Design and optimization of a telescopic wing regarding the applicability of the AZ31B-F magnesium alloy under multiaxial cyclic loading

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To my mother, my brother, my sister and my grandparents.
Acknowledgments

This thesis means the end of my graduation, therefore I must be grateful to a lot of persons who helped me and supported me throughout my long journey at Instituto Superior Técnico.

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**Resumo**

O magnésio e as suas ligas estão a tornar-se cada vez mais usadas na indústria aeroespacial e automóvel devido às suas propriedades mecânicas. A investigação de tecnologias relacionadas com materiais e ligas sofreu uma grande evolução permitindo às ligas de magnésio terem uma performance mecânicas e protecção contra a corrosão próximas das ligas de alumínio. Com o desenvolvimento de novos materiais foi possível desenvolver novos conceitos e novas ideias relativamente a asas adaptativas (morphing wings).

O presente trabalho foi desenvolvido com o objectivo de realizar a otimização de uma asa telescópica e de caracterizar o comportamento mecânico da liga de magnésio AZ31B-F quando sujeita a fadiga multiaxial. Após este estudo estar concluído, o próximo passo seria considerar a hipótese de integrar o magnésio num mecanismo capaz de actuar a asa telescópica.

O software MATLAB, em conjunto com outras ferramentas de cálculo, foram utilizados para criar um algoritmo capaz de obter a solução numérica com a configuração optimizada da asa adaptativa. Por outro lado, o estudo da liga de magnésio foi desenvolvido com recurso a ensaios experimentais realizados com provetes tubulares, sujeitos a carregamentos proporcionais e não proporcionais.

Para finalizar, os resultados obtidos para a asa telescópica demonstram que existe uma clara relação entre a fase de voo e a dimensão, estendida da asa. Em relação ao magnésio e com os dados obtidos nos ensaios experimentais, foi possível obter a função ssf (stress scale factor), em função do carregamento axial e de corte.

**Palavras-chave:** Fadiga Multiaxial, Liga de Magnésio, Asa Adaptativa, Solução numérica, Otimização da asa
Abstract

Magnesium and its alloys are becoming increasingly common in the aerospace and automotive industries. In terms of mechanical performance, magnesium alloys are reaching a level similar to aluminium alloys, which are the main competitors. With the development of new materials it has been possible to develop new concepts and new ideas regarding morphing wings.

This work was developed with the aim of accomplish the optimization of a telescoping wing and to characterize and understand the mechanical behaviour of magnesium alloy AZ31B-F, when subjected to multiaxial fatigue. After the study is complete, the next step would be to consider the possibility of integrating the magnesium in a mechanism capable of operating the telescopic wing.

MATLAB® software in association with other design tools were used to create an algorithm capable of obtaining the numerical solution for the optimal configuration of the telescopic wing. On the other hand, the study of the magnesium alloy was developed using experimental tests, carried out with tubular specimens, under proportional and non-proportional loading paths.

Finally, the results obtained for the telescopic wing establish that there is a clear relationship between the phase of flight and the span size of the wing. Regarding the magnesium and the data obtained in the experiments, it was possible to obtain a stress scale factor, ssf, function using as arguments the axial stress amplitude and the stress amplitude.

Keywords: Multiaxial Fatigue, Magnesium alloy, Morphing Wing, Numerical Solution, Wing Optimization
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Nomenclature

Greek symbols
\(\alpha\) Angle of attack.
\(\gamma\) Climb angle.
\(\nu\) Kinematic Viscosity.
\(\rho\) Density.
\(\sigma\) Normal stress.
\(\tau\) Shear stress.
\(\theta\) Twist angle.

Roman symbols
\(b\) Wing span.
\(c\) Chord.
\(Cd\) Drag coefficient.
\(Cl\) Lift coefficient.
\(Cm\) Moment coefficient.
\(D\) Drag.
\(h\) Altitude.
\(L\) Lift.
\(N_t\) Number of cycles.
\(T\) Thrust.
\(V\) Velocity.
\(W\) Weight.
\(x\) Telescopic wing span.
<table>
<thead>
<tr>
<th>Subscripts</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Initial values.</td>
</tr>
<tr>
<td>a</td>
<td>Applied.</td>
</tr>
<tr>
<td>eqv</td>
<td>Equivalent.</td>
</tr>
<tr>
<td>root</td>
<td>Wing root.</td>
</tr>
<tr>
<td>tip</td>
<td>Wing tip.</td>
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### Glossary

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>AISI</td>
<td>American Iron and Steel Institute</td>
</tr>
<tr>
<td>ASME</td>
<td>American Society of Mechanical Engineers</td>
</tr>
<tr>
<td>ASTM</td>
<td>American Society for Testing and Materials</td>
</tr>
<tr>
<td>BCC</td>
<td>Body Centred Cubic</td>
</tr>
<tr>
<td>EAP</td>
<td>Electro Active Polymers</td>
</tr>
<tr>
<td>HCF</td>
<td>High-Cycle Fatigue</td>
</tr>
<tr>
<td>HCP</td>
<td>Hexagonal Close-Packed</td>
</tr>
<tr>
<td>HV</td>
<td>Hardness</td>
</tr>
<tr>
<td>IST</td>
<td>Instituto Superior Técnico</td>
</tr>
<tr>
<td>LCF</td>
<td>Low-Cycle Fatigue</td>
</tr>
<tr>
<td>MAT</td>
<td>Morphing Aircraft Technology</td>
</tr>
<tr>
<td>MCC</td>
<td>Minimum Circumscribed Circle</td>
</tr>
<tr>
<td>MCE</td>
<td>Minimum Circumscribed Ellipse</td>
</tr>
<tr>
<td>R&amp;D</td>
<td>Research and Development</td>
</tr>
<tr>
<td>SMA</td>
<td>Shape Memory Alloys</td>
</tr>
<tr>
<td>SSM</td>
<td>Semi-Solid Metal</td>
</tr>
<tr>
<td>SWT</td>
<td>Smith, Watson and Topper</td>
</tr>
<tr>
<td>UAV</td>
<td>Unmanned Aerial Vehicle</td>
</tr>
<tr>
<td>sar</td>
<td>Stress Amplitude Ratio</td>
</tr>
<tr>
<td>ssf</td>
<td>Stress Scale Factor</td>
</tr>
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</table>
Chapter 1

Introduction

Magnesium is a very abundant metal. In fact, it constitutes about two percent of the Earth’s crust, and it is the third most plentiful element dissolved in seawater although it is never found free in nature [1].

In terms of mechanical performance, magnesium alloys are reaching a level similar to aluminium alloys, which are the principal competitors of magnesium alloys in the aerospace and automobile industries. Magnesium alloys belong to the category of light alloys and this is based on the fact that its crystalline structure is Hexagonal Close-Packed (HCP).

The magnesium production is increasing, and this tendency seems to be generalized in the world. The reduced weight and the possibility of recycling are two of the main reasons for that demand. These parameters have become crucial in the implementation of life cycle procedures where the modern mechanical design has a fundamental role [2].

The definition of the Morphing Aircraft Technology concept has not been well established yet. Morphing is short for metamorphose and has its root in the Greek words ‘meta’ (change) and ‘morpheme’ (form). In the aeronautical field, morphing is adopted to define a set of technologies that increase a vehicle’s performance, by manipulating certain characteristics to better match the vehicle state to the environment and task at hand. There is neither an exact definition, nor an agreement, between researchers, about the type or the extent of the geometrical changes necessary to qualify an aircraft for the title shape morphing [3].

The performance and dynamic efficiency of an aircraft are significantly influenced by the aircraft shape and configuration. Therefore, the wing load response in terms of drag and lift has been given increasing attention through morphing technology [4].
1.1 Magnesium Alloys Overview

The development of magnesium alloys and associated technologies is, nowadays, a subject of intensive research. In the past, magnesium alloys have been studied extensively, in order to determine fatigue properties under constant amplitude loadings [2].

Bentachfine et al. [5] studied a lithium–magnesium alloy under proportional and non-proportional loading paths, under low-cycle and high cycle fatigue regime, observing the deformation mode evolution and plasticity behavior. It was stated that the phase shift angle in the non-proportional loading paths, decreases the material fatigue strength. The comparative parameter used to correlate experimental data was the von Mises equivalent stress/strain. However, with this approach the material under non-proportional loadings, keeps a constant equivalent stress. This way, no change in the material occurs along each loading cycle [6].

In table 1.1 the mechanical properties of AZ31B-F magnesium alloy are presented.

<table>
<thead>
<tr>
<th>Mechanical Properties</th>
<th>Microstructure type</th>
<th>Young’s modulus (GPa)</th>
<th>Poisson’s ratio</th>
<th>Density (Kg/m$^3$)</th>
<th>Hardness (HV)</th>
<th>Tensile strength (MPa)</th>
<th>Yield strength (MPa)</th>
<th>Elongation (%)</th>
<th>$\sigma'_{f}$ Fatigue strength coefficient (MPa)</th>
<th>$b$ Fatigue strength coefficient</th>
<th>$\epsilon'_{f}$ Fatigue ductility coefficient</th>
<th>$c$ Fatigue ductility exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HCP</td>
<td>45</td>
<td>0.35</td>
<td>1770</td>
<td>86</td>
<td>290</td>
<td>203</td>
<td>14</td>
<td>450</td>
<td>-0.12</td>
<td>0.26</td>
<td>-0.71</td>
</tr>
</tbody>
</table>

1.2 Motivation

Although there are a lot of publications regarding multiaxial fatigue behavior of steels, there are very few studies regarding multiaxial fatigue of magnesium alloys [7].

Recently, Yu et al. [8], have also studied in-phase and out-phase behavior under strain controlled tests on AZ61A extruded magnesium alloy, using tubular specimens. The conclusions were similar to Bentachfine et al. [5] - the presence of the phase shift angle leads to a decrease in the fatigue strength compared with in-phase cases for the same equivalent strain amplitude. At low-cycle fatigue regime, it was reported a kink in the strain life curve which is a typical behavior for uniaxial fatigue regime in magnesium alloys. Furthermore, the effect of compressive mean stress was evaluated, concluding that a compressive mean stress increases fatigue life [9].
A key point on the multiaxial fatigue damage interpretation is to find out the relationship between the damage caused by the axial and shear stress components and quantify their combined effect, which is very different from the effect of a single stress component [10]-[11].

The majority of multiaxial fatigue models somehow tries to establish this damage relationship between axial and shear stress components [12]-[13].

### 1.3 Objectives

A study of the magnesium alloy AZ31B, under high cycle fatigue regime and under proportional and non-proportional loading conditions, is presented in order to map the stress scale factor (ssf).

In this work, hour-glass specimens for HCF made of the magnesium alloy, AZ31B-F, were used with the goal of being sensitive to the loading path type and stress level, considering experimental fatigue data, under proportional loading conditions, with different stress amplitude ratios (sar).

Consequently, it is intended to analyse and quantify the ssf between the axial and shear stresses, as well as their combined effect in proportional and non-proportional multiaxial loading conditions, in order to be used as a tool in fatigue life estimations.

The ssf is commonly used in multiaxial fatigue models to describe the combined axial and shear damage effect on the fatigue process. The objective is to reduce the axial and shear stresses to the same stress space. Prof. Vítor Anes et al. [14] made a similar study using a high strength steel 42CrMo.

Another objective is to develop a mechanism for a telescopic wing, using the magnesium alloy AZ31B-F.

Magnesium alloys have distinct properties that can enable better performance when applied to a morphing wing.

### 1.4 Morphing Aircraft Technologies: Concept Definition

The ability of a wing surface to change its geometry, during flight, has interested researchers and designers over the years as this reduces the design compromises required.

Geometrical parameters that can be affected by morphing solutions can be categorized into: planform alteration (span, sweep, and chord), out-of-plane transformation (twist, dihedral/gull, and span-wise
bending), and airfoil adjustment (camber and thickness).

Historically, morphing solutions always led to penalties in terms of cost, complexity, or weight, although, in certain circumstances, these were overcome by system-level benefits. The current trend for highly efficient and ‘green’ aircrafts makes such compromises less acceptable, calling for innovative morphing designs able to provide more benefits and fewer drawbacks.

The challenge is to design a structure that is capable of withstanding the prescribed loads, and, at the same time, be able to change its shape: ideally, there should be no distinction between the structure and the actuation system.

Furthermore, any successful wing morphing system must overcome the weight penalty due to the additional actuation systems [3].

Morphing aircrafts are flight vehicles that change their shape to effect both a change in the mission of the aircraft and to perform flight control without the use of conventional control surfaces. Aircrafts constructed with morphing technology promise the distinct advantages of being able to fly multiple types of missions, to perform radically new manoeuvres (not possible with conventional control surfaces), to be more fuel efficient and to provide a reduced radar signature. The key to morphing aircraft is the full integration of the shape control into the wing structure: a truly smart structure [15].

Mechanisms such as deployable flaps provide the current standard of adaptive aerofoil geometry, although this solution places limitations on manoeuvrability and efficiency, and produces a design that is non-optimal in many flight regimes. The development of new ‘smart’ materials, together with the always present need for better unmanned aerial vehicle performance is increasingly prompting designers towards the concept of morphing aircraft. These aircrafts possess the ability to adapt and optimize their shape to achieve multi-objective mission roles efficiently and effectively [15].

Airplanes fly under a wide range of temperature, density and wind conditions. They also have to perform different flight manoeuvres during a flight: take off, landing, cruise, climb, coordinated turns and other manoeuvres. To perform efficiently in these conditions, the aircraft is required to have different configurations. The aircrafts that will be able to operate in optimal conditions, throughout the entire flight envelope, will increase their fuel efficiency and manoeuvring capabilities, for that reason, are called ‘morphing aircrafts’.

The most notorious morphing technology application can be found in the F-14 TomCat fight aircraft 1.1. This plane changes its sweep angle to strike a balance between range and speed by delaying the rise in drag for higher speeds [16].

Different forms of aerodynamic shape control can be achieved by retractable landing gears, flaps and
other devices that are deployed when needed. The majority of aircrafts use these kinds of devices for specific flight phases, e.g. flaps are deployed for take-off and landing to increase lift at low speeds, by increasing the wing surface and changing its camber line: ailerons change wing twist angle to provide roll control and are used for all types of turns [16].

Whatever the type and age of the technology and whatever materials, actuators and structures it may use, the morphing aircraft technology significantly expands the aircraft flight envelope and/or improves its performance parameters at some, or all, flight conditions, when compared to a baseline conventional aircraft [17].

Morphing aircrafts can bring many advantages to aeronautical industries. Therefore, a lot of research in this area has been done. The latest research has been made on unmanned aerials vehicles with morphing technology and drones. These airplanes are easier to build, lighter and less complex, making them the perfect candidates for experiments, therefore increasing the interest and demand to universities and students.

This type of aircraft is also suitable for research in the field of new material. As a result, testing of new structures and actuation mechanisms is in order, since they are lighter, reliable and stiffer [16].

Traditionally, Morphing Aircraft Technologies (MAT) are described, and divided, according to the morphing concept to which they are related to, which in turn are generally related to a particular geometrical change in the wing. Studies were made on the following morphing concepts/geometric parameters: twist, sweep, dihedral (folding and winglets), chord, span and camber. Major literature reviews can be found in Barbarino et al. [3] and Gomez et al. [18].

