

Optimization of routes for road surface inspection of a portuguese national road network

The Case Study of Infraestruturas de Portugal S.A.

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Abstract

The company Infraestruturas de Portugal S.A. (IP) is responsible for managing part of the national road network, with a total of 14,000 km. The pavements of this set of roads are inspected annually by an equipment designated Road Surface Tester Laser (RST Laser). This activity is an expensive procedure, resulting in the need to create an optimization model to present the optimal circuit of inspection, respecting the existing technical constraints and with the objective of minimizing the total cost of the process. The problem was characterized as an Arc Routing Problem, the Rural Postman Problem, rarely addressed in the literature but with many practical applications. After defining the problem, it was conducted a literature review and then an optimization model was developed, along with the collection and processing of the data of the national road network (with a geographic information system - ArcGIS software).

The model uses binary decision variables and positive integer auxiliary variables. Three models were developed (for the undirected, directed and mixed variants of the Rural Postman Problem), that were implemented in GAMS modelling system and resolved with the CPLEX solver.

Based on the results, which were compared with those presented in literature references, it was concluded that the optimization model has a good performance (resolution times). Regarding the case study (district of Bragança), it was possible to reduce the total distance travelled during the inspection in 44% (considering the model closest to the real system, the Mixed Rural Postman Problem).

Keywords: Rural Postman Problem, Road Inspection Routing, Routing, Linear Programming, Geographical Information Systems (GIS).

1. Introduction

Infraestruturas de Portugal, S.A. (IP) is a Portuguese institution with the responsibility of managing the rail and part of the road network infrastructures in Portugal.

Every year, an inspection must be performed on the roads that are managed by IP, which include national roads and some motorway segments, in an approximate total of 14,000 km. The inspection is performed based on a direct registration of a set of parameters related with pavement geometry and conditions, as well as other relevant road

events, using a vehicle equipped with a device called Road Surface Tester (RST) Laser (Figure 1). The use of direct pavement parameter registration to incorporate data in Pavement Management Systems is a current practice in many countries.



Figure 1 - Road Surface Tester Laser

The Laser RST consists of a computer system and measuring lasers, which collects data about the geometry and characteristics of the pavement surface, tracing the cross-sectional and longitudinal profile of the road. The readings collected by the Laser RST are transmitted in real time to the computers aboard the inspection vehicle. This functionality allows an operator to check data consistency in real time, as well as manually introducing additional information (through a simple keyboard specific for such function).

The inspection process is carried out continuously and completely integrated in the traffic, and must follow practical restrictions. The inspection operation should be performed in summer months, in order to avoid the existence of wet sections. As the collection of data is based on a process of emission-reception of a beam emitted by a laser, the presence of water can influence the reflection and consequently the information collected.

In order to reduce the work involved in the process, the inspection is done in only one of the directions of the road, and on roads that have more than one lane per direction, the process is performed in the rightmost one. Over the years, the inspected lane is always the same (in the same direction as the previous year), in order to allow the comparison with historical data. On motorway sections, the inspection is performed in both directions, keeping the monitoring only on the rightmost traffic lane. The inspection with the RST Laser represents an expensive procedure, on which circuits for the vehicle have to be planned. This way, it was decided to develop an optimization model for the definition of the road network inspection circuits, respecting the existing constraints. The objective function of such model represents the minimization of the total costs involved in the operation which are not only related with the extension of the circuits but also with other expenses, such as the costs of lodging and equipment maintenance.

In the development of a model for this problem, the objective is to inspect only the roads under IP concession, which only need one yearly visit. However, the entire national

road network is considered, since the use of sections that are not under IP concession and act as shortcuts in the overall plan can contribute to the production of a better global solution. Likewise, it will also be possible to travel the same section more than once. At the end of the work day, there is the possibility of returning the vehicle to one of the company headquarters (Lisbon or Coimbra) or to stay in a certain place (possibly near the place where the inspection ended that day).

Currently, IP inspections are based on district-level plans, on which the total area of Portugal is divided into clusters (districts) and the sections contained in that district are visited in the same work stage. These plans are designed so that the day starts and ends in Lisbon or Coimbra.

