Buckling behaviour of graphene sheets

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Abstract
Graphene is a single layer of carbon atoms, in an hexagonal lattice, and one of the most interesting nano-structures of today, given its excellent mechanical, electrical and optical properties, with applications in almost every field of engineering. Taking this into account, this work aims to build a finite element model of a graphene sheet that allows the study of its structural stability, verifying this model with results from other investigators. This model was built following the atomic-continuum approach, using the AMBER inter-atomic potential. This work also shows a parametric study of the buckling of graphene sheets for different combinations of loads and supports, making a critical assessment of the variation of the buckling load and the corresponding buckling mode with respect to geometric parameters. The finite element method has proven to be a good solution for the research of the mechanical properties, with results in agreement with the available literature. This study also demonstrates that the buckled graphene sheet behaves like the classic plate theory predicts.

Keywords: Graphene, nano-materials, buckling, finite element, atomic-continuum approach

1. Introduction
Graphene is an atomic-scale honeycomb lattice composed of carbon atoms (Figure 1), and it was only obtained in a stable crystalline form in 2004 by Geim and Novoselov [1], rewarded with the 2010 Nobel Prize for their discoveries, concluding that graphene is not only the thinnest material ever (one atom thick), but also the stronger. The excellent properties of this nanomaterial, not only mechanical [2], but also electrical and optical [3], makes it useful in many fields of engineering, being one of the most studied nanostructures of the last decade. Some examples of applications are: using graphene as reinforcement in composites materials, like graphene-ceramic composites that improve brittle fracture behaviour of the ceramics used in jet engines without compromising heat resistance [4], or graphene-metal and graphene-polymer nanocomposites that are stiffer, stronger and lighter than most of the composites available today using other materials as reinforcement [5] [6] [7]; in electronics, graphene has been under research to build faster and smaller transistors [8], and also to build flexible touchscreens [9] for mobile devices; graphene also proved to be useful to produce and store energy, with investigators producing a graphene-based supercapacitor with improved characteristics comparing with existent technology [10], and also building solar panels with the capacity of, not only retrieve energy from the sun, but also from the impact of raindrops on the panel [11].

Concerning the mechanical properties of graphene, Lee [2] obtained experimentally a Young modulus of 1 TPa and an intrinsic strength of 130 GPa for graphene, using an atomic force microscope. Multiple efforts have been made in order to develop computational simulations in order to predict graphene’s mechanical properties. Tsai [13] used molecular dynamics (MD) simulations on graphene to obtain a Young modulus of 0.912 TPa, a Poisson coefficient of 0.26, and a shear modulus of 0.358 TPa. Liu [14] adopted ab initio calculations using DFTP (a perturbation based variation of Density Functional Theory (DFT)) to achieve a Young modulus of 1.05 TPa and a strength of 110-121 GPa for a fracture strain of 0.19-0.266. The thickness of graphene sheets isn’t yet clearly defined, but the scientific community has accepted the value $t = 0.34nm$, the interlayer adhesion
distance between layers in graphite [15]. Another problem associated with graphene is, because of its small thickness, it is susceptible to buckling under compression, flexion or torsion. With this in mind, and considering that the above referenced computational methods require a great amount of computational effort and time investment, it is presented in this study a methodology based in the Finite Element Method (FEM), available and widely used in engineering applications, to study with precision and effectiveness the mechanical buckling of graphene sheets. Some authors have already used this method, originally devised by Odegard [16] and Li [17] with success.

2. Background
2.1. Buckling of thin plates
Timoshenko and Gere [18] studied the theoretical buckling behaviour of plates, determining the buckling force and corresponding buckled shape (buckling mode) of thin plates for many boundary conditions (BC). For different BC, different values of buckling force and mode are obtained, as seen in figure 2.

2.2. Buckling of graphene sheets - experimental
Researchers studied the buckling of graphene sheets by measuring in detail both stress and compression buckling strain in single flakes of different geometries. In all cases the mechanical response is monitored by simultaneous Raman measurements through the shift of either the G or 2D phonons of graphene [20]. The estimated critical buckling strain has been found to depend on size and geometry as would do any thin plate in an Euler buckling regime.

