DM
DEPARTAMENTO DE MATEMÁTICA
TÉCNICO LISBOA

Introduction to Stochastic Processes

LEIC-A, LMAC, MECD, MEIC-A, MMA

2nd. Semester – 2020/2021
2021/07/14 – 9:50AM
Test 2 (*Recurso*)

Duration: 90 minutes

- Please justify all your answers.
- This test has TWO PAGES and THREE GROUPS. The total of points is 20.0.

Group 1 — Renewal Processes — 2.5 points

The rate a certain insurance company charges its policyholders alternates between r_1 and r_0. A new policyholder is initially charged at a rate of r_1 per time unit. When a policyholder paying at rate r_1 has made no claims for the most recent s $(s>0)$ time units, then the rate charged switches to r_0 per time unit. The rate charged remains at r_0 until a claim is made, at which time it reverts to r_1. (2.5)

Suppose that a given policyholder lives forever and makes claims at times chosen according to a Poisson process with rate λ.

After describing a convenient alternating renewal process, find P_i, the proportion of time that the policyholder pays at a rate of r_i in the long-run, for $i=0,1$. Determine the long-run average amount paid per time unit.

- **R.v.**

  X_i = time between claims $i-1$ and i, $\quad i \in \mathbb{N}$

  $X_i \stackrel{i.i.d.}{\sim} X \sim \text{exponential}(\lambda)$

  $F_X(x) = 1 - e^{-\lambda x}, \quad x \geq 0$

- **Up and down times**

  U = time a policyholder pays at a rate of r_1 (system is UP)

  $$U = \min\{X, s\} = \begin{cases} X, & X < s \\ s, & X \geq s \end{cases}$$

  D = time a policyholder pays at a rate of r_0 (system is DOWN)

- **Duration of the up-down cycle**

  $X = U + D$

- **Alternating renewal process**

  $\{X(t) : t \geq 0\}$

  $$X(t) = \begin{cases} 1, & \text{if the policyholder is paying at a rate of } r_1 \text{ (system is UP) at time } t \\ 0, & \text{if the policyholder is paying at a rate of } r_0 \text{ (system is DOWN) at time } t \end{cases}$$

- **Expected duration of the up time**

  Integrating by parts, we obtain

  $$\begin{aligned} E(U) &= E[\min\{X,s\}] \\ &= \int_0^s x\, dF_X(x) + \int_s^{+\infty} s\, dF_X(x) \\ &= \int_0^s x \times \lambda e^{-\lambda x}\, dx + s \times P(X > s) \\ &= -x \times e^{-\lambda x}\Big|_0^s + \int_0^s e^{-\lambda x}\, dx + s \times [1 - F_X(s)] \\ &= -s \times e^{-\lambda s} - \frac{e^{-\lambda x}}{\lambda}\Big|_0^s + s \times e^{-\lambda s} \\ &= \frac{1 - e^{-\lambda s}}{\lambda}. \end{aligned}$$

Furthermore,

$$E(X) \; = \; E(U + D) = \frac{1}{\lambda}.$$

- **Long-run proportion of time spent paying at a rate r_i, $i = 0, 1$**

  $E(X) < \infty$ and c.d.f. of X is nonlattice, thus the long-run proportion of time spent being charged at a rate of r_1 and r_0 are

  $$\begin{aligned} P_1 &= \lim_{t \to +\infty} P[X(t) = 1] \\ &= \frac{E(U)}{E(U) + E(D)} \\ &= \frac{\frac{1 - e^{-\lambda s}}{\lambda}}{\frac{1}{\lambda}} \\ &= 1 - e^{-\lambda s} \\ P_0 &= \lim_{t \to +\infty} P[X(t) = 0] \\ &= \frac{E(D)}{E(U) + E(D)} \\ &= 1 - P_1 \\ &= e^{-\lambda s}, \end{aligned}$$

  respectively.