The design of adaptive mechanisms and structures, along with the development of smart materials that allow the imitation of biological configurations of aircraft, is highly desired in the near future. Piezoelectric actuation, Shape Memory Alloy (SMA) actuation and Electro Active Polymers (EAP) are the most relevant actuation systems in morphing technology and still under heavy research and development.
1.4.1 Telescopic Wing

During take-off and landing the high lift airfoils are extended at the wing tips. When transitioning to a high speed cruise, they are retracted in flight to leave a high-speed low drag wing capable of withstanding high ‘g’ loads.

The mechanical innovations lead to aerodynamic advantages. The telescopic wing is an alternative to sophisticated high lift devices (e.g. slotted fowler flaps). It is used at only low speeds and has a maximum speed limit just as a conventional flap [20].

The increased aspect ratio at low speeds improves efficiency. Unlike conventional flaps, the retracted configuration has much higher stiffness and resistance to bending and twist (i.e. flutter resistant) than a conventional wing and the retractable wing is lighter than a conventional wing with the same stall speed, maximum speed, and strength requirements. These are a few of the aerodynamic advantages of the telescopic wing [20].

In terms of safety, numerous innovations make this a very safe aircraft. For this type of morphing technology, the extended wing lowers the stall speed, take off and landing distance, and increases the
climb rate, climb angle, ceiling, range, and endurance.

1.5 Goals and Inspiration

Aircrafts built with morphing technology have the distinct advantages of being able to fly many types of missions, to perform radically new manoeuvres (not possible with conventional control surfaces), to be more fuel efficient and to provide a reduced radar signature. The key concept is the full integration of the control shape of wing structure with a truly intelligent structure. The design of these vehicles must take full account to the aerodynamic loads, and must carefully consider the power requirements for shaping control, in order to ensure an overall performance benefit [21].

Some of the main challenges are concerned with environmental issues, such as reducing noise and greenhouse emissions [22]. Efficiency increase and consequence reduced burned fuel has a multiple effect when addressing this issues.

So far, new concepts never have gone further than the experimental state due to the high complexity of structures, the lack of energy efficiency as well as the weight efficiency of the actuation devices. Recently, new technologies and the creation of advanced materials made possible the design of morphing wings that can adapt to a specific flight condition, with the aim to improve the aircraft performance. Morphing structures are complex and require several actuation devices, which increase aircraft weight. Manufacturing is, thus, harder than in fixed wings. As a result, it’s necessary to balance the improvement of performance with the increase of complexity and weight [16].

Other topic of discussion is the role of the aircraft in which the morphing technology is supposed to be introduced, whether it is frequently changed or it remains the same during the aircraft lifetime. A multi-role aircraft is more likely to benefit from morphing technologies than an aircraft which remains in the same flight conditions throughout the majority of its service time and is optimized for those flight conditions [17].

The present work focuses on the study, design and optimization of a variable-span morphing wing to be fitted to a UAV.

The design of adaptive mechanisms and structures, along with the development of smart materials that allow bio-mimetic configurations of aircraft, is highly desired in the near future. The new concepts and technologies developed up to now resulted from a constant attempt to enhance the overall flight performance of aircraft, enabling new approaches to the aircraft’s design and improving multi-mission flexibility. This performance enhancement capability was clearly demonstrated by Tidwell et al. [23].
1.6 Thesis Outline

The remaining Chapters of this thesis are organized as follows:

Chapter 2 describes the theoretical background for magnesium alloys and morphing aircrafts. Also presents the most significant engineering applications, the issue of recycling and respective comparison in terms of extraction costs, given its main competitor, aluminum. To end this chapter, a mathematical model is explained that served as based for the calculation algorithm in following chapters.

Chapter 3 presents the overview of the AZ31B-F magnesium alloy used in this study, a detailed explanation regarding the specimen geometry, a brief overview of the test equipment and the methodology to conduct the experimental tests.

Chapter 4 introduces a brief description of numerical methods used in this work, necessary to characterize the morphing wing.

Chapter 5 presents and discusses the results obtained for the various approaches.

Chapter 6 presents the conclusions drawn from this study as well as some proposals for future development.
Chapter 2

Bibliographic review

2.1 Introduction

This chapter presents a theoretical background that covers several aspects related to both magnesium and morphing.

It presents a brief explanation on pure magnesium production as well as alloy nomenclature, crystal structure, component manufacturing processes, some of the most significant applications and finally, the issue of recycling this material. Regarding the topic of fatigue, a historical summary is presented while the remaining of the chapter is devoted to explaining fundamental concepts regarding the stress scale factor (ssf), which are key to present the theoretical background of the work performed.

It is also summarized the method used for aerodynamic shape optimization. One fundamental objective of this morphing wing is to be able to fly with a constant lift, equal to aircraft weight, with the lowest drag possible. Furthermore, relevant principles are introduced in terms of the Lifting Line Theory.

2.2 Magnesium vs Aluminium

This section presents the comparison between magnesium and his major competitor aluminum.

The main disadvantage of magnesium alloys were the difficulties inherent to technological process, in particular production, and due to problems associated with corrosion. However, some of these difficulties have been overcome through the development of new alloys with improved corrosion resistance; by careful addition of alloying elements, in particular manganese [24].

The mechanical properties of pure magnesium are not sufficient to enable it to be used in normal technical applications, [25]. Thus, in the aircraft industry, magnesium alloys are not used in structural applications, [26]. Regarding the automotive industry, magnesium alloys are used in very specific appli-
cations, a few are presented in section 2.2.3. As such, magnesium should be used as alloying element to other metals, so it can be used in engineering applications [25]. One of the most common elements in the magnesium alloy is aluminum, so as one of the main competitors [24]. The main differences between these two materials can be seen in Table 2.1.

<table>
<thead>
<tr>
<th>Property</th>
<th>Mg</th>
<th>Al</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atomic number</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>Density when solid (at 20°C), [g/cm³]</td>
<td>1.74</td>
<td>2.70</td>
</tr>
<tr>
<td>Density when liquid (at 651°C), [g/cm³]</td>
<td>1.59</td>
<td>2.38</td>
</tr>
<tr>
<td>Melting point [°C]</td>
<td>649</td>
<td>660</td>
</tr>
<tr>
<td>Boiling point[°C]</td>
<td>1090</td>
<td>2520</td>
</tr>
<tr>
<td>Thermal conductivity (at 0-100°C), [W/(mK)]</td>
<td>155.5</td>
<td>238</td>
</tr>
<tr>
<td>Specific heat (at 20°C), [J/(kgK)]</td>
<td>1022</td>
<td>917</td>
</tr>
<tr>
<td>Coefficient of thermal expansion (at 0-100°C), [10⁻⁶K⁻¹]</td>
<td>26</td>
<td>23.5</td>
</tr>
<tr>
<td>Electrical resistivity (at 20°C), [µΩ]</td>
<td>4.2</td>
<td>2.67</td>
</tr>
<tr>
<td>Temperature coefficient of resistivity (at 0-100°C), [10⁻⁴K⁻¹]</td>
<td>4.25</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Magnesium alloys have been replaced in many engineering applications for aluminum alloys and polymers, [28]. In some applications magnesium alloys can compete with polymers. Magnesium density is substantially lower when compared with aluminum, 1.74 g/cm³ against 2.7 g/cm³, respectively, which represents about one third lower. There are some interesting characteristics of magnesium and aluminum in regard to the extraction and production. The energy required for extracting magnesium from the ore it is higher than that required to extract other comparable materials such as aluminum [29]. An advantage of magnesium compared with aluminum resides in the fact that the energy required to melt magnesium alloys is about 70% less than that required to melt the aluminum alloy [28], [29].

Currently, magnesium alloys are being R&D target in the industrialized countries. Magnesium alloys have enormous potential, reflecting the interest of aerospace and automotive industry, as well as help protect the environment [30].

### 2.2.1 Magnesium metal and alloy components manufacturing processes

Production of magnesium metal is usually performed through one of two paths: it is either done by a thermal reduction process or by an electrolytic process. However, there are variations depending on the manufacturer and also depending on the actual method within the process. A third way of producing magnesium, is by means of recycling [31],[32].

In the thermal reduction process, dolomite ore (a mineral composed of calcium magnesium carbonate) is crushed and put in a thermally insulated chamber in order for the mineral to go through a process called calcining, which produces a mixture of magnesium and calcium oxides. After obtaining the oxides, the magnesium oxide should be reduced, for the reduction to take place, ferrosilicon is used. Ferrosilicon is then crushed and mixed with the oxides, and finally made into briquettes that are loaded into a
reactor. The reaction takes place under low pressure and between 1200 to 1500 °C temperature range. The conditions mentioned produce magnesium as a vapour, which is condensed by cooling to about 850 °C in steel-lined condensers, and afterwards removed and cast into ingots.

For the electrolytic process two stages are required: first pure magnesium chloride should be produced from seawater or brine and only then, can the electrolysis of fused magnesium chloride take place. If magnesium chloride is produced from seawater, it must be treated with mixed oxides (obtained from dolomite), inducing the precipitation of magnesium hydroxide, which when heated will form magnesium oxide. To obtain magnesium chloride, the oxide should be heated while mixed with carbon, in a stream of chlorine at high temperature in an electric furnace. Obtaining magnesium chloride from brines requires evaporation stages to remove impurities. The product of the evaporation stages has to go through a final stage of dehydration, which requires hydrogen chloride to be present in gaseous form, in order to avoid hydrolysis of the magnesium chloride. Finally the magnesium chloride obtained is subjected to electrolysis, where it is continuously fed into electrolytic cells, which in turn are at temperatures high enough to melt it. Figure 2.1 describes de process. This operation produces magnesium and chlorine. The molten magnesium is then removed and cast into ingots.

Figure 2.1: Illustrating the production of magnesium chloride from magnesium oxide, [32].

Although magnesium alloys’ mechanical properties are currently slightly lower than its main competitors, they are still widely used in a variety of industries. For such uses, the alloys are mainly divided into two categories: casting alloys and wrought alloys. Casting alloys can be manufactured into magnesium alloy components through some conventional casting methods [33].

Sand casting can be used to manufacture components without much change in usual practices re-
lated to this process, there are however, a few particularities related to the characteristics of magnesium (physical and chemical) that should be investigated in order to produce a given part through this process. Die casting has similarities to the plastic injection moulding process and is commonly used for high production rates. This process can achieve high dimensional accuracy, produce parts with thin walls and improve productivity.

Another process that can be used is designated squeeze casting, which combines forging and casting processes. This process can be defined as direct or indirect depending on the method used to produce the actual part. In direct squeeze casting, molten magnesium is poured into a die at slow speeds, once the die cavity is filled a punch is brought down, applying pressure until the metal solidifies. In indirect squeeze casting the molten magnesium is poured into an encasement, after that, the speed of the molten metal flowing into the mould is controlled by a plunge[5].

Production of components by casting can also be done by means of Semi-Solid Metal (SSM) casting. Within SSM casting, there is one method that is more commonly used with magnesium alloys, this method is called Thixomolding and it was introduced in 1990 by Dow engineers. Thixomolding is quite alike plastic injection moulding, only in thixomolding, the feeder is filled with magnesium chips, taking the chips into a heated screw that starts heating the chips by rotating, while pushing them simultaneously. The heat and shear forces produced by the screw, generate a semi-solid slurry, which is injected into the mould to obtain the desired component [33].

Components made from magnesium alloys may also be obtained from wrought products like, extrusions, forgings, sheet and plate. Regardless of way a component has been produced, there is always room to give shape by means of machining.

Magnesium is a material with high machinability [34], this implies that the relative power required for a certain operation is lower than for other metals. However, there is a drawback in the midst of these characteristics, specifically, machining magnesium might present a fire hazard.

The machining operations can be performed by conventional manually-operated machine tools, or purpose-built, automatized machine tools [34]. The fact that magnesium alloys have good machinability allows for heavy cuts at high cutting speeds and feeds, which implies reduced operating times. Besides the previously stated, high thermal conductivity and low cutting pressure let the generated heat dissipate quickly, and thus improve tool life. The tools used in machining operations on magnesium alloys, should be chosen with great care, generally, regular carbon steel tools, can be used with satisfactory service lives. However, carbide-tip or diamond-tip tools can be used as well, especially if very fine finishes are required [13]. Independently of the tool, these should be kept sharp and smooth at all times to avoid poor surface finish, excessive heat, formation of long chips with burnished surfaces and the occurrence of flashing or sparking at the tool edge.
There are certain characteristics that the tools used for machining magnesium alloys should have, such as large peripheral relief angles, large chip spaces, few blades (for certain milling cutters) and small rake angles. Large relief and clearance angles are important in order to avoid excessive heating [34].

Machining of magnesium alloys is often done without cutting fluids, due to the material's thermal conductivity and resistance to galling, cooling and lubrication are seldom needed [34]. Although magnesium is mostly machined without recurring to cutting fluids, in certain cases, usage of said fluids might be required, particularly in operations that combine very high feeds and cutting speeds (higher than recommended), or in scenarios where the part must be cooled to avoid part distortion and prevent ignition of chips.

### 2.2.2 Metal alloys and crystal structure

Magnesium, as a metal obtained through the methods stated previously, is not suitable for mechanical applications. However, when alloyed to other elements, its properties improve significantly. Magnesium is the lightest structural metal available, with a density lower than aluminium's by about a third, and close to that of fibre reinforced plastics. The physical properties of magnesium are shown in Table 2.1 [27]. Magnesium is often alloyed with other elements, in order to improve its properties and become suitable for a wider range of applications in several industries.

The type of magnesium alloy mentioned is easily identified by its designation. These designations are usually comprised by two letters, followed by two numbers, and when applicable a third letter, and/or a fourth part consisting of a letter followed by a number, separated from the third part of the designation by a hyphen. The first two letters indicate the two main alloying elements, arranged in order of decreasing percentage, or alphabetically in case the percentages are equal. The two numbers that follow, express the percentages of the two main alloying elements. The letter in the third part distinguishes alloys with slightly different compositions within the same main designation. The fourth part of the designation indicates that the alloy has undergone some treatment [35], [36]. The correspondence between the letters used in the first part of the designation is as follows:

<table>
<thead>
<tr>
<th>Letter</th>
<th>Designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A – Aluminum</td>
<td>B – Bismuth</td>
</tr>
<tr>
<td>N – Nickel</td>
<td>P – Lead</td>
</tr>
<tr>
<td>T – Tin</td>
<td>W – Yttrium</td>
</tr>
</tbody>
</table>

Regarding the fourth part of the designation, the code used is presented in Table 2.3

Magnesium has a hexagonal close-packed (HCP) crystal structure being the main feature in terms of its metallurgy. The packing factor, which indicates the portion of volume in a crystal structure that is occupied by the atoms that constitute said volume, is 0.74. The layering of this type of crystal structure
alternates between two equivalent shifted positions, arranged in an \textit{aba} sequence as shown in Figure 2.2 [37].

