LiDAR-based Perception and Control for Rotary-wing UAVs in GPS-denied Environments

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Abstract

This thesis explores the problem of Attitude Estimation and Control of a quadcopter in a GPS denied environment. This is particularly important in cases such as bridge inspections. The sensor used for this purpose was a LiDAR which allows the estimation of the relative position and attitude of the vehicle relative to a pier. Firstly, the problem of attitude estimation was addressed, using vector measurements expressed in the body frame and in the inertial frame, fusing this information with the one obtained from the gyroscopes. The presented observers can estimate a constant bias in the angular velocity with a few changes. Both versions were developed and the cases were they can be used were analysed. Experimentally the observers were tested using vector measurements in the body and inertial frames obtained from the LiDAR, where those measurements correspond to edges of a reference pier. Position and attitude controllers were also developed in order to keep the Quadcopter facing the pier. These were then experimentally tested using the Rotation Matrix from the IMU, the estimator and the estimated position from the LiDAR. The results obtained for this system were promising and can be used when GPS information is unavailable.

Keywords: Attitude Estimation,Nonlinear Estimation, Nonlinear Control, LiDAR, Bias Estimation

1. Introduction

The motivations for this work can be divided in three interrelated topics, Navigation in GPS denied environments, Attitude Estimation and Feedback Control.
The first topic is important because in several environments it is not possible to use GPS for navigation , overcoming this challenge means new kinds of navigations are possible, such as bridge inspections and indoor navigation.
The second topic, attitude estimation is a well known problem and has been studied for several decades. In theory the attitude can be obtained by integrating the rotation matrix kinematics using the data given by the gyroscopes. This approach is not applicable in practice as it ignores the existence of bias and sensor noise.
This thesis explores an alternative which is to determine the attitude using vector measurements and fusing that data with the one obtained by the gyroscopes. This topic, in particular, had a lot of interest in the last decade, specifically fusing those measurements with the angular velocity of the gyroscope to obtain a better estimate of the attitude.
Finally, the third topic is feedback control for RUAVs, the motivation here is particularly with using non-linear controllers, as they can make more robust controllers which work under more conditions. Additionally, it is interesting to test the interaction between this kind of controllers and estimators as this has been an ongoing research topic for control.

1.1. Attitude Estimation

Attitude reconstruction is a estimation scheme where the vector measurements are used to calculate the orientation, usually it is reconstructed from the observation of at least two non-collinear vectors, in the two respective frames,and solved as an optimization problem. The first algorithm developed was the TRIAD algorithm. However when there is measurement noise, the attitude matrix is not guaranteed to remain in the rotation group SO(3) making further projection necessary. Second the algorithm is inflexible regarding the number of vector measurements and does not take into account their reliability.
These problems disappear with the Wahba's problem, however they are computationally more expensive and were not well suited to real-time applications until Davenports q-method and the

numerical technique QUEST appeared. In all algorithms there is a trade-off between computational time and precision. Furthermore the information contained in measurements of past attitudes is not preserved [4].

To tackle this problem several methods have been developed to estimate the attitude, these include the MEKF (Multiplicative Extended Kalman filter) which is the standard for low cost IMUs , however this method is still computational expensive and difficult to tune which caused the development of deterministic non-linear observers [1] .

A lot of research also considers the problem of angular velocity bias. The vector measurements can be used directly for the attitude estimation and almost global asymptotic convergence is proven even when there is a constant bias in the angular velocity, assuming the measurements are constant in the inertial frame [10]. For a similar case, [6] generalizes [10] and notices it's parallelism with the MEKF.

In [11] almost global asymptotic convergence is proven for the case where there is only one vector measurement if there is permanent input excitation and without angular velocity bias. In [8] it was proven that if the one vector measurement is time varying, this permanent input excitation isn't needed. [3] shows that semi global stability can be proven for time varying vectors with bias estimation.

1.2. Attitude and Position Control

In a general way attitude control systems can be divided between linear and non-linear control.

In the first case, the system's dynamics are approximated around the desired flight condition, the advantage of the control methods lie on their simplicity.

The advantage of the second method is that it can yield controllers with a larger domain of stability and enhanced robustness [5]. However, it might present undesired dynamics when the modelling is not accurate enough, plus the interaction between the estimation and the controller has been an ongoing research topic.

Non-linear control applied to RUAV, the specific case relevant to this thesis, can be divided in 2 large categories. In the first one, the thrust vector is considered as a state variable and differentiated until one obtains 3 independent control variables, allowing exact linearisation of the translational dynamics. The second one however uses a two stage architecture with a fast inner loop and a slower outer loop. In most cases one uses Lyapunov for control design, however there are cases with sliding mode and predictive control.

The structure for this document is, first observers are derived, analysed and experimentally tested with the LiDAR. Then controllers are derived and finally they are tested experimentally using the LiDAR for estimating the position.