- **Requested long-run average amount paid per time unit**

  $$\begin{aligned} r_0 \times P_0 + r_1 \times P_1 &= r_0 \times e^{-\lambda s} + r_1 \times (1 - e^{-\lambda s}) \\ &= r_1 - (r_1 - r_0) \times e^{-\lambda s}. \end{aligned}$$

Group 2 — Discrete time Markov chains — 9.5 points

1. Consider a DTMC $\{X_n : n \in \mathbb{N}_0\}$, with state space $\mathscr{S} = \{1,2,3,4\}$ and TPM

$$\mathbf{P} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

(a) Draw the associated transition diagram, identify and classify the communicating classes of this DTMC. (1.5)

- **DTMC**

  $\{X_n : n \in \mathbb{N}_0\}$

- **State space**

  $\mathscr{S} = \{1,2,3,4\}$

- **TPM**

  $$\mathbf{P} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

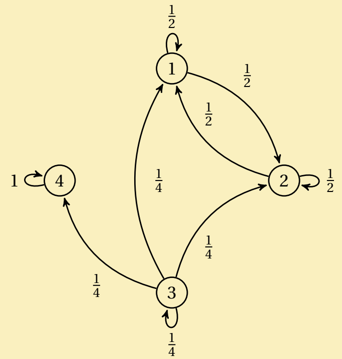
- **Communicating classes and classification of the states of the DTMC**

  Judging by the transition diagram, the communicating classes of this Markov chain are $\{1,2\}$, $\{3\}$, and $\{4\}$. Hence, the state space is not a single communicating class therefore we are dealing with a reducible DTMC.

  Since state 4 is an absorbing state, that is, $P_{44} = 1$, no other state is accessible from it. Needless to say that $\{4\}$ is a closed communicating class with a recurrent state.

Note that while states 1 and 2 are accessible from state 3, the reverse is not true: $3 \to i$ $(i = 1,2)$, but $i \not\to 3$ $(i = 1,2)$. $\{3\}$ is not a closed communicating class. (The same holds with state 4: $3 \to 4$ but $4 \not\to 3$.)
Furthermore, once the DTMC enters class $\{1,2\}$ it never leaves, thus this is a closed communicating class with two recurrent states.
We can also add that state 3 is transient, because given that the DTMC starts at state 3, there is a non-zero probability that it will never return to this state.

- **Transition diagram**



(b) Admit that the initial state X_0 has p.f. $\underline{\alpha} = [0.1\ \ 0.2\ \ 0.3\ \ 0.4]$ and compute $P(X_2 = 3)$. (1.0)

- **Initial state**
  X_0
  $\underline{\alpha} = [P(X_0 = i)]_{i\in\mathscr{S}} = [0.1 \quad 0.2 \quad 0.3 \quad 0.4]$
- **Requested probability**

$$\begin{aligned}
\underline{\alpha}^2 &= [P(X_2 = i)]_{i\in\mathscr{S}} \\
&\overset{form.}{=} \underline{\alpha} \times \mathbf{P}^2 \\
P(X_2 = 3) &= \underline{\alpha} \times \mathbf{P} \times \text{3rd. column of } \mathbf{P} \\
&= \underline{\alpha} \times \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ \frac{1}{4} \\ 0 \end{bmatrix} \\
&= [0.1 \quad 0.2 \quad 0.3 \quad 0.4] \times \begin{bmatrix} 0 \\ 0 \\ \frac{1}{16} \\ 0 \end{bmatrix} \\
&= 0.3 \times \frac{1}{16} \\
&= 0.01875.
\end{aligned}$$

(c) Calculate $f_{ij}^n = P(X_n = j, X_{n-1} \neq j, \ldots, X_1 \neq j \mid X_0 = i)$, for $i, n = 1,2,3$ and $j = 2$. (2.0)

