An integrated economic-energy computable general equilibrium model of Portugal

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A creative man is motivated by the desire to achieve, not by the desire to beat others.

Ayn Rand
Acknowledgments

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I wouldn’t have made it this far without you.
Abstract

In recent years Portugal witnessed a major energy transition towards a high penetration rate of renewables, motivated by a policy of Feed-In Tariffs (FITs) and other subsidies. This strategy is part of the European environmental targets for 2020.

In this thesis, a Computable General Equilibrium (CGE) static model was developed to assess the impact of that renewable energy policy in the Portuguese economy.

Portugal was modeled as an open economy whose two primary factors (Capital and Labor) were treated as imperfect markets. For Capital, a new piecewise function was introduced to create inefficiencies in its sector mobility. Furthermore, each industry is capable of producing multiple goods – a rare feature for CGEs.

In order to calibrate the model, a Social Accounting Matrix with data from the Portuguese National Statistics Institute was built and then altered using information from the National Energy Balance to include 13 Energy sectors, that represent specific technologies.

A custom MATLAB project was used to solve the model, whose features, such as the production structure, behavioral parameters and the amount of goods, industries or factors can be altered freely.

Comparing the benchmark year with an alternative scenario without the FIT mechanism revealed that FITs were largely responsible by the shift towards renewable electricity, but they also induced a small loss in Gross Domestic Product. Hence, the model is able to quantify the tradeoff between the environmental benefits (resulting in better quality of life) and the loss of economic welfare caused by the FITs.

Keywords

Static Computable General Equilibrium, Renewable, Energy, Feed-In Tariff (FIT), Portugal
Resumo

Nos últimos anos, Portugal testemunhou uma grande alteração energética com a elevada taxa de energias renováveis motivada pela política de Feed-In Tariffs (FITs) e outros subsídios. Esta estratégia esta alinhada com os objectivos ambientais europeus para 2020.

Esta tese consistiu na elaboração de um modelo Computacional de Equilíbrio Geral (CGE) estático para avaliar o impacto económico do mecanismo de FITs existente em Portugal.

Portugal foi modelado como uma economia aberta cujos factores primários de produção (capital e trabalho) não são completamente utilizados. Para o capital, desenvolveu-se uma função por ramos de forma a introduzir ineficiências na sua mobilidade através de sectores. Para além disso, cada indústria é capaz de produzir vários tipos de bens – uma característica rara em CGEs.

Para calibrar o modelo, construiu-se uma Matriz de Contabilidade Social usando dados do Instituto Nacional de Estatística e, posteriormente, introduziram-se os fluxos energéticos provenientes do Balanço Energético Nacional de forma a incluir 13 sectores energéticos que representam tecnologias específicas.

Elaborou-se um projeto em MATLAB para encontrar os novos estados de equilíbrio, cujas características fundamentais, como a quantidade de bens e sectores, são flexíveis.

A simulação sem o mecanismo de FIT demonstra que essas políticas foram largamente responsáveis pelo aumento do uso de energias renováveis para gerar eletricidade em Portugal. Na realidade, este cenário sem apoios à energia renovável aponta para um Produto Interno Bruto menor enquanto os indicadores de bem-estar sugerem uma melhor qualidade de vida, no entanto estes últimos não incluem melhoramentos de vida intangíveis, como a saúde.

Palavras Chave

Modelo estático CGE, Renovável, Energia, Feed-In Tariff (FIT), Portugal
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Abbreviations

A Assets

BaU Business as Usual

CES Constant Elasticity of Substitution

CGE Computable General Equilibrium

DGEG General Directorate for Energy and Geology

E Energy Goods

EU European Union

FF Fossil Fuel

FIT Feed-In Tariff

GAMS General Algebraic Modeling System

GDP Gross Domestic Product

GE General Equilibrium

GTAP Global Trade Analysis Project

INE (Portuguese) National Statistics Institute

IO Input-Output

K Capital

L Labor

M Material Goods

MIBEL Iberian Electricity Market

MSW Municipal Solid Waste

OECD Organization for Economic Co-operation and Development

PF Production Function
RES Renewable Energy Source

RoW Rest of the World

SAM Social Accounting Matrix

TTM Trade and Transport Margin

U Utility

Y Sector Activity
Introduction

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1.1 Motivation

Given the contemporary concern with climate change, the European Union (EU) decided to include environmental targets in Europe’s 2020 agenda (European Commission, 2013). The main goal is to reduce CO₂ emissions by 20% (or 30% if possible) compared to 1990 mostly through the introduction of Renewable Energy Sources (RESs) and also by increasing Energy Efficiency thus reducing the total energy demand. Portugal has opted to introduce Feed-In Tariffs (FITs) in order to promote the growth of RESs, which should produce 31% of the total consumed energy and represent at least 45% of used electricity by 2020.

According to the FIT policy, Rede Eléctrica Nacional¹ (REN), the Portuguese grid operator, is required to purchase all the energy from RESs at a preset price regardless of the price of electricity in the Iberian Electricity Market (MIBEL). The FIT price depends on the used technology, for instance, Wind Parks are paid around 75€/MWh while Photovoltaic installations receive on average 257€/MWh. According to Direcção Geral de Energia e Geologia (Directorate General for Energy and Geology – DGEG), these payments are guaranteed for a certain time frame, which is often 15 years, but which can go up to 35, or until a certain production level is reached, so that the investment in the RESs becomes profitable. The formula that calculates the FIT for each technology also includes a premium based on production capacity (warranty of available capacity) and a special regime for micro-production and co-generation (combined heating systems) of renewable energy, designed to promote energy efficiency.

However, as energy consumption and economic growth are often strongly correlated (Fuinhas and Marques, 2012), Europe’s environmental objectives may impair the wealth of its member states so the EU let each one chose how to reach its individual goals. The recently introduced FITs resulted in a drastic change of the Portuguese electricity mix and the economic structure. The major economic and environmental impacts were a slight decrease in Gross Domestic Product (GDP) and a large reduction in carbon emissions (Proença and St. Aubyn, 2013). Also, since the rest of the economy purchases goods, namely electricity, from the energy sector, the other sectors also suffered some structural change.

The goal of this thesis is to build a hybrid static Computable General Equilibrium (CGE) of the Portuguese economy, with emphasis on the energy sector, that is capable of analyzing the simultaneous effects of the FIT mechanism on the economy and the environment, as well as other indirect outcomes.

1.2 Previous Work

A CGE model combines abstract General Equilibrium (GE) economic theory, expressed through equations, with realistic economic data to determine numerically the impact of an exogenous shock (such as the introduction of a FIT) in a set of endogenous variables such as prices, unemployment or green house gas emissions. The CGE static model has an initial scenario, often called Business as Usual (BaU) that represents reality without the introduction of the policy. Then, by changing one or some parameters, action known as introducing a shock, both scenarios can be compared to check the policy inefficiencies and how it can interfere with other measures (Dixon and Jorgenson, 2013).

¹National Electricity Grid
CGE models are widely employed by international institutions such as the World Bank or the IMF, by consulting firms and academic researchers, to assess the the economy-wide impacts of policies, for instance the introduction of a tax.

GE models are also appropriate to study environmental policies: the SNOW model was used to evaluate alternative designs for tariffs on embodied carbon and the OECD has ENV-LINKAGES to assess different approaches for leveling carbon prices in a world with fragmented carbon markets (Böhringer et al., 2012).

In 2009, the government of Ontario approved a FIT policy that, besides promoting investment in renewable energy, had the purpose of creating 50,000 green jobs. Böhringer et al. (2013) studied the impact of that FIT policy using a CGE. The results point to an economic loss of $1.7 billion and despite the creation of 12,400 jobs in the energy sector, the job loss in the rest of the economy inflates the unemployment rate from 8% to 8.32%. This study doesn’t invalidate the policy because there are unaccounted welfare gains in health and the environment. In addition, the domestic production of some products will lead to technological change which is beneficial in the long run.

There are several studies in the literature which use CGE models to study the impact of FITs, some of them applied to Portugal.

Proença and St. Aubyn (2013) provide an empirical assessment of the economic and environmental effects of Portugal’s FITs policy to promote renewable electricity generation at a level which ensures compliance with the national target of 45% of RES electricity in 2010. They have created a static CGE modeling framework to appraise the interactions between economy, energy and environment in an integrated and consistent manner. Although the model predicts correctly a 45% fraction of green electricity and estimates a 31% reduction in CO\textsubscript{2} emissions, the authors point out that their work can be improved in several ways, namely the study of alternative policies.

In Portugal, other interesting cases include Pereira and Pereira (2012), where a Dynamic CGE model of the Portuguese Economy was developed. It has been used to evaluate the impact of tax policy and social security reform in Portugal and, in more recent applications, to deal with energy and environmental policy issues. It is a unique work as it captures relevant climate policy issues, for example budgetary concerns, public debt accumulation and even it even features traits of endogenous economic growth.

Furthermore, Fortes et al. (2014) combined the Portuguese TIMES model and the GEM\textsuperscript{3}-E3 to examine a possible CO\textsubscript{2} tax and also renewable energy subsidies. Their modeling platform allows the computation of policy economic impacts while considering technological detail and expliciting the mechanisms through which energy demand evolves.

Both the Dynamic CGE model developed by Pereira and Pereira and the GEM-E3 are very robust, as they were based on other works. On the other hand, Proença and St. Aubyn created a new model specifically designed to evaluate the Portuguese FIT policy. In this thesis, a similar model was assembled, but different characteristics of CGE models were explored, which are detailed in Section 1.3.

\textsuperscript{2}Statistics Norway, Oslo
\textsuperscript{3}General Equilibrium Model, same as CGE
1.3 Original Contribution

The purpose of this dissertation is to build a solid CGE modeling framework and use it to analyse the impact of the FIT policy implemented in Portugal.

Constructing such a model requires the development of a system of equations capable of describing reality from an economic perspective. GE economic theory usually does not include non-clearing (or incomplete) markets, for instance unemployment rate and unused Labor. To introduce those concepts, a wage curve was incorporated into the model, as it was by Böhringer et al. (2013), which had not yet been included into a CGE to study energy policies in Portugal. Moreover, a piecewise function was developed to account for Capital losses when it moves across production sectors. Also, unlike the works mentioned in Section 1.2, the production sectors are capable of simultaneously producing several different goods.

Bearing in mind that CGE models are calibrated to a benchmark period using actual data, a Social Accounting Matrix (SAM) was built to represent Portugal in 2010, with a total of 82 industries producing 88 goods. Then, the conventional energy sectors were replaced by 13 specific technologies and, after being converted into monetary flows, national energy interactions were added to the SAM. Nevertheless, due to computational constraints, both the non-Energy sectors and goods were reduced each to 10 items only.

In order to solve the set of equations that compose the model, i.e. to find new equilibria, a computational project was developed using MATLAB. This project is also capable of processing the large quantities of data necessary to calibrate the model and execute the calibration process according to the user’s definitions. It is quite customizable as it can deal with a variable number of sectors, commodities and even the nesting structure can be easily specified.

1.4 Thesis Outline

Chapter 2 starts with an outline of CGE models’ historical background, up to its popularity sprout in 1987. Then, Hybrid models are introduced and Section 2.5 reviews both strengths and weaknesses inherent to CGE models.

In Chapter 3, some essential concepts of GE theory are explained, such as Production Functions, nesting and aggregate goods. These notions are fundamental to understand the model, which is thoroughly described in Chapter 4.

Chapter 5 begins explaining how the SAM was built, which data was gathered and how it was processed. Then, the sources of the elasticity parameters are detailed in Section 5.2. Lastly, an in-depth description of the code, including some configuration details, is provided.

Chapter 6 presents the results of two simulations. The first was performed to evaluate the CGE model by attempting to reproduce the year 2011 and comparing the results with real data. Then, bearing in mind the flaws of the model, follows the assessment of the FIT mechanism. The Chapter ends with a sensitivity analysis of the most relevant elasticity parameters.

Finally, Chapter 7 offers some final remarks and suggests directions for future research.
## Historical Background

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Chapter 2 starts with a brief description the history of neoclassical economy until the recent developments of CGE models. Then, Section 2.4 describes how top-down and bottom-up models are combined into hybrid models – such as the one built in this thesis. Finally, the last Section discusses the main weaknesses intrinsic to CGEs.

2.1 General Equilibrium Theory

In the 18th century, classical economists, such as Smith, Ricardo, Mill, and Marx, developed the cost-of-production theory of value propelled by the developing industrial sector, mostly focused on how the cost of inputs affected the output price.

Inputs can be either factors, namely Labor, Capital and Resources or intermediate goods, which are outputs from other activities. Producers combine those items into goods, which they try to sell at a profit. However, since they will try to sell at the lowest value possible when competing with each other, classical economists believed that in the long term profit drops to zero and the price of a good only covers its production cost (with capital rents included in this cost) (Smith et al., 1843).

It was Cournot who introduced the role of demand by drawing supply and demand curves, which led to the concept of market equilibrium in microeconomics.

![Supply and Demand curves for a certain good.](image)

Figure 2.1: Supply and Demand curves for a certain good.

Figure 2.1 shows the partial equilibrium of a given commodity. For producers, the higher the unitary price $P$, the more quantity $Q$ they are willing to create and sell; whilst consumers are bound to buy more when the prices are lower. Therefore, the price and quantities will balance so that supply equals demand.

In an attempt to model the real economy, Léon Walras increased the complexity of the problem by trying to model simultaneous markets, i.e. several activity sectors and commodities, resulting in the basis of GE theory (Walras, 1874), yet it was neglected for almost 50 years.

The Swedish economist Cassel recovered the Walrassian theory in his book *The Theory of Social Economy*, but it only gained popularity in 1954, when Arrow and Debreu published a paper that proved the existence of one GE solution for an economy without loss of generality by analyzing Walras’s assumptions and adding a few more constraints.
2.2 Development of Computational Methods

The Computable component that would serve as base for CGE, originated in Input-Output (IO) models and Linear Programming algorithms. The IO analysis tracks the flow of goods and money in the economy and granted Leontief a Nobel Prize in Economics, as he was the first to use a matrix representation of a national or regional economy.

\[ x = Ax + y \]  

(2.1)

The quantity of each good produced in the economy is represented in vector \( x \), while \( y \) contains the final (consumer) demand for that same commodity. Nevertheless, producing a good requires intermediate inputs, which are proportional to the output through the Leontief (fixed) coefficients, or matrix \( A \) (Miller and Blair, 2009).

\[ x = (I - A)^{-1}y \]  

(2.2)

IO analysis is essentially a demand driven production theory that optimizes supply considering only one possible course of action, as one specifies the final demand \( y \) and obtains industry output using Equation 2.2 – it does not provide the whole picture of the economy.

The set of objects \( A, y, x \) and \( v \) (the latter being primary inputs) defines an object known as an IO table, which represents the series of flows involving production sectors in the economy. In the modern system of national accounts (SNA, 2008), this concept was expanded to an object known as a SAM, which takes into account the set of all flows in the economy involving not only production sectors but also economic agents (households, government), products and financial accounts. SAMs are developed to calibrate computational GE models.

2.3 Complementarity Problem Approach

In 1985, Mathiesen proposed a new approach to Arrow and Debreu’s GE model – he interpreted the model as a Complementarity Problem with three sets of variables: the commodity prices, the sector’s activity levels and the agents’ incomes.

Alongside the sets of variables are three main premises:

1. Zero profits: firms make no profit;
2. Market Clearing: supply equals demand;

Though formally the models remain the same, in more recent papers by Ferris and Kanzow (1998) and Böhringer (1998) a new approach has been discussed – the Mixed Complementarity Problems – that can handle a larger range of assumptions for the variables.

Initially models were built individually and solved by a program customized for that particular case, which resulted in a high barrier to outsiders. Fortunately the World Bank invested in the development of the General Algebraic Modeling System (GAMS), which was released to the public in 1987, and enabled a widespread use of CGE models.
2.4 Hybrid Models

The use of CGE models quickly became popular after the launch of GAMS. As it might have been expected, many different entities adopted this type of models, greatly boosting the richness and diversity of CGEs.

CGE models are typically split into top-down and bottom-up. Though the first offers a better overview of the economy, it does not provide much detail into the technologies used by the production sectors, since they are only represented by smooth functions. Otherwise, bottom-up approaches have a detailed technical description of the productive technology through rigid coefficients, preventing any input replacement (Böhringer and Löschel, 2006).

Recently this problem has been addressed by creating hybrid models which comprise the advantages of both techniques: for instance, the present thesis will have a detailed portrayal of the Portuguese Energy sector with fixed coefficients, and at a more aggregate level smooth production functions will be used. This division is beneficial as the FIT policy affects directly the Energy sector, which is more developed than the rest of the economy, without losing the overall macroeconomic picture provided by the top-down approach.

However, some inconsistencies may emerge due to incompatibilities between these two approaches. For example: from a top-down perspective, electricity is an indistinguishable good, as 1kWh has the same use whether it was produced by RESs or through Fossil Fuels (FFs), so producers should simply sell the 1kWh with the lowest marginal cost. Yet, the bottom-up view is that both productions coexist at different marginal costs, which means there is a discriminated demand (Wing, 2006).

2.5 Criticism and Extensions

Taking into consideration that models attempt to describe reality, they need to be based on real economic data. The static procedure simply uses the benchmark values obtained from a SAM of the region in study. The other possibility is to use sets of data to econometrically estimate the parameters. During this process the model’s parameters are adjusted so that it yields as a solution the BaU, or benchmark scenario. The calibration process is often appointed as a flaw since it impairs the falsifiability of the model (Scricciu, 2007).

It is important to denote that in CGE models the production factors are combined into a single industry output. Since production is not rigid, it is possible to substitute factors, i.e. a factory can have more employees (more Labor) and cheaper non-automated machinery (less Capital), up to a certain point. The degree of replacement between two inputs is dictated by an adimensional parameter: the elasticity of substitution. These elasticity parameters are present in several components of the model other than production: consumer demand, imports and exports. High elasticity means high substitutability, so it is easy to replace expensive products by cheaper ones and retaining the same final use. Nonetheless, determining elasticity parameters is an arduous feat. Recovering the preceding example of electricity, from a top-down perspective the elasticity of substitution of electricity demand is infinite as electricity from all sources is the same, but the bottom-up interpretation requires a finite
value. Wing (2006) solved such problem by choosing 10 for the elasticity parameter.

Besides the data treatment, the fundamental assumptions of the model also have a large impact on the outcomes. Specifically, the behavioral assumptions can be made from a micro or a macroeconomic perspective. Model closure, which is the choice between that variables that are fixed (exogenous) and those that are solved by the model (endogenous), can lead to disagreements. In fact, Scrieciu (2007) claims that there is a confirmation bias since the modeler chooses his assumptions based on beliefs and the model returns the outputs he was already expecting. In addition, Scrieciu points out that static models don’t show the adjustment process, only the final equilibrium, so to distinguish between short run and long run, the modeler must select the appropriate closure premises.

As stated before, the CGE models are based on a neoclassical (or GE) economic theory that assumes agents behave rationally and that reality converges to equilibrium conditions. Though allowing simultaneously explaining of agents’ income and expenditure, representing market interactions, whole economy effects and interferences with other policies, CGE models’ conjectures may be far from reality.

Actually, as DeCanio (2003) details, there are several shortcomings in GE theory. Human manners are so complex that the ideology fails to embody many social and ethical issues and general institutional agreements, for instance the spending on healthcare, culture or education that are usually made available for free or below their cost. Overall, GE ideas focus on a self-interest materialistic utility, which is a very limited representation of human conduct.

Another theoretical flaw is the static equilibrium hypothesis – another misrepresentation of reality, since there isn’t an equilibrium state of the world, and at best a natural equilibrium is dynamic. Real market imbalances, such as unemployment, and other structural barriers are abundant and often lacking representation. Some models include a wage curve that correlates the wage level with an unemployment tax (Böhringer et al., 2013; Böhringer and Rutherford, 2013), yet other gaps remain.

Technological change is also absent from most models or it is imposed externally (Scrieciu, 2007), although non-disruptive evolution of technology can be represented endogenously as a function of cumulative production (learn-by-doing); a few models even consider knowledge accumulation (Böhringer and Löschel, 2006).

Scrieciu (2007) also mentions another relevant issue: CGE models tend to have aggregate sectors and regions with average effects while environmental problems tend to be local: the pollution of a lake, high concentration of toxic gases, etc. Then again, disparities between the model and reality are unavoidable – as in any scientific subject, one must be aware of the limitations of our work. There are still variables which are not incorporated into the problem (known unknowns) and there are even matters beyond our knowledge (unknown unknowns).

In general, CGE models are a great tool to develop policies as they provide useful estimates, at least the order of magnitude, of several economic and environmental indicators. CGE models can be specialized to analyze emissions of other green house gases, include market distortions and technical change and even build in social-economic analysis by including different households instead of a single representative agent. As long as the models are used with an understanding of their limitations and complemented by other instruments, opinions from different fields and distinct perspectives, they
provide plenty of value-added.
Theoretical Background

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Prior to detailing the CGE used during this work to describe Portugal, in both economic and energy terms, there are some essential concepts that must be described. Specifically, Production Functions (PFs) and elasticities of substitution that are essential to model the industrial sectors and consumer preferences. The concepts of aggregate goods and utility are also introduced.

3.1 Production Functions

A production function is a functional form that describes how a set of inputs is combined to generate a single output (Dixon and Jorgenson, 2013). For example, to build a paper clip we need metallic wire, a pressing machine to bend the clip and a person to operate that machine. Assuming that this machine can produce up to 3000 clips during a 24h day and each clip requires one piece of wire, then the production function would represent the transformation of inputs into clips as

\[ \#\text{Clips} = \min(\#\text{wire pieces}, 3000 \#\text{machines}) \] (3.1)

It is also necessary to find workers to operate the machines, which only work 8h shifts and produce 1000 clips during that period. Considering this constraint, the number of clips produced each day is

\[ \#\text{Clips} = \min(\#\text{wire pieces}, 3000 \#\text{machines}, 1000 \#\text{workers}) \] (3.2)

Even if there is enough wire to produce more clips, without sufficient machines or workers, the output will not increase. This production function does not allow any substitution between its inputs, i.e. it is not possible to compensate the lack of workers with more machines. However, there is more than a way of producing paper clips. If the machines are too expensive and employing workers is cheap, maybe it is more efficient to buy pliers and hire more labor. We are now substituting inputs, so we need a more flexible production function, such as the Constant Elasticity of Substitution (CES) Production Function.

The CES is generally formulated as

\[ y(\mathbf{x}) = A \left( \sum_i a_i x_i^\alpha \right)^{1/\alpha} \] (3.3a)

\[ \alpha = \frac{\sigma - 1}{\sigma} \] (3.3b)

\[ \sum_i a_i = 1 \] (3.3c)

The output \( y \) is a function of the input vector \( \mathbf{x} \), \( A \) and \( a_i \) are shift and share parameters used to calibrate the function and the constant \( \sigma \) is the elasticity of substitution, which indicates how flexible the production is - higher elasticities indicate that is easier to replace one input with others (\( \alpha \) was defined to simplify the notation). The elasticity is constrained to a non-negative interval: \( \sigma \geq 0 \). The particular cases \( \sigma = 0 \) and \( \sigma = 1 \) lead to different functional forms:

1. Leontief (\( \sigma = 0 \))

\[ y(\mathbf{x}) = A \min(a_1 x_1, \ldots, a_n x_n) \] (3.4)
2. Cobb-Douglas ($\sigma = 1$)

\begin{align*}
y &= A \prod x_i^{a_i} \\
\sum a_i &= 1
\end{align*}  \hspace{1cm} (3.5a)  \hspace{1cm} (3.5b)

Using isoquants, i.e. seeing which different sets of $x_i$ and $x_j$ yield the same output $y$ according to the elasticity, we get Figure 3.1. The elasticity parameter ranges from $0$, the Leontief production function (fixed proportions), to $\infty$, which corresponds to the linear production function (output is the sum of inputs).

