Application of Dilation Invariance Principle in Eddy-current Nondestructive Testing

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Abstract: The objective of this thesis is to provide a proof of concept on the possibility to measure the same magnetic fields, by the principle of dilation invariance, using eddy current nondestructive testing technique. According to this principle, dilation of the dimensional and electrical parameters of a probe and a plate with crack reproduces the same magnetic field (measured at every dilated point) which was produced by a probe and a plate with crack, without dilation. This implies that, in order to evaluate the performance of eddy current method in a specific industrial case study, one can carry out tests in a scaled model in the laboratory and then, extend the conclusions to the real situation by using dilation invariance principle. The experimental and simulation results show a good agreement of crack detection by using two eddy current probes of which one is the dilated version of the other. The work has been analytically supported by considering a simpler case of an axially symmetric geometry, without a crack. The resulting equations of the magnetic flux density are evaluated numerically using stratified Monte Carlo numerical integration technique.

Keywords: ECT; eddy current testing; nondestructive evaluation; dilation invariance principle; Maxwell equations; magnetic field; magnetic vector potential; eddy-currents.

I. Introduction

Eddy current testing (ECT) is one of the most used techniques to detect and characterize defects in electrically conductive materials [1]. This technique is a fast and effective method not only to detect defects in metallic plates caused by corrosion or stress, but also to identify other anomalies, like magnetic permeability changes, that occur due to extreme conditions of heat and pressure.

Eddy current testing is a potential technique to detect the anomalies that occur during aging in the catalytic tubes of reformer furnaces [2]. In petrochemical plants, furnace tubes are exposed to extremely high temperatures and the materials of the tubes become deteriorated and may break [3-5]. Such anomalies include surface and subsurface cracks, inclusions, voids, macro and microstructural degradation. The inspection of those tubes in a nondestructive way to assess the emergence of anomalies is very important and yet it is not used on a regular basis [6-7].

In this thesis ECT is presented as a technique that can be used to determine from the outer surface of a furnace tube defects that appear in the inner surface. This inspection is not a trivial task. Two limit conditions exist: one, is the thickness of the tube, which can vary from 10 to 20 mm surpassing the capacity of detection of currently existing ECT commercial equipment and the other is related to the capacity of detection of the microstructural changes that can lead to tube failures that initiate within the tube wall some two thirds of the way through from the outer surface [2,6].

To demonstrate that the ECT method can be used, this thesis reports tests performed with two probes of which one probe is a scaled version of the other, to characterize two cracks which were machined on aluminum (Al1050) plates. Tests showed that, when geometrical parameters and electrical quantities obey the dilation invariance principle, then the magnetic field obtained in the two situations is the same. Thus, by applying the principle of dilation invariance, one may calculate the dimension and electrical parameters of the probe required to inspect from the outer surface a material that has a thickness more than the one that conventional ECT based devices can detect, as it happens in the furnace tubes.

The work done is divided into six sections. After this Introduction, section 2 describes in detail the situations to be tested. The dilation principle theory presented in [8] will be revisited by highlighting the main outcomes. Section 3 describes the experimental setup used in the tests and the results obtained experimentally. Section 4 presents a brief description of the application of a finite element program and the results obtained with the simulation. Section 5 deals with the analytical aspects of the dilation invariance principle. Finally section 6 concludes the work done.

II. Description of Case Studies

Experiments involved two case studies [12]. In the first study (study1), a plate (p1) with a notch (nc1) was tested with a probe (pb1). The probe pb1 was excited with a sinusoidal current of amplitude $I_1$ with a frequency $f_1$. The probe had $N_1$ turns. A giant magnetoresistor (GMR) sensor [9-10]
integrated in probe pb1, was used to measure the magnetic field. The second study (study2) involved a plate (p2) with a notch (nc2) being tested with a probe (pb2) with pb2 being excited with a sinusoidal current of amplitude I2 and frequency f2. A GMR identical to the one in pb1 is integrated in pb2.

