Helmholtz resonator with multi-perforated plate

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November 2016

Abstract

The present work focuses on evaluating the acoustic behavior of a Helmholtz resonator when a thin metal plate, with small perforations, is introduced on the top. The basic geometry of the Helmholtz resonator, which consists of a volume of air connected to a small neck, can be found in many different applications, examples are such as a car with an open window, fake ceilings, water bottles, industrial burners present in furnaces and heaters, etc. Due to the presence of a plate with small perforations on the neck, an extension to the acoustic theory concerning Helmholtz resonators is proposed. In the mathematical model, the plate on the top of the resonator acts as an additional impedance in the system. It was observed that the resonance frequency, when the plate is introduced, decreases according to the characteristics of the plate, such as the porosity and perforation ratio. The diameter of the necks ranged from $7 - 25.4\ mm$, and the volume of the cavity of the resonators ranged from $90 - 250 * 10^{-6}\ m^3$. The plates have a thickness of $0.5\ mm$ with varying geometry configurations for the perforated holes, with hole diameters ranging from $0.6 - 6\ mm$ and hole spacing from $1.5 - 5\ mm$. The results demonstrate that the mathematical model predicts the experimental results.

Keywords: Helmholtz, resonator, perforated, plates, acoustics

1. Introduction

Helmholtz resonators have been used for more than 100 years in many different applications, but mainly to effectively reduce high wavelength sounds, i.e. attenuate the narrow-band low-frequency noise. The classic lumped theory approximates an Helmholtz resonator to a mass-spring-damper system, and yields the expressions for the resonator frequency and the transmission loss [1],[2],[3],[4]. The Helmholtz resonator consists essentially of a cavity connected to an outer opening. Modifications to the geometry of the cavity [1],[5],[6], and resonator neck [7],[8],[9] have also been studied. These resonators find most applications in acoustics to amplify or attenuate sound waves, namely in musical instruments to amplify certain frequencies to produce a desired tone and internal combustion engines to attenuate the high noise levels produced by the combustion engine (ICEs)[10]. However they are also present in different scenarios, such as a car with an open window where the air inside the car acts as the cavity in the Helmholtz resonator analog, fake ceilings where the end goal is attenuate sound noise produced inside the room by perforating small openings backed with a large cavity in the ceilings, and the most common example is an empty bottle where the air inside the neck vibrates when you blow air across the top, generating the typical low sound. The burners presented in this work have the same configuration as the Helmholtz resonator. The burners have this design because the initial concept of a burner was essentially a premixing cavity for the fuel and air followed by a small exit for the flame, which ended having the geometry of a Helmholtz resonator. These burners are usually present in industrial furnaces, domestic stoves (single or multiple openings), heaters, etc. Thermoacoustic instabilities occur in these burners where some flames are silent and other flames are noisy. There are a few different ways to prevent such instabilities, namely by either changing the burning conditions (air-fuel ratio, Reynolds number, etc.) or by changing the geometry of the burner (which changes the acoustics). However the focus of this work is to study the acoustics of Helmholtz burners which produce very small flames, which can be achieved by placing a small perforated plate on the burner exit. The instability regimes have not been studied as deeply as in the previous case. However by introducing the metal plate on the top, the acoustics of the system will change according to the characteristics of the plate, which is the main objective of this work.

2. Mathematical model

2.1. *Helmholtz resonator model*

In order to model the acoustics of the Helmholtz resonator, a brief description of the mathematical model is presented, which was taken from the book of Kinsler [3]. It predicts the resonant frequency of the Helmholtz resonator, which is a starting point to understand the influence of the geometry. Consider the system based on a volume *V* with a neck of length *l* and area *S*, has shown in Figure 1.

