams' lunch break

All teams must have a lunch break. This was modelled imposing all teams to visit the day care center replica corresponding to the lunch break.

$$\sum_i x_{ijat} = 1, \qquad \forall\, a \in A, \forall\, t \in T, \forall\, j \in V_L \quad (4)$$

Weekly loyalty constraint

Each patient node cannot be visited by two different teams in different days of the week

$\sum_j \boldsymbol{x_{ijat}} + \sum_j \boldsymbol{x_{ijez}} \leq \mathbf{1}, \quad \forall \boldsymbol{a}, \boldsymbol{e} \in \boldsymbol{A}: \boldsymbol{a} \neq \boldsymbol{e}, \forall\, \boldsymbol{t}, \boldsymbol{z} \in \boldsymbol{T}, \forall\, \boldsymbol{i} \in \boldsymbol{V_P} \quad (\mathbf{5})$

Daily loyalty preference

For each patient with more than one visit per can be visited by more than one team

$\sum_j \boldsymbol{x_{ijat}} + \boldsymbol{a_{iat}} = \sum_j \boldsymbol{x_{(i+1)jat}} + \boldsymbol{b_{(i+1)at}} \quad \forall \boldsymbol{i} \in \boldsymbol{R_i}, \forall \boldsymbol{a} \in \boldsymbol{A}, \forall \boldsymbol{t} \in \boldsymbol{T} \quad (\mathbf{6})$

with $(i+1)$ representing the node flowing node *i* in set R_i. Notice that if one team cannot perform all the day visits, variable b will be one and will introduce a penalization in the objective functio**n.**

All teams must depart from the day care center.

$\sum_{j \in V_P \cup V_L} \boldsymbol{x_{ijat}} = \mathbf{1}, \ \forall \boldsymbol{a} \in \boldsymbol{U}, \forall\, \boldsymbol{t} \in \boldsymbol{T}, \forall\, \boldsymbol{i} \in \boldsymbol{V_R} \ (\mathbf{7})$

All teams must arrive to the day care center.

$\sum_{i \in V_P \cup V_L} \boldsymbol{x_{ijat}} = \mathbf{1}, \ \forall \boldsymbol{a} \in \boldsymbol{U}, \forall\, \boldsymbol{t} \in \boldsymbol{T}, \forall\, \boldsymbol{j} \in \boldsymbol{V_F} \ \ (\mathbf{8})$

Route continuity

Equation (9) states that the team that enters a node must also leave from it.

$\sum_i \boldsymbol{x_{ijat}} - \sum_i \boldsymbol{x_{jiat}} = \mathbf{0}, \quad \forall \boldsymbol{a} \in \boldsymbol{U}, \forall\, \boldsymbol{t} \in \boldsymbol{T}, \forall \boldsymbol{j} \in \boldsymbol{A_p} \cup \boldsymbol{A_L} \quad (\mathbf{9})$

Lunch distribution

Equation (10) assures that exactly K teams should be available for lunch distribution.

$\sum_a \sum_i \boldsymbol{x_{ijat}} = \boldsymbol{K}, \ \forall\, \boldsymbol{t} \in \boldsymbol{T}, \forall \boldsymbol{j} \in \boldsymbol{V_D} \quad (\mathbf{10})$

Visit starting time

Constraint (11) assures that if the same team visits nodes i and j, the corresponding starting times allow the travelling between nodes (D_{ij}) and the working time at node i. Notice that if team a does not visits these two nodes, M_{ijt} is a value large enough to make the constraint redundant.

$\boldsymbol{s_{iat}} + \boldsymbol{D_{ij}} + \boldsymbol{w_{it}} + \boldsymbol{M_{ijt}}(\mathbf{1} - \boldsymbol{x_{ijat}}) \leq \boldsymbol{s_{jat}}, \quad \forall \boldsymbol{a} \in \boldsymbol{U}, \forall\, \boldsymbol{t} \in \boldsymbol{T}, \forall \boldsymbol{i}, \boldsymbol{j} \in \boldsymbol{V} \quad (\mathbf{11})$

Teams daily work time

Constraint (12) assures to each team the daily working time of H minutes is respected.