2. Attitude Estimation Methods

2.1. Problem Statement

This problem follows on the work of [2], as such one will often use the same notation. Consider two frames, the Inertial frame $\{I\}$, and a body frame $\{B\}$, attached to the rigid body. Let $R \in SO(3)$ be the rotation matrix that transforms the vectors expressed in $\{B\}$ to $\{I\}$ and $\omega \in \mathbb{R}^3$ the rigid body angular velocity expressed in $\{B\}$. The rigid body attitude kinematics is described by the differential equation

$$\dot{R} = RS(\omega).$$

The angular velocity measurements $\omega_r \in \mathbb{R}^3$ are assumed to be corrupted by a constant unknown bias term $b \in \mathbb{R}^3$ so that

$$\omega_r = \omega + b.$$

For the estimation one assumes, vector measurements (Q_i) are obtained from sensors which are then normalized ($v_i = \frac{Q_i}{||Q_i||}$) in order to have a problem in the form [9]. Let $\hat{R}$ be an estimate of R, therefore an estimate of the normalized vector in $\{B\}$ frame can be written as $\hat{v_i}^B = \hat{R}^T {v_i}^I$ or also as $\hat{v_i}^B = \tilde{R} {v_i}^B$, the vector ${v_i}^B$ can also be written as ${v_i}^B = R^T {v_i}^I$, also from here on they will be written as ${v_i}^B = v_i$ and ${v_i}^I = v_{i0}$.

In the following subsections, several observer solutions will be derived for cases with different assumptions.

2.2. Simple Case

In this case an observer without bias in the angular velocity measurements is being considered. The dynamics are given by

$$\dot{\hat{R}} = \hat{R}S(\hat{\omega})$$

with $\hat{\omega}$ taking the form

$$\hat{\omega} = \omega + k_P \omega_{error}.$$

The innovation term for the angular velocity can be expressed as

$$\omega_{error} = \sum_{i=1}^{n} k_i v_i \times \hat{R}^T v_{i0}.$$

For simplicity, writing $M = \sum_{i=1}^{n} v_i v_i^T$,using some algebraic manipulation

$$S(\omega_{error}) = \sum_{i=1}^{n} \hat{v}_i v_i^T - v_i {\hat{v}_i}^T = \tilde{R}M - M\tilde{R}^T \quad (1)$$

where $S()$ is the skew operator.Defining $\tilde{R} = \hat{R}^T R$ as the rotation matrix error, the error dynamics is given by

$$\dot{\tilde{R}} = -S(\hat{\omega})\tilde{R} + \tilde{R}S(\omega) = [\tilde{R}, S(\omega)] - k_p S(\omega_{error})\tilde{R}. \tag{2}$$

Using as candidate Lyapunov function

$$V = tr(I - \tilde{R}).$$

It can be seen that this function is bounded $0 < V < 4$. Differentiating this function, plugging equation (2) and using the property that states the trace of a commutator is zero, results in

$$\dot{V} = -tr(\dot{\tilde{R}}) = tr(k_p(\tilde{R}M - M\tilde{R}^T)\tilde{R}).$$

Using properties of the trace, plugging equation (1) and using Rodrigues rotation formula, the previous equation can be rearranged as

$$\dot{V} = -2k_p \sin(\theta)^2 \sum_{i=1}^{n} ||S(\lambda)v_i||^2.$$

Since $\sin(\theta)^2 > 0 \ \forall\theta \in]0, \pi[$ and using the assumption that there are at least two non-colinear vectors at all times in both frames, $\sum_{i=1}^{n} ||S(\lambda)v_i||^2 > 0$, finally using Theorem 4.9 from [7], it is concluded that with this observer, $\tilde{R} = I$ is uniformly asymptotically stable in the region $\tilde{R} = rot(\lambda, \theta) \in SO(3) : 0 \leq \theta < \pi$.

2.3. Single vector measurement

If there is only one available measurement, it is still possible to prove the stability of the previous observer under certain conditions, as discussed in [11], specifically if the signal is persistently exciting, what this means will be explained further on. Here the stability proof will be shown based on that work.

Consider the same observer as before, where this time ω_{error} is given by

$$\omega_{error} = v_i \times \hat{v}_i = S(v_i)\hat{R}^T v_{i0}.$$

Assuming that v_{i0} is persistently exciting, that is

$$\int_t^{t+T} S(\Lambda)v_{i0}dt \geq c, \forall c(c > 0).$$

This means the dynamics of v_{i0} can be written as

$$\dot{v}_{i0} = S(\Lambda)v_{i0}$$

where Λ is the orientation velocity of v_{i0} with respect to the inertial frame. In this case the alternative error for the rotation matrix $\bar{R}$ as $\hat{R}R^T$.The error dynamics for this error function is given by

$$\dot{\bar{R}} = k_p \hat{R}S(\omega_{error})R^T.$$

The Lyapunov function candidate is given by

$$V = tr(I - \bar{R}).$$

Following the same procedure as in last section we get

$$\dot{V} = -k_P(1 - v_{i0}^T \bar{R}^2 v_{i0}) \leq 0.$$

Since $\dot{V}$ is negative semidefinite, V is monotonically nonincreasing and lower bounded by zero, implying that it has a bounded limit. Since $\dot{V}$ and $\ddot{V}$ are bounded, $\dot{V}$ is uniformly continuous and Barbalat lemma tells us that $\dot{V}$ converges to zero and consequently $v_i^T \tilde{R}^2 v_i$ converges asymptotically to 1. Since $tr(I - \bar{R}) < 4$, the only possible solution is $\tilde{R}v_i = v_i$. Since $v_i \to \hat{v}_i$ then