![Figure 3.1: Isoquants for different PF: linear $\sigma = \infty$, Cobb-Douglas $\sigma = 1$ and Leontief $\sigma = 0$](image)

### 3.1.1 Cost Optimization

Production functions describe how firms process their inputs, yet they convey no information regarding the price $p_i$ of each input or the budget limit. Here we follow the line of reason of Rutherford’s lecture notes to obtain the calibrated expressions of the previously mentioned production functions:

**CES**

\[
y = y \left[ \sum_i \theta_i \left( \frac{x_i}{\bar{x}_i} \right)^\alpha \right]^{1/\alpha}
\]  \hspace{1cm} (3.6)

**Leontief**

\[
y = \frac{y}{x_i}
\]  \hspace{1cm} (3.7)

**Cobb-Douglas**

\[
y = y \prod \left( \frac{x_i}{\bar{x}_i} \right)^{\theta_i}
\]  \hspace{1cm} (3.8)

Remembering the zero profit hypothesis, we know that revenue balances cost

\[
R - C = 0
\]

\[
R - \sum_i x_i p_i = 0
\]  \hspace{1cm} (3.9)

Using the Lagrangian, it is possible to find the optimal set of inputs $\mathbf{x}$ that maximize $y$ and comply with the zero profit constraint.

\[
\mathcal{L} = y(\mathbf{x}) - \lambda (R - C)
\]  \hspace{1cm} (3.10)
Solving with respect to a particular input

\[
\frac{\partial \mathcal{L}}{\partial x_i} = a_i \left( \frac{x_i A^{1/\alpha}}{y} \right)^{\alpha - 1} + \lambda p_i = 0
\]  

(3.11)

Then

\[
x_i = \frac{y}{A^{1/\alpha}} \left( -\frac{\lambda p_i}{a_i} \right)^{\frac{1}{\alpha - 1}}
\]  

(3.12)

Now we eliminate the Lagrange multiplier \( \lambda \) using the budget restriction

\[
C = \sum_j p_j x_j
\]

\[
= \sum_j p_j \frac{y}{A^{1/\alpha}} \left( -\frac{\lambda p_j}{a_j} \right)^{\frac{1}{\alpha - 1}}
\]

\[
= (-\lambda)^{\frac{1}{\alpha - 1}} \frac{y}{A^{1/\alpha}} \sum_j p_j \left( \frac{p_j}{a_j} \right)^{\frac{1}{\alpha - 1}}
\]

(3.13)

So the optimal demand for each input is

\[
x_i = \frac{C}{\sum_j p_j \left( \frac{p_j}{a_j} \right)^{\frac{1}{\alpha - 1}}} \left( \frac{p_i}{a_i} \right)^{\frac{1}{\alpha - 1}}
\]

\[
= \frac{C}{\sum_j p_j \left( \frac{a_j}{p_j} \right)^{\frac{1}{\alpha}}} \left( \frac{a_i}{p_i} \right)^{\sigma}
\]

(3.14)

Yet, it is still necessary to find values for the share and the elasticity parameters. In GE it is assumed that the collected data represents an equilibrium state of the economy, so the measured inputs \( \bar{x}_i \) at benchmark prices \( \bar{p}_i \) and output \( \bar{y} \) can be used to calibrate the production functions.

\[
\bar{x}_i = \frac{\bar{C} \left( \frac{\bar{p}_i}{\bar{a}_i} \right)^{\frac{1}{\alpha - 1}}}{\sum_j \bar{p}_j \left( \frac{\bar{p}_j}{\bar{a}_j} \right)^{\frac{1}{\alpha - 1}}}
\]

(3.15)

Through algebraic manipulation it is possible to calibrate the share parameter as

\[
a_i = \frac{\bar{p}_i \bar{x}_i}{\bar{C} \left[ \sum_j \bar{p}_j \left( \frac{\bar{p}_j}{\bar{a}_j} \right)^{\frac{1}{\alpha - 1}} \right]^{\alpha - 1} \left( \frac{\bar{x}_i}{\bar{C}} \right)^{\alpha}}
\]

\[
= \frac{\left[ \sum_j \bar{p}_j \left( \frac{\bar{p}_j}{\bar{a}_j} \right)^{\frac{1}{\alpha - 1}} \right]^{1 - \alpha} \theta_i}{C^\alpha \bar{x}_i^{\alpha}}
\]

(3.16)

Where we introduced the parameter \( \theta_i \) that corresponds to the cost share of input \( i \)

\[
\theta_i = \frac{\bar{p}_i \bar{x}_i}{\sum_i \bar{p}_i \bar{x}_i}
\]

(3.17a)

\[
\sum_i \theta_i = 1
\]

(3.17b)
Placing it back into equation 3.3a

\[ y(x) = A \left( \sum_i \left[ \frac{\sum_j \bar{p}_j \left( \frac{\bar{x}_j}{a_j} \right)}{C^{\alpha}} \right] \frac{1-\alpha}{\theta_i x_i^{\alpha}} \right)^{1/\alpha} \]  

(3.18)

\[ = \frac{A}{C} \left[ \sum_j \bar{p}_j \left( \frac{\bar{x}_j}{a_j} \right) \right]^{1-\alpha/\gamma} \left[ \sum_i \theta_i \left( \frac{x_i}{\bar{x}_i} \right)^{\alpha} \right]^{1/\alpha} \]

Evaluating the function at the benchmark year yields

\[ \tilde{y}(\bar{x}) = \frac{A}{C} \left[ \sum_j \bar{p}_j \left( \frac{\bar{x}_j}{a_j} \right) \right]^{1-\alpha/\gamma} \left[ \sum_i \theta_i \left( \frac{x_i}{\bar{x}_i} \right)^{\alpha} \right]^{1/\alpha} \]

(3.19)

Dividing Equation 3.18 by 3.19, the CES production function can be elegantly written as

\[ y(x) = \tilde{y} \left[ \sum_i \theta_i \left( \frac{x_i}{\bar{x}_i} \right)^{\alpha} \right]^{1/\alpha} \]

(3.20)

Likewise, combining Equations 3.14 and 3.16 reveals that the cost \( C \) only depends the inputs’ prices and the benchmark cost

\[ C(p) = \tilde{C} \tilde{y} \left[ \sum_i \theta_i \left( \frac{p_i}{\bar{p}_i} \right)^{1-\sigma} \right]^{\sigma/\gamma} \]

(3.21)

The CES production function is very convenient, however the Leontief and Cobb-Douglas cases must be treated individually.

### 3.1.2 Leontief Production Function

In the paper clips example at the beginning of the chapter the productive factors were used in fixed proportions as the elasticity of substitution \( \sigma \) was 0. We were using a Leontief production function, which is generally calibrated as

\[ y(x) = \bar{y} \min \left( \frac{x_1}{\bar{x}_1}, \ldots, \frac{x_n}{\bar{x}_n} \right) \]

(3.22)

The optimal use implies that there is no exceeding input, so for any input \( i \)

\[ \frac{y}{y} = \frac{x_i}{\bar{x}_i} \]

(3.23)

And the total cost is

\[ C = y \sum_i \bar{p}_i \frac{\bar{x}_i}{y} \]

(3.24)

\[ = \tilde{C} \tilde{y} \sum_i \theta_i \frac{p_i}{\bar{p}_i} \]

(3.25)
3.1.3 Cobb-Douglas Production Function

The Cobb-Douglas production is heavily used in aggregate economic analyses since, under competition, the factor income shares are independent of relative factor prices (Miller, 2008). This property of the Cobb-Douglas production function is often consistent with the data as, historically, the real wage has increased by a factor of ten or twenty while the rental price of capital and factor income shares have remained roughly constant (Miller, 2008). Formally, it is presented as

\[ y = A \prod x_i^{a_i} \]

(3.26a)

\[ \sum a_i = 1 \]

(3.26b)

Following once again the Lagrangian optimization method

\[ L = y(x) - \lambda (R - C) \]

(3.27)

Setting equal to 0 the partial derivatives relative the input \( x_i \) we obtain

\[ \frac{\partial L}{\partial x_i} = a_i \frac{y}{x_i} + \lambda p_i = 0 \]

(3.28)

\[ x_i = -\frac{a_i y}{\lambda p_i} \]

(3.29)

From the budget restriction we extract the multiplier:

\[ \lambda = -\frac{y}{C} \]

(3.30)

So the input demand \( x_i \) is easy to determine

\[ x_i = \frac{a_i C}{p_i} \]

(3.31)

Finding the share parameters is equally simple

\[ a_i = \frac{\bar{x}_i \bar{p}_i}{\bar{C}} = \theta_i \]

(3.32)

So the calibrated Cobb-Douglas function takes the form

\[ y = \bar{y} \prod_i \left( \frac{x_i}{\bar{x}_i} \right)^{\theta_i} \]

(3.33)

And the total cost is

\[ C = \bar{C}\bar{y} \prod_i \left( \frac{p_i}{\bar{p}_i} \right)^{\theta_i} \]

(3.34)

3.1.4 Shepard’s Lemma

Shepard’s Lemma (Shephard, 1953) simply states that given a expenditure function \( C \), differentiable at \( (p, y) \) and \( p_i > 0 \) for \( i = 1, \ldots, n \) then the demand for input \( i \) is

\[ x_i(p, y) = \frac{\partial C(p, y)}{\partial p_i} \]

(3.35)

For example, we can compute the input demand of the CES Production Function without evaluating Equation 3.14

\[ x_i = \frac{\partial C}{\partial p_i} = \bar{x}_i \bar{y} \left( \frac{C/\bar{C}}{p_i/\bar{p}_i} \right) \]

(3.36)
3.2 Production Structure

Despite the convenience of the CES, this function alone does not grasp the complexity inherent to the production structure of the sectors we want to represent in the model. In this Section we describe how different PFs are combined in larger nesting structures that make up the production sectors.

In the paper clips example, while the machines – which correspond to the capital investment – could be replaced by other tools and more workers, the metallic wires are always required to produce clips. Thus, when producing the output $Y$ there is some elasticity of substitution between Capital ($K$) and Labor ($L$) but there is no method to replace the Material Goods ($M$) with other inputs.

This production structure can be represented in diagrams, such as the one in Figure 3.2. The capital-labor aggregation is a CES with elasticity $\sigma_{KL}$ (oblique lines) that is nested with $M$ within a Leontief Production Function (whose static coefficients are represented through orthogonal lines).

\[ KL(\alpha) = \bar{K}L \left[ \sum_i \theta_i \left( \frac{x_i}{\bar{x}_i} \right)^\alpha \right]^{1/\alpha} \]  
(3.37)

and

\[ Y = \bar{Y} \min \left( \frac{KL}{\bar{K}L}, \frac{M}{\bar{M}} \right) \]  
(3.38)

However, it is not practical to determine $\bar{K}L$ or to use the minimum function that constitutes the Leontief PF. To simplify the process, we compute the unitary output cost ($c_Y$), i.e. the price of producing one unit of output $Y$. From that function, we can determine the demand for each input using Shepard’s Lemma, regardless of the complexity of the production structure.

We start with the budget constraint

\[ R = C \]

\[ p_y Y = p_K K + p_L L + p_M M \]  
(3.39)

Dividing the equation by the benchmark budget equation

\[ \frac{p_y Y}{R} = \frac{p_K K + p_L L + p_M M}{C} \]  
(3.40)
To simplify the expression, the share costs are represented by \( \theta \)

\[
\theta_M = \frac{\bar{p}_M \bar{M}}{C} \quad \theta_{KL} = 1 - \theta_M \\
\theta_K = \frac{\bar{p}_K \bar{K}}{\bar{p}_K \bar{K} + \bar{p}_L \bar{L}} \quad \theta_L = \frac{\bar{p}_L \bar{L}}{\bar{p}_K \bar{K} + \bar{p}_L \bar{L}} 
\]

(3.41)

So Equation 3.40 becomes

\[
\frac{p_Y}{\bar{p}_Y} = \theta_{KL} \left[ \theta_K \left( \frac{\bar{p}_K}{\bar{p}_K} \right) + \theta_L \left( \frac{\bar{p}_L}{\bar{p}_L} \right) \right] + \theta_M \frac{p_{KL} M}{p_{KL} M} 
\]

(3.42)

From 3.36 we know

\[
\frac{K}{K} = KL \left( \frac{C \bar{p}_K}{C \bar{p}_K} \right)^\sigma \quad \frac{L}{L} = KL \left( \frac{C \bar{p}_K}{C \bar{p}_K} \right)^\sigma 
\]

(3.43)

Which we plug back into the main expression

\[
\frac{p_Y}{\bar{p}_Y} = \theta_{KL} \left[ \theta_K \left( \frac{\bar{p}_K}{\bar{p}_K} \right)^{1-\sigma} + \theta_L \left( \frac{\bar{p}_L}{\bar{p}_L} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} + \theta_M \frac{p_{KL} M}{p_{KL} M} 
\]

(3.44)

And using the equation 3.23 we can simplify the previous expression to draw a relationship exclusively between input and output prices

\[
\frac{p_Y}{\bar{p}_Y} = \theta_{KL} \left[ \theta_K \left( \frac{\bar{p}_K}{\bar{p}_K} \right)^{1-\sigma} + \theta_L \left( \frac{\bar{p}_L}{\bar{p}_L} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} + \theta_M \frac{p_{KL} M}{p_{KL} M} 
\]

(3.45)

Where the price of the aggregate \( KL \) is also present

\[
\frac{p_{KL} M}{p_{KL} M} = \left[ \theta_K \left( \frac{\bar{p}_K}{\bar{p}_K} \right)^{1-\sigma} + \theta_L \left( \frac{\bar{p}_L}{\bar{p}_L} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} 
\]

(3.46)

These formulations, also known as unitary expenditure functions, represent the cost of producing one unit of output of a given industry

\[
c_Y = \frac{p_Y}{\bar{p}_Y} \equiv \frac{C/\bar{C}}{y/\bar{y}} 
\]

(3.47)

(3.48)

They can be broadly defined according to the underlying production function

\[
\text{Leontief} \quad c_Y = \sum_i \theta_i \frac{p_i}{\bar{p}_i} 
\]

(3.49a)

\[
\text{CES} \quad c_Y = \left[ \sum_i \theta_i \left( \frac{p_i}{\bar{p}_i} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} 
\]

(3.49b)

\[
\text{Cobb-Douglas} \quad c_Y = \prod_i \left( \frac{p_i}{\bar{p}_i} \right)^{\theta_i} 
\]

(3.49c)

Where \( c_Y \) represents the aggregated price of what goes into the production of one unit of output – be it material goods or primary factors. These are the expressions that are used to incorporate the PFs in the model, they are included in the Profit equations and their derivatives are used to compute each sector’s input demand.
This is particularly interesting when trying to build a flexible code framework, as more nesting levels can be included at will.

The structure in Figure 3.3-a) represents the following set of expressions\(^1\)

\[
c^M = \left[ \sum_{i \in M} \theta_i \left( \frac{p_i}{\bar{p}_i} \right)^{1-\sigma_M} \right]^{\frac{1}{1-\sigma_M}}
\]

\[
c^E = \left[ \sum_{i \in E} \theta_i \left( \frac{p_i}{\bar{p}_i} \right)^{1-\sigma_E} \right]^{\frac{1}{1-\sigma_E}}
\]

\[
c^{KL} = \left[ \theta_K \left( \frac{p_K}{\bar{p}_K} \right)^{1-\sigma_{KL}} + \theta_L \left( \frac{p_L}{\bar{p}_L} \right)^{1-\sigma_{KL}} \right]^{\frac{1}{1-\sigma_{KL}}}
\]

\[
c^{KLE} = \left[ \theta_{KL} \left( c^{KL} \right)^{1-\sigma_{KLE}} + \theta_E \left( c^E \right)^{1-\sigma_{KLE}} \right]^{\frac{1}{1-\sigma_{KLE}}}
\]

\[
c^{KLEM} = \left[ \theta_{KLE} \left( c^{KLE} \right)^{1-\sigma_{KLEM}} + \theta_M \left( c^M \right)^{1-\sigma_{KLEM}} \right]^{\frac{1}{1-\sigma_{KLEM}}}
\]

Incidentally, Okagawa and Ban (2008) tested how compositions 3.3-a) and 3.3-b) would fit compared to real data and the outcome was that the quality of the elasticity parameters was more important than the structure itself.

### 3.3 Constant Elasticity of Transformation

Just like a production process typically requires a variety of inputs, it often also generates a mixture of different commodities, which can be split into final goods to be sold either in the domestic market or abroad. After choosing the production process that minimizes costs, a sector’s output is partitioned into different goods to maximize revenue. The Constant Elasticity of Tranformation is the function responsible for this process, much like the CES manages the combination of inputs, yet with a different

---

\(^1\)If the elasticity in a given nest is 0, the CES becomes a Leontief PF, by if it is 1 the expression must be replaced by Equation 3.49c.
concavity.

\[ X_1 \cdots X_n \]

\[ \eta \]

\[ Y \]

**Figure 3.4:** Diagram of the disaggregation of a firms’ output \( Y \) into a vector consumable goods \( X \). \( \eta \) is the elasticity of output transformation parameter.

The mathematical formulation is identical, except for the range of possible values for \( \eta \), which must be non-positive (\( \eta \leq 0 \)).

\[
Y = \bar{Y} \left[ \sum_i \theta_i \left( \frac{X_i}{X_j} \right)^{1-\eta} \right]^{1/1-\eta} \tag{3.51}
\]

The upper bound limit, \( \eta = 0 \), corresponds to a linear transformation, i.e. by reducing the production of good \( i \) it is possible to increase the output of good \( j \) by the same amount. Usually there are some inefficiencies (\( \eta < 0 \)) and the production possibilities follow the pattern in Figure 3.5. When the elasticity of transformation tends to \(-\infty\), we fall into the Leontief case and firms always produce the goods in the same proportion.

**Figure 3.5:** Constant Elasticity of Transformation isoquants: Linear (\( \eta = 0 \)), Leontief (\( \eta \rightarrow \infty \)).

### 3.4 Utility and Consumer Behaviour

In a similar fashion, production functions can be used to model how an agent, such as Families or Government, behaves from an economic point of view. That is, what goods it consumes based on their prices and the agent’s budget. This thesis follows neoclassical economic theory (Armington, 1969),
where the preferences are portrayed through utility functions, particularly the CES.

\[ U^C = \bar{U}^C \left[ \sum_i \theta_i \left( \frac{X^C_i}{X^C} \right)^{1-\sigma_i} \right]^{\frac{1}{1-\sigma_i}} \quad (3.52) \]

Equation 3.52 is equivalent to 3.20, although the output is Utility \((U)\) instead of a good, or an aggregate of commodities. That being so, it is possible to use the same optimization method to determine which set of goods maximizes the Utility within the Family or Government budget.

Following from Equation 3.52 and using a fictitious sector that aggregates goods \(X^C_i\) into the composite good \(C\), according to the consumption preferences expressed by the \(\theta\)s that are computed according to the benchmark consumption pattern

\[ \theta_i = \frac{\bar{p}_i \bar{X}^C}{\sum_j \bar{p}_j X^C_j} \quad (3.53) \]

And the unitary cost is

\[ c^C = \left[ \sum_i \theta_i \left( \frac{p_i}{\bar{p}_i} \right)^{1-\sigma_i} \right]^{\frac{1}{1-\sigma_i}} \quad (3.54) \]

So this fictitious sector has the following unitary profit equation, which is 0 in equilibrium

\[ \Pi^C = \frac{pC}{\bar{p}_C} - \left[ \sum_i \theta_i \left( \frac{p_i}{\bar{p}_i} \right)^{1-\sigma_i} \right]^{\frac{1}{1-\sigma_i}} \quad (3.55) \]

For instance, families spend their consumption budget \(M\) purchasing the aggregate good \(C\)

\[ M = p_C C \quad (3.56) \]

Finally, in the model, the consumption of each individual good \(X^C_i\) is derived from Shepard’s lemma

\[ X^C_i = C \frac{\partial c^C}{\partial p_i} = -C \frac{\partial \Pi^C}{\partial p_i} \quad (3.57) \]

While the consumer preferences (included in the set of \(\theta\)s) and the price variations determine the proportion of consumed goods, the amount is specified by the budget \(M\) that constrains \(C\).

### 3.5 International Trade

The region that is being modeled is not isolated from the outside world, in fact, it trades with other regions. Although it may seem reasonable to argue that if a product is available at a lower price in another country then consumers will completely forego their domestic equivalents, this is not an accurate description of reality. Clearly the domestic and the imported varieties of a commodity are imperfect substitutes that once again can and must be merged into an compound version before being consumed.

The Armington assumption states that first that consumers pick the basket of goods they are going to purchase and afterwards, for each good within that basket, they chose the proportion of domestic and imported varieties in order to minimize cost while maximizing utility. The underlying hypothesis is that those choices are independent from one another (Armington, 1969).
Figure 3.6: Armington Aggregation: 1 – producers sectors and other consumers elaborate their basket of commodities; 2 – they select the ratio of domestic and imported varieties.

So most of the equations that have been discussed so far must be corrected to include the Armington aggregate good $A_i$. Furthermore, we assume that Portugal is a small region in the world so it has no effect on the price of the imported and exported goods, which are defined outside of the model (exogenous variables), although those prices affect the amount of imports and exports. Also, the Rest of the World (RoW) is always capable of supplying the demanded goods and absorbing the exported ones. In the model there is going to exist an Armington sector for each commodity that aggregates the domestic $X_{iD}$ and the imported $X_{iM}$ versions of the good and returns a single aggregate commodity.

\[
\Pi_i^A = \frac{p_i^A}{\bar{p}_i^A} - \left[ \theta_i^D \left( \frac{p_i^D}{\bar{p}_i^D} \right)^{1-\sigma_A} + \theta_i^M \left( \frac{p_i^M}{\bar{p}_i^M} \right)^{1-\sigma_A} \right]^{\frac{1}{1-\sigma_A}}
\]  

(3.58)
4

Model Description

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In this Chapter we develop the mathematical formulation that describes an approximation of the Portuguese economy, using concepts and assumptions from GE theory, which were illustrated in Chapter 3.

### 4.1 Computational Problem

Large systems, such as the CGE model developed in this thesis, must be solved numerically. Hence, we created a computational project that could read and process the data from Chapter 5 to calibrate the equations detailed in this Chapter and, finally, find a new equilibrium.

![Figure 4.1: Schematics underlying the computational project.](image)

The set of Equations that make up the model can be interpreted as

\[
y = F(x, \theta, \sigma) = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix} = 0
\]  

(4.1)

Here, each \( f_i \) represents an equation and finding a new equilibrium is equivalent to solving \( F(x) = 0 \).

