Control System for a Vehicle Exhaust Waste Heat Recovery System
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Abstract

Most motor vehicles currently operate with an internal combustion engine (ICE), responsible for the emission of polluting substances which directly influence the chemical composition of the air. The waste heat recovery (WHR) system, for the thermal energy contained in the exhaust gases through an Organic Rankine cycle (ORC), is a promising approach to reduce fuel consumption as well as exhaust emissions from the engines of motor vehicles.

In this work, the $\mathcal{H}_\infty$ and $\mathcal{H}_2$ control strategies were developed with the goal of rejecting any effects triggered by disturbances in the thermal energy available for the ORC, commonly due to variations of mass flow and exhaust gas temperature. Thus, control is essential to ensure a stable performance of the system and to avoid the deterioration of the same.

The controllers developed in this work are able to reject some of the effects caused by small amplitude disturbances. Moreover, due to a superior performance $\mathcal{H}_\infty$ control has proven to be more suitable than $\mathcal{H}_2$ control.

Keywords: $\mathcal{H}_\infty$ control, $\mathcal{H}_2$ control, Organic Rankine Cycle, Thermodynamic Model Identification, Waste Heat Recovery

1 Introduction

Most motor vehicles currently operate with an internal combustion engine, responsible for the emission of polluting substances and directly influencing the chemical composition of the surrounding air. The WHR system through an ORC, for the thermal energy contained in the exhaust gases, is a promising approach to reduce fuel consumption as well as exhaust emissions from the engines of motor vehicles [1].

Fig. 1 is the operation scheme of the ORC modeled by [2]. In this context, the used ORC dynamic model comprises only the dynamic modeling of the evaporator. The remaining components are represented by steady state models. This option is justified, in the case of the Expander and the bomb, by the notion that any angular momentum variations of these are negligible given the dynamic that occurs in the evaporator, considered as the dominant dynamic [3]. The condenser has also a stationary design, once intended to focus on the influence that the dynamics of heat source exerts on the Rankine cycle.

The template in analysis can be seen as a system having a single controlled input with multiple output variables, presented in Table 2. As the exhaust gases are the heat source, its characterization (composition, variations of mass flow and temperature as a function of time) are important to better understand any disturbances in this model. The conditions considered for exhaust gases are listed in Table 1.
The working fluid used in the ORC modeling was the organic fluid R245fa. This fluid is commonly referred to as the most viable option for this type of cycles for the application in question [5]. The R245fa fluid presents a lower thermal inertia when compared to water, an advantage that assumes a greater importance when considering control implementation within the cycle.

2 Linear Approximation of the Model

For the development of the controllers it was necessary to linearize the ORC’s numeric model presented in the previous section. Table 2 presents the variables considered in the development of the linear model of the ORC herein presented. The model has three input variables, and while one is manipulated the remaining two are disturbances. Furthermore, seven output variables were taken into account for the design of the cycle:

For the model linearization herein performed, the working fluid flow is maintained identical to the inlet and outlet of the evaporator. Such simplification was necessary because 1) it has not been possible to identify the isolated behavior for each variable and 2) any other attempt resulted in errors in the numerical nonlinear model. The operating condition around which the model was linearized was number 3 in Table 1 as suggested in [2]. Step variations were...
applied to each of the input variables individually, keeping the remaining ones in their initial values and corresponding to the 3 operating conditions (see Table 3).

<table>
<thead>
<tr>
<th>Input</th>
<th>Initial value</th>
<th>Step</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \dot{m}_g )</td>
<td>g/s</td>
<td>21</td>
</tr>
<tr>
<td>( T_{g, in} )</td>
<td>K</td>
<td>829.7</td>
</tr>
<tr>
<td>( \dot{m}_f )</td>
<td>g/s</td>
<td>40</td>
</tr>
</tbody>
</table>

As mentioned above, throughout the identification of system behavior, this underwent a step in an input and disturbance at a time, keeping the remaining ones their nominal values for the 3rd operating condition. The step is applied in the 50th second of the numerical model simulation in order to stabilize it before the application of the step.