In study1, the geometrical (i.e. dimensions) parameters and the electrical parameters (i.e. frequency and current amplitude) of study1 were “dilated”. Measurement of the magnetic field from both the cases at every normalized measuring point (the measuring points are normalized with respect to the dilation factor) resulted in the same signature.

The dilation factor (k) was considered to be 4. According to [3], if the geometrical parameters are dilated k times, the electrical parameters like frequency should be dilated k² and the magnetomotive force (mmf) by a factor k (in this study, the number of turns has been dilated k times and the current amplitude is not dilated), in order that the magnetic field remains an invariant at every normalized point in the space.

For the study1 a notch of dimensions 0.5 × 12.5 × 0.5 mm (w × l × t) was machined in the aluminium plate p1 of 1 mm thickness. The probe pb1 had an axially symmetric geometry with a rectangular cross section of dimension 5 × 9 mm, and an inner diameter of 10 mm. The probe was excited with a sinusoidal current of amplitude, I1 = 50 mA and an excitation frequency of 10 kHz. This value was chosen, since the crack was superficial with a thickness depth of 0.5 mm and considering the that skin effect for the Al1050 at 10 kHz the standard penetration depth is 0.44×10⁻³ m.

The parameters for both the case studies are specified in the following comparison tables (table 1 and table 2).

<table>
<thead>
<tr>
<th>dimensional parameters</th>
<th>study1(k=1)</th>
<th>study2(k=4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>crack dimension</td>
<td>0.5 × 12.5 × 0.5 mm³</td>
<td>0.5 × 50 × 2 mm³</td>
</tr>
<tr>
<td>plate thickness</td>
<td>1 mm</td>
<td>3 mm</td>
</tr>
<tr>
<td>probe inner diameter</td>
<td>10 mm</td>
<td>40 mm</td>
</tr>
<tr>
<td>probe cross-section area</td>
<td>45 mm²</td>
<td>720 mm²</td>
</tr>
</tbody>
</table>

In study2 the crack width was not dilated, as because most of the eddy currents flow is perpendicular to the notch length, the effect of eddy current perturbation due to the width, is very less.

### III. Experimental Work

The experimental setup [11] used for the experiments is depicted in Fig. 1.

![Fig. 1. Experimental setup – two-axis positioning system.](image)

It includes a Fluke 5700 calibrator working as an AC-source, a USB DAQ for signal acquisition, and a ROTRA X-Y positioning system. The instruments are remotely controlled using an application developed in LabVIEW and installed in a desktop PC that communicates with the instruments through RS232 (XY scanner), GPIB (Fluke 5700) and USB (NI USB DAQ 6251).

The current applied to the excitation coil of the probe is produced by the calibrator functioning as a current generator. The voltage signal obtained from the GMR [9-10] (V_{GMR}) is amplified by a low noise instrumentation amplifier stage that was implemented using an INA118 with a gain value set to 101. The output signal is applied to the analogue inputs of the 16-bits multifunction board working with a maximum acquisition rate of 1.2 MHz. The number of samples per position was fixed to 8000 (for 100 periods). So depending on the excitation frequency, the sampling rate changes.

For each position of the probe the output signal is acquired during some periods of the excitation current. The estimation of the signal amplitude and phase is obtained using the MATLAB “tone measurement function” that is based on a sine-fitting algorithm.

In both probes the induction field is measured using a GMR (NVE-AA002) sensor coplanar with the sample and positioned at the excitation coil axis. The axis of sensitivity is normal to the excitation coil symmetry axis. It has a linear operation range between 1.5 Oe to 10.5 Oe, and sensitivity in the range between 3.0 mV/V Oe and 4.2 mV/V-Oe. It is powered by a Calex BPS4000 power supply (not depicted in Fig.1). A permanent magnet is used to assure the GMR biasing, providing a constant magnetic field in the sensitive direction. The output of the GMR, being a voltage signal, can be converted to the magnetic field using the conversion

