Muticriteria Structural Optimization of Flotation Circuits

Application of a Genetic Algorithm

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Abstract

The concentration processes comprise the separation of valuable minerals (ore) from the non-valuable ones (gangue). Froth flotation stands out for its versatility and it is widely accepted as the most important of all concentration methods. However, the separation of the different mineral species is seldom successful if the process relies on only one stage of concentration, wherefore, the use of a set of stages connected by a net of streams (circuit) is a common practice, in order to make sure certain criteria, related with the separation products quality (grade) and quantity (recovery), are satisfied.

Given that concentration circuits have many possible configuration, their optimal synthesis is a combinatorial problem with exponential growth of complexity as more stages are added to it. One of the most recent approaches to solve this problem is the application of metaheuristics (more specifically, evolutionary algorithms), which allow the finding of a "good enough" solution, faster than the evaluation of all possible configurations or even exact optimization algorithms.

Therefore, the main purpose of this dissertation is the development of an evolutionary algorithm a genetic algorithm in MATLAB ®, to solve the problem of multicriteria structural optimization of flotation circuits and to analyze its performance, comparing the produced solutions with the global optimal solutions (exact). This problem consists in the formulation and resolution of two problems: the determination of the separation equipment's configuration (circuit's layout) that maximizes/minimizes a given objective function that expresses the technical-economic criteria, and the parametric optimization that consists in the determination of a set of values for the project constraints, corresponding to the regulation of the separation equipment (for example, the pulp's density or the cell's volume), that allows the optimization of the same objective function.

Keywords: Circuit Design; Multicriteria Optimization; Mineral Processing Circuits; Froth Flotation;

Genetic Algorithm;

1. Introduction

The purpose of mineral concentration processes is the separation of the of minerals in two products, sending the ore particles (valuable) to the concentrate product and the gangue particles (non-valuable) to the tailings product [1].

This process has some basic terminology that is important to consider. In this dissertation, two main concepts will be highlighted: grade and recovery. Considering that a solid stream is usually composed by many mineralogical species and that each of these is also composed by more than one substance, it is possible to obtain the grade and the recovery of a given substance in a product by formulating the following: let **m** be the number of mineralogical classes in a concentrate product; let $\mathbf{c_j}$ be the grade of a certain substance in the mineralogical class **j**, and $\mathbf{C_j}$ and $\mathbf{M_{F,j}}$ the stream of solids of the mineralogical class **j** in the concentrate product and the feed of the process, respectively. Then, the computation of the grade (**G**) and recovery (**η**) of that substance in the concentrate product is given throw equations (1)(2), respectively.

$$G = \frac{\sum_{j=1}^{m} C_j c_j}{\sum_{j=1}^{m} C_j} \quad \textbf{(1)}$$

$$\eta = \frac{\sum_{j=1}^{m} C_j c_j}{\sum_{j=1}^{m} M_{F,j} c_j} \quad \textbf{(2)}$$

These two criteria are conflictive because, it is seldom possible to perfectly liberate the minerals of interest during the comminution processes, which means that an increase in recovery implies a greater harnessing of “less pure” (mix) particles, reducing the grade in valuable substance in the concentrate product. Nevertheless, it is possible to maximize both if some multicriteria method is used.

However, separation of the different mineral species is seldom successful if the process relies on only one stage of concentration, wherefore, the use of a set of stages connected by a net of streams (circuit) is a common practice, in order to make sure certain criteria, related with the separation products quality (grade) and quantity (recovery), are satisfied.

This raises the need to engineer ways to deal with the design of these circuits. The design comprises the choice of the number of stages to use and the way they interact with each other (their configuration), in order for the products' characteristics to be in line with what is needed. Given that concentration circuits have many possible configuration, their optimal synthesis is a combinatorial problem with exponential growth of complexity as more stages are added to it. Thus, in the last decades there have been some important breakthroughs and approaches to the problem that use computer aided optimization techniques to efficiently deal with the combinatorial nature of the problem, producing optimal solutions (or very close) in an acceptable amount of time.