<table>
<thead>
<tr>
<th></th>
<th>F – As fabricated</th>
<th>O – As annealed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>H10 and H11 – Slightly strain hardened</td>
<td>H23, H24 and H26 – Strain hardened and partially annealed</td>
</tr>
<tr>
<td>T4 – Solution heat treated</td>
<td>T5 – Artificially aged only</td>
<td></td>
</tr>
<tr>
<td>T6 – Solution heat treated and artificially aged</td>
<td>T8 – Solution heat treated, cold worked and artificially aged</td>
<td></td>
</tr>
</tbody>
</table>

![Figure 2.2: Layering of a Hexagonal close-packed structure, [37]](image)

This structure has implications regarding the behaviour of the material, since it is a non-cubic lattice and that means slippage does not occur easily, which in turn makes the material less deformable at room temperature, however, it can be deformed by conventional methods at higher temperatures (in a range about 200 to 225 °C). At room temperatures, the only deformation mechanisms are gliding and twinning.

### 2.2.3 Applications

The alloy category to be used depends totally on the component to be manufactured, which in turn depends on the application within the industry where it is to be applied. Magnesium alloys are used in a wide range of industries, such as: Aerospace, Automotive, Medical, Electronic, Sports and others [26], [33], [38]. From the aforementioned branches of application, the main consumers of magnesium alloys are the aerospace and automotive industries.

**Aerospace Industry**

Magnesium alloys have been used in the aerospace industry for quite some time, mainly used in military aircrafts such as the Sikorsky S-56 [39], the Lockheed F-80C [40], the Convair B-36 Peacemaker [41]).
or even the Tupolev TU-95MS [42].

Figure 2.3: Military aircraft applications that employ Magnesium (a) Sikorsky S-56 [39]; (b) Lockheed F-80C [40]; (c) Convair B-36 Peacemaker [41]; (d) Tupolev TU-95MS [42]

In the Sikorsky helicopter, magnesium alloys were found in the fuselage and the housing of the main gearbox [39]. The Lockheed F-80C, was completely built with magnesium [26], [43]. The American bomber, Convair B-36 had an impressive 8600 kg of magnesium [33], [26]. A considerable amount of magnesium could also be found on the Tupolev TU-95MS aircraft, with about 1550 kg of magnesium [26].

Nowadays magnesium alloys are broadly used in the aerospace industry, maintaining its presence in the branch of defence, particularly in the manufacturing of UAV’s [44] (Unmanned Aerial Vehicle) also known as Drones, or in other non-structural aircraft application such as cast transmission housings for example [26].

Automotive Industry

The usage of magnesium alloys in the automotive industry comprises the motor sport branch and also motorcycle manufacturers, since these fields are also using magnesium parts presently. The first noteworthy use of magnesium alloys in the automotive industry, dates back to 1938, with its use in the Volkswagen Beetle, which was designed by Ferdinand Porsche, Figure 2.4 a). This vehicle had more than 20 kg of magnesium made up from the transmission housing, the crankcase and other smaller parts, all obtained through casting [33]. A few other significant applications of magnesium alloys in the automotive industry came in the 1950’s, specifically with the Allard sports car Figure 2.4 b), and also the 1955 Mercedes-Benz 300 SLR Figure 2.4 c) [33].
The main application for the latter two examples was in body components, mainly made out of magnesium alloy sheet which contributed considerably to weight reduction, and in the case of the Allard, the global weight of the body with doors and bonnet reached 64 kg. Magnesium alloys are becoming more and more common in the automotive industry and there are a few applications in which this can be observed. There is a strong presence of magnesium alloy-based parts in the drivetrain, with components such as gearbox housings, intake manifolds, crankcases, cylinder head covers and even engine blocks as shown in Figure 2.5.

Also, magnesium alloys are used for interior components, such as steering wheel armatures, and other steering system components, seat frames, instrument panel trims and console frames. Body parts, such as doors, tailgates, roofs or bonnets can also be manufactured in magnesium alloy. Some of these examples are shown in Figure 2.6.
Magnesium alloys are found very useful in motorsport and other high performance road vehicles, namely in Formula 1, where the wheels should be manufactured in magnesium alloys (AZ70 or AZ80) as specified in the technical rule book for the 2015 season [45], and also Italian high performance motorcycle manufacturer MV Agusta, currently employs magnesium alloys in quite a few drivetrain components, and has used it in swingarms in past models.

Other applications

Magnesium alloys can be found in many other fields, particularly medical, electronic and sports. The main reasons behind the presence of magnesium alloys in the fields previously mentioned, is due to its characteristics, specifically its density, heat dissipation and improved mechanical resistance when compared to plastics that are replaced or compete with magnesium. Medical applications of magnesium alloys consist mainly of implants, due to the fact that magnesium emulates bone behaviour consistently. In electronic devices like laptops, cellular phones and other handheld products, magnesium is used more commonly to replace plastics, providing better mechanical properties, while maintaining if not improving weight savings. In sports, magnesium alloys present a strong alternative once again due to its low density, which allows for lighter equipment to be produced, specifically bicycle frames, tennis rackets and golf clubs [38].

2.2.4 Recycling

Metals complete the recycling cycle very easily, in the sense that they can be recovered after the end of the life cycle of their applications. The possibilities are virtually unlimited, provided that there is availability and economic advantage over other materials.

The recycling is possible due to the casting of alloys, as well as the subsequent reuse of scrap. With this process it is possible to produce a new component with quality identical to a root produced component [46], [47].

The magnesium production, mineral as well as seawater origin is relatively expensive [25]. Although is less expensive than aluminium. Therefore being recyclable is an important characteristic of this type of element. The industry, in general, also aims to minimize the cost of the life cycle, energy consumption and CO₂ emissions of their products in order to minimize the ecological footprint.
The automotive industry is constantly improving the process of recycling through systematic improvement of casting, as well as post-consumer scrap [25]. Energy requirements for melting scrap and recycle are only 5% of the energy required to produce the same amount of primary material [25].

2.3 Fatigue

Fatigue history is presented here by a chronological date. The majority of the cases presented in this history were based on Walter Schütz’s paper [48] but also a few others can be found in [49] and [50]. Focus is made in multiaxial fatigue and general fatigue advancements.

In 1837 Albert, a German mining administrator, published the first fatigue test results. To do the tests he constructed a test machine for the conveyor chains which had failed in service at the Clausthal mines.

In 1854 Braithwaite, an Englishman, used for the first time the term “fatigue” in his published papers where he describes many service fatigue failures of brewery equipment, water pumps, propeller shafts, crankshafts, railway axles, levers, cranes, etc.

Between 1858 and 1870 Whöler, a German engineer, measured service loads of railway axles with self developed deflection gages at 1858 and 1860. In 1860 Whöler published the results of fatigue tests with railway axles. In 1870 he presented a final report concluding: “material can be induced to fail by many repetitions of stresses, all of which are lower than the static strength. The stress amplitudes are decisive for the destruction of the cohesion of the material. The maximum stress is of influence only in so far as higher it is, the lower are the stress amplitudes which lead to failure”. According to Whöler, the stresses amplitudes are the most important parameter for fatigue life, but a mean stress also has a significant influence. Whöler represented his test results in form of tables but never plotted them, i.e., the stress vs life cycles number or S-N curve. His successor Spangenberg will have done it.

In 1881 Bauschinger, a German engineer, named the Bauschinger effect. This effect is “the change of the elastic limit by often repeated stress cycles” by author’s words.

In 1886 Bauschinger, professor of mechanics at the currently named Technical University of Munich, named the Bauschinger effect, which in his own words is “the change of the elastic limit by often repeated stress cycles”.

In 1898 Kirsch calculated a stress concentration factor of 3.0 for a cylindrical hole in an infinite plate.

In 1903 Ewing and Humfrey, from England, observed the slip bands on the surface of rotating bending specimens. This was probably the first metallurgical description of the fatigue process.
In 1910 Basquin, an American, represented the type of Whöler data table in the form $\log(\sigma_a)$ on the ordinate and $\log(N)$ on the abscissa. He describes the plotted data by the following formula, which is still used nowadays. In a large table Basquin presented values for $C$ and $n$ based on Whöler data.

$$\sigma_a = CN^n$$

In 1917 Haigh, an Englishman, mentioned the first time the term "corrosion fatigue".

In 1918 The first full-scale fatigue test with a large aircraft component was carried out at the Royal Aircraft Establishment in the U.K. and the analysis was published: "Methods employed at the Royal Aircraft Establishment for the experimental determination of the ultimate strength of aeroplane structures".

In 1920 Griffith, a Welshman, working in the Royal Aircraft Establishment in the U.K., developed the basis of fracture mechanics. He showed by tests on the brittle material glass that small cracks like scratches considerably reduced the breaking strength and that the crack size also had an influence. He described this process by a formula.

In 1921 Mason did a study of cyclic deformations in combined bending and torsion tests with a steel material. The first work considering the out of phase angle between loadings appears this year due to Mason and Delaney.

In 1924 Gough's book "The Fatigue of Metals", mentions the influence of surface roughness on fatigue limit and also the stress concentration factors of V shaped notches based on the results obtained by Coker. Also in 1924, Palmgren authored a well-known paper that contained the Palgrem-Miner rule and a four-parameter equation extending from the tensile strength to the fatigue limit for the SN curve.

In 1937 Neuber published the first comprehensive book covering the theoretical calculation of stress concentration factors, and fatigue stress concentration factors.

Between 1939 and 1945 Gassner defined the topic of operational fatigue strength (Betriebsfestigkeit in his native German), which consists of dimensioning a component for finite, but sufficient fatigue life under variable loads.

In 1954 Coffin and Manson defined the field of Low-Cycle Fatigue (LCF) by describing the behaviour of metallic materials under cyclic inelastic strain amplitudes through a four-parameter formula.

In 1955 Sines published a report in which he reviews experimental data available in alternated biaxial loadings and combined static and alternated stresses. He concluded that for brittle material, in spite of yield occurring near maximum theoretical normal stress, the failure could not be caused by normal
stresses; the shear cyclic stress appears to be the fatigue failure cause. Because of the last statement
Sines analyzed the static stresses influence in geometric planes where cyclic shear stresses are bigger
[51].

In 1956 Findley reviewed many experimental test in fatigue. He concluded that none previous result
is in contradiction with a shear stress limit. He investigated the Aluminum alloy 7075-T6 fatigue behavior
subjected to combined bending and torsion. All the loadings were in-phase. Findley extended some
yield known criteria to fatigue analysis. Trying to determine a better criterion for multiaxial fatigue analy-
sis, he observed in the plane of maximum shear stress amplitude the influence of normal tension.

In 1962 Paris in his Ph.D. Thesis, stated that fatigue crack propagation could be described by an
equation, nowadays known as Paris’ Law, which relates the crack growth rate, with the stress intensity
factor.

In 1967 Miller presents a modified octahedral shear stress criterion, which takes into account the
effects of principal axis rotation in non-proportional loadings [51].

In 1968 Elber, in his Ph.D. Thesis, observed that after a high traction load the crack closed before
the load return to zero. Today this phenomenon is known as crack closure.

Between 1969 and 1974 The American Society of Mechanical Engineers (ASME) debated about
applying Tresca criterion to the project of pressurized reservoirs involving multiaxial fatigue. Because of
its simplicity, conservatism and flexibility, Tresca criterion gained advantage to von Mises criterion. In
1974 it was chosen to estimate multiaxial fatigue life based on a procedure code.

In 1970 Smith, Watson and Topper (SWT) presented a fatigue damage parameter for materials that
generally fail in mode I. In this mode crack nucleates in shear directions and fatigue life is controlled by
the crack growth in perpendicular planes to maximum principal stress and maximum principal strain.

In 1973 Brown and Miller proposed a theory in which both cyclic shear strain and normal strain on
the plane of maximum shear must be considered, since according to this theory, cyclic shear strain will
help nucleate cracks, and normal strain will assist crack growth. This theory also suggests the terms
Case A and Case B cracks, depending on the nucleation and growth of the cracks [6].

In 1975 Grubisic and Simburger observed the effects of out-of-phase loading on the biaxial fatigue
strength of carbon steel, using thin-walled cylindrical specimens, and the results showed that phase
difference between shear and normal stresses can have a large influence in fatigue life [51].

In 1976 Blass and Zamrik subjected specimens made of AISI (American Iron and Steel Institute)
304 stainless steel to simultaneous tension-compression and alternating torsional loads at different tem-
peratures and with different shear strain ratios. They concluded that a fatigue failure criterion based on shear and normal strains acting on the plane of maximum shear strain would be more suitable than other criteria based on equivalent strain and other common measures [52].

In 1977 Kanazawa et al. studied LCF considering phase angle in multiaxial loadings. Steel 1% Cr-MoV specimens were subjected to combined torsion and axial loadings. They concluded that fatigue cracks initiate in the planes of maximum shear stresses subjected to all loading conditions and that fluctuations around this plane can be found in some cases; Tresca criterion and von Mises are not conservative under out of phase cyclic loadings; the multiaxial combined tension and torsion out of phase loadings produce more damage than the in phase loadings, specifically the 90° out of phase loading [53].

In 1979 Kanazawa et al. [54] continued on the work presented in 1977, and analysed the cyclic deformation under out-of-phase loads of the same 1% Cr-Mo-V steel, subjecting the specimens to combined axial and torsional loads. The results obtained showed that the hysteresis loop for the out-of-phase cyclic loads is quite different from the in-phase one.

In 1981 Garud proposed a new form to approach the multiaxial fatigue analysis and presented a failure criterion to multiaxial fatigue based on an energetic model. He suggested applying the concept of uniaxial cyclic hysteresis energy (of Morrow) to multiaxial fatigue. Naturally plastic work per cycle is the fatigue life (crack nucleation) parameter. The calculations were made with the help of a plasticity model. The author obtained good correlations for proportional and non proportional loadings applied to a 1% CrMoV steel. After this work Garud concluded that traction work causes more damage than shear work. Because of that he applied a weight factor to the term representing shear work. This model however is not suitable for HCF because the work per cycle is very small and so it is very difficult to calculate accurately [51].

In 1988 Fatemi and Socie presented a multiaxial fatigue damage parameter based on Brown and Miller work. They replaced the normal deformation term by normal stresses. The physical justification is that normal stresses make the surfaces of the crack to deviate, reducing the friction forces. Accounting the stresses instead of deformations makes the damage parameter calculus more suitable [55].

In 1989 Dang Van presented an endurance limit criterion based on the microstresses in a critical volume. Fatigue crack nucleation is a local process and starts in grains that have suffered plastic deformation and have formed slip bands. Due to this, Dang Van suggests that microscopic shear and hydrostatic stresses are an important parameter [6].

In 1993 Liu and Zenner presented a criterion based on a double integral. Reviewing the previous works they concluded that there were two ways to formulate a multiaxial fatigue damage parameter, i.e., by an integral formulation or by a critical plane formulation. The advantage of integral formulation is that
damage is calculated at all planes of a critical volume. The critical plane formulation only considers the plane where the damage parameter is maximum.

Between 1995 and 1997 Papadopoulos presented a microscopic integration model in 1995. In 1997 he presented the critical plane model named the Minimum Circumscribed Circle (MCC) to estimate the shear stress amplitude [56].

In 1997 Palin-Luc and Morel working in multiaxial fatigue concluded that for HCF a model based on a critical plane is not enough to explain all the experimental observations. Because of that they proposed a model based on the analysis of the volume around a critical point (that influences crack propagation initiation). The damage parameter is calculated per cycle and it is the energy density of volumetric elastic deformation that exceeds a limit value. This limit value depends on the material and it is based on a new fatigue limit which is less than the considered normal fatigue limit. This is because (according to the authors) conventional considered fatigue limit is not a limit to damage initiation, but a limit of not propagation of damage. The presented model is based on Papadopoulos mesoscopic criterion. There are problems with the determination of the volume because it is needed a good computational equipment and the model was still in development [57].