There is no algorithm used in the definition of the circuits to be travelled, only the knowledge and experience of the employees that define them manually.

Considering the data for 2014, with an average speed of approximately 55 km/h, it took 65 days of operation to carry out the inspection of the entire national road network. Figure 2 displays the map of the road network to be inspected by IP (in red) as well as connecting roads that might be used (in black).

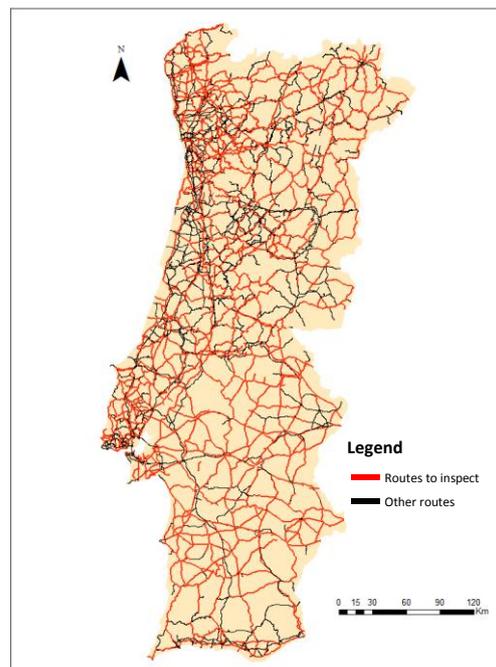


Figure 2 – National road network to be inspected by IP (red) and other roads (black) (IP)

It is considered that this work is necessary and important, and may represent a contribution to the scientific area and an improvement to the results of IP, and consequently in the performance of other companies that may adopt the presented method.

In order to be able to test and evaluate the performance of the model, it will be applied to a case study concerning a zone of Portugal, the district of Bragança. In the end, a comparison will be made between the inspection circuit used by the IP and the circuit obtained through this study, and a significant reduction in the total costs of the operation is expected.

2. Methodology

To achieve the proposed goals, it is necessary to define an appropriate methodology. This way, the main activities performed during this study after getting acquainted with the problem were the following:

- Elaborate a literature review on routing problems related with the road inspection problem faced by IP, specially by focusing on the types, characteristics, modelling and solving algorithms;
- Collect and processing of data on the road network under IP concession using the ArcGIS 10.0 geographical information system (ESRI, 2011);
- Define a mathematical programming model that adapts to the IP road inspection;
- Implement, solve and analyse the model within the GAMS (General Algebraic Modelling System) modelling software system;
- Produce, present and analyse a solution;
- Compare the solution of the inspection circuit proposed in the study and the inspection circuit used in the inspection of 2014 for the case study;
- Learn conclusions from the study.

3. Routing problems

Routing Problems are a family of mathematical problems studied in Graph

Theory that deal with the optimization of paths modelled by graphs. In general, the aim is to define routes in conformity with a set of constraints that approximates to some real problem. Included in the Routing Problems family are Node Routing Problems – which contain the class of Vehicle Routing Problems – and Arc Routing Problems, including the Chinese Postman Problem and the Rural Postman Problem (Figure 3).

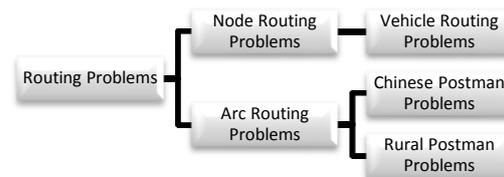


Figure 3 – Routing Problems classes

Considering a graph $G = (V, A)$, Routing Problems are used to determine a minimum cost path that uses a subset of desired vertices Q contained in V or a subset of desired arcs / edges R contained in A . Vertices are specific geographic points (e.g. shop, city), while arcs and edges are the connection between those vertices. If the connection has a direction (for example, it is only possible to travel from the vertex A to B), it is called arc, otherwise the link is designated as an edge.

The Vehicle Routing Problems comprise a set of problems in which the objective is to output a route or a set of routes that set up a plan that allows one or more vehicles (fleet), with one or more places of departure (deposits), to satisfy a certain number of geographically dispersed vertices (typically customers). These routes should satisfy the demand of all the customers involved, with the lowest possible cost (e.g. less distance travelled, less number of vehicles needed), where each route starts at the same point where it ends (Caric and Gold, 2008).