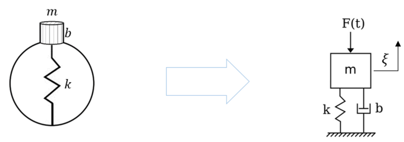

Figure 1: Helmholtz Resonator Model

The displacement ξ in an one degree-of-freedom mass-spring system with damping is determined by the solution of an ordinary second order differential equation:

$$m\frac{d^2\xi}{dt^2} + b\frac{d\xi}{dt} + k\xi = SPe^{i\omega t} \quad (1)$$

where P represents the pressure amplitude of the sound wave at time t with an angular velocity ω.

Assuming that the response to this type of solicitation is also harmonic then $\xi(t) = \xi_0 e^{i\omega t}$. Using equation (1) and knowing that $\frac{d^2\xi}{dt^2} = -\omega^2\xi_0 e^{i\omega t}$ gives:

$$(-m\omega^2 + k + i\omega b)\xi_0 e^{i\omega t} = SPe^{i\omega t} \quad (2)$$

By solving the previous equation (2), the maximum of the displacement is:

$$\xi_0 = \frac{1}{i\omega}\frac{SP}{b + i(\omega m - \frac{k}{\omega})} \quad (3)$$

The formula for the pressure fluctuations inside the volume is given by [3]:

$$P_c = \frac{\rho_0 c^2 S}{V}\xi_0 \quad (4)$$

With equations (4) and (3) it is possible to obtain the transfer function - ratio of the acoustic pressure amplitude within the cavity to the external driving pressure amplitude of the incident wave - which is:

$$\frac{P_{in}}{P} = \left(\frac{\rho_0 c^2 S^2}{i\omega V}\right)\left[b + i\left(\omega m - \frac{k}{\omega}\right)\right]^{-1} \quad (5)$$

With the transfer function it is possible to obtain the magnitude and phase plots for a Helmholtz resonator with a certain geometry, which gives the resonant frequency. Note that the second term of the previous equation is known, in the literature, as the impedance of the resonator:

$$Z_{HR} = \left[b + i\left(\omega m - \frac{k}{\omega}\right)\right]^{-1} \quad (6)$$

2.2. *Helmholtz resonator with multi-perforated plate (MPP) model*

To predict the new resonant frequency of the Helmholtz resonator with multi-perforated plate (MPP), changes must be done to the model, which is given by equation (5). It was found that a change in the impedance of the system was necessary, to adjust the frequency behavior influenced by the MPP. In the previous case where the impedance was given just by the impedance of the resonator, the introduction of the MPP acts as an additional impedance to the system. The impedance of the metal plate can be taken from micro-perforated panel theory initially proposed by Maa [11]:

$$Z_{MPP} = Z_0(R_{MPP} + iX_{MPP}) \quad (7)$$

where $Z_0 = c/S$ is the specific acoustic impedance.

In equation (7) the real part is the normalized specific acoustic resistance given by

$$R_{MPP} = \frac{32 t_p \mu}{\sigma \rho_0 c d_p^2} \left(\sqrt{1 + \frac{k_p^2}{32}} + \frac{\sqrt{2}}{32} k_p \frac{d_p}{t_p} \right) \quad (8)$$

and the imaginary part is the normalized specific acoustic reactance given by

$$X_{MPP} = \frac{\omega}{c} \frac{t_p}{\sigma} \left(1 + \frac{1}{\sqrt{9 + \frac{k_p^2}{2}}} + 0,85 \frac{d_p}{t_p} \right) \quad (9)$$

where $k_p = (d_p/2)\sqrt{\rho_0 \omega / \mu}$ is proportional to the ratio of the radius to the viscous boundary layer thickness inside the orifice, ρ_0 is the equilibrium density, c is the speed of sound, μ is absolute viscosity coefficient, ω is the angular radian frequency, t_p is the thickness of the MPP, d_p is the hole diameter and σ is the porosity, which defined by as the area of all holes divided by the area of the neck. However, since the current application of this theory demands different units for the impedance, Z_0 will be modified in order to obtain the dimensions of kg/s, and it becomes $Z_0 = \rho_0 c S$.