In 2000 Freitas, Li and Santos proposed a new damage parameter based on the MCC of Papadopoulos. They proposed the Minimum Circumscribed Ellipse (MCE) parameter to take into account the non-proportional loading effects that are not considered in the MCC model [58].

In 2003 Reis, Li and Freitas analysed the effect of non-proportional loading in 42CrMo4 alloy steel, and concluded that the loading path has great influence in fatigue life [59]:

In 2004 Reis, in his PhD thesis [60] studying steels behaviours subjected to axial-torsion non-proportional and proportional loadings, concluded that the model which showed better results in accordance with experimental data was the minimum circumscribed ellipse model, MCE.

In 2005 Wang and Yao, concluded that for a case of multiaxial load, with the same equivalent von Mises stress, fatigue life was shorter with the increase of non-proportionality between loads, finding the minimum fatigue life with 90° out of phase loads. The conclusions presented were based on experimental tests performed on LY12CZ Aluminium alloy specimens. From this study, the authors proposed a new critical plane damage parameter based on shear stress range and normal stress range which acts perpendicularly to the critical plane [61].

In 2006 Hasegawa et al. [62], presented the results of their work on stress controlled, uniaxial Low-Cycle Fatigue (LCF) tests, performed on extruded AZ31 Magnesium alloy, which allowed them to conclude that compression yielding is easy due to twinning which leads to asymmetric hysteresis
curves; and the specimens also tend to deform quasi-elastically during unloading from compression, which makes the plastic strain amplitude smaller to the maximum one in the hysteresis curve.

In 2008 Tsushida et al. studied the relation between grain size and fatigue strength, for the AZ31 Magnesium alloy, concluding that twinning under the fatigue test depends on the grain size, and it affects the fatigue life of the alloy [63].

Also in 2008 Begum et al. noticed the asymmetrical cyclic behaviour when performing strain controlled axial tests on an AZ31 Magnesium alloy [64].

In 2009 Tokaji et al. studied fatigue crack propagation and fracture mechanics for wrought AZ31 and AZ61 magnesium alloys, in different environments, namely laboratory air, dry air and distilled water. The fractography analysis allowed them to conclude that the fracture mechanisms that operated in laboratory air and in distilled water were different, possibly due to hydrogen embrittlement and anodic dissolution, respectively [65].

In 2011 Albinmousa et al. published more work directed at the behaviour of AZ31B magnesium alloy. One of the studies presented by the authors, investigated the multiaxial cyclic behaviour of extruded AZ31B magnesium alloy using tubular specimens machined from large extruded sections, subjected to two loading conditions: axial and torsional. They concluded that twinning has a large influence in deformation under multiaxial loading and non-proportionality has no significant influence on fatigue life [66]. The other work presented by Albinmousa et al. consisted of pure cyclic axial and pure cyclic torsional behaviour characterization, through testing of tubular specimens machined from extruded AZ31B magnesium alloy. The authors concluded that the material in question, experiences significant cyclic hardening and plastic strain reduction when subjected to cyclic axial loading while the cyclic shear hardening is less pronounced [67].

Also in 2011 Zeng et al. published their investigation on the influence of frequencies on fatigue crack propagation rates of two magnesium alloys: AZ80 and AZ61. The results obtained allowed them to conclude that the fatigue crack propagation rates on both alloys would increase with a reduction of the frequency and that the cyclic loading frequency has a significant impact on the strain rate which leads to a change in the mechanical properties of the specimens [68].

In 2012 Anes et al. presented the study conducted to evaluate the mechanical behaviour of AZ31 magnesium alloy subjected to low-cycle fatigue. AZ31B cylindrical specimens were subjected to a cyclic uniaxial load and several total strain amplitudes. The authors observed material softening at tension and hardening at compression, for lower total strain amplitudes [69].

In 2013 Anes et al. published the results of their investigation on crack path evaluation for two diff-
ferent microstructures: BCC, body-centered cubic, and HCP, performing tests on two different materials, specifically 42CrMo4 with a BCC microstructure, and magnesium alloy AZ31B-F with a HCP microstructure. They observed that for multiaxial loading conditions the loading path trajectory had a significant influence on stress concentration factors [70]. In the same year, Anes et al. also presented a new approach to determine stress scale factors (ssf) for multiaxial fatigue loadings on any materials, with an algorithm based on S-N results from specific loading paths [14].

Also in 2013 Itoh et al. proposed a method to determine the principal stress and strain ranges along with mean stress and strain under proportional and non-proportional loading in 3D stress and strain space [71].

Still in 2013 Shamsaei and Fatemi's study on small fatigue crack growth under multiaxial stresses. The authors carried out experimental tests on 1045 and 1050 steels, 304L stainless steel and Inconel 718. The authors observed that a compressive normal stress on the maximum shear plane contributes to the deceleration of crack growth, while a tensile normal stress accelerates crack growth and also that crack surface roughness resulted in friction-induced closure [72].

In 2014 Cláudio et al. presented the results of a study regarding in plane biaxial fatigue of cruciform aluminium specimens. The authors concluded that most of the criteria used, yielded non-conservative results with the exception of the MCE (Minimum Circumscribed Ellipse) method, which provided better results [73].

Also in 2014 Baptista et al. presented the results of the study performed to optimize the design of cruciform specimens for in-plane biaxial fatigue testing [74].

Still in the year of 2014 Anes et al. proposed a new approach to evaluate non-proportionality in multiaxial loading. The authors carried out tests on three different steels, specifically Ck45, 42CrMo4 and AISI 303. The results allowed to conclude that a constant damage scale factor between axial and shear stress is not suitable to quantify different damage mechanisms in proportional and non-proportional loading paths. The proposed factor, Y factor, allowed achieving good results in fatigue life correlations [75].

Another work of interest, presented by Anes et al. demonstrated the application of previously developed models (MCE and ssf) to experimental data obtained by other research groups. The results obtained for the aforementioned models were very acceptable [76]. Finally, Anes et al. also proposed a new cycle counting method and a fatigue life evaluation criterion in 2014. The proposed models were compared with other well-known models and were correlated with fatigue data, yielding acceptable results [77].

In 2015 Anes et al. presented their investigation on damage accumulation under variable amplitude loading conditions, employing Palmgren-Miner’s rule, Morrow’s rule and the ssf criterion as a damage parameter to compare the results [78].
Still in 2015 Li et al. proposed a new fatigue life prediction model composed of three parts: multiaxial fatigue life surface, a new path-dependent factor for multiaxial high cycle fatigue and a material parameter that takes into account the material sensitivity to non-proportional loading [79].

### 2.3.1 Multiaxial Fatigue

Generally, most of the engineering components found in every field of application are subjected to fatigue loadings, and in most cases the loadings are multiaxial. The multiaxial states of stress that arise from the loading on a structure/component, present a much more difficult assessment of fatigue life for said structure/component. When a component is subjected to cyclic stresses, usually both the orientation of the principal axes and the magnitude of the stresses change with time, and due to this, the study of multiaxial fatigue becomes more difficult and less predictable. Multiaxial loadings can be classified as proportional, and non-proportional, depending on a combination of factors.

**Proportional Loading**

In a proportional loading case the cyclic stresses are applied in-phase, and may or may not have the same amplitude. In Figure 2.7 [6], the concept of proportional loading is illustrated considering a shaft subjected to in-phase axial and shear cyclic stresses. If a new coordinate system, $X'-Y'$, is defined so that $\sigma_{x'} = \sigma_1$, and it is kept fixed relatively to the shaft’s axes $X-Y$, one can observe that the $X'$ axis always coincides with the principal normal stress axis. Quoting Socie and Marquis [6], “proportional loading is defined as any state of time varying stress where the orientation of the principal stress axes remained fixed with respect to the axes of the component”.

![Figure 2.7: Proportional multiaxial loading](6)
Non-proportional Loading

In the case of non-proportional loadings, the cyclic stresses are applied out-of-phase, or as in Figure 2.8 [6], one of the stresses (axial in this case) is kept constant, while the applied shear stress is cyclic. Considering once again a $X'$ axis, fixed relatively to $X$, so that $\sigma_{x'} = \sigma_1$ at point A.

It is possible to observe that the orientation of $X'$ does not coincide at all times with the principal normal stress axis, therefore this is a non-proportional loading. Once again quoting Socie and Marquis, [29], it is a "state of time varying stress in which the orientation of the principal stress axes changes with respect to the axis of the component".

![Figure 2.8: Non-proportional multiaxial loading, [6].](image)

2.4 Stress Scale Factor (ssf)

Stress Scale Factor (ssf) is commonly used in multiaxial fatigue models in order to describe the combined axial and shear damage effect on the fatigue process. The objective is to reduce the axial and shear stresses to the same stress space [14].

2.4.1 von Mises yield criterion

The von Mises yield criterion is based on the assumption that only shear stress causes plastic flow, considering that the hydrostatic stress causes no effect on the yield process [80].

Considering the variation in time it is possible to measure an equivalent stress amplitude which can be used as a multiaxial fatigue parameter. This equivalent stress has known shortcomings, produces a lower equivalent stress range, for some conditions particularly in out-of-phase loading cases, leading to overestimate the fatigue life, which is in contradiction with experimental results [14].
2.4.2 Determining the ssf based on material fatigue strength

The ssf is determined based on the fatigue strength on pure axial, bending and pure shear loading conditions [14].

However, some of them can be applied in the finite fatigue life regime under the assumption that the S–N slopes in pure shear and pure axial loading cases are the same. In these cases, the damage parameter can be understood as an equivalent stress, because the ssf under these conditions, remains constant for all loading conditions. In previous works [81],[82] and [83], authors find out that the loading path causes a strong influence on the material fatigue strength, not only due to the stress level influence but also due to the relation between axial and shear stress amplitudes, i.e. stress amplitude ratio (sar); which can be directly related with the ssf concept. This observation leads to conclude that ssf is not constant and varies with the loading type, i.e. proportional or non-proportional; in order to clarify that, it was implemented a series of loading paths with a different amplitude ratio under proportional loading conditions aiming to avoid other fatigue factors such as the non-proportionality or mean stress effect. This does not mean that the ssf cannot be applied to non-proportional loading situations.

In order to determine the ssf factor it was considered the follow strategy: in Figure 2.9 it is shown in the same graph the axial and shear stress amplitudes of a biaxial loading under fatigue failure condition. Furthermore, it is shown the pure shear S-N trend line which will be used as a reference case in terms of fatigue life estimation [14].

![Illustrative case of multiaxial and uniaxial reference case, [14]](image_url)

The stress amplitude ratio (see Fig. 2.9) is given by dividing the shear stress amplitude $AB$ by the axial stress amplitude $AD$, i.e. $AB/AD$; this ratio remains constant along all fatigue life. However, considering the pure shear stress amplitude (point C) the $AB/AC$ and $AC/AD$ ratios may vary along fatigue life; indicating a non-linear effect on fatigue damage due to stress magnitudes. The stress amplitude ratio variation will also vary the $AB/AC$ and $AC/AD$ ratios allowing to capture different material
fatigue strengths due to the axial and shear stress combined effect. Since the trends lines represented in 2.9 are related to fatigue failure condition, the pure shear stress amplitude (point C) at 1E5 cycles for instance is equivalent to the combined contribution of AB and AD. Furthermore, the AB shear stress amplitude is too low to cause failure being necessary an AC–AB shear stress increment to cause fatigue failure at 1E5 cycles; thus, this increment must be equal to the AD axial stress amplitude. Under this approach, the AD axial stress amplitude must be corrected to a shear stress space through a ssf which is given by ssf = BC/AD. Considering that the ssf varies with stress amplitude ratio and also with fatigue life, which is a dependent variable on the normal stress amplitude, thus the ssf can be defined through two variable function.

From this approach, an equivalent shear stress can be represented as:

\[ \tau_{eqv} = \max(\tau_a + ssf(\lambda, \sigma_a) \cdot \sigma_a) \] (2.1)

where \( \tau_a \) is the shear stress amplitude, \( \sigma_a \) is the normal stress amplitude and \( \lambda \) is defined by the following equation:

\[ \lambda = \tan^{-1}\left(\frac{\tau_a}{\sigma_a}\right) \] (2.2)

### 2.5 Morphing Aircraft

Mankind has always been intrigued by what happens in nature. Stories have been made; legends have been created with many successful and unsuccessful attempts at reaching the skies. From the legend of Icarus’ flight with bird wings, to the successful gliding flights of Otto Lilienthal [84] and reaching the first powered flight by the Wright brothers [85], nature was always the inspiration that moved men to look further ahead.

![Figure 2.10: Otto Lilienthal's bird inspired glider in 1894, [84].](image)

The process of adjusting flight surfaces to increase performance thus originated a concept known as morphing. It is a concept without worldwide agreement but is more commonly associated with large changes in flight surfaces with the goal of increasing efficiency in a flight envelope. Ever since the Wright brothers first flight there have been evidences of morphing; in their case, they developed a mechanism.
that allowed the wings of their airplane to warp during flight, thus increasing the roll control of the aircraft.

Morphing, however, depends vastly on the advances in technology and materials in order to truly improve the flight of an aircraft. Some of the materials that are being explored and that have a large potential for success when it comes to morphing are the Shape Memory Alloys (SMA). These materials can change form by going from an austenitic state to a martensitic state. Through the application of heat they can revert back to their austenitic state with the same shape that they had prior to the transformation.

As for actuation techniques the one that is most explored nowadays is the piezoelectric actuation. The actuators have the capability of transforming electrical energy in mechanical, thereby allow the movement of a component whose weight can be supported by them, [86] and [17]

However, some of the technologies and materials that exist today do not yet make it possible to develop some revolutionary ideas but, hopefully, developments will appear in the near future that enable airplanes to better mimic the flights of the species in nature in order to increase their performance and reduce maintenance and operation costs.

Some of the information presented in this chapter is given in references [87], [88], [89], [90] and [91].

2.5.1 Lift Line Theory

The Prandtl lifting-line theory is essentially a mathematical model that is a three-dimensional extension of the Kutta-Joukowski Theorem. This theory applies to large aspect ratio unswept wings at small angle of attack and was developed by Prandtl and Lanchester during the early 20th century. Its relevance endures since it is use for obtaining analytic results for simple wings, as basis of much of modern wing theory (e.g. helicopter rotor aerodynamic analysis, extends to vortex lattice method) as well as basis of much of the qualitative understanding of induced drag and aspect ratio.

