MEEC

Controlo em Espaço de Estados

2020/2021

Exam 2 – 9th July 2021

Duration: 3 hour

No consultation of any kind allowed

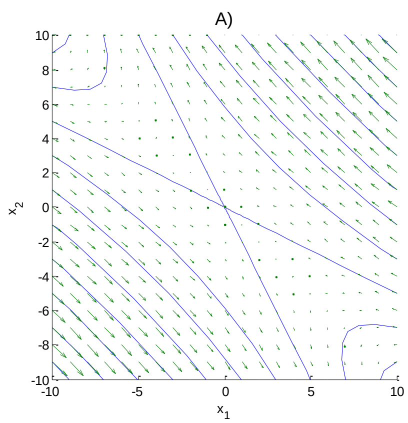
Grades: P1-6 **P2**a) 2 b) 2 c) 1 d) 2 **P3** a)1 b)1 c)0,5 **P4**a) 0,5 b)0,5 c)0,5 **P5** 3

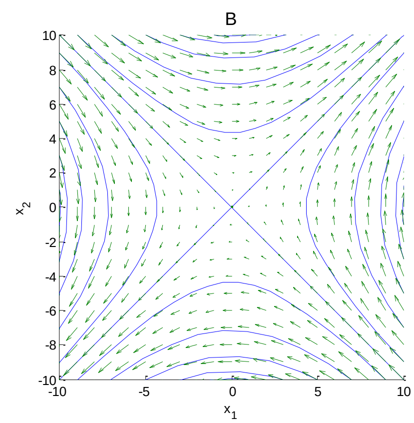
P1. Consider six linrat dynamical systems with zero input, described by $\dot{x} = Ax$ where the matrices "A" are

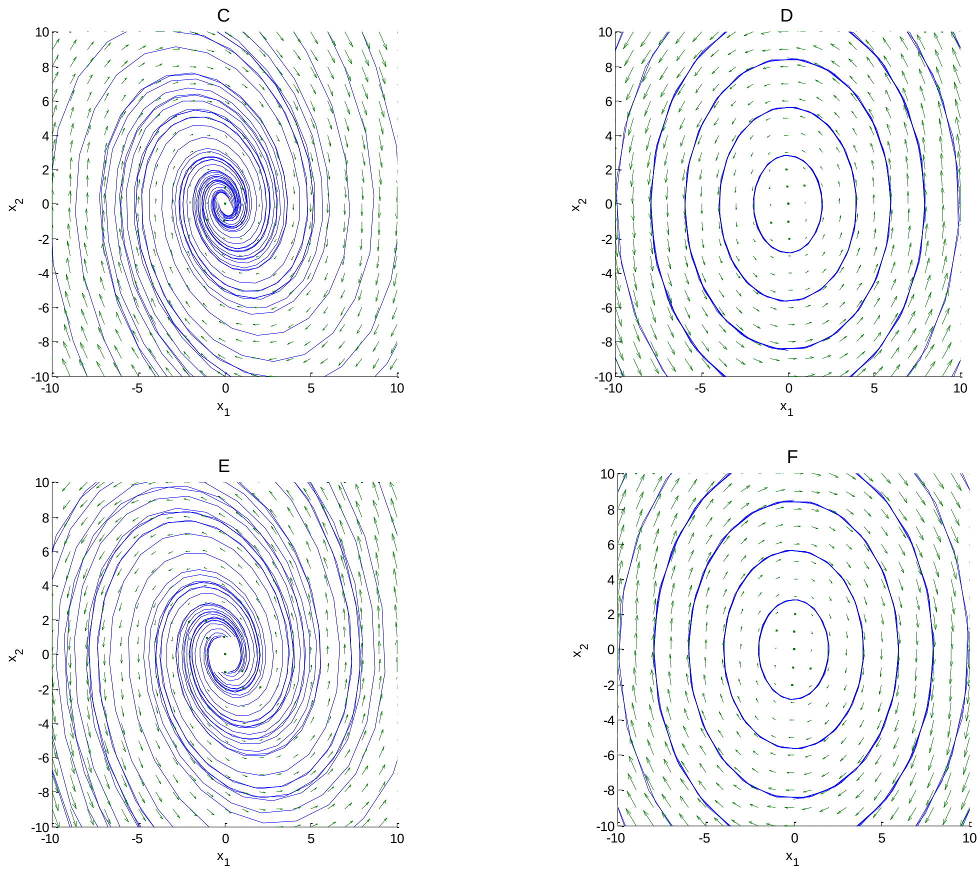
$$A_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad A_2 = \begin{bmatrix} 0 & 1 \\ -2 & -0.6 \end{bmatrix} \quad A_3 = \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} -\frac{5}{3} & -\frac{4}{3} \\ \frac{4}{3} & \frac{5}{3} \end{bmatrix} \quad A_5 = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} \quad A_6 = \begin{bmatrix} 0 & -1 \\ 2 & 0.6 \end{bmatrix}$$

Consider also the following phase portraits, identifyed by the letters A, B, C, D, E, F. Say, justifying your answer on the eigenvalues and, if needed, the eigenvectors and derivative signs whet is the correspondence between the matrices and the phase portraits.