One of the most recent approaches consists in the application of metaheuristics techniques (in particular, evolutionary algorithms) to solve the structural optimization problem. Although these techniques do not guarantee the convergence to a global optimal solution, they usually allow to obtain a "good enough" solution faster than the evaluation of all possible configuration or even exact optimization algorithms.

There are many types of concentration processes but froth flotation stands out for its versatility, being widely accepted as the most important of all concentration methods [1]. Therefore, it is reasonable to choose this process as an example for the structural optimization problem. This process takes place in cells, where three phases coexist: solids (minerals), liquids (water) and gas (air). Flotation cells are usually fed by a pulp, composed by grinded minerals and water. In mechanically agitated cells, the pulp is agitated and mixed with a stream of air bubbles that drag the hydrophobic particles to the surface, while the hydrophilic particles do not bond with the air bubbles and stay in the water [2].

2. Structural Optimization of Flotation Circuits

The issue of structural optimization of flotation circuits consists in finding the configuration (connection layout) of the separation equipment that optimizes a given objective function that expresses one or more technical-economic criteria. On the other hand, there is another kind of optimization that consists in finding the set of values for the project variables (parameters/regulation of the separation equipment – as for the pulp's density or cell's volume) that maximizes the same objective function – the parametric optimization. These two concepts cannot be separated because each circuit configuration requires solving a parametric optimization problem, while the structural optimization searches for the best configuration, in terms of performance of the objective function criteria, which can only be obtained by solving the parametric optimization problem. The procedure for structural optimization of circuits, as any optimization problem, incorporates two main stages: problem formulation and resolution.

2.1. Problem Formulation

The problem formulation covers the structural model, the unit operation (of separation) model and the objective function. The structural model can be coded using the concept of an oriented graph, represented by a set of nodes and a set of ordered pairs of nodes. There are three types of nodes: the froth flotation unit operations (separation process); the stream junction nodes; and the binary stream division nodes. On the other hand, the ordered pairs of nodes represent the streams that are established between nodes and are represented by adjacency matrices of the three main classes of solid streams (in the form of pulp): circuit new feed, concentrate product (froth) and tailings product (sunk). The entries to these matrices are represented by $\boldsymbol{\beta_{k,i}}$ (concentrate from cell **k** that flows to cell **i**), $\boldsymbol{\delta_{k,i}}$ (tailings from cell **k** that flows to **i**) e $\boldsymbol{\delta_{F,i}}$ (new feed flowing to cell **i**). If a stream leaves the circuit (to the outside) as a final product, then it is treated as if it goes to "cell zero", with **i = 0** [3]. The entries of this binary sparse matrices are equal to one every time a stream connects an **i** node to a **j** node, and equal to zero otherwise. The possible directions for streams are shown in Figure 1, using the example of a circuit with two cells.

