Code Verification

Its goal is: check that the code is bug-free

- Does the numerical error tend to zero when the grid refinement / time step tends to zero?

$$e(\phi) = \phi_i - \phi_{exact} \cong e_o + \alpha h_i^p$$

- Is the observed order of grid convergence in agreement with the "theoretical order" of the code?

- If not, find out why?!

Code Verification

- Error described by a Power Series Expansions

$$e(\phi) = \phi_i - \phi_{exact} \cong e_o + \alpha h_i^p$$

ϕ_i → Local or functional flow quantity for grid i

ϕ_{exact} → Exact solution of local or functional flow quantity

e_o → Error for cell size zero (supposed to be "0")

α → Constant

h_i → Typical cell size

p → Observed order of grid convergence

If $e_o \cong 0, \ \log\left(e(\phi)\right) = p \log\left(h_i\right) + \log\left(\alpha\right)$

Code Verification

- Geometrical Similarity

  - Typical cell size definition, h_i

$$\Lambda = \begin{cases} l \Leftarrow n_{space} = 1 \\ S \Leftarrow n_{space} = 2, \\ V \Leftarrow n_{space} = 3 \end{cases} \quad \Lambda_1 = \frac{\sum_{i=1}^{N_{cells}} \Lambda_i}{N_{cells}}, \quad \Lambda_2 = \sqrt{\frac{\sum_{i=1}^{N_{cells}} \Lambda_i^2}{N_{cells}}}, \quad \Lambda_{\text{mode}}$$

$$\left(\frac{h_i}{h_1}\right)_N = \left(\frac{(N_{cells})_1}{(N_{cells})_i}\right)^{\frac{1}{n_{space}}}, \quad \left(\frac{h_i}{h_1}\right)_1 = \left(\frac{(\Lambda_1)_i}{(\Lambda_1)_1}\right)^{\frac{1}{n_{space}}}$$

$$\left(\frac{h_i}{h_1}\right)_2 = \left(\frac{(\Lambda_2)_i}{(\Lambda_2)_1}\right)^{\frac{1}{n_{space}}}, \quad \left(\frac{h_i}{h_1}\right)_{\text{mode}} = \left(\frac{(\Lambda_{\text{mode}})_i}{(\Lambda_{\text{mode}})_1}\right)^{\frac{1}{n_{space}}}$$

Code Verification

- Geometrical Similarity (a 1-D example)

  - Definition of a typical cell size (single parameter) requires sets of geometrically similar grids

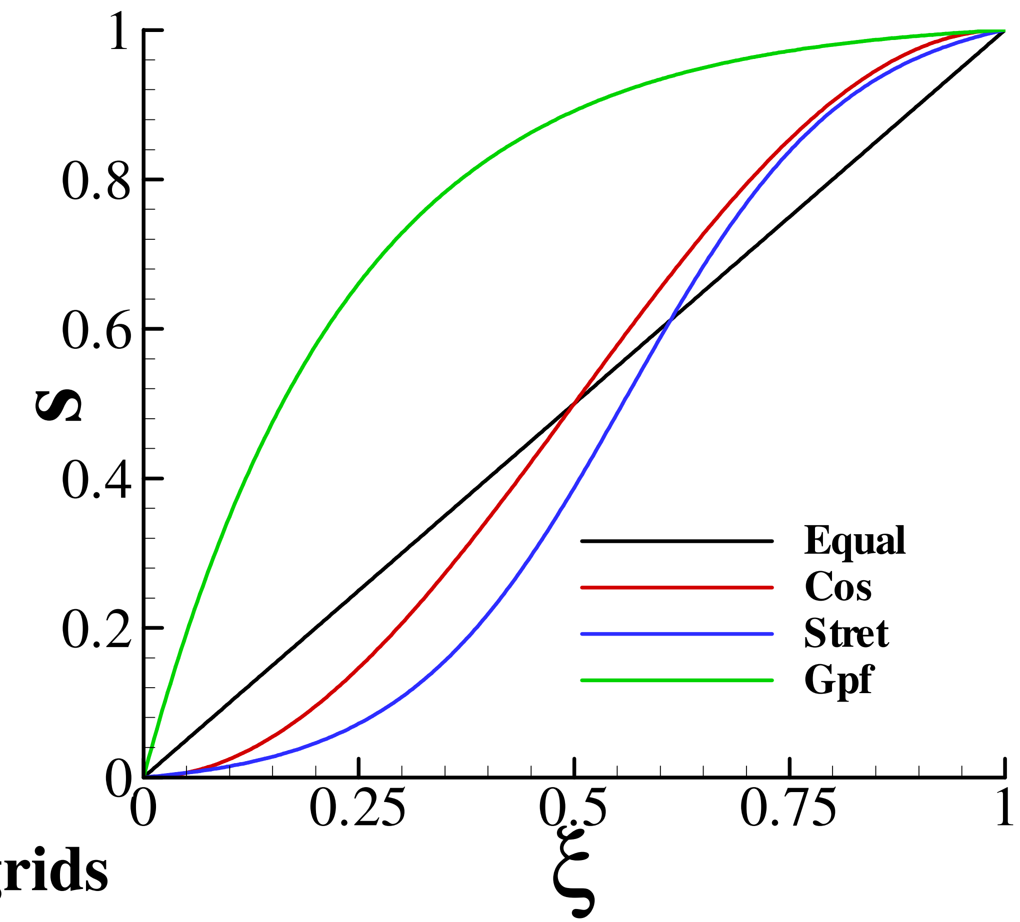
  - Consider a 1-D problem for $0 \leq x \leq 1$

  - Geometrically similar grids have a single definition of the dimensionless distance (*s*) as a function of the "dimensionless" node counter (*x*)

$$s = \frac{x - x_{\min}}{x_{\max} - x_{\min}} = x, \quad \xi = \frac{i-1}{N_x - 1}$$

Code Verification

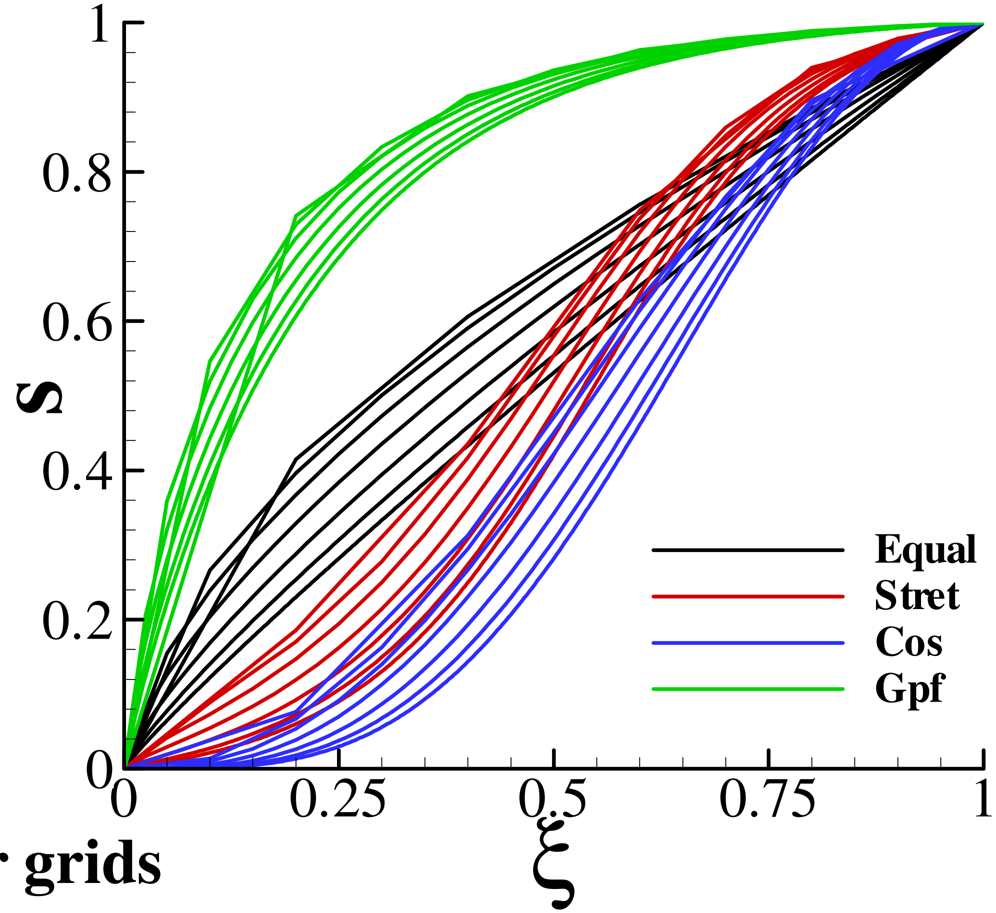
- Geometrical Similarity (a 1-D example)