Each of the variations was fitted by a linear model of first or second order with or without zero (1).

\[
\begin{align*}
'P1Z' &= \frac{K(1 + T_z s)}{1 + T_{p1} s} \\
'P1' &= \frac{K}{1 + T_{p1} s} \\
'P2Z' &= \frac{K(1 + T_z s)}{(1 + 2 \zeta T_w s + (T_w s)^2)} \\
'P2' &= \frac{K}{(1 + 2 \zeta T_w s + (T_w s)^2)}
\end{align*}
\]

where \( K \) is the proportional gain, \( \zeta \) the damping constant and \( T_w \) the time constant of the pair of complex conjugate poles. \( T_z \) is the time constant of the zero and \( T_{p1} \) is the time constant for the real pole.

The estimated parameters for models (1), used in the linearization of the numerical model, are obtained using the `procest()` function of MATLAB for a coarse approximation, and then by adjusting the curve through the use of `fminsearch()` function, evaluated through the following performance indices:

\[
\text{VAF} = 1 - \frac{\sigma^2(y - \hat{y})}{\sigma^2(y)} \\
\text{EE} = 100 \times \frac{\hat{y}_{t \to \infty} - y_{t \to \infty}}{|y_{t \to \infty}|}
\]

The following table presents the initial study from which the appropriate template for every curve was chosen, having also considered the selected entry that was subject to a variation in step. Average VAF was chosen as a performance index for the steps considered in each of the three input variables (see Table 3). The best performances (in bold) will later be used for the linear model.
2.1 Linear Model Performance

The linear model that approximates the nonlinear model around the 3rd operating condition was obtained through an average of the identified linear models. The evaluation of the linear model (Table 5) was obtained by comparison with the nonlinear model, during the various steps applied on the input variables.

<table>
<thead>
<tr>
<th>Input</th>
<th>Model</th>
<th>( P_{\text{evap}} )</th>
<th>( T_{\text{evap}} )</th>
<th>( \Delta T_{\text{sob}} )</th>
<th>Eff_1</th>
<th>Eff_2</th>
<th>( W_{\text{net}} )</th>
<th>( T_{g,\text{out}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_g )</td>
<td>P1</td>
<td>0.9955</td>
<td>0.9964</td>
<td>0.9964</td>
<td>0.9939</td>
<td>0.8769</td>
<td>0.9961</td>
<td>0.9940</td>
</tr>
<tr>
<td></td>
<td>P1Z</td>
<td>0.9960</td>
<td>0.9964</td>
<td>0.9961</td>
<td>0.9940</td>
<td>0.9876</td>
<td>0.9965</td>
<td><strong>0.9941</strong></td>
</tr>
<tr>
<td></td>
<td>P2</td>
<td>0.1816</td>
<td>0.9983</td>
<td>0.9977</td>
<td>0.9950</td>
<td>0.9118</td>
<td>0.9984</td>
<td>0.9940</td>
</tr>
<tr>
<td></td>
<td>P2Z</td>
<td><strong>0.9989</strong></td>
<td><strong>0.9987</strong></td>
<td><strong>0.9978</strong></td>
<td><strong>0.9951</strong></td>
<td>0.9891</td>
<td><strong>0.9995</strong></td>
<td>0.9941</td>
</tr>
<tr>
<td>( T_{g,\text{in}} )</td>
<td>P1</td>
<td>0.9945</td>
<td>0.9983</td>
<td>0.9984</td>
<td>0.8562</td>
<td>0.8452</td>
<td>0.9962</td>
<td>0.9938</td>
</tr>
<tr>
<td></td>
<td>P1Z</td>
<td>0.9954</td>
<td>0.9982</td>
<td>0.9982</td>
<td><strong>0.9607</strong></td>
<td><strong>0.9887</strong></td>
<td>0.9967</td>
<td>0.9983</td>
</tr>
<tr>
<td></td>
<td>P2</td>
<td>0.9981</td>
<td>0.9988</td>
<td>0.9983</td>
<td>0.8913</td>
<td>0.8535</td>
<td>0.9986</td>
<td>0.9961</td>
</tr>
<tr>
<td></td>
<td>P2Z</td>
<td><strong>0.9992</strong></td>
<td><strong>0.9993</strong></td>
<td><strong>0.9987</strong></td>
<td><strong>0.9607</strong></td>
<td>0.9887</td>
<td><strong>0.9996</strong></td>
<td><strong>0.9997</strong></td>
</tr>
<tr>
<td>( m_f )</td>
<td>P1</td>
<td>0.9962</td>
<td>0.9876</td>
<td>0.9836</td>
<td>0.9883</td>
<td>0.9488</td>
<td>0.9681</td>
<td>0.9887</td>
</tr>
<tr>
<td></td>
<td>P1Z</td>
<td>0.9960</td>
<td>0.9981</td>
<td>0.9942</td>
<td>0.9883</td>
<td>0.9732</td>
<td>0.9952</td>
<td>0.9887</td>
</tr>
<tr>
<td></td>
<td>P2</td>
<td>0.9964</td>
<td>0.9900</td>
<td>0.9807</td>
<td>0.9881</td>
<td>0.9524</td>
<td>0.9842</td>
<td>0.9886</td>
</tr>
<tr>
<td></td>
<td>P2Z</td>
<td><strong>0.9968</strong></td>
<td><strong>0.9990</strong></td>
<td><strong>0.9957</strong></td>
<td><strong>0.9884</strong></td>
<td><strong>0.9873</strong></td>
<td><strong>0.9957</strong></td>
<td><strong>0.9887</strong></td>
</tr>
</tbody>
</table>