$$\bar{R}v_{i0} - v_{i0} \to 0. \tag{3}$$

Differentiating this results in

$$k_P \hat{R}S(\omega_{error})R^T v_{i0} + \bar{R}S(\Lambda)v_{i0} - S(\Lambda)v_{i0} \to 0$$

which simplifies to

$$\bar{R}S(\Lambda)v_{i0} - S(\Lambda)v_{i0} \to 0. \tag{4}$$

Using the properties of rotation matrices, the first term can be rewritten as

$$\bar{R}S(\Lambda)v_{i0} = \bar{R}S(\Lambda)\bar{R}^T\bar{R}v_{i0} = S(\bar{R}\Lambda)\bar{R}v_{i0}.$$

Using the result from equation (3) in the previous one, equation (4) becomes

$$S(\Lambda - \bar{R}\Lambda)v_{i0} \to 0.$$

This means Λ is an eigenvector of $\bar{R}$ associated with the eigenvalue equal 1. Consequently $\bar{R}$ converges to I provided that Λ is persistently exciting and $\bar{R} = I$ is locally asymptotically stable, with a basin of attraction containing almost all initial conditions , i. e. all initial conditions such that $tr(I - \tilde{R}) < 4$. [11]

2.4. Bias Correction

The proposed observer, which is based on [10], takes the form $(\hat{R}, \hat{b})$, with $k_I, k_P \in \mathbb{R}_0^+$, its dynamics are given by

$$\begin{cases} \dot{\hat{R}} = \hat{R}S(\hat{\omega}) \\ \dot{\hat{b}} = -k_I\omega_{error} \end{cases} \tag{5}$$

where it is assumed that there is a constant angular velocity bias.The estimation of the angular velocity is given by

$$\hat{\omega} = \omega_r - \hat{b} + k_P\omega_{error}. \tag{6}$$

Using the definition of antisymmetric part of a matrix as well as the fact M is symmetric, $S(\omega_{error})$ can be written as

$$S(\omega_{error}) = 2P_a(\tilde{R}M).$$

Defining the errors

$$\begin{cases} \tilde{R} = \hat{R}^T R \\ \tilde{b} = b - \hat{b} \end{cases}$$

and assuming the bias is constant, the error dynamics can be written, after replacing the results from equations (5) and (6), as

$$\begin{cases} \dot{\tilde{R}} = [\tilde{R}, S(\omega)] - S(\tilde{b} + k_p \omega_{error})\tilde{R} \\ \dot{\tilde{b}} = k_I \omega_{error}. \end{cases}$$

Define the Lyapunov function as

$$V = \sum_{i=1}^{n} k_i - tr(\tilde{R}M) + \frac{\tilde{b}^T \tilde{b}}{2k_I}.$$

To differentiate the Lyapunov function, the terms will be broken down. Considering the assumption that the measurements are constant in the Inertial Frame., the first term can be written, after simplifications, as

$$\frac{d}{dt}(tr(\tilde{R}M)) = -2k_P tr((P_a(\tilde{R}M))^2) - tr(S(\tilde{b})\tilde{R}M). \tag{7}$$

$$\frac{d}{dt}\frac{\tilde{b}^T \tilde{b}}{2k_I} = -tr(\tilde{R}MS(\tilde{b})). \tag{8}$$

Substituting equations (7) and (8) in the derivative of V, plus using the cyclic permutation of the trace results in

$$\dot{V} = 2k_P tr((P_a(\tilde{R}M))^2).$$

Since the square of antisymmetric matrix is negative definite, the only way for $\dot{V}$ to be 0 would mean $\tilde{R}$ is equal to the identity (no error in the rotation matrix). Computing the second derivative of V yields

$$\ddot{V} = 4k_P tr(P_a(\tilde{R}M)P_a(\frac{d}{dt}(\tilde{R}M))).$$

From the previous calculations, it can be seen that $\ddot{V}$ is composed by bounded terms and therefore $\ddot{V}$ is bounded. One can now apply Barbalat's lemma [7] which implies that

$$\tilde{R}M = M\tilde{R}^T.$$

This result is only true when $\theta = \pi$ and the axis of rotation is an eigenvector of M. If M is such that its eigenvalues are all distinct, it can be shown that the undesired equilibrium points (with $\theta = \pi$) are unstable [10], thus one can conclude that almost all solutions converge to the identity, so that $\tilde{R} = I$ is almost globally asymptotically stable.