Similar grids

Code Verification

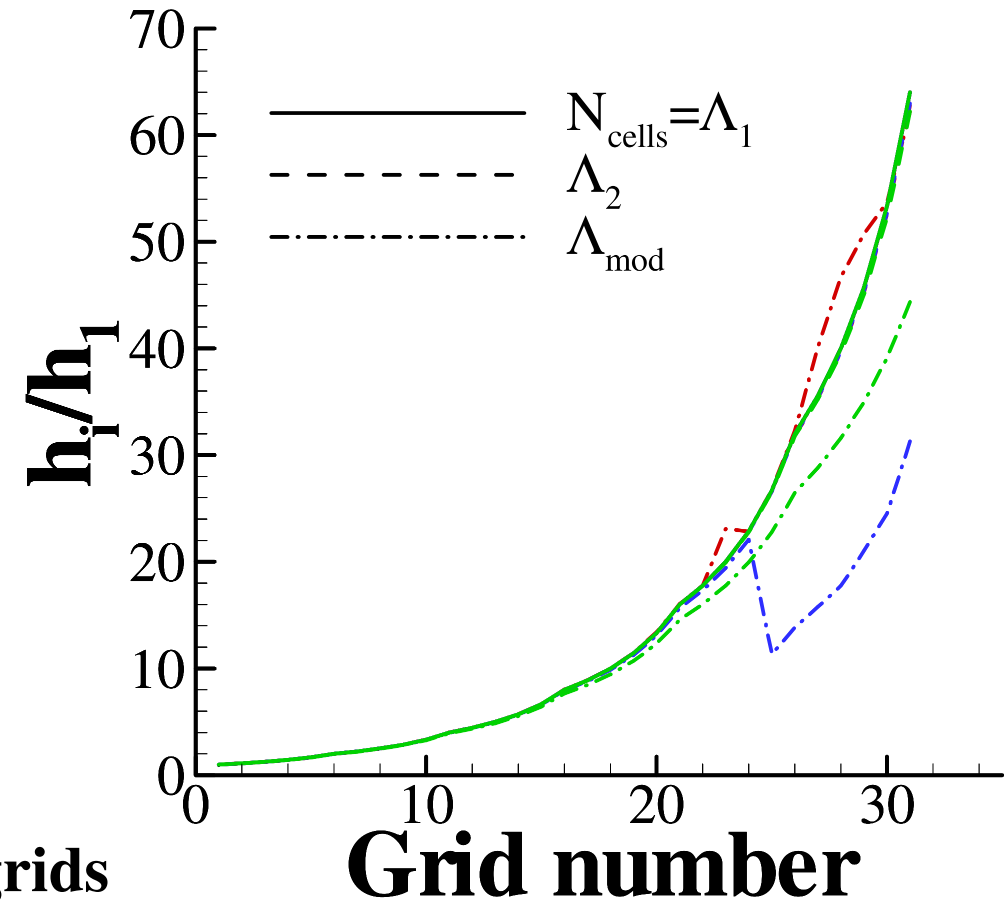
- Geometrical Similarity (a 1-D example)



Unsimilar grids

Code Verification

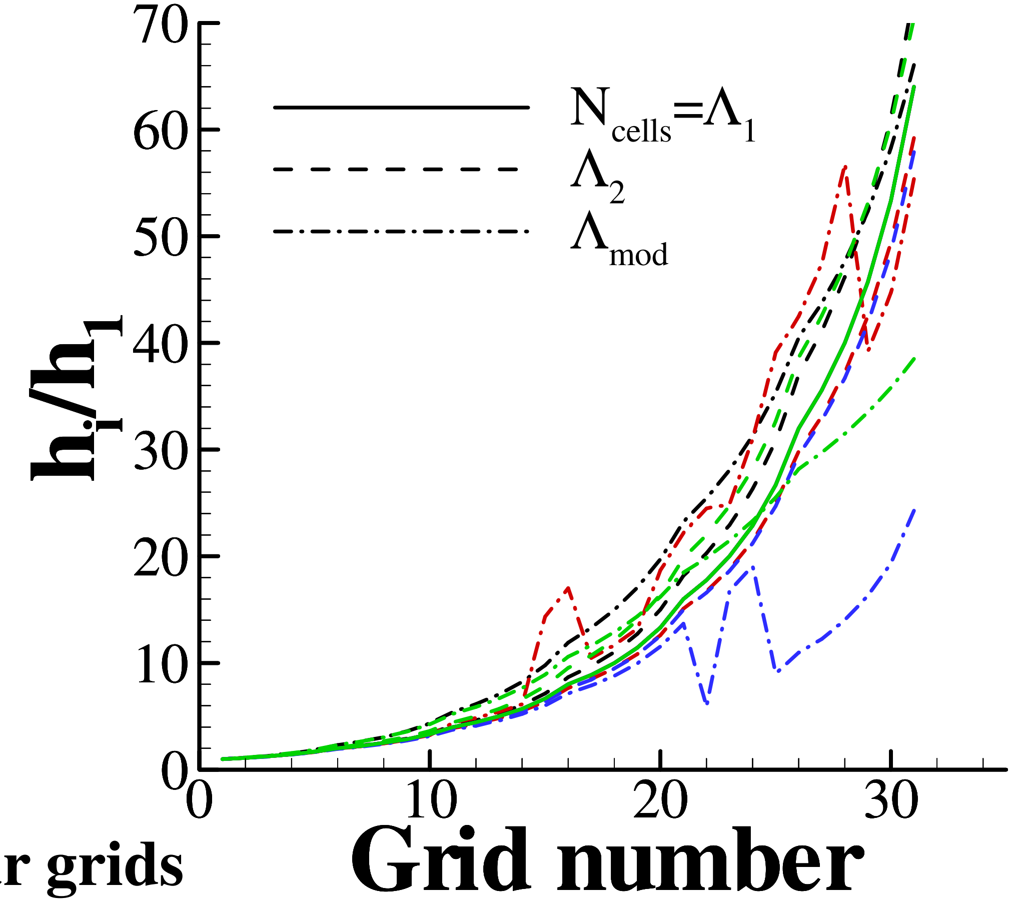
- Geometrical Similarity (a 1-D example)



Similar grids

Code Verification

- Geometrical Similarity (a 1-D example)



Unsimilar grids

Code Verification

- Geometrical Similarity (a 1-D example)

$$\frac{d^2\phi}{dx^2} = a\phi \quad \text{for} \quad 0 < \mathrm{x} < 1$$

$$\phi(0) = 1 \ \wedge \ \phi(1) = 0$$

$$\phi_{exact} = -\frac{e^{\sqrt{a}}}{e^{-\sqrt{a}} - e^{\sqrt{a}}} e^{-x\sqrt{a}} + \frac{e^{-\sqrt{a}}}{e^{-\sqrt{a}} - e^{\sqrt{a}}} e^{x\sqrt{a}}$$

$$(a = 5)$$

Sets of 31 grids with $5 \leq$ Ncells ≤ 320

Code Verification

- Geometrical Similarity (a 1-D example)

\- Error norms:

$$L_{\infty}[e(\phi)] = \max\left(\phi_j - \phi_{exact}\right)$$

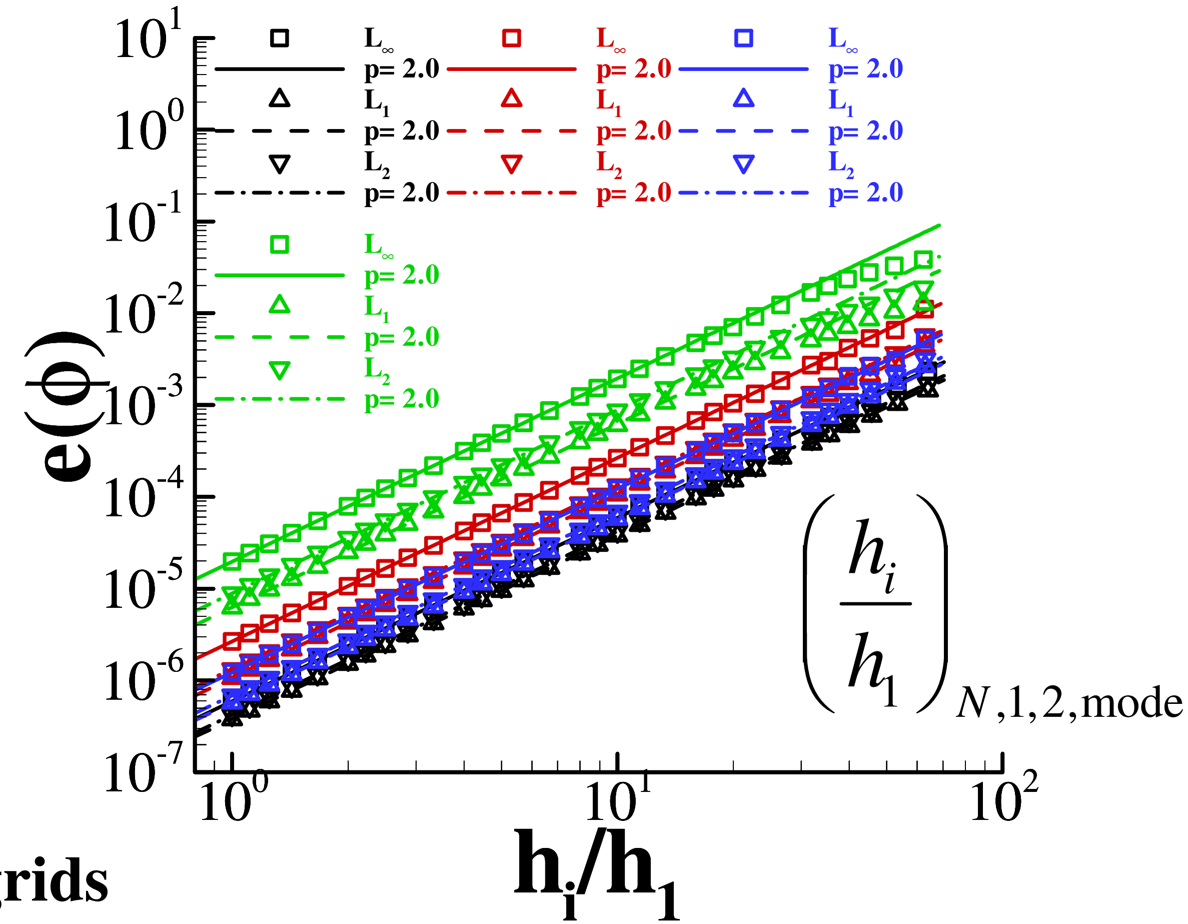
$$L_1[e(\phi)] = \frac{\sum_{j=2}^{N_x-1}\left|\phi_j - \phi_{exact}\right|}{N_x - 2}$$

$$L_2[e(\phi)] = \sqrt{\frac{\sum_{j=2}^{N_x-1}\left(\phi_j - \phi_{exact}\right)^2}{N_x - 2}}$$