The impedance described by Maa [11] as two contributions. By looking at the term in parenthesis in the previous equations, the first term corresponds to the oscillating mass of air within the neck (with the influence of the boundary layer) and the second term represents the end correction terms due to the oscillating mass of air around the neck. This characteristic is also found on the impedance of the Helmholtz, where the oscillating air on the neck influences the air around. Adding this impedance Z_{MPP} to the impedance of the Helmholtz resonator Z_{HR} gives:

$$Z_{Total} = Z_{HR} + Z_{MPP} \quad (10)$$

Starting with the original formula for the Helmholtz resonator given by equation (3) and adding the impedance of the MPP the response of the system becomes:

$$\xi_0 = \frac{1}{i\omega} \frac{SP}{b + i(\omega m - \frac{k}{\omega}) + Z_{MPP}} \quad (11)$$

The transfer function for the system with the metal plate impedance can be obtained from equation (11) which gives:

$$\frac{P_{in}}{P} = \left(\frac{\rho_0 c^2 S^2}{i\omega V} \right) [Z_{HR} + Z_{MPP}]^{-1} \quad (12)$$

2.3. *MPP characterization*

The metal plate sheets are defined by the porosity σ and the perforation ratio ϕ. Porosity is defined as the total area of the perforated holes divided by the area of the neck. Since the resonators have different necks the same plate will have different values for each case. Perforation ratio is the area of a single hole divided by the polygon area where it is inserted, which gives the influence between hole interaction.

$$\sigma = \frac{n S_f}{S} \quad (13)$$

$$\phi = \frac{S_f}{S_a} \quad (14)$$

where n is the numbers of holes of the plate, S_f is the area of a single hole, S_a is the area of the polygon where it is inserted in and S is the neck area of the resonator, as shown in the following figure 2:

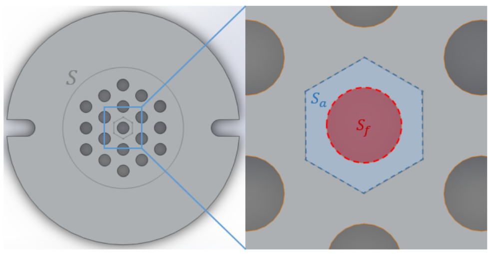

Figure 2: Metal plate schematics

3. Experimental Setup

3.1. *Installation description*

A computer was used for handling the programs of electronic instruments and for processing all the data obtained from experiments. The oscilloscope is a model '*Tektronix TDS 1001C-EDU*'. The signal generator is a model '*Tektronix AFG 3021b*'. The amplifier is a model '*ROTEL R8-890*' with a power of 150 W which amplifies the electric signal before entering the speaker. The Microphone device is a model '*Bruel and Kjaer type 2250*'. It is connected to the microphone and converts the pressure fluctuations into electric signals with a default gain. The default gain on the device is 10 dB but it can be adjusted. The data acquisition board is a model '*Data Translation DT9841-VIB-SB*' and it receives, processes and sends signals obtained from experiments between the different instruments and has an acquisition frequency of 10000 Hz.

In order to obtain experimental data, a microphone model '$B\&K\ Type\ 4189$' attached to a microphone amplifier model '$B\&K\ Type\ ZC0020$' was used to capture the acoustic perturbations by a speaker.

The structural apparatus is presented in the following figure 3:

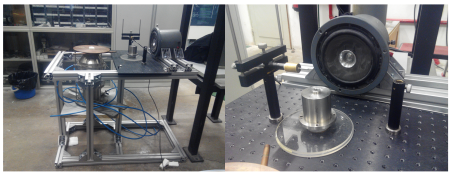

Figure 3: Structural Apparatus. Left - Full structure; Right - Testing area

The structure where the resonators, speaker and microphone are tested is built from aluminum bars, connected along bolted stainless steel joints. The installation was projected to withstand the 'big burner' presented in the picture and the other components. This installation was modeled in CAD using Solidworks and then built in the laboratory using an aluminum saw.