2.3. Buckling of graphene sheets - DFT
Researchers also characterized angstrom-scale periodic buckling structures in free-standing graphene bilayers produced by liquid-phase processing in the absence of specific substrates, and observed through TEM that the buckling spontaneously chooses the direction of the lowest buckling energy matching its wavelength [21]. These researchers also performed first-principles calculations of the buckling structures using the density functional theory (DFT) [22]. The ab initio graphene structures of the lowest deformation energy under compression buckle exactly along the experimentally observed directions; the entire atomic configurations closely match the 3D buckling structures reconstructed from the Z contrast of the TEM micrographs. The consistency between experimental observations, ab initio calculations and Euler buckling theory implies that the geometry rules are intrinsic properties of graphenes buckling at this fine level. It was also shown that the electronic structure of graphene changes when in the buckled shape.

2.4. Buckling of graphene sheets - MD
Ansari and Sahmani [23] studied the buckling behaviour of graphene sheets under biaxial compression, using MD [24], comparing the obtained results with different plate theories, including the classical plate theory [18]. These investigators concluded that the MD results were in agreement with the various plate theories.

3. Methodology
In this section the finite element model development methodology is described. In the first subsection, the connection between molecular mechanics and the equivalent beam model is made, to simulate the behaviour of the C-C bond. The second subsection describes how the model’s geometry was built, and how material models and element type were chosen.

3.1. Molecular mechanics
An important component in molecular mechanics calculations is the description of the forces between individual atoms. This description is characterized by a force field. In the most general form, the total inter-atomic potential energy in a molecule, $U_{total}$, for a nano-structured material is described by the sum of many individual energy contributions related to the interactions between linked atoms, and non linked atoms, in the molecular lattice, as shown in the equation [1]

$$U_{total} = \sum U_r + \sum U_\theta + \sum U_\phi + \sum U_\omega + \sum U_{vw}$$  \hspace{1cm} (1)$$

in which the first four terms represent the interactions between bonded atoms and the fifth term,
$U_{\text{vdW}}$ represents the van der Waals interaction between any non bonded atoms, often described by the Lennard-Jones potential [24], but here is despised due to its weak influence on the mechanical properties of graphene. The first four terms describe the atoms motion and position lattice. $U_r$ is related to the axial deformation on the molecular link, $U_\theta$ is the "bending" term and it describes angular motion between three atoms and finally, $U_\omega$ are the in-plane and out-of-plane torsion terms. These interactions are easily interpreted from a basic molecule scheme depicted in figure 3:

![Figure 3: Existing molecular interactions](image)

Obtaining accurate parameters for a force field amounts to fitting a set of experimental or empirical data to the assumed functional form, specifically, the force constants and equilibrium structure of the molecule. In this article the interatomic potential used is AMBER [26]. It may be represented in its harmonized form in equation 2:

$$
U_r = \frac{1}{2} k_r (\delta r)^2 , \quad (2a) \\
U_\theta = \frac{1}{2} k_\theta (\delta \theta)^2 , \quad (2b) \\
U_\omega = U_\phi + U_\omega = \frac{1}{2} k_\omega (\delta \phi)^2 . \quad (2c)
$$

where $k_r$ is the bond stretching force constant, $k_\theta$ is the bond bending force constant and $k_\omega$ is the equivalent bond torsion term, and $\delta r$, $\delta \theta$ and $\delta \phi$ are the bond stretching increment, bond angle variation and angle variation of bond twisting, respectively.

In order to determine the elastic moduli of the beam elements that will compose graphene’s structure, relations between the sectional stiffness parameters in structural mechanics and the force-field constants in molecular mechanics need to be obtained. For simplicity reasons, the sections of the bonds are assumed to be identical and circular. The elastic properties that need to be obtained are Youngs modulus $E$, Poisson’s ratio $\nu$ and the shear modulus $G$. The deformation of a space-frame results in changes of strain energies. Thus, the elastic moduli can be determined through the equivalence of the energies due to the interatomic interactions presented in equation 2a and the elastic energy that results from the deformation of the space-frame structural elements. As each of the energy terms of equations 2 represents deformations in specific degrees of freedom, the strain energies of structural elements under the same deformations in equivalent degrees of freedom will be considered. According to the Euler-Bernoulli beam model from classical structural mechanics, the strain energy $U_A$ of a uniform beam of length $L$ and cross-section $A$ under pure axial force $N$ is:

$$
U_A = \frac{1}{2} \int_0^L N^2 \frac{E A}{L} \, dx = \frac{1}{2} \frac{E A}{L} (\delta L)^2 \quad (3)
$$