The Kutta-Joukowski theorem relates two-dimensional lift to the “circulation” of the flow. The lift generated in 2 dimensions is proportional to how much the flow is made to turn:
\[ L' = \rho_{\text{inf}} V_{\text{inf}} \Gamma \quad (2.3) \]

Lifting-line theory integrates the two-dimensional lift along the wingspan (in aeronautics, this is conventionally called the y-direction) so that:

\[ L(y) = \rho_{\text{inf}} V_{\text{inf}} \Gamma(y) \quad (2.4) \]

Defining support and Kutta-Joukowski theorem, we have, for the local profile \( y_0 \),

\[ L' = \frac{1}{2} \rho_{\infty} V_{\infty}^2 c(y_0) \alpha = \rho_{\infty} V_{\infty} \Gamma(y_0) \quad (2.5) \]

\[ C_L = \frac{2\Gamma(y_0)}{V_{\infty}c(y_0)} \quad (2.6) \]

Also, for the local profile \( y_0 \) it is possible to obtain the equation for induced angle of attack along finite wing in terms of \( \Gamma(y) \):

\[ \alpha_i(y_0) = \tan^{-1} \left( -\frac{w(y_0)}{V_{\text{inf}} y_0} \right) \quad (2.7) \]

Considering small angles, the next approximation can be made:

\[ \alpha_i(y_0) = \frac{-w(y_0)}{V_{\infty}} \quad (2.8) \]

\[ \alpha_i(y_0) = \frac{1}{4\pi V_{\infty}} \int_{-b/2}^{b/2} \frac{d\Gamma}{dy} dy \quad (2.9) \]

Therefore, the lift coefficient for the local profile \( y_0 \) can be obtain as follow:

\[ C_l = a_0 [\alpha_{\text{eff}}(y_0) - \alpha_{L=0}] = 2\pi [\alpha_{\text{eff}}(y_0) - \alpha_{L=0}] \quad (2.10) \]

Effective angle of attack seen locally by airfoil:

\[ \alpha_{\text{eff}} = \alpha_{\text{eff}}(y_0) \quad (2.11) \]

Relating previous expression, Equation 2.6 and 2.10, and solving for \( \alpha_{\text{eff}} \):

\[ \alpha_{\text{eff}} = \frac{\Gamma(y_0)}{\pi V_{\infty} c(y_0)} + \alpha_{L=0} \quad (2.12) \]

When the lift changes from one point along the span to another, this is modeled as a change in the circulation \( \Gamma(y) \), and the change in circulation is modeled as a vortex of strength \( \frac{d\Gamma}{dy} \) which is shed from that point along the span.

Lifting-line theory models changes in lift as vortices being shed. It gives a model for only the induced
drag (not other drag components such as form drag) but this is okay once induced drag is usually the dominant effect. Induced drag is a function of the change in effective angle of attack of the wing caused by the downwash from these shed vortices. Therefore, everything is derived from $\Gamma(y)$, which is a function of the wing geometry.

The mapping of angle ($\theta$) to span ($b$) position is done using Fourier series and allows variation of the model to suit different geometries. Replacing finite wing ($span = b$) with bound vortex filament extending from $y = -\frac{b}{2}$ to $y = -\frac{b}{2}$ and origin located at center of bound vortex (center wing) [92]. Figure 2.12.

\[ y = -\frac{b}{2} \cos \theta; 0 \leq \theta \leq \pi \]  

(2.13)

The span-wise lift distribution is assumed to be approximately elliptical with a small modifications due to wing plan-form geometry. The vortex line strength can thus be modelled using the following Fourier series approximation,

\[ \Gamma(\theta) = 2bV_\infty \sum_{n=1}^{N} A_n \sin(n\theta) \]  

(2.14)

Differentiating the above equation 2.14 in order to $y$,

\[ \frac{d\Gamma}{dy} = \frac{d\Gamma}{d\theta} \frac{d\theta}{dy} = 2bV_\infty \sum_{n=1}^{N} nA_n \cos(n\theta) \frac{d\theta}{dy} \]  

(2.15)

The required strength of the distribution coefficients ($A_n$) for a given geometry and set of free-stream conditions can be calculated by applying a surface flow boundary condition. The equation used is based on the usual condition of zero flow normal to the surface.

For 3-D wings, this condition is applied at several span-wise locations. By matching flow and surface angles, the normal velocity boundary condition can be restated as a requirement for the flow angles at the section to be in balance. Unlike 2-D section flow where flow angles are set by freestream direction and surface angles only, in 3-D wing flow an extra flow angle component is introduced by the shed vortices that are produced and trail behind the wing. This trailing vortex sheet produces a downwash.
Assuming that higher order coefficients become increasingly small and make negligible contribution to the result, one method of solution is to truncate the series at term $A_N$. By applying the boundary condition at $N$ span locations a set of simultaneous linear equations can be constructed. This set can be solved for coefficients $A_1$ to $A_N$.

A cosine distribution of span-wise locations should be used for the boundary conditions to match the assumed wing loading distribution. Clearly, the number of coefficients used will determine the accuracy of the solution. If the wing loading is highly non-elliptical then a larger number of coefficients should be included. This occurs when analysing wings with part span flaps. This type of geometry causes a discontinuity in the spanwise loading and, hence, requires a much larger number of coefficients to accurately describe the distribution. Where the the wing loading is symmetric about the wing root, the contribution of even functions will become zero. Coefficients $A_2, A_4, A_6, ...$ are all zero and can be dropped from the analysis.

When $\Gamma_\theta$ is known, the lift coefficient of the finite wing can be express as:

$$C_L = \frac{2}{V_\infty S} \int_{-b/2}^{b/2} \Gamma(y) dy = \frac{2b^2}{S} \sum_{n=1}^{N} A_n \int_{0}^{\pi} \sin(n\theta) \sin(\theta) d\theta$$

(2.16)

The integral of the equation 2.16 is solved using,

$$\int_{0}^{\pi} \sin(n\theta) \sin(\theta) d\theta = \begin{cases} \pi/2, & n = 1 \\ 0, & n \neq 1 \end{cases}$$

(2.17)

Finally,

$$C_L = A_1 \frac{b^2}{S} = A_1 \pi AR$$

(2.18)

It can be seen that $C_L$ is dependent upon the first coefficient of the Fourier series and the aspect ratio $AR$, is equal to $b^2/S$ where $S$ is the wing area.

A consequence of the downwash flow is that the direction of action of each section’s lift vector is rotated relative to the free-stream direction. The local lift vectors are rotated backward, give rise to a lift induced drag. While the overall governing equations are potential flow and, consequently, do not give rise to friction or pressure drag, this lift induced drag will be a significant component of the overall drag of the wing. The downwash velocity induced at any span location can be calculated once the strength of the wing loading is known. The variation in local flow angles can then be found.

By integrating the component of section lift coefficient that acts parallel to the free-stream across the span, the induced drag coefficient can be found which produces the following induced drag coefficient,
\[ C_{Di} = \frac{2}{V_\infty S} \int_{-b/2}^{b/2} \Gamma(y) \alpha_i(y) dy = \frac{2b^2}{S} \int_0^\pi \left( \sum_n A_n \sin(n \theta) \right) \alpha_i(\theta) \sin(\theta) d\theta \] (2.19)

Substituting the equation 2.20 into the equation 2.19 a final equation is obtained for the induced drag coefficient 2.21,

\[ \alpha_i(y_0) = \sum_n n A_n \frac{\sin n \theta}{\sin \theta} \] (2.20)

\[ C_{Di} = \frac{2b^2}{S} \int_0^\pi \left( \sum_n A_n \sin(n \theta) \right) \left( \sum_n A_n \sin(n \theta) \right) d\theta \] (2.21)

From standard integration techniques,

\[ \int_0^\pi \sin(m \theta) \sin(k \theta) d\theta \begin{cases} 0, & m \neq k \\ \pi/2, & m = k \end{cases} \] (2.22)

Can be concluded that the multiplication of different index \((m \neq k)\) will become zero (e.g. \(A_1 A_2, A_2 A_4, \ldots\)). So,

\[ C_{Di} = \frac{2b^2}{S} \left( \sum_n n A_n^2 \right) \frac{\pi}{2} = \pi A \sum_n n A_n^2 \] (2.23)

\[ C_{Di} = \pi A (A_1^2 + \sum_n n A_n^2) = \pi A A_1^2 \left[ 1 + \sum_n \frac{A_n^2}{A_1^2} \right] \] (2.24)

The information presented in this section can be found in more detail, in [93].
Chapter 3

Material, Equipment and Methods

In this chapter firstly an analysis of the material properties of the extruded magnesium alloy AZ31B-F is made. After that is presented the equipment used for the experimental tests as well as brief description of the test performed. The experimental component took place on the laboratory of mechanical engineering department of Instituto Superior Técnico – Universidade Técnica de Lisboa (DEM-IST-UTL).

3.1 Material

In this study was used the magnesium alloy AZ31B-F, whose crystalline structure is hexagonal compact (HC). Samples were machined from magnesium extruded rods in order to obtain the standard shape. The magnesium alloy was not subject to any treatment after extrusion, i.e., its final state is as produced.

3.1.1 Chemical Composition

The extruded magnesium alloy AZ31B-F which according to the explanation previously stated (section 2.2 corresponds to an alloy whose main alloying elements are Aluminium and Zinc, with around 3% and 1% respectively. The letter B indicates that this alloy was the second to be manufactured. The meaning of the letter F can be found in Table 2.3.

<table>
<thead>
<tr>
<th>Element</th>
<th>Al</th>
<th>Zn</th>
<th>Mn</th>
<th>Fe</th>
<th>Ni</th>
<th>Cu</th>
<th>Ca</th>
<th>Si</th>
<th>Mg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight (%)</td>
<td>3.1</td>
<td>1.05</td>
<td>0.54</td>
<td>0.0035</td>
<td>0.0007</td>
<td>0.0008</td>
<td>0.04</td>
<td>0.1</td>
<td>Balance</td>
</tr>
</tbody>
</table>

In this alloy can be concluded that aluminum increases the strength, manganese reduces the solubility of iron, produces relatively innocuous compounds and improves corrosion resistance, iron is a very
harmful impurity since it reduces the corrosion resistance and finally zinc which improves the mechanical strength.

### 3.1.2 Mechanical Properties

The mechanical properties of AZ31B-F magnesium alloy was determined following the standard procedures, namely following ASTM E8 and ASTM E606 standards, and are presented in Table 3.2.

<table>
<thead>
<tr>
<th>Property</th>
<th>AZ31B-F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Microstructure type</td>
<td>HC</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.35</td>
</tr>
<tr>
<td>Density (Kg/m³)</td>
<td>1770</td>
</tr>
<tr>
<td>Hardness (HV)</td>
<td>86</td>
</tr>
<tr>
<td>Tensile strength (MPa)</td>
<td>290</td>
</tr>
<tr>
<td>Yield strength (MPa)</td>
<td>203</td>
</tr>
<tr>
<td>Elongation (%)</td>
<td>14</td>
</tr>
<tr>
<td>Young’s modulus (GPa)</td>
<td>45</td>
</tr>
<tr>
<td>a Fatigue strength coefficient (MPa)</td>
<td>450</td>
</tr>
<tr>
<td>b Fatigue strength coefficient</td>
<td>-0.12</td>
</tr>
<tr>
<td>c Fatigue ductility coefficient</td>
<td>0.26</td>
</tr>
<tr>
<td>c Fatigue ductility exponent</td>
<td>-0.71</td>
</tr>
</tbody>
</table>

### 3.1.3 Specimen Geometry

The specimen was measured using a caliper rule and then numbered. Figure 3.1 illustrate the dimensions and geometry of the specimens used in the multiaxial fatigue test load control.

![Specimen geometry (mm).](image)

The companies that provided the magnesium extruded rods are certified companies in order to ensure chemical composition, thus minimizing the metallurgical imperfections.

### 3.2 Equipment

The Instron biaxial servo-hydraulic machine model 8874 is a bi-axial tabletop servohydraulic testing system providing a combined axial and torsion dynamic actuator in the upper crosshead. With a precision
aligned twin-column frame and a lower t-slot table, the 8874 meets the challenging demands of a varied range of both static and dynamic testing requirements. Combined with the advanced features of the 8800 digital controller, and the Instron® patented Dynacell™, these table model systems are compact and flexible enough to be located anywhere within a laboratory, saving valuable space. Console software provides full system control from a PC: including waveform generation, calibration, limit set up, and status monitoring. Add WaveMatrix™ block loading software for simple and advanced multi-axial tests on materials or components [95].

Figure 3.2 shows the machine used in the fatigue tests and Figure 3.3 a scheme of the machine mechanism.

![Figure 3.2: Instron biaxial servo-hydraulic machine, model 8874.](image)

![Figure 3.3: Scheme of a servo-hydraulic machine [96].](image)

Some of the main features, [95]:

- Up to $\pm 25kN$ ($5620lb_f$) axial force capacity;
- $\pm 100Nm$ ($880in - lb$) torque capacity;
- Integral T-slot base to accommodate the needs of many biomedical and component testing applications;
• Patented Dynacell load cell featuring compensation for inertial loads caused by heavy grips and fixtures;
• Standard or extra-height frame options;
• Wide range of axial-torsional grips, fixtures and accessories.

The servo-hydraulic machine used during the tests, Figure 3.2, with a control system console, which is connected by cables to a digital computer, two degrees of freedom, and the axial rotation being controlled by computer program. During the course of the tests, the load paths are represented on the computer screen. The efforts are transmitted to the specimen by means of two jaws that hold rigidly the specimen and by means of actuators which incorporate sensors to ensure the desired efforts. It proved to be very sensitive, very good stability and high reliability. The operation of this servo-hydraulic machine is based on servos.

3.3 Experimental Methods

The objective of this chapter is to add value and information to the investigation done by Anes et al. [70], testing the alloy under different cyclic conditions. Figure 3.4 shows the loading paths performed in this study represented in the von Mises stress space.

Fatigue tests were carried out through a servo-hydraulic machine (Figure 3.2) under stress control at room temperature; the testing frequency was 6 Hz. In order to perform the ssf mapping it was selected five different proportional loading paths with different stress amplitude ratios, cases 1–5 loading paths, pure uniaxial axial cyclic tension test (0°), pure shear loading (90°), proportional biaxial loading of 30°, proportional biaxial loading of 45° and proportional biaxial loading of 60°, respectively. One additional case was considered, the loading case 6, which is a non proportional 45° biaxial loading with 90° of phase shift loading path, in order to correlate and evaluate the achieved ssf with experimental data.
Figure 3.5 shows the amplitude stress levels at $R = -1$ used on each loading path. Despite the stress amplitude ratio $\tau_a/\sigma_a$ being an important fatigue variable, the stress level also has a huge influence on the fatigue damage mechanisms. These two variables will be used in the ssf mapping. For each loading case, the stress amplitude ratio remains constant for all tested stress levels, however, in loading case 6, a non-proportional case, the stress levels shown in Figure 3.5 are related to the maximum stress amplitudes verified during the loading period for each stress component, not indicating that the stress amplitude ratio remains constant on this loading case.

![Figure 3.5: Experimental stress amplitudes used in each loading cases.](image)

A brief description of the parameters that differed between tests is presented in Table 3.3.