\- $e_o(\phi)$, p and a obtained in the least squares sense from the data of the six finest grids

Code Verification

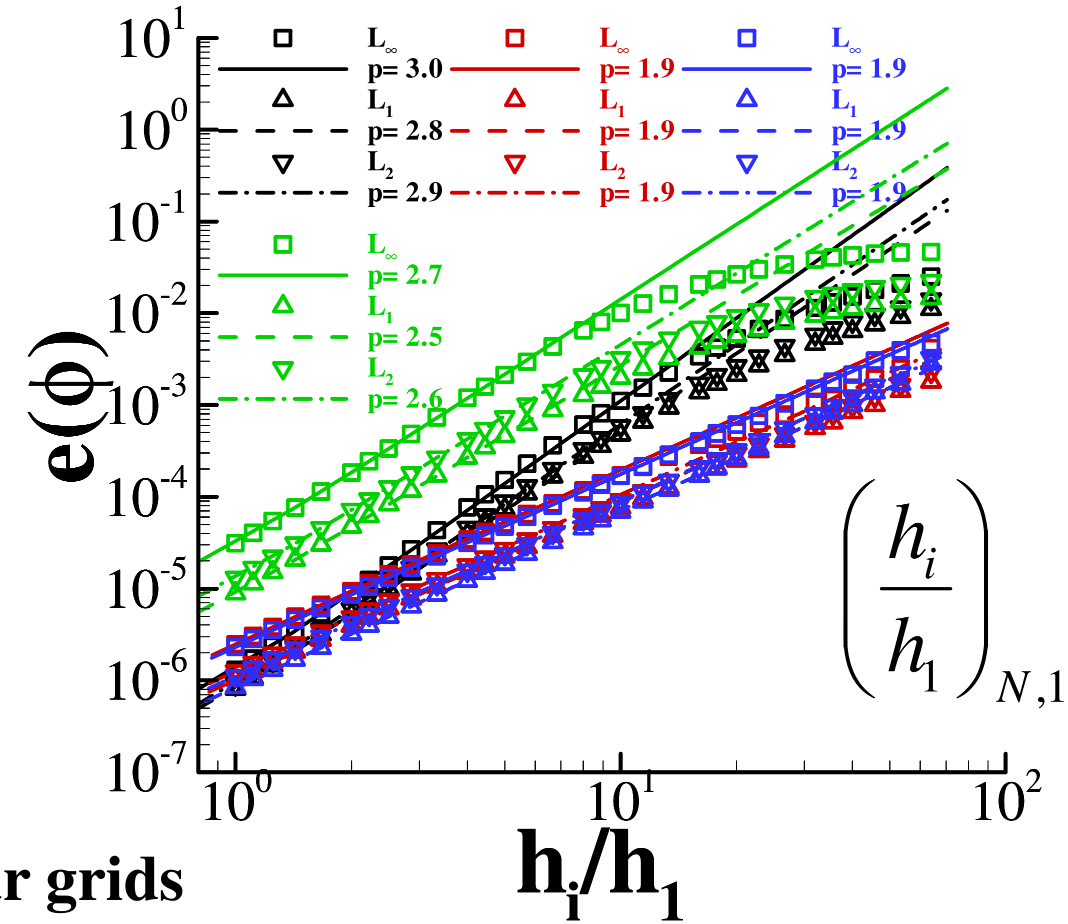
- Geometrical Similarity (a 1-D example)



Similar grids

Code Verification

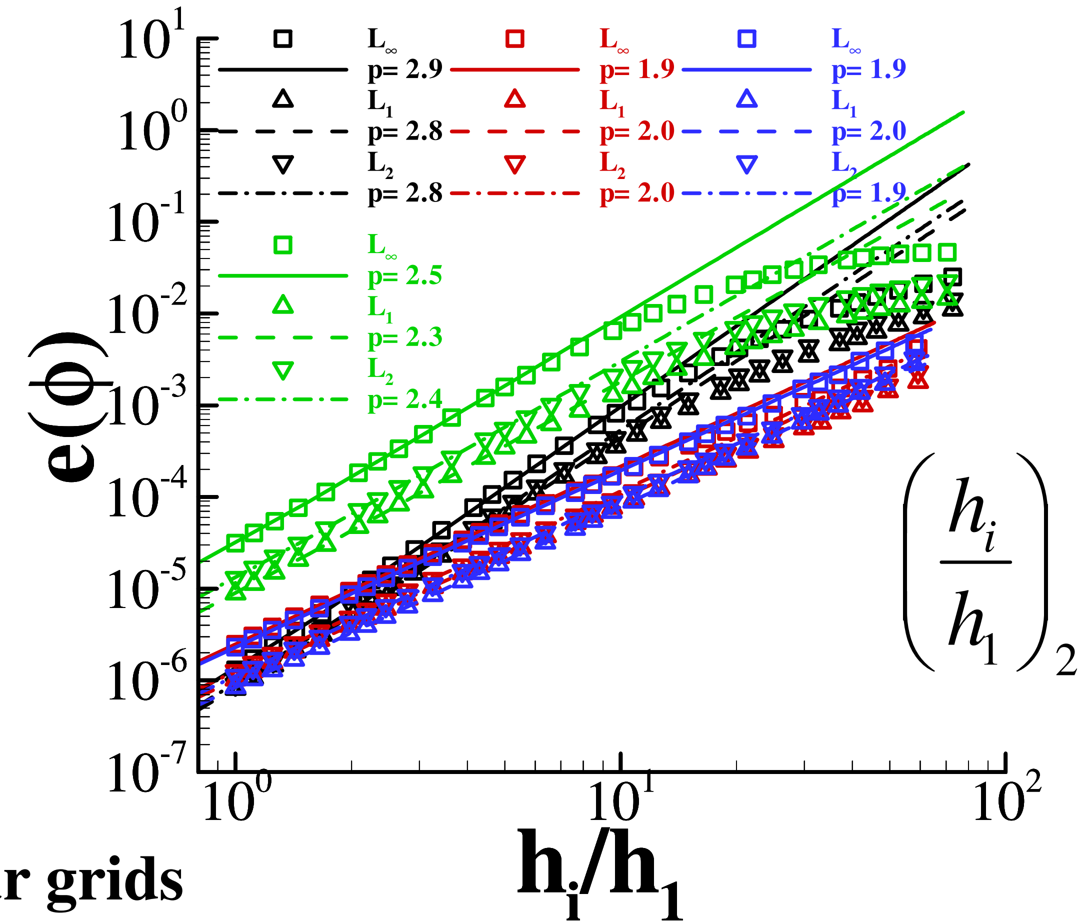
- Geometrical Similarity (a 1-D example)