The strain energy $U_M$ of a uniform beam with a moment of inertia $I$, under pure bending moment $M$ is:

$$
U_M = \frac{1}{2} \int_0^L M^2 \frac{E I}{L} \, dx = \frac{1}{2} \frac{E I}{L} (2\alpha)^2 . \quad (4)
$$

where $\alpha$ denotes the rotational angle at the ends of the beam. The strain energy $U_T$ of a uniform beam under pure torsion $T$ is:

$$
U_T = \frac{1}{2} \int_0^L T^2 \frac{G J}{L} \, dx = \frac{1}{2} \frac{G J}{L} (\delta \beta)^2 . \quad (5)
$$

where $\delta \beta$ is the relative rotation between the ends of the beam and $J$ the polar moment of inertia.

The equivalence of bond stretch equation 2a and axial beam equation 3, bond and beam bending equations 2b, 4 and bond and beam torsion equations 2c, 6 serves as foundation of the equivalent atomic-beam model that is used in the analysis of graphene’s linear mechanical behaviour. This equivalence is presented in a more compact form in the following equations:

$$
\frac{E A}{L} = k_r , \quad \frac{E I}{L} = k_\theta , \quad \frac{G J}{L} = k_\omega . \quad (6)
$$

The system of equations 6 establishes the foundation for the application of the theory of structural mechanics in modelling of graphene or other similar fullerene structures, for linear elastic simulation. The length of the C-C bond used was $L = 0.142nm$, based on the length of the covalent bond of carbon in a benzene ring [15].

3.2. Reference model

A finite element model was built in the FEM software ANSYS APDL. First, the hexagonal geometry of graphene, shown in figure 3 was generated through Matlab scripts. This script was built...
to determine the coordinates of the carbon atoms, and which atoms are connected with each other, and writes the APDL code with the necessary commands to build a graphene column of any height. Then, using ANSYS®, this column may be copied and replicated to build a graphene sheet of any length. This method allowed to build graphene sheets of many different sizes.

![Graphene sheet built using ANSYS®](image)

Regarding the properties of the beams that represent the C-C bonds, they were determined using the AMBER force field, and the element section was taken as circular, as noted in the previous section. Applying the force field in the equations for the properties of the beams were obtained and are expressed in table 1.

![Deformed geometry for the zig-zag uniaxial test](image)

### Table 1: Force field constants $k$ derived from AMBER interatomic potential and the results from equation 6 for $E$, $G$ and section diameter

<table>
<thead>
<tr>
<th>Interatomic force fields</th>
<th>AMBER</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_x \left[ \frac{nN}{nm} \right]$</td>
<td>$6.52 \times 10^{-7}$</td>
</tr>
<tr>
<td>$k_y \left[ \frac{nN}{mm} \right]$</td>
<td>$8.76 \times 10^{-10}$</td>
</tr>
<tr>
<td>$k_{\theta} \left[ \frac{nN}{mm \cdot rad^2} \right]$</td>
<td>$2.78 \times 10^{-10}$</td>
</tr>
</tbody>
</table>

To find the Young modulus and the buckling tension regarding the aspect ratio of the sheet. To find the Young modulus and the buckling tension, it is necessary to consider a thickness $t$ for the graphene sheet. Two values of thickness were considered: $t = 0.34nm$, most accepted in the scientific community, and $t = 0.147nm$, that correspond to the model thickness, given by the diameter of the beams.

### 3.3. Verification

This section discusses the tests made in order to determine which beam element type and thickness should be used in order to model the graphene sheet with the best results.

#### 3.3.1 Young modulus

As discussed previously in section 3, in order to determine the Young modulus of a single sheet of graphene, two uniaxial traction test were made, one in the $X$ direction, and other in the $Y$ direction.

