# Formulation of a Lumped-Parameter Thermal Model for an Induction Electric Machine with a Spherical Rotor

João Miguel Lança Martinho, Instituto Superior Técnico

Abstract— This master thesis development contributes for the study and development of a new topology for the spherical electromagnetic converters, taking as advantage the extra degrees of freedom that a spherical geometry allows when compared to the conventional solutions.

The lumped-parameter thermal model for an induction electric machine with a spherical rotor allows us to analyse the effect of the spherical geometry in the operating temperature that constitute a limiting factor in the operation of the electromagnetic converters. The rise in temperature can lead to a change in material properties, therefore it is critical knowing the machine operating limits in terms of current or current density in which the machine operates as expected. The lumped-parameter thermal model allows us to know with great accuracy the temperature achieved in all the materials that constitute the machine. This model is validated through a finite element analysis.

With this thermal model it is possible to optimize the dimensions and properties of the machine for all the desired scenarios, allowing a deeper study of the geometry implications in temperature and in machine operation.

In the end of this work a tool that compiles all steps of the process, and that is easy to upgrade the formulas used is going to be developed, allowing the easier use of the model to anyone that wishes to keep the study of this geometry.

Keywords— Electrical machines, Multi-degree of freedom actuator, Spherical induction motor, Thermal model, Lumped-Parameter thermal model, Thermal analysis.

#### I. INTRODUCTION

THIS paper aims to describe the work develop in the thermal analysis of an electrical machine.

Thermal models are important tools to determine the temperatures reached by the machine in several conditions. Temperature represents the major limitation factor for the machine operation, due to the power losses machine temperature rises and therefore a model capable of representing that machine operation can be extremely useful to determine the nominal power of the machine. The losses are dissipated by the machine to the exterior environment following a gradient of temperature.

Materials can change their properties when the temperature rises so it is mandatory that the machine operates only inside the interval where the materials changes do not compromise the functionality of the machine. The Joule power losses are the main source of all power losses and inside a machine the critical point in terms of temperature is in the copper insulation witch is classified according to its maximum temperature.

The lumped-parameter thermal model allows us to achieve with great accuracy for all the problems that have a possible analytical solution. The main advantage of lumped-parameter models in comparison with finite element analysis is that lumped-parameter models require a lot less computational power with results not as accurate as the finite element analysis but accurate enough for its purpose. The analytic analysis of this problems gives us lots of information about all the physics in this process.

#### II. HEAT TRANSFER

The heat behavior is similar to the current behavior in an electric circuit. It always goes for the path with less resistance, and in this work we will assume that the external environment is able to absorb all the heat without changing its own temperature.

In order to study the thermal behavior of the machine an electrical to thermal correspondence is imperative.

TABLE I
ELECTRICAL AND THERMAL CORRESPONDENCE

Thermal Parameters	Electrical Parameters
Power Losses	Current
Temperature	Voltage
Thermal Resistance	Electric Resistance
Thermal Capacitance	Electric Capacitance

Conductive heat transfer normally occurs in solids being neglected for gases except when the gap is very small. Convective heat transfer normally occurs in liquids and gases being neglected in solids. And the heat transfer by radiation is neglected in this scenario because it has very low values when compared with the other two types of heat transfer.

# III. LUMPED-PARAMETER THERMAL ANALYSIS

The type of machines being studied have a spherical geometry and as some of its layer might not be completely spherical or a spherical segment, those layers have to be simplified in order that the model can give proper results.

#### A. Thermal Conduction Resistances

The thermal resistance  $R_{cond}$  models the flow of heat P between parts (nodes) of the machine, being proportional to the temperature difference  $\Delta T$  between the two nodes [1]:

$$R_{cond} = \frac{\Delta T}{P} \quad \text{K/W} \tag{1}$$

Due to the fact that the machine has a spherical geometry it is possible to define all resistances as spherical segments. Assuming that z is the cosines' part of the projection of the arc that describes that layer in the xx axis, conductive resistance can all be modelled by:

$$R_{cond} = \frac{1}{z \pi K} \left( \frac{1}{r_i} - \frac{1}{r_2} \right) \tag{2}$$

In which K represents the thermal conductivity and r the radius of the inner and outer part of the layer.

#### B. Thermal Convection Resistances

Newton's cooling law [2] defines that heat transfer in fluids is given by:

$$P = h \cdot A \cdot \Delta T \tag{3}$$

With A being the area and h the convective heat transfer coefficient.

The heat convection is not a linear phenomenon because it depends on the type of flux and the properties of the fluid that normally change a lot with temperature. The convective heat transfer coefficient is given by:

$$h = \frac{K \cdot Nu}{d} \tag{4}$$

Nu representing the Nusselt number and the convective heat transfer is given:

$$R_{conv} = \frac{1}{h \cdot A} \quad \text{K/W} \tag{5}$$

Prandtl (6), Grashof (7), Reyleigh (8) and Reynolds (9) numbers [3] are crucial for the calculation of all the convective type that exists, but in this case it will only be addressed the natural and forced laminar flow.

$$P_{r} = \frac{\mu \cdot c}{K} \tag{6}$$

$$G_r = \frac{d^3 \cdot \rho^2 \cdot \Delta T \cdot \beta \cdot g}{\mu^2} \tag{7}$$

$$R_a = P_r \cdot G_r \tag{8}$$

$$R_e = \frac{\rho \cdot v \cdot d}{u} \tag{9}$$

Where  $\mu$  is the dynamic viscosity, c the specific thermal capacity, d the distance,  $\rho$  the density,  $\beta$  the thermal expansion coefficient, g the gravity acceleration and v the speed of the border wall.