2.5. Bias Correction and time varying vectors in inertial frame

The difference here is that the vectors are time varying in the inertial frame. The motivation for this is that in many real case scenarios, the assumption made in the previous subsection of the vectors being constant in the inertial frame is not valid.
The errors are also defined in the same way as before, if the lyapunov function from the previous analysis was used, the result would be to conservative. In order to obtain a less conservative result,

$$V = tr(I - \tilde{R}) + \frac{\tilde{b}^T \tilde{b}}{2k_I} - k_c tr(\tilde{R}S(\tilde{b})).$$

First it needs to be proven that positive functions can be found in order to bound V,using Rodrigues Rotation formula and some algebraic manipulation leads to

$$V = 2\frac{\sin(\theta)^2}{1 + \cos(\theta)} + \frac{\tilde{b}^T \tilde{b}}{2k_I} - k_c \sin(\theta) tr(S(\lambda)S(\tilde{b})).$$

Since the last term can be bounded by

$$k_c |\sin(\theta) tr(S(\lambda)S(\tilde{b}))| < 2k_c \sin(\theta)||\tilde{b}||$$

V can be bounded by positive functions W_1 and W_2, provided that $k_c^2 K_I < \frac{1}{8}$,. Its derivative is given by

$$\dot{V} = -tr(\dot{\tilde{R}}) + \frac{\tilde{b}^T \dot{\tilde{b}}}{k_I} - \frac{d}{dt}(tr(\tilde{R}S(\tilde{b}))).$$

Expanding $\dot{V}$ will result in many terms so they will be omitted, the complete proof appears on the thesis. After bounding them and collecting them

$$\dot{V} \leq -\begin{bmatrix} |\sin(\theta)| & ||\tilde{b}|| \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix} \begin{bmatrix} |\sin(\theta)| \\ ||\tilde{b}|| \end{bmatrix}$$

a_{11},a_{12},a_{22} are given by

$$\begin{cases} a_{11} = 2(k_p - k_c k_I) \sum ||S(\lambda)v_i||^2 + \frac{k_c}{1+\cos(\theta)} ||S(\lambda)\tilde{b}||^2 \\ a_{12} = -(2k_c||M|| + ||M|| + 1 + k_c \frac{B_\omega}{2}) \\ a_{22} = 2k_c \cos(\theta) \end{cases}.$$

To make the matrix negative definite

$$\begin{cases} a_{11} > 0 \\ a_{11}a_{22} - a_{12}^2 > 0 \end{cases}.$$

This translates in setting up the gains k_P, k_I and the constant k_c. By obeying this conditions, uniform asymptotic stability of the observer errors can be guaranteed, using Theorem 4.9 from [7]. Since for this stability proof θ needed to be less than $\frac{\pi}{2}$, the domain is smaller than for the previous section case, nonetheless, the simulation results suggest that the observer can perform very well in the presence of angular velocity bias, even over a larger interval.

3. Observer Results

To test the nonlinear observers LiDAR is used.Two blocks are used for the conversion of between the LiDAR distances measured and the vectors needed for the observer, these were already developed at DSOR prior to this work. One block is responsible for clustering the data points and selects the one whose centroid is the closest from the origin in $\{B\}$.
For the second block, after receiving the clustered data points, it estimates the dimensions of the pier, removes the outliers and calculates the edges. Estimating the dimensions of the pier is the main difference with respect with [2], which relied heavily on knowing beforehand the edge dimensions.
The second block requires as input, besides the cluster information (number of points, distances,angles), the absolute height at which the quadcopter is.
To determine the height of the quadcopter, mirrors were used which reflect the LiDAR rays into the ground. Figure 1 shows the correspondence between the Data points and their usage, due to using some laser points for the height estimation, some points inbetween the height estimation and the edges calculation need to be ignored.

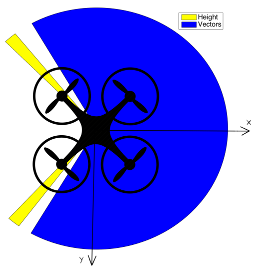

Figure 1: Data point usage

Below is shown the discretized version of the observer that also estimates the bias where the h is the sampling time. The other observer formula is obtained by simply ignoring the second equation

$$\begin{cases} \hat{R}^T_{k+1} = e^{-S(\hat{\omega}_k)h} \hat{R}^T_k \\ \hat{b}_{k+1} = -k_I \omega_{error\,k} h + \hat{b}_k \end{cases} .$$

3.1. Observer without Bias

The test was conducted at a sampling rate of 40Hz, with $k_P = 0.8$ and $\hat{R}_0 = I$. In the time response between 20 and 25 seconds, a strange response is seen for the IMU.This might be due to the magnetometer data which is used, through sensor fusion, by the IMU. It can be seen from Figure 2 that the Roll, Pitch and Yaw responses for the Filter are similar to the ones obtained from the IMU.

As was stated before, sometimes there is only one measurement available. To test for this case, one used the same real data as with the previous experiment, however, here only one is used, even if two are available, in order to mimic this case. Again, a value of $k_p = 0.8$ is used for the proportional gain and $\hat{R}_0 = I$ for the initial rotation matrix.

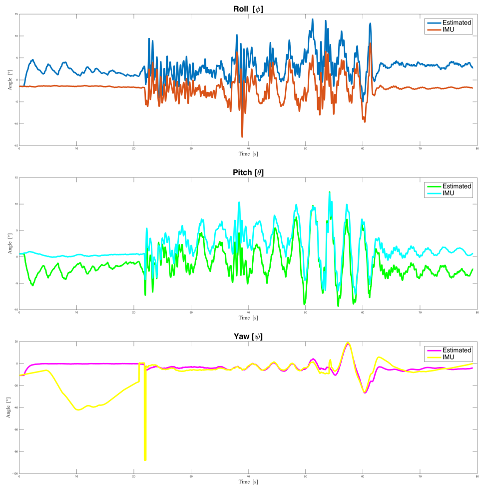

Figure 2: Estimated Roll vs IMU Roll, using multiple vector measurements, $k_P = 0.8$

From Figure 3, it can be seen that the responses for the angles of the observer, the first 20 seconds, have smaller oscillations than the ones for the case with multiple vector measurements. In addition, it can be seen that the peak values for the Roll and Pitch angles are smaller than for the previous case counterparts.