The Arc Routing Problems aim to determine a route that includes a certain set of arcs and/or edges, respecting imposed constraints and minimizing the associated costs.

Two important Arc Routing Problems are distinguished based on the characteristics of the subsets Q and R defined above (Eiselt *et al.*, 1995a): the **Chinese Postman Problem**

(CPP), where $Q = \emptyset$ and $R = A$; and the **Rural Postman Problem (RPP)**, where $Q = \emptyset$ and R is a predefined subset of A .

In the Chinese Postman Problem the objective is to create a route that contains all the arcs or edges of the graph. If it is not necessary to provide the service in a given arc or edge, then it can not be used only for vertex movement and is not contemplated in the graph.

Contrary to the modelling used in the Chinese Postman Problem, in most real-world problems only a few arcs or edges needs to be served in the network, while the remaining are used for connection purposes in moving between successive vertices. This way, the Rural Postman Problem (RPP) emerged as a model that fits the studied problem, as the set of roads to inspect is a subset of the entire network.

The RPP, based on the CPP, was introduced by Orloff in 1974. Two years later, Lenstra and Rinnooy Kan (1976) proved that it was an NP-hard problem. Over the years, this problem has been used to model many real-world tasks, such as municipal garbage collection, reading electricity meters, salt distribution on the roads to snow melting, distribution of newspapers and mail, school bus routes, inspection of power lines and roads.

Christofides *et al.* (1981) created the first Integer Linear Programming (ILP) formulation to solve the Undirected RPP through an exact algorithm. However, this algorithm is of exponential complexity, that is, only small size problems can be solved in acceptable computational times. The complexity of the problem and the need to solve larger instances led to the exploration of approximate methods of resolution. One of the best known heuristics was proposed by Frederickson (1979), based on a Travelling Salesman Problem (TSP) heuristic. Córdoba *et al.* (1998) presented a heuristic to solve the TSP using the methods used in Monte Carlo Simulation. The idea is to simulate a vehicle that moves randomly in a certain graph, limited to some restrictions. In each step, the destination is chosen based on probabilities and the vehicle returns to the starting node once all the

desired arcs/edges have been visited. This process is repeated a number of times and the result is the route that presents the lowest cost (e.g. total distance travelled). In 2009, Aráoz *et al.* presented an alternative way of addressing the Rural Postman Problem. In this paper it is proposed that the first visit to each of the desired edges contributes positively to the objective function, replacing the use of a subset of desired edges. Later, Monroy-Licht *et al.* (2014) presented three proposals of mathematical formulation for the Problem of the Undirected Rural Postman Problem with Time Windows.

4. Data collection and processing

In order to obtain a solution that could be applicable in the real system, it is necessary to provide the correct input data. Hence, it was necessary to verify that the available data was transformable and compatible with the various software packages used in the work. IP provided the data in two types of files: shapefile (a geographic dataset in vector format with the road features, with a binary classification of being or not being a road to inspect, among other attributes) and a spreadsheet with the operations diary log.

Knowing the format that the data should present in the modelling software system that was later used, it was needed to process the original datasets in the area of the Bragança district using a GIS (ArcGIS 10.0): the first step was a spatial data processing and editing that was necessary to get a representative model of the geometry and topology of the road network. Examples of the various editing operations include the union of road sections which were split by district boundaries, transformation of all the segments of a roundabout into a single vertex and representation of two parallel road segments in one road centerline. It was verified that the changes resulting from this editions were irrelevant to the distance values involved in the problem. The edition operations were made manually and the correction of the approach was validated through an internal data structure, designated network dataset, built using the

ArcGIS Network Analyst module. Data needed by the modelling software system are the values of distances between pairs of nodes in table format. This data was not provided by IP, which implied its extraction from the available data. The collection of the coordinates of the 264 nodes in the district of Bragança was made using again ArcGIS, while the 69.696 edges distance collection and validation was made using ArcGIS simultaneously with Google Maps. The cost matrix was written in a spreadsheet (Microsoft Excel), and data was validated in order to check if there were errors in the manual insertion of data (e.g. conditional formatting, sampling). In the transition of the data from the spreadsheet to the GAMS software, it was needed an additional processing stage.