$\boldsymbol{s_{iat}} - \boldsymbol{s_{jat}} \leq \boldsymbol{H} \ \ \forall \boldsymbol{a} \in \boldsymbol{U}, \forall\, \boldsymbol{t} \in \boldsymbol{T}, \forall\, \boldsymbol{i} \in \ \boldsymbol{V_F}, \boldsymbol{j} \in \ \boldsymbol{V_R} \quad$ **(12)**

Non-negativity

$\boldsymbol{s_{iat}} \geq \mathbf{0}, \ \ \forall \boldsymbol{a} \in \boldsymbol{U}, \forall\, \boldsymbol{t} \in \boldsymbol{T}, \forall \boldsymbol{i} \in \boldsymbol{V} \quad$ **(13)**

Binary Variables

$\boldsymbol{x_{ijat}} \in \{\mathbf{0}, \mathbf{1}\}, \quad \forall \boldsymbol{a} \in \boldsymbol{U}, \forall\, \boldsymbol{t} \in \boldsymbol{T}, \forall\, \boldsymbol{i}, \boldsymbol{j} \in \boldsymbol{V} \quad (\mathbf{14})$

$\boldsymbol{a_{iat}} \in \{\mathbf{0}, \mathbf{1}\}, \quad \forall \boldsymbol{a} \in \boldsymbol{U}, \forall\, \boldsymbol{t} \in \boldsymbol{T}, \forall\, \boldsymbol{i} \in \boldsymbol{V} \quad (\mathbf{15})$

$\boldsymbol{b_{iat}} \in \{\mathbf{0}, \mathbf{1}\}, \ \forall \boldsymbol{a} \in \boldsymbol{U}, \forall\, \boldsymbol{t} \in \boldsymbol{T}, \forall\, \boldsymbol{i} \in \boldsymbol{V} \quad$ **(16)**

E. SOLUTION METHODOLOGY

VRP are NP-hard problems, which implies that achieving an optimal solution is computationally intractable for large-scale instances. The instances provided by APOIO are of medium size. However, since routes need to be planned for 6 teams visiting daily approximately 57 nodes, for every day of the week (a lessened number of nodes on weekends). Consequently, in order to overcome the experienced computational difficulties, a solution approached based on the mathematical formulation was designed. The procedures constituting the solution methodology are discussed bellow.

Solution methodology

The first strategy was to decompose the model in two dimensions: patient typology and days of the week. The division into dimensions result from analyzing the case study characteristics. By dividing into days of the week, the solved instance becomes much smaller since data provide to the model concerns one single day (at most 57 nodes to be visited). In regards to the dimension of patient typology, the division allowed the differentiation of the teams used. When the model is run for the bedridden typology, the teams have to be composed of two caregivers while for autonomous only one assistant performs the visits.

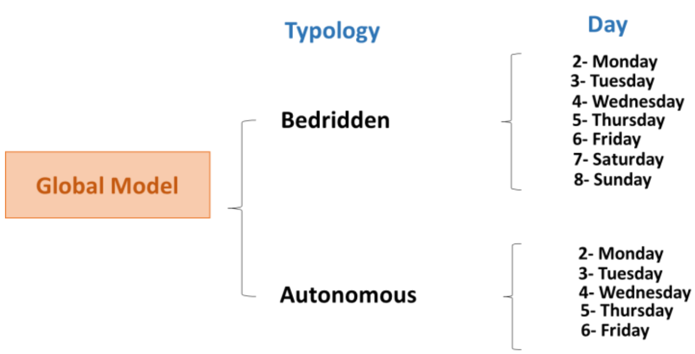


Figure 1: Schematic of the dimensions considered the model.

A schematic representation of the decomposition strategy is depicted in Figure 1. Moreover, it is noteworthy that the dimensional division previously mentioned automatically assures one of the requests made by APOIO, namely, assigning teams of two assistants to bedridden patients and those teams of one to semi-dependent patients.

The model was run in accordance with the dimensional division, for each of the objective functions. Nonetheless, there was one aspect about the operational preferences of the institution that was would not become assured with this solution: the weekly loyalty preference, which states that every visit to a patient within the same week should be assigned to one single team. This separation of the weekdays implies that equation (5) is cannot be implemented by the model (It is run for each day at a time). In order to account for that preference, the following procedure was designed.

Weekly loyalty procedure

After solving each day independently, optimize routes for each day of the week are obtained. However, a patient served by team one on day 1 may be assigned to team 3 on day 2. This aspect is not relevant for bedridden patients, since the services to be performed are the same on weekdays, varying only on weekends. The weekly loyalty was assured by fixing the patients to the teams of the weekday, and solving the problem only for weekends.