Unsimilar grids

Code Verification

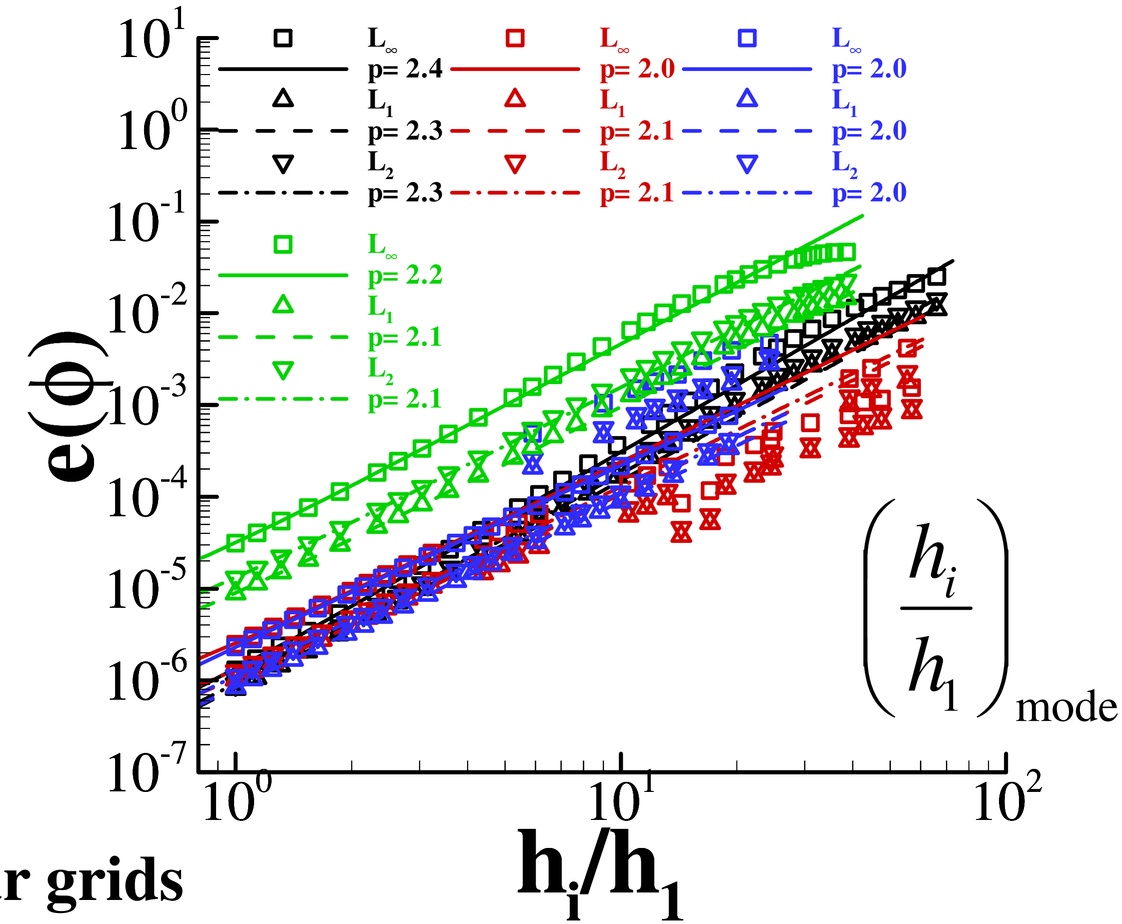
- Geometrical Similarity (a 1-D example)



Unsimilar grids

Code Verification

- Geometrical Similarity (a 1-D example)



Unsimilar grids

Code Verification

- Code Verification

  - In some practical applications, there are no exact solutions available. For example, the Reynolds-averaged Navier-Stokes equations supplemented by a turbulence model do not have any known exact solutions

  – The Method of Manufactured Solutions (MMS) is a viable alternative to perform Code Verification when analytical solutions are not available.

Code Verification

- Method of Manufactured Solutions:

1. Choose the calculation domain

2. Choose the analytical solution for the dependent variables of the problem

3. Substitute the chosen solution in the differential equations to determine source terms that guarantee that the selected exact solution satisfies the “new” system of differential equations

Solution Verification

- Solution/Calculations Verification
  - Exact solution is not available
  - Numerical error estimation usually assumes that the discretization error is dominant (this requires an iterative error at least two orders of magnitude smaller)
  - Methods based on grid refinement studies are one of the alternatives for the estimation of the discretization error/uncertainty
  - Mathematical problem

Solution Verification

– Estimate the uncertainty, U, of a numerical solution of a flow quantity ϕ for which the exact solution is not known

Objective:

$$\phi - U(\phi) \leq \phi_{exact} \leq \phi + U(\phi)$$

with a confidence level of 95%

$$U(\phi) = F_S e(\phi)$$

$F_S \rightarrow$ Safety factor

$e(\phi) \rightarrow$ Error estimate

Solution Verification

- Error described by a Power Series Expansions

$$e(\phi_i) = \phi_i - \phi_o = \delta_{RE} = \alpha h_i^p$$

ϕ_i → Local or functional flow quantity for grid i

ϕ_o → Estimate of the exact solution of local or functional flow quantity

δ_{RE} → Estimate of the error

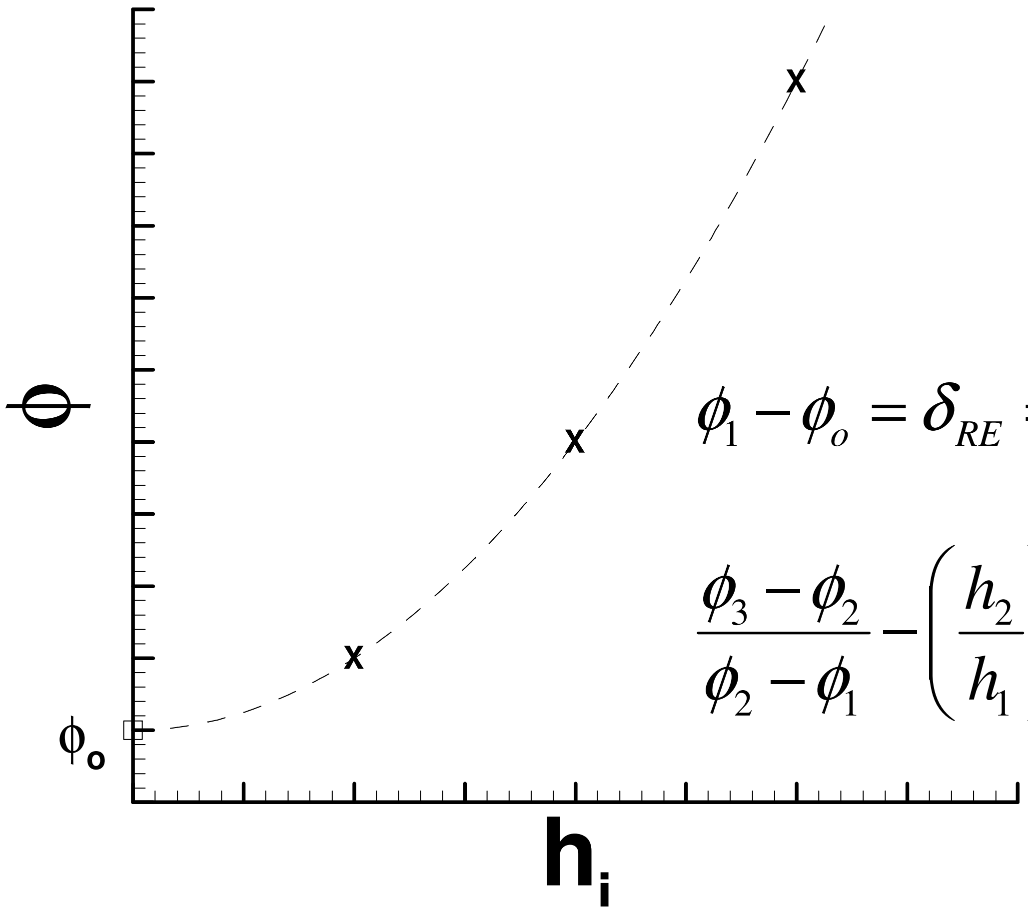
α → Constant

h_i → Typical cell size

p → Observed order of grid convergence

Solution Verification

- At least 3 grids are required to determine ϕ_o, α, p



$$\phi_1 - \phi_o = \delta_{RE} = \frac{\phi_2 - \phi_1}{(h_2/h_1)^p - 1}$$

$$\frac{\phi_3 - \phi_2}{\phi_2 - \phi_1} - \left(\frac{h_2}{h_1}\right)^p \frac{(h_3/h_2)^p - 1}{(h_2/h_1)^p - 1} = 0$$

Solution Verification

- Apparent convergence or divergence for a grid triplet with $h_2/h_1=h_3/h_2$!

Convergence ratio : $$R = \frac{\phi_2 - \phi_1}{\phi_3 - \phi_2}$$

$0 < R < 1$ → Monotonic Convergence

$-1 < R < 0$ → Oscillatory Convergence

$R > 1$ → Monotonic Divergence

$R < -1$ → Oscillatory Divergence

- Previous equation can only be used for Monotonic Convergence

Solution Verification

- Example I

$$\frac{d^2\phi}{dx^2} = a\phi \quad \text{for} \quad 0 < x < 1$$

$$\phi(0) = 1 \;\wedge\; \phi(1) = 0$$

Sets of 31 grids with 5 ≤ Ncells ≤320

Geometrically similar grids require

$$s = \frac{x - x_{\min}}{x_{\max} - x_{\min}} = f(\xi), \quad \xi = \frac{i-1}{N_x - 1}$$

Solution Verification

- Example I

$$\frac{d^2\phi}{dx^2} = a\phi \quad \text{for} \quad 0 < x < 1 \qquad \phi(0) = 1 \ \wedge \ \phi(1) = 0$$