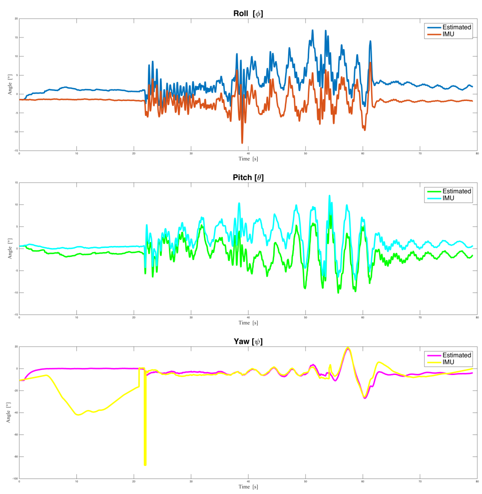

Figure 3: Estimated euler angles vs IMU euler angles, single vector measurement, $k_P = 0.8$

3.2. Observer with Bias

For this case one used the same data as in the previous cases, since there was no bias in the angular velocity (at least no appreciable one), A bias of $b^T = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$ was artificially set, by adding this vector to the angular velocity.
The integral gain k_I for the bias estimate was chosen to be 0.3 so that the setting time for the bias was 20 seconds, the reason for this being to not filter the dynamics, at the same time, the proportional gain k_P value was kept the same from the previous observer with a value of 0.8. The initial estimates were set as $\hat{R}_0 = I$ and $\hat{b}_0 = [0\ 0\ 0]^T$.
As it can be seen on Figure 4 after 20 seconds the responses for the observer are similar to the ones for the previous one, as desired. Furthermore one can see in Figure 5 that the response for bias in the x and y directions converge to the true value around 20 seconds, while experiencing a slight overshoot. The bias response in the z direction converges to the true value faster, at approximately 12 seconds.This shows experimentally that this observer works for cases where the vectors are time varying in the Inertial Frame.

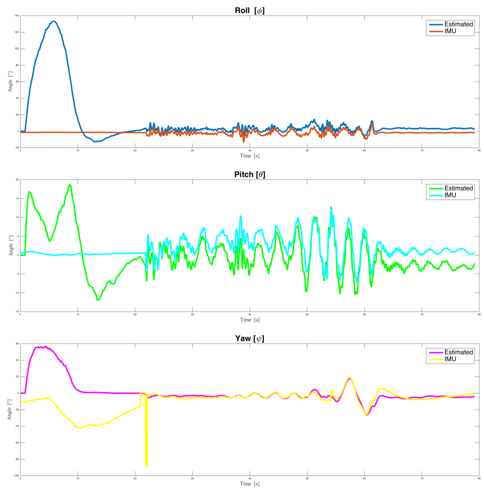

Figure 4: Estimated euler angles vs IMU euler angles, estimating bias, $k_P = 0.8, k_I = 0.3$

4. Position and Attitude Control

The developed controller belongs to the class of Hierarchical Controllers, where the Attitude Control loop (Inner Loop) is faster then the Position Control Loop (Outer Loop), it follows on the work in [12].

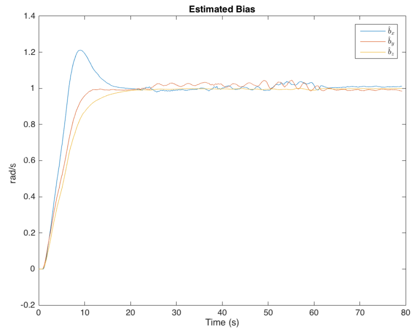

Figure 5: Estimated angular velocity bias, $k_P = 0.8, k_I = 0.3$

4.1. Outer Loop - Position Control

The system dynamics of a quadcopter are considered to be given by the system of equations

$$\begin{cases} \dot{x} = v \\ \dot{v} = ge_3 - \frac{T}{m} r_3 \end{cases}$$

where T is the scalar thrust applied to the vehicle and r_3 as the third column vector of the rotation matrix from $\{B\}$ to $\{I\}$, g is the acceleration due to gravity, and e_3 is third versor that defines the coordinate system. The error dynamics of the system to be controlled are then given by

$$\begin{cases} \dot{\tilde{x}} = \tilde{v} \\ \dot{\tilde{v}} = ge_3 - \frac{T}{m} r_3 - \dot{v}_d \end{cases} . \tag{9}$$