- **Requested probabilities**
  Let:
  i) $f_{ij}^n = P(X_n = j, X_{n-1} \neq j, \ldots, X_1 \neq j \mid X_0 = i)$ be the probability of reaching state j for the first time starting from state i, for $i, j \in \mathscr{S}$ and $n \in \mathbb{N}$;

i) $\underline{f}_j^n = [f_{ij}^n]_{i\in\mathscr{S}}$ be the associated vector, for fixed $j \in \mathscr{S}$ and $n \in \mathbb{N}$.
According to the formulae,

$$\underline{f}_j^n = \begin{cases} \underline{f}_j^1 = [P_{ij}]_{i\in\mathscr{S}}, & n = 1 \\ {}^{(j)}\mathbf{P} \times \underline{f}_j^{n-1} = \left[{}^{(j)}\mathbf{P}\right]^{n-1} \times \underline{f}_j^1, & n = 2,3,\ldots, \end{cases}$$

where ${}^{(j)}\mathbf{P}$ is obtained by setting all the entries of the j^{th} column of $\mathbf{P}$ equal to 0.
When $j = 2$, we get

$$\begin{aligned}
{}^{(2)}\mathbf{P} &= \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
\underline{f}_2^1 &= [P_{i2}]_{i\in\mathscr{S}} \\
&= \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{4} \\ 0 \end{bmatrix} \\
\underline{f}_2^2 &= {}^{(2)}\mathbf{P} \times \underline{f}_2^1 \\
&= \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{4} \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} \frac{1}{4} \\ \frac{1}{4} \\ \frac{3}{16} \\ 0 \end{bmatrix} \\
\underline{f}_2^3 &= {}^{(2)}\mathbf{P} \times \underline{f}_2^2 \\
&= \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \frac{1}{4} \\ \frac{1}{4} \\ \frac{3}{16} \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} \frac{1}{8} \\ \frac{1}{8} \\ \frac{7}{64} \\ 0 \end{bmatrix}.
\end{aligned}$$

(d) Determine the expected transitions until the DTMC reaches states 1, 2 or 4, given that $X_0 = 3$. (1.0)

- **Initial state**
  $X_0 = 3$
- **Important**
  To obtain the expected number of transitions until the DTMC reaches states 1, 2 or 4, given $X_0 = 3$, we have to consider another DTMC where states 1 and 2 are absorbing, as state 4, and state 3 is transient. The associated TPM in block form and state space $\mathscr{S}' = \{3, 1-2-4\}$ is

$$\mathbf{P}' = \begin{bmatrix} \frac{1}{4} & \frac{3}{4} \\ 0 & 1 \end{bmatrix}.$$

- **Requested expected value**
  Let $\mathbf{Q} = \left[\ \frac{1}{4}\ \right]$ be the substochastic matrix governing the transitions between the only transient state in $T = \{3\}$ of this new DTMC, and $\tau = \inf\{n \in \mathbb{N}_0 : X_n \notin T\}$ be the number of transitions until the DTMC reaches state $1-2-4$ is simply obtained by

$$
\begin{aligned}
[E(\tau \mid X_0 = i)]_{i\in T} &= (\mathbf{I}-\mathbf{Q})^{-1} \times \underline{1} \\
&= \left(\begin{bmatrix} 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{4} \end{bmatrix} \right)^{-1} \times 1 \\
&= \begin{bmatrix} \frac{3}{4} \end{bmatrix}^{-1} \times 1 \\
&= \frac{4}{3}.
\end{aligned}
$$

2. Let $\{X_n : n \in \mathbb{N}_0\}$ be a branching process, where X_n represents the size of generation n. Admit that $X_0 = 1$ and the number of offspring per individual has p.f. $P(Z = z) = (1-p)^z\, p,\ z \in \mathbb{N}_0$, where $\frac{1}{2} < p < 1$.

(a) Determine the expected value of the total number of individuals that ever existed. (1.5)

- **Branching process**
  $\{X_n : n \in \mathbb{N}_0\}$
  X_n = size of generation n a colony of cells
  $X_0 = 1$
  $X_n = \sum_{l=1}^{X_{n-1}} Z_l,\ n \in \mathbb{N}$
- **Number of offspring per individual and its p.g.f.**
  $Z_l \equiv Z_{l,n}$ = number of offspring of the l^{th} individual of generation n
  $Z_l \overset{i.i.d.}{\sim} Z, \quad l \in \mathbb{N}$
  $Z \sim \text{geometric}^\star(p), \quad \frac{1}{2} < p < 1$
- **Expected number of offspring per individual**

$$
\begin{aligned}
\mu &= E(Z) \\
&\overset{form.}{=} \frac{1-p}{p}
\end{aligned}
$$