In contrast, the services paid to autonomous patients are considerably variable each day. To assure the week loyalty, a procedure was established and is defined as follows. Firstly, the days of the weekdays are sorted decreasingly with respect to the number of patients to visit. Then, in the second step, the MILP model is resolved for the first day (d1), establishing the first partial patients assignment to the teams. Afterwards, in the third step, the model is resolved for the second day (d2) with the partial patients' assignment fixed in beforehand. The preceding step guarantees that the assignments attained in d1 are maintained in d2 and that the patients that are not served in day d1 are one of the model solution. Finally, the procedure described in the third step is repeated sequentially for the other days, assigning the totality of the patients to the teams. A diagram of the procedure is depicted in Figure 2.

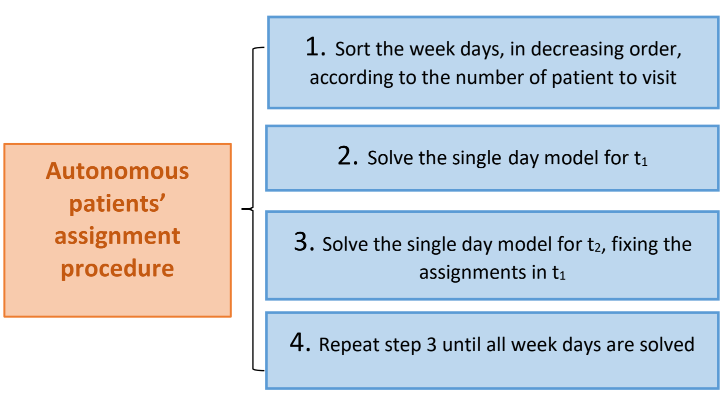


Figure 2: Diagram of the steps constituting Autonomous patients' assignment procedure to the teams.

F. Case Study Results and Discussion

Considering the two objective functions and the solution methodology, it was considered that the analysis of the results should be carried out in four scenarios. These are characterized by two components: the objective function that is under scrutiny and the application of the weekly loyalty constraint. For each scenario, a solution is obtained and analyzed then, they are compared among each other so that the functionalities of the model can be more accurately understood and validated. A schematic of the scenario's organization can be found in Figure 3.

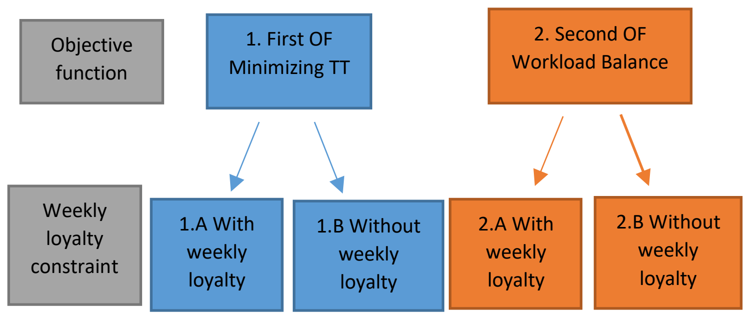


Figure 3: Scenarios analyzed.

A. *Indicators*

The analysis of the four scenarios considered five indicators, defined as:

Travelling time indicator:

$$iTT_{at} = \sum_i D_{ij} * x_{ijat} \quad (17)$$

Service time indicator:

$$iST_{at} = \sum_i w_{it} + \sum_i D_{it}. \quad (18)$$

Waiting time indicator:

$$iWT_{at} = s_{aDFt} - s_{aDRt} - iST_{at}, \quad (19)$$

with DF is the arriving day care center node and the DR is the departing day care center node.

Indicator of route balance:

$$iRB_t = Max(iST_{at}) - Min(iST_{at}) \quad (20)$$

Indicator of route duration balance:

$$iRDB_t = Max(s_{aDFt} - s_{aDRt}) - Min(s_{aDFt} - s_{aDRt}) \quad (21)$$

B. Parameter Determination

Most of the parameters were determined by collecting data directly from APOIO, through the analysis of work sheets. However, some parameters were obtained rather differently. The D_{ij} was obtained by inserting the patient's address into a google API which determined the distance, that was afterwards converted into a car time distance. The big M was calculated resorting to the expression:

$$M_{ijt} = l_{it} - e_{jt} + w_{it} + D_{ij} \ . \ (22)$$

The last relevant parameter was the penalty value. In order to determine a value for each of the objective functions, a sensibility analysis was performed. According to the solution methodology, For the bedridden the analysis was preformed for Monday, whereas for the autonomous, the chosen day for the analysis was Friday. The conclusions were that the values for the P were to be kept constant for each of the OF, independently of the team type. For the first OF this value 100 minutes, whereas for the second OF the value was fixed as 20 minutes.