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>Phase Shift [°]</th>
<th>Load amplitude [KN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>AZ31B-01</td>
<td>30°</td>
<td>3.16</td>
</tr>
<tr>
<td>AZ31B-02</td>
<td>30°</td>
<td>3.30</td>
</tr>
<tr>
<td>AZ31B-03</td>
<td>30°</td>
<td>3.45</td>
</tr>
<tr>
<td>AZ31B-04</td>
<td>30°</td>
<td>3.59</td>
</tr>
<tr>
<td>AZ31B-05</td>
<td>30°</td>
<td>3.74</td>
</tr>
<tr>
<td>AZ31B-06</td>
<td>60°</td>
<td>1.6</td>
</tr>
<tr>
<td>AZ31B-07</td>
<td>60°</td>
<td>1.7</td>
</tr>
<tr>
<td>AZ31B-08</td>
<td>60°</td>
<td>1.8</td>
</tr>
<tr>
<td>AZ31B-09</td>
<td>60°</td>
<td>2.0</td>
</tr>
</tbody>
</table>
Chapter 4

Magnesium alloy AZ31B-F fatigue results

It is well-know that axial and shear stresses have different damage scales [97], [14]. This is so because in experiments it is obtained different fatigue lives regarding the same stress amplitude in both uniaxial loading conditions (shear and axial). Usually, for structural materials, the uniaxial axial S-N curve stands above the shear one. Thus, to obtain the same fatigue life in both loading conditions it is required higher axial stress amplitude comparatively to the shear stress amplitude. Under multiaxial loading conditions, fatigue damage results from the combination of both axial and shear loading components of a multiaxial loading where it is necessary to account the axial and shear damage contribution to the overall damage. To do that, it is necessary to have both axial and shear damages in the same damage scale. For example, the von Mises equivalent stress uses the constant $\sqrt{3}$ to reduce shear damage to the axial one. This procedure can be found in a wide range of multiaxial fatigue criteria being a common practice to account combined fatigue damage [98].

In table 4.1 the results of the experimental tests are presented. As expected the number of cycles depends on the applied load. For cases where the number of cycles is equal to $1E+06$ we can conclude that the specimen did not break.

4.1 S-N experimental results

Figure 4.1 shows the fatigue life results for the loading cases 1 and 2. These results show the material fatigue strength under uniaxial loading conditions.

Similarly to the procedure performed in [14], in order to analyze and compute the fatigue data a trend line approach was adopted to characterize the uniaxial and multiaxial stress components inherent to each loading case; this trend line is represented in graphs as dashed lines. Despite the axial and shear fatigue stresses have different degradation mechanisms, the uniaxial loading cases can be directly
Table 4.1: Magnesium alloy AZ31B-F fatigue results.

<table>
<thead>
<tr>
<th>Loading Case</th>
<th>Normal Stress (MPa)</th>
<th>Shear Stress (MPa)</th>
<th>$N_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1- Pure uniaxial cyclic tension test, 0°, PT</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>140</td>
<td>0</td>
<td>13164</td>
</tr>
<tr>
<td></td>
<td>135</td>
<td>0</td>
<td>22873</td>
</tr>
<tr>
<td></td>
<td>130</td>
<td>0</td>
<td>38102</td>
</tr>
<tr>
<td></td>
<td>120</td>
<td>0</td>
<td>62352</td>
</tr>
<tr>
<td></td>
<td>105</td>
<td>0</td>
<td>721573</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0</td>
<td>1000000</td>
</tr>
<tr>
<td>2- Pure shear, 90°, PS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>75</td>
<td>88871</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>69</td>
<td>128769</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>64</td>
<td>227808</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>59</td>
<td>388236</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>53</td>
<td>1000000</td>
</tr>
<tr>
<td>3-Proportional biaxial loading, 30°, PL30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>112.58</td>
<td>37.53</td>
<td>65318</td>
</tr>
<tr>
<td></td>
<td>108.25</td>
<td>36.08</td>
<td>84432</td>
</tr>
<tr>
<td></td>
<td>103.92</td>
<td>34.64</td>
<td>170311</td>
</tr>
<tr>
<td></td>
<td>99.59</td>
<td>33.2</td>
<td>366799</td>
</tr>
<tr>
<td></td>
<td>95.26</td>
<td>31.75</td>
<td>1000000</td>
</tr>
<tr>
<td></td>
<td>106</td>
<td>61</td>
<td>16800</td>
</tr>
<tr>
<td></td>
<td>92</td>
<td>53</td>
<td>46878</td>
</tr>
<tr>
<td></td>
<td>78</td>
<td>45</td>
<td>138986</td>
</tr>
<tr>
<td></td>
<td>74</td>
<td>43</td>
<td>242685</td>
</tr>
<tr>
<td></td>
<td>71</td>
<td>41</td>
<td>353718</td>
</tr>
<tr>
<td></td>
<td>67</td>
<td>39</td>
<td>1000000</td>
</tr>
<tr>
<td>4- Proportional biaxial loading, 45°, PP45</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>60</td>
<td>52110</td>
</tr>
<tr>
<td></td>
<td>55</td>
<td>55</td>
<td>94116</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>50</td>
<td>191187</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>45</td>
<td>1000000</td>
</tr>
<tr>
<td>5- Proportional biaxial loading, 60°, PL60</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>106</td>
<td>61</td>
<td>7182</td>
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<td>95</td>
<td>55</td>
<td>8595</td>
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<td>78</td>
<td>45</td>
<td>11986</td>
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<td></td>
<td>74</td>
<td>43</td>
<td>167525</td>
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<tr>
<td></td>
<td>73</td>
<td>42</td>
<td>576336</td>
</tr>
<tr>
<td></td>
<td>71</td>
<td>41</td>
<td>1000000</td>
</tr>
</tbody>
</table>

compared. However, in biaxial loading conditions, where the fatigue loading has two components, the relation to the reference curve, i.e. uniaxial fatigue data, is not direct. In these loading cases that relation is entirely different from the one verified between cases 1 and 2. Under multiaxial loading conditions an appropriate equivalent stress is required.

The fatigue data for these two loading cases are represented in the same graph, Figure 4.1, but it is known that the stresses nature in both cases are quite distinct; considering that case 1 and 2, S-N results represents a fatigue failure condition for the tensile–compression and pure shear stress amplitudes and regarding a specific fatigue life, it can be concluded that different stress amplitudes in tensile and shear loading conditions can lead to the same fatigue life. This lead to conclude that, if it is possible to achieve the same fatigue life with different stress natures, axial and shear, so they are in some way equivalent because the final result is the same number of cycles to fail. Determining that relation between the stress components by establishing a stress scale factor to transform axial stress nature to shear one or vice versa allows to use a unique uniaxial S–N curve to estimate multiaxial fatigue live [14].
Figures 4.2, 4.3 and 4.4 display the fatigue data results for loading cases 3, 4 and 5, respectively. In each loading case the biaxial loading is represented through two trend lines, one representing the axial stress component and the other the shear stress component. In addition, it is considered the pure shear results, loading case 2, as the reference case. In this study, the pure shear case is considered as the reference case in order to quantify the stress scale factor between tensile and shear stresses and also to perform fatigue life estimations.

From the plotted results in Figures 4.2, 4.3 and 4.4, it can be concluded that the shear stress amplitude from each biaxial loading is clearly insufficient to create a fatigue failure, compared with the reference line, i.e. the shear trend lines from case 3, 4 and 5 are below the reference case, the pure shear case number 2. The missing damage contribution is done through the axial component. Also the axial trend line in loading cases 3 and 4 are above the case 2 reference line, indicating that, if only this load component was used in the reference trend line equation to estimate fatigue lives the result would be shorter than the experimental results. In loading case 5, the opposite is observed, the axial trend line of the biaxial loading is below the reference case.

Table 4.2 presents the trend line equations obtained from experimental fatigue tests. For each loading case it is correlated the axial and shear stress amplitudes with the experimental fatigue data conducting to establish two trend lines per each multiaxial loading case. The trend lines have a power-law format, which typically fits well the fatigue behavior, with acceptable $R^2$. 

![Figure 4.1: Case 1 (PT) and 2 (PS), S-N results.](image-url)
4.2 Stress scale factor (ssf) determination based on experimental results

The stress scale factor was determined based on experimental results, i.e. the S–N trend lines represented in Table 4.2 were used to compute ssf. The computed results for the selected proportional loading cases are shown from Tables 4.3 - 4.6.
Figure 4.4: Loading case 5, S-N results.

Table 4.2: Axial and shear fatigue trend lines from fatigue failure results for each loading case.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\sigma_a$</th>
<th>$\tau_a$</th>
<th>Trend line (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\infty$</td>
<td>0</td>
<td>$\sigma_a = 283.93 \cdot N_f^{-0.075}$</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>$\infty$</td>
<td>$\tau_a = 0$</td>
</tr>
<tr>
<td>3</td>
<td>0.33</td>
<td>$\sigma_a$</td>
<td>$\tau_a = 365.14 \cdot N_f^{-0.141}$</td>
</tr>
<tr>
<td>4</td>
<td>0.56</td>
<td>$\sigma_a$</td>
<td>$\tau_a = 211.66 \cdot N_f^{-0.05}$</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>$\sigma_a$</td>
<td>$\tau_a = 322.22 \cdot N_f^{-0.117}$</td>
</tr>
</tbody>
</table>

In all tables, the first column is filled with a specific fatigue life range from 1000 to 1000000 in order to estimate the necessary stress amplitudes, through the use of the trend lines inherent to each case. On the second and third column it was determined the pure tension and pure shear amplitudes for the selected fatigue life range; on the fourth and last column the ssf is determined.

Figure 4.5 presents for each loading case the axial stress amplitude versus the shear stress increment ($ssf \cdot \sigma_a$); this increment represents the axial stress amplitude contribution in shear stress space accordingly to the methodology previously presented.

Considering the results for loading cases 1 and 3, the data curve slopes for both cases are similar, which is acceptable because those loading cases are also very alike, the shear stress component on case 3 is much smaller than the axial one.
Table 4.3: Case 1, values in MPa

<table>
<thead>
<tr>
<th>(N_f)</th>
<th>(\sigma_a)</th>
<th>(\tau_a)</th>
<th>ssf</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>169.13</td>
<td>0</td>
<td>0.82</td>
</tr>
<tr>
<td>10000</td>
<td>142.3</td>
<td>0</td>
<td>0.7</td>
</tr>
<tr>
<td>50000</td>
<td>126.12</td>
<td>0</td>
<td>0.63</td>
</tr>
<tr>
<td>100000</td>
<td>119.73</td>
<td>0</td>
<td>0.6</td>
</tr>
<tr>
<td>500000</td>
<td>106.12</td>
<td>0</td>
<td>0.54</td>
</tr>
<tr>
<td>1000000</td>
<td>100.74</td>
<td>0</td>
<td>0.52</td>
</tr>
</tbody>
</table>

Table 4.4: Case 3, values in MPa

<table>
<thead>
<tr>
<th>(N_f)</th>
<th>(\sigma_a)</th>
<th>(\tau_a)</th>
<th>ssf</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>141.79</td>
<td>47.24</td>
<td>0.64</td>
</tr>
<tr>
<td>10000</td>
<td>124.06</td>
<td>41.34</td>
<td>0.47</td>
</tr>
<tr>
<td>50000</td>
<td>113.01</td>
<td>37.65</td>
<td>0.37</td>
</tr>
<tr>
<td>100000</td>
<td>108.55</td>
<td>36.17</td>
<td>0.33</td>
</tr>
<tr>
<td>500000</td>
<td>98.88</td>
<td>32.94</td>
<td>0.25</td>
</tr>
<tr>
<td>1000000</td>
<td>94.98</td>
<td>31.65</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Table 4.5: Case 4, values in MPa

<table>
<thead>
<tr>
<th>(N_f)</th>
<th>(\sigma_a)</th>
<th>(\tau_a)</th>
<th>ssf</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>143.6</td>
<td>82.1</td>
<td>0.39</td>
</tr>
<tr>
<td>10000</td>
<td>109.69</td>
<td>63.14</td>
<td>0.33</td>
</tr>
<tr>
<td>50000</td>
<td>90.86</td>
<td>52.56</td>
<td>0.3</td>
</tr>
<tr>
<td>100000</td>
<td>83.78</td>
<td>48.57</td>
<td>0.28</td>
</tr>
<tr>
<td>500000</td>
<td>69.4</td>
<td>40.43</td>
<td>0.24</td>
</tr>
<tr>
<td>1000000</td>
<td>64</td>
<td>37.35</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Table 4.6: Case 5, values in MPa

<table>
<thead>
<tr>
<th>(N_f)</th>
<th>(\sigma_a)</th>
<th>(\tau_a)</th>
<th>ssf</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>84.91</td>
<td>84.91</td>
<td>0.62</td>
</tr>
<tr>
<td>10000</td>
<td>68.22</td>
<td>68.22</td>
<td>0.46</td>
</tr>
<tr>
<td>50000</td>
<td>58.55</td>
<td>58.55</td>
<td>0.36</td>
</tr>
<tr>
<td>100000</td>
<td>54.82</td>
<td>54.82</td>
<td>0.31</td>
</tr>
<tr>
<td>500000</td>
<td>47.05</td>
<td>47.05</td>
<td>0.22</td>
</tr>
<tr>
<td>1000000</td>
<td>44.05</td>
<td>44.05</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Figure 4.5: Shear stress increment versus axial stress amplitude for each loading case.

In loading case 4, the data curve slope changes being much less steep than cases 1 and 3, as a result of the axial stress represented in the shear stress space.

4.3 Stress scale factor surface (ssfs)

For the proportional loading cases considered in this study the ssf values were determined as explained previously; however, it is not feasible to determine that value for all possible stress amplitude ratios and stress levels through experimental tests. The selected loading cases were chosen in order to be representative of the material fatigue strength under specific stress amplitude ratios in order to create a ssf mapping. To do that it was considered two representative variables, the stress amplitude ratio and the axial stress level. The stress amplitude ratio is sensitive to the loading path trajectory and the axial stress level is sensitive to fatigue life. Table 4.7 shows the data collected to perform the ssf regression.
In the first column, the axial stress amplitude for each loading case is considered, at second column is displayed the stress amplitude ratio arc tangent and lastly on the third column it is shown the determined ssf for each loading case. Due to the stress amplitude ratio definition an infinite value is reached for the uniaxial tension–compression case.

In order to avoid an undefined region with the consequent information loss it was performed a variable transformation using the arctangent function. This transformation does not affect the variable physical meaning. As a result of the regression process Eq. 2.2 was achieved from which it is possible to estimate the ssf for all stress amplitude ratios and stress level, for the selected material.

This equation is a combination of polynomials with two arguments. The goodness of fit it is given by $R^2 = 0.92$, which is acceptable since the $R^2 = 1$ indicates that the regression fits utterly the experimental data.

$$ssf(\sigma_a, \lambda) = a + b \cdot \sigma_a + c \cdot \sigma_a^2 + d \cdot \sigma_a^3 + e \cdot \sigma_a^4 + f \cdot \sigma_a^5 + g \cdot \lambda + h \cdot \lambda^2 + i \cdot \lambda^3 + j \cdot \lambda^4 \quad (4.1)$$

where $a = -2.96; b = 0.14; c = -2.07 \times 10^{-3}; d = 1.48 \times 10^{-5}; e = -4.67 \times 10^{-8}; f = 4.87 \times 10^{-11}; g = -0.5; h = -1.34; i = 3.32; j = -0.88$

Figures 4.6 and 4.7 represents the ssf surface, where the ssf variation with the stress amplitude ratio and the axial stress amplitude level can be observed.

The ssf damage surface, given by the polynomial function shown in Equation 4.1, was obtained for the AZ31B-F material and translates its cyclic behaviour under different stress amplitude ratios and stress levels. Therefore, the ssf fatigue estimates for other materials must be done using their ssf damage maps, which must be previously obtained by experiments.