The calibration process consists of using the information in the SAM to compute the parameters in the model (set of \( \theta \)) and to fill benchmark values for every variable (vector \( \bar{x} \)). It is also necessary to assign values to the elasticities (set of \( \sigma \)). To simulate a certain scenario, one must shock the model by giving values for a fixed amount of variables – the exogenous variables (\( x_X \)).

\[
F(x, \theta, \sigma) = 0 \rightarrow F(x_E, x_X, \theta, \sigma) = 0
\]

(4.2)

Now, there are as many variables in \( x_E \) as there are equations in \( F \). Afterwards, we find the set of endogenous variables (\( x_E \)) that meets the requirements of Equation 4.2, in other words, we solve the system. Thus, we have a solution \( x \) for the Scenario.

### 4.2 Model Overview

Portugal is the main region being modeled in this CGE. It has 2 agents: 1 representative Household and 1 Government. They supply two primary production factors: Labor (L) and Capital or Assets (K).
or A), receiving money in return. There are a total of $s$ production sectors that create up to $g$ different commodities. Goods and industries are both divided into 2 groups: Energy Goods (E) and Material (or Non-Energy) Goods (M), since they are going to be modeled into an slightly different manner.

Figure 4.2 has a stylized illustration of the model. The large square represents Portugal as an open economy, capable of interacting with the RoW.

The following Sections contain a detailed description of the model, including the behavior of the previously mentioned agents.

![Diagram of Macroeconomic structure of Portugal](image)

**Figure 4.2:** Macroeconomic structure of Portugal.

In order to simplify the notation, all benchmark prices, including those related to imports and exports, were normalized to 1. Also, index $i$ always refers to goods and sums over $1,\ldots,s$ and index $j$ does the same for the production sectors $(1,\ldots,g)$.

### 4.3 Production Sectors

The Production Sectors, Firms or Industries transform inputs, a combination of goods and factors, into goods and services. For a certain production structure, each firm tries to maximize its profits, which means it will try to produce as much output as possible while complying with a budget constraint following the optimization path shown in the previous section.

After considering several options, the structure chosen for the group of sectors that are not involved in energy production is represented in Figure 4.3.

Following the structure presented in Figure 3.3-a), $K$ and $L$ were combined using a CES whose elasticity value depends on the sector. The non-Energy $M$ are also aggregated using a CES, but its elasticities are usually much smaller or even 0 in most CGE models, so a Leontief function was chosen. The $E$ nest will be merged using a CES, but its elasticity is not relevant for most sectors as they only consume electricity. Finally, both the added-value-energy and the material composite goods are used in fixed proportions.

Figure 4.4 represents the production structure of the Energy sectors. For these technologies, $\sigma_{KL} = 1$, $\sigma_k = 1$ and every other elasticity is zero. The unitary cost of sector $j$’s output is computed as shown
Figure 4.3: Production structure of the non-Energy sectors.

Figure 4.4: Production structure of the Energy sectors.
in Equation 4.3.

\[ c_j^{KL} = \left( \theta_L (p_L)^{1-\sigma_{KL}} + (1 - \theta_L) (p_K)^{1-\sigma_{KL}} \right)^{-\frac{1}{\sigma_{KL}}} \]

\[ c_j^E = \left( \sum_{i \in E} \theta_j^E (p_i^A)^{1-\sigma_E} \right)^{-\frac{1}{\sigma_E}} \]

\[ c_j^{KLE} = \left[ \theta_{KL} (c_j^{KLE})^{1-\sigma_{KLE}} + (1 - \theta_{KL}) (c_j^E)^{1-\sigma_{KLE}} \right]^{rac{1}{1-\sigma_{KLE}}} \]

\[ c_j^Y = \left[ \theta_j^M \left( \sum_{i \in M} \theta_j^IN (p_i^A)^{1-\sigma_M} \right)^{1-\sigma_M} + (1 - \theta_j^M) (c_j^{KLE})^{1-\sigma_Y} \right]^{rac{1}{1-\sigma_Y}} \]

Finally, their output is split into several commodities and then in exports or domestic varieties, as seen previously in Figure 3.4.

\[ r_j^Y = r_j^Y \left[ \sum_i \theta_j^OUT \left( \theta_i^X (p_i^X)^{1-\sigma_X} + (1 - \theta_i^X) (p_i^D)^{1-\sigma_X} \right)^{1-\sigma_X} \right]^{1-\sigma_{OUT}} \]

Resulting in

\[ \text{Revenue}_j = Y_j r_j^Y \] (4.5)

Besides the cost of inputs, both goods and productive factors, the production sectors also invest money to improve their productivity and compensate for the degradation of their Capital stock. Assuming that the Depreciation of Capital in each industry is proportional to the total stock in the economy, then:

\[ \delta K_j = \delta \bar{K}_j \frac{KS}{\bar{K}S} \]

\[ \delta KS = \sum_j \delta K_j \] (4.6)

Taxes on production are charged as a fraction of cost, therefore the total costs of firm \( j \) amount to

\[ \text{Costs}_j = Y_j c_j^P (1 + t_j^P) + \delta K_j \] (4.7)

From equations 4.5 and 4.7, the profit of sector \( j \) is

\[ \Pi_j^Y = Y_j r_j^Y - Y_j c_j^P (1 + t_j^P) - \delta K_j \] (4.8)

Which implies

\[ r_j^Y = 1 + t_j^P + \frac{\delta \bar{K}_j}{Y_j} \] (4.9)

The demand for each input can be drawn from the profit equation

\[ X_j^{IN} = \left. \frac{\partial \Pi_j^Y}{\partial p_i^A} \right|_{1 + t_j^P} \] (4.10a)

\[ L_j = \left. \frac{\partial \Pi_j^Y}{\partial p_L} \right|_{1 + t_j^P} \] (4.10b)

\[ K_j = \left. \frac{\partial \Pi_j^Y}{\partial p_K} \right|_{1 + t_j^P} \] (4.10c)
The Capital resources that the firms demand aren’t met by their own stock, instead they borrow assets from Households, the Government or the Rest of the World, such as property, money or others, which grant them a mixed income of interests and rents. Nevertheless, Capital reallocation has losses – not every unit of Capital can be repurposed and some turns into waste. This shall be portrayed using variable that representst the percentage of unused capital.

\[(K_C + K_G)(1 - u_K) + \dot{K}_{RoW} = \sum_j K_j \]  \hspace{1cm} (4.11)

So, if the borrowed Capital in a firm is smaller than the benchmark value part of it is going to be destroyed.

\[u_K = \frac{\text{Lost Capital}}{K_C + K_G} \]  \hspace{1cm} (4.12)

With

\[\text{Lost Capital} = \sum_j \frac{\dot{K}_j - K_j}{e^{K_{mob}}} \text{ if } \dot{K}_j > K_j \]  \hspace{1cm} (4.13)

Likewise, the production of goods, both Domestic \(X_{ij}^D\) and Exported \(X_{ij}^X\), come from the same profit function.

\[X_{ij}^D = \frac{\partial Y_j}{\partial p_i^D} \]  \hspace{1cm} (4.14a)

\[X_{ij}^X = \frac{\partial Y_j}{\partial p_i^X} \]  \hspace{1cm} (4.14b)

As for the Formation of Fixed Capital, the investment in each commodity \(i\) can be modeled according to its price

\[I_i = \frac{\tilde{p}_i^A \tilde{I}_i}{p_i^A I} \]  \hspace{1cm} (4.15)

Then the net change of capital stock in the total economy is the difference between investment and depreciation

\[\Delta K = I - \delta K \]

\[K_S = \dot{K}_S + \Delta K \]  \hspace{1cm} (4.16)

### 4.4 Households

Families also have their own budget equation, where income must balance consumption \(M\) and savings \(S\). There are three main sources of income: wages and rents from lending assets and a foreign spending term that compensates any trades performed with the Rest of the World. The latter is defined in the Trade subsection and can be negative. Furthermore, a Current Transfers \(\bar{CT}_C\) parameter is introduced to represent other flows which are not being modeled.

#### 4.4.1 Wage Curve

Wage income is obtained by renting Labor to the production sectors, however, due to unemployment, there is a percentage of the Labor force that doesn’t receive any money. The unemployment rate is directly related to the price of Labor, or wage rate, through a wage curve. The parameter \(\psi\),
usually negative, represents the wage elasticity with respect to the unemployment rate: an increase in wages leads to a change in unemployment proportional to 

\[
\frac{p_L}{\bar{p}_L} = \left( \frac{u_R}{\bar{u}_R} \right) ^ \psi
\]  

(4.17)

And the labor used by all the production sectors must equal the employed labor.

\[
L(1 - u_r) = \sum_j L_j + L_{RoW}
\]  

(4.18)

So, discounting both unemployment and taxes, the total wages are

\[
\text{Income} = p_i L(1 - u_r) + p_K K_C (1 - u_K)
\]  

(4.19)

### 4.4.2 Consumer Budget

Therefore, the budgetary equilibrium for the consumers is defined as follows

\[
M_C + S_C = \left[ p_i L(1 - u_r) + p_K K_C (1 - u_K) \right] (1 - t^F) + \bar{C} T_C
\]  

(4.20)

And the ratio between spending and savings is called the savings rate

\[
s_C = \frac{S_C}{S_C + M_C}
\]  

(4.21)

The consumption is spent acquiring the aggregate consumer good

\[
p_C C = M_C
\]  

(4.22)

With the following profit equation based on acquisition (or Armington) prices:

\[
\Pi^C = p_C - \left[ \sum_i \theta_i^C \left( p_i A_i \right)^{1 - \sigma_C} \right] \frac{1}{\sigma_C}
\]  

(4.23)

So the individual demand for commodity \(i\) within the family’s consumption basket is

\[
X_i^C = -C \frac{\partial \Pi^C}{\partial p_i^C}
\]  

(4.24)

### 4.5 Government

The Government collects taxes on several transactions present in the economy, such as:

1. Goods: imported, exported or produced domestically
2. Revenues of each industry
3. Household Income

When being taxed, the domestically produced goods also include the exported variety.

\[
T = \sum_i \left[ p_i^D X_i^D + p_i^X X_i^X + p_i^M X_i^M t_i^M \right] t_i^G
\]

\[
+ \sum_j Y_j^P \Delta_j^P t_j^P
\]

\[
+ \left[ p_i L(1 - u_r) + p_K K_C (1 - u_K) \right] t^F
\]  

(4.25)
The government also accumulates savings

\[ s_G = \frac{S_G}{S_G + M_G} \]  
(4.26)

So the government’s budget balance is the following

\[ S_G + M_G = T + p_KK_G + C_T G \]  
(4.27)

And \( M_G \) indicates the value of its consumption basket

\[ p_G G = M_G \]  
(4.28)

Whose profit equation is

\[ \Pi^G = p_G - \left[ \sum_i \theta_i^G (p_i^A)^{1-\sigma_G} \right]^{\frac{1}{1-\sigma_G}} \]  
(4.29)

So the Government demand for good \( i \)

\[ X_i^G = -G \frac{\partial \Pi^G}{\partial p_i^G} \]  
(4.30)

### 4.6 International Trade and Armington Goods

As discussed above, the Armington good is a combination of both the imported and the domestic variety of a certain good. However there is a difference between the production price of a good, its base price, and the price for which it is sold, the consumer’s price, due to taxes levied on that good and the cost of transporting a commercializing that commodity – Trade and Transport Margin (TTM). Taxes are charged as a percentage of the base price, and margins are included through a Leontief production function as transportation and the good itself are not substitutable.

![Diagram](image-url)

**Figure 4.5:** Combination of Domestic and Imported Goods \( k \) into its Armington aggregate, including taxes and TTM.
Recalling the Armington profit equation from 3.58, one can rewrite it including the adjustments described above.

\[ \Pi_i^A = p_i^A - \left[ \theta_i^D \left( \theta_i^{Mar} p_{TTM} + (1 - \theta_i^{Mar}) \rho_i^D \frac{1 + \gamma_i^G}{1 + \rho_i^G} \right)^{1-\sigma_A} + (1 - \theta_i^D) \left( \rho_i^M \frac{1 + \gamma_i^G}{1 + \rho_i^G} \right)^{1-\sigma_A} \right]^{\frac{1}{1-\sigma_A}} \] (4.31)

The TTM used in the Armington aggregation are provided by another sector which only uses of commodities as inputs, and thus has the following profit:

\[ \Pi_{TTM} = p_{TTM} - \left[ \sum_i \theta_i^{TTM} (p_i^A)^{1-\sigma_{TTM}} \right]^{\frac{1}{1-\sigma_{TTM}}} \] (4.32)

This sector is supplied with TTM\(_i\) from each commodity \(i\) and produces the compound good TTM.

\[ TTM_i = TTM \frac{\partial \Pi_{TTM}}{\partial p_i^A} \] (4.33)

The total consumption of the Armington aggregate good \(k\)

\[ A_i \frac{\partial \Pi_i^A}{\partial p_i^A} = \sum_j X_{ij}^{IN} + X_{ij}^C + X_{ij}^G + I_i + TTM_i \] (4.34)

Which must balance in the production of domestic good \(k\) equations

\[ X_{iD} = \sum_j X_{ij}^{D} \] (4.35a)

\[ X_{iD} = -\frac{A_i \frac{\partial \Pi_i^A}{\partial p_i^D}}{1 + \rho_i^G} \] (4.35b)

And the amount of imported good \(k\)

\[ X_{iM} = -\frac{A_i \frac{\partial \Pi_i^A}{\partial p_i^M}}{1 + \rho_i^G} \] (4.36)

For each Armington Aggregated Good \(i\), the used TTM are

\[ X_{iTTM} = -\frac{A_i \frac{\partial \Pi_i^A}{\partial p_{TTM}}}{1 + \rho_i^G} \] (4.37)

That also balance with the supply of TTM.

\[ \sum_i X_{iTTM} = TTM \] (4.38)

To transform this model into a system with \(n\) equations and variables, it is necessary to set \(s + 3g + 7\) variables as exogenous parameters defined outside the models’ equations.
Data and Methods

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5.3 Code Workflow .................................................. 38
As it has been stated, data from a benchmark year, alongside the elasticity parameters in Table 5.5, is used to calibrate the model. In this case, the representative year is 2010 and the most important source is the (Portuguese) National Statistics Institute (INE), which provides information regarding the production sectors, the commercialized goods and also the Government and the Households. Trades with other region are placed in the RoW account.

This macroeconomic data was organized into a SAM – a table that contains all the transactions within the economy.

5.1 Social Accounting Matrix

The SAM is convenient as it keeps record of both the spending and the invoices of every agent, sector, good or factor. Each one has one row and column which represent their sources of income and expenditure respectively. For example, when the representative Household buys a car, this transaction appears in the Household’s column, as it is an expenditure, and in the car’s row as it counts as income. In a SAM, money flows from columns to rows and goods in the opposite direction.

The SAM in Table 5.1 was organized as follows:

- Products: a block with all the goods and also the TTM
- Production sectors: a block with all the industries
- Factors: Labor and Assets
- Domestic Accounts: a block with the Households, the Government and Corporations
- Financial Account: interactions with the banking sector
- Capital Account: gross capital formation
- Rest of the World: interactions with other regions
- Discrepancy: difference between income and expenditure

Outside the SAM, there are two relevant variables: the unemployment rate and the capital stock. The latter information was provided by AMECO – Annual Macro-Economic database for the European Commission.

The first lines contain the consumption of goods from each entity: goods use Margins, the production sectors have Intermediate Consumption, Government and Household consumption goes in the current account, then there is the Gross Capital Formation. Lastly the Exported commodities go to the Rest of the World’s expenses, in contrast with the Imports that are considered as a product expense.

The Industries’ line is composed of only the Make Table that represents how the output of a certain sector is disaggregated into several goods.

The third block contains use of Factors (Labor (L) and Assets (A)) by the Industries and the RoW region. The income generated by those Factors is distributed by their respective columns.
The block of Current Accounts has in fact three agents: Corporations (both financial and non-financial), Government and to conclude Households and Non-Profit Institutions Serving Households, which were merged into a single agent – Families. The Corporations are not included in the model, however they are needed to guarantee data coherence. Besides Assets, the Government raises funds by collecting taxes on the products and on production. The interaction between all of the agents is known as the ‘whom-to-whom’ matrix, still the model is only focused in two of them, thus the other values will be considered fixed current transfers.

The capital account represents the flows of physical capital. Firms support the depreciation of the capital stock and the discrepancy between the formation and depreciation is its net change. On the other hand, the financial account represents mostly interactions in the bank: the most important are the Savings accumulated by the Households and the Government.

5.1.1 Aggregation Levels

INE follows the Statistical Classification of Economic Activities in the European Community Rev. 2 guidelines, therefore its data is usually organized in either 10, 38 or 88 products and activities. Nevertheless, the classification with an aggregation level of 82 activity branches (A82) was created by the Portuguese System of National Accounts, deriving from the classification established in the implementation Eurostat manual with an aggregation level of 88 activity branches (A88). Additionally, there are two items that are not available at the highest disaggregation level: taxes on activities and capital depreciation by industry. It is possible, though, to estimate them by using the A38 available data. So, the SAM created to represent Portugal has 82 industries and 88 goods. Due to time constraints in the computational aspect of the CGE developed in this thesis, the aggregation used in the simulation only had 10 non-Energy Sectors and 10 non-Energy goods. A schematic version of the SAM with the energy goods and sectors can be seen in Table 5.1 whereas the remaining values are in Appendix B.

5.1.2 Energy Goods and Activities

The preceding SAM portrays all of the economic Portuguese activities, nonetheless in the context of this thesis, the energy commodities and activities require special emphasis. Using the National Energy Balance from 2010, a document elaborated by the DGEG containing the Portuguese energy fluxes – the energy equivalent of a SAM – it was possible to separate the energy activities and goods from the rest. A set of specific Energy related sectors and commodities replaced the following activity branches: Electricity, gas, steam and air conditioning supply, Manufacture of coke and refined petroleum products and Waste management, as they were related to the energy fluxes.

Energy Products

- Coal: mineral coal and lignite
- Petroleum: crude oil and non-fuel derivatives, lubricants, asphalts, paraffin’s, solvents and propylene
<table>
<thead>
<tr>
<th>Item</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
<th>2014</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exports</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Household Consumption</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industrial Products</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Financial Account</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grand Total</td>
<td>411,081</td>
<td>323,625</td>
<td>87,065</td>
<td>47,174</td>
<td>13,966</td>
</tr>
</tbody>
</table>

Table 5.1: Portuguese SAM in 2010
- Natural Gas: mostly methane, butane and also Liquified Petroleum Gas
- Refined Oil: Oil used as fuel to generate electricity
- Distributed Electricity: all electrical energy supplied by the Grid regardless of source
- Electricity from: Thermo, Hydro, Wind, Photovoltaic, Geothermal and Biomass (separate goods)
- Heat: heat from combined cycles
- Biomass: woods, organic residues, briquettes, pellets, Municipal Solid Waste (MSW), biogas and biofuels

Energy Sectors

- Oil refinery
- Thermoelectric
- Hydro power
- Wind power
- Solar power
- Geothermal
- Electric Grid
- Biomass

The oil refinery processes the imported fuel which is partially sold to the Thermoelectric sector and burnt to generate electricity. The Biomass sector yields both electricity and biomass (product). The Electric Grid is the electricity retail sector that combines electricity from all of the other sources into a final homogeneous good. It was necessary, though, to convert the energy fluxes into monetary ones.

For the RESs, the Table 5.2 indicates the average FIT value, which provides an accurate conversion rate since for most technologies all of the produced energy was purchased under the FIT agreement.

<table>
<thead>
<tr>
<th>Technology</th>
<th>Energy (GWh)</th>
<th>Average FIT (€/MWh)</th>
<th>Total FITs (M€)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind</td>
<td>7794</td>
<td>91.07</td>
<td>709.80</td>
</tr>
<tr>
<td>Mini Hydric</td>
<td>885</td>
<td>88.70</td>
<td>78.50</td>
</tr>
<tr>
<td>Biogas</td>
<td>58</td>
<td>111.20</td>
<td>6.45</td>
</tr>
<tr>
<td>Biomass</td>
<td>590</td>
<td>113.40</td>
<td>66.91</td>
</tr>
<tr>
<td>Solar and Others¹</td>
<td>83</td>
<td>344.77</td>
<td>28.62</td>
</tr>
<tr>
<td>Geothermal</td>
<td>196</td>
<td>270.00</td>
<td>53.30</td>
</tr>
<tr>
<td>Thermal</td>
<td>1588</td>
<td>83.60</td>
<td>132.76</td>
</tr>
<tr>
<td>MUW</td>
<td>445</td>
<td>80.90</td>
<td>36.00</td>
</tr>
<tr>
<td>Co-Generation</td>
<td>3441</td>
<td>83.80</td>
<td>288.36</td>
</tr>
<tr>
<td>Micro-Generation</td>
<td>14</td>
<td>587.00</td>
<td>8.22</td>
</tr>
</tbody>
</table>

Table 5.2: Feed-In Tariff premium received by energy technology in 2010

Large scale Hydro power is already a mature RES so it competes directly in the regular MIBEL. For the FFs, the imported and exported energy provides an acceptable estimation for production and

¹Wave and tidal Energy
consumer prices.

<table>
<thead>
<tr>
<th>Fuel</th>
<th>Imports (GWh)</th>
<th>Price (€/MWh)</th>
<th>Exports (GWh)</th>
<th>Price (€/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal</td>
<td>19777</td>
<td>193</td>
<td>9.76</td>
<td>819</td>
</tr>
<tr>
<td>Gas</td>
<td>52611</td>
<td>1151</td>
<td>21.88</td>
<td>49111</td>
</tr>
<tr>
<td>Oil</td>
<td>176408</td>
<td>6705</td>
<td>38.01</td>
<td>49111</td>
</tr>
<tr>
<td>Electricity</td>
<td>4350</td>
<td>176</td>
<td>40.46</td>
<td>1717</td>
</tr>
<tr>
<td>Biomass</td>
<td>181</td>
<td>3</td>
<td>16.61</td>
<td>1082</td>
</tr>
</tbody>
</table>

Table 5.3: Imports and Exports of Fossil Fuels, Electricity and Biomass used to establish the conversion from energy to monetary fluxes.

Table 5.4 displays the Capital (K) and Labor (L) shares of RES and FF technologies that were retrieved from Proença and St. Aubyn (2013), who claim to have determined their cost shares using data from the Markal-TIMES model of Portugal. Although we are forced to admit these values are somewhat limited and will affect the quality of the results, due to the discrepancies between similar technologies such as gas and coal, it is the available data. Also, it was assumed that geothermal energy has costs similar to solar power.