- Determination of solution at $x=0.5$

- Three grid sets of 21 grids with $5 \leq$ Ncells ≤ 80

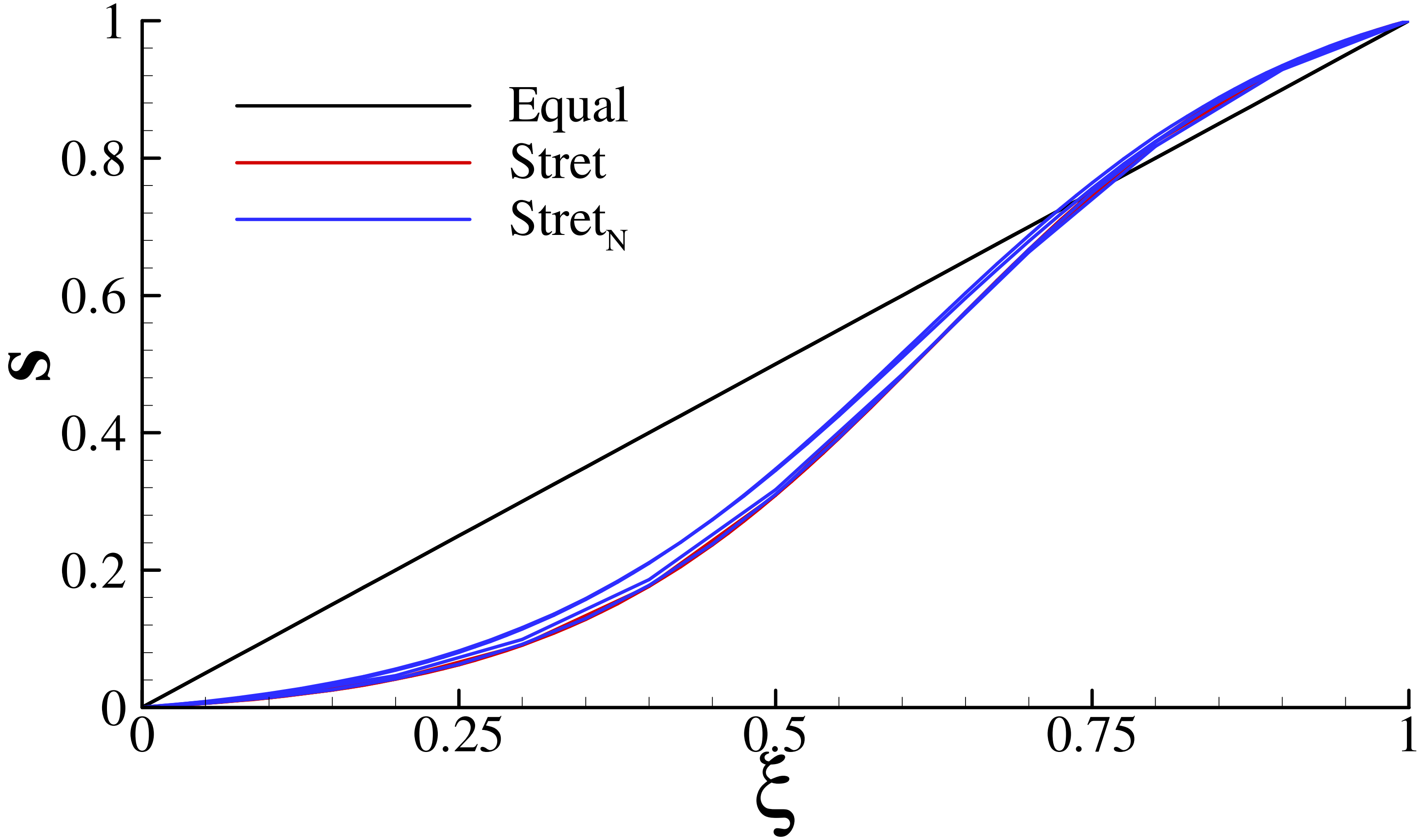
a) Equally-spaced geometrically similar grids, Equal