C. Discussion

Weekly analysis of the scenarios

The analysis of the weekly travelling time grants a holistic view of the overall benefit of the solutions found, taking into account the greatest worry of the institution: reducing the TT. In Figure 4, two solutions are shown to have improved weekly TT, when compared with the current solution, namely the two originated from scenarios 1.B and 1.A. They both represent a decrease of 7%. Considering that not only the time is reduced, but all of the constraints proposed by the institution are accounted for, these are promising result. Notice that in the current solution bedridden patients may be visited by teams for the autonomous patients. On the other hand, however, the solutions found for scenarios 2.A and 2.B are worse than the common practices at the moment. For scenario 2.A there is an increase of 8% of the weekly TT, and for the last scenario the raise in quantity of total traveled time is of about 17%.

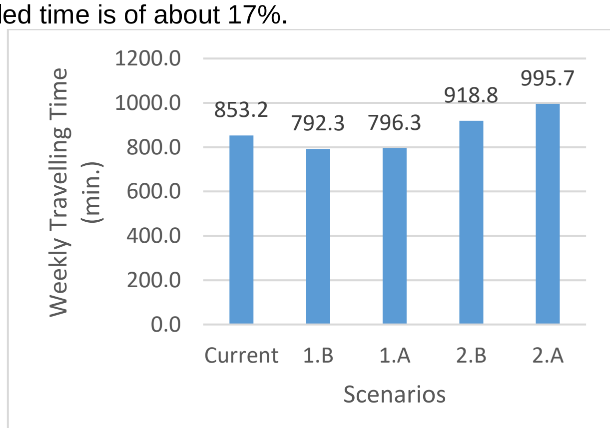


Figure 4: Weekly Travelling Time, for each of the four scenarios

Workload balance

The indicator of route balance confirms the correct functioning of the second OF. In Figure 5, where the WLC is not repected, when the indicators from both the first solution and the second are compared, there is always a decrease in the value of the indicator.

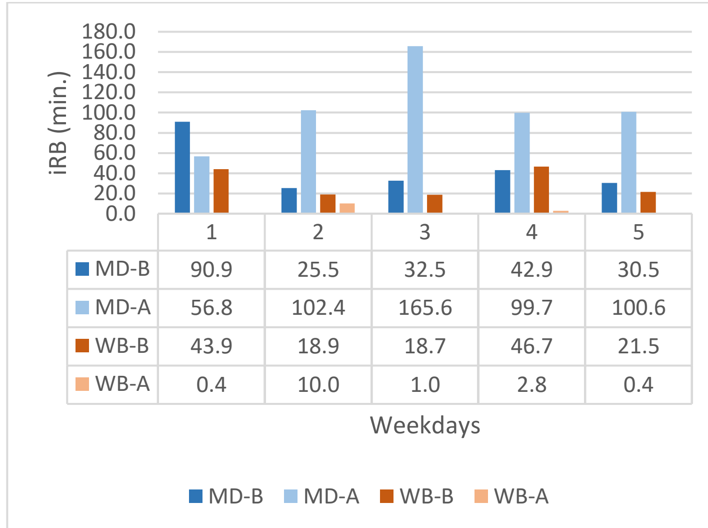
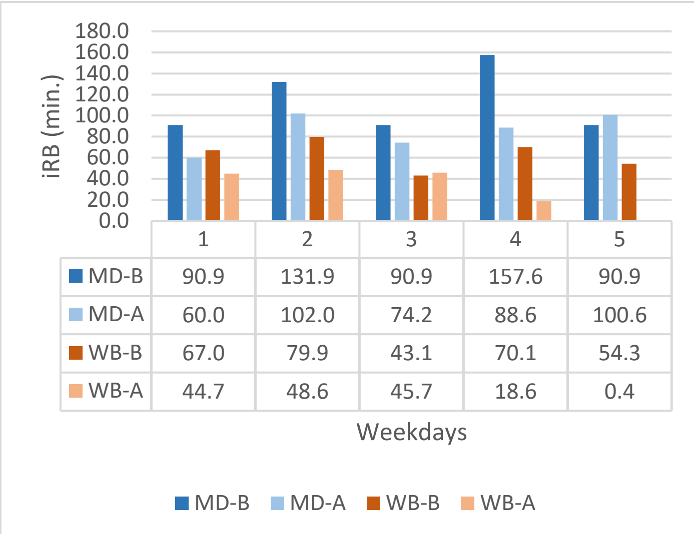