<table>
<thead>
<tr>
<th>Technology</th>
<th>Labor (L)</th>
<th>Capital (K)</th>
<th>Fuel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal</td>
<td>0.29</td>
<td>0.32</td>
<td>0.39</td>
</tr>
<tr>
<td>Gas</td>
<td>0.11</td>
<td>0.10</td>
<td>0.79</td>
</tr>
<tr>
<td>Oil</td>
<td>0.08</td>
<td>0.09</td>
<td>0.83</td>
</tr>
<tr>
<td>Hydro</td>
<td>0.08</td>
<td>0.92</td>
<td></td>
</tr>
<tr>
<td>Wind</td>
<td>0.21</td>
<td>0.79</td>
<td></td>
</tr>
<tr>
<td>Solar</td>
<td>0.11</td>
<td>0.89</td>
<td></td>
</tr>
<tr>
<td>Biomass</td>
<td>0.25</td>
<td>0.33</td>
<td>0.43</td>
</tr>
</tbody>
</table>

Table 5.4: Cost shares for energy technologies

5.2 Elasticity Parameters

Due to the difficulty of determining elasticity parameters, it is common practice in CGE models to use values from relevant literature. Bearing this in mind, table 5.5 contains a summary of the parameters and their sources.

The sector specific values of $\sigma_{KL}$ and $\sigma_{KLE}$ as well as the Armington trade elasticities $\sigma_A$ were gathered from the Global Trade Analysis Project (GTAP) database, the transformation elasticities come from Proença and St. Aubyn (2013) and Böhringer et al. (2013) also provided guidance when choosing the remaining values. The wage curve elasticity $\psi$ was given by Robalo Marques et al. (2010).

5.3 Code Workflow

Naturally, there are already general software solutions to solve CGE, such as the Mathematical Programming System for General Equilibrium which is a subsystem of GAMS, yet I chose to elaborate
<table>
<thead>
<tr>
<th>Elasticity</th>
<th>Value(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_m$</td>
<td>Aggregation of Material inputs</td>
</tr>
<tr>
<td>$\sigma_E$</td>
<td>Aggregation of Energy inputs</td>
</tr>
<tr>
<td>$\sigma_{K,L}$</td>
<td>Aggregation of Capital and Labor</td>
</tr>
<tr>
<td>$\sigma_{K,LE}$</td>
<td>Aggregation of Added-value and Energy</td>
</tr>
<tr>
<td>$\sigma_V$</td>
<td>Final aggregation of inputs</td>
</tr>
<tr>
<td>$\eta_V$</td>
<td>Disaggregation of industry good into several goods</td>
</tr>
<tr>
<td>$\eta_X$</td>
<td>Disaggregation into domestic and foreign varieties</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>Armington aggregation of Domestic and Imported varieties</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Wage curve elasticity</td>
</tr>
<tr>
<td>$\sigma_G$</td>
<td>Consumer basket elasticity</td>
</tr>
<tr>
<td>$\sigma_{G}$</td>
<td>Government basket elasticity</td>
</tr>
<tr>
<td>$K_{mob}$</td>
<td>Capital Mobility</td>
</tr>
</tbody>
</table>

Table 5.5: Values of the external parameters (mostly elasticities) used in the model.

a customized MATLAB\textsuperscript{2} project that reads the benchmark data and other parameters from an Excel file. The code framework can be adapted to different CGE projects. This section contains a description of the code that was developed for this thesis. Its purpose is to create the system described on Chapter 4 and solve it after the introduction of an external shock, i.e., a change in the exogenous or control variables.

![Figure 5.1: Main scripts in the code.](image)

There are two main scripts in this project: Main.m must be executed once to configure the system and then Simulate.m finds a new equilibrium after reading the control variables – see Figure 5.1. To use this project, there must be a readable Excel file whose path is specified in the Main script (variable folder) – referenced as ‘Data.xlsx’ from now on. This file is the only source of information needed. The main page is called ‘Configuration’ and it has general configurations and also informations on how other pages should be read.

5.3.1 Main.m

Here we specify the functions called by the script Main.m and how those function read data from the Excel file:

\textsuperscript{2}Version R2013a (8.1.0.604)
• create_code_filepath
• system_config
• system_read_data
• system_vars
• system_pars
• create_eq_list
• system_topology

Although only the datafile needs to be in a specific path, the function create_code_filepath includes all subfolders of folder in MATLAB’s search path to ensure that all the necessary functions can be found by MATLAB.

The function system_config is responsible for reading the ‘Configuration’ page in ‘Data.xlsx’. The first lines read should have the following organization:

```
number Sets
136 Goods
133 Sectors
  2 Factors
  1 Households
  1 Government
...
size Set Name
...
```

**Figure 5.2:** First lines of page ‘Configuration’ containing a description of Sets in the model

`number` states how many lines are describing sets (itself included). In the default configuration there are 6 sets, the ones listed above, and each one of them must have a page in the Excel file with its name, to describe the set. For instance, the above lists indicates that the page ‘Goods’ has 136 lines (items).

<table>
<thead>
<tr>
<th>Aggregation Level</th>
<th>Item Index</th>
<th>Parent Index</th>
<th>Label</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>Goods</td>
<td>Gds</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>0</td>
<td>Services</td>
<td>Srs</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>2</td>
<td>Chemical Products</td>
<td>CE</td>
</tr>
</tbody>
</table>

**Figure 5.3:** Page: ‘Goods’

As mentioned in Section 5.1, commodities and industries have several aggregation levels. The item Chemical Products is the 6th of its level (3) and its parent item is the 2nd of level 2. The code column indicates how the production function recognize this item. Items on level 1 have no parent item as they are at the top of the aggregation chain and items with level 0 are not aggregated at all (so they don’t require indices) – see the scheme in Table 5.6

The remaining lines of the page ‘Configuration’ can be used change the default value of variables in the system.config structure, which are listed in Table 5.7
Then *system_read_data* uses the strings *SAM*, *Others*, *VNames*, *Elast* and *PF* to read the respective pages and saves the information into *system.data*. While the page *SAM* contains a replica of Table 5.1 and each text entry is the name of the page where the corresponding matrix or vector is, the page *Others* has a list of variables that do not fit into the *SAM*.

In the page *VNames* the user must name the variables in the system. The indices *i* and *j* are used to number variables across rows and columns respectively. For example:

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>dK</td>
<td>Capital Depreciation in sector <em>i</em></td>
</tr>
<tr>
<td>p.M</td>
<td>Price of Imported Good <em>j</em></td>
</tr>
</tbody>
</table>

*Figure 5.4:* Example lines from the page ‘VarNames’

And finally, page *PF* describes the industry production structure from Figures 4.3 and 4.4 whereas *Elast* lists the elasticity values for each sector or good, according to the case. The former has two columns that represent the nesting structure of the production function. In a given nest, for instance Capital-Labor (KL), it is necessary to pair each input (Capital K and Labor L) with the parent aggregate good, as shown in the default configuration of Figure 5.5.

The character *Y* is determined by the variable *output* in the ‘Configuration’ page. The page *Elast* complements this information with the elasticities of substitution in each nest. In this page, the first eight lines are bound to their descriptions and each must have either one elasticity or vector of elasticities for each good or sector at the highest disaggregation level. The 8th line corresponds to the mixture of TTM’s and the domestic goods, as shown if Figure 4.5, which was simply assumed to be 0.
- or a Leontief PF.

The remaining lines correspond to the nests that were created in page PF and are referenced by the code of compound good in each nest. Again, the system needs one elasticity for each sector at the lowest aggregation level. However, the remaining descriptions must correspond to the codes used in page PF. So, line KL contains the elasticity of substitution of the aforementioned aggregation of Capital and Labor, for each production sector.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\eta_Y)</td>
<td>Output Disaggregation (per sector)</td>
<td>-1 -1 -1 ... -1 -1</td>
</tr>
<tr>
<td>(\eta_X)</td>
<td>Export/Domestic Disaggregation (per good)</td>
<td>-2 -2 -2 ... -2 -2</td>
</tr>
<tr>
<td>(\sigma_A)</td>
<td>Armington Import (per good)</td>
<td>3 2.5 1.25 ... 1 1</td>
</tr>
<tr>
<td>(\psi)</td>
<td>Wage Curve</td>
<td>-0.1</td>
</tr>
<tr>
<td>(\sigma_C)</td>
<td>Consumption Basket</td>
<td>0.2</td>
</tr>
<tr>
<td>(\sigma_G)</td>
<td>Government Basket</td>
<td>0.2</td>
</tr>
<tr>
<td>(\sigma_{TTM})</td>
<td>Margins Sector</td>
<td>1</td>
</tr>
<tr>
<td>Add TTM in the Armington Sector</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>(\sigma_M)</td>
<td>M</td>
<td>0 0 0 ... 0 0</td>
</tr>
<tr>
<td>(\sigma_E)</td>
<td>E</td>
<td>10 10 10 ... 10 10</td>
</tr>
<tr>
<td>(\sigma_{KL})</td>
<td>KL</td>
<td>0.24 0.2 0.2 ... 1.12 1.12</td>
</tr>
<tr>
<td>(\sigma_{KLE})</td>
<td>KLE</td>
<td>0.6 0.6 0.6 ... 0.6 0.6</td>
</tr>
<tr>
<td>(\sigma_Y)</td>
<td>Y</td>
<td>0 0 0 ... 0 0</td>
</tr>
</tbody>
</table>

Table 5.8: Page Elast

In the next step, the function `system_vars` creates the structure `vars` with the benchmark endogenous and exogenous variables and a similar variable with its names instead of values.

The function `system_pars` creates the structure `system.pars` which contains all the parameters used in the model: the elasticities, the \(\theta_s\) and other relevant values that were stored in `system.data`.

The variable `list` is a cell array that contains strings with the names of the functions that represent the model and their respective arguments. Every model variable must be introduced by the structure `vars`, while other arguments do not have restrictions. The functions must yield column vectors as output. The format of `list` is the following:
<table>
<thead>
<tr>
<th>field</th>
<th>dimension</th>
<th>field</th>
<th>dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>array c×1</td>
<td>KC</td>
<td>scalar</td>
</tr>
<tr>
<td>xIN</td>
<td>array g×c</td>
<td>KG</td>
<td>scalar</td>
</tr>
<tr>
<td>L</td>
<td>array c×1</td>
<td>dK</td>
<td>array c×1</td>
</tr>
<tr>
<td>K</td>
<td>array c×1</td>
<td>dKt</td>
<td>scalar</td>
</tr>
<tr>
<td>A</td>
<td>array g×1</td>
<td>KS</td>
<td>scalar</td>
</tr>
<tr>
<td>xD</td>
<td>array c×g</td>
<td>DK</td>
<td>scalar</td>
</tr>
<tr>
<td>xDt</td>
<td>array 1×g</td>
<td>Lt</td>
<td>scalar</td>
</tr>
<tr>
<td>xX</td>
<td>array c×g</td>
<td>Ur</td>
<td>scalar</td>
</tr>
<tr>
<td>xM</td>
<td>array g×1</td>
<td>xTTM</td>
<td>array g×1</td>
</tr>
<tr>
<td>xC</td>
<td>array g×1</td>
<td>TTM</td>
<td>array g×1</td>
</tr>
<tr>
<td>xG</td>
<td>array g×1</td>
<td>TTMt</td>
<td>scalar</td>
</tr>
<tr>
<td>xI</td>
<td>array g×1</td>
<td>tax.</td>
<td>structure</td>
</tr>
<tr>
<td>M</td>
<td>structure</td>
<td>F</td>
<td>scalar</td>
</tr>
<tr>
<td>C</td>
<td>scalar</td>
<td>P</td>
<td>array c×1</td>
</tr>
<tr>
<td>G</td>
<td>scalar</td>
<td>G</td>
<td>array g×1</td>
</tr>
<tr>
<td>S</td>
<td>structure</td>
<td>p.</td>
<td>structure</td>
</tr>
<tr>
<td>C</td>
<td>scalar</td>
<td>D</td>
<td>array 1×g</td>
</tr>
<tr>
<td>G</td>
<td>scalar</td>
<td>A</td>
<td>array 1×g</td>
</tr>
<tr>
<td>Sr.</td>
<td>structure</td>
<td>M</td>
<td>array 1×g</td>
</tr>
<tr>
<td>C</td>
<td>scalar</td>
<td>X</td>
<td>array 1×g</td>
</tr>
<tr>
<td>G</td>
<td>scalar</td>
<td>C</td>
<td>scalar</td>
</tr>
<tr>
<td>C</td>
<td>scalar</td>
<td>G</td>
<td>scalar</td>
</tr>
<tr>
<td>G</td>
<td>scalar</td>
<td>TTM</td>
<td>scalar</td>
</tr>
<tr>
<td>T</td>
<td>scalar</td>
<td>L</td>
<td>scalar</td>
</tr>
<tr>
<td>I</td>
<td>scalar</td>
<td>K</td>
<td>scalar</td>
</tr>
<tr>
<td>LK</td>
<td>scalar</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.9: Structure system.vars

list = {
    {'Investment', 'vars.p.A', 'vars.xI', 'vars.I', 'bench'},
    ...
    {function name, argument 1, argument 2, ... }
}

This way, function system_topology composes the vector \( \mathbf{F} \) by connecting each function to \( f_i \) as in Equation 4.1 and grasping its output size. This information is used when building the Jacobian and stored in system.topg.

### 5.3.2 Simulate.m

As Figure 5.6 illustrates, the script Simulate.m reads the list of control variables, introduces them in the system and then finds a new equilibrium.

Inside fsolve_my_fun there is a nested function: \( \mathbf{F}(\mathbf{x}) \), computed using call_eq_list and also a mechanism that provides the Jacobian matrix (using call_Jacobian). Other than call_Jacobian the functions mentioned are quite straightforward and are all written in Appendix C. This function is interesting as it uses the information in system.list and system.topg to calculate the Jacobian efficiently.

The variable system.list specifies that the function ‘Investment’ (Equation 4.15) only depends on the variables: 1. Armington prices (p.A), 2. formation of fixed Capital (xI) and 3. total investment.
Simulate.m

- `system_exog` reads the each control variable individually
- creates a structure with all the control variables

- `fsolve_my_fun` reduces `system.vars` to a vector of endogenous variables
- calls `fsolve` to solve the system (returns a vector)
- converts solution vector back to a structure

Figure 5.6: Description of the script `Simulate.m`.

\[
\begin{bmatrix}
    f_1(x) \\
    \vdots \\
    \text{Investment} \\
    \vdots \\
    f_n(x)
\end{bmatrix}
\begin{bmatrix}
    x_1 & \ldots & x_I & I & p.A & \ldots & x_n
\end{bmatrix}
\]

Figure 5.7: Scheme of the Jacobian matrix

(1). So `call_Jacobian` knows that the matrix lines corresponding to the derivatives of 'Investment' can only have values in the positions marked in Figure 5.7. It then checks if there is a function with the name `Deriv_Investment_1` that contains the derivative of Equation 4.15 to the first argument – pA. If there is no such function, it computes the derivative numerically. If the user provides every function of the Jacobian, its computation time decreases significantly.
6

Results

Contents

6.1 Scenario 2011 ........................................ 46
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6.3 Sensitivity Analysis .................................. 53
In this Chapter we presents the results of obtained after performing two simulations. The first was used to analyse the strengths and weaknesses of the CGE model, as it attempts to reproduce the year 2011. Since we have real data from that year, we can see if the model is producing accurate results. In the second simulation, the shock consisted in the removal of the FITs. By comparing the results of this simulation (without FITs) with the benchmark data (with FITs), it is possible to assess the impact of this mechanism in the Portuguese economy. The Chapter ends with a sensitivity analysis of the most relevant elasticity parameters.

6.1 Scenario 2011

In order to assess the validity of the model, the first scenario attempts to describe the year 2011. By comparing the simulated version (labeled model) with real data from that year, it should be possible to assess the GE model.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Benchmark (M€)</th>
<th>Scenario (M€)</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Household Capital Stock</td>
<td>26,476</td>
<td>27,062</td>
<td>2.2%</td>
</tr>
<tr>
<td>Government Capital Stock</td>
<td>-3,842</td>
<td>-2,822</td>
<td>-26.5%</td>
</tr>
<tr>
<td>Household Savings</td>
<td>2,862</td>
<td>1,922</td>
<td>-32.8%</td>
</tr>
<tr>
<td>Government Savings</td>
<td>-15,771</td>
<td>-12,253</td>
<td>-22.3%</td>
</tr>
<tr>
<td>Available Labor</td>
<td>97,191</td>
<td>97,384</td>
<td>0.2%</td>
</tr>
<tr>
<td>Import and Export prices</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tax Revenue</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taxes on Goods</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taxes on Activities</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>FITs</th>
<th>Benchmark (€/MWh)</th>
<th>Scenario (€/MWh)</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydro Power</td>
<td>91.07</td>
<td>88.88</td>
<td>-2.4%</td>
</tr>
<tr>
<td>Wind Power</td>
<td>88.70</td>
<td>90.54</td>
<td>2.1%</td>
</tr>
<tr>
<td>Photovoltaic(^2)</td>
<td>344.77</td>
<td>304.94</td>
<td>-11.6%</td>
</tr>
<tr>
<td>Geothermal(^3)</td>
<td>344.77</td>
<td>304.94</td>
<td>-11.6%</td>
</tr>
<tr>
<td>Biomass</td>
<td>113.40</td>
<td>106.63</td>
<td>-6.0%</td>
</tr>
</tbody>
</table>

Table 6.1: Control variables used in the first scenario – year 2011

The control variables used for this scenario are described in Table 6.1. The numeraire is the consumer price index \(p_c\) and a new equilibrium was found with \(|y_i| = 0.2823\).

First of all, Table 6.2 displays the Portuguese electricity mix, excluding imported electricity, in 2010, then the model’s projections for 2011 and finally the real data from 2011.

In the imposed shock, only the wind power FIT was increased which should result in a growth of wind generated electricity. In fact, its projected share in the energy mix rose by 0.5%, matching the real share. On the other hand, hydroelectricity saw its production increased by 0.2% despite the FIT

\(^1\)The benchmark year is 2010.
\(^2\)Also includes Tidal energy.
\(^3\)ERSE combines the FIT for Photovoltaic and Geothermal electricity, but the data from the National Energy Balance provided by DGEG separates those two RESs, so they were processed as different technologies with the same FIT.
<table>
<thead>
<tr>
<th></th>
<th>2010</th>
<th>Model</th>
<th>2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal</td>
<td>13.7%</td>
<td>13.6%</td>
<td>24.5%</td>
</tr>
<tr>
<td>Natural Gas</td>
<td>31.6%</td>
<td>31.5%</td>
<td>27.8%</td>
</tr>
<tr>
<td>Oil</td>
<td>1.0%</td>
<td>1.0%</td>
<td>0.1%</td>
</tr>
<tr>
<td>FFs</td>
<td>46.3%</td>
<td>46.1%</td>
<td>52.4%</td>
</tr>
<tr>
<td>Hydro Power</td>
<td>30.6%</td>
<td>30.8%</td>
<td>23.1%</td>
</tr>
<tr>
<td>Wind Power</td>
<td>17.0%</td>
<td>17.5%</td>
<td>17.5%</td>
</tr>
<tr>
<td>Photovoltaic</td>
<td>0.40%</td>
<td>0.32%</td>
<td>0.53%</td>
</tr>
<tr>
<td>Geothermal</td>
<td>0.36%</td>
<td>0.25%</td>
<td>0.40%</td>
</tr>
<tr>
<td>Biomass</td>
<td>5.4%</td>
<td>5.1%</td>
<td>6.1%</td>
</tr>
<tr>
<td>RESs</td>
<td>53.7%</td>
<td>53.9%</td>
<td>47.6%</td>
</tr>
</tbody>
</table>

Table 6.2: Portuguese Electricity mix for 2010, the model’s results for 2011 and also that year’s real values for comparison.

reduction as it is still a relatively cheap energy source. This projection is incorrect as the benchmark year was characterized by high rainfall and the model doesn’t take pluviosity into account.

The FIT for photovoltaic and geothermal power suffered the highest cut, resulting in expected drops of 0.08% and 0.11% in their respective electricity share, which is contradicted by the real data that shows a small increase in electricity production from both sources. There are several explanations for this discrepancy: the already installed capacity was not reduced as it is difficult to repurpose the equipment; the FIT was already higher than the break-even point or there was a reduction of cost of installing these technologies due to technological improvements.

Besides generating electricity, the biomass sector also transforms waste into biomass\(^4\) for other industries, so it profited from the waste management activities and heat-electricity cogeneration despite the small reduction in the FIT, resulting in a rise of biomass generated electricity. Yet, in the model, due to the smaller FIT the price of biomass generated electricity grew by 4% causing a reduction of 0.3% in its electricity share.

In 2011, coal consumption soared as it was used to compensate the lack of hydroelectricity. However, the simulated scenario only foresaw a small increase in RES Electricity accompanied by an equivalent reduction in the use of Fossil Fuels to generate electricity and completely missed this shift. Furthermore, the trends in the FF mix could not be predicted correctly as they depend on external factors not included in the model.

After the shock, the overall electricity\(^5\) consumption increased by 0.65% and its price went down by more than 3%. These results are contradicted by DGEG’s report of a 3% decline in consumption and a price hike of 3.8% in electricity tariffs. Even though some FITs were reduced and the total green electricity production was lower, the electricity price was increased to compensate the ‘tariff deficit’. The conjunction of higher prices and austerity measures caused the electricity demand reduction whereas the model expected the opposite results.

Regarding the use of Labor and Capital, Table 6.3 provides an overview. The increased availability

---

\(^4\)According to the DGEG Biomass includes dry woods, organic residuals, briquettes, pellets, MSW, sulfitic liquors, biogas and biofuels.

\(^5\)See Table 6.5, Code: EL.
of both factors was imposed in the shock, following the INE’s data of 2011. Despite the growth of available Labor, the actual use of this factor dropped significantly since the unemployment rate grew by 3.4%, increasing more than the real value: 12.7% in 2011. Also, with such a large rise in the amount of available Assets, the rents obtained from those were 2.3% lower.

Table 6.3: Use of Capital and Labor in the 2011 Scenario, including relative changes to the benchmark year.

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Model</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Available Labor</td>
<td>97,191</td>
<td>97,384</td>
<td>0.2%</td>
</tr>
<tr>
<td>Unemployment</td>
<td>10.8%</td>
<td>14.2%</td>
<td>3.4%</td>
</tr>
<tr>
<td>Wages</td>
<td>1</td>
<td>1.007</td>
<td>0.7%</td>
</tr>
<tr>
<td>Available Assets</td>
<td>22,634</td>
<td>24,240</td>
<td>7.1%</td>
</tr>
<tr>
<td>Rents</td>
<td>1</td>
<td>0.977</td>
<td>−2.3%</td>
</tr>
</tbody>
</table>

Since the total tax revenue was kept constant, the tax on Household income decreased by 2.55% mostly through indirect effects: with smaller subsidies, the market distortions are lessened and the other sectors are more active – providing more tax money. The reduction in electricity costs was quite relevant. The direct subsidies to RESs actually increased from 870 to 1087€ as wind and hydroelectricity provided more energy than during benchmark year.

Table 6.4 shows that the GDP decreased by 1% relative to 2010, while the model foresaw a −1.79% reduction. Still, in CGE, the overall impact on welfare is usually assessed using two indicators:

- Compensating Variation: a measure of change in utility, computed using the aggregated consumer good C that increased by 0.30%;
- Real Consumption Change: consumption change at constant prices, also increased, by 0.32%.

However, these parameters don’t incorporate the impact of future investments or the health benefits drawn from the lower CO₂ emissions.