- **Requested expected number of offspring per individual**
  Since $\frac{1}{2} < p < 1$, we get $\mu = \frac{1-p}{p} < 1$. Moreover, $E(X_n \mid X_0 = 1) \overset{form.}{=} \mu^n$, thus the expected value of the total number of individuals that ever existed is given by

$$
\begin{aligned}
E\left(\sum_{n=0}^{+\infty} X_n \mid X_0 = 1 \right) &= \sum_{n=0}^{+\infty} E(X_n \mid X_0 = 1) \\
&= \sum_{n=0}^{+\infty} \mu^n \\
&\overset{\mu<1}{=} \frac{1}{1-\mu} \\
&= \frac{1}{1-\frac{1-p}{p}} \\
&= \frac{p}{2p-1}.
\end{aligned}
$$

(b) Now, admit that $0 < p < \frac{1}{2}$ and calculate the extinction probability. (1.0)

- **P.g.f. of Z**
  $P(s) = E(s^Z) = \sum_j s^j \times P(Z = j) \overset{form.}{=} \frac{p}{1-(1-p)s}, \quad s \in [0,1]$
- **Probability of extinction**
  Since $E(Z) = \frac{1-p}{p} > 1$, for $0 < p < \frac{1}{2}$, the probability of extinction, $\pi \overset{\text{form.}}{=} \lim_{n\to+\infty} P(X_n = 0 \mid X_0 = 1)$, is the smallest positive number satisfying

$$
\begin{aligned}
s &\overset{form.}{=} \sum_{j=0}^{+\infty} s^j \times P_j \\
&= P(s) \\
s &= \frac{p}{1-(1-p)s} \\
(1-p)s^2 - s + p &= 0 \\
s &= \frac{1 \pm \sqrt{1-4(1-p)p}}{2(1-p)} \\
&= \frac{1 \pm \sqrt{(1-2p)^2}}{2(1-p)} \\
&= \frac{1 \pm (1-2p)}{2(1-p)},
\end{aligned}
$$

hence $\pi = \frac{1-(1-2p)}{2(1-p)} = \frac{p}{1-p}$.

(c) Consider $p = 0.3$ and compute the mean and the standard deviation of the size of generation 5. (1.5)

- **Requested variance**
  $\mu = \frac{1-p}{p} = \frac{7}{3} \neq 1$ and $\sigma^2 \overset{form.}{=} \frac{1-p}{p^2} = \frac{70}{9}$, $E(X_n) = \mu^n$, and

$$
V(X_n) \overset{form.}{=} \sigma^2 \mu^{n-1} \times \frac{\mu^n - 1}{\mu - 1}
$$

yield

$$
\begin{aligned}
E(X_5) &= \left(\frac{7}{3}\right)^5 \\
&\simeq 69.165 \\
SD(X_5) &= \sqrt{V(X_5)} \\
&\overset{form.}{=} \sqrt{\frac{70}{9} \times \left(\frac{7}{3}\right)^{5-1} \times \frac{\left(\frac{7}{3}\right)^5 - 1}{\frac{7}{3} - 1}} \\
&\simeq \sqrt{11786.446341} \\
&\simeq 108.565.
\end{aligned}
$$

Group 3 — Continuous time Markov chains	8.0 points

1. Admit that a company uses two robots in the manufacture of circuit boards. Moreover: each robot breaks down after an exponentially distributed time with parameter λ; the company has two repair people to do service when robots fail (one repair person per robot); the repair time for each robot is exponentially distributed with mean μ^{-1}.

Let $X(t)$ represent the number of robots being repaired at time t and admit that $\{X(t) : t \geq 0\}$ is a CTMC.

(a) Identify the state space and the infinitesimal generator $\mathbf{R}$ of this birth and death process. (1.5)
Draw the associated rate diagram.