The errors for position and velocity are defined as $\tilde{x} = x - x_d$ and $\tilde{v} = v - v_d$, respectively. The desired rotation matrix vector r_{3d} is given by

$$r_{3d} = \frac{ge_3 - u}{||ge_3 - u||}$$

where $u = -k_p\tilde{x} - k_v\tilde{v} + \dot{v}_d$.Furthermore the desired thrust satisfies

$$m(k_p\tilde{x} + k_v\tilde{v} + ge_3 - \dot{v}_d) = T_d r_{3d}.$$

Defining $T = T_d r_{3d}^T r_3$, replacing this into (9) and adding and subtracting $\frac{T_d r_{3d}}{m}$, the error dynamics can be rewritten as $\dot{\tilde{v}} = -(k_p\tilde{x} + k_v\tilde{v}) + \tilde{u}$ where $\tilde{u} = -\frac{T_d S(r_3)^2 r_{3d}}{m}$.
Therefore the error dynamics can be written in the following conventional form

$$\dot{\tilde{X}} = A\tilde{X} + \tilde{U}$$

where $\tilde{X} = [\tilde{x}, \tilde{v}]$, $\tilde{U} = B\tilde{u}$ and $B = [0_{3\times3}\ I_{3\times3}]^T$ and $A = \begin{bmatrix} 0_{3\times3} & I_{3\times3} \\ -k_p I_{3\times3} & -k_v I_{3\times3} \end{bmatrix}$. Define real symmetric positive definite matrices $P, Q \in \mathbb{R}^{6\times6}$, such that the Lyapunov equation is given by

$$\frac{1}{2}(A^T P + PA) + Q = 0$$

is satisfied. Consider the Lyapunov function candidate to be used in the sequel is given by

$$V_p = \frac{1}{2}\tilde{X}^T P \tilde{X}.$$

4.2. Inner Loop - Attitude Control
Let another Lyapunov function be defined by

$$V_a = 1 - {r_{3d}}^T r_3.$$

The derivative of r_3 can be written as $\dot{r}_3 = -S(r_3)R\omega$ and the derivative of r_{3d} as $\dot{r}_{3d} = \Pi_{r_{3d}} \frac{\dot{q}}{||q||}$, where $q = ge_3 - u$. Define the angular velocity control as ω which is given by

$$\omega = R^T \left(k_{r3} S(r_3) r_{3d} - \frac{m}{T_d} S(r_{3d}) \ddot{v}_d \right).$$

Considering the sum of the previous two Lyapunov candidate functions, $V = V_a + V_p$, it can be seen that the time derivative $\dot{V} = \dot{V}_a + \dot{V}_p$ can be bounded by the sum of inequalities, the steps were omitted but are presented in the thesis, which can be written in matrix form as

$$\dot{V} \leq -z^T \begin{bmatrix} k_{r_3} + (r_{3d}^T r_3) k_v & \frac{n_1}{2} & \frac{n_2}{2} \\ \frac{n_1}{2} & \lambda_{min}(Q) & 0 \\ \frac{n_2}{2} & 0 & \lambda_{min}(Q) \end{bmatrix} z$$

where $z^T = \left[||S(r_3) r_{3d}|| \quad ||\tilde{x}|| \quad ||\tilde{v}|| \right]$, $n_1 = -\left(k_v k_p \frac{m}{T_d} + \lambda_{max}(P) \frac{T_d}{m} \right)$ and $n_2 = -\left(\lambda_{max}(P) \frac{T_d}{m} + |k_p - k_v^2| \frac{m}{T_d} \right)$. Selecting the gains, k_p, k_v, k_{r3}, and matrix Q such that previous matrix in is positive definite, it can be seen that there is a function $W_3(z)$ such that $\dot{V} \leq W_3(z) < 0$. Therefore, as the conditions of Theorem 4.9 from [7] are satisfied, it can be shown that the origin of the error dynamics ($\tilde{p} = 0, \tilde{v} = 0$, and $r_3 = r_{3d}$) is asymptotically stable.

5. Inner Loop to Control R1

In addition to the previously developed controller one decided to test a novel non-linear heading controller.

For this controller the problem turns out to be moving the r_1 vector of the rotation matrix R to a desired vector r_{1d}, this vector is defined as $r_{1d} = \frac{-\Pi_{r_3} p}{||\Pi_{r_3} p||}$, which is the projection of the vector pointing to the center of the pier (r_3). Vector p denotes the position of the vehicle in $\{I\}$, assuming the pier is located at the center of $\{I\}$.The derivative of r_{1d} can be written as $\dot{r_1}_d = \Pi_{r_{1d}} \frac{\dot{q}}{||q||}$, where in this case $q = -\Pi_{r_3} p$. The derivative of r_1 can be written as $\dot{r_1} = -S(r_1)R\omega$.

To prove the stability, the we use as candidate Lyapunov function

$$V_{r_1} = 1 - r_1^T r_{1d}.$$

After simplifications and reworking terms in the derivative of V_{r_1} results in

$$\dot{V}_{r_1} = r_{1d}^T S(r_1) r_3 (\omega_z - \phi)$$

where $\phi = \frac{r_{1d}^T}{||q||} \left(\left(-\omega_x S(r_3) + \omega_y I \right) S(r_1) p + S(r_3) v \right)$. In order to have only one negative definite term, the control action is defined as

$$\omega_z = \phi - k_{r_1} r_{1d}^T S(r_1) r_3.$$

This results in

$$\dot{V}_{r_1} = -k_{r_1} (r_{1d}^T S(r_3) r_1)^2 < 0.$$

Selecting a positive gain k_{r_1}, it can be seen that the lyapunov function derivative satisfies $\dot{V}_{r_1} \leq -W_{r_1}(z) = -k_{r_1}(r_{1d}^T S(r_3) r_1)^2 < 0$. Therefore, as the conditions of Theorem 4.9 from [7] are satisfied, it can be shown that the origin of the error dynamics ($r_1 = r_{1d}$) is asymptotically stable.