5. Model definition and implementation

Based on the problem description and the literature review, a mathematical programming model was developed. Considering the computational results and characteristics of each of the formulations studied in the literature, the choice fell in a formulation based on the work by Monroy-Licht *et al.* (2014) – a model "on the nodes", using the sub-route elimination proposed by Miller *et al.* (1960). According to Pataki (2003), the formulation of Miller *et al.* (1960) presents a mathematical formulation option of small size that only requires a number of extra variables that is the same as the number of nodes and about half the square of that number in additional constraints. Additionally, this formulation is flexible regarding the order in which sections are visited in the network (for example, adding to the objective function the term $-\alpha u_i$ for $\alpha > 0$, it is possible to prioritize the order in which certain sections of the network are visited). These reasons, together with the level of complexity regarding the implementation of the mathematical model in GAMS language, led to the choice of applying the Miller *et al.* (1960) formulation to the elimination of sub-routes in the resolution.

Using this formulation, it is necessary to add one node in the beginning and end of each segment of the road network, i.e., when an

intersection occurs between one or more required segments, it is necessary to divide and consider them as independent, adding one node to each one of its extremities. The mathematical programming model used to represent the procedure described above is presented below.

a) Indexes

i – Visited vertex

j – Vertex to visit

b) Parameters

c_{ij} – Travel cost between the nodes i and j

n – Total number of nodes

R – Subset of required edges

N_1 – Subset of required nodes

c) Decision variables

x_{ij} – Binary variable that assumes the value 1 if vertex j is visited after vertex i , 0 otherwise.

u_i – Auxiliary integer variable

u_j – Auxiliary integer variable

Objective function:

$$\min \sum_{ij} c_{ij} \times x_{ij} \quad (1)$$

Subject to:

$$x_{ij} + x_{ji} = 1, \quad \forall (i, j) \in R \quad (2)$$

$$\sum_{j \in N_1, j \neq i} x_{ij} = 1, \quad \forall i \in N_1 \quad (3)$$

$$\sum_{i \in N_1, i \neq j} x_{ij} = 1, \quad \forall j \in N_1 \quad (4)$$

$$u_i = 1 \quad (5)$$

$$2 \leq u_i \leq n, \quad \forall i \neq 1 \quad (6)$$

$$u_i - u_j + 1 \leq (n - 1)(1 - x_{ij}), \quad (7)$$

$$\forall i \neq 1, \forall j \neq 1$$

The objective function of this model (1) represents the minimization of the total cost of the road inspection (distance). In order to assure that each one of the required arcs/edges are visited, in only one direction, equation (2) was used. Constraints (3) and (4) impose that each node should be visited exactly one time. For sub-route elimination, the constraints (5), (6) and (7) were added. This formulation defines that for each edge

(i,j), u_j has to be greater or equal than $u_i + 1$. If the solution contains more than one route, at least one of the sub-routes doesn't have the node $i = 1$, which would result in a situation where u_i had to increase to infinity in that route. Therefore, the only possible value to u_i is the position of node i in the route.

After the mathematical programming model definition, it was necessary to translate it into a model using GAMS language, so it is then possible to run the model and obtain solutions. Over time, some tests and improvements were made to the model, going through various forms of programming, until a final solution was obtained.

The model was then solved using the software GAMS (GAMS Development Corporation, 2013) and the solver IBM ILOG CPLEX (GAMS 24.4.6; CPLEX 12.6.2.0).

6. Results

Table 1 presents some information regarding the solution of the Undirected Rural Postman Problem (URPP). The relative gap represents the difference between the presented solution and the optimal solution, and the number of iterations is associated to the branch and bound algorithm.