The model did not foresee any noticeable change in capital formation patterns: the expected total investment receded by less than 0.1% while in reality the reduction was close to 9%, despite the rise in Government savings.

Table 6.4: GDP comparison: benchmark year (2010), model projection and INE’s provisional value for 2011.

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Model</th>
<th>2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP (M€)</td>
<td>172,859</td>
<td>169,771</td>
<td>171,126</td>
</tr>
<tr>
<td>Change</td>
<td>−1.79%</td>
<td>−1.00%</td>
<td></td>
</tr>
</tbody>
</table>

For more detailed information, Tables 6.5 and 6.6 disclose the relative acquisition price and sector activity changes that occurred in this scenario – codes can be mapped to their respective goods and sectors in Appendix B.

To sum up, despite the lack of a parameter to control the availability of resources, the changes in a FIT has a direct impact on the technology it represents. Nevertheless, the model doesn’t predict correctly the impact of the policy in the electricity price and demand, an effect that is propagated to
<table>
<thead>
<tr>
<th>Good(^6)</th>
<th>Price (%)</th>
<th>Good</th>
<th>Price (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.04</td>
<td>OL</td>
<td>-1.04</td>
</tr>
<tr>
<td>2</td>
<td>0.08</td>
<td>NG</td>
<td>-0.02</td>
</tr>
<tr>
<td>3</td>
<td>-0.15</td>
<td>DE</td>
<td>-3.09</td>
</tr>
<tr>
<td>4</td>
<td>-1.10</td>
<td>TE</td>
<td>-0.50</td>
</tr>
<tr>
<td>5</td>
<td>-0.38</td>
<td>HE</td>
<td>-1.83</td>
</tr>
<tr>
<td>6</td>
<td>-0.76</td>
<td>WE</td>
<td>-3.98</td>
</tr>
<tr>
<td>7</td>
<td>0.23</td>
<td>PV</td>
<td>21.39</td>
</tr>
<tr>
<td>8</td>
<td>0.97</td>
<td>GT</td>
<td>44.91</td>
</tr>
<tr>
<td>9</td>
<td>1.65</td>
<td>BE</td>
<td>4.39</td>
</tr>
<tr>
<td>10</td>
<td>0.41</td>
<td>HT</td>
<td>-0.73</td>
</tr>
<tr>
<td>CO</td>
<td>-0.03</td>
<td>BM</td>
<td>-1.19</td>
</tr>
<tr>
<td>PL</td>
<td>-1.23</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 6.5:** Relative commodity price changes from the benchmark year in the 2011 Scenario.

<table>
<thead>
<tr>
<th>Sector(^7)</th>
<th>Activity (%)</th>
<th>Sector</th>
<th>Activity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-33.90</td>
<td>OR</td>
<td>37.3</td>
</tr>
<tr>
<td>2</td>
<td>-6.03</td>
<td>TE</td>
<td>26.5</td>
</tr>
<tr>
<td>3</td>
<td>1.31</td>
<td>HP</td>
<td>40.1</td>
</tr>
<tr>
<td>4</td>
<td>9.56</td>
<td>WP</td>
<td>43.2</td>
</tr>
<tr>
<td>5</td>
<td>2.08</td>
<td>SP</td>
<td>-9.0</td>
</tr>
<tr>
<td>6</td>
<td>5.87</td>
<td>GT</td>
<td>-36.1</td>
</tr>
<tr>
<td>7</td>
<td>-1.05</td>
<td>EG</td>
<td>22.0</td>
</tr>
<tr>
<td>8</td>
<td>-7.65</td>
<td>BM</td>
<td>-5.7</td>
</tr>
<tr>
<td>9</td>
<td>-15.43</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>-3.28</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 6.6:** Relative activity changes from the benchmark year in the 2011 Scenario.
other economic sectors. As for other economic indicators, such as GDP and unemployment, the results are coherent with the real data from 2011. Concerning investment and capital depreciation, there were no relevant changes in the variables, which means the model is not accurately representing this aspect of the economy.

6.2 Scenario No FITs

Since the FIT mechanism had already been implemented in the benchmark year, in order to assess the impact of the FITs the second scenario simulates the Portuguese economic and energy sectors without those tariffs. The system could not reach a new equilibrium under the desired tolerance with the parameters used in the previous scenario – namely, the low Energy aggregation elasticity, so this parameter was increased to 10, in compliance with Wing (2006).

<table>
<thead>
<tr>
<th>Variables</th>
<th>Benchmark</th>
<th>Scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>Household Capital Stock</td>
<td>No change</td>
<td></td>
</tr>
<tr>
<td>Government Capital Stock</td>
<td>No change</td>
<td></td>
</tr>
<tr>
<td>Household Savings</td>
<td>No change</td>
<td></td>
</tr>
<tr>
<td>Government Savings</td>
<td>No change</td>
<td></td>
</tr>
<tr>
<td>Available Labor</td>
<td>No change</td>
<td></td>
</tr>
<tr>
<td>Import and Export prices</td>
<td>No change</td>
<td></td>
</tr>
<tr>
<td>Tax Revenue</td>
<td>No change</td>
<td></td>
</tr>
<tr>
<td>Taxes on Goods</td>
<td>No change</td>
<td></td>
</tr>
<tr>
<td>Taxes on Activities</td>
<td>No change</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.7: Control variables used in the No FITs scenario.

match The control variables were the same as in the previous scenario but all of them retained their benchmark values, excluding the FITs, that were completely removed. The consumer price index $p_c$ remained the *numéraire* and the new equilibrium was found with $|y_i| = 0.6813$.

Without any FIT wind, photovoltaic and geothermal power would have almost no representation in the electricity sector, in the same way those technologies were not used in the early 2000s. The projected decrease in hydroelectricity corresponds to the dismantlement of many small to medium sized hydric power plants ($< 10$ MW) as only large hydro power is viable without a FIT – matching once again the portuguese energy mix before the introduction of this policy. As for biomass, its share in the electricity mix would also be higher since before the introduction of the FITs, biomass and co-generation were already used to produce electricity, so a large decline was never expected. Overall, the price of electricity would have decreased by 1.74% resulting in a consumption growth of 13.38% –
Table 6.8: Portuguese Electricity mix in the scenario without FITs and differences from the benchmark year 2010.

still, it is important to recall that these variables were not correct in the previous scenario.

In this scenario, the use of Labor would increase as the expectations for the unemployment rate are a 0.1% fall, accompanied by a small wage increase of 0.1% inducing a raise in Household income. Assets would also provide higher earnings in this scenario as their rental price increased by 0.5%.

Table 6.9: Comparison of the use of Capital and Labor in the No FITs scenario and the benchmark year.

The tax burden on Households decreased by 3.13%, which is expected since the Government was no longer supporting the FIT scheme and kept the same level of tax revenue. This increase in income led to improved Welfare: compensating variation (or utility) increased by 2.35% and the real consumption grew 2.33%. Still, GDP would have decreased by -6% as Portugal would be much more dependent on imported Fossil Fuels. This reliance in external energy sources would have made Portugal more susceptible to the spikes in Brent oil prices that started in 2003 and could have aggravated the effects of the 2008 financial crisis.

Table 6.10: Comparison of the GDP in the benchmark year (2010) and according to the model’s projection without the FITs.

---

8Sector code: 1
rise of 21.2%. On the other hand, retail, commerce and transportation\(^9\) are the most damaged activity (down by 12.5%) as the latter relies a lot on imported fuels and the increasing demand created a price hike.

<table>
<thead>
<tr>
<th>Good</th>
<th>Price (%)</th>
<th>Good</th>
<th>Price (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.07</td>
<td>OL</td>
<td>309.65</td>
</tr>
<tr>
<td>2</td>
<td>4.35</td>
<td>NG</td>
<td>60.98</td>
</tr>
<tr>
<td>3</td>
<td>0.61</td>
<td>DE</td>
<td>13.39</td>
</tr>
<tr>
<td>4</td>
<td>-4.22</td>
<td>TE</td>
<td>18.49</td>
</tr>
<tr>
<td>5</td>
<td>0.38</td>
<td>HE</td>
<td>-52.20</td>
</tr>
<tr>
<td>6</td>
<td>2.67</td>
<td>WE</td>
<td>-99.93</td>
</tr>
<tr>
<td>7</td>
<td>1.26</td>
<td>PV</td>
<td>-99.55</td>
</tr>
<tr>
<td>8</td>
<td>2.10</td>
<td>GT</td>
<td>-99.51</td>
</tr>
<tr>
<td>9</td>
<td>0.95</td>
<td>BE</td>
<td>16.93</td>
</tr>
<tr>
<td>10</td>
<td>2.18</td>
<td>HT</td>
<td>39.14</td>
</tr>
<tr>
<td>CO</td>
<td>70.88</td>
<td>BM</td>
<td>134.60</td>
</tr>
<tr>
<td>PL</td>
<td>362.36</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 6.11:** Relative commodity price changes from the benchmark year in the No FITs Scenario.

<table>
<thead>
<tr>
<th>Sector</th>
<th>Activity (%)</th>
<th>Sector</th>
<th>Activity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21.16</td>
<td>OR</td>
<td>552.8</td>
</tr>
<tr>
<td>2</td>
<td>7.77</td>
<td>TE</td>
<td>30.9</td>
</tr>
<tr>
<td>3</td>
<td>0.55</td>
<td>HP</td>
<td>-56.5</td>
</tr>
<tr>
<td>4</td>
<td>-12.45</td>
<td>WP</td>
<td>-100.0</td>
</tr>
<tr>
<td>5</td>
<td>-6.65</td>
<td>SP</td>
<td>-100.0</td>
</tr>
<tr>
<td>6</td>
<td>2.57</td>
<td>GT</td>
<td>-100.0</td>
</tr>
<tr>
<td>7</td>
<td>-0.27</td>
<td>EG</td>
<td>26.3</td>
</tr>
<tr>
<td>8</td>
<td>7.73</td>
<td>BM</td>
<td>27.8</td>
</tr>
<tr>
<td>9</td>
<td>0.62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>5.21</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 6.12:** Relative sectorial activity changes from the benchmark year in the No FITs Scenario.

\(\text{\textsuperscript{9}}\text{Sector code: 4}\)
6.3 Sensitivity Analysis

The sensitivity analysis is used to test the impact of key parameters in the model and in the results. Two of the most relevant parameters are the aggregation of energy inputs elasticity ($\sigma_E$) and the capital mobility ($K_{mob}$), so their impact on the 2011 scenario will be thoroughly examined. Furthermore, the impact of the Armington trade (import and export) elasticities will be examined, and the wage curve elasticity too. Finally, there will be a simulation with $\sigma_C = 0$ and another $\sigma_C = 0.4$ to assess how the consumer preferences affect the results.

6.3.1 Testing Capital Mobility and Energy Elasticity

Table 6.13 displays the electricity mix for several pairs of capital mobility and energy aggregation elasticity – the original scenario had $K_{mob} = 1$ and $\sigma_E = 1$.

The hydro and the wind power shares increase in all the considered possibilities. As the contrains become more relaxed with higher the parameters, the more their share increases. Similarly, biomass, photovoltaic and geothermal power have their share reduced in all cases. On the other hand, the behaviour of FFs is simple to understand: either they all increase or they all decrease in proportion to their original share. The main scenario was chosen for its forecast of the wind power share, which is the closest to the real value of 2011.

Regardless of those parameter values, the price of electricity decreased and its consumption increased – more details in Table 6.14. As for the unemployment rate, it grew in every scenario, ranging from 13.20% to 14.79%. The same happened to the welfare indicators and the change in GDP. Overall, just by changing these two parameters there is no large impact on the results described in Section 6.1.

Table 6.13: Sensitivity analysis of capital mobility and energy aggregation elasticity, by comparing the electricity mix of scenario 2011 for different values of capital mobility and energy aggregation elasticity.

<table>
<thead>
<tr>
<th>Capital Mobility ($K_{mob}$)</th>
<th>2010</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>1</th>
<th>2</th>
<th>1</th>
<th>2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy Elasticity ($\sigma_E$)</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Oil</td>
<td>1.02</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.98</td>
<td>1.01</td>
<td>1.02</td>
<td>1.02</td>
<td>0.89</td>
<td>0.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Natural Gas</td>
<td>31.61</td>
<td>31.55</td>
<td>31.50</td>
<td>31.46</td>
<td>31.41</td>
<td>31.81</td>
<td>31.72</td>
<td>31.67</td>
<td>30.25</td>
<td>27.81</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FFs</td>
<td>46.31</td>
<td>46.21</td>
<td>46.14</td>
<td>46.08</td>
<td>46.00</td>
<td>46.59</td>
<td>46.47</td>
<td>46.40</td>
<td>44.31</td>
<td>52.38</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hydro Power</td>
<td>30.59</td>
<td>30.62</td>
<td>30.72</td>
<td>30.80</td>
<td>30.89</td>
<td>30.71</td>
<td>30.95</td>
<td>31.06</td>
<td>31.23</td>
<td>23.09</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wind Power</td>
<td>16.97</td>
<td>17.42</td>
<td>17.45</td>
<td>17.47</td>
<td>17.50</td>
<td>17.91</td>
<td>17.99</td>
<td>18.02</td>
<td>19.24</td>
<td>17.46</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Photovoltaic</td>
<td>0.40</td>
<td>0.32</td>
<td>0.32</td>
<td>0.32</td>
<td>0.32</td>
<td>0.00</td>
<td>0.01</td>
<td>0.01</td>
<td>0.16</td>
<td>0.53</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Geothermal</td>
<td>0.36</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.00</td>
<td>0.01</td>
<td>0.01</td>
<td>0.10</td>
<td>0.40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Biomass</td>
<td>5.36</td>
<td>5.19</td>
<td>5.13</td>
<td>5.08</td>
<td>5.03</td>
<td>4.77</td>
<td>4.58</td>
<td>4.50</td>
<td>4.95</td>
<td>6.14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RESs</td>
<td>53.69</td>
<td>54.69</td>
<td>53.86</td>
<td>53.92</td>
<td>54.00</td>
<td>53.41</td>
<td>53.53</td>
<td>53.60</td>
<td>55.69</td>
<td>47.62</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Regardless of those parameter values, the price of electricity decreased and its consumption increased – more details in Table 6.14. As for the unemployment rate, it grew in every scenario, ranging from 13.20% to 14.79%. The same happened to the welfare indicators and the change in GDP. Overall, just by changing these two parameters there is no large impact on the results described in Section 6.1.

Table 6.14: Sensitivity analysis of capital mobility and energy aggregation elasticity comparing electricity price and demand for different values of capital mobility and energy aggregation elasticity (in the first Scenario).
6.3.2 Testing Trade Elasticities

While the high sensitivity simulation was made by increasing every Arminton elasticity by 33%, the low version was executed by decreasing every trade elasticity parameter by 33%. Table 6.15 lists some of the major variables in Scenario 2011 and their respective high and low counterparts. The trading section has a relatively large impact on some variables: noticeably wages, rents and the price of electricity, with repercussions to the rest of the system. Still, the electricity mix, the GDP and the welfare indicators maintain their regular trends.

<table>
<thead>
<tr>
<th>Variable</th>
<th>low</th>
<th>Model: No FITs</th>
<th>high</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment</td>
<td>10.38%</td>
<td>10.66%</td>
<td>11.04%</td>
</tr>
<tr>
<td>Wages</td>
<td>0.40%</td>
<td>0.13%</td>
<td>-0.22%</td>
</tr>
<tr>
<td>Rents</td>
<td>-0.33%</td>
<td>0.53%</td>
<td>1.15%</td>
</tr>
<tr>
<td>GDP (M€)</td>
<td>161,400</td>
<td>162,488</td>
<td>165,593</td>
</tr>
<tr>
<td>Electricity Price</td>
<td>2.8%</td>
<td>-1.7%</td>
<td>-3.2%</td>
</tr>
<tr>
<td>Electricity Domestic Use</td>
<td>0.9%</td>
<td>13.4%</td>
<td>17.2%</td>
</tr>
<tr>
<td>Compensating Variation</td>
<td>0.77%</td>
<td>2.35%</td>
<td>1.35%</td>
</tr>
<tr>
<td>Real Consumption Change</td>
<td>0.77%</td>
<td>2.33%</td>
<td>1.30%</td>
</tr>
</tbody>
</table>

Table 6.15: Simulations of the scenario No FITs with different values of Armington Import and Export elasticities.

6.3.3 Evaluating the Wage Curve

As it is perceivable by looking at Table 6.16, changes in the wage curve elasticity do not cause any significant alterations in the unemployment rate or in wage alterations.

<table>
<thead>
<tr>
<th>Wage Curve Elasticity ψ</th>
<th>-0.15</th>
<th>-0.1</th>
<th>-0.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment</td>
<td>10.72%</td>
<td>10.66%</td>
<td>10.59%</td>
</tr>
<tr>
<td>Wage Relative Change</td>
<td>0.12%</td>
<td>0.13%</td>
<td>0.10%</td>
</tr>
</tbody>
</table>

Table 6.16: Simulations of the scenario No FITs with different wage curve elasticities to test the sensitivity of the wage curve elasticity.

6.3.4 Assessing the Impact of Consumer Preferences

Other than changing the proportions of the goods in the consumer basket, $σ_c$ doesn’t seem to have any major repercussions on other variables. When this elasticity is higher, Households have more freedom to choose which goods they purchase and can obtain more utility within the same budget, hence both the compensating variation and the real consumption change increase with the consumer basket aggregation elasticity.
Table 6.17: Sensitivity analysis of capital mobility and energy aggregation elasticity by running the scenario No FITs with different consumer basket aggregation elasticities.

<table>
<thead>
<tr>
<th>$\sigma_c$</th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment</td>
<td>10.67%</td>
<td>10.66%</td>
<td>11%</td>
</tr>
<tr>
<td>Wages</td>
<td>0.13%</td>
<td>0.13%</td>
<td>0.12%</td>
</tr>
<tr>
<td>Rents</td>
<td>0.46%</td>
<td>0.53%</td>
<td>0.55%</td>
</tr>
<tr>
<td>GDP (M€)</td>
<td>162,107</td>
<td>161,400</td>
<td>162,512</td>
</tr>
<tr>
<td>Electricity Price</td>
<td>-1.7%</td>
<td>-1.7%</td>
<td>-2.1%</td>
</tr>
<tr>
<td>Electricity Domestic Use</td>
<td>12.9%</td>
<td>13.4%</td>
<td>14.8%</td>
</tr>
<tr>
<td>Compensating Variation</td>
<td>2.2%</td>
<td>2.3%</td>
<td>2.4%</td>
</tr>
<tr>
<td>Real Consumption Change</td>
<td>2.2%</td>
<td>2.3%</td>
<td>2.4%</td>
</tr>
</tbody>
</table>
Conclusions
Climate change is one of the key issues of the XIX\textsuperscript{st} century. To prevent it, we are heading towards sustainable development, so that we can meet the needs of the present without compromising the ability of future generations to meet their own needs. Aligned with the EU’s goals, Portugal has shifted towards green electricity with the introduction of a FIT mechanism, but what was the economic impact of its policies?

In this thesis a CGE model was built with the purpose of assessing that impact. There are three basic pillars to CGE modelling: the set of equations, parameter calibration and finding a numerical solution. In Chapter 6, the first scenario, a small leap from the benchmark year (2010) to the shock year (2011) was used to assess the qualities and flaws of the model.

In term of equations, the Capital mobility piecewise function successfully controlled this primary factor. In Table 6.13, we can see that higher Capital Mobility led to lower shares of Photovoltaic and Geothermal in the electricity mix as they became less attractive with the FIT reduction and lower Capital mobility prevented the Capital from reallocating as easily.

The Labor market included a wage curve, which was not present in other CGEs applied to Portugal. Indeed, the introduction of the unemployment rate in the model suggests it would be slightly lower without the FITs.

Also, the depreciation of capital is usually supported by the investors instead of the industrial sectors and this experiment did not demonstrate any additional insights from that alteration as most values did not change. These expressions (Equation 4.6) should be either revisited or removed entirely from the model.

Still, the system that was created has its merits: it’s a hybrid CGE that can process sectors with multiple outputs, whereas most models have each sector producing only one type of output and require more data treatment. The introduction of a piecewise function to model Capital mobility was a success, yey the depreciation of capital failed at presenting interesting results.

As a fundamental part of the modelling tool’s construction, plenty of data was gathered and built into a SAM matching the model’s equations. The energy fluxes from the National Energy Balance were transformed into monetary flows and incorporated into the SAM in order to represent with detail 13 energy technologies. The elasticity parameters were obtained through a literary review, as is commonplace in the field. Calculating new Portuguese specific elasticities would be enough work for a separate thesis.

However, the computational side of this work restricted the ambitious goals set in the beginning, considering that there is enough data to simulate 77 economic sectors, producing 82 non-Energy goods, and also 13 energy sectors that represent different technologies, but due to computation time restraints the non-Energy sectors and goods were only represented in the $10 \times 10$ aggregation.

The project that was developed to calibrate and solve the model was able to serve its purpose, although the code can be improved in several aspects. It was built with some flexibility, so it can be expanded to include more sectors, goods, factors and agents. Even the customization of the production structure is quite simple. Also, the algorithm created to compute the Jacobian numerically using analytical expressions succeeded in greatly reducing the computation time – one of the largest problem
faced when trying to find new equilibria.

In terms of results, the analysis was performed using a static CGE model, so it does not incorporate important dynamic effects, such as the ‘Tariff Deficit’ or technological development. And even as a static model, it could be improved in several fronts. One of the major issues in the first scenario, which was used for validation, was the lack of control over pluviosity, solar or wind availability. This problem can be solved by including more specific factors in the production of renewable electricity, yet these require careful calibration and are hard to grasp in economical terms. Further testing could have been made to estimate the potential impact of this missing feature in the final results.

The simulation of the Portuguese economy without the FIT mechanism in place shows that these policies were largely responsible by the shift towards using RES to generate electricity. Without them, the Government could have saved directly more than 4,000M€, but the dissociation from FFs also provided some economies. In fact, the results pointed to a smaller GDP in the scenario without the FITs. In contrast, the welfare indicators are biased towards higher well-being without the FIT policy in place, however they do not account for health improvements and other intangible enhancements in the quality of life obtained from it.

Despite the similarities, Proença and St. Aubyn’s work predicts a smaller impact on the Portuguese economy. Major differences arise from the description of Portugal’s production structure: whereas in this thesis we propose sectors capable of generating multiple outputs, Proença and St. Aubyn (2013) follows a more traditional approach of mapping one commodity to each sector. This difference is deepened further by the pre and post economic crisis context of the benchmark years used in both projects. Furthermore, the inclusion of incomplete markets, particularly the introduction of unemployment, also contributes to a wider gap.
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van der Werf, E.

Varian, H.

Walras, L.
Wing, I. S.

List of Equations and Variables
There are $3sg + 4s + 10g + 18$ equations in the model and $3sg + 5s + 13g + 25$ variables identified over the following tables.

