$$\bar{E}_p = -\frac{d}{d\beta}\left[\ln\left[\frac{1}{mg\beta}\left(1-e^{-\beta mgL}\right)\right]\right] = -\left[\frac{1}{mg\beta^2}\cdot\left(1-e^{-\beta mgL}\right) +\right.$$

$$\left. + \frac{1}{mg\cancel{\beta}}\left(-e^{-\beta mgL}\, mgL\right)\right]\cdot\frac{\cancel{mg\beta}}{\left(1-e^{-\beta mgL}\right)} =$$

$$= \frac{1}{\beta} + \frac{mgL\, e^{-\beta mgL}}{\left(1-e^{-\beta mgL}\right)} = \frac{1}{\beta} + \frac{mgL}{\left(e^{\beta mgL} - 1\right)}$$

Dividir acima e abaixo por $e^{-\beta mgL}$

$$\boxed{\bar{E}_p = kT + \frac{mgL}{\left(e^{\frac{mgL}{kT}} - 1\right)}}$$

$$-\frac{d}{d\beta}\left[\ln\left(\int_{-\infty}^{\infty} e^{-\beta E_x}\,dp_x\right)\right] = -\frac{d}{d\beta}\left[-\frac{1}{2}\ln\beta \cancel{+\ln\left(\int_{-\infty}^{\infty} e^{-by^2}\,dy\right)}\right] =$$

$$= +\frac{1}{2}\frac{1}{\beta} = \frac{1}{2}kT$$

$\boxed{\bar{E}_x = \frac{1}{2}kT}$ → Teorema de equipartição da energia. É geral, vale para todas as situações nas que:

a) A energia separa-se numa soma de termos independentes.

b) O termo ε_i é quadrático na coordenada que toca.

$$\bar{E}_c = \underbrace{\frac{1}{2}kT}_{p_x} + \underbrace{\frac{1}{2}kT}_{p_y} + \underbrace{\frac{1}{2}kT}_{p_z} = \boxed{\frac{3}{2}kT}$$

b) $\bar{E}_p$?

$$\bar{E}_p = \frac{\int e^{-\beta\left(\frac{p^2}{2m}+mgz\right)}\cdot E_p\, d^3\vec{p}\, d^3\vec{r}}{\int_{-\infty}^{\infty} e^{-\beta\left(\frac{p^2}{2m}+mgz\right)}\, d^3\vec{p}\, d^3\vec{r}} = \frac{\int_0^L e^{-\beta mgz}\, mgz\, dz}{\int_0^L e^{-\beta mgz}\, dz} =$$

Parte cinética cancela acima e abaixo (desacopla)
coordenadas x, y também

$$= \frac{-\frac{d}{d\beta}\int_0^L e^{-\beta mgz}\,dz}{\int_0^L e^{-\beta mgz}\,dz} = -\frac{d}{d\beta}\left[\ln\left(\int_0^L e^{-\beta mgz}\,dz\right)\right]$$

$$\int_0^L e^{-\beta mgz}\,dz = -\frac{1}{\beta mg}\,e^{-\beta mgz}\Big]_0^L = -\frac{1}{mg\beta}\left(e^{-\beta mgL} - 1\right) =$$

$$= \frac{1}{mg\beta}\left(1 - e^{-\beta mgL}\right)$$

3.14

gravidade g

a) Energia cinética média / partícula?

$$\mathcal{E}_c = \frac{\vec{p}^{\,2}}{2m} \qquad \mathcal{E}_g = mgz$$

$$\underbrace{\mathcal{E} = \frac{p^2}{2m} + mgz}$$

situação muito comum:
Um termo quadrático, um outro independente.

E = soma de dois termos.

Equilíbrio térmico (reservatório)
Distribuição canônica

$$\bar{\mathcal{E}}_c = \frac{\int_{-\infty}^{\infty} e^{-\beta \mathcal{E}_p} \mathcal{E}_c \, d^3\vec{p}\, d^3\vec{z}}{\int_{-\infty}^{\infty} e^{-\beta \mathcal{E}_p} d^3\vec{p}\, d^3\vec{z}} = \frac{\int_{-\infty}^{\infty} e^{-\beta \mathcal{E}_c} \cdot \mathcal{E}_c \, d^3\vec{p} \cdot \cancel{\int_{-\infty}^{\infty} e^{-mgz\beta} d^3\vec{z}}}{\int_{-\infty}^{\infty} e^{-\beta \frac{p^2}{2m}} d^3 p \cancel{\int_{-\infty}^{\infty} e^{-\beta mgz} d^3\vec{z}}}$$

$$\bar{\mathcal{E}}_c = \frac{\frac{d}{d\beta}\left[\int_{-\infty}^{\infty} e^{-\beta \mathcal{E}_c} d^3\vec{p}\right]}{\int_{-\infty}^{\infty} e^{-\beta \mathcal{E}_c} d^3\vec{p}} = -\frac{d}{d\beta}\left[\ln\left(\int_{-\infty}^{\infty} e^{-\beta \mathcal{E}_c} d^3\vec{p}\right)\right] =$$

$$= -\frac{d}{d\beta}\left[\ln\left(\int_{-\infty}^{\infty} e^{-\beta \frac{p_x^2}{2m}} dp_x\right) + \ln\left(\int_{-\infty}^{\infty} e^{-\beta \frac{p_y^2}{2m}} dp_y\right) + \ln\left(\int_{-\infty}^{\infty} e^{-\beta \frac{p_z^2}{2m}} dp_z\right)\right] =$$

<u>Geral:</u> uma coordenada:

$$\int_{-\infty}^{\infty} e^{-\beta b p_x^2} dp_x \underset{\uparrow}{=} \beta^{-1/2} \int_{-\infty}^{\infty} e^{-by^2} dy \Rightarrow \ln \int_{-\infty}^{\infty} e^{-\beta b p_i^2} dp_i = -\frac{1}{2}\ln\beta + \underbrace{\ln \int_{-\infty}^{\infty} e^{-by^2} dy}_{\text{não envolve } \beta!} \Rightarrow$$

Mudança de variável

$$y = \beta^{1/2} p_i \Rightarrow dy = \beta^{1/2} dp_i$$