Table 1 – GAMS model numeric characteristics (URPP)

CPU time (min)	Relative Gap	Absolute Gap (km)	# Iterations	# variables	# equations
2	16.20%	273	32,330	70,225	71,148
10	9.37%	148	80,884		
30	4.48%	67	202,708		
60	0.88%	13	649,203		

Observing Table 1, it is verified that after 1 hour of computational time, a relative gap lower than 1% is reached, representing a difference between the solution obtained and the optimal solution of just 13 km.

At this point, it was concluded that this variant of the model, main objective of the study, could be reformulated in order to improve the benefits for IP, namely the maintenance of previous visiting directions

of road segments, for reasons related with data comparison with previous inspections. For that reason, it was developed the Directed Rural Postman Problem (DRPP). The numeric characteristics of the model resolution are presented in Table 2.

Table 2 - GAMS model numeric characteristics (DRPP)

CPU time (min)	Relative Gap	Absolute Gap (km)	Number of Iterations	Number of variables	Number of equations
0.11	0.0%	0	0	70,225	71,148

It is verified that this variant of the RPP is less complex, achieving the optimal solution in just 6.76 seconds.

During the development of the undirected variant, it was verified that the “theoretical” optimal solution resulted in a solution that could not be applied by IP, due to the existence of motorways: contrary to the rest of the roads, motorways are represented in the model by two different segments, each representing one traffic direction. Using the undirected variant, it is not possible to define the direction of an edge, which could result in a route inspection solution that has a motorway edge that couldn't be inspected in the direction presented in the model solution. This is a detail that represents a variation in the total distance of the solution of 1% (84 km). Despite being a small variation, it was considered important to approximate the model to reality by creating a mixed variant. Data regarding the GAMS resolution of this model is presented in Table 3 and relates with the Mixed Rural Postman Problem (MRPP).

Table 3 - GAMS model numeric characteristics (MRPP)

CPU time (min)	Relative Gap	Absolute Gap (km)	# Iterations	# variables	# equations
2	27.39%	530	4,112	70,225	71,148
10	5.19%	78	44,113		
30	4.64%	70	87,839		
60	0.72%	10	1,107,154		

For the mixed variant, it is possible to obtain a solution within a 0.72% relative gap after 1 hour of CPU time, which represents a solution worse than the optimal in 10 km. Due to the requirements of the real problem,

the implementation of this solution is absolutely acceptable.

The solution of the Mixed Rural Postman Problem (MRPP), the variant most similar to the real problem under study, was represented in the ArcGIS geographic information system (Figure 4). Such solution has a total distance of 1,454 km. Red lines represent the roads to inspect, while the blue segments are roads used in the solution where no inspection was made and were used in connecting the inspected segments.

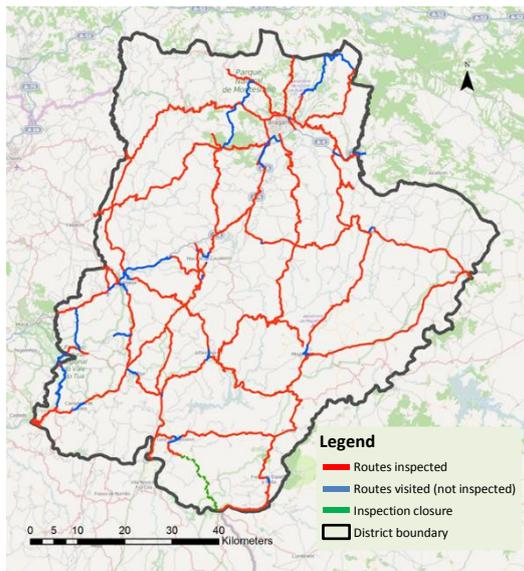


Figure 4 – Representation in ArcGIS of the solution of the problem - MRPP variant (IP, OpenStreetMap)

To assure the quality of the developed models, several analyses were performed for the three variants of the RPP, comparing the undirected variant (main objective of the present dissertation) with results of similar problems in the literature.

Using the undirected variant of the RPP, the results achieved by this model were compared with some references in the literature (in the comparisons made, it was not possible to use the same test examples). The proposed model for the URPP presents a superior capacity comparing with that presented by Christofides *et al.* (1981), who created the first formulation of Integer Linear Programming, successfully applied to instances of the problem with 9 to 84 nodes,

13 to 184 edges and 4 to 78 edges required (to be inspected).