By analyzing the above table, it was possible to observe that the linear model developed shows a high level of performance for the desired application. In any of the evaluated cases, the EE never exceeded 10% and had a mean absolute value equal to 1.7, ensuring a good approximation of the model in the steady state. The VAF had an average value of 0.9789, which assures a good approximation between the dynamic and stationary regimes. It was concluded that the linear model reproduces the nonlinear model with the accuracy. The linearization range was also validated.

2.2 Model Reduction

The order reduction in the linear model was achieved through the implementation of the following methods: the singular values of Hankel through the function `balancecmr()` and the conservation of the steady gain using the function `balred()`. In Table 6, various models are presented and sorted through the infinite norm of the standard error between the models. The selected reduced model is the 24th order model obtained through the method of Hankel. This model was chosen because it preserves the abilities of full-order model while being, however, a reasonable reduction of the same. In Fig. 2 it is possible to compare the singular values of the two models depending on the frequency [6].
3 \( \mathcal{H}_\infty \) and \( \mathcal{H}_2 \) Control

3.1 \( \mathcal{H}_\infty \) and \( \mathcal{H}_2 \) Norms

\( \mathcal{H}_\infty \) and \( \mathcal{H}_2 \) norms are used to quantify the \( G(s) \) system size. Usually, we mean a positive, scalar number that is a measure of the size of \( G(s) \) over all points in the complex \( s \)-plane [7].

The \( \mathcal{H}_\infty \) norm of \( G(s) \), denoted \( \| G(s) \|_\infty \), is defined as

\[
\| G \|_\infty = \sup \sigma_{\max}[G(j\omega)]
\]

and it classifies the greatest increase in energy that can occur between the input and output of a given system.

The \( \mathcal{H}_2 \) norm of \( G(s) \), denoted \( \| G(s) \|_2 \), is defined as

\[
\| G \|_2 = \left( \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{trace}[G(j\omega)G^*(j\omega)]d\omega \right)^{\frac{1}{2}} = \left( \frac{1}{2\pi} \int_{-\infty}^{\infty} \sum_{i=1}^{r} \sigma_i^2[G(j\omega)]d\omega \right)^{\frac{1}{2}}
\]

and gives a precise measure of the power or signal strength of the output of a system driven with unit intensity white noise.