6. Controller Results

The Quadcopter used for the Experiment is a Mikrokopter Quadro XL, equipped with Onboard computer *Gumstix Overo Fire COM* (ARM Cortex-A8 CPU at 720MHz with 256MB of RAM), an Attitude Heading Reference System (AHRS) *3DM-GX3* and a LiDAR *Hokuyo UTM-30LX*.

The Control is done outside, using a Mini PC *Intel Kit NUC6I5SYH* (Intel Core i5-6260U at 1,9GHz with 16GB of RAM).

The Nick, Roll and Gier correspond to the desired angular velocities in y, x and z respectively. Those values are integers saturated between $-127°$ and $127°$. The gas, is an integer between 0 and 255, this value directly related with Thrust, however one needs to convert the controller thrust into this desired quantity. Since the LiDAR works at 40Hz and this value cannot be changed, this value was chosen as the sampling rate for the experiment.

In order to do the position control one needs some sort of sensor which gives the position of the quadcopter in the inertial frame. The block that was previously developed at DSOR for the determination of the edges of the pier also estimates the position of the center of the pier in the body frame. This can then be converted to the position in the inertial frame through the transformation

$$p_{body}^I = p = -R p_{pier}^B.$$

The chosen gains for the controllers developed in the previous section were limited by the sampling time, the data communication delay and inaccuracies in the Rotation Matrix estimation. In addition, instead of considering them as constant, different weights were given to the x, y components and to the z component defined as

$$K_p = \begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 3 \end{bmatrix}, K_v = \begin{bmatrix} 1.3 & 0 & 0 \\ 0 & 1.3 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}.$$

The gains related to the attitude control loops were $k_{r_3} = 2$ and $k_{r_1} = 2.5$.

6.1. Using the Rotation Matrix from the IMU

For the following analysis, the trajectory was chosen to have take off at 10 seconds until its $0.5m$ above the initial position, then making a circular trajectory around the center point counterclockwise, holding the final position for 10 seconds, then doing the same clockwise, holding the position for 10 seconds, again going counterclockwise, holding the position for 10 seconds and then land.

Figures 6 and 7 show the desired control action sent to the Mikrokopter, notice that since its reference frame is not the NED, the sign of the Nick and Roll responses had to be changed, they actually have opposite sign to the one in NED. The input responses are only shown after 10 seconds as it was chosen for it to not work before that, for safety reasons. The angular velocity never goes above 30 degrees per second, which is within the limits of operation of the quadcopter. The Gier angular speed is higher than the other ones as it is supposed to keep the quadcopter always facing the pier. In terms of the GAS, it is always within the operation limits as well.

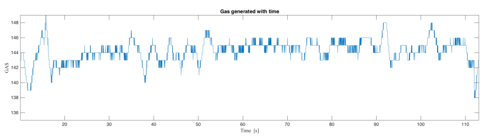

Figure 6: Desired Gas sent to the Mikrokopter

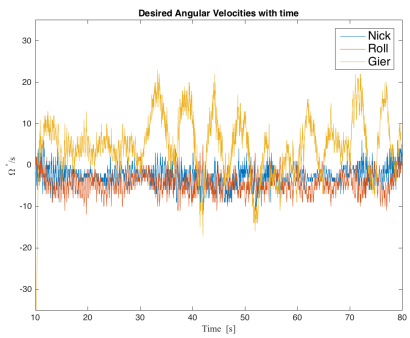

Figure 7: Desired Angular velocity response sent to the Mikrokopter in its reference frame

In Figure 8 it can be seen the response obtained for the Position Control, using the Rotation Matrix from the IMU. The trajectory following in the z direction has smaller oscillations, this is mainly due to the chosen gains which are higher, however it was seen experimentally that higher gains in the other directions would decrease the stability.

On other side, the z direction has more irregular oscilation, this is probably due to inaccuracies in the height estimation which are then propagated to the control.

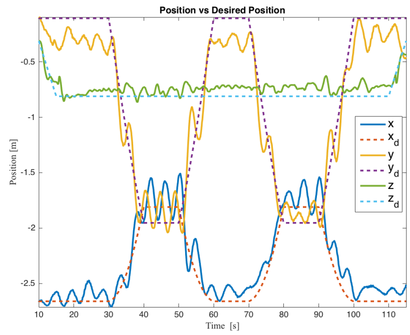

Figure 8: Position vs Desired Position using the IMU

6.2. Using the Rotation Matrix from the Estimator

After testing the Estimator, one realized estimated roll and pitch angles were not accurate enough for the control. Still, with some algebraic manipulation, one could still use the estimated yaw, contained in the Rotation Matrix vector r_1. This is done creating a new rotation matrix whose vectors will have a subscript f before the number indicating their direction. The rotation matrix from the estimator will have a subscript e and the one from the IMU will have a subscript s.