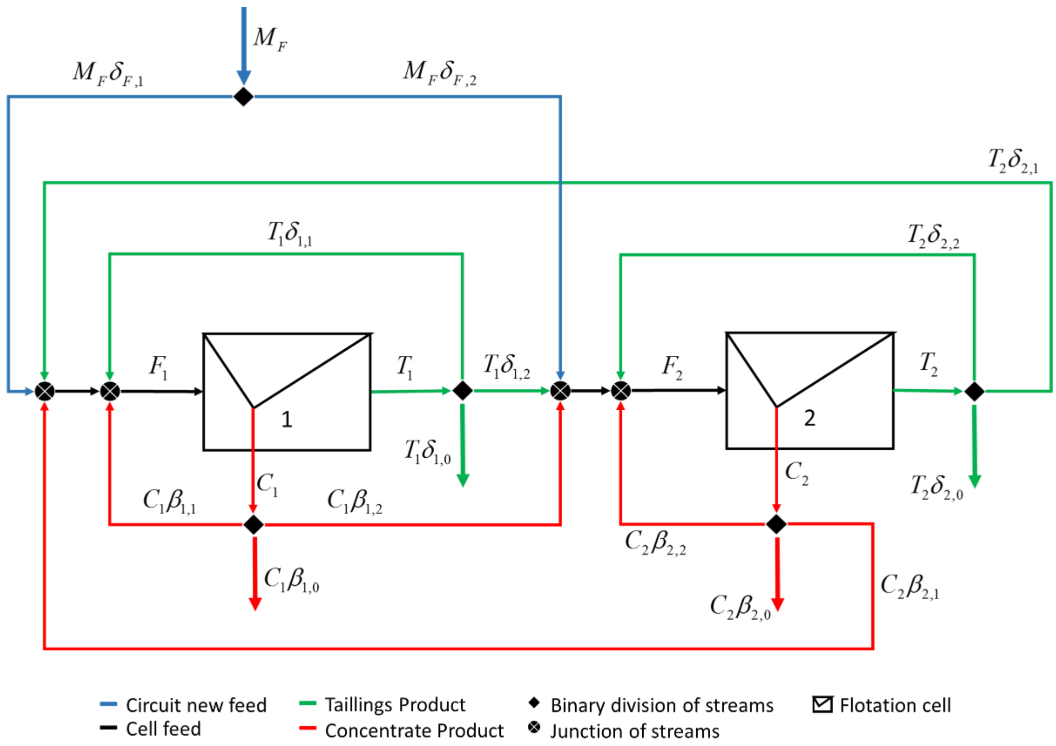

Figure 1 – Generalized structural model for a circuit with two cells. Adapted from [3].

As the froth flotation unit operations represent one of the nodes in the structural model, it is also important to proceed with the mathematical modelling of the unit operation. To keep the problem simple, the linear kinetic model is widely used to describe flotation unit operations [4]. It allows the computation of the mass balance inside a flotation cell, through the assumption that the flow rate of concentrate product (froth) is directly proportional to the total mass that is inside the cell. The proportionality constant is known as flotation kinetic constant (**k_j**), and it is relative to each mineralogical species. It is also assumed that the flotation unit operations work in continuous mode and the pulp is fully mixed j. Therefore it is possible to compute the partition coefficients (**$\alpha_{j,i}$**) relative to the mineralogical class **j** in the tailings product of the cell **I**, as a function of the kinetic constant (**k_j**) and the mean residence time of the particles in the cell (**τ_i**), through the following equation [1]:

$$\alpha_{j,i} = \frac{1}{1 + k_j \tau_i}, \qquad i = 1,2, \dots, n; \; j = 1,2, \dots, m \tag{3}$$

This equation allows to establish a relationship between the tailings product (**$T_{j,i}$**), and the cell's feed (**$F_{j,i}$**), relative to the cell **i** and of the mineralogical class **j**. The relationship is shown through the equation (4):

$$T_{j,i} = F_{j,i}\alpha_{j,i}, \qquad i = 1,2, \dots, n; \; j = 1,2, \dots, m \tag{4}$$

On the other hand, this partition coefficient represents the mass fraction (from the feed) of the mineralogical class **j** that goes to the tailings product. Therefore, by the conservation of mass, the partition coefficient for the concentrate product is given by one (1) minus $\boldsymbol{\alpha_{j,i}}$.

However, the industrial process is usually designed to work with streams with high values of flow. Therefore, it is frequent to make use of series of cells instead of "giant" cells. The generalization of the mathematical model to a bank on **N** cells is done by introducing a modification in the equation (3), and the repartition coefficient of the bank is then given by:

$$\alpha_{j,i} = \left(\frac{1}{1 + k_j \tau_i}\right)^N , \qquad i = 1,2, \dots , n; \ j = 1,2, \dots , m \tag{5}$$

As for solid (mass) balance of the circuit, it revolves around the equation (6), which refers to the assumption of the steady-state of the process:

$$F_{j,i} = C_{j,i} + T_{j,i}, \qquad i = 1,2, \dots , n; j = 1,2, \dots , m \tag{6}$$