![Deformed geometry for the armchair uniaxial test](image)

In these uniaxial tests, a force of $1nN$ was applied to the nodes of one end of the sheet, while...
restricting the movement of the other edge, resulting on the deformed structures seen in figures 5 and 6. For the uniaxial tests under the X direction:

\[
E_x = \frac{\sigma_x}{\varepsilon_x}, \quad \varepsilon_x = \frac{\Delta a}{a}, \quad \sigma_x = \frac{\sum F_x}{A_{\text{section}}} \tag{7}
\]

where \(E_x\) is the Young moduli for the direction of the test, \(\sigma_x\) is the normal stress, \(\varepsilon_x\) is the strain, \(\Delta a\) is the displacement occurred determined with ANSYS, \(a\) is the size of the sheet’s edge (Figure 4), \(F_x\) the nodal force applied and \(A\) is the section area of the sheet, in this case given by \((a \times t)\). The process to determine \(E_x\) is similar.

Using the equations (7) it is possible to determine the Young modulus of the sheet. In table 2 are the obtained results for both beam elements considering both sheet thickness discussed in the previous section, comparing them with results obtained from various studies (MD, DFT, FEM, Monte Carlo) presented in the literature.

The observation of the results presented in table 2 shows that graphene has a slightly orthotropic behaviour, as the Young’s moduli in the armchair and zig-zag directions is not equal, but as the difference being so narrow (2% - 4%) graphene can be easily accounted as an isotropic material. The models using BEAM4 with \(t = 0.34\text{nm}\) and BEAM188 with \(t = 0.147\text{nm}\) resulted in a graphene sheet with a Young modulus of around 147TPa, which is in agreement with other investigations, and that shows that these two models are fit to model graphene sheets under axial deformations. Also in the table are presented the results from the present work and the ones found in the literature, in order to verify the methodology applied in this study. The literature results come from similar methods that use the finite element method, from MD simulations, and from DFT and other statistical physics simulations like the Monte Carlo method [33]. The results from MD and DFT theory are more accurate than the present ones, in principle due to the inclusion of quantic, thermodynamic and many other parameters involved in the atomic scale. Some experimental studies are also presented, even if they don’t provide the full spectrum of elastic properties, their results are the most accepted in the scientific community (mainly the results from Lee [2]). Finally, we can observe that the results from the finite element method have a greater variability due the different modeling procedures: Alzebdeh [28] used an equivalent continuum framework after using the atomistic-beam model and Reddy [32] minimized the potential energy of the beams to find the equilibrium structure of graphene and its stiffness. The simple fact of considering one or another type of beam element has changed the properties determined for the graphene sheet, and that is proof of how important the modeling process is. This way, the two models that proved to model correctly the graphene sheet will now be subjected to a buckling analysis.

### 3.3.2 Buckling tension

The two models that passed the axial deformation test, were then subjected to a buckling analysis under four different BC’s:

- **SET 1** - Two parallel edges simply supported and uniaxial compression perpendicular to the supported edges;

- **SET 2** - Two parallel edges clamped and uniaxial compression perpendicular to the clamped edges;

- **SET 3** - All edges supported and uniaxial compression;

- **SET 4** - All edges simply supported and biaxial compression.

These BC’s were applied to graphene sheets of multiple sizes, in order to evaluate the change of the buckling force with the aspect ratio (AR) \(\frac{b}{a}\) (a and b defined in figure 4). The results are shown in figures 7, 8, 9, and 10.

The results show that both models produce satisfactory results, with the model built with BEAM188 and thickness \(t = 0.147\text{nm}\) showing the best agreement with the results obtained by Tserpes [37], that tested two models built with FEM, using the continuum and the atomic-continuum approach, and got results in agreement with classic plate theory [18]. This classical plate behaviour of graphene was observed using other methods, like DFT [21] and MD [23], which indicate that this model is a good FEM model to study the stability of graphene sheets. The model using BEAM4 and thickness \(t = 0.34\text{nm}\) showed some deviations within the obtained results, mainly in set 3, which allows the conclusion that this type of beam element is not very good with out-of-plane deformations, even though it was the the model that produced the best result for in-plane deformation of graphene.