<table>
<thead>
<tr>
<th>Description</th>
<th>Equation(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector $j$ Profit</td>
<td>$\Pi_j^g = 0$</td>
</tr>
<tr>
<td>Cap. Depreciation in Sector $j$</td>
<td>$\delta K_j = \delta \bar{K}_j K_j$</td>
</tr>
<tr>
<td>Intermediate Consumption</td>
<td>$X_{ij}^{IN} = \frac{\partial Y_j}{\partial y_{ij}} - 1$</td>
</tr>
<tr>
<td>Demand for Labor (Sector $j$)</td>
<td>$L_j = \frac{\partial Y_j}{\partial y_{j}} - 1$</td>
</tr>
<tr>
<td>Demand for Capital (Sector $j$)</td>
<td>$K_j = \frac{\partial Y_j}{\partial y_{j}} - 1$</td>
</tr>
<tr>
<td>Domestic production of good $i$ by sector $j$</td>
<td>$X_{ij}^{D} = \frac{\partial Y_j}{\partial y_{j}}$</td>
</tr>
<tr>
<td>Exports of good $i$ by sector $j$</td>
<td>$X_{ij}^{X} = \frac{\partial Y_j}{\partial y_{j}}$</td>
</tr>
</tbody>
</table>

Table A.1: Equations that model the Production Sectors.

<table>
<thead>
<tr>
<th>Description</th>
<th>Equation(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Family Budget</td>
<td>$MC + SC = p_L L(1 - u_R) + p_K K_C(1 - t^F) + CT_C$</td>
</tr>
<tr>
<td>Savings Rate</td>
<td>$SC = SC/(SC + MC)$</td>
</tr>
<tr>
<td>Consumption</td>
<td>$p_C C = MC$</td>
</tr>
<tr>
<td>Consumption Basket Profit</td>
<td>$Q^C = 0$</td>
</tr>
<tr>
<td>Demand for Good $i$</td>
<td>$X_{i}^F = -C \frac{\partial C}{\partial y_{i}}$</td>
</tr>
</tbody>
</table>

Table A.2: Equations that model Households.

<table>
<thead>
<tr>
<th>Description</th>
<th>Equation(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax Revenue</td>
<td>$T = \sum (p_i^D X_{i}^D + p_i X_{i}^X + p_i^M X_{i}^M) t_F$</td>
</tr>
<tr>
<td></td>
<td>$+ \sum p_j P_j Y_{j} t_F + [p_L L(1 - u_R) + p_K K_C] t_F$</td>
</tr>
<tr>
<td>Savings Rate</td>
<td>$S_C = S_C/(S_C + M_C)$</td>
</tr>
<tr>
<td>Budget</td>
<td>$S_C + M_C = T + p_K K_C + C T_C$</td>
</tr>
<tr>
<td>Consumption</td>
<td>$p_C G = M_G$</td>
</tr>
<tr>
<td>Consumption Basket Profit</td>
<td>$\Pi_G = p_G - \left[ \sum_1^g \theta_i^G \left( \mu_i^A \right)^{1-\sigma_G} \right] \frac{1}{1-\sigma_G}$</td>
</tr>
<tr>
<td>Demand for Good $i$</td>
<td>$X_{i}^G = -G \frac{\partial G}{\partial y_{i}}$</td>
</tr>
</tbody>
</table>

Table A.3: Equations that model Government.
### Table A.4: Equations that model Trade (Armington Sector)

<table>
<thead>
<tr>
<th>Description</th>
<th>Equation(s)</th>
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</thead>
<tbody>
<tr>
<td>Armington Profit of Good $i$</td>
<td>$\Pi^A_i = 0$</td>
</tr>
<tr>
<td>Armington Aggregation of Good $i$</td>
<td>$A_i \partial \Pi^A_i / \partial p_i = \sum_j X^G_{ij} + X^C_i + X^G_i + I_i$</td>
</tr>
<tr>
<td>Total Domestic Production of good $i$</td>
<td>$X^D_i = \sum_j X^D_{ij}$</td>
</tr>
<tr>
<td>Consumption of Domestic Good $i$</td>
<td>$X^D_i = -A_i \partial \Pi^A_i / \partial p_i$</td>
</tr>
<tr>
<td>Consumption of Imported Good $i$</td>
<td>$X^M_i = -A_i \partial \Pi^A_i / \partial p_i^M$</td>
</tr>
<tr>
<td>Consumption of TTM in Good $i$</td>
<td>$X^T_{iTTM} = -A_i \partial \Pi^A_i / \partial p_{iTTM}$</td>
</tr>
<tr>
<td>Profit of the TTM Sector</td>
<td>$\Pi^T_{TTM} = 0$</td>
</tr>
<tr>
<td>Consumption of Good $i$ by the TTM Sector</td>
<td>$TTM_i = TTM \partial \Pi^T_{TTM} / \partial p_i$</td>
</tr>
<tr>
<td>Total TTM</td>
<td>$\sum_i X^T_{iTTM} = TTM$</td>
</tr>
</tbody>
</table>

### Table A.5: Equations that model Investment and Market Clearing Conditions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Variable</th>
<th>Description</th>
<th>#</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_j$</td>
<td>Output of Sector $j$</td>
<td>$t^P_j$</td>
<td>Net Tax on Sector $j$</td>
<td>2s</td>
</tr>
<tr>
<td>$X^I_{ij}$</td>
<td>Input Good $i$ in Sector $j$</td>
<td>$L_j$</td>
<td>Labor use in Sector $j$</td>
<td>$sg + s$</td>
</tr>
<tr>
<td>$K_j$</td>
<td>Capital borrowed by Sector $j$</td>
<td>$\delta K_j$</td>
<td>Capital Consumption in Sector $j$</td>
<td>2s</td>
</tr>
<tr>
<td>$X^D_{ij}$</td>
<td>Domestic good $i$ by Sector $j$</td>
<td>$X^X_{ij}$</td>
<td>Exports of Good $i$ in Sector $j$</td>
<td>$2sg$</td>
</tr>
<tr>
<td>$MC$</td>
<td>Household Consumption</td>
<td>$MG$</td>
<td>Government Consumption</td>
<td>2</td>
</tr>
<tr>
<td>$SC$</td>
<td>Household Savings</td>
<td>$SG$</td>
<td>Government Savings</td>
<td>2</td>
</tr>
<tr>
<td>$sC$</td>
<td>Household Saving Rate</td>
<td>$s_G$</td>
<td>Government Saving Rate</td>
<td>2</td>
</tr>
<tr>
<td>$C$</td>
<td>Aggregate Consumer Good</td>
<td>$G$</td>
<td>Aggregate Government Good</td>
<td>2</td>
</tr>
<tr>
<td>$pr$</td>
<td>Price of Agg. Consumer Good</td>
<td>$pc$</td>
<td>Price of Agg. Government Good</td>
<td>2</td>
</tr>
<tr>
<td>$X^G_i$</td>
<td>Household Demand for Good $i$</td>
<td>$X^G_i$</td>
<td>Government Demand for Good $i$</td>
<td>2g</td>
</tr>
<tr>
<td>$T$</td>
<td>Tax Revenue</td>
<td>$K$</td>
<td>Capital Stock</td>
<td>2</td>
</tr>
<tr>
<td>$I_i$</td>
<td>Investment in good $i$</td>
<td>$t^G_i$</td>
<td>Net taxes on Goods</td>
<td>2g</td>
</tr>
<tr>
<td>$I^A$</td>
<td>Net Income Tax</td>
<td>$I$</td>
<td>Total Investment</td>
<td>2</td>
</tr>
<tr>
<td>$L$</td>
<td>Available Labor</td>
<td>$KC$</td>
<td>Household Capital Stock</td>
<td>2</td>
</tr>
<tr>
<td>$u_R$</td>
<td>Unemployment Rate</td>
<td>$KC$</td>
<td>Government Capital Stock</td>
<td>2</td>
</tr>
<tr>
<td>$X^P_i$</td>
<td>Domestic consumption of Good $i$</td>
<td>$X^M_i$</td>
<td>Imports of Good $i$</td>
<td>2g</td>
</tr>
<tr>
<td>$\Delta K$</td>
<td>Net Capital Accumulation</td>
<td>$u_K$</td>
<td>Rate of Unused Capital</td>
<td>2</td>
</tr>
<tr>
<td>$\delta K$</td>
<td>Total Depreciation of Capital</td>
<td>$\Pi_{TTM}$</td>
<td>Price of TTM</td>
<td>2</td>
</tr>
<tr>
<td>$A_i$</td>
<td>Armington Good $i$</td>
<td>$X^T_{iTTM}$</td>
<td>TTM used by Good $i$</td>
<td>2g</td>
</tr>
<tr>
<td>$TTM_i$</td>
<td>Use of Good $i$ in the TTM Sector</td>
<td>$TTM$</td>
<td>Total TTM</td>
<td>$g + 1$</td>
</tr>
<tr>
<td>$pr^L$</td>
<td>Price of borrowed Capital</td>
<td>$pl$</td>
<td>Price of Labor</td>
<td>2</td>
</tr>
<tr>
<td>$p^D_i$</td>
<td>Domestic Price of Good $i$</td>
<td>$p^M_i$</td>
<td>Import price of Good $i$</td>
<td>2g</td>
</tr>
<tr>
<td>$p^X_i$</td>
<td>Armington Price of Good $i$</td>
<td>$p^X_i$</td>
<td>Export price of Good $i$</td>
<td>2g</td>
</tr>
</tbody>
</table>

### Table A.6: List of variables. Total: $3sg + 5s + 13g + 25$

67
Social Accounting Matrix (SAM)
<table>
<thead>
<tr>
<th>Code</th>
<th>Industry</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Agriculture, Forestry and Fishing</td>
</tr>
<tr>
<td>2</td>
<td>Mining, Quarrying, Manufacturing and Waste Related Activities</td>
</tr>
<tr>
<td>3</td>
<td>Construction Activities</td>
</tr>
<tr>
<td>4</td>
<td>Retail, Transportation, Accomodation and Food and Beverage Services</td>
</tr>
<tr>
<td>5</td>
<td>Information Services</td>
</tr>
<tr>
<td>6</td>
<td>Insurance Financial Services</td>
</tr>
<tr>
<td>7</td>
<td>Real Estate Activities</td>
</tr>
<tr>
<td>8</td>
<td>Professional, Scientific, Technical and Business Support Activities</td>
</tr>
<tr>
<td>9</td>
<td>Public Administration, Education and Health</td>
</tr>
<tr>
<td>10</td>
<td>Other Activities, such as Sports, Arts and Social Work</td>
</tr>
<tr>
<td>CO</td>
<td>Coal and Lignite</td>
</tr>
<tr>
<td>PL</td>
<td>Petroleum</td>
</tr>
<tr>
<td>NG</td>
<td>Natural Gas</td>
</tr>
<tr>
<td>OL</td>
<td>Refined Oil</td>
</tr>
<tr>
<td>DE</td>
<td>Distributed Electricity</td>
</tr>
<tr>
<td>TE</td>
<td>Termoelectricity</td>
</tr>
<tr>
<td>HE</td>
<td>Hydroelectricity</td>
</tr>
<tr>
<td>WE</td>
<td>Wind Electricity</td>
</tr>
<tr>
<td>PV</td>
<td>Photovoltaic Electricity</td>
</tr>
<tr>
<td>GT</td>
<td>Geothermal Electricity</td>
</tr>
<tr>
<td>BE</td>
<td>Biomass Electricity</td>
</tr>
<tr>
<td>HT</td>
<td>Heat</td>
</tr>
<tr>
<td>BM</td>
<td>Biomass</td>
</tr>
</tbody>
</table>

Table B.1: List of Goods

<table>
<thead>
<tr>
<th>Code</th>
<th>Industry</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Agriculture, Forestry and Fishing</td>
</tr>
<tr>
<td>2</td>
<td>Mining, Quarrying, Manufacturing and Waste Related Activities</td>
</tr>
<tr>
<td>3</td>
<td>Construction Activities</td>
</tr>
<tr>
<td>4</td>
<td>Retail, Transportation, Accomodation and Food and Beverage Services</td>
</tr>
<tr>
<td>5</td>
<td>Information Services</td>
</tr>
<tr>
<td>6</td>
<td>Insurance Financial Services</td>
</tr>
<tr>
<td>7</td>
<td>Real Estate Activities</td>
</tr>
<tr>
<td>8</td>
<td>Professional, Scientific, Technical and Business Support Activities</td>
</tr>
<tr>
<td>9</td>
<td>Public Administration, Education and Health</td>
</tr>
<tr>
<td>10</td>
<td>Other Activities, such as Sports, Arts and Social Work</td>
</tr>
<tr>
<td>OR</td>
<td>Oil Refinery</td>
</tr>
<tr>
<td>TE</td>
<td>Thermoelectric Power</td>
</tr>
<tr>
<td>HP</td>
<td>Hydro Power</td>
</tr>
<tr>
<td>WP</td>
<td>Wind Power</td>
</tr>
<tr>
<td>SP</td>
<td>Solar Power</td>
</tr>
<tr>
<td>GT</td>
<td>Geothermal</td>
</tr>
<tr>
<td>EG</td>
<td>Electric Grid</td>
</tr>
<tr>
<td>BM</td>
<td>Biomass</td>
</tr>
</tbody>
</table>

Table B.2: List of Sectors
| Good | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | CO | PL | NG | OL | DE | TE | HE | WE | PV | GT | BE | HT | BM |
|------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|
|      | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| TTM | 2,876 | 72,117 | 0 | -31,644 | 574 | 0 | 0 | 0 | 0 | 3 | 84 | 2,233 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 216 |
| Taxes | -1 | 8,699 | 8,241 | 1,667 | 1,035 | 1,105 | 25 | 2,264 | 63 | 1,040 | 63 | 4,102 | 0 | 274 | 231 | -78 | 710 | -78 | 2.7 | 6.5 | 10.0 |
| Imports | 17,278 | 7,676 | 30,047 | 1,308 | 1,181 | 1,018 | 219 | 1,181 | 1,018 | 219 | 1,181 | 1,018 | 219 | 1,181 | 1,018 | 219 | 1,181 | 1,018 | 219 | 1,181 | 1,018 | 219 |

Table B.3: Goods supply: margins, total output, taxes and imports in M.
<table>
<thead>
<tr>
<th>Good</th>
<th>Households</th>
<th>Government</th>
<th>Investment</th>
<th>Exports</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4,546</td>
<td>0</td>
<td>481</td>
<td>922</td>
</tr>
<tr>
<td>2</td>
<td>45,002</td>
<td>1,718</td>
<td>11,791</td>
<td>33,727</td>
</tr>
<tr>
<td>3</td>
<td>469</td>
<td>38</td>
<td>17,352</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>18,048</td>
<td>731</td>
<td>177</td>
<td>6,583</td>
</tr>
<tr>
<td>5</td>
<td>5,188</td>
<td>205</td>
<td>2,273</td>
<td>934</td>
</tr>
<tr>
<td>6</td>
<td>5,991</td>
<td>0</td>
<td>0</td>
<td>489</td>
</tr>
<tr>
<td>7</td>
<td>12,149</td>
<td>22</td>
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<td>8</td>
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<tr>
<td>8</td>
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<td>10</td>
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<td>50</td>
<td>103</td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>PL</td>
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<td>0</td>
</tr>
<tr>
<td>HE</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>WE</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>PV</td>
<td>0</td>
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<td>0</td>
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**Table B.5: Demand For Goods**
Table B.6: Intermediate Consumption in M Ä.

| Sector | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | OR | TE | HP | WP | SP | GT | EG | BM |
|--------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|----|----|----|----|----|----|----|
| Inputs | 4,387| 50,428| 20,047| 32,979| 7,665| 6,420| 3,145| 10,159| 15,642| 3,083| 8,104| 1,916| 88| 83| 2| 3| 5,820| 1,002|
| Labor  | 1,047| 12,129| 6,495| 20,582| 2,616| 4,786| 524| 5,988| 27,734| 3,454| 183| 246| 86| 257| 4| 7| 497| 169|
| Assets | 2,355| 4,959| 1,922| 9,703| 1,297| 4,793| 2,243| 2,231| 2,507| 672| 97| 253| 984| 968| 36| 56| 22| 102|
| Taxes  | -751| 114| 15| 224| 0| 27| 589| -41| -870| -298| 8| 0| 0| 0| 0| 0| 937| -54|
| Depreciation | 816| 3,857| 1,032| 5,200| 1,602| 769| 9,505| 1,937| 3,452| 519| 213| 0| 0| 0| 0| 1,200| 287|

Table B.7: Industry Expenses: material inputs, labor, assets (or borrowed capital), taxes and capital depreciation in M Ä.

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Table B.8: Goods Industries

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</table>
Code
C.1 Main.m

% Main Script
% -> Locate File Path
% -> Calls other functions
% - System Config
% - System Read Data
% - System Variables
% - System Parameters

% Create filepaths
folder = '/Users/joaoromao/Dropbox/Tese-Romao/Calc';
system = create_code_filepath(folder);

% System Configuration
% Reads general information about the Model
% Indicates where/how to read the data
system.config = system_config(system);

% System Variables
% Read Data
system.data = system_read_data (system);

% Calculate initial solution (sol) vector
% [system.vars, system.names] = system_vars(system);
% system.bench = system.vars;

% Determine parameters
system.pars = system_pars(system);

% Create List of Equations
system.list = create_eq_list();

% Define System Topology (Eqs and Vars Ranges)
% [system.topg, system.y0] = system_topology(system);

function [ system ] = create_code_filepath ( folder )
% Add folder to 'system' structure
system.folder = folder;
% Add Path and SubPath
addpath(genpath(folder));
% Select Data filepath
system.rfp = fullfile(folder, 'Data.xlsx');

end

function [ gen ] = system_config (system)
% System Configuration
% Reads general information about the Model
% Indicates how to read the rest of the data
% Read general Sets (Set of sets) from Excel
% [config_n, config_txt] = xlsread(system.rfp,'Configuration');
%
% Determine number of sets
% gen.nset = config_n(1,1);

% Get information about each Set
% set = cell(gen.nset, 1);
% for i = 1 : (gen.nset - 2)
% set[i].length = config_n(i);
% set[i].name = config_txt(i,2);
%
% Read list of items in each set
% [page_n, ~, page_all] = xlsread(system.rfp, set[i].name);
% range = 1 : set[i].length;
% item.num = page_n(range, 1);
% item.parent = page_n(range, 3);
item.label = page_all(range, 4); % Item name / description
item.code = page_all(range, 5); % Item Code

% Amount of aggregation levels
set{i}.maxl = max( page_n(range,1) );
max_num = zeros(set{i}.maxl, 1);
for k = 1 : set{i}.maxl
  max_num(k) = max(item.num(item.level == k));
end

% Replace 0-level items
% With copies at every level (1 : set{i}.maxl)
for j = 1 : length(item.level)
  if isnumeric(item.code{j})
    item.code{j} = num2str(item.code{j});
  end
  if item.level(j) == 0
    max_num = max_num + 1;
    item.level = [item.level; (1 : set{i}.maxl)'];
    item.num = [item.num; max_num];
    item.parent = [item.parent; 0];
    new_names = repmat(item.label{j}, set{i}.maxl, 1);
    item.label = [item.label; new_names];
    new_codes = repmat(item.code{j}, set{i}.maxl, 1);
    item.code = [item.code; new_codes];
  end
end

% Remove original 0-level items
delete_items = (item.level == 0);
item.level(delete_items) = [ ];
item.num(delete_items) = [ ];
item.parent(delete_items) = [ ];
item.label(delete_items) = [ ];
item.code(delete_items) = [ ];

% Item List is now complete
set{i}.item = item;

% Amount of Items at the lowest level
set{i}.size = max( page_n(range,2) );

% Default Aggregation is the maximum possible
set{i}.level = set{i}.maxl;

end

end

% Other configurations %

% Read list of from the 'Configurations' Page
for i = (gen.nset + 1) : length(config_txt(:,1))
  if ~isfield(gen,config_txt{i,1})
    gen.(config_txt{i,1}) = [ ];
  end % if
  if isnan(config_n(i,2))
    gen.(config_txt{i,1}) = [gen.(config_txt{i,1}); config_txt(i,2)];
  else
    gen.(config_txt{i,1}) = [gen.(config_txt{i,1}); config_n(i,2)];
  end % if
end % for

% Default settings
% - Spacing used when calculating derivatives
if ~isfield(gen, 'h')
  gen.h = 10^-6;
end

% - Tolerance for error in the Newton Method
if ~isfield(gen, 'tol')
  gen.tol = 10^-6;
end

% - Max fsolve iterations
if ~isfield(gen, 'cycles')
  gen.cycles = 100;
end

% - multiplier for profit equations
if ~isfield(gen, 'm_profit')
112   gen.m_profit = 10^4;
113   end
114
115   % Define set aggregation levels (default is maxl)
116   % 1: Full aggregation
117   % 2: P/A10 disaggregation
118   % 3: P/A38 disaggregation
119   % 4: P88/A80
120
121   % Goods/Products
122   if isfield(gen, 'levelGood')
123     set{2}.level = min(gen.levelGood, set{2}.maxl);
124   else
125     set{2}.level = set{2}.maxl;
126   end
127
128   % Activities/Industries
129   if isfield(gen, 'levelSector')
130     set{3}.level = min(gen.levelSector, set{3}.maxl);
131   else
132     set{3}.level = set{3}.maxl;
133   end
134
135   % Factors
136   if isfield(gen, 'levelFactor')
137     set{4}.level = min(gen.levelFactor, set{4}.maxl);
138   else
139     set{4}.level = set{4}.maxl;
140   end
141
142   % Create iteration ranges
143   gen.g = sum(set{2}.range{set{2}.level});  \% N of Goods
144   gen.c = sum(set{3}.range{set{3}.level});  \% N of sectors
145   gen.f = sum(set{4}.range{set{4}.level});  \% Factors
146
147   % Assign Information about Set to the gen (General Information Structure)
148   gen.set = set;
149
150 end

function [ data ] = system_read_data (system)

3   % System Variables
4   % Creates a structure with the benchmark variables
5   g = system.config.set{2}.size;  \% N of Goods
6   c = system.config.set{3}.size;  \% N of sectors
7   f = system.config.set{4}.size;  \% N of Factors
8
9   % Read SAM table
10   [~,~, SAM] = xlsread(system.rfp, system.config.SAM{:});
11
12   % Store SAM
13   data.SAM = SAM;
14
15   % Remove Headers
16   SAM{1,1} = {};  \% N of Goods
17   SAM{1,2} = {};  \% N of Factors
18   SAM = cellfun(@(nan20, SAM, 'UniformOutput', false);
19
20   size_SAM = size(SAM);  \% N of Goods
21   for i = 1 : size_SAM(1)
22     for j = 1 : size_SAM(2)
23       if isnan(SAM(i,j))
24         SAM(i,j) = 0;
25     end
26   end
27
28
29   % Read Intermediate Consumption Array From an Excel page
30   data.xIN = xlsread(system.rfp, SAM{1,2});
31   data.xIN = nan20(data.xIN[1:1,1:1]);
32   data.xOUT = xlsread(system.rfp, SAM{2,1});
33   data.xOUT = nan20(data.xOUT[1:1,1:1]);
34
35   % Read Output Array From an Excel page
36   data.xG = nan20(data.xIN[1:1,1:1]);
37   data.xM = nan20(data.xIN[1:1,1:1]);
38
39   % Read eXports and iMports Array From an Excel page
40   data.KX = nan20(data.xIN[1:1,1:1]);
41   data.KM = nan20(data.xIN[1:1,1:1]);
42
43   % Read Final Demand Array From an Excel page
44   data.xG = nan20(data.xIN[1:1,1:1]);
45   data.xC = nan20(data.xIN[1:1,1:1]);
data.xI = nan20(xlsread(system.rfp, SAM{1,8}));