Figure 5: Indicator of Route Balance for the two objective functions without the WLC. MD-B – Value of iRB for the solution Minimizing the Distance for the Bedridden patients; MD-A – Value of iRB for the solution Minimizing the distance for Autonomous patients; WB-B - Value of iRB for the solution optimizing Workload Balance for the Bedridden patients; WB-A - Value of iRB for the solution optimizing Workload Balance for the Autonomous patients

Respecting the WLC, introduced by the solution methodology, also increases the value of iRB. When the third step of the solution methodology is preformed for the second time, some of the patients and replicas have already been assigned to a team, restricting the problem. The latter situation significantly diminishes the ability of the model to balance the workload of the three teams. For the autonomous patients, the day with which the solution methodology is initiated is Friday. Consequently, this week day presents the lowest value for the indicator. In order to enhance the support to this reasoning, Figure 6 is presented, in which the values for the same indicator are displayed, except that, in turn, this are the values obtained when the weekly loyalty constraint is disregarded. In it, the maximum value for the iRB is 158 minutes.



	1	2	3	4	5
MD-B	90.9	131.9	90.9	157.6	90.9
MD-A	60.0	102.0	74.2	88.6	100.6
WB-B	67.0	79.9	43.1	70.1	54.3
WB-A	44.7	48.6	45.7	18.6	0.4

Figure 6: Indicator of Route Balance for the two objective functions with the WLC. MD-B – Value of iRB for the solution Minimizing the Distance for the Bedridden patients; MD-A – Value of iRB for the solution Minimizing the distance for Autonomous patients; WB-B - Value of iRB for the solution optimizing Workload Balance for the Bedridden patients; WB-A - Value of iRB for the solution optimizing Workload Balance for the Autonomous patients

Comparing Figures 5 and 6 confirms the logic used to justify the connotation of the weekly loyalty constraint as a factor with major impact in the indicator of route balance, diminishing the model's capacity for producing routes with a more equitable distribution of workload. The IRB has a higher value for the first OF, practically every day of the week and for both team types.

Working Schedule and iRDB

Generally, there appears to be a tendency for the iRDB value to decrease from the solutions to the first OF to the solutions of the second. However, this indicator might be misleading, especially when remembering that the schedules will be put into practice in an organization. Comparing between the difference in schedules is common amongst coworkers, and despite not being considered to assess the quality of the routes obtained, its analysis possesses relevance.

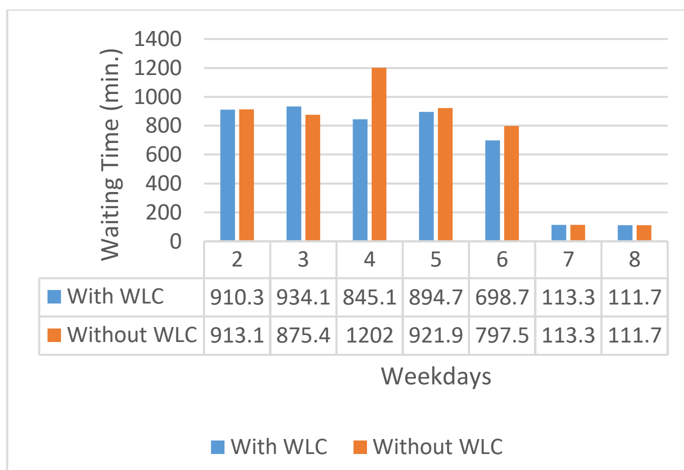
Two opposite situations that reveal how it can be deceiving are the cases of the Monday B-Teams, for the traveling time OF, and the Friday A-Teams, regarding the workload OF. In the first situation, the value of the indicator of route duration balance is 15,2 minutes, leading the workers to believe that the work is quite well distributed, however, revisiting the table present in Figure 6 it is possible to verify that the difference in service time is of 90 minutes, representing one extra hour of service time. On the other hand, for the second objective function, Friday presents a dreadful value for the indicator (138 minutes), one of the worst found for either the first or the second objective functions. Nevertheless, in Figure 6 is possible to observe that the values of service time for each of the teams is exactly the same, fixed at 273 minutes, within the proposed routes regarding workload.