### 4. Parametric study on buckling of graphene sheets

Now that the model is built appropriately, with the right beam element, some other buckling tests using different BC’s were made to observe the behaviour of the graphene sheet under buckling. In this section will be presented 4 cases of grand total of 32 tests. The results shown in this section will be the variation of buckling force with respect to the geometric parameter \(b\), set in figure 4 and
Table 2: Results obtained in this study, with other results obtained by other researchers using different methods

<table>
<thead>
<tr>
<th>Researchers</th>
<th>$E_{armchair}$ [TPa]</th>
<th>$E_{zigzag}$ [TPa]</th>
<th>$E$ [TPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(FEM)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Presente (BEAM4/$t = 0.34nm$)</td>
<td>1.035</td>
<td>1.047</td>
<td>-</td>
</tr>
<tr>
<td>Presente (BEAM4/$t = 0.147nm$)</td>
<td>2.394</td>
<td>2.423</td>
<td>-</td>
</tr>
<tr>
<td>Presente (BEAM188/$t = 0.34nm$)</td>
<td>0.369</td>
<td>0.353</td>
<td>-</td>
</tr>
<tr>
<td>Presente (BEAM188/$t = 0.147nm$)</td>
<td>0.854</td>
<td>0.818</td>
<td>-</td>
</tr>
<tr>
<td>Pedrosa [27]</td>
<td>0.749</td>
<td>0.763</td>
<td>-</td>
</tr>
<tr>
<td>Alzebdeh [28]</td>
<td>0.990</td>
<td>0.100</td>
<td>-</td>
</tr>
<tr>
<td>Scarpa [29]</td>
<td>1.957</td>
<td>1.379</td>
<td>-</td>
</tr>
<tr>
<td>Reddy [30]</td>
<td>0.670</td>
<td>0.814</td>
<td>-</td>
</tr>
<tr>
<td>Huang [31]</td>
<td>-</td>
<td>-</td>
<td>2.690</td>
</tr>
<tr>
<td><strong>(MD)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tsai [13]</td>
<td>-</td>
<td>-</td>
<td>0.912</td>
</tr>
<tr>
<td>Ni [32]</td>
<td>1.050</td>
<td>1.130</td>
<td>-</td>
</tr>
<tr>
<td><strong>(DFT e Monte Carlo)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zakharchenko [33]</td>
<td>-</td>
<td>-</td>
<td>1.040</td>
</tr>
<tr>
<td>Zhao [34]</td>
<td>-</td>
<td>-</td>
<td>0.910</td>
</tr>
<tr>
<td>Liu [14]</td>
<td>-</td>
<td>-</td>
<td>1.050</td>
</tr>
<tr>
<td><strong>(Experimental)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lee [2]</td>
<td>-</td>
<td>-</td>
<td>1.010</td>
</tr>
<tr>
<td>Frank [35]</td>
<td>-</td>
<td>-</td>
<td>1.000</td>
</tr>
<tr>
<td>Blakslee [36]</td>
<td>-</td>
<td>-</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Figure 7: Variation of the buckling tension with respect to the AR - Set 1
Figure 8: Variation of the buckling tension with respect to the AR - Set 2
Figure 9: Variation of the buckling tension with respect to the AR - Set 3
Figure 10: Variation of the buckling tension with respect to the AR - Set 4
also the corresponding buckling mode. The cases under study will be set 3 and 4, i.e., uniaxial and biaxial compression with all edges supported, and also two new ones, which will evaluate the effect of shear stress, and bending, with all the edges supported as well. All compressions presented here are in the $Y$ direction and the following results were obtained for $a = 7.14 \text{nm}$, and then varying the size of $b$, in order to obtain AR’s from 0.3 to almost 5. The graphs give the buckling force variation to the parameter $b$ for pinned and clamped supports, presenting also the associated buckling mode for both supports. The buckling modes shown were obtained for an AR=3 ($b = 21.17 \text{nm}$).

The graphs give the buckling force variation to the parameter $b$ for pinned and clamped supports, presenting also the associated buckling mode for both supports. The buckling modes shown were obtained for an AR=3 ($b = 21.17 \text{nm}$).