The main focus of the ssf analysis has been on the idea that the damage scale of normal and shear stresses of a given multiaxial loading is not constant \[99\]. This damage scale is strongly dependent on the stress level and stress amplitude ratio and can be evaluated with the so-called ssf damage surface. From experiments can be conclude that the axial damage is substantially different from the shear one.
The shear stress level required to cause a cyclic failure is less than the one needed when it is considered an axial stress, thus it can be concluded that there exists a sort of damage scale between shear and axial damages.

Using the ssf damage surface it is also possible to obtain the boundary conditions where the axial component have influence.

Figure 4.6: Ssf damage surface.

Figure 4.7: Top view ssf damage surface.
Chapter 5

Aerodynamic Shape Optimization

This chapter describes the wing optimization process for the aircraft wing. The aim was to optimize the wing geometry and aerodynamic parameters, under specific flight conditions.

The optimization is done in three steps. First, the UAV aircraft main characteristics and mission flight are selected, Table 5.1. Then, a few airfoils are analysed and the one that best fits the flight conditions is chosen 5.2. Finally, a lifting line method algorithm is used to obtain the lift and induced drag coefficients. The lift and the induced drag are obtained by integrating the lift and induced drag coefficients corresponding to all local angles of attack. The aerodynamic shape optimization is carried out with a function, available in MATLAB® software, that attempts to find the minimum of a function subject to linear, nonlinear or bound constraints. This function and algorithm will be presented in more detail in section 5.3.

5.1 UAV characteristics and mission profile

In table 5.1 the UAV characteristics are presented. Geometric dimensions such as, chord and span, will be obtain using the optimization algorithm. Some initial consideration were made, for instance, weight of approximate 50 kg, cruise speed between 140 km/h and 150 km/h and the use of an electric motor. Using this information and correlating with the UAV characteristics in [100] it was possible to achieve the final characteristics.

<table>
<thead>
<tr>
<th>Table 5.1: UAV Characteristics [100].</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight (N) = 500</td>
</tr>
<tr>
<td>Payload (kg) = 10</td>
</tr>
<tr>
<td>Endurance (hr) = 4</td>
</tr>
<tr>
<td>Range (km) = 100</td>
</tr>
<tr>
<td>Ceiling (m) = 4000</td>
</tr>
<tr>
<td>Rate of Climb (m/s) = 5</td>
</tr>
<tr>
<td>Engine : Electric Motor</td>
</tr>
</tbody>
</table>

A typical general aviation mission consists of the following flight phases: start up at the origin airport,
taxi out to the active runway, take-off, climb, cruise to the vicinity of the destination airport, descend, hold in a holding pattern to await a landing slot if so instructed by Air Traffic Control, approach the destination airport, land, taxi in to the ramp or hangar, and shut down. The mission phases selected for optimization are illustrated in the following mission profile, Figure 5.1. More about flight phases can be found in [101].

![Mission Profile](image)

Figure 5.1: Mission profile [102].

In section 5.3 each of the mission phases will be presented in more detail.

### 5.2 Airfoil profile selection

An airfoil-shaped body moved through a fluid produces an aerodynamic force. The component of this force perpendicular to the direction of motion is called lift. The component parallel to the direction of motion is called drag.

Subsonic flight airfoils have a characteristic shape with a rounded leading edge, followed by a sharp trailing edge, often with a symmetric curvature of upper and lower surfaces. The lift on an airfoil is primarily the result of its angle of attack and shape. When oriented at a suitable angle, the airfoil deflects the oncoming air (for fixed-wing aircraft, a downward force), resulting in a force on the airfoil in the direction opposite to the deflection. This force is known as aerodynamic force and can be resolved into two components: lift and drag. Most airfoil shapes require a positive angle of attack to generate lift, but cambered airfoils can generate lift at zero angle of attack [103] and [104].

An example of an airfoil profile can be found in figure 5.2.
Some important parameters to describe an airfoil’s shape are its camber and its thickness. For example, an airfoil of the NACA 4-digit series such as the NACA 2415 (to be read as 2 - 4 - 15) describes an airfoil with a camber of 0.02 chord located at 0.40 chord, with 0.15 chord of maximum thickness.

In Table 5.2 six airfoils from different NACA series (4-digits, 5-digits and 6-digits) are displayed. Using the Airfoil Tools application [106] and searching in the database for each series, filtering for a low range value of Reynolds number that provides the max $\frac{C_l}{C_D}$, the best airfoils are selected.

<table>
<thead>
<tr>
<th>Table 5.2: Airfoils selected by series.</th>
</tr>
</thead>
<tbody>
<tr>
<td>NACA 4 digits:</td>
</tr>
<tr>
<td>2408</td>
</tr>
<tr>
<td>2412</td>
</tr>
<tr>
<td>2415</td>
</tr>
<tr>
<td>4412</td>
</tr>
<tr>
<td>NACA 5 digits:</td>
</tr>
<tr>
<td>25112</td>
</tr>
<tr>
<td>NACA 6 digits:</td>
</tr>
<tr>
<td>64A210</td>
</tr>
</tbody>
</table>

After the airfoils are chosen, the next step is to obtain the values of $\frac{C_l}{C_D}$ for a given angle of attack ($\alpha$) and lift coefficient ($C_l$) for each series, and compare the results.

In aeronautics, the study of airfoils and their properties is very important. JavaFoil is an application based on Java, that aims to provide all the necessary instruments for getting an insight in airfoil design and analysis. Through a simple interface you can access all the functions of the program and design, modify and analyse airfoils. Using this tool it is possible to obtain the values that needed.

The first step is to introduce the profile shape. In JavaFoil, the first area of the main window is reserved for the geometry analysis. Menu ’Geometry’ allows the user to create the intended airfoil in two different ways, by selecting family, number of points and respective position for max thickness and max camber, in percentage, from the leading edge or by copying the data coordinates from Airfoil Tools. In this paper the latest was the applied.

When it comes to analyse the selected geometry, it is required to use the dedicated tab inside JavaFoil called ’Polar’. This menu allows to obtain polar diagrams. These show the change in lift coef-
ficient \((Cl)\), drag coefficient \((Cd)\) and pitching moment \((Cm)\) with angle of attack \((\alpha)\).

The \(Cl vs \alpha(\degree)\) plot shows the lift coefficient \(Cl\), plotted versus the angle of attack \(\alpha\). Also, it is possible to find the maximum lift coefficient of the airfoil and the corresponding angle of attack.

The \(Cm vs \alpha(\degree)\) plot is similar to the previous plot, but shows the moment coefficient \(Cm\) of the airfoil section instead of the lift coefficient versus the angle of attack \(\alpha\).

There is also a graph of lift coefficient \((Cl)\) against drag coefficient \((Cd)\) which gives the theoretical glide angle of the airfoil. To find the best glide ratio, you have to draw a line from the origin \((Cl = 0/Cd = 0)\) tangentially to the curve.

The data in these diagrams are valid for the two dimensional airfoil. To find the characteristics of a wing, the geometry of the wing has to be taken into account. Nevertheless, the airfoil polars are a useful means of screening possible candidates for a certain application. More information about polar diagrams can be found in [107].

To be able to obtain the range of values that best suits our problem, a few assumption needed to be done. The most substantial is for the Reynolds number. \textit{JavaFoil} asks for first, last and step value for the Reynolds number and angle of attack. The Reynolds number \((Re)\), can be calculated using equation 5.1.

\[
Re = \frac{V \tau}{\nu}
\]  

Where:

- \(V\) = Velocity of the fluid;
- \(\tau\) = Mean aerodynamic chord;
- \(\nu\) = Kinematic viscosity of the fluid.

The assumed values are velocity equal to 40 m/s, \(\tau\) equal to 0.3 m and \(\nu\) equal to 1.45E-0.5 \(m^2/s\). The consider value for \(Re\) was 800000. With this value it is possible, using the \textit{JavaFoil} tool, to obtain de values of \(\frac{L}{D}\) for the corresponding lift coefficient. It was also assumed that the first angle of attack is equal to \(-10\degree\), the last \(10\degree\) and the step increase in each iteration of \(1\degree\).

5.3 Numerical model

This chapter is basically the description of the numerical implementation of the model explained in Chapter 2 section 2.5.1, into every flight phase consider.
For the optimization the MATLAB® \texttt{fmincon} function is used. This function starts at initial value, $x_0$, and attempts to find a minimizer, $x$, of a constrained nonlinear multi variable function.

A total of eight design variables are adopted in this problem. The design variables are the chord length at root and tip, $c_{\text{root}}$ and $c_{\text{tip}}$, the span of the wing, $b$, the angle of attack, $\alpha$, the twist at the tip, $\theta_{\text{tip}}$, the velocity, $V$, the altitude $h$ and finally the span variation due to the telescopic wing, $x$.

The Lift line theory will be utilized, in each flight phase, for the calculation of all the aerodynamic angles ($\alpha$, $\alpha_{\text{induced}}$ and $\alpha_{\text{effective}}$), as well as lift and drag coefficient.

## 5.3.1 Cruise

The objective of this morphing wing, under cruise conditions, is to be able to fly with a constant lift, equal to aircraft weight, with the lowest drag possible, which means the maximum value of $\frac{L}{D}$.

In this flight phase almost every design variables is allowed to change, trying to analyse all possible wing planforms between the smallest area wing and the maximum area wing. For instance, the span is allowed to vary between 1 and 4 meters, the chord length is limited to a minimum of 0.1 and a maximum of 0.5 m for either root and tip, the twist angle at the tip between -5 to 5 degrees and the angle of attack between -15 and 15 degrees. The weight of the aircraft is considered equal to 500 N, the velocity equal to 40 m/s and lastly, the altitude equal to 1000 m.

Next in order is to constrain the value of the effective angle of attack, $-10 \leq \alpha_{\text{effective}} \leq 10$, due to stall condition. Stalls in fixed-wing flight are often experienced as a sudden reduction in lift as the pilot increases the wing’s angle of attack and exceeds its critical angle of attack.

![Figure 5.3: Wing optimization geometry for cruise.](image)

## 5.3.2 Take-off

After the fixed wing dimensions are determined it is time to optimize the telescopic wing.
The objective is to minimize the take-off velocity which afterwards will be considered the stall velocity for this aircraft. The total span of the telescopic wing is determined in this condition.

In take-off conditions the design variables are angle of attack, $\alpha$, telescopic span variation, $x$ and velocity, $V$. Similarly to the previous section, $\alpha$, can alter its value, in each iteration, between -15 and 15 degrees, $x$ between 0 and 0.6 meters (0 m stands for no need for telescopic wing) and velocity limited to a minimum of 0 and a maximum of 30 m/s.

The weight of the aircraft is the same as in cruise flight, W=500 N, and the altitude is equal to 0 m.

Next in order is to constrain the value of the effective angle of attack, $-10 \leq \alpha_{\text{effective}} \leq 10$, due to stall condition.

### 5.3.3 Climb to cruise

Climb to cruise is the next flight phase, in the mission profile (Fig. 5.1). During this phase and until reach of the cruise altitude, the telescoping wing has to retract completely. The $f_{\text{mincon}}$ algorithm will be able to obtain the values of $x$, regarding altitude and speed.

For this to be possible the design variables are $x$, $\alpha$ and thrust, $T$. The first two feature the same minimum and maximum values as previous. The last one, $T$, can alter its value between 0 and 500 N.

Figure 5.4 shows applied forces and angles in the aircraft climb phase.

![Figure 5.4: Climb performance [108].](image)

As a result a new set of equation needs to be considered, 5.2 - 5.5.

\[
L = W \cdot \cos(\gamma) \quad (5.2)
\]

\[
T - W \cdot \sin(\gamma) - D = m \cdot a \quad (5.3)
\]

\[
a = \frac{V}{t} (m/s^2) \quad (5.4)
\]
\[ t = \frac{h}{\text{RoC}} (s) \]  \hspace{1cm} (5.5)

It is assumed that the aircraft is in a steady climb. Climb angle, \( \gamma \), can be calculated using equation 5.6. \( \text{RoC} \), Rate of Climb is the rate of positive altitude change with respect to time.

\[ \gamma = \frac{\text{RoC}}{V} (\text{rad}) \]  \hspace{1cm} (5.6)

### 5.3.4 Loiter

The last phase in our mission profile, Figure 5.1, is loiter. In loiter mode the plane will circle around the point where the loiter was started, holding altitude. It occurs, for general aviation, generally at the end of the flight plan, normally when the plane is waiting for clearance to land.

MATLAB\textsuperscript{®} function \texttt{fmincon} has the tasks to minimize the drag but also to verify the two main constraints, \( L = W \) and \(-10 \leq \alpha_{\text{effective}} \leq 10\).

The calculation method is very similar to cruise 5.3.1. The main differences are in the altitude, 250 m, and speed. Loiter speed will be consider to have a 10% increase take off speed. Moreover this phase is considered to be a cruise phase at low altitude and speed.
Chapter 6

Morphing wing results

In this chapter the wing optimization results are presented. Chapter 5 describes the main considerations being made in each flight path. Using this information, the next step is to implement in the MATLAB® algorithm, in order to calculate the final wing configuration, the lift and drag, and other significant results such as bending and torsion moment.

An aerodynamic shape optimization code was developed by the present author. This code uses a lifting-line algorithm to determine the total lift of the wing and a second algorithm is used to solve a drag minimization problem to determine the optimal values of wing span for the whole vehicle's flight speed envelope, while subjected to geometric constraints.

6.1 Cruise segment - Fixed wing geometry optimization

Section 5.2 illustrates table 5.2 with six airfoil profiles and the Reynolds number to be $8E5$. Next in order and applying the previous data in the JavaFoil program, is to calculate for each airfoil the values of $\frac{L}{D}$ relative to $C_l$.

The results are presented in tables 6.1-6.6.

<table>
<thead>
<tr>
<th>Table 6.1: NACA 2415 airfoil</th>
<th>Table 6.2: NACA 2412 airfoil</th>
</tr>
</thead>
<tbody>
<tr>
<td>NACA 2415</td>
<td>NACA 2412</td>
</tr>
<tr>
<td>Camber: 2%</td>
<td>Camber: 2%</td>
</tr>
<tr>
<td>Thickness: 15%</td>
<td>Thickness: 12%</td>
</tr>
<tr>
<td>$C_l$</td>
<td>$C_l$</td>
</tr>
<tr>
<td>$\frac{L}{D}$</td>
<td>$\frac{L}{D}$</td>
</tr>
<tr>
<td>$\alpha(\circ)$</td>
<td>$\alpha(\circ)$</td>
</tr>
<tr>
<td>0.140 13.178 -1</td>
<td>0.150 18.602 -1</td>
</tr>
<tr>
<td>0.263 24.549 0</td>
<td>0.268 34.352 0</td>
</tr>
<tr>
<td>0.386 35.827 1</td>
<td>0.388 49.823 1</td>
</tr>
<tr>
<td>0.509 46.654 2</td>
<td>0.507 63.733 2</td>
</tr>
<tr>
<td>0.631 56.435 3</td>
<td>0.624 72.848 3</td>
</tr>
<tr>
<td>0.752 64.140 4</td>
<td>0.740 73.134 4</td>
</tr>
<tr>
<td>0.869 84.156 5</td>
<td>0.853 78.212 5</td>
</tr>
</tbody>
</table>
The airfoil chosen was the NACA 2408. It is the one that presents higher values of $\frac{L}{D}$, when comparing the 4-digit and 5-digit NACA series, for the smallest values of $C_l$.