3.2. *Signal acquisition process*

It is necessary to obtain two pressure data points from the resonators as shown in figure 4: one inside the cavity and one outside it. The inside data is obtained by coupling a pressure probe into the microphone tip in order to introduce the microphone into the cavity of the resonator. The probe is a small tube that measures pressure in the center. Several probes were available, and the longest ones - with around 10 cm - have shown larger dissipation and therefore more noise in the signal, which implies an increase in gain of the microphone for a better signal output. A 'good' signal obtained from the microphone is around 5-10 V_{p-p} (volts peak-to-peak), being 10 V_{p-p} the maximum the microphone can withstand. Due to this the shortest probe was used since it captures the least background noise using the default gain.

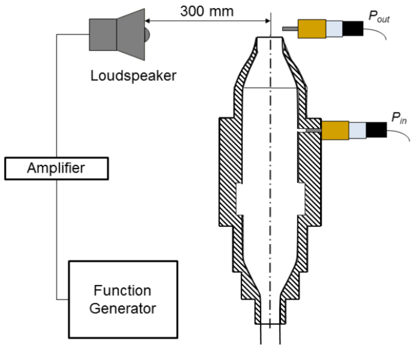

Figure 4: Resonator testing diagram. Adapted from [12]

3.3. *Resonator samples*

Since this work focused on studying small resonators it was decided to (mainly) vary the neck size between them, so it is possible to understand it's influence. The resonators tested and modeled in CAD are presented in figure 5. Resonators 4 and 5 are essentially resonators 2 and 3 but with an additional coupled volume below the resonator to increase the volume of the cavity. This volume as an approximate value of $150ml$.

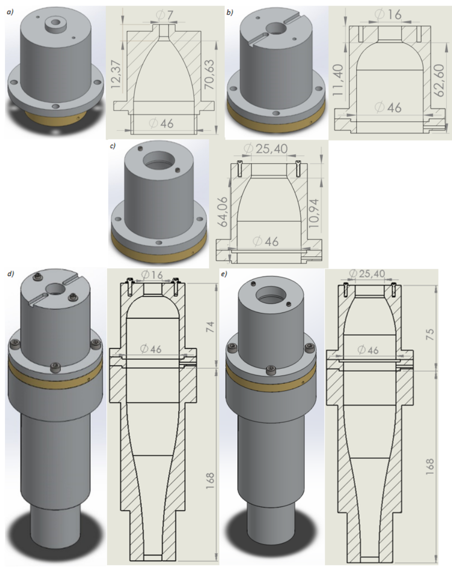

Figure 5: a) Resonator 1; b) Resonator 2; c) Resonator 3; d) Resonator 4; e) Resonator 5

3.4. *MPP Samples*

The plates tested are presented in the following figure 6:

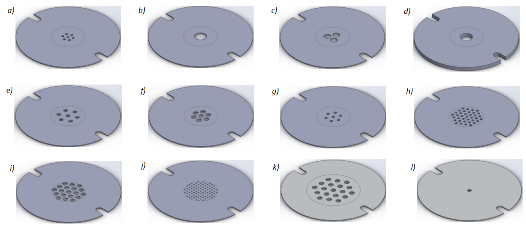

Figure 6: a) Plate 1; b) Plate 2; c) Plate 3; d) Plate 4; e) Plate 5; f) Plate 6; g) Plate 7; h) Plate 8; i) Plate 9; j) Plate 10. k) Plate 11; l) Plate 12.