% Read More data on Goods From an Excel page
data.xTax = nan20(xlsread(system.rfp, SAM{6,1}));
data.pTax = nan20(xlsread(system.rfp, SAM{6,2}));

% Read More data on Factor use From an Excel page
data.L = nan20(xlsread(system.rfp, SAM{3,2}));
data.K = nan20(xlsread(system.rfp, SAM{4,2}));

% Read Info on Physical Capital From an Excel page
data.xK = nan20(xlsread(system.rfp, SAM{8,2}));
data.DK = SAM{8,11};

% Read More data on TT. Margins Array From an Excel page
data.TTM = nan20(xlsread(system.rfp, SAM{1,1}));

% Read More data on Agents (Gov & HH) From an Excel page
data.H.M = sum(data.xC);
data.H.S = SAM{9,7};
data.H.taxes = SAM{7,6} - SAM{6,7};
data.H.L = SAM{7,3};
data.H.K = SAM{7,4};
data.G.M = sum(data.xG);
data.G.S = SAM{9,6};
data.G.T = sum(data.xTax) + sum(data.pTax) - data.H.taxes;
data.G.CT = SAM{5,6} + SAM{10,6} - SAM{6,5} - SAM{6,10};
data.G.L = 0;
data.G.K = SAM{6,4};

% Factors
for i = 1 : f
    code = system.config.set{4}.item.code{i};
data.(code) = xlsread(system.rfp, SAM{2+i, 2});
data.row.(code) = SAM{10, 2+i} - SAM{2+i, 10};
data.oth.(code) = SAM{f+3,2+i};
end % for

% Sector Output
data.Y = sum(data.xIN,1)' + data.K + data.L;

% Read Page: Others (remaining variables)
[-~, data.others] = xlsread(system.rfp, system.config.Others{:});
data.others = cell2struct(data.others(:,2),data.others(:,1));

% Read Page: Vars Names
[-~, data.var_names] = xlsread(system.rfp, system.config.VNames{:});

% Read the structure of the Production Functions
[-~, data.PF] = xlsread(system.rfp, system.config.PF{:});
[data.pars, ~, data.pars_all] = xlsread(system.rfp, system.config.Elast{:});
data.pars_all(1:9,:) = [];
data.elast = cell2struct(data.pars_all(:,2), data.pars_all(:,1));

function [ val, names ] = system_vars( system )
% System Variables
% Creates a structure with the benchmark variables
% Based on the data structure and also their names
data = system.data;
c = system.config.c;
g = system.config.g;
goods = system.config.set{2};
sects = system.config.set{3};

% Sector Output
val.Y = set_aggregate(data.Y, sects);

% Intermediate Consumption
val.xIN = set_aggregate(data.xIN', sects);
val.xIN = set_aggregate(val.xIN', goods);

% Labor Input
val.L = set_aggregate(data.L, sects);

% Capital Input
val.K = set_aggregate(data.K, sects);

% Depreciation of Capital
val.dK = set_aggregate(data.dK, sects);

% Total Depreciation of Capital
val.dKt = sum(val.dK);

% OUTPUT
xOUT = set_aggregate(data.xOUT', goods);
xOUT = set_aggregate(xOUT', sects);
xXt = set_aggregate(data.xXt, goods);

% Split Output into Exports & Domestic production
xD = zeros(c, g);
line = 1 - ((xXt') ./ sum(xOUT));
line(isnan(line)) = 0;
line(isinf(line)) = 0;
for j = 1 : c
    xD(j,:) = xOUT(j,:) .* line;
end

% Separate values that are too small
val.xD = xD;
val.xx = xOUT - val.xD;
small_values = val.xx;
small_values(val.xx > system.config.roundErr) = 0;
val.xx = val.xx - small_values;
val.xD = val.xD + small_values;
val.xDt = sum(val.xD, 1);

% Consumption: HH & Gov
val.M.C = data.H.M;
val.M.G = data.G.M;

% Savings: HH & Gov
val.S.C = data.H.S;
val.S.G = data.G.S;

% Saving Rate: HH & Gov
val.Sr.C = data.H.S / (data.H.S + data.H.M);
val.Sr.G = data.G.S / (data.G.S + data.G.M);

% Aggregate Goods: HH & Gov
val.C = val.M.C;
val.G = val.M.G;

% HH & Gov Demand for Goods
val.xC = set_aggregate(data.xC, goods);
val.xG = set_aggregate(data.xG, goods);

% HH & Gov Capital Stock
val.KC = data.H.K;
val.KG = data.G.K;

% Tax Revenue
val.T = data.G.T;

% Investment
val.xI = set_aggregate(data.xI, goods);

% Total Investment
val.I = sum(val.xI);

% Capital Stock
val.KS = data.others.KS;

% Net Capital Accumulation
val.DK = data.DK;

% Available Labor
val.Lt = data.others.Lt;

% Unemployment Rate
val.Ur = data.others.Ur;

% Lost Capital Rate
val.LK = 0;

% Imports of good i
val.xM = set_aggregate(data.xM, goods);

% Trade and Transport Margins
val.xTTM = set_aggregate(data.TTM, goods);
val.TTM = - val.xTTM;

% Clear Negative values
val.xTTM(val.xTTM < 0) = 0;

% TTM supply
val.TTM(val.TTM < 0) = 0;

% Total TTM
val.TTM = sum(val.TTM);

% Taxes
% On production
val.tax.P = set_aggregate(data.pTax, sects) ./ (val.Y);
val.tax.P(isnan(val.tax.P)) = 0;

% On goods
val.tax.G = set_aggregate(data.xTax, goods);
val.tax.G = val.tax.G ./ (val.xM + val.xD');
val.tax.G(isnan(val.tax.G)) = 0;

% On HH income
val.tax.F = data.H.taxes / (data.H.L + data.H.K);
val.tax.F(isnan(val.tax.F)) = 0;

% Armington Good
val.A = (val.xD' + val.xM) .* (1 + val.tax.G) + val.xTTM;

% Create price structure (Benchmark prices are all 1)
% Domestic
val.p.D = ones(1, g);
% Armington
val.p.A = ones(1, g);
% Export
val.p.X = ones(1, g);
% Import
val.p.M = ones(1, g);

% Aggregate Goods: HH & Gov
val.p.C = 1;
val.p.G = 1;
val.p.TTM = 1;

% Factors: Labor and Capital
val.p.L = 1;
val.p.K = 1;

% Create list of number strings 'i'
num = cell(max(c,g),1);
for i = 1 : max(c,g)
    num{i} = ['','num2str(i)];
end

% Convert Variable Names to structure
% Go through the list of variable names
% Change 'i' and 'j' to numbers
for i = 2 : length(data.var_names(:,1))
    sub_names = strsplit(data.var_names{i,1}, '.');
    if length(sub_names) > 1
        size_nam = size(val.(sub_names{1}).(sub_names{2}));
        nam = cell(size_nam);
        for j = 1 : size_nam(1)
            for k = 1 : size_nam(2)
                nam{j,k} = regexprep(data.var_names{i,2}, 'i(?=|$)|', num{j});
                nam{j,k} = regexprep(nam{j,k}, 'j(?=|$)|', num{k});
            end
        end
        names.(sub_names{1}).(sub_names{2}) = nam;
    else
        size_nam = size(val.(sub_names{1}));
        nam = cell(size_nam);
        for j = 1 : size_nam(1)
            for k = 1 : size_nam(2)
                nam{j,k} = regexprep(data.var_names{i,2}, 'i(?=|$)|', num{j});
                nam{j,k} = regexprep(nam{j,k}, 'j(?=|$)|', num{k});
            end
        end
        names.(sub_names{1}) = nam;
    end
end % for i
end % if
end % for
end % function

function [ pars ] = system_pars(system)
% Call common variables
% N of Goods
g = system.config.g;
% N of Sectors
c = system.config.c;
% N of Factors
f = system.config.f;
data = system.data;
vars = system.vars;
% variables
goods = system.config.set{2};
sects = system.config.set{3};
data = system.data;
vars = system.vars;
% Set of Goods
goods = system.config.set{2};
% Set of Sectors
goods = system.config.set{2};
% Convert PF page into structure
PF = struct;
for i = 1 : length(data.PF)
    % Extract code (2nd column)
    code = num2str(data.PF{i,2});
    % Check corresponding item
    item_index = cellfun(@(x)strcmp(x,code), goods.item.code);
    if any(item_index) == 0
        % It's a good, so store the number
        PF.(data.PF{i,1}) = code;
    else
        % ELSE: Item is found
        if isfield(PF, data.PF{i,1})
           PF.(data.PF{i,1}) = {PF.(data.PF{i,1}), code};
        else
            PF.(data.PF{i,1}) = code;
        end
    end
end
PF2 = struct;
pf_names = fieldnames(PF);
for i = 1 : length(pf_names)
    if ~isnumeric(PF.(pf_names{i}))
        PF2.(pf_names{i}) = g + cellfun(@sum, PF.(pf_names{i}));
    else
        PF2.(pf_names{i}) = [PF.(pf_names{i})];
    end
end
end
end
PF2 = PF;
for i = 1 : f
    code = system.config.set{4}.item.code{i};
    PF2.(code) = g + i;
end
range = system.config.output(:);
while any(isfield(PF2, range))
    range = cellfun(@(x)isfield_out(PF2, x)), range, 'UniformOutput', false);
end
for i = 1 : f
    code = system.config.set{4}.item.code{i};
    pars.range.(code) = range == (i+g);
end
range(range > g) = [];
pars.range.goods = [];
for i = 1 : g
    pars.range.goods = [pars.range.goods, find(range == i)];
end
pars.Ur = data.others.Ur;
pars.rY = 1 + vars.tax.P + vars.dK ./ vars.Y;

% Benchmark output
%
% Input Thetas
theta = struct;
prod = struct;

names = fieldnames(PF);
for k = 1 : length(names)
    vector = zeros(length(PF.(names{k})), c);
    for i = 1 : length(PF.(names{k}))
        for j = 1 : c
            if iscellstr(PF.(names{k})(i))
                vector(i, j) = vars.(PF.(names{k})(i))(j);
            elseif isfield(theta.in, PF.(names{k})(i))
                vector(i, j) = sum(theta.in.(PF.(names{k})(i))(1:j));
            end
        end
    end
end

% Output Thetas
theta.OUT = zeros(g, c);
for i=1:c
    theta.OUT(:,i) = ( vars.xD(i,:) + vars.xX(i,:) )' / sum( vars.xD(i,:) + vars.xX(i,:) );
end

% Exports - Theta X
theta.X = sum(vars.xX,1) ./ sum(vars.xD + vars.xX,1);
theta.X = [theta.X; 1];

% Imports (Armington) production
base = (vars.xD' .* (1 + vars.tax.G)' + vars.xTM');
theta.Mr = [vars.xTM' ./ base;
            vars.xD' .* (1 + vars.tax.G)' ./ base];
theta.Mr = [theta.Mr; 1];

% TTM Sector
theta.TTM = vars.TTM ./ sum(vars.TTM);

% Agent Consumption
theta.C = vars.xC / sum(vars.xC);
theta.G = vars.xG / sum(vars.xG);

% Make sure there are no Inf or NaN in the array
theta_names = fieldnames(theta);
for i = 1 : length(theta_names)
    field = theta_names{i};
    if ~isfield(theta.(field))
        theta.(field) = zeros(1, length(theta.(field)));
    elseif ~isfield(theta.in.(field))
        theta.in.(field) = zeros(1, length(theta.in.(field)));
    end
end

% Normalize Input thetas
theta_names = fieldnames(theta.in);
for i = 1 : length(theta_names)
    field = theta_names{i};
    if ~isstruct(theta.in.(field))
        line = sum(theta.in.(field));
    for j = 1 : length(theta.in.(field))
        theta.in.(field)({j,1}) = theta.in.(field)({j,:}) ./ line;
    end
end

% Assign thetas to the parameters (pars) structure
pars.theta = theta;
% Create Make and Intermediate Consumption sparse indices

roundErr = system.config.roundErr;
[pars.X.sects , pars.X.goods ] = find(vars.xX > roundErr);
[pars.D.sects , pars.D.goods ] = find(vars.xD > roundErr);
[pars.IN.sects , pars.IN.goods ] = find(vars.xIN > roundErr);

% Create Exclusion Indices
pars.exclude.X = vars.xX(:) <= roundErr;
pars.exclude.D = vars.xD(:) <= roundErr;
pars.exclude.IN = vars.xIN(:) <= roundErr;

% Factors: from RoW and Corporations
for i=1:f
    code = system.config.set{4}.item.code{i};
    pars.(code) = data.row.(code) + data.oth.(code);
end

% Assign elasticities to the elasticity structure
elast.out(1:c) = set_aggregate(data.pars(1,1:sects.size)', sects, @mode)';
elast.exp(1:g) = set_aggregate(data.pars(2,1:goods.size)', goods, @mode)';
elast.imp(1:g) = set_aggregate(data.pars(3,1:goods.size)', goods, @mode)';
elast.psi = data.pars(4,1);
elast.C = data.pars(5,1);
elast.G = data.pars(6,1);
elast.TMgood = data.pars(7,1);
elast.TM = data.pars(9,1);
elast.prod = prod;

pars.elast = elast;

% Assign other necessary values
pars.g = g;
pars.c = c;
pars.f = f;
pars.output = system.config.output;
pars.h = system.config.h;
pars.epsilon = system.config.epsilon;
pars.m_profit = system.config.m_profit;
end

% function [ topology, y0 ] = system_topology (system)

function [ topology, y0 ] = system_topology (system)

% EQ_RESIDUALS - Equation Residuals
% This function verifies numerically all the equations in the model
% Compute: F(vars struct) = out {column vector}

% Call Main Structures
% Variables: has all the variables in the model
vars = system.vars;

% Parameters: contains information about the system
% - thetas
% - elasticities
% - production structure
pars = system.pars;

% Benchmark: Variables in the initial state
bench = system.bench;

% Compute Residuals Using functions in 'list'
list = system.list;
_n_list = length(list);
out = cell(_n_list, 1);
eqs_range = out;

for i = 1 : _n_list
    % Get Function Name
    f_name = str2func(list{i}{1});

    % Create Cell with Arguments
    _n_args = length(list{i}{1}) - 1;
    f_args = cell(1,_n_args);
    for j = 1 : _n_args
        f_args(j) = eval([list{i}{1}(j+1)]);
    end % for j
end % for i
```matlab
% Calculate Function
out{i} = feval(f_name, f_args{:});
% Add size to range
if i == 1
    eqs_range(i) = (1 : length(out{i}))';
else
    eqs_range(i) = max(eqs_range(i-1)) + ((1 : length(out{i})));
end
end % for i
jacob_height = max(eqs_range{i});
% Create a Range for Each Variable (in the Jacobian)
vars_names = fieldnames(vars);
max_i = length(vars_names);
var_range = vars;
index = 0;
for i = 1 : max_i
    sub_vars = vars.(vars_names{i});
    sub_vars_names = fieldnames(sub_vars);
    max_j = length(sub_vars_names);
    for j = 1 : max_j
        new_name = [vars_names{i},sub_vars_names{j}]
        if isfield(var_range, vars_names{i})
            var_range = rmfield(var_range, vars_names{i});
        end
        var_range.(new_name) = index + (1 : numel(sub_vars.(sub_vars_names{j})));
        index = max(var_range.(new_name));
    end
    else
        var_range.(vars_names{i}) = index + (1 : numel(sub_vars));
        index = max(var_range.(vars_names{i}));
    end
end
% Merge exclusion lists
exclude = [ var_range.xX(pars.exclude.X) ... var_range.xD(pars.exclude.D) ... var_range.xIN(pars.exclude.IN) ];
% Assign ranges to topg structure
topology.eqs_range = eqs_range;
topology.var_range = var_range;
topology.jacob_size = [jacob_height, index];
topology.exog = index - jacob_height - length(exclude);
topology.exclude = exclude;
% Create reference y0
y0 = cell2mat(out);
% Display: amount of exogenous/control variables
display(['The system requires ' num2str(topology.exog) ' exogenous variables'])
end % function

function [ out ] = isfield_out ( Structure, Field )
% FUNCTION IsField Out
% If Structure has Field, returns Structure.Field
% Else, returns Field
% Arguments
% - Structure (struct)
% - Field (string)
% Output is Field (default)
out = Field;
% If Structure has Field, change Output to Structure.Field
if isfield(Structure, Field)
    out = Structure.(Field);
end
end % Function

function [ matrix ] = nan20( matrix )
```

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% FUNCTION nan20 – NaN to Zero
% Replaces every 'NaN' inside a matrix to '0'
% Arguments
% matrix – matrix that gets the NaNs replaced
% matrix(isnan(matrix)) = 0;
end

function [ var ] = set_aggregate( var, set, fun )
% FUNCTION Set_Aggregate
% Transforms variables into the chosen aggregation level
% Arguments:
% var (system.vars structure)
% set (Goods, Sectors or Factors)
% fun (function handle)
% The default fun is @sum
% It is used to aggregate quantities (such as Y)
if nargin < 3
fun = @sum;
end

% Initialize cycle variables
lev = set.maxl;
while lev > set.level
% Get parent indexes
indexes = set.item.parent(set.range{lev});
% Initialize for variables
n_columns = length(var{1,:});
B = zeros(max(indexes), n_columns);
% Merge child items into parents using fun
for i = 1 : n_columns
B(:,i) = accumarray(indexes, var{1,i}, [], fun);
end % for i
% Update variables
var = B;
lev = lev - 1;
end % while
end % function

function [ output ] = struct2vect( input )
% FUNCTION Struc2Vect
% Transforms a structure into a vector with all the values inside
% The structure cannot have strings, cells, tables
% or function handles.
% Arguments
% input (structure with other structures or arrays inside)
% Create Output vector
output = [];
% Read field names
fnames = fieldnames(input);
% Iterate over every field
for i = 1 : length(fnames);
% IF structure, recall function
if isstruct(input.(fnames{i}))
vector = struct2vect(input.(fnames{i}));
else
vector = input.(fnames{i});
end % if
end % for
output = [output; vector];
end % function
function [ output, vector ] = struct_add( input, vector )
% FUNCTION Struct_add (Struct Add)
% Goes through all the values inside structure 'input'
% and adds the corresponding value in vector
% Arguments
%   input (structure with other structures or arrays inside)
%   vector (line or column array)
output = input;
fnames = fieldnames(input);
for i = 1 : length(fnames)
    if isempty(vector)
        return;
    elseif isstruct(input.(fnames{i}))
        [output.(fnames{i}), vector] = struct_add(input.(fnames{i}), vector);
    else
        v_size = size(input.(fnames{i}));
        range = 1 : prod(v_size);
        output.(fnames{i}) = output.(fnames{i}) + reshape(vector(range), v_size);
        vector(range) = [];
    end
end
end % function

function [ output ] = struct_cmp( a, b )
% FUNCTION Struct_Cmp (Structure Compare)
% Transforms structure 'a' into a vector, but changes the values to 0
% unless 'b' also has the same values in the same field
% Arguments
%   a (leading structure)
%   b (reference structure)
% Create Output vector
output = [];
a_fields = fieldnames(a);
for i = 1 : length(a_fields)
    sub_a = a.(a_fields{i});
    if isfield(b, a_fields{i})
        sub_b = b.(a_fields{i});
        if isstruct(sub_a)
            output = [output, struct_cmp(sub_a, sub_b)];
        else
            is_end = cellfun(@isempty, sub_b);
            c = numel(sub_a) - numel(is_end);
            if c > 0
                c = zeros(1, c);
            else
                c = [];
            end
            output = [output, -is_end, c];
        end
    else
        if isstruct(sub_a)
            output = [output, struct2vect(sub_a)' * 0];
        else
            output = [output, zeros(1, numel(sub_a))];
        end
    end
end % if
end % function

function [ output, vector ] = vect2struct( input, vector )
% FUNCTION Vect2Struc
% Undoes Struc2Vector
% Transforms a vector back into a structure with the shape of 'input'
% retaining all the values of vector.
% Arguments
%   input (structure with other structures or arrays inside)
%   vector (vector of values)
end % function
output = input;
fnames = fieldnames(input);

for i = 1 : length(fnames);
    if isstruct(input.(fnames{i}));
        output.(fnames{i}), vector = vect2struct(input.(fnames{i}), vector());
    else
        v_size = size(input.(fnames{i}));
        range = 1 : prod(v_size);
        output.(fnames{i}) = reshape(vector(range), v_size);
        vector(range) =[];
    end
end

C.2 Solver

function [ out ] = call_eq_list( system )
    %
    % FUNCTION Call_Eq_List
    % System list to compute numerically all the equations in the model: F<vars struct> = out <column vector>
    %
    % Call Main Structures
    % Variables: has all the variables in the model
    vars = system.vars;
    % Parameters: contains information about the system
    pars = system.pars;
    % Benchmark: Variables in the initial state
    bench = system.bench;
    % For call each function in list a place output in a cell
    n_list = length(system.list);
    out = cell(n_list, 1);
    for i = 1 : n_list
            f_name = str2func(system.list{i}{1});
            n_args = length(system.list{i,:}) - 1;
            f_args = cell(1,n_args);
            for j = 1 : n_args
                    f_args{j} = eval(system.list{i}{j+1});
            end
            out{i} = feval(f_name, f_args{:});
    end
end

function [ Jacob ] = call_Jacobian( system )
    %
    % FUNCTION Call_Jacobian
    % Use system.list to compute the analytical Jacobian in the model: J<vars struct> = Jacob <square matrix>
    %
    % Call Main Structures
    % Variables: has all the variables in the model
    vars = system.vars;
    % Parameters: contains information about the system
    pars = system.pars;
    % Benchmark: Variables in the initial state
    bench = system.bench;
    % List of equation variables
    list = system.list;
    % Block ranges
    eqs_range = system.topg.eqs_range;
    var_range = system.topg.var_range;
    % Declare Jacobian as a sparse object
    % Size was predicted in system_topology
Jacob = sparse(system.topg.jacob_size(1), system.topg.jacob_size(2));

% Compute Jacobian
% Go through all of the Equations (lines) on list
n_list = length(list);
for i = 1 : n_list
    % Create cell with arguments
    n_args = length(list{i,:}) - 1;
    f_args = cell(1, n_args);
    for j = 1 : n_args
        f_args{j} = eval(list{i}[j+1]);
    end % for j

    % Calculate Jacobian Block
    % Derivative: Function(j) to args(j)
    for j = 1 : n_args
        % Skip unless argument is a variable ('vars___')
        if ~isempty(regexp(list{i}[j+1], '^\{vars\}\b.*$'))
            % Retrieve variable name
            deriv_to = regexp(list{i}[j+1], '\{vars\\b.*$');
            % IF the argument is a structure
            if isfield(f_args{j}, deriv_to) && isstruct(f_args{j}.