For the Frederickson (1979) heuristic, one of the most common in the literature of this area, consider the computational results for some instances of Christofides *et al.* (1981) and Corberán (1998), presented in the work of Hertz *et al.* (1999). Table 4 enables a comparison between the results of the Frederickson heuristic, the Frederickson heuristic associated with the 2-opt algorithm and the proposed model.

Table 4 - Comparison between the proposed model and the heuristic of Frederickson and Frederickson + 2-opt¹

# edges	# edges required	Frederickson Heur.		Frederickson Heur. + 2-opt		Proposed model	
		Rel. Gap	CPU Time (s)	Rel. Gap	CPU Time (s)	Rel. Gap	CPU Time (s)
110 (4,950)	55 (50)	8.89%	0.01	0%	21.14	0%	6.25
216 (19,900)	82 (100)	6.76%	0.2	0.28%	3271	0%	40.20
228 (19,900)	108 (100)	5.33%	0.2	1.62%	5225	0%	40.20
186 (44,850)	132 (150)	0%	0.1	0%	1056	0%	13.97

The test cases used on the proposed model were not based on the same instances; however, it can be concluded that the proposed model finds the optimal solution for problems of higher complexity (similar number of edges to be inspected and a higher number of edges). In general, solving times of the proposed model are lower than those of the remaining heuristics that were presented.

To compare the performance of the proposed model with the computational results of more recent studies, Table 5 presents data regarding the heuristic of Pérez-Delgado (2007a) for some instances of Christofides *et al.* (1981). Through the analysis of Table 5 and comparing with the heuristic of Pérez-Delgado (2007a), it can be stated that the proposed model is able to find the optimal solution for problems of greater complexity, presenting lower CPU times.

¹ The values presented between brackets are the number of edges and edges required for the proposed model.

Table 5 - Comparison between the proposed model and the Heuristic of Pérez-Delgado¹

# edges	# edges required	Pérez-Delgado Heuristic		Proposed model	
		Relative Gap	CPU Time (s)	Relative Gap	CPU Time (s)
79 (4,950)	31 (50)	0%	1.82	0%	6.25
110 (4,950)	67 (50)	7.92%	8.12	0%	6.25
184 (19,900)	74 (100)	5.96%	15.8	0%	40.20

Based on the results obtained in the test examples solved by the proposed model and in some existing references in the literature, it is concluded that the model presents a good performance (mainly visible at the level of the capacity supported and the presented computational times).

In order to approximate the model to the reality of the problem under study, two scenarios were analysed for each of the four existing paths (solutions from the three types of models developed and the heuristic solution used by the IP in 2014). The scenarios are based on two options: (i) the vehicle returns to the headquarter after the inspection; or (ii) it stays overnight near the place where the daily inspection ended. To carry out this analysis, data referring to the inspection performed in 2014 were used and some assumptions were considered (e.g. average values of fuel price, average daily wage).

For option (i), it was considered the average distance to travel from Bragança to Coimbra and return to Bragança, 540 km. For option (ii), staying in Bragança, was considered the average distance between the periphery and the center of the district, 80 km. Thus, while for the option 1 an average speed of 100 km/h is assumed (due to the type of roads in which the vehicle is traveling during the course), for option 2, the average speed value is 70 km/h. Based on the average speed and distance, the time spent in each scenario is calculated. Considering a working day of 8 hours, removing the hours used in travelling out of inspections, it is obtained the period of time available for inspection. With the time available for inspection and the overall average speed

(64 km/h), the distance travelled during a day, and consequently the number of days required to complete the inspection work is calculated. With the number of days required, the number of trips, the amount of wages to be paid and the cost of stay are defined (for option 1, return to Coimbra, it is considered that there is no cost of stay). Through the analysis of Table 6, the first conclusion is that option 2, stay in Bragança, is economically more attractive than the option to return to Coimbra. The option to stay in Bragança (for any path considered) represents a reduction of 72% comparing with the total distance travelled in option 1, while the cost is 59% lower than the alternative to return to Coimbra.