b) Stretched geometrically similar grids, Stret

c) Stretched non-geometrically similar grids, Stret_N

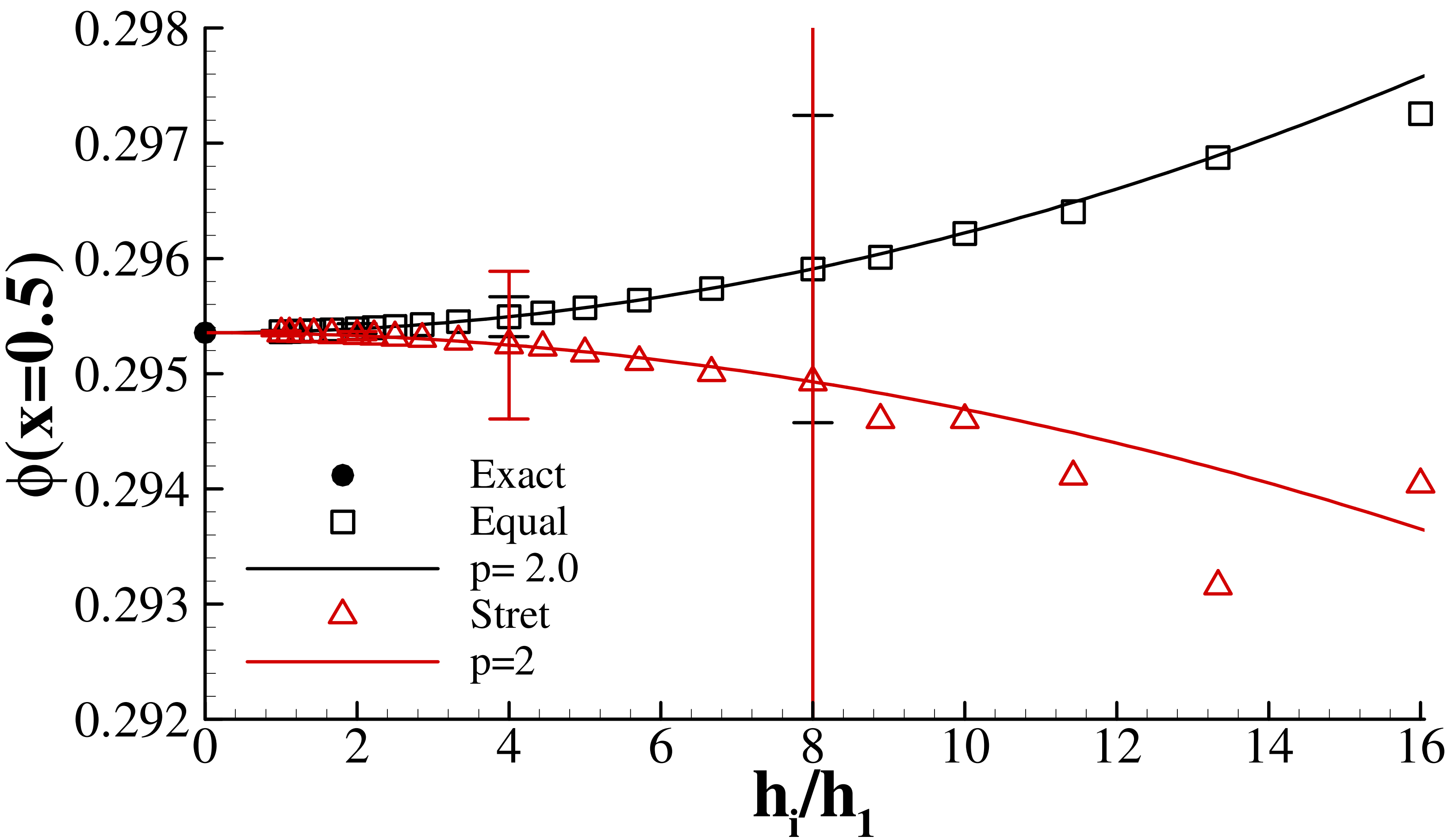
Solution Verification

- Example I



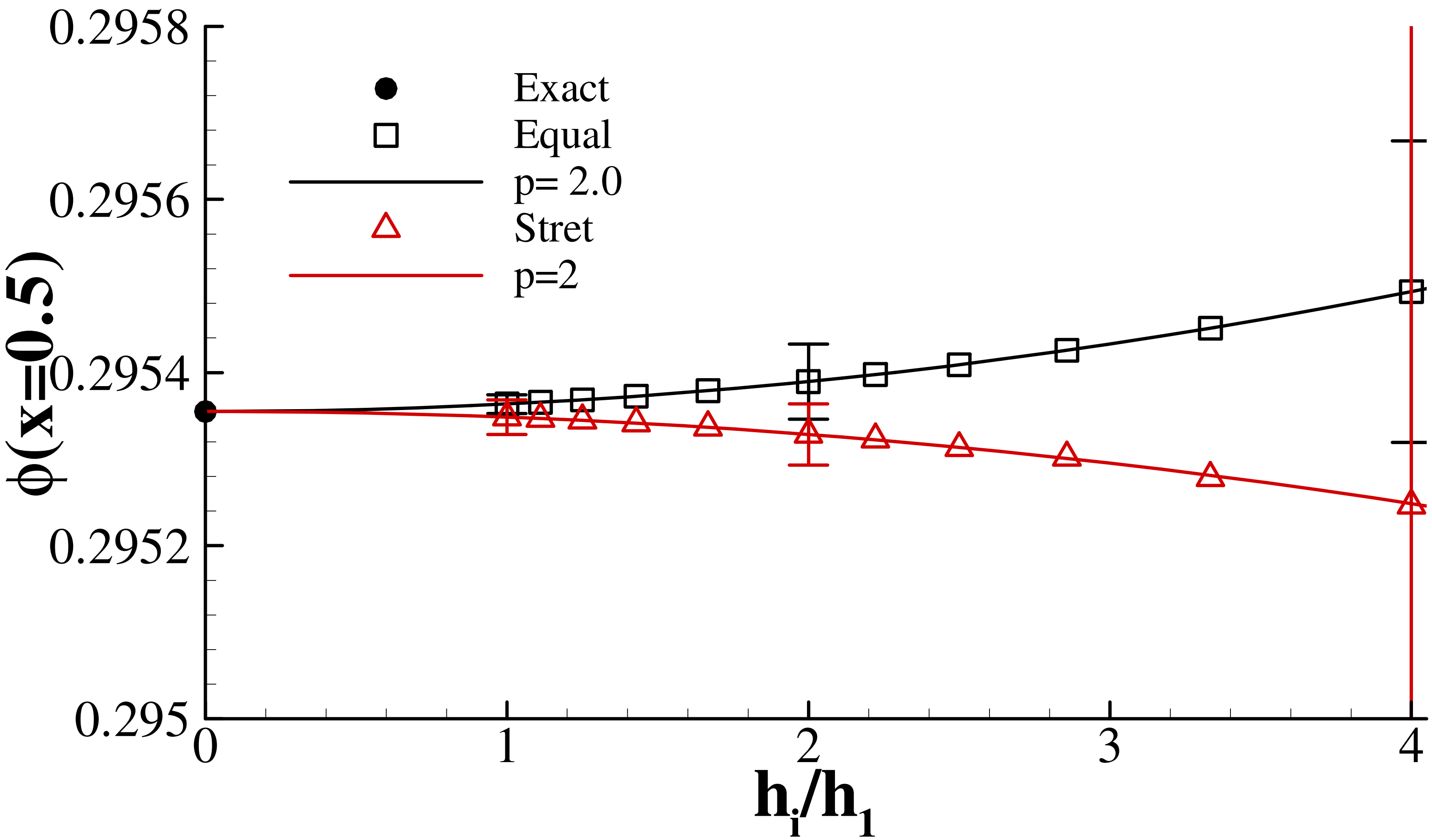
Solution Verification

- Example I



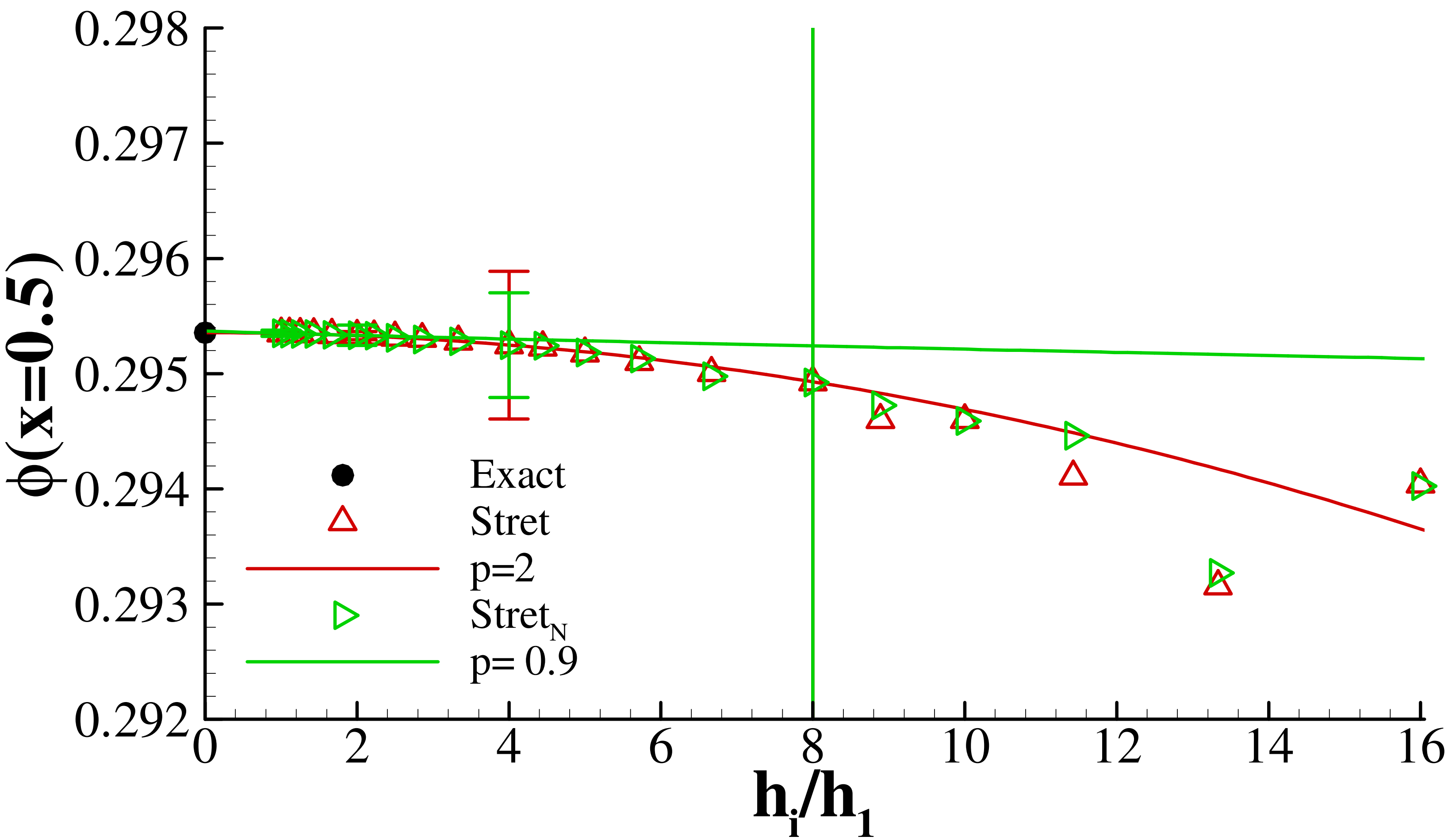
Solution Verification

- Example I



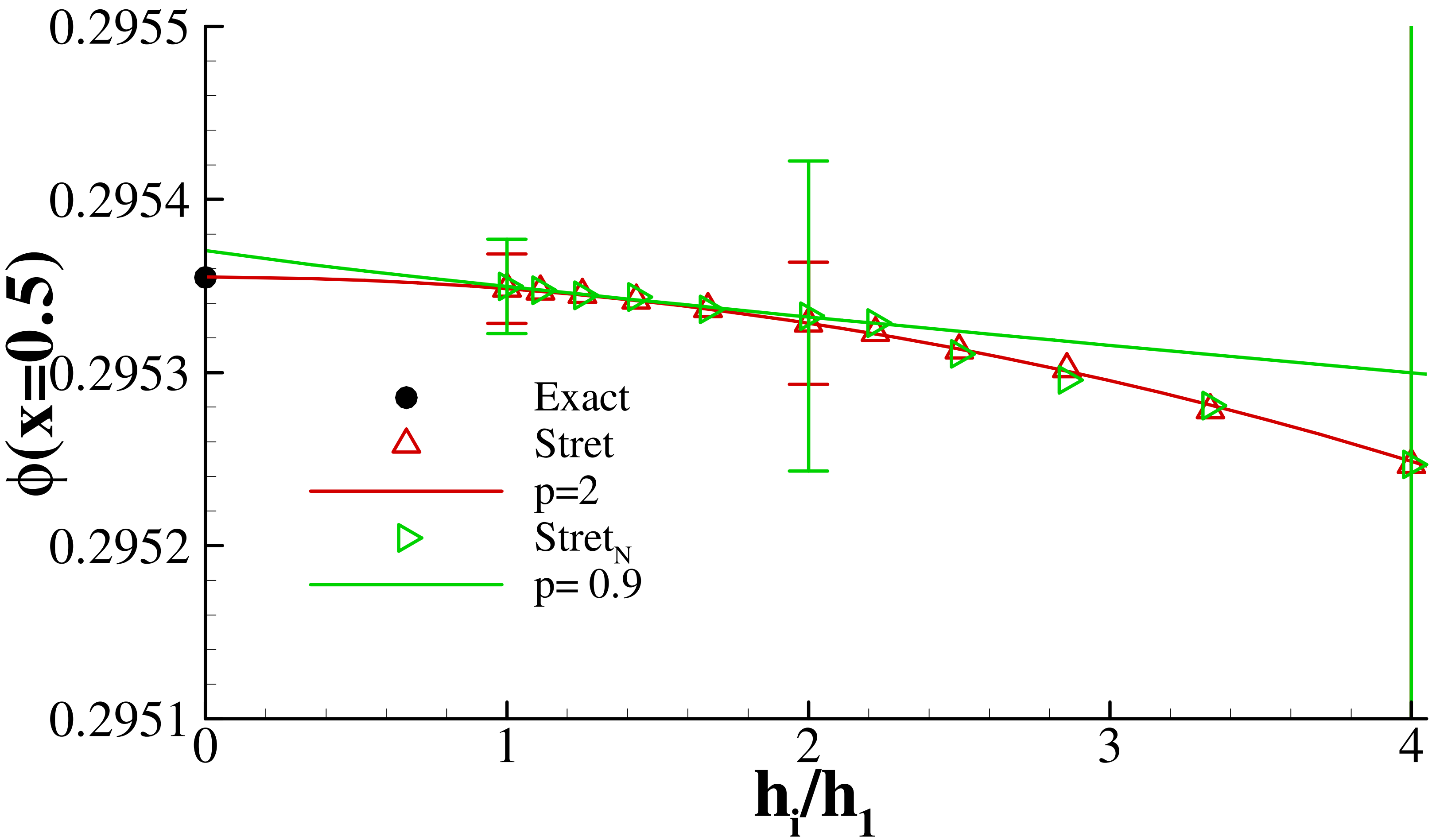
Solution Verification

- Example I



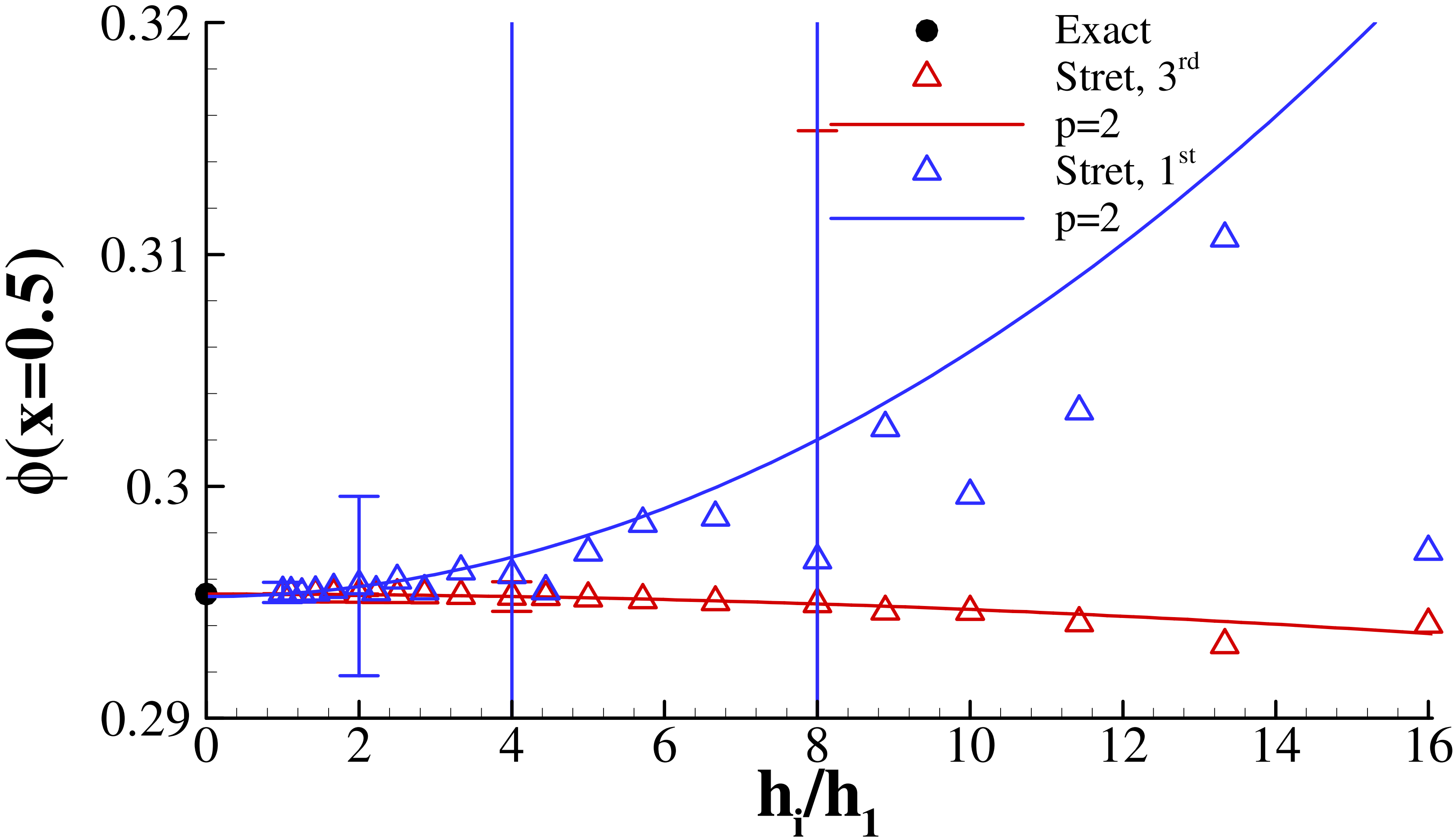
Solution Verification

- Example I



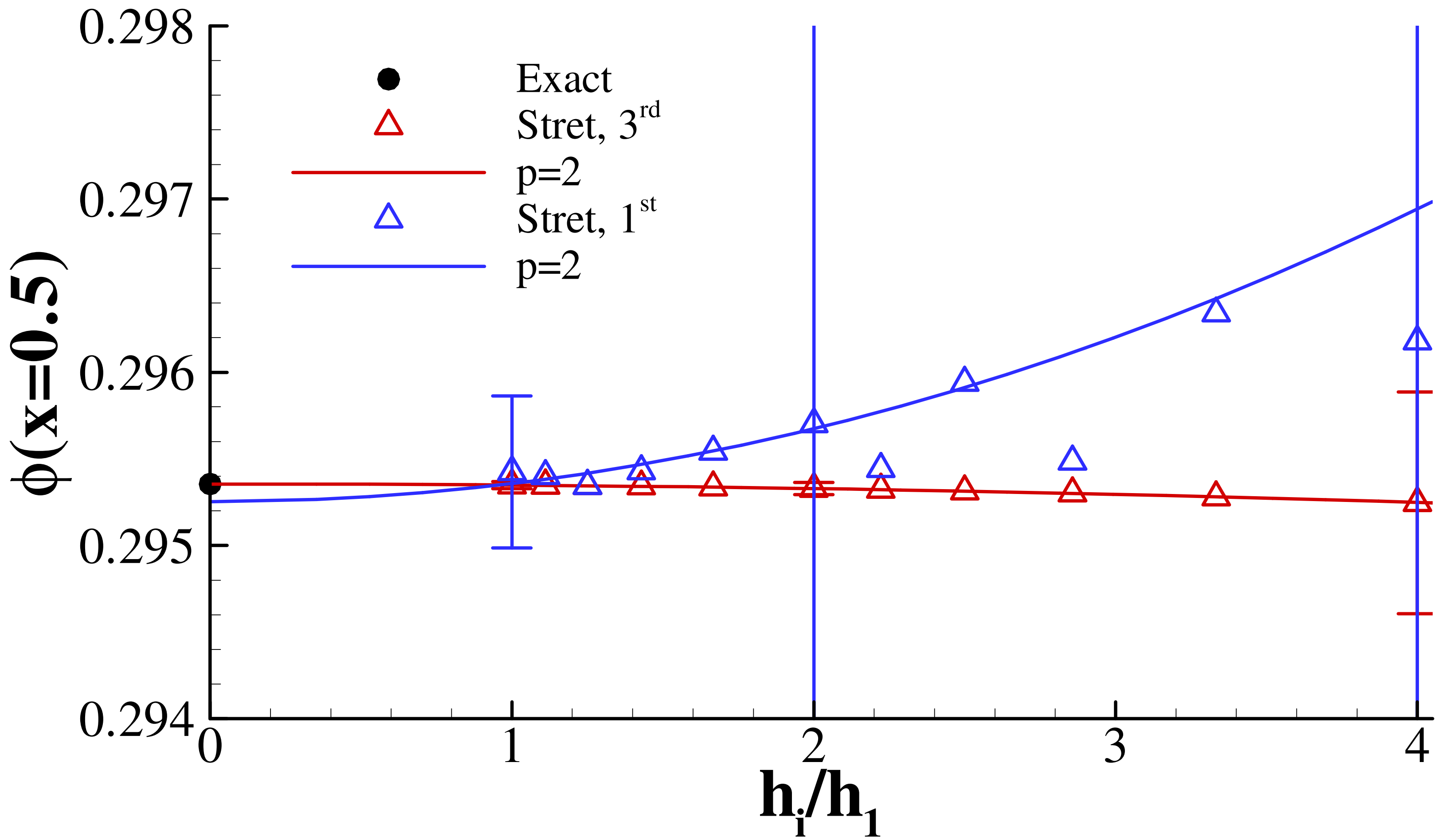
Solution Verification

- Example I