3.2 Control Strategy

The main goal of the controller is to minimize the influence of disturbances induced by the heat source, in order to ensure a high yield of ORC and to prevent deterioration of its mechanical components. To dismiss the effects of disturbances, reference values for temperature and evaporating pressure were defined based on stationary conditions of the ORC.

3.3 Closed-Loop Controller Design Specifications

The controller design is carried out taking into account the linearized model, which considers the following variables:
\[ y = \begin{bmatrix} P_{\text{evap}} \\ T_{\text{evap}} \end{bmatrix}, \quad u = [\dot{m}_f] \] (4)

The process can then be represented by:

\[ G = \begin{bmatrix} G_w & G_u \end{bmatrix} \quad y = G_w w + G_u u \] (5)

where \( G_w(s) \) are the system transfer functions in relation to disturbances and \( G_u(s) \) is the array of transfer functions corresponding to the control signals. The block diagram of the open-loop system is presented in Fig. 3, where are the interconnections between the system \( G(s) \) and the weighting functions \( W_p(s) \) e \( W_u(s) \) that will shape the transfer function of the closed-loop \( T_{zw}(s) \). \( \mathcal{H} \) control has the goal to minimize the functions presented in (6). \( T_{zw}(s) \) is the transfer function in closed-loop that relates the disturbances \( w \) with outputs to minimize \( z \).

![Fig. 3 – Block-diagram of the open-loop system with performance specifications](image)

The system has two types of inputs: references \( r \) and disturbances \( w \), and two output signals \((e_u e_y)\). The transfer function \( G(s) \) represents the linearized ORC. The weighting functions \( W_u \) and \( W_p \) are used to define the relative importance of this signal to the frequency ranges required on system performance. Thus, the objective of system performance can be reshaped with the possibility of a more conservative approach:

\[ T_{zw}(s) = \begin{bmatrix} W_p S_o G_u K & W_p S_o G_w \\ W_u S_i K & -W_u K S_o G_w \end{bmatrix} \] (6)

where \( S_i = (I + KG)^{-1}, \quad S_o = (I + GK)^{-1} \) are the sensitivities of input and output, respectively. In (6) is exploited the fact that \( S_i K = K S_o \). This formulation is similar to that of a typical problem of mixed S/KS sensitivity optimization and takes into account the criteria of stability and performance. The problem formulation is reflected in the minimization of the four functions listed in Table 7.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_p S_o G_u K )</td>
<td>Weighted closed-loop system sensitivity</td>
</tr>
<tr>
<td>( W_p S_o G_w )</td>
<td>Weighted disturbance sensitivity</td>
</tr>
<tr>
<td>( W_u S_i K )</td>
<td>Weighted control effort due to reference</td>
</tr>
<tr>
<td>(-W_u K S_o G_w )</td>
<td>Weighted control effort due to disturbances</td>
</tr>
</tbody>
</table>
The closed-loop system would be able to achieve the intended performance if condition (4) were satisfied, i.e. $\|T_{zw}(s)\|_{\infty}$ for $H_\infty$ control and $\|T_{zw}(s)\|_{2}$ for $H_2$ control and with the considered rule always depending on the required control.

$$\begin{bmatrix} W_p S_o G_u K & W_p S_o G_w \\ W_u S_1 K & -W_u KS_o G_w \end{bmatrix} < 1$$  \hspace{1cm} (7)