P2. The transfer function of a permanent magnet direct current motor that drives a robot arm joint is

$$G(s) = \frac{Y(s)}{U(s)} = \frac{10}{s(s+1)}$$

where U is the Laplace transform of the electric tension applied to the motor, that corresponds to the input signal u, and Y is the Laplace transform of the robot arm joint angle, that corresponds to the output signal y.

a) Take as state variables $x_1 = y$ and $x_2 = \dot{y}$ and write the corresponding state equations in matrix form.

b) Compute the gains K_1 and K_2 such that $u = -K_1 x_1 - K_2 x_2$ places the roots of the controlled system at the roots of $\alpha_c(s) = s^2 + 3s + 9$.

c) Design a state estimator such that the estimation errors have poles at the roots of the polynomial $\alpha_o(s) = s^2 + 15s + 225$.

d) Find the transfer function of the controller obtained by the combination of the two precedent questions. Indicate numerical values for the coefficients of the numerator and denominator.

P3. HIV-1 virus infects the T-CD4+ cells of the imune system leading, if proper treatment is not followed to AIDS. This infection can be represented by the nonlinear state model

$$\dot{x}_1 = s - dx_1 - \alpha x_1 x_2$$
$$\dot{x}_2 = \alpha x_1 x_2 - \mu x_2$$

where x_1 is the plasma concentration of healthy T-CD4+ cells (number of cells per unit volume), x_2 is the concentration of infected cells, and s, d, α and μ are constant parameters that depend on the individual. In this problem they have the values

$$s = 10,\ d = 0.02,\ \alpha = 0.001,\ \mu = 0.24$$

Answer the following questions:

a) Compute all the equilibrium points.

b) Obtain the linearized models around each of the equilibrium points and classify the stability of the equilibrioum points of the **nonlinear system** based on the linearization. Consider all the equilibrium points.

c) On the basis of the previous ellements ando n the signs of the derivatives at points that you consider relevant , sketch the state trajectories arouind each of the equilibroium points. Justifify the direction of the state evolution on the state trajectories for the lines you draw.

Help: Jacobian matrix

$$\frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}$$

P4. Consider the block diagram of a feedback servomechanism shown on figure P4-1. In this position control system, the input signal u of a direct current motor is generated by an actuator whose characteristics is described by a nonlinear function f whose argument is the tracking error.

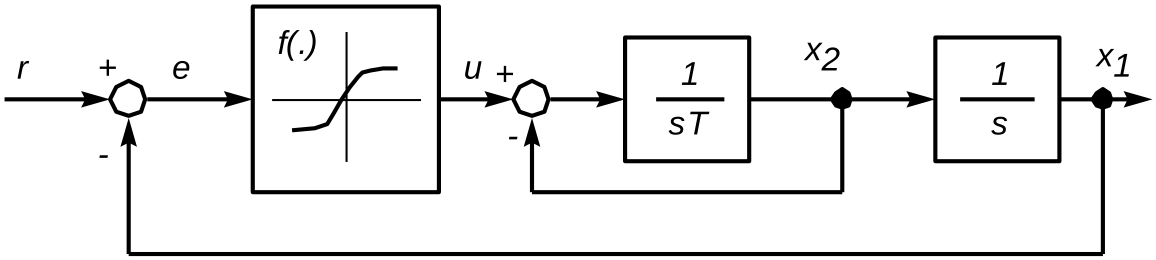


Fig. P4-1 A feedback servomechanism with a nonlinear actuator.

It is known that this function is such that

$$f(e) > 0 \text{ for } e > 0; \quad f(e) = 0 \text{ for } e = 0; \quad f(e) < 0 \text{ for } e < 0$$

This means that:

$$\int_0^e f(\sigma)d\sigma > 0 \quad \text{for} \quad e \neq 0$$

The reference r is constant in time.

The variables x_1 and x_2 are, respectively, the angular position and the angular velocity of the motor shaft. The parameter$T > 0$ is the motor dominant time constant.

Answer the following questions:

a) Consider the state defined by the tracking error e and the angular velocity x_2. Write the corresponding nonlinear state equations.

b) Show that

$$V(e, x_2) = \frac{T}{2}x_2^2 + \int_0^e f(\sigma)d\sigma$$

Is a Lyapunov function for the origin of the system described in a). What can you conclude about the stability of this equilibrium point using the Lyapunov Theorem?

c) What conclusions can you draw for the same problem by applying the invariant set theorem?

P5. The velocity v of a vehicle is related, in a normalized set of coordinates, with the rate of fuel consumption u through

$$\frac{dv}{dt} = -v + u$$

Using Pontryagin maximum principle, find the optimal function $u(t)$ in the time interval between $t = 0$ and $t = 10$ that transfers the velocity from $v(0) = 0$ to $v(10) = 70$, minimizing

$$J(u) = \frac{1}{2}\int_0^{10} u^2(t)dt$$

Help:

L represents the Laplace transform and a is a constant):

$$L(e^{at}) = \frac{1}{s-a} \qquad L\left(\frac{dv}{dt}\right) = sV(s) - v(0)$$

$$\frac{dx}{dt} = f(x,u) \qquad x(0) = x_0 \qquad J(u) = \Psi(x(T)) + \int_0^T L(x,u)dt$$

$$-\left(\frac{d\lambda}{dt}\right)' = \lambda'(t)f_x(x(t),u(t)) + L_x(x(t),u(t)) \qquad \lambda'(T) = \Psi_x(x(T))$$

$$H(\lambda,x,u) = \lambda' f(x,u) + L(x,u)$$

$$f_x = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} \qquad L_x = \begin{bmatrix} \frac{\partial L}{\partial x_1} & \frac{\partial L}{\partial x_2} \end{bmatrix} \qquad \Psi_x = \begin{bmatrix} \frac{\partial \Psi}{\partial x_1} & \frac{\partial \Psi}{\partial x_2} \end{bmatrix}$$