<table>
<thead>
<tr>
<th>electrical parameters</th>
<th>study1(k=1)</th>
<th>study2(k=4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of turns (N)</td>
<td>130</td>
<td>520</td>
</tr>
<tr>
<td>current amplitude (I)</td>
<td>50 mA</td>
<td>50 mA</td>
</tr>
<tr>
<td>excitation frequency (f)</td>
<td>10 kHz</td>
<td>625 Hz</td>
</tr>
</tbody>
</table>
factor that was calculated (by using, sensitivity: 3.6mV/V) to be 101808 $V_T^{-1}$. A photo of the probes to be used is presented in Fig. 2.

![Probes used for the study](image)

**Fig. 2:** Probes used for the study

The experiment consists of two scans for each study; horizontal (parallel to crack length i.e. along y axis) and vertical (perpendicular to the crack length i.e. along x axis). The axis of sensitivity of the GMR sensor is aligned parallel to x-axis, thus detecting the field along that axis. The schematic of the procedure is shown in Fig.3 a) and Fig. 4 a).

These studies were carried out using the previously described ‘two-axis positioning system’ which positions the probe at a desired point above the plate. It has a movement resolution of 0.1mm. For study1 the probe was scanned along the y axis over the crack at a step size of 0.1 mm with lift-off of 0.23 mm and a scan along the x axis through the centre of the crack at a step size of 0.1 mm with lift-off of 0.23 mm. For study2 the same procedure was repeated but with a step size of 0.4 mm (0.1 mm × 4) and lift-off of 0.92 mm (≈ 0.23 mm × 4). The output voltage signal from the GMR sensor was amplified using an instrumentation amplifier and was converted to the corresponding magnetic field value by knowing the sensor sensitivity. It was plotted in an application developed in MATLAB. The value x = 0 is considered to be the middle point of the defect.

![Horizontal scan direction and signature of the Bx](image)

**Fig. 3.** Horizontal scan: a) Scan direction b) Scan result

In a similar way the vertical scan direction and the signature of the $B_x$ for both the case studies is shown in Fig. 4.

![Vertical scan direction and signature of the Bx](image)

**Fig. 4.** Vertical scan: a) Scan direction b) Scan result

It can be observed that if the geometries are scaled by 4 times, mmf by 4 times and frequency by 1/16 times, the GMR response remains invariant for either case. There are some discrepancies in the detected fields as shown in Fig.5.

![Discrepancies in the detected magnetic field](image)

**Fig. 5.** the discrepancies in the detected magnetic field

These discrepancies can be attributed to the misalignment of the sensor inside the probe or the non-positioning of the probe at the exact dilated point. It could as well have happened due to the inexact winding of the coils. Not positioning the probe exactly vertical (a small tilt) above the plate could also have caused these discrepancies. These discrepancies have been overcome by simulating the studies.

### IV. Simulation

The problems involving eddy currents can be verified by simulating them using a numerical simulation program. For an isotropic, non magnetic medium, the field equation (that needs to be solved) can be represented using the magnetic vector potential ($A$) as:

$$\nabla^2 A + j\omega \mu_0 \sigma A = \mu_0 J_s$$

This is the differential equation which needs to be solved when we are using low frequency ($\omega$) excitation (few kHz). The solution to such problems doesn’t exist in a closed form manner. So to solve such a system, the finite element method is used. Softwares like COMSOL Multiphysics, Ansys, FEMM and Maxwell performs solution to these problems based on the geometry of the problem assigned to it by the user. In this case the geometry
is a plate of specific dimension, with a crack and a probe of specific dimension.

The situations of the experiments reported in the previous sections have been simulated using commercial finite element simulation software, COMSOL, to confirm the dilation invariance [12].