Here the reasoning is, the r_{s3} accurately represents the roll and pitch, so one wants to keep it the same. To keep the yaw from the Estimator one needs project r_{e1}

$$r_{f1} = \frac{\Pi_{r_{s3}} r_{e1}}{||\Pi_{r_{s3}} r_{e1}||}.$$

r_{f2} is the cross product between the other 2 vectors

$$r_{f2} = S(r_{s3}) r_{f1}.$$

In this case the trajectory was half of the one in the previous case, followed by the landing. Again the control (Figure 10) is shown in the Mikrokopter reference frame. It can be seen in Figure 11 that the position response is similar to the previous case. This proves the feasibility of the approach, still this was expected as the r_1 vector does not have much impact on the control.

7. Conclusions

The main conclusion with this work is that it is possible to control the attitude and position of a

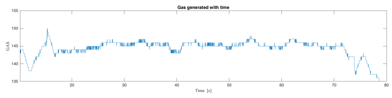

Figure 9: Desired Gas sent to the Mikrokopter

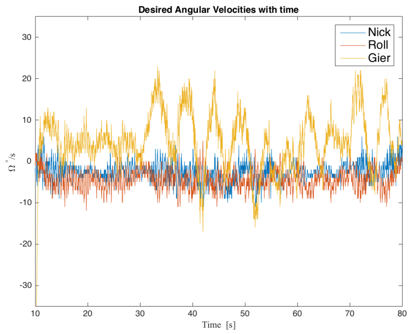

Figure 10: Desired Angular velocity response sent to the Mikrokopter in its reference frame

Figure 11: Position vs Desired Position using the new Rotation Matrix

quadcopter in a GPS denied environment using LiDAR in the position and attitude estimation.
It was theoretically proven that the observer works for with multiple vector measurements as well as when there is only a time varying vector measurement. In addition it was also proven that the observer with Bias Correction, can indeed estimate attitude correctly.
Experimentally, using a quadcopter equipped with LiDAR it was seen that using only one vector measurement presents a larger error. This might be due to the theory not having into consideration the existence of noise in the vector measurement or needing more time for convergence.
The complete developed controller, which includes a nonlinear hierarchical controller for position and attitude and the nonlinear heading controller, is stable and can be used with LiDAR data for position control relative to a pier.
The controllers were successfully implemented in a Mikrokopter Quadro XL, using the rotation matrix given by the IMU and yielded a good trajectory following considering the limitations. Using the observer for the control, it was not possible to achieve satisfying results since the errors in the estimation for the pitch and roll were to high. It is possible however to still use just the yaw estimation part of the observer for the control with some mathematical manipulation.

References

[1] J. L. Crassidis, F. L. Markley, and Y. Cheng. Survey of nonlinear attitude estimation methods. *Journal of Guidance, Control, and Dynamics*, 30(1):12–28, 2007.

[2] A. Gomes. Laser-based control of rotary-wing uavs. Master's thesis, Instituto Superior Técnico, 2015.

[3] H. F. Grip, T. Fossen, T. A. Johansen, A. Saberi, et al. Attitude estimation using biased gyro and vector measurements with time-varying reference vectors. *Automatic Control, IEEE Transactions on*, 57(5):1332–1338, 2012.

[4] M.-D. Hua, G. Ducard, T. Hamel, and R. Mahony. Introduction to nonlinear attitude estimation for aerial robotic systems. *Aerospace Lab*, pages AL08–04, 2014.

[5] M.-D. Hua, T. Hamel, P. Morin, and C. Samson. Introduction to feedback control of underactuated vtolvehicles: A review of basic control design ideas and principles. *IEEE Control Systems*, 33(1):61–75, 2013.

[6] K. J. Jensen. Generalized nonlinear complementary attitude filter. *Journal of Guidance, Control, and Dynamics*, 34(5):1588–1593, 2011.

[7] H. K. Khalil and J. Grizzle. *Nonlinear systems*, volume 3. Prentice hall New Jersey, 1996.

[8] A. Khosravian and M. Namvar. Globally exponential estimation of satellite attitude using a single vector measurement and gyro. In *Decision and Control (CDC), 2010 49th IEEE Conference on*, pages 364–369. IEEE, 2010.

[9] R. Mahony, T. Hamel, and J.-M. Pflimlin. Complementary filter design on the special orthogonal group so (3). In *Decision and Control, 2005 and 2005 European Control Conference. CDC-ECC'05. 44th IEEE Conference on*, pages 1477–1484. IEEE, 2005.

[10] R. Mahony, T. Hamel, and J.-M. Pflimlin. Nonlinear complementary filters on the special orthogonal group. *Automatic Control, IEEE Transactions on*, 53(5):1203–1218, 2008.

[11] R. Mahony, T. Hamel, J. Trumpf, and C. Lageman. Nonlinear attitude observers on so (3) for complementary and compatible measurements: A theoretical study. In *Decision and Control, 2009 held jointly with the 2009 28th Chinese Control Conference. CDC/CCC 2009. Proceedings of the 48th IEEE Conference on*, pages 6407–6412. IEEE, 2009.

[12] R. Santos. Load transportation using rotary-wing uavs. Master's thesis, Instituto Superior Técnico, 2015.