            % Then we compute the derivatives numerically
            if isfield(f_args{j}, deriv_to) && isstruct(f_args{j})
                % Computes f_index(x)
                y0 = feval(list{i}[1], f_args{:});
                % Identifies structure variable
                svar = f_args{j};
                % Get all of its fieldnames
                v_fields = fieldnames(svar);
                % Iterate over all of its field names
                for k = 1 : length(v_fields)
                    % Allocate space for D
                    D = sparse(numel(eqs_range{i}, numel(var_range.

                    % IF it's an Exogenous variable
                    if system.exog.pos(var_range.

                    % Skip it if it's an Exogenous variable
                    if system.exog.pos(var_range.

                    % Acceptable difference: to make sure the
                    % derivative takes place in a linear part of
                    % the function
                    ok_dif = abs(f_args{j}.

                    while max(abs(a + b - 2 * y0)) > ok_dif
                        % Compute step h
                        h = abs(f_args{j}.

                        % Make sure 'h' is not 0
                        if h == 0, h = h_deriv; end % if

                        % Add h to var(m)
                        f_args{j}[1].v_fields[k] = f_args{j}.

                        % Compute 'a'
                        a = feval(list{i}[1], f_args{:});
                        % Remove h to var(m)
                        f_args{j}[1].v_fields[k] = f_args{j}.

                        % Compute 'b'
                        b = feval(list{i}[1], f_args{:});
                        % Return var(m) to original value
                        f_args{j}.v_fields[k] = f_args{j}.

                        % Make h_deriv shorter until function is
                        % approx. linear
                        h_deriv = h_deriv / 10;
                    end % while

                    % Compute column 'm' of D
                    D(:,m) = (a - b) ./ (2 * h);
                end % for k
            end % Else
            % No call the function Deriv_'Function'_farg
        end % if

        % Write new function name
        f_name = ['Deriv_'; list{i}[1], '_' num2str(j)];
        % Call the function with its corresponding arguments
        % And store it in Jacobian
        Jacob(eqs_range{i}, var_range.(deriv_to)) = feval(f_name, f_args{:});
    end % if
end % for j
end % for i
% Remove columns with the Control Variables
% Or columns that are Excluded because they represent 0 = 0 equations
system.exog.pos(system.topg.exclude) = 1;
Jacob(i, system.exog.pos == 1) = [];

function [ sol, res, exit_flag, config ] = fsolve_my_fun(system, Jacob_on)

% With the Levenberg-Marquardt algorithm
% Transform vars (structure) into vector
x0 = struct2vec(system.vars);

% Remove variables that do not change
x0(system.remove) = [];

% Configure method
setup = optimset('Display', 'iter-detailed', 'Algorithm', 'levenberg-marquardt', ...
'TolX', 10000 * eps, 'TolFun', 10000 * eps, 'ScaleProblem', 'Jacobian');

if Jacob_on
    setup.Jacobian = 'on';
    setup.MaxIter = system.config.cycles;
else
    setup.MaxIter = system.config.cycles;
end

% Use fsolve to find solution
[sol, res, exit_flag, config] = fsolve(@my_fun, x0, setup);

function [ y, J ] = my_fun(vector)

% Create full variable vector
vars_vector = zeros(size(system.exog.pos));

% Update values
vars_vector(~system.remove) = vector;

% Transfer back to structure
system.vars = struct2vec(system.vars, vars_vector);

% Reaply exogenous vars
system.vars = struct_set_exog(system.vars, system.exog, 1);

% Compute residuals
y = call_eq_list(system);

% Compute Jacobian (if needed)
if nargout > 1, J = call_Jacobian(system); end

end

C.3 Model

function [ list ] = create_eq_list()

% Creates Equation List (system.list)
% Creates a cell (30,1)
% With a list of all the equations in the model
% Each line has:
% {"Function Name", "Argument 1 Name", "Argument 2" ... }

list = {
    'pars', 'bench'},
    {'Intermediate_Consumption', 'vars.XIN', 'vars.p', 'vars.Y', 'pars'}
};
function { profit } = Profit_firms(Y, prices, taxP, dK, pars)
% Profit_firms - Firms Profit
% Computes profit for each firm (should be 0)
% Profit = Revenues - Costs
% Arguments:
% Y - Output Aggregate Good Vector (col)
% prices - structure with all prices (struct)
% taxP - Production Tax vector (col)
% dK - Capital Depreciation vector (col)
% Compute Revenues
% Revenue = Y * c'Y
revenue = Y .* unit_revenue(prices, pars);
% Compute Cost
% Cost = Y * c'Y + (1 + taxP) * dK
cost = Y .* unit_cost(prices, pars) .* (1 + taxP) + dK;
% Compute Profit
profit = revenue - cost;
end

function [ out ] = Armington_Profit(pA, pD, pM, pTM, taxG, pars, bench)
% Armington Profit
% Compute profit of the Armington Aggregation Sector
% Arguments:
% pA - Armington price (line)
% pD - Domestic price (line)
% pM - Import Price (line)
% pTM - TTM price (scalar)
% taxG - tax on Goods Vector
% pars - parameters (structure)
% bench - benchmark values (structure)
% Output: Armington Profit (col vector g+1)
% pr = Subs( TTM and Domestic )
% out = p_Armington - Subs( pr, Imported )
% Call number of goods
g = pars.g;
% Create TTM Elasticity vector
eTTM = pars.elast.TTM * ones(1,g);

% Call Parameters:
% Domestic vs. Imported Share
theta = pars.theta.D;
% Armington Import Elasticity
elast = pars.elast.imp;
% Margin Share
tTTM = pars.theta.Mar;

% Tax modification of prices
pD = pD .* (1 + taxG') ./ (1 + bench.tax.G');
pM = pM .* (1 + taxG') ./ (1 + bench.tax.G');

% Compute D+TTM prices
pr = subs_vector(tTTM, [pTTM * ones(1, g); pD], eTTM, ones(2,1));
pr(pr == 0) = 1;

% Compute profit: output
out = (pA' * subs_vector(theta, [pr; pM], elast, ones(2,1))') * pars.m_profit;
out(isinf(out)) = 0;

end %function

function [ out ] = Intermediate_Consumption(xIN, prices, Output, pars)
% FUNCTION Intermediate consumption
% This function verifies the IC balance:
% xIN_i = \frac{\partial \gamma}{\partial \beta_i}

% Arguments
% xIN - Intermediate Consumption matrix
% prices - Price structure
% Output - Sector output vector
% pars - parameters structure

% Compute derivative
derivative = deriv_unit_cost( prices, pars, 'goods' );

% Verify equality
out = xIN - (Output * ones(1, pars.g) .* derivative)';

% Transform output from matrix into vector
out = out(:);
% Exclude equations with benchmark xIN == 0
out(pars.exclude.IN) = [];
end % function

function [ output ] = Factor_Demand (factor_name, factor_use, prices, Y, pars)
% Factor_Demand
% Computes Residuals for Factor (K or L) Demand

% Arguments
% factor_name - 'K' or 'L'
% factor_use - vector of factor demand for each sector (col)
% prices - price structure
% Y - Industry output (col)
% taxP - Production Taxes sector (col)
% dK - Capital Depreciation vector (col)
% pars - parameters structure

% Output: Residuals (col)
derivative = deriv_unit_cost( prices, pars, factor_name );
output = factor_use - Y .* derivative;
end %function

function [ output ] = Firm_Capital_Depreciation (dK, KS, bench)
% FUNCTION Firm_Capital_Depreciation
% Computes residuals for Firm Capital Depreciation:
% dK / benchmark = KS / benchmark
% Arguments
% - dK - Capital Depreciation by sector (vector)
% - KS - Capital Stock (scalar)
% - bench - benchmark variables structure
% Compute output:
output = dK * bench.KS - bench.dK * KS;
end % function

function [ out ] = Firm_Output ( goods, type, prices, Y, pars)
% FUNCTION Firm_Output
% Computes Residuals for production equations:
% x_ij = Y dc_j / dp_i
% Arguments:
% goods - either X or XD (domestic or exported production)
% type = 'X' or 'D' for Exports or Domestic
% prices - price structure
% Y - Industry output (col)
% pars - parameters structure
% Output: Residuals (col)
out = goods * Y * ones(1, pars.g) .* deriv_unit_revenue(prices, pars, type);
% Reshape output into vector
out = out(:);
% Remove equations with benchmark X or XD == 0
out(pars.exclude.(type)) = [];
end % function

function [ out ] = Investment(pA, xI, I, bench)
% Investment: Compute Investment Residuals:
% out = pA * xI * \bar{I} - \bar{xI} * I
% Arguments:
% pA - Armington price vector (line)
% xI - Gross Capital Accumulation vector (col)
% I - total Investment (scalar)
% bench_xI - benchmark of xI (col)
% bench_I - benchmark of I (scalar)
% Output: Residuals (col)
out = pA' .* xI .* bench.I - bench.xI * I;
end

function [ out ] = Check_Total( Parcels, Total )
out = sum(Parcels) - Total;
end

function [ out ] = Capital_Stock(dK, dKt, I, DK, KS, bench)
% Capital_stock
% Calculates the residuals of 3 Capital Stock equations
% a = Sum(Sector Capital Dep.) - Total Cap. Dep. = 0
% b = Investment - Total Cap. Dep + Net Cap. Change = 0
% c = New Capital Stock - Bench KS + Net Cap. Change = 0
a = sum(dK) - dKt;
b = I - dKt - DK;
c = KS - bench.KS + bench.DK - DK;
out = [a; b; c];
end

end % function
function [ out ] = Basket_Goods (agent, good_Ag, p_Ag, budget, p_Arm, pars)
% Family Profits:
% Compute consume price index
out = rand(2, 1);
theta = pars.theta.(agent)';
elast = pars.elast.(agent);
out(1) = good_Ag * p_Ag / budget;
out(2) = (p_Ag - subs_vector(theta', p_Arm', elast, 1)) * 10^3;
end

function [ out ] = SavingRate(SavingR, Savings, Consumption)
% Saving Rate
% Sr * (S + M) - S
out = SavingR * (Savings + Consumption) - Savings;
end

function [ budget ] = Family_Budget(wages, rents, labor, unemploy, assets, ...
% Family Budget Residuals
% Difference between Household Income and money Use
% Arguments
% wages - Price of Labor use (scalar)
% rents - Price of Borrowed Assets (scalar)
% labor - Total Labor (scalar)
% unemploy - Labor Unemployment rate (scalar)
% assets - Total Borrowed Assets (scalar)
% consmp - money spent on goods (scalar)
% savings - money Saved by Households (scalar)
% taxF - Income Tax Rate (scalar)
% lostAssets - percentage of 'lost' Assets (scalar)
% tnftrs - other transfers (scalar)
% Output: residual budget
= Consumption + Savings - Income + Transfers
% Income = Wages * Labor * (1 - Unemployment) +
% + Rents * Assets * (1 - lost Assets)
% budget = consmp + savings - income - (1 - taxF) + CT;
end

function [ out ] = Agent_Consumption(goods, Ag_Good, agent, p_Arm, pars)
% Agent_Consumption
% Computes the agent's demand vector
% Arguments:
% goods - Demand for Good i (col)
% Ag_Good - Aggregate Agent good C/G (scalar)
% agent - Consumer ('C') or Government ('G') (char)
% p_Arm - Armington Prices (line)
% pars - Parameters (structure)
% Output: (col vector g)
out = Ag_Good * D[Pi^A, pA] - Goods;
end % function
function [ out ] = Numeraire( price )
% Numeraire
% Makes sure the selected price is 1
% Arguments:
% price = Numeraire Price (scalar)
% Output:
% out = Price - 1
out = (price - 1) * 10^5;
end

function [ budget ] = Government_Budget(taxes, rents, assets, consmp, ...

Function [ budget ] = Government_Budget(taxes, rents, assets, consmp, ...

Government_Budget Residuals
% Difference between Government's cashflows
% Arguments:
% taxes = Total Tax Revenue (scalar)
% rents = Price of Borrowed Assets (scalar)
% assets = Total Borrowed Assets (scalar)
% consmp = money spent on goods (scalar)
% lostAssets = percentage of 'lost' assets (scalar)
% trnfrs = other transfers (scalar)
% Output:
% residual budget
% = Consumption + Savings - Income + Transfers
% Income = Taxes + Rents + Assets + (1 - lost Assets)
% budget = consmp + savings - income + trnfrs;
end

function [ out ] = Domestic_Flux(InterCnsmp, GovDem, HouseDem, Invest, Margins, Armington)
% Domestic_Flux
% Check flow of domestic Goods
% Arguments:
% InterCnsmp = Intermediate Consumption (Matrix g*c)
% GovDem = Government Demand (col g)
% HouseDem = Household Demand (col g)
% Invest = Gross Fixed Cap. Formation (col g)
% Margins = Goods used as TTM (col g)
% Armington = Aggregate Armington good (col g)
% Output:
% out = Sum_j xIN_ij + xC_i + xG_i + xI_i + TTM_i = A_i
out = sum(InterCnsmp, 2) + HouseDem + GovDem + Invest + Margins - Armington;
end

function [ out ] = Armington_Consumption(ArmGood, prices, type, In_Good, taxG, pars, bench)
% Armington_Consumption
% Computes Armington aggregation of Domestic and Imported Goods
% Arguments:
% ArmGood = Armington aggregate Good i (col)
% prices = All Prices (structure)
% type = 'M' for imported and 'D' for domestic (char)
% In_Good = Total Domestic/Imported Good i (col/line)
% taxG = VAT tax rate of good i (col)
% pars = Parameters (structure)
% bench = Benchmark values (structure)
function [ out ] = Armington_TTM( ArmGood, Margins, prices, taxG, pars, bench )

% UNTITLED2 Summary of this function goes here
% Detailed explanation goes here

derivative = dPiA_dp('TTM', prices.D, prices.M, prices.TTM, taxG, pars, bench);
out = Margins * ArmGood .* derivative;
end

function [ profit ] = TTM_Profit ( pTTM, pArm, pars )

% TTM_Profit
% Computes Profits for the Margin Sector.
% Profits must be 0.
% Arguments:
% pTTM = Price of Output (scalar)
% pArm = Armington Price (line g-vector)
% pars = Parameters (structure)
% Output: (scalar)
% profit = pTTM - ProductionFunction(pArm, pars)
% = pTTM - subs_vector(theta_TTM, pArm, elast_TTM)
% profit = pars.m_profit * (pTTM - subs_vector(pars.theta.TTM, pArm', pars.elast.TTMgood, 1));
end

function [ out ] = TTM_Supply(TTM, TotalTTM, pArm, pars)

% TTM_Supply
% Supply of 'Margins' from the Armington Goods to the Margin Sector. In other words, the inputs of that sector
% Arguments:
% TTM = Sector Inputs (col g-vector)
% TotalTTM = Sector Output (scalar)
% pArm = Armington Price (line g-vector)
% pars = Parameters (structure)
% Output: (column vector)
% out = TTM + TotalTTM * D[TTM_Profit, pA_i]
% out = TTM - TotalTTM * deriv_sub_vector(pars.theta.TTM, pArm', pars.elast.TTMgood, 1)';
end

function [ out ] = Labor_Balance(AvailLab, UnemplRate, UsedLab, wage, pars)

% Labor_Balance
% Computes balance between Labor Supply and Demand
% Computes Unemployment Rate
% Arguments:
% AvailLab = Total Available Labor (scalar)
% UnemplRate = % of unused Labor (scalar)
function [ out ] = Asset_Balance(UsedAsst, GovAsst, HouseAsst, LostCap, pars, bench)

end

function [ out ] = subs_vector ( x, y, e, f)

end

function [ out, c ] = deriv_subs_vector ( x, y, e, f)
function [ output ] = deriv_unit_cost( prices, pars, type)
 % FUNCTION Deriv_Unit_Cost:
 % dc/dp (matrix)
 % Arguments
 % prices - price structure
 % pars - parameters structure
 c = pars.c;
 PF = pars.PF;
 PF_fields = fieldnames(PF);

 % Convert price structure into price matrix
 Prices per sector
 p = struct;
 price_types = fieldnames(prices);
 for i = 1 : length(price_types)
 code = price_types{i};
 p.(code) = transpose(ones(c, 1) * prices.(code));
 dp.(code) = ones(c, 1);
 end

 % Transform Production Function <structure>
 for i = 1 : length(PF_fields)
 field = PF_fields{i};
 if isnumeric(PF.(field))
 prices = p.A(PF.(field),:);
 else
 prices = cell2mat(cellfun(@(x)(p.(x)'), PF.(field), ...UniformOutput', false))';
 end

 % Clear invalid entries
 prices(isfinite(prices) == 0) = 0;

 % Fetch thetas
 t_theta = pars.theta.in.(field);
 q = length(t_theta(:,1));

 % Fetch Elasticity
 t_elast = pars.elast.prod.(field);

 % Compute new aggregate price
 p.(field) = subs_vector(t_theta, prices, t_elast, ones(q,1));

 % Compute Derivative
 dp.(field) = deriv_subs_vector(t_theta, prices, t_elast, ones(q,1));

 if ~isnumeric(PF.(field))
 t_dp = cell(1, length(PF.(field)));
 for j = 1 : length(PF.(field))
 l_size = size(dp.(PF.(field){j}));
 t_dp{1, j} = dp.(field)(:,j) * ones(1, l_size(2)) * dp.(PF.(field){j});
 end
 dp.(field) = cell2mat(t_dp);
 end

 % Return as output the appropriate derivatives
 if strcmp('goods',type) == 1
 factors = fieldnames(pars.range);
 range = 0;
 for i = 1 : pars.f
 range = range | pars.range.(factors{i});
 end
 dp.(pars.output{:})(:,range) = [];
 end
 else
 output = dp.(pars.output{:})(:,pars.range.(type));
 end

 end % function

function [ deriv ] = deriv_unit_revenue( p, pars, type)
 % FUNCTION Deriv Firm Revenue:
 % dr_j/dp_i
 % Arguments
 % p - price structure
 % pars - parameters structure
 % Parameters
 thetaX = pars.theta.X;
 thetaOUT = pars.theta.OUT;
 sigmaX = pars.elast.exp ;

% The rest of the code
sigmaOUT = pars.elast.out;
[r, c] = deriv_subs_vector(thetaX, [p.X; p.D], sigmaX, ones(2,1));
c = c' * ones(1, pars.c);
drevenue = deriv_subs_vector(thetaOUT, c, sigmaOUT, ones(pars.g, 1));
switch(type)
case 'X' range = 1;
case 'D' range = 2;
end
deriv = (pars.rY * r(:,range)' * drevenue);
end

function [ cost ] = unit_cost( prices, pars)

    % FUNCTION unit_cost: compute unitary cost of every sector (vector)
    c = pars.c;
    PF_fields = fieldnames(pars.PF);
    p = struct;
    price_types = fieldnames(prices);
    for i = 1 : length(price_types)
        code = price_types{i};
        p.(code) = transpose(ones(c, 1) * prices.(code));
    end
    for i = 1 : length(PF_fields)
        field = PF_fields{i};
        if isnumeric(pars.PF.(field))
            prices = p.A(pars.PF.(field),:);
        else
            prices = cell2mat(cellfun(@(x)(p.(x)'), pars.PF.(field), 'UniformOutput', false))';
        end
        % Clear invalid entries
        prices(isfinite(prices) == 0) = 0;
        % Fetch thetas
        t_theta = pars.theta.in.(field);
        q = length(t_theta(:,1));
        % Fetch Elasticity
        t_elast = pars.elast.prod.(field);
        % Compute new aggregate price
        p.(field) = subs_vector(t_theta, prices , t_elast, ones(q,1));
    end
    % Final cost price (output)
    cost = p.(pars.output{:})';
end

function [ revenue ] = unit_revenue( p, pars)

    % FUNCTION unit_revenue: compute unitary revenue of every sector (vector)
    output: r_} (column)
    r = subs_vector(thetaX, [p.X; p.D], sigmaX, ones(2,1));
    r = r' * ones(1, pars.c);
    revenue = subs_vector(thetaOUT, r, sigmaOUT, ones(pars.g, 1))' * pars.rY;
function [ out ] = dPiA_dp(type, pD, pM, pTTM, taxG, pars, bench)

% dPiA_dp:
% Compute derivative of 'Armington Profit' with respect to price of type
% type: 'TTM', 'D' or 'M'

% Arguments
% type - derivative selector
% pD - Domestic price (line)
% pM - Import Price (line)
% pTTM - TTM price (scalar)
% taxG - tax on Goods Vector (col)
% pars - parameters (structure)
% bench - Benchmark values (structure)

% Output (col vector g*1):
% out = D[Armington Profit_i, p.(type)_i]

% Call number of goods
g = pars.q;

% Create TTM Elasticity vector
eTTM = pars.elast.TTM * ones(1,g);

% Call Parameters:
% Domestic vs. Imported Share
theta = pars.theta.D;
% Armington Import Elasticity
elast = pars.elast.imp;
% Margin Share
tTTM = pars.theta.Mar;

% Tax modification of prices
pD = pD .* (1 + taxG') ./ (1 + bench.tax.G');
pM = pM .* (1 + taxG') ./ (1 + bench.tax.G');

% Aggregate D & TTM prices
pD_TTM = subs_vector(tTTM, [pTTM * ones(1, g); pD], eTTM, ones(2,1));
dpD_TTM = deriv_subs_vector(tTTM, [pTTM * ones(1, g); pD], eTTM, ones(2,1));
pD_TTM(pD_TTM == 0) = 1;

% Compute final derivative
dp_int = deriv_subs_vector(theta, [pD_TTM; pM], elast, ones(2,1));
dp_out = [dpD_TTM .* (dp_int(:,1) * ones(1, 2)), dp_int(:,2)];

% Select output column according to type
switch(type)
    case 'TTM'
        out = dp_out(:,1) ./ tax_n;
    case 'D'
        out = dp_out(:,2) ./ tax_n;
    case 'M'
        out = dp_out(:,3) ./ tax_n;
end
end %function

function [ result ] = dPiAgent_dpA(agent, p_Arm, pars)

% Partial Derivative
dPiA_i
% To compute family/government consumption x_k

theta = pars.theta.(agent);
elast = pars.elast.(agent);
result = deriv_subs_vector(theta, p_Arm', elast, 1);
end

C.4 Simulate.m
% Simulate Script
% --> Requires variable system {run Main.m}
% --> Apply Exogenous Variables
% --> Solve System by calling fsolve

% Create exogenous variables structure
system.exog = system_exog(system);

% Apply changes in control variables
system.vars = struct_set_exog(system.vars, system.exog, 1);

% Create list of constant variables
remove = system.exog.pos;
remove(system.topg.exclude) = 1;
system.remove = (remove == 1);

% Initialized cycle constants
y = call_eq_list(system);
Num_jacob: use numerical Jacobian
Num_jacob = false;

% Main cycle
% While |residuals| < tolerance
while norm(y) > system.config.tol

% Use fsolve
[sol_v, y, exit_flag, solver] = solve_my_fun(system, ~Num_jacob);

% Convert resize vector to include constant variables
vars_v = zeros(size(system.exog.pos));
vars_v(~system.remove) = sol_v;

% Transform vector back to structure
sol = vect2struct(system.vars, vars_v);
sol = struct_set_exog(sol, system.exog, 1);

% Update system.vars with sol
system.vars = sol;

% If convergence is slow, change to numerical Jacobian (more accurate)
if solver.stepsize < (max(abs(y)))
    Num_jacob = true;
end

end