![Figure 11: Variation of the buckling tension with respect to the AR for uniaxial compression - Pinned buckling mode on the left and clamped buckling mode on the right](image1)

![Figure 12: Variation of the buckling tension with respect to the AR for biaxial compression - Pinned buckling mode on the left and clamped buckling mode on the right](image2)

From the analysis of these figures, there are some joint conclusions we can make about all of the cases: It can be seen that the clamped support always results in a higher buckling force, no matter the BC’s. It can also be observed that, when $b \gg a$ there is no change in the buckling force, with this value remaining constant. The buckling mode does not show that behaviour, with the number of semi-waves becoming higher with the increasing size of $b$, except for the biaxial case, which is a bit more complex. It is also noticeable that clamped edges produce more, but smaller semi-waves. When under biaxial compression, the buckling shape of the sheet will be either dominated by the longitudinal or the transversal compression. It can be seen in figure 12 that the change of the pinned support to a clamped one changes the domain of compression from transversal to longitudinal. Now, looking at figure 13 there are some interesting things happening. Only the left side of the sheet is under compression in the $Y$ direction, while the other half is under traction, resulting in bending. Since only the left side is under compression, the buckling deformation only appears on that side of the sheet, producing similar semi-waves to the uniaxial compression test (figure 11), but only on half of the sheet. When on clamped edges, the buckling mode is a
bit different, with the semi-waves showing a curve. This happens because of the bending deformation of the sheet. Finally, analysing the case with shear forces (Figure 14), the buckling shape is again similar to the uniaxial compression test, but with the semi-waves displaying in a diagonal direction due to the shear forces. This case, together with the case of bending, were the ones which produced the highest values of buckling force, while the biaxial produced the lowest. Sheets with other sizes of $a$ were tested and different buckling forces were determined, which shows the importance of the AR in buckling of graphene sheets. It is of relevance to show that the obtained buckling modes are in agreement with what was expected form the classic plate theory (Figure 2), which is another proof of the validity of this FEM model.

5. Conclusions
This article was a extended resume of the thesis that was set out to infer the buckling behaviour of graphene sheets with the development of a consistent FEM model. The methodology employed in the present work was proved to be a simple, fast and accessible method to the engineering industry to study and analyse the mechanical properties of graphene, and other nano-materials, without a relevant loss of precision. This is needed due to graphene many prospective applications in many fields, which will require a deep understanding of the mechanical behaviour of graphene, so it can be used in the advanced technological components of the future.

The main findings of this thesis can be summarized in brief:

- The atomic-beam equivalent model in conjunction with the finite element method were proven to be a useful methodology in the simulation of the mechanical behaviour of graphene [16], [17]. With low computational cost and processing, the results obtained for the mechanical properties of this nano-material had a satisfactory precision, in comparison with the results in the literature;
- BEAM188 (beam element following Timoshenko beam theory) showed to be the best beam element to investigate out-of-plane deformations, when comparing with BEAM4 (beam element following Euler-Bernoulli beam theory). Even so, when in axial deformation, BEAM4 obtained great results;
- Buckled graphene follows a classic plate behaviour [15], something concluded by this investigation and others too, not only by computational analysis [21], [23], but also through experimental trials [20];
- Graphene under biaxial compression may show different buckling modes depending on the type of support: When the support is pinned, there is a domain of transversal compression, but when the support is clamped, the longitudinal compression is the dominating one;
- Bending and shear forces produce the most high buckling force values, whereas the biaxial compression results in the lowest values of buckling force;
- Clamped supports always produce higher buckling forces;
- When all edges are supported, both the parameters $a$ and $b$ have to be taken into account, since the AR plays a major role on the buckling force and mode;
- The buckling force assumes a constant value after $b/a = 1$. While $b < a$, the buckling force is very sensitive to dimensions $a$ and $b$.
- Different BC’s lead to different buckling forces and modes. Since buckling may lead to structure failure and has also been shown that graphene electronic properties change when buckled [21], the study of this phenomenon is of the utmost importance for graphene to be introduced to nowadays devices.

After the development of this thesis, it is believed that there are still many improvements to be done within the framework of the present work, as well as other future developments. These include:

- Use the FEM model in multi-physics analysis, in order to simulate the variation of the mechanical properties with temperature, or in the presence of electrical currents through the material;
- Use different force fields, like the Morse interatomic potential, with non-linear characteristics;
- Create a nano-composite FEM model, which uses the sheet modelled in the present work as a reinforcement element in a dispersion of nano-platelets in a polymeric matrix, and conduct studies on the mechanical behaviour of said nano-composite.
- Study the behaviour of the graphene sheet in post – buckling, i.e., what happens once the sheet assumes its buckled shape.
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