In order to find accurate results with this method, it was necessary to follow several guidelines. These guidelines are:

- The magnetic flux was computed in a region with high density mesh, whatever the position of the probe to avoid numerical “noise”.
- A fine mesh was applied in the part of the solid conductor below the probe. Eddy currents are created in this zone. An extrusive mesh was used to control the number of mesh layers in the plate and adapt it to the skin depth (two or three layers of elements in the skin depth).
- The lift-off region was very narrow and the air boundary was very large. So an additional layer was created and was given the properties of air. This layer was meshed with high density mesh.
- The air boundary on either sides of the plate and probe was kept at a sufficiently large distance so that the Dirichlet boundary condition of the magnetic vector potential at the boundaries was sufficiently zero.
- The probe being cylindrical, should have a low curvature resolution, so that it closely resembles a circle.

For the case of simulation two kinds of scans (same as that of experiment) were considered. In those scans, the probe position was kept constant and the crack was moved. It is because the point of interest (evaluation point) which is located at the axial center of the probe, doesn’t change. However in the case of simulation both the \( B_x \) and the \( B_z \) has been evaluated [12].

The results of the magnetic field for the horizontal scan is shown in Fig.6.

![Fig.6: Horizontal scan measurement of a)\( B_x \), b) \( B_z \)](image)

The results of the magnetic field for the vertical scan is shown in Fig.7. It can be seen that the obtained results from Fig.6 and Fig.7 have a good match between the 2 case studies. The discrepancies that arose in the experimental results are now resolved. Also simulation of the problem helped us to find the solution of the other components of the magnetic field, namely the \( B_z \) component.

![Fig.7: Vertical scan measurement of a)\( B_x \), b) \( B_z \)](image)

However even in the simulation study we could observe some errors. These could be attributed to the meshing of domain that is under study. The computation time for the solution, when the mesh is very fine, is very long. So some regions has to be meshed coarsely and some regions densely. The region between the probe and the plate being very small, might cause inverted mesh element error, which leads to not so accurate solutions. It is difficult to avoid these conditions. So the only way out is to repeat the simulations many times and identify which gives better results.

A simplified version of the 3D problem, which has been considered until now, could be reduced by considering that the probe and plate have a symmetry along the z-axis. This is a 3D problem but with 1 less variation i.e. Azimuthal component. In this case consider that the electric field and the magnetic vector potential have just 1 component (i.e. along \( \phi \)). So the magnetic flux density must be present in the plane perpendicular to it, having 2 components (\( B_r \) and \( B_z \)). The surface plot of the two components are depicted in Fig. 8.

![Fig.8 a) Radial component of magnetic field b) Axial component of magnetic field](image)
V. Analytical Study

The analytical study involves in proving the dilation invariance principle analytically by solving the differential equation for the 2D axi-symmetric geometry and by considering the case of absence of cracks. The four basic equations of Maxwell is shown to be below:

\[ \nabla \times H = J \]  
\[ \nabla \times E = -\frac{\partial B}{\partial t} \]  
\[ \nabla \cdot B = 0 \]  
\[ \nabla \cdot D = \rho \]

From equation 1 to equation 4, the variables that are involved are, H is the magnetic field intensity, E is the electric field intensity, D is the Displacement field density, \( \rho \) is the charge density. For the simple case of the excitation current being a sinusoid, \( e^{j\omega t} \), any time derivative involved such as equation 4 could be re-written as

\[ \nabla \times E = -j\omega B \]

The Helmholtz decomposition of the vector field is that, any vector field consists of 2 components. One term is solenoidal and the other term a curl free term. Applying this to the magnetic field, yields 2 components. In this case the curl free term is 0 because of equation 4. So it has just 1 term and is shown as:

\[ B = \nabla \times A \]

Here the vector \( A \) is called as the magnetic vector potential. To arrive at the field equation, we consider initially the solution of magnetic vector potential for a filamentary loop (delta function coil), which is the scalar Green’s function [13]. The full solution is obtained by superposition theorem i.e. solution of the other filamentary loop present in the coil is added up (integrated, because it is a continuous case) [13].

