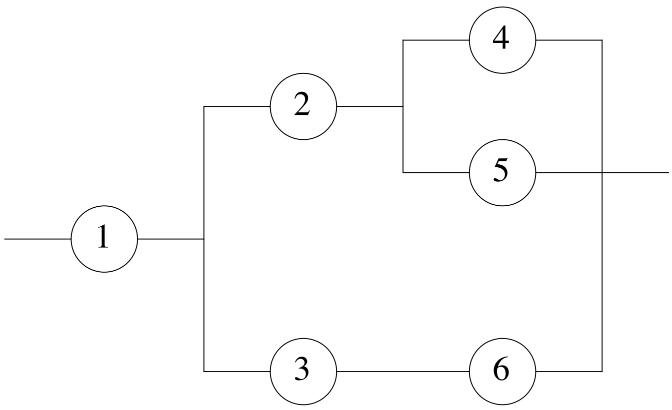
Exam

Question 1

(Reliability block) diagram

```
ClearAll["Global`*"]
Show[Graphics[{
   Circle[{0, 0}, 1],

   Circle[{6, 4}, 1],
   Circle[{6, -4}, 1],

   Circle[{12, 6}, 1],
   Circle[{12, 2}, 1],
   Circle[{12, -4}, 1],

   Text["1", {0, 0}],

   Text["2", {6, 4}],
   Text["3", {6, -4}],

   Text["4", {12, 6}],
   Text["5", {12, 2}],
   Text["6", {12, -4}],

   Line[{{-3, 0}, {-1, 0}}],
   Line[{{1, 0}, {3, 0}}],
   Line[{{3, 4}, {5, 4}}],
   Line[{{3, -4}, {5, -4}}],
   Line[{{7, 4}, {9, 4}}],
   Line[{{7, -4}, {11, -4}}],
   Line[{{9, 6}, {11, 6}}],
   Line[{{13, 6}, {15, 6}}],
   Line[{{9, 2}, {11, 2}}],
   Line[{{13, 2}, {15, 2}}],
   Line[{{13, -4}, {15, -4}}],
   Line[{{15, 2}, {17, 2}}],

   Line[{{3, -4}, {3, 4}}],
   Line[{{9, 2}, {9, 6}}],
   Line[{{15, -4}, {15, 6}}]
  }]]
```



1(b)

```
p = .99;
ℛ = ReliabilityDistribution[
    x1
     ⋀
     ((x2 ⋀ (x4 ⋁ x5))
       ⋁ (x3 ⋀ x6)),
    {{x1, BernoulliDistribution[p]},
     {x2, BernoulliDistribution[p]},
     {x3, BernoulliDistribution[p]},
     {x4, BernoulliDistribution[p]},
     {x5, BernoulliDistribution[p]},
     {x6, BernoulliDistribution[p]}
    }];
InputForm[Mean[ℛ]]
```

0.989801039601

```
p = .99;
```

$$(1 - (1 - p)) \times \left(1 - (1 - p)^2\right)^2 \times \left(1 - (1 - p)^3\right)^2$$

$$p^3$$

0.9898

0.970299

Question 2

2(a)

```
ClearAll["Global`*"]
```

$$\lambda[t\_] = \left(\frac{t}{4}\right)^3;$$

$$R[t\_] = \mathrm{Exp}\left[-\int_0^t \lambda[s]\, ds\right]$$

$$\int_0^\infty R[t]\, dt$$

```
Round[%, 0.001]
dist = WeibullDistribution[4, 4.];
Mean[dist]
R[%]
```

$$e^{-\frac{t^4}{256}}$$

$$\mathrm{Gamma}\left[\frac{1}{4}\right]$$

3.626

3.62561

0.509172

2(b)

```
𝒟 = dist;
dists = {{x, 𝒟}, {y, 𝒟}, {z, 𝒟}, {w, 𝒟}};
ℛ = ReliabilityDistribution[BooleanCountingFunction[{3, 4}, {x, y, z, w}], dists];
FullSimplify[SurvivalFunction[ℛ, t]]
```