### 3.4 Performance Functions

The weighting functions $W_u$ and $W_p$, that guaranteed the best performance, were found using the meta-heuristics Particle Swarm Optimization (PSO) developed by Biswas [8]. Equations (8) are the functions of performance used in the design of controllers,

$$W_p = \begin{bmatrix} 1 \times 10^{-16}(s + 1 \times 10^5) \\ s + 1 \\ 0 \\ 7.455 \times 10^{-7}(s + 10) \\ s + 1 \times 10^{-5} \end{bmatrix}, \quad W_u = \frac{10(s + 1 \times 10^{-5})}{s + 1} \hspace{1cm} (8)$$

### 4 Results with the Nonlinear Model

In this subsection, the performance of controllers with the nonlinear model were evaluated. It was also presented the PI controller performance developed by [2] as a basis for comparison with the controllers developed in this work. In Fig. 5 is depicted the control action for all three controllers.

In Fig. 6, it is possible to observe the response of the nonlinear model for all the three controllers. The behavior of the $H_\infty$ and $H_2$ controllers differ from that presented in the linear model, with $H_\infty$ control rejecting the disturbance effects on evaporating pressure. As the evaporating temperature has a steady error of 3 K, the variations in the temperature of evaporation are within the acceptable limits and do not represent a risk for the physical system.
Furthermore, the performance of controllers in the nonlinear model subject to the variations in the exhaust gas conditions higher than those used in the linearization of the model was also assessed. The $\mathcal{H}_2$ controller presented an oscillatory behavior that led the nonlinear model to become unstable. Therefore, this controller has been discarded from the following analysis because it is not valid. Fig. 7 shows the perturbation imposed on the system that represents a successive step variation to the conditions of operation $\{3, 2, 3, 4, 5, 6, 3, 4, 5, 8, 9, 10, 7, 8, 9, 12, 13\}$.

Fig. 7 – Disturbance input

Fig. 8 – Control action for input in Fig. 7
Fig. 9 shows that the evaporation pressure of the nonlinear model controlled with $\mathcal{H}_\infty$ control remains almost constant. The evaporating temperature decreases with increased mass flow and exhaust gas temperature, suffering a deviation of more than 20 K. Therefore, it is expected that this performance must be lower because the amplitude of these disorders is far beyond the range of validity of the linearization.

![Fig. 9 – Controller comparison for input in Fig. 7](image)

5 Conclusions

This work regards the development of $\mathcal{H}_\infty$ e $\mathcal{H}_2$ control strategies in order to reject the effects caused by the variation of heat energy available in the heat of an ORC used as WHR system in motor vehicles. The referred energy variations are due to the variation of flow and exhaust gas temperature, from where thermal energy is recovered.

The study was conducted using the numerical model of an ORC developed by Elias [2]. During the development of the controllers a linearization of the numerical model was needed for performance purposes. After the identification and linearization of the model by selecting the best model through the performance indices VAF and EE and by comparison of the linear model with a nonlinear model. The linear model was validated for various scenarios of variation of the conditions of the exhaust gases having these variations been applied in different amplitudes step flow and exhaust gas temperature. Comparing the linear model and nonlinear model, VAF presents a mean value equal to 0.9789 and EE presents an mean absolute value equal to 1.7 ensuring a good approximation of the dynamics of the models and the stationary value. It was concluded that the linear model presents an acceptable performance for the intended study by also validating the linearization range chosen.

In the non-linear model, two sets of disturbances were assessed: the first, identical to the studied for the linear model, by comparing the performances in both templates, and, in the
second set, by applying successive variations in the exhaust conditions, thus surpassing the variations used in the linearization of the model. In the analysis of the performance of controllers in the linear model is also presented the PI controller developed by [2] as a basis for comparison.

The controllers herein developed showed a good rejection of the effects caused by small amplitude disturbances, that is, within the limit of the linear model. When subjected to disturbances of higher amplitudes, as in the case of nonlinear model, the performance is lower than even for in the case of $\mathcal{H}_2$ controller to unsettle the system. However, for the $\mathcal{H}_\infty$ control the variations in pressure and temperature of evaporation are within acceptable limits and do not represent a risk to the physical system.

Reference