In order to simplify the notation, it is introduced the notion of "enrichment" coefficient of the cell **i** [5], of the mineralogical species **j**. This coefficient is given by the equation (7):

$$g_{j,i} = \frac{C_{j,i}}{T_{j,i}} = \frac{1 - \alpha_{j,i}}{\alpha_{j,i}} \ i = 1,2, \dots , n; j = 1,2, \dots , m \tag{7}$$

It is also possible to generalize the enrichment coefficient to a bank of flotation cells through the equation (8):

$$\left(g_{j,i}\right)_{[N]} = \left(\left(1 + g_{j,i}\right)^N - 1\right) \ i = 1,2, \dots , n; j = 1,2, \dots , m \tag{8}$$

Finally, it is possible to present the general equation for the solids balance in a circuit with **n** banks of flotation cells through the following equation:

$$F_{j,i} = M_{F,j}\delta_{F,i} + \sum_{\substack{k=1 \\ k \neq i}}^{n} \left[\left(\left(g_{j,i}\right)_{[N]}\beta_{k,i} + \delta_{k,i}\right) T_{j,k}\right] , \qquad i, k = 1,2, \dots , n; j = 1,2, \dots , m \tag{9}$$

After the establishment of the solids balance, it's possible to deduce the equations that relate the mass balance to the volume of the cells and the percentage of solids by weight of the pulp in the cell **i** ($\mathbf{S_{p,i}}$). This relationship is made clear by the equation (10), where $\mathbf{M_{s,I}}$ refers to the mass of solids inside the cell **i** at a given instant.

$$M_{s,i} = \tau_i \sum_{j=1}^{m} T_{j,i}\,, \qquad i = 1,2,\dots,n;\ j = 1,2,\dots,m \tag{10}$$

Therefore, given the specific mass of the water (**ρ_L**) and the solids from the mineralogical class **j** (**ρ_j**), it is also possible to compute the specific mass of the totality of solids inside the cell **i** (**$\rho_{s,i}$**), through the following equation:

$$\rho_{s,i} = \frac{\sum_{j=1}^{m} T_{j,i}}{\sum_{j=1}^{m} \frac{T_{j,i}}{\rho_j}}, \qquad i = 1,2,\dots,n;\ j = 1,2,\dots,m \tag{11}$$

It is finally possible to express the volume of the cells through the equation (12):

$$V_{P,i} = \rho_L M_{s,i}\left(\frac{100 - \mathrm{S_{p,i}}}{\mathrm{S_{p,i}}} + \frac{\rho_L}{\rho_{s,i}}\right), \qquad i = 1,2,\dots,n \tag{12}$$

Now that the structural and the flotation unit operation models are set, it is important to clarify the concept of multicriteria optimization, which is intimately related with the nature of the objective function. As mentioned before, the grade and recovery are conflicting criteria, which means that it's not possible to hope for the maximum values (100%) for both. Therefore, the basic notion of optimal solution must be abandoned, introducing the concept of "efficient solution" or Pareto optimal solution. A given solution is efficient if the values of its criteria are not dominated by any other solution in the possible solutions set, meaning that if all the solutions that allow the upgrade of one criteria imply exclusively de degradation of the other. This means that in a multicriteria problem with conflicting criteria, there is not an optimal solution but a set of efficient solutions with the same merit in terms of optimality [6].

One of the simplest approaches to this problem is the weighted sum of the criteria, representing a multicriteria problem with a monocriteria objective function (the sum). It is also possible to use penalty based methods like goal programming, where goals are established for each criteria and, if the goals are not achieved, there is a penalty determined by the "distance" between the value of the criteria and the goals. In this case, the objective function consists in the sum of the deviations between each criterion value and its goal. Naturally, the optimization problem will be formulated in order to minimize the sum of the deviations. Finally, it is important to consider that goal programming is not a method to find efficient solutions but it can be if the goals are set to the maximum (or minimum) values of the criteria, as for a grade and recovery of 100% both.