- **CTMC**
  $\{X(t) : t \geq 0\}$
  $X(t)$ = number of robots being repaired at time t
- **State space**
  $\mathcal{S} = \{0, 1, 2\}$
- **Birth/death rates**
  We can interpret a break down of a robot as an arrival (to the repair shop) or a *birth*. Also note that as soon as a reparation of a robot is concluded, we can say that a departure (from the repair shop) or *death* has occurred. Moreover, there are two repair people or servers.
  [By the way, we are dealing with an $M/M/2$ queueing system with a finite customer population, namely 2.]

Thus,

$$\lambda_i = (2-i)\lambda, \quad i = 0,1$$
$$\mu_i = i\,\mu, \quad i = 1,2.$$

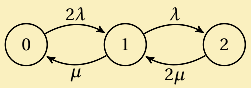
- **Infinitesimal generator**
  This matrix has entries

$$r_{ij} = \begin{cases} q_{ij}, & i \neq j \\ -\nu_i = -\sum_{m\in\mathscr{S}} q_{im}, & j = i \end{cases}$$

and in this case $\mathbf{R} = [r_{ij}]_{i,j\in\mathscr{S}}$ is equal to

$$\begin{bmatrix} -2\lambda & 2\lambda & 0 \\ \mu & -(\lambda+\mu) & \lambda \\ 0 & 2\mu & -2\mu \end{bmatrix}.$$

- **Rate diagram**
  [Recall that the rate diagram of a CTMC is a directed graph — with no loops — in which each state is represented by a node and there is an arc going from node i to node j (if $q_{ij} > 0$) with q_{ij} written on it. These rates coincide with the birth and death rates...]



(b) Obtain the equilibrium probabilities $P_j = \lim_{t\to+\infty} P[X(t) = j \mid X(0) = 0]$. (2.0)

- **Equilibrium probabilities** $P_j = \lim_{t\to+\infty} P_j(t)$
  Since this CTMC has a finite state space, we only need to deal with $\rho = \frac{\lambda}{\mu} < +\infty$ to guarantee the existence of equilibrium probabilities $P_j = \lim_{t\to+\infty} P_j(t)$.

$$\begin{aligned} P_0 &= \left[1 + \sum_{n=1}^{+\infty}\left(\prod_{i=0}^{n-1}\frac{\lambda_i}{\mu_{i+1}}\right)\right]^{-1} \\ &= \left(1 + \frac{\lambda_0}{\mu_1} + \frac{\lambda_0}{\mu_1}\frac{\lambda_1}{\mu_2}\right)^{-1} \\ &= \left(1 + \frac{2\lambda}{\mu} + \frac{2\lambda}{\mu}\frac{\lambda}{2\mu}\right)^{-1} \\ &= \left(\frac{2\mu^2 + 4\lambda\mu + 2\lambda^2}{2\mu^2}\right)^{-1} \\ &= \frac{\mu^2}{(\lambda+\mu)^2} \end{aligned}$$

$$P_j = P_0 \times \prod_{i=0}^{j-1}\frac{\lambda_i}{\mu_{i+1}}, \quad j = 1,2$$

$$\begin{aligned} P_1 &= P_0 \times \frac{\lambda_0}{\mu_1} \\ &= P_0 \times \frac{2\lambda}{\mu} \\ &= \frac{2\lambda\mu}{(\lambda+\mu)^2} \end{aligned}$$

$$\begin{aligned} P_2 &= P_0 \times \frac{\lambda_0}{\mu_1}\frac{\lambda_1}{\mu_2} \\ &= P_0 \times \frac{2\lambda}{\mu}\frac{\lambda}{2\mu} \\ &= \frac{\lambda^2}{(\lambda+\mu)^2}. \end{aligned}$$

(c) Compute the fraction of repair people that are idle in the long-run. (1.0)