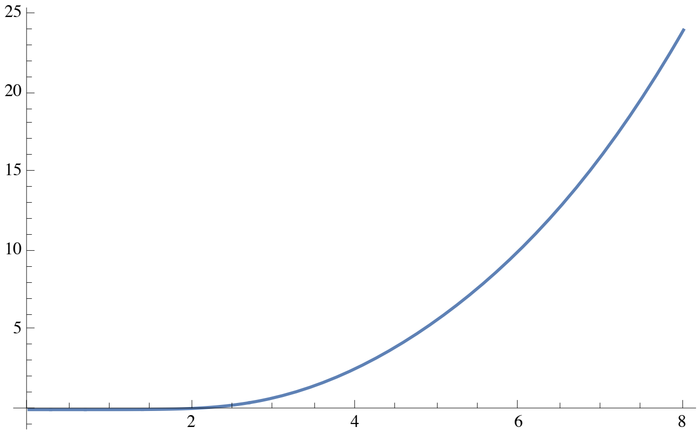
$$4.\, R[t]^3 - 3\, R[t]^4$$

$$\begin{cases} 1 & t \le 0 \\ -3\, e^{-0.015625\, t^4} + 4\, e^{-0.0117188\, t^4} & \text{True} \end{cases}$$

$$-3\, e^{-\frac{t^4}{64}} + 4.\, e^{-\frac{3\, t^4}{256}}$$

2(c)

```
h[t_] = HazardFunction[ℛ, t];
Plot[h[t], {t, .001, 8}]
```



2(d)

```
Mean[ℛ]
```

$$\int_0^\infty \left(4.\, R[t]^3 - 3\, R[t]^4\right) dt$$

$$\frac{4}{3^{\frac{1}{4}}} \text{Mean[dist]} - \frac{3}{4^{\frac{1}{4}}} \text{Mean[dist]}$$

$$\int_0^\infty \left(R[t]^3\right) dt$$

$$\frac{\text{Mean[dist]}}{3^{1/4}}$$

```
3.32839

3.32839

3.32839

2.75487

2.75487
```

Question 3

Type I/item censoring with replacement

```
ClearAll["Global`*"]
n = 24;
t0 = 74 000.;
data = {2760., 3700., 7100., 17 220., 29 500., 48 400., 52 600., 65 000.};
r = Length[data]
ξ = n * t0
λhat = r/ξ
dist = ExponentialDistribution[λhat];
2 000 000 * CDF[dist, 24 000]
Round[2 000 000 * (1 - Exp[-24 000 * λhat]), 0.001]
```

8

1.776×10^{6}

4.5045×10^{-6}

204 939.

204 939.

Question 4

4(a)

```
ClearAll["Global`*"]
p0 = 0.15;
n = 125;
```

$$p_0 - 3 \times \sqrt{\frac{p_0 (1 - p_0)}{n}}$$

$$p_0 + 3 \times \sqrt{\frac{p_0 (1 - p_0)}{n}}$$

0.0541877

0.245812

4(b)

```
p0 = 0.15;
n = 20;
n * p0
n * p0 (1 - p0)
LCL = Ceiling[Max[0, n * p0 - 3 × Sqrt[n * p0 (1 - p0)]]]
UCL = Floor[ n * p0 + 3 × Sqrt[n * p0 (1 - p0)] ]
ξ[δ_] = 1 - (CDF[BinomialDistribution[n, p0 + δ], UCL] -
      CDF[BinomialDistribution[n, p0 + δ], LCL - 1]);
Round[1 / ξ[0], 0.001]

(* with tables *)
Round[1 / (1 - (0.9941 - 0)), 0.001]
```

```
3.

2.55

0

7

168.886

169.492
```

Question 5

5(b)

```
ClearAll["Global`*"]
CDF[GeometricDistribution[p], x]
```

$$\begin{cases} 1 - (1 - p)^{1+\text{Floor}[x]} & x \geq 0 \\ 0 & \text{True} \end{cases}$$

```
p0 = 0.001;
α = 0.005;
probsignalunbiased[p_, L_, U_, gammaLCL_, gammaUCL_] =
  CDF[GeometricDistribution[p], L - 1] + 1 - CDF[GeometricDistribution[p], U] + gammaLCL ×
    PDF[GeometricDistribution[p], L] + gammaUCL × PDF[GeometricDistribution[p], U];
ARLunbiased[p_, L_, U_, gammaLCL_, gammaUCL_] =
  1 / probsignalunbiased[p, L, U, gammaLCL, gammaUCL];
LCLunbiased = 4;
UCLunbiased = 7428;
γL = 0.415872;
γU = 0.349557;

Round[1 / ARLunbiased[0.9 * p0, LCLunbiased, UCLunbiased, γL, γU], 0.000001]
Round[1 / ARLunbiased[p0, LCLunbiased, UCLunbiased, γL, γU], 0.000001]
Round[1 / ARLunbiased[1.1 * p0, LCLunbiased, UCLunbiased, γL, γU], 0.000001]

Round[ARLunbiased[0.9 * p0, LCLunbiased, UCLunbiased, γL, γU], 0.001]
Round[ARLunbiased[p0, LCLunbiased, UCLunbiased, γL, γU], 0.001]
Round[ARLunbiased[1.1 * p0, LCLunbiased, UCLunbiased, γL, γU], 0.001]
```

```
0.005213

0.005

0.00513

191.833

200.

194.95
```