In APOIO teams are meant to rotate amongst schedules, this will reduce the impact of the inequality in terms of total route duration. However, when schedules are allocated to a real team, some human resources conflicts might arise. In the studied problem, the difference in terms of working schedule, within the same type of teams and considering a single day of the week, varies between 0 hours and up to 2 hours and 52 minutes. Three hours have a very significant impact on the life of any worker therefore, from a worker's perspective, it would be important to decrease this discrepancy.

The origin of the divergence on the routes durations heavily relies on the third type of time associated to the routes, the waiting times. Although this time is not modeled in any of the objective functions, it is analyzed as an indicator in the following subsection.

Waiting Time

Route's waiting times are frequently not considered in optimization problems. However, since the waiting times are a component of the routes resulting from the need to respect the patient's time windows availabilities, it greatly influences the duration of the route, as approached in the previous subsection. A general notion of the time spent on waiting by the teams can be obtained by analyzing the Figure 7, derived from the first OF and Figure 8 elaborated with the results obtained from the second OF.



	2	3	4	5	6	7	8
With WLC	910.3	934.1	845.1	894.7	698.7	113.3	111.7
Without WLC	913.1	875.4	1202	921.9	797.5	113.3	111.7

Figure 6.21: Daily Waiting Times for the First Objective function.

For the solutions attained from the first OF, the one respecting the weekly loyalty constraint is constituted by routes which, aggregately, amount to a total waiting time of 4596 minutes, representing for about 30% of the total time available to the teams. In turn, the solution that ignores that same constraint has the value of 4935 minutes, corresponding to approximately 32% of the 15360 minutes available for the teams to develop their activities.

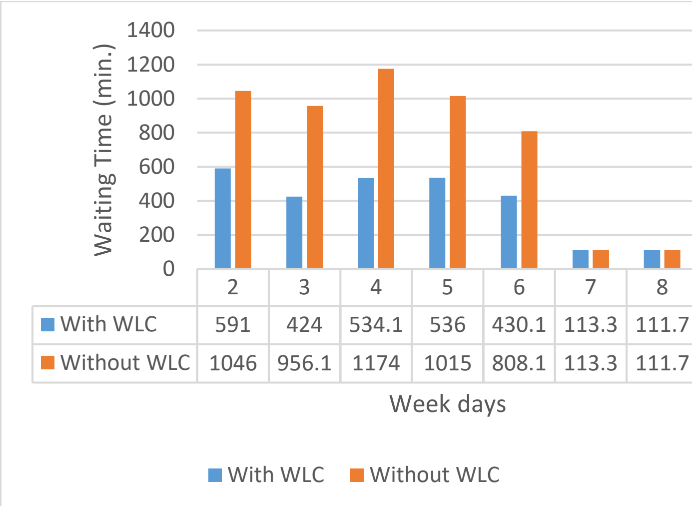


Figure 6.22: Daily Waiting Time for the Second Objective function.

Regarding the solutions associated to the second OF, an increase in waiting time is obvious when the model does not respect the WLC. The main reason is the fact that without the restriction the model is free to balance the workload regardless of the times waited so that the total duration of each route is reduced. In turn, the solution respecting the WLC, some of the patients to visit have already been preassigned and the freedom of the model to add waiting times is, therefore, limited.

Reconsidering the differences in duration and the indicator of route duration, for the A-Teams on Friday, the difference of 138,8 is due, in a considerable part, to waiting times. It is possible to verify that the difference between the waiting times of the team with the highest and lowest route durations is of 138 minutes. This result shows that the total difference in duration is associated to waiting times, rendering this indicator as very relevant when designing routes with balanced durations.

G. CONCLUSIONS

One of the most significant global megatrends responsible for shaping nowadays society is population aging. Accompanying aging is the increase in the prevalence of non-communicable chronic diseases that considerably interfere in the patients' ability to perform the activities of daily living autonomously. The solution to this problem frequently resides in the requisition of services to institutions that provide HHC. Generally, institution's administration staff design the routes manually. The aim of these study is thus to propose and validate a mathematical model to be used as a tool to support route design.