Plate #	n	d_p [mm]	t_p [mm]	dlt [mm]	Perf.Ratio [%]
1	7	1.5	0.5	2.5	43.53
2	1	6	0.5	-	-
3	3	3.5	0.5	5	59.25
4	1	6	2	-	-
5	7	2.5	0.5	4.5	37.32
6	7	2.5	0.5	3.5	61.69
7	7	1.25	0.5	3.5	15.42
8	37	1.5	0.5	2.5	43.53
9	19	2.5	0.5	3.5	61.69
10	85	0.6	0.5	1.5	19.35
11	19	2.5	0.5	4.5	37.32
12	1	1	0.5	-	-

Table 1: n = number of holes; d_p = hole diameter; t_p = thickness; dlt = spacing between holes

4. Results and Discussion

4.1. *Helmholtz resonator results*

The results presented in the following figures are the amplitude and phase plots for each of the resonators. The theoretical curves plotted in black lines are obtained from the transfer functions, in this case for the Helmholtz resonator given by equation (5), and the black squares represent the experimental data for each case. Since the purpose of this work is to model the acoustics of these resonators, the frequency response gives the location of the resonance frequency, which from the thermoacoustics point of view is the instability frequency. Resonator 1 has a resonance peak at 283 Hz, resonator 2 has a resonance peak at 523 Hz, and resonator 4 has a resonance peak at 295 Hz. The losses are very small in these types of resonators, and the maximum difference between the damped and non-damped resonant frequency is about 0.2%. The resonance peak is higher with the increase of neck area and is lower with the increase in volume. From these results one can conclude that these resonators follow Helmholtz resonator theory.