$$1 - (1 - 1 \times 0.001)^{1+4-1} + (1 - 1 \times 0.001)^{1+7428} + 0.415872 * (1 - 1 \times 0.001)^{4} * 1 \times 0.001 + 0.349557 * (1 - 1 \times 0.001)^{7428} * 1 \times 0.001$$

$$1 - (1 - 1.1 \times 0.001)^{1+4-1} + (1 - 1.1 \times 0.001)^{1+7428} + 0.415872 * (1 - 1.1 \times 0.001)^{4} * 1.1 \times 0.001 + 0.349557 * (1 - 1.1 \times 0.001)^{7428} * 1.1 \times 0.001$$

```
1 /
 %
```

```
0.005

0.00512952

194.95
```

Question 6

```
ClearAll["Global`*"]
n = 9;
μ0 = 350.;
σ0 = 2.;
α = 1 / 370.4;

distmu = NormalDistribution[0, 1];
γmu = Quantile[distmu, 1 - α/2];
γmu = 3.;
ξmu[δ_, θ_] = 1 - (CDF[distmu, (γmu - δ)/θ] - CDF[distmu, (-γmu - δ)/θ]);
ξmu[0, 1]

ARLmu[δ_, θ_] = 1/ξmu[δ, θ];
ARLmu[0, 1]

distsigma = ChiSquareDistribution[n - 1];
γsigma  = Round[Quantile[distsigma, 1 - α], 0.000001]
ξsigma[θ_] = 1 - CDF[distsigma, γsigma/θ^2];
ξsigma[1];
ARLsigma[θ_] = 1/ξsigma[θ];
ARLsigma[1]
0.0026998

370.398

23.5746

370.4

shiftmu = 0.25;
shiftsigma = Sqrt[γsigma/21.95];
Round[(γmu - shiftmu)/shiftsigma, 0.01]
Round[(-γmu - shiftmu)/shiftsigma, 0.01]
1 - CDF[distmu, Round[(γmu - shiftmu)/shiftsigma, 0.01]] + CDF[distmu, Round[(-γmu - shiftmu)/shiftsigma, 0.01]]
1 - (0.9960 - (1 - 0.999155))
2.65

-3.14

0.00486933

0.004845
```

```
quantileorder = {0.25};
MatrixForm[
 Table[Round[Log[1 - quantileorder[[i]]] / Log[1 - ξmu[shiftmu, shiftsigma]], 0.001], {i, 1, Length[quantileorder]}]]
MatrixForm[Table[Round[Log[1 - quantileorder[[i]]] / Log[1 - 0.004845], 0.001],
  {i, 1, Length[quantileorder]}]]
```

(59.31)

(59.233)

Question 7

7(a)

```
ClearAll["Global`*"]
p1 = 0.01; (* AQL *)
α = 0.05; (* producer's risk *)
p2 = 0.05; (* LTPD *)
β = 0.1; (* consumer's risk *)

gdist = NormalDistribution[0, 1];
Φ[x_] := CDF[gdist, x];
Ω[x_] := Quantile[gdist, x];

((Ω[1 - α] - Ω[β]) / (Ω[p2] - Ω[p1]))^2
nσ = Ceiling[((Ω[1 - α] - Ω[β]) / (Ω[p2] - Ω[p1]))^2];
kσ = (Ω[p2] × Ω[1 - α] - Ω[p1] × Ω[β]) / (Ω[β] - Ω[1 - α])
PVar[n_, p_] = Φ[Sqrt[n] × (-kσ - Ω[p])];

i = nσ;
While[PVar[i, p1] < 1 - α || PVar[i, p2] > β, Print["Do not use sample size nσ=", i,
  " because Pa[p1]=", PVar[i, p1], "<", 1 - α , " or Pa[p2]=", PVar[i, p2], ">", β];
 i++]

Print["Use sample size nσ=", i, " and acceptance constant ", kσ,
 " because Pa[p1]=", PVar[i, p1], "≥", 1 - α , " and Pa[p2]=", PVar[i, p2], "≤", β]
```

18.4393

1.9433

Use sample size n_σ=19 and acceptance constant 1.9433 because $P_a[p_1]$=0.952508≥0.95 and $P_a[p_2]$=0.096648≤0.1