To be able to know exactly what are the convective

resistances for each type, it is needed to know the Nusselt number for each case and therefore it is possible to know all the convective resistances.

The natural convection in the air-gap [4] and stator [3] is given by (10) and (11):

$$N_u = 2 + 0.14 R_a^{1/3} \tag{10}$$

$$N_{u} = 2 + \frac{\left[0.589 \left(P_{r} G_{r}\right)^{1/4}\right]}{\left[1 + \left(\frac{0.469}{P_{r}}\right)^{9/16}\right]^{4/9}}$$
(11)

The forced convection is given by the next Nusselt number [5]:

$$N_u = 2 + \left(0.4R_e^{1/2} + 0.06R_e^{2/3}\right)P_r^{0.4} \tag{12}$$

# C. Thermal Capacitance

The thermal capacitance C represents the ability of a node to absorb heat for a given change in temperature. It is a function of the mass m and its heat capacity c [6]:

$$C = m \cdot c \quad \text{K/J} \tag{13}$$

#### D. Heat Sources

The major heat sources are the losses by Joule effect that appear in the copper and aluminum sections of the machine and with their specific geometry they are represented by formula (14):

$$P = \frac{2\pi}{3} z \cdot \rho \cdot \left[ r_2^3 - r_1^3 \right] \cdot \left( \frac{J}{\sqrt{2}} \right)^2$$
 (14)

where J is the current density.

#### E. Slots

The machine can have the presence of slots and for the matter of helping the simplification of the layers affected by the slots to their spherical segment geometries the total angular length of the slots is:

$$\delta_{S-Total} = \frac{\delta_S}{\frac{\pi}{2 \cdot n_{pp}}} \tag{15}$$

where  $\delta_s$  is the angular length of each slot and  $n_{pp}$  is number of pairs of poles in the machine.

# F. Stator Angle

The stator angle  $\theta_e$  allows this model to be able to compute the thermal analysis for a shell-like stator. This angle has a direct influence in the variable z on the formulas behind. It is calculated as:

$$z = 1 - \sin\left(\theta_e\right) \tag{16}$$

#### G. Lumped-Circuit Model

A lumped-circuit model is a thermal equivalent circuit witch consists of thermal resistances and capacitances. In steady-state analysis all capacitances can be neglected but for transient analysis it is imperative that the capacitances are present

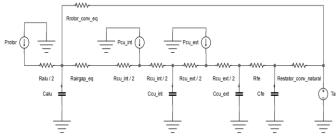


Fig. 1. Lumped-parameter thermal model.

João Miguel Lança Martinho is concluding the Master degree in Electrical and Computer Engineering in Instituto Superior Técnico, with the thesis "Formulation of a Lumped-Parameter Thermal Model for an Induction Electric Machine with a Spherical Rotor".

### IV. RESULTS

The lumped-parameter thermal model created have been tested for validation against a finite element tool and the maximum absolute error has been under 5% for several scenarios for both steady-state and transient analysis. Because of that the model displays a good accuracy and is a good tool to analyse the thermal behavior of spherical machines.

#### V. CONCLUSION

The thermal model has a very important role in the development and optimizing processes, but a finite element analysis requires big computational power and time, so in order for a quick but still precise way of doing the thermal analysis the lumped-parameter thermal model is the model to take into considerations, as it is fast, does not require many computational power and is still very accurate.

This thermal model takes into consideration several scenarios and therefore allows for different materials, angles and configurations allowing a great number of cases in which this thermal model gives good and important results.

There is not a lot of information about this type of machines and this works comes to help filling that gap in the spherical rotor electrical machines

# VI. REFERENCES

- [1] Fernandes, João F. P.: "Design Concept, Development and Prototyping of a Shell-like Spherical Induction Electrical Machine", Instituto Superior Técnico, June 2015, In development.
- [2] Holfman, J. P., "Heat Transfer", McGraw.Hill Series in Mechanical Engineering, 10th Edition, McGraw-Hill, 1997.
- [3] Martynenko, Oleg G.; Kharmtsov, Pavel P.: "Free-Convective Heat Transfer, With Many Photographs of Flows and Heat Exchange", IX edition, Springer, 2005.
- [4] Barelko, V. V.; Shtessel, A: "Heat transmission by natural convection in cylindrical and spherical interlayers", Institute of Chemical Physics, Academy of Sciences of the USSR, Moscow, January, 1973.
- [5] Dandy, David S.; Dwyer, Harry A.: "A sphere in shear flow at finite Reynolds number: effect of shear on particle lift, drag, and heat transfer", Journal of Fluid Mechanics, Cambridge University Press, 1990.
- [6] Soong, W. L.: "Thermal Analysis of Electrical Machines: Lumped-Circuit, FE Analysis and Testing", University of Adelaide, Australia, May 2008.