Reliability and Quality Control

LMAC, MMA

2nd. Semester – 2023/2024
2024/03/05 — 17:10
MAP30 #1

Duration: **30** minutes

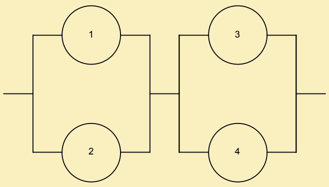
- Write your number and name below.
- Add your answers to this and the following page.
- Please justify all your answers.
- This test has ONE PAGE and TWO QUESTIONS. The total of points is 5.0.

Number: **Name**:

1. Consider an aircraft with two engines on each wing. Suppose this a four-engined aircraft needs a least one engine operating on each wing. Assume that the four engines operate independently and their reliabilities are equal to p_i $(i = 1, \ldots, 4)$.

(a) Draw a (reliability block) diagram of this system. **(2.0)**
Provide expressions for its: i) structure function; ii) reliability; iii) the reliability importance of engine 1.

- **(Reliability block) diagram**



- **Structure function**

$$\begin{aligned}\phi(\underline{X}) &= \min\{\max\{X_1, X_2\}, \max\{X_3, X_4\}\}\\ &= [1-(1-X_1)\times(1-X_2)]\times[1-(1-X_3)\times(1-X_4)]\\ &= (X_1+X_2-X_1\,X_2)\times(X_3+X_4-X_3\,X_4)\end{aligned}$$

- **Reliability**

$$\begin{aligned}r(\underline{p}) &= E[\phi(\underline{X})]\\ &= E[(X_1+X_2-X_1\,X_2)\times(X_3+X_4-X_3\,X_4)]\\ &\overset{X_i \sim_{i.i.d.} Ber(p_i)}{=} (p_1+p_2-p_1\,p_2)\times(p_3+p_4-p_3\,p_4).\end{aligned}$$

- **Reliability importance of component 1**

$$\begin{aligned}I_r(1) &= \frac{\partial r(\underline{p})}{\partial p_1}\\ &= \frac{\partial[(p_1+p_2-p_1\,p_2)\times(p_3+p_4-p_3\,p_4)]}{\partial p_1}\\ &= (1-p_2)\times(p_3+p_4-p_3\,p_4)\end{aligned}$$

(b) Identify the minimal path sets and the minimal cut sets; use them to provide expressions for two upper bounds on the reliability of this system, assuming that $p_i = p$ $(i = 1, \ldots, 4)$. Which bound is stricter when $p = 0.9$? **(1.5)**

- **Minimal path sets**
$\mathscr{P}_1 = \{1,3\}$, $\mathscr{P}_2 = \{1,4\}$, $\mathscr{P}_3 = \{2,3\}$, $\mathscr{P}_4 = \{2,4\}$, $p^* = 4$ minimal path sets

- **Minimal cut sets**
$\mathscr{K}_1 = \{1,2\}$, $\mathscr{K}_2 = \{3,4\}$, $q = 2$ minimal path sets

- **Upper bounds**
[Since we are dealing with a coherent system with components operating independently (thus, also in a positively associated manner), we can apply theorems 1.68 and 1.70:]

$$r(\underline{p}) \overset{Th., 1.68}{\leq} 1 - \prod_{j=1}^{p^*}\left[1 - \prod_{i\in\mathscr{P}_j} p_i\right] \overset{p_i=p,\, p^*=4, \#\mathscr{P}_j=2}{=} 1-\left(1-p^2\right)^4$$

$$r(\underline{p}) \overset{Th., 1.70}{\leq} \min_{j=1,\ldots,q}\left[1 - \prod_{i\in\mathscr{K}_j}(1-p_i)\right] \overset{p_i=p,\, q=2, \#\mathscr{K}_j=2}{=} \min\{1-(1-p)^2, 1-(1-p)^2\} = 1-(1-p)^2$$

- **Comparing the upper bounds**
  When $p = 0.9$, $1-\left(1-p^2\right)^4 = 0.998697$ (Th. 1.68) and $1-(1-p)^2 = 0.99$ (Th. 1.70). Since $0.99 < 0.998697$, Theorem 1.70 provides a stricter upper bound.

2. Admit that the times to failure of the engines of the aircraft are independent and exponentially distributed with parameter $\lambda = 1$. **(1.5)**

Write down the time to failure of the aircraft (T) in terms of the times to failure of its engines. Derive expressions for $R_T(t)$ and $E(T)$.

- **Time to failure** (engines)
  T_i = time to failure of component i
  $T_i \overset{i.i.d.}{\sim} \exp(\lambda = 1), \quad i = 1,\ldots,5$
  $$R_i(t) = P(T_i > t) = R(t) = \begin{cases} e^{-t}, & t \geq 0 \\ 1, & t < 0 \end{cases}$$

- **Time to failure** (aircraft)
  $T = \min\{\max\{T_1, T_2\}, \max\{T_3, T_4\}\}$

- **Reliability function**

$$\begin{aligned}
R_T(t) &= P(T > t) \\
&= R_{\max\{T_1,T_2\}}(t) \times R_{\max\{T_3,T_4\}}(t) \\
&\overset{R_i(t)=R(t)}{=} \{1-[1-R(t)]^2\}^2 \\
&\overset{R(t)=e^{-t}}{=} (1-1+2e^{-t}-e^{-2t})^2 \\
&= (2e^{-t}-e^{-2t})^2 \\
&= 4e^{-2t}-4e^{-3t}+e^{-4t}, \quad t > 0 \\
[&\equiv r(R(t),\ldots,R(t)) = r(e^{-t},\ldots,e^{-t}).]
\end{aligned}$$

- **Expected value of T**

$$\begin{aligned}
E(T) &\overset{(2.10)}{=} \int_0^{+\infty} R(t)\,dt \\
&= \int_0^{+\infty} \left(4e^{-2t}-4e^{-3t}+e^{-4t}\right) dt \\
&= \frac{4}{2}\int_0^{+\infty} 2e^{-2t}\,dt - \frac{4}{3}\int_0^{+\infty} 3e^{-3t}\,dt + \frac{1}{4}\int_0^{+\infty} 4e^{-4t}\,dt \\
&= 2\int_0^{+\infty} f_{exp(2)}(t)\,dt - \frac{4}{3}\int_0^{+\infty} f_{exp(3)}(t)\,dt + \frac{1}{4}\int_0^{+\infty} f_{exp(4)}(t)\,dt \\
&= 2\times 1 - \frac{4}{3}\times 1 + \frac{1}{4}\times 1 \\
&= \frac{11}{12}.
\end{aligned}$$