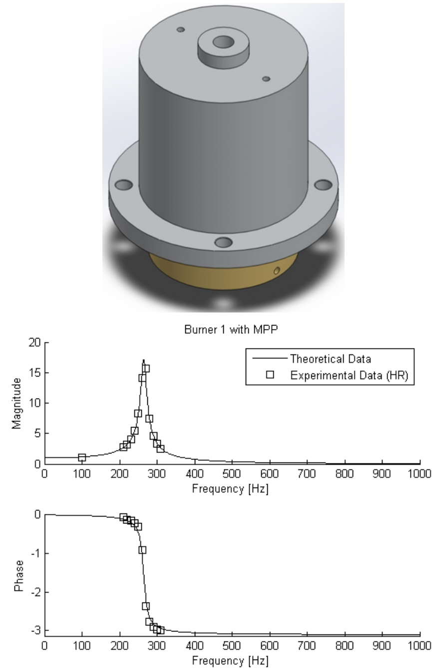

Figure 7: Resonator 1 Results

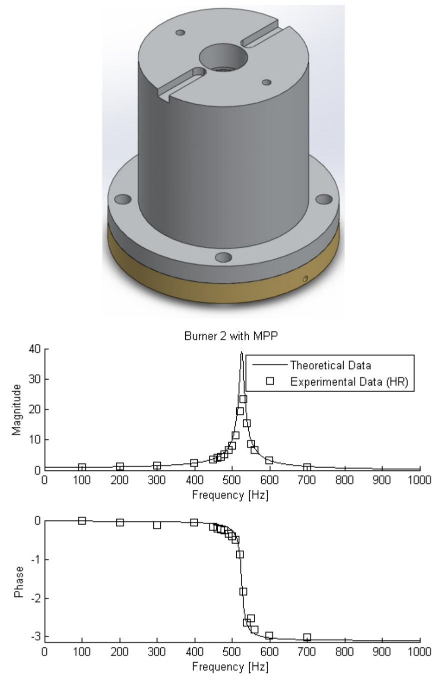

Figure 8: Resonator 2; Left: CAD image; Right: Results

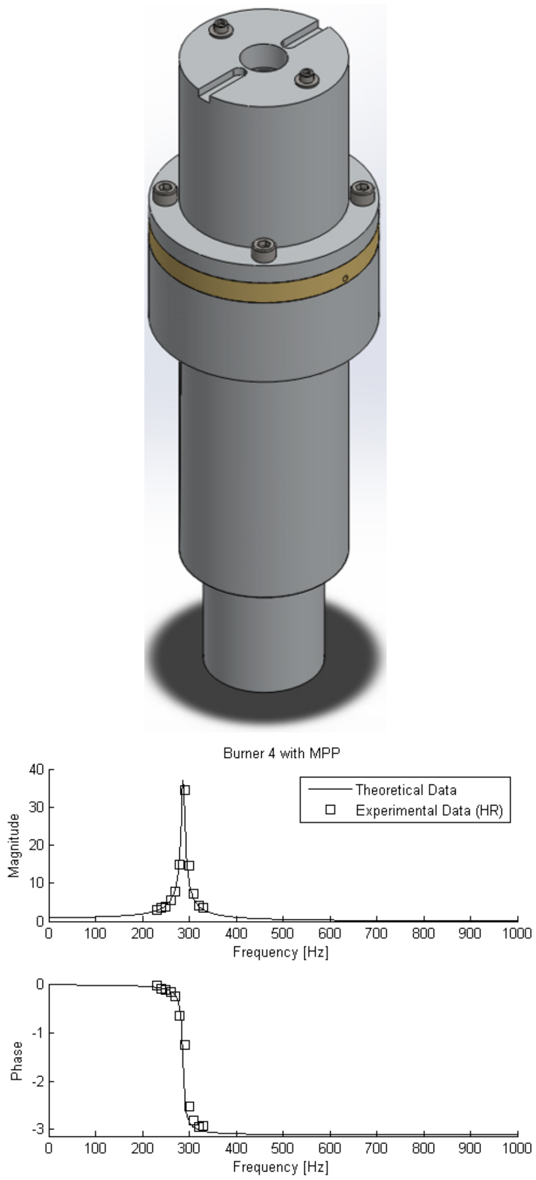

Figure 9: Resonator 4; Left: CAD image; Right: Results

4.2. *Helmholtz resonator with MPP results*

Following the methodology from the previous section, the transfer function for this case is now equation (12) due to the presence of the MPP. It is expected for the resonance frequency to always be lower, independently of the metal plate chosen, since the impedance of the MPP is always positive. Figures 10, 11 and 12 show the behavior of the modified resonator, and it can be seen that the resonant frequency lowered slightly but with a considerable magnitude peak reduction. The experimental data matches very well with the theoretical curve, proving that the theory for Helmholtz resonators with MPP is valid.