Regarding the three models created, both have an estimated cost reduction of more than 40% when compared to the heuristic path of IP in 2014. The directed model is the one with the smallest reduction (the optimization potential is limited by the imposition of the inspection direction). For the undirected and mixed models, the percentage reduction is similar, which is justified by the small number of motorway sections in the case study of Bragança. However, regardless of the results obtained for the case study, the mixed model is the one that provides the most correct solution to apply in the real world problem.

Table 6 - Comparative analysis regarding the 2 scenarios created and the circuits in study

Comparative analysis			
Option 1: return to Coimbra			
	km	€	% Reduction €
Inspection route of IP in 2014	11,747	3,188	0%
Undirected RPP	6,066	1,646	48%
Directed RPP	6,565	1,782	44%
Mixed RPP	6,150	1,669	48%
Option 2: stay in Bragança			
	km	€	% Reduction €
Inspection route of IP in 2014	3,281	1,308	0%
Undirected RPP	1,694	676	48%
Directed RPP	1,834	731	44%
Mixed RPP	1,718	685	48%

7. Conclusion

In this document, a brief introduction to the problem placed by Infraestruturas de Portugal (IP) and the methodology used on its resolution were presented. Based on the problem description and the literature review, a study on the most appropriate Routing Problem to model the studied problem was made. Specific procedures for data collection and processing were needed to adapt data for the various software-based steps until a model was able to be developed.

The problem was modelled and solved with three different variants. The undirected variant is the one that provides the best solution (1,646 km in a closed path), but directed and mixed variants were developed due to the requisites placed by IP; results for these indicate a reduction of the total distance of the closed path in 44% and 48%, respectively. At this point, it is necessary to understand the priority of the company: reduce the total distance in 44% and keep the historical directions of road segments inspection, or reduce 48% with the loss of the possibility to compare the results with historical data. Comparing directly the two models, and considering only the district of Bragança, the mixed model has a total distance (closed path) inferior to the directed model in 6%. Considering the district of Bragança, 6% may seem not very representative, but at the scale of an entire national road network it gains relevance. To each of the variants, two scenarios were created: (i) the inspection vehicle returns to the closest headquarter at the end of the day – Coimbra; or (ii) the vehicle stays in Bragança at the end of the day. Through the results of the analysis, it is concluded that the option in which the vehicle stays in Bragança is the most effective, presenting an estimated cost reduction of 59%, when compared with option (i).

Analyses were made to test the quality of the developed models and have proved that the model exhibits a good performance. Comparing the Undirected Rural Postman Problem proposed in this study with the Integer Linear Programming presented in Christofides *et al.* (1981), it is concluded that

the proposed model has the capacity to solve larger problems in shorter computational times. Regarding both the heuristic and the heuristic 2-opt of Frederickson (1979), the proposed formulation finds the optimal solution for more complex problems, usually in less computation time. Using recent references, the model was compared with the work performed by Pérez-Delgado (2007). Once again, the model proposed here is capable of finding the optimal solution for more complex problems, usually in less CPU time. It is necessary to explain that the comparative analysis was not made using the same test examples: such analysis was based only on the dimension of the problems solved, in the relative gap and the computational times obtained.

A future development of this work is its extension to the totality of the national road network. However, to assure that this scale improvement is successful, it is important that the data collection and processing be optimized and automated. The model should be able to create daily inspection plans and the optimization software must be integrated with the technology of the company IP, improving the data compatibility and transference processes. In order to make the model more dynamic, the optimization software could be integrated with the system onboard the vehicle, providing the best solution in real time (using information not available in the beginning of the inspection). This study also contributes to the academic field through the development of three variants for the Rural Postman Problem: undirected, directed and mixed, which were modelled and implemented in GAMS language. This study was presented to the academic community in IO 2015 – XVII Congresso da APDIO (September 2015, Portugal), ICORES 2016 – International Conference on Operations Research and Enterprise Systems (January 2016, Italy) and II Conferência Nacional de Geodesição (May 2016, Portugal). Regarding the contribution to Infraestruturas de Portugal, this study is a proposal with potential to reduce costs and optimize the inspection process of the national road network and represents the first step towards the

development of a decision support system for the institution.

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