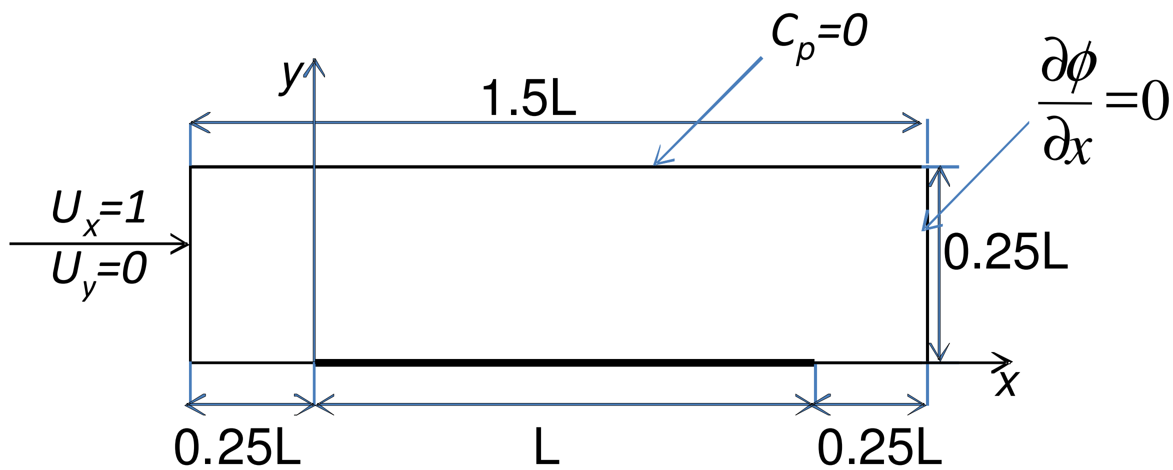
Solution Verification

- Example I



Solution Verification

- Example II
  - Flow over a flat plate at Reynolds number 10^7



  - Time-averaged Navier-Stokes equations supplemented with eddy-viscosity models

Solution Verification

- Example II
  - Flow over a flat plate at Reynolds number 10^7
  - Three eddy-viscosity turbulence models
    1. SST k-ω two-equation model (SST)
    2. k-$\sqrt{k}$L two-equation model (KSKL)
    3. Spalart & Allmaras one-equation model (SPAL)
  - Quantity of interest:
    Friction resistance coefficient of the plate