It is important to mention that the airfoil NACA 64A210, shows better results, which means higher values of $\frac{L}{D}$, but with the increase of $C_l$, over a certain value, the result of $\frac{L}{D}$ drastically decreases.

Data from JavaFoil is exported to an excel file. The solution that corresponds to $Re$ equal to 800000 is selected and the table 6.7 display the results.

<table>
<thead>
<tr>
<th>Name = NACA 2408</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reynolds Number = 800000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\alpha(\degree)$</th>
<th>$C_l$</th>
<th>$C_d$</th>
<th>$\alpha(\degree)$</th>
<th>$C_l$</th>
<th>$C_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10</td>
<td>-0.595</td>
<td>0.0706</td>
<td>1</td>
<td>0.366</td>
<td>0.0068</td>
</tr>
<tr>
<td>-9</td>
<td>-0.548</td>
<td>0.0613</td>
<td>2</td>
<td>0.478</td>
<td>0.0073</td>
</tr>
<tr>
<td>-8</td>
<td>-0.489</td>
<td>0.0533</td>
<td>3</td>
<td>0.588</td>
<td>0.0078</td>
</tr>
<tr>
<td>-7</td>
<td>-0.512</td>
<td>0.0455</td>
<td>4</td>
<td>0.696</td>
<td>0.0083</td>
</tr>
<tr>
<td>-6</td>
<td>-0.410</td>
<td>0.0105</td>
<td>5</td>
<td>0.802</td>
<td>0.0119</td>
</tr>
<tr>
<td>-5</td>
<td>-0.304</td>
<td>0.0099</td>
<td>6</td>
<td>0.903</td>
<td>0.0129</td>
</tr>
<tr>
<td>-4</td>
<td>-0.195</td>
<td>0.0094</td>
<td>7</td>
<td>0.987</td>
<td>0.0140</td>
</tr>
<tr>
<td>-3</td>
<td>-0.085</td>
<td>0.0090</td>
<td>8</td>
<td>1.043</td>
<td>0.0158</td>
</tr>
<tr>
<td>-2</td>
<td>0.027</td>
<td>0.0078</td>
<td>9</td>
<td>0.897</td>
<td>0.0579</td>
</tr>
<tr>
<td>-1</td>
<td>0.139</td>
<td>0.0064</td>
<td>10</td>
<td>0.920</td>
<td>0.0680</td>
</tr>
<tr>
<td>0</td>
<td>0.251</td>
<td>0.0064</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In figure 6.1 the lift coefficient, $C_l$, is plotted versus the angle of attack $\alpha$ and in figure 6.2 versus the drag coefficient, $C_d$. 

Figure 6.1: $C_l$ vs $\alpha(\degree)$ plot.

Figure 6.2: $C_l$ vs $C_d$ plot.
After obtaining all the necessary results the next step is to use the MATLAB® function `fmincon` to optimize the design variables. Different initial values for the set of all the design variables, \( x_0 \), will be considered. Using the excel file data and the mathematical algorithm from the lift line theory, it is possible to calculate the lift and drag coefficients and afterwards the total lift and drag. This will allow the function `fmincon` to optimize the parameters that better validate the conditions previously imposed.

Analysing the results, the max \( \frac{L}{D} \) value occurs in the optimization presented in table 6.8. This indicates the best geometry of the fixed wing.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( c_{\text{root}} )</th>
<th>( c_{\text{tip}} )</th>
<th>( b )</th>
<th>( \theta_{\text{tip}} )</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.335</td>
<td>0.155</td>
<td>1.694</td>
<td>-1.19</td>
<td>6.90</td>
<td></td>
</tr>
</tbody>
</table>

From this point forward, the dimensions of the fixed wing do not change and will be considerer in the remain flight phases, Figure 6.3.
6.2 Take-off segment - Velocity and telescopic span optimization

The conditions consider in this section are the same as presented in section 5.3.2.

As mention before the fixed wing is already fully defined. The only design variables that are approved to change are angle of attack, $\alpha$, telescopic span variation, $x$ and velocity, $V$. The weight of the aircraft it is the same as in cruise flight, $W = 500N$, and the altitude is equal to 0 m.

Once more, the objective is to minimize de take-off velocity. The take off velocity will be consider to be the stall velocity.

Table 6.9 shows the results for the design variables, obtained using the algorithm with $fmincon$ function, considering the initial condition, $x_0$. It is possible to confirm that the condition, lift equal to the weight, was checked.

<table>
<thead>
<tr>
<th>$x$ (m)</th>
<th>$\alpha$ (°)</th>
<th>$V_{stall}$ (m/s)</th>
<th>Lift (N)</th>
<th>Drag (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_0$</td>
<td>0.6</td>
<td>15</td>
<td>10</td>
<td>-</td>
</tr>
<tr>
<td>$x$</td>
<td>0.6</td>
<td>11.16</td>
<td>26.654</td>
<td>499.500</td>
</tr>
</tbody>
</table>

The max value for $x$, 0.6 meters, was determined by considering nearly 70% of half the span. Take off velocity is equal to 26.654 m/s and angle of attack is 11.16°. In this optimization, performing a change in the initial point, $x_0$, did not change the final value.

Figure 6.4 is an illustration of the fully extended morphing wing during the take off phase.

![Figure 6.4: Wing optimization geometry for take-off.](image)

6.3 Climb segment - Thrust optimization

For the thrust optimization the conditions are the same as introduced in section 5.3.3.

As previously mentioned, during the phase of climb and until reach of the cruise altitude, the telescoping wing has to retract completely. So, the main objective for the MATLAB® algorithm is to minimize thrust while obtaining the values of $x$, regarding altitude, speed and time.
The design variables that are allowed to change its value in each iteration are $x$, $\alpha$ and thrust, T. For geometric limitations, telescopic wing span can’t have a higher value than 0.6 meters, as result from take off phase.

Table 6.10 shows the optimization results.

<table>
<thead>
<tr>
<th>Altitude, h (m)</th>
<th>100</th>
<th>150</th>
<th>162.5</th>
<th>175</th>
<th>200</th>
<th>250</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>V (m/s) 27.488</td>
<td>28.322</td>
<td>28.531</td>
<td>28.739</td>
<td>29.156</td>
<td>29.990</td>
<td>30.658</td>
<td></td>
</tr>
<tr>
<td>time (s)</td>
<td>12.5</td>
<td>25</td>
<td>28.125</td>
<td>31.25</td>
<td>37.5</td>
<td>50</td>
<td>60</td>
</tr>
<tr>
<td>Thrust, T (N)</td>
<td>184.901</td>
<td>157.845</td>
<td>138.495</td>
<td>134.773</td>
<td>128.932</td>
<td>120.008</td>
<td>113.696</td>
</tr>
<tr>
<td>$x$ (m)</td>
<td>0.60</td>
<td>0.60</td>
<td>0.56</td>
<td>0.54</td>
<td>0.52</td>
<td>0.46</td>
<td>0.42</td>
</tr>
<tr>
<td>$\alpha$ (°)</td>
<td>9.47</td>
<td>8.41</td>
<td>6.65</td>
<td>6.66</td>
<td>6.67</td>
<td>6.68</td>
<td>6.70</td>
</tr>
<tr>
<td>Lift (N)</td>
<td>491.250</td>
<td>491.728</td>
<td>492.341</td>
<td>492.445</td>
<td>492.659</td>
<td>493.049</td>
<td>493.382</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Altitude, h (m)</th>
<th>350</th>
<th>400</th>
<th>450</th>
<th>500</th>
<th>750</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>V (m/s) 31.325</td>
<td>31.992</td>
<td>32.660</td>
<td>33.327</td>
<td>36.663</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>time (s)</td>
<td>70</td>
<td>80</td>
<td>90</td>
<td>100</td>
<td>150</td>
<td>200</td>
</tr>
<tr>
<td>Thrust, T (N)</td>
<td>108.751</td>
<td>104.686</td>
<td>101.26</td>
<td>98.207</td>
<td>86.932</td>
<td>78.973</td>
</tr>
<tr>
<td>$x$ (m)</td>
<td>0.38</td>
<td>0.34</td>
<td>0.31</td>
<td>0.27</td>
<td>0.12</td>
<td>0</td>
</tr>
<tr>
<td>$\alpha$ (°)</td>
<td>6.71</td>
<td>6.73</td>
<td>6.74</td>
<td>6.76</td>
<td>6.83</td>
<td>6.78</td>
</tr>
<tr>
<td>Lift (N)</td>
<td>493.648</td>
<td>493.911</td>
<td>494.132</td>
<td>494.365</td>
<td>495.351</td>
<td>495.599</td>
</tr>
</tbody>
</table>

Figure 6.5 and 6.6 illustrate the lift and telescopic span variation relative to the altitude.

![Lift vs altitude plot](image.png)
We can conclude that with increasing altitude the lift also increases, approaching to the value of $L = W$ in cruise conditions. Moreover, the span variation is decreasing and approaching zero. The results are consistent with what would be expected.

### 6.4 Loiter segment - Drag optimization

The last optimization is for the drag, $D$. Minimizing $D/L$ is the main condition for the loiter flight phase. Since the loiter phase is in levelled flight, thus with constant lift, the last optimization is for the drag, $D$. More information can be found in section 5.3.4. For this optimization the loiter altitude is 250 m and the loiter speed was considered to be 10% over the take off speed.

Table 6.11 presents the minimum drag value for the given conditions.

<table>
<thead>
<tr>
<th>$x$ (m)</th>
<th>$\alpha$ (°)</th>
<th>Drag (N)</th>
<th>Lift (N)</th>
<th>$h_{loiter}$ (m)</th>
<th>$V_{loiter}$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_0$</td>
<td>0.6</td>
<td>15</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$x$</td>
<td>0.524</td>
<td>6.66</td>
<td>6.957</td>
<td>499.982</td>
<td>250</td>
</tr>
</tbody>
</table>
Figure 6.7 is an illustration of the morphing wing during the loiter phase.

![Figure 6.7: Wing optimization geometry for loiter.](image)

### 6.5 Moment calculation results

After all the optimization are conclude it is possible to calculate the bending and torsion moment for each phase of the mission profile. Introducing the following equations 6.1, 6.2 and 6.3, in the lifting line mathematical method it is possible to obtain the values for the bending and torsion moment.

\[
BM = \int_{0}^{\text{span}} L(x) x \, dx
\]  
(6.1)

\[
TM = \int_{0}^{\text{span}} L(x) y(x) \, dx
\]  
(6.2)

In equation 6.1 the \( x \) variable, present in equations 6.1 and 6.2, allows to perform the integration along the wingspan of the aircraft. Due to simplification purposes it was only considered half of the total span calculated previously. For the torsion moment equation 6.2 the variable \( y(x) \) can be obtained as follows.

\[
y(x) = y_{\text{beam}} - y_{\frac{1}{4}c}(x)
\]  
(6.3)

Where:

\[
y_{\text{beam}} = 0.3 \cdot c_{\text{root}}
\]  
(6.4)

\[
y_{\frac{1}{4}c}(x) = 0.25 \cdot c(x)
\]  
(6.5)

The variable \( c(x) \) is the chord variation in each section along the spanwise direction.

Since all the optimizations have been performed and all the important values are determined, next in order is to calculate the bending and torsion moment values. In this section will just be considered the
following flight phases: Take-off, Climb at 250 m, Climb at 500 m, Cruise and Loiter.

In table 6.12 the results can be found.

<table>
<thead>
<tr>
<th>Flight Phase</th>
<th>Bending Moment [Nm]</th>
<th>Torsion Moment [Nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Take off</td>
<td>92.3</td>
<td>5.3</td>
</tr>
<tr>
<td>Climb 250 m</td>
<td>85.5</td>
<td>5.4</td>
</tr>
<tr>
<td>Climb 500 m</td>
<td>75.6</td>
<td>5.6</td>
</tr>
<tr>
<td>Cruise</td>
<td>59.1</td>
<td>5.8</td>
</tr>
<tr>
<td>Loiter</td>
<td>90.0</td>
<td>5.5</td>
</tr>
</tbody>
</table>

In the next graphs 6.8 and 6.9, it is illustrated the bending an torsion moment regarding half the wing span (semi-span).

Figure 6.8: Bending moment variation for Flight phase: Cruise.

Figure 6.9: Torsion moment variation for Flight phase: Cruise.

After performing the calculations for the bending and torsion moments, the next step would be to
establish the relation between the moments and the uniaxial (shear and normal) stresses. Afterwards, verify in the ssf damage surface under which conditions it would end up in a fracture.
Chapter 7

Conclusions

The sixth and final chapter presents the conclusions drawn from this study as well as present topics for future development.

This document studies the influence of multiaxial loading conditions and fatigue life on magnesium alloy AZ31B-F. From the experimental and theoretical work carried out with this material, some remarks can be drawn:

Under multiaxial fatigue loading conditions two kinds of stresses are involved in the fatigue damage process. Shear and axial stresses perform on the material different damage mechanisms with different damage scales. To quantify the combined damage it is necessary to have both stresses on the same space stress. In order to achieve that, a stress scale factor (ssf) is used.

Generally, multiaxial fatigue models use a constant stress scale factor. In this study was shown that the ssf is not constant and that it is dependent of the loading path. The stress scale factor proposed in this study is a function that takes into account the material’s response in terms of stress intensity level and loading paths trajectory and must be experimentally determined to correctly establish the adequate stress space to compute the multiaxial fatigue damage.

The used method was based on the S–N results from specific loading paths. As a result, it was obtained a stress scale factor surface (ssfs), for the AZ31B-F magnesium alloy, in a shear stress space domain, which is a two variable polynomial function with axial stress and stress amplitude ratio as arguments.

This master thesis also analysed the optimization process for a telescopic wing. From the results obtained some conclusions can be drawn:

The fixed wing has been developed for high speed, particularly for cruise conditions. In the other
hand, the telescopic wing was developed for low speed flight condition.

For the take off phase, the morphing wing increases its size to the maximum allowed by the developed algorithm. After take off and during climb phase, the morphing wing will, as expected, decrease its size with the increase of altitude. When the UAV reaches the cruise altitude, the telescopic wing is completely retracted.

In loiter, the aircraft is very close to the ground. The results show that the telescopic wing contributes, considerably, for the total lift since it presents extended almost entirely.

It is possible to conclude that this type of wing can bring great benefits for this type of mission. When considering a telescopic wing, the slower the velocity, in each phase, the better.

7.1 Future Work

For future work there are three areas that can be developed.

First and in the materials’ area, performing further fatigue tests with the magnesium alloy AZ31B would allow to verify and validate the results previously obtained with much more support. Another great development would be the creation of a computational or numerical method that allows to obtain the elastoplastic behaviour for this type of materials.

Another possible work to be developed, in this case, related to aerodynamics, is concerned to the airfoil profile. In this work it was only considered NACA airfoil profiles. As known, in today’s market many other airfoils are available with equal, or better, features that can be implemented to a wing with these characteristics.

Finally, the analysis regarding the applied loads in extreme conditions, such as, high velocity and rough angle of attack changes or considering a condition with higher ‘g’ loads, could not be performed due to lack of time. For this reason and in the mechanical field, optimization and development of a mechanism for the telescopic wing, using the magnesium alloy AZ31B, would be an interesting approach.
Bibliography