■ Using tables

$$\left(\frac{1.6449 - (-1.2816)}{(-1.6449) - (-2.3263)}\right)^2$$

$$\frac{(-1.6449) * 1.6449 - (-2.3263) * (-1.2816)}{(-1.2816) - 1.6449}$$

$\text{Round}\left[\sqrt{19} \times (-1.943305 - (-2.3263)), 0.01\right]$

$\text{Round}\left[\sqrt{19} \times (-1.943305 - (-1.6449)), 0.01\right]$

18.4456

1.94331

1.67

−1.3

```
nσ = 19;
kσ = 1.943305;
u = 3 × nσ × (kσ^2 - 2) + 8
v = 3 × nσ^2 × kσ^2
ns = nσ + (u + √(u^2 + 24 v))/12
ks = √((3 × ns - 4)/(3 × ns - 3)) × kσ

PVarDesc[n_, k_, p_] = Φ[(Ω[1 - p] - k × √((3×n-4)/(3×n-3)))/√((1+(3×n×k^2)/(6×n-8))/n)];

(* PVarDesc[n_,k_,p_]=CDF[NoncentralStudentTDistribution[n-1,√n ×Ω[p]],-√n ×k]; *)
i = Ceiling[ns];
While[PVarDesc[i, √((3 × i - 4)/(3 × i - 3)) × kσ, p1] < 1 - α || PVarDesc[i, √((3 × i - 4)/(3 × i - 3)) × kσ, p2] > β,
 Print["Do not use sample size ns=", i, " and acceptance constant ks=",
  √((3 × i - 4)/(3 × i - 3)) × kσ, " because Pa[p1]=", PVarDesc[i, √((3 × i - 4)/(3 × i - 3)) × kσ, p1],
  "<", 1 - α , " or Pa[p2]=", PVarDesc[i, √((3 × i - 4)/(3 × i - 3)) × kσ, p2], ">", β];
 i++]

Print["Use sample size ns=", i, " and acceptance constant ks=",
 √((3 × i - 4)/(3 × i - 3)) × kσ, " because Pa[p1]=", PVarDesc[i, √((3 × i - 4)/(3 × i - 3)) × kσ, p1],
 "≥", 1 - α , " and Pa[p2]=", PVarDesc[i, √((3 × i - 4)/(3 × i - 3)) × kσ, p2], "≤", β]
```

109.257

4089.88

55.7551

1.93738

Do not use sample size n_s=56 and acceptance constant k_s= 1.93741 because $P_a[p_1]$=0.958031<0.95 or $P_a[p_2]$=0.104762>0.1

Do not use sample size n_s=57 and acceptance constant k_s= 1.93751 because $P_a[p_1]$=0.959324<0.95 or $P_a[p_2]$=0.102561>0.1

Do not use sample size n_s=58 and acceptance constant k_s= 1.93761 because $P_a[p_1]$=0.960574<0.95 or $P_a[p_2]$=0.100414>0.1

Use sample size n_s=59 and acceptance constant k_s= 1.93771 because $P_a[p_1]$=0.961783≥0.95 and $P_a[p_2]$=0.0983205≤0.1

```
ns = 59;
ks = 1.937713;
```

$$\text{Round}\left[\frac{\Omega[1-p_1]-k_s\times\sqrt{\frac{3\times n_s-4}{3\times n_s-3}}}{\sqrt{\frac{1+\frac{3\times n_s\times k_s^2}{6\times n_s-8}}{n_s}}},\ 0.01\right]$$

```
CDF[NormalDistribution[0, 1], %]
```

$$\text{Round}\left[\frac{\Omega[1-p_2]-k_s\times\sqrt{\frac{3\times n_s-4}{3\times n_s-3}}}{\sqrt{\frac{1+\frac{3\times n_s\times k_s^2}{6\times n_s-8}}{n_s}}},\ 0.01\right]$$

```
CDF[NormalDistribution[0, 1], %]
```

1.77

0.961636

-1.29

0.0985253

```
59. / 19
```

3.10526

7(b)

```
n = 59;
dist = NormalDistribution[22.8, 1];
data = Sort[RandomVariate[dist, n]]
Mean[data]
```

$$\sqrt{\text{Variance[data]}}$$

```
U = 22.8;
```

$$\frac{\text{U} - \text{Mean[data]}}{\sqrt{\text{Variance[data]}}}$$

{20.3943, 20.5876, 20.6208, 20.8758, 21.0931, 21.4309, 21.4959, 21.5489, 21.6618,
21.9066, 22.025, 22.0338, 22.0475, 22.1982, 22.2045, 22.2372, 22.339, 22.3949,
22.4577, 22.4921, 22.5467, 22.5807, 22.6088, 22.6239, 22.6558, 22.6577, 22.7478,
22.7598, 22.7626, 22.9843, 22.9913, 23.0445, 23.0524, 23.0839, 23.0855,
23.1004, 23.1393, 23.1589, 23.3116, 23.3416, 23.4056, 23.4158, 23.4617,
23.4795, 23.5617, 23.5978, 23.5996, 23.7058, 23.8337, 23.8832, 23.9604,
23.9615, 24.0249, 24.4578, 24.6766, 24.8325, 25.0852, 25.1159, 25.3972}

22.8769

1.11628

-0.0689274