2.2. Problem Resolution

Generally, the application of a genetic algorithm to solve the problem of structural optimization of flotation circuits consists in three main points: synthesis of the best circuit by approximate structural optimization (with the genetic algorithm); determination of optimum values for the volumes of the cells, solving the problem of parametric optimization (subjected to restrictions upon the variables' limits) with a general non-linear optimization method like Sequential Quadratic Programming, Augmented Lagrangian o General Reduced Gradient; finally, computation of the mean residence times through the resolution of non-linear equation systems and computation of the solids balance (linear equations systems).

Genetic algorithms make use of biology, genetics and evolution concepts to find an optimal solution (or as close as possible). This is possible through the implementation of five main operations: generation of an initial population of feasible circuits, evaluation of their fitness, selection, crossover (mating) and mutation. This operations “guide” the initial population through the evolutionary process, producing fitter solutions as the number of generations increases. Therefore, it is possible for the algorithm to obtain optimal solutions for the problem, as long as it iterates for a sufficient number of generations [7].

3. Methodology

It was possible to obtain the evaluation of the fitness (objective function) value for every configuration of the circuits with 2, 3 and 4 cells. This analysis allowed to obtain the exact optimal solution for both cases of objective function, which was used to compare the genetic algorithm results.

The implementation of the genetic algorithm incorporated the five main operations mentioned above, being tested for both kinds of objective function (weighted sum of the criteria and goal programming), 30 times each for different numbers of configurations for the initial population and of generations (iterations). Both 3 cell and 4 cell circuits were tested under this circumstances, being the 2 cell circuit left out because the number of feasible circuits was too low (8). The 3 cell circuit was tested for 10, 20 and 30 initial configurations and the 4 cell circuit for 20, 30 and 40. The influence of the number of generations (iterations) was studied using a stopping criteria based on the number of iterations on which the value of the best fitness did not change. Therefore, the genetic algorithms were tested with two different stopping criteria: stop when the best solution does not change for 3 and 7 repeated iterations. Therefore, the algorithm was tested for both stopping criteria, every initial population number, both circuits and both multicriteria objective functions

3.1. Generation of the Initial Population

The codification of the configuration followed a successive display of rows of the adjacency matrices into a line vector. As an example, the equation (13) shows the method of codification of the adjacency matrices into a vector, for a circuit with **n** cells.

$$\begin{gathered} v(n) = [\beta_{1,i} \quad \beta_{2,i} \quad \cdots \quad \beta_{n,i} \quad \delta_{1,i} \quad \delta_{2,i} \quad \cdots \quad \delta_{n,i} \quad \delta_F], \qquad i = 0,1,2 \dots n \\ com: \beta_{1,i} = [\ \beta_{1,0} \quad \beta_{1,1} \quad \cdots \quad \beta_{1,n}], \qquad (analogous\ for\ all) \\ \dim(v) = [2 * n * (n+1)] + n \end{gathered} \tag{13}$$

Process-based rules were considered in the generation of the initial population in order to restrain the search space. These rules were the following: each cell has to have at least one feed, concentrate and tailings stream; It is not allowed stream splitting in any of the streams (purely binary); The self recirculations are eliminated; Both the products (concentrate and tailings) from a given cell cannot flow simultaneously into only one cell; The circuit has to have, at least, one final concentrate and one final tailings; The cell that receives new feed has to interact with as many cells as the necessary to avoid short-circuiting; If there is only a final concentrate, then the cell that produces it cannot be fed exclusively by concentrate products.

3.2. Fitness Evaluation

The evaluation of the fitness of each configuration in the population included the parametric optimization task, as well as the computation of the solids balance, outputting a value of the chosen objective function. The parametric optimization problem was solved using an implementation of a Sequential Quadratic Programming[1] and the non-linear equation systems using a direct substitution method, as it converged (generally) quite fast. It is important to note that the goal programming objective function comprises a minimization problem while the weighted sum comprised a maximization problem. The values of the weights used were 0.5/0.5 and the goals 100/100 for the criteria grade/recovery.