- **Requested fraction**
  Since 2, 1, 0 of the two repair people are idle at time t if and only if $X(t) = 0,1,2$, respectively. Hence, the fraction of repair people that are idle in the long-run equals

$$\begin{aligned} f_{idle} &= \frac{2\times P_0 + 1\times P_1 + 0\times P_2}{2} \\ &= \frac{1}{2}\times\left[2\times\frac{\mu^2}{(\lambda+\mu)^2} + \frac{2\lambda\mu}{(\lambda+\mu)^2}\right] \\ &= \frac{2\mu(\lambda+\mu)}{2(\lambda+\mu)^2} \\ &= \frac{\mu}{\lambda+\mu}. \end{aligned}$$

2. Surveys have shown that: people living in a large western metropolitan area tune in and view a TV channel according to a Poisson process with rate equal to 100000 per hour; viewing times are exponentially distributed with an average of 1.5 hours. Admit that at $t = 0$ no one has tuned in or is watching the TV channel.

(a) After having chosen a suitable queueing system, obtain the probability that there are more than 149900 people watching the TV channel at 12:00 (of the first day). (2.0)

- **Arrival process/rate**
  $PP(\lambda)$
  $\lambda = 100000$ viewers per hour
- **Service times/rate**
  S_i = viewing time of the i^{th} person who tuned in, $i \in \mathbb{N}$
  $S_i \overset{i.i.d.}{\sim} S \sim \text{exponential}(\mu^{-1} = 1.5)$
  $\mu = \frac{2}{3}$
- **Birth and death queueing system**
  Assuming that each viewer tunes in and watches the TV channel using his/her own TV set, we can assume that this is a self-service system, namely a
  $M/M/\infty$ queueing system
- **R.v.**
  $(X(t) \mid X(0) = 0)$ = number of viewers watching the TV channel at time t, given that the system is initially empty
  $(X(t) \mid X(0) = 0) \sim \text{Poisson}(\lambda(1 - e^{-\mu t})/\mu)$
- **Requested probability**

$$\begin{aligned} P[X(12) > 149900 \mid X(0) = 0] &= 1 - P[X(12) \leq 149900 \mid X(0) = 0] \\ &= 1 - F_{Poisson(100000\times(1-e^{-\frac{2}{3}\times 12})/\frac{2}{3})}(149900) \\ &\simeq 1 - F_{Poisson(149949.680606)}(149900) \\ &\simeq 1 - \Phi\left(\frac{149900 - 149949.680606}{\sqrt{149949.680606}}\right) \\ &= 1 - \Phi(-0.13) \\ &= \Phi(0.13) \\ &\overset{tables}{=} 0.5517. \end{aligned}$$

(b) Obtain the average number of viewers of this TV channel in the long-run. Compare it with the corresponding expected value if the viewing times (in hours) have instead a $\chi^2_{(3)}$ distribution. (1.5)

- **Performance measure of the original queueing system**
  $L_s^{M/M/\infty}$ = number of viewers watching the TV channel in the long-run in the $M/M/\infty$ system
  $L_s^{M/M/\infty} \sim \text{Poisson}(\lambda/\mu = 100\,000/\frac{2}{3} = 150\,000)$ (see formulae)
  $E\left(L_s^{M/M/\infty}\right) = 150\,000$
- **Alternative queueing system**
  $M/G/\infty$
  $\lambda = 3$
  $\mu_G = \frac{1}{E(Service)} = \frac{1}{E[\chi^2_{(3)}]} \stackrel{form.}{=} \frac{1}{3}$
- **Performance measure of the alternative queueing system**
  $L_s^{M/G/\infty} = \lim_{t\to+\infty}(X(t) \mid X(0) = 0)$
  = number of viewers watching the TV channel in the long-run in the $M/G/\infty$ system
  $L_s^{M/G/\infty} \sim \text{Poisson}(\lambda/\mu_G = 100\,000/\frac{1}{3} = 300\,000)$
  $E\left(L_s^{M/G/\infty}\right) = 300\,000$
- **Comment**
  Since $E\left(L_s^{M/G/\infty}\right) = 300\,000 > 150\,000 = E\left(L_s^{M/M/\infty}\right)$, $\chi^2_{(3)}$ distributed viewing times lead to a larger expected number of viewers.