The validation of the mathematical model proposed is attained by resorting to the instances provided by APOIO, the partner institution. The model provides as outputs both the sequence of the visits that should be performed to patients and the times at which each visit should start. The model respects a series of institutional operational constraints, namely:

- •One hour lunch breaks which can begin anywhere between 12 p.m. and 2p.m.;
- •Lunch distribution, occupying 90 minutes and starting precisely at 12 p.m.;
- •Modeling a labor time inferior to the functioning time of the institution;
- •Preference of the assignment of a single team to a patient, whenever possible (daily-loyalty).

The MP-VRPTW model is extended to accommodate the aforementioned constraints. In order to do so, it is characterized by hard time windows, which obliges the visit to nodes to be initiated within the time interval it defines. Actually, all constraints are hard constraints, except for the one modelling the daily-loyalty. The reason for being modelled as a soft-constraint resides in the fact that there might be cases in which the difference in time between the first and last visit time windows may be large that the working time of a team is not sufficient to perform both visits. Such situations result in the attribution of two teams to one patient.

The MILP was implemented in GAMS and solved resorting to CPLEX. Due to the heavy computational memory requirement, it was imperative to use a heuristic to solve the model. The first part of the heuristic was to run the model separately for the two typologies of teams: the bedridden patients' teams and the autonomous patients' teams. Then, within each team type, the model was run for every day of the week. In that sense, it was assured that the bedridden patients were always assigned to teams of two patients, as asked by the institution. Interestingly, since the patients' typologies distinguish them to the point of being either on the bedridden side or on the autonomous side, it was almost as if the model had been run for two problems with different characteristics: in the bedridden side, there are less patients, but with more replicas, when compared with the autonomous side. For that reason, the effects of the broken daily loyalty constraint are more relevant in the bedridden side. The second part of the heuristic involved the assignment of patients to teams. The assignment consisted of four steps process in which the model was run for the days, which had previously been ordered according to the number of patients to visit, and the patients are fixed to the teams. Whenever a patient is fixed to a team, the allocation transits to all the other days ranked below. This second stage of the heuristic is actually the function that ought to be realized by weekly loyalty constraint, the fourth equation in the model's constraints. However, due to the previous stage, the only constraint that has to receive inputs from different days of the week cannot be used.

Despite the possibility of the obtaining sub-optimal results, the solutions obtained are quite satisfying. For the scenarios associated with the first objective function, which aims ate reducing weekly traveled time, both with and without the WLC, there is a reduction on the weekly traveled time. The scenario with WLC presents a reduction of 7% of the TT, equal to that of the scenario without WLC. The latter was an interesting result, since it was expected that the addition of a constraint would worsen the value of the reduction. Actually it does, the soft-constraint which penalizes the objective function worsens the solution's value but, in turn, permits the amount of weekly TT to be slightly increased (A-Teams) or even reduced (B-teams). The optimization of the TT was the objective of the institution, and it was optimized when compared with the current solution.

The results regarding the second objective function are of a more difficult assessment. It was not possible to have access to the current solution's service time. As such, the two were compared in terms of weekly TT, for which the results verified and increase of 8% without WLC and an increase of 17% when the same constraint is enforced. Nonetheless, the objective function aims at balancing the workload of the teams, thus making the comparison unsuitable to assess the correct functioning and utility of the function. It was then decided that a new indicator should be created, discriminating the difference regarding ST. The results clearly show that there is an improvement in the values of the indicator from the first to the second objective function. In addition, there is also an improvement in the indicator when the WLC is enforced.

7.2 Future work

The first suggestion of work regards the model presented. A study of a form to implement the second function based on the indicator of route balance being defined as the sum of the differences among teams' service times. Of course, it would involve modulus, because the difference between the teams' service time can be either positive of negative, and that quantity should be minimized. However, I am familiar with the fact that it is not easy to model the idealized objective function in GAMS, what would probably require some time to investigate strategies to transform the equation into something that the GAMS can work with.

The second suggestion arises from the knowledge that APOIO is currently implementing a software that enables the insertion of arrivals and departures to patients' homes directly into a database, enhancing the accessibility of the data, which will no longer be stored in paper records. This also permits the monitoring of patients and of the uncertainty to them associated. The newly available type of details is exactly what is needed to develop a Stochastic HHC routing and scheduling problem. In spite of being common practice in other areas of problems formulated as VRPs, in HHC there is very little work done in the area as is stated by Fikar and Hirsch (2017).

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