3.3. Selection

The selection operation was based on the Tournament Selection mode (suggested by [7]). This operation aims the selection of individuals for the crossover/mutation operations. It consists in the choice of a given number of configurations and compare their fitness function and, just like in a competition tournament, the winner (selected) is the one who performs best in terms of fitness function values.

3.4. Crossover

As an analogy to the mating process between lining beings, the crossover operation is takes place after the selection of some of the most fit individuals and its main purpose is to force them to share “genes”, meaning some features of their configuration (exchange of adjacency matrices lines between configurations. This method produces two children (clones of each parent) and is based on a probability of a given block (row) of the coded vector for the adjacency matrices, to be swapped between children. The intention is to make good configuration share “good” features with each other, in the search for the improvement of the children’s “genome”.

3.5. Mutation

The mutation operation is very similar to the crossover but, instead of swapping characteristics between induvial, is own characteristics are swapped, meaning that there is a probability for a given characteristic

[1] Authorship of Dr. Fernando de Oliveira Durão

(block of the adjacency matrix) of an individual, to be chosen and swapped with another of its characteristics.

3.6. Results and Discussion

The evaluation of every configuration allowed to understand that, even though the generation of configurations was highly restricted, there is a possibility of reducing the amount of feasible configurations of circuits with **n** cells, by a factor of **n!**.This is related with the fact that for every circuit with a different number of cells, there are **n!** equivalent configurations because the order of the cells do not matter. The exact optimal circuits obtained represent precisely the classical type of flotation circuit, known as roughening-scavenge-cleaning circuit.

Generally, the results from the genetic algorithm application show that, for the 3 cell circuit, the increase of initial population does improve de efficacy in the search for the optimal solution, while a higher number of generations does nor demonstrate such high influence. However, for the circuit with 4 cells, it is the contrary. A higher number of generations do promote an increase in the efficacy of the algorithm in terms of the solutions produced. This can be related with the difference between the dimension of the feasible population of configurations when compared to the dimension of the initial population. While the circuit with 3 cells has 276 feasible configurations, the circuit with 4 cells has 26964. So, when values like 10, 20, 30 or 40 are compared to 276, they are not that low, but when compared to 26964, they are really low, therefore it’s plausible for the initial population to have a small influence on the 4 cell circuit.

There is also an apparent difference between the efficacy of the genetic algorithm that comprises the goal programming method and the one that comprises the weighted sum. However, the obtained results do not suggest any clear reason for this disparity.

Finally, it is important to refer that the performance of the genetic algorithm was remarkable, especially because it was able to find optimal (or very close) solutions in a matter of minutes. Therefore, it is possible to claim that this methodology, based on the restriction of the space of feasible configurations by topologic/empiric process-based rules, shows a great potential on the resolution of this problem. On the other hand, the robustness and efficiency shown make clear why this kind of algorithms have been revisited so many times.

3.7. Final Considerations

The main purpose of this work was the development of a genetic algorithm in MATLAB®, in order to solve the problem of structural optimization of circuits, using a topologic/empiric process-base rules methodology to restrain the space of feasible configurations.

The evaluation of all of the configurations allowed to understand that it is possible to go further in the restriction of the feasible circuits space. It was also possible to attain that the exact optimal configurations obtained represented precisely the classic circuit of roughening-scavenge-cleaning.

The genetic algorithm solved this problem of structural optimization of flotation circuits, showing a remarkable robustness (for an approximate method) in obtaining the optimal (or very close) solutions and in the sort computation time verified.

A more detailed study about the topology and/or more empirical rules to be included in the process can reduce significantly the search space. On the other hand, it is suggested a detailed study of the influence of the type of multicriteria objective function given the disparities observed between methods that should, by principle, be equivalent (the way they were applied).

Finally, this problem is not exclusively associated with genetic algorithms. Therefore, it could be interesting to try another approaches in the field of metaheuristics. Methods like simulated annealing or ant colony system could be used to solve this combinatorial hard problem.

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