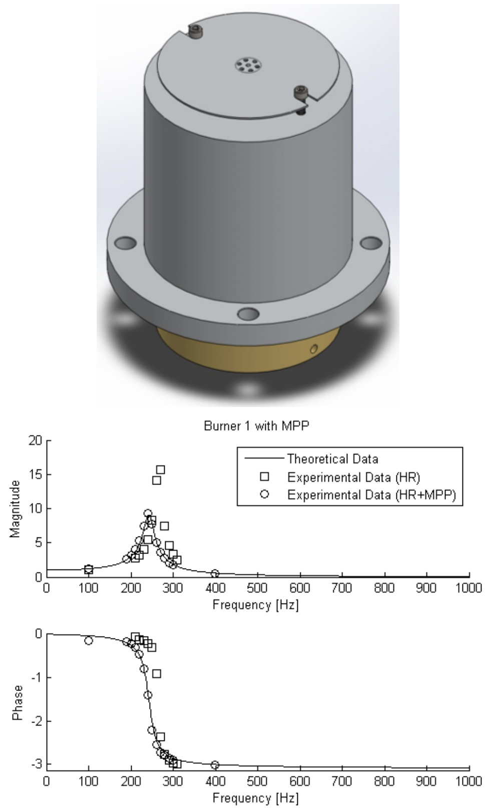

Figure 10: Resonator 1 with plate 1 Results

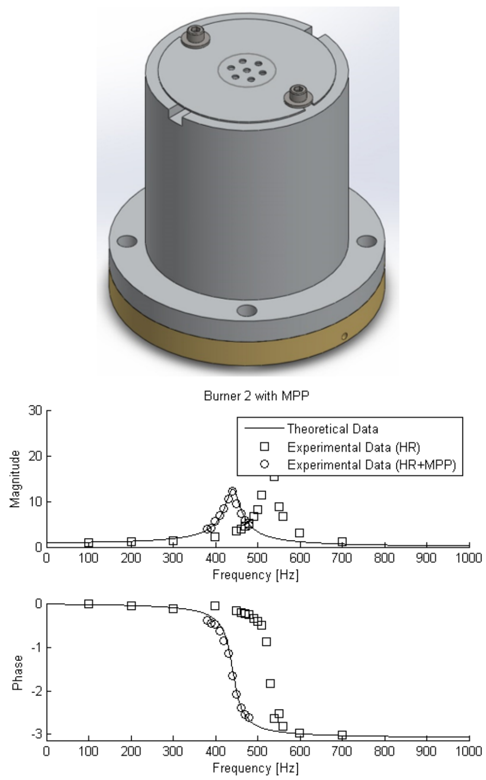

Figure 11: Resonator 2 with plate 5 Results

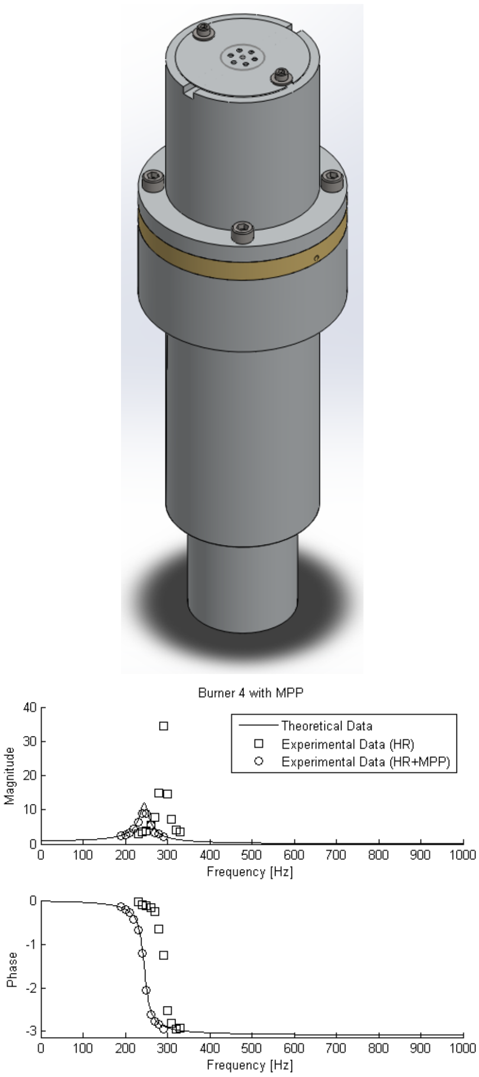

Figure 12: Resonator 4 with plate 5 Results

5. Conclusions

This study focused on the influence of the introduction of a metal plate on the acoustic properties of the Helmholtz resonator. Different geometries were used in order to understand the influence of neck and volume on acoustics of the resonator. An experimental installation was developed to test the resonators, which were tested by obtaining two time signals from a microphone, one obtained inside the resonator cavity and one outside as a reference signal. Both signals were pressure fluctuations caused by a speaker that was mounted on the installation. After these signals were obtained, they were processed using FFT method to produce magnitude and phase data. The theoretical results for the Helmholtz resonator without MPP show very good agreement with experiments. Then the metal plate was introduced on top of the resonators to study the impact on resonant frequency, which was the original objective of this work. After some more study in the literature on how to model the metal plate mathematically, a theoretical model is proposed which matches with experimental results. The final model combines two models from different authors to simulate the whole resonator. The experimental results for the resonator with the MPP also show very good agreement with the theoretical model, and they were obtained using the same method for the previous case.

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