For the case of the delta function coil, the field equation could be derived by using equations 1 to 6 and is shown in equation 7.

\[ \nabla^2 A - j\omega \mu \sigma A = -\mu \delta(r - r_0, z - L) \]  

The geometry of the problem under consideration is shown in Fig. 9

Fig.9: A delta function coil placed over 2 conducting samples

The solution approach would be to compute the first order Hankel transform.

\[ A(r, z) \xrightarrow{\text{Hankel}} \Psi(s, z) \]  

The differential equation represented in equation 7, has been solved by computing the Hankel transform. The resulting equation in the transformed domain is:

\[ \frac{\partial^2 \Psi}{\partial s^2} - (s^2 + j\omega \mu \sigma)\Psi = 0 \]

The solution to this differential equation is shown in equation 10:

\[ \Psi^i(s, z) = x^i(s)e^{-siz} + y^i(s)e^{siz} \]

Where, \( s^2 = s^2 - k^2 \).

There are 2 boundary conditions considered in our case [13]. The Electric field, which has only the azimuthal component, is continuous at the boundary. So the magnetic vector potential is also continuous (since, \( E = -j\omega A \)) at the boundary. The other boundary condition being the Magnetic field. We know that \( B_r(-\partial A/\partial z) \) is discontinuous at the boundary, due to discontinuity of permeability. But in our case, we have assumed that \( \mu_r = 1 \) everywhere in the domain. So the normal derivative of \( A \) with respect to \( z \) is continuous. There is a discontinuity caused by the current source (delta function coil), So we have to apply an Amperian loop at that region and make the equation at the boundary to be continuous.

Solving the system using the boundary condition, we can obtain the Green’s function of the magnetic vector potential. To get the full solution, we need to superpose the solution of the delta function coils, in the region in which the coil is defined. This is mathematically represented as:

\[ A(r, z) = \int_{r_1}^{r_2} \int_{L_1}^{L_2} A(r, z, r_0, L)dLdr_0 \]

The current in the filamentary loop, should be replaced by the current density of an N turn coil [13]. Having known the total magnetic vector potential, the magnetic flux density [15] could be computed by considering equation 6. For the case of a single conducting plate (region 2 is taken as air), the radial and axial component of the magnetic field has been computed using stratified Monte Carlo numerical integration [14] technique. The results are displayed. For the case of k=1, the magnetic field is shown in fig. 10

Fig.10: Case k=1 – a) Radial component of magnetic field b) Axial component of magnetic field
Similarly for the case of $k=4$, the dilated sample, the magnetic field is shown in fig. 11.

![Fig.11: Case k=4](image)

(a) Radial component of magnetic field

(b) Axial component of magnetic field

In order to see that the magnetic field has been invariant upon the dilation of dimensional and electrical parameters, their ratio is taken and displayed in fig. 12.

![Fig.12: Ratio of](image)

(a) Radial component of magnetic fields

(b) Axial component of magnetic fields

It can be seen that the ratio is not exact 1, but there is a small variation of 2% around 1, which could be attributed to the error due to numerical integration. But we can say that the magnetic field remains invariant upon dilation of the dimensional and electrical parameters.

### VI. Conclusion

This thesis proves the possibility to inspect a material in a stretched geometry with an eddy current method (ECM) using a formulation of the scale invariance principle theory. This thesis demonstrates that in the scale transform domain, the stretch factor of signals can be manipulated in order to compute quantities invariant to changes in that stretch factor.

The experimental and simulation studies confirmed that, the signature of the magnetic field produced by both the dilated and non-dilated versions match. Hence it is proved that in order the magnetic field to remain an invariant on dilating the dimensions of the plate and the crack by ‘$k$’ times, the dimensions of the probe needs to be dilated ‘$k$’ times, mmf by ‘$k$’ times and the operation frequency by ‘$k^{-2}$’ times. The analytical work, which has been carried out for a simplified geometry, shows a good match for both the case studies. Thus this articles serves as a proof of concept for applying the dilation invariance theory in eddy current testing for industrial specimen.

### References