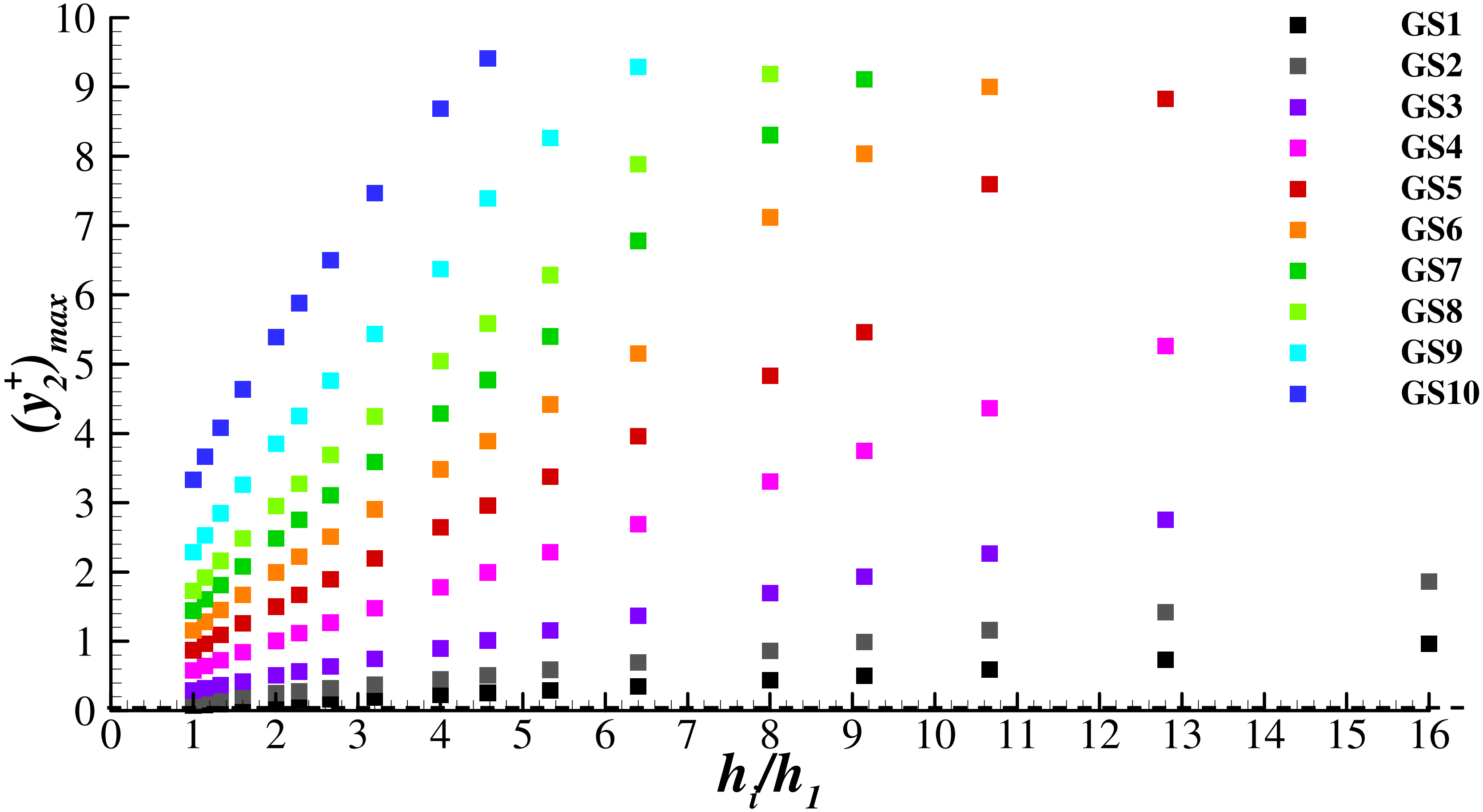
$$C_F = \frac{\int_0^L \tau_w dx}{\frac{1}{2}\rho U_\infty^2 L} = \frac{\int_0^L \mu \left(\frac{\partial U_x}{\partial y}\right)_{y=0} dx}{\frac{1}{2}\rho U_\infty^2 L}$$

Solution Verification

- Example II
  - Flow over a flat plate at Reynolds number 10^7
  - 10 sets of 17 geometrically similar orthogonal grids with clustering of grid nodes at the leading and trailing edges and at the wall
  - Same number of cells (1152 to 294912, $1 \leq h_i/h_1 \leq 16$) and horizontal grid line spacing for all sets
  - Different stretching parameters for the definition of the size of the near-wall cells
  - Negligible iterative and round-off errors

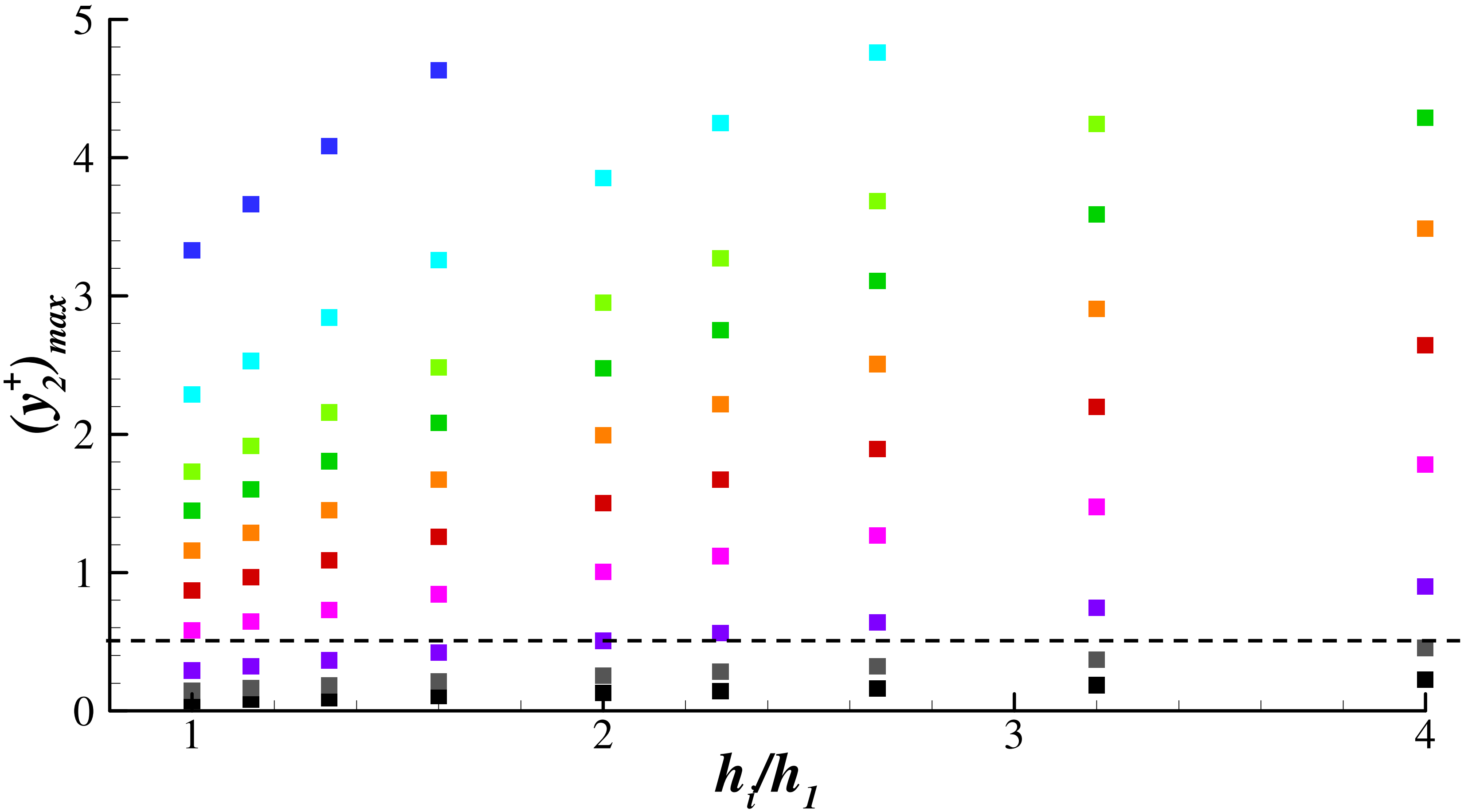
Solution Verification

- Example II
  - Flow over a flat plate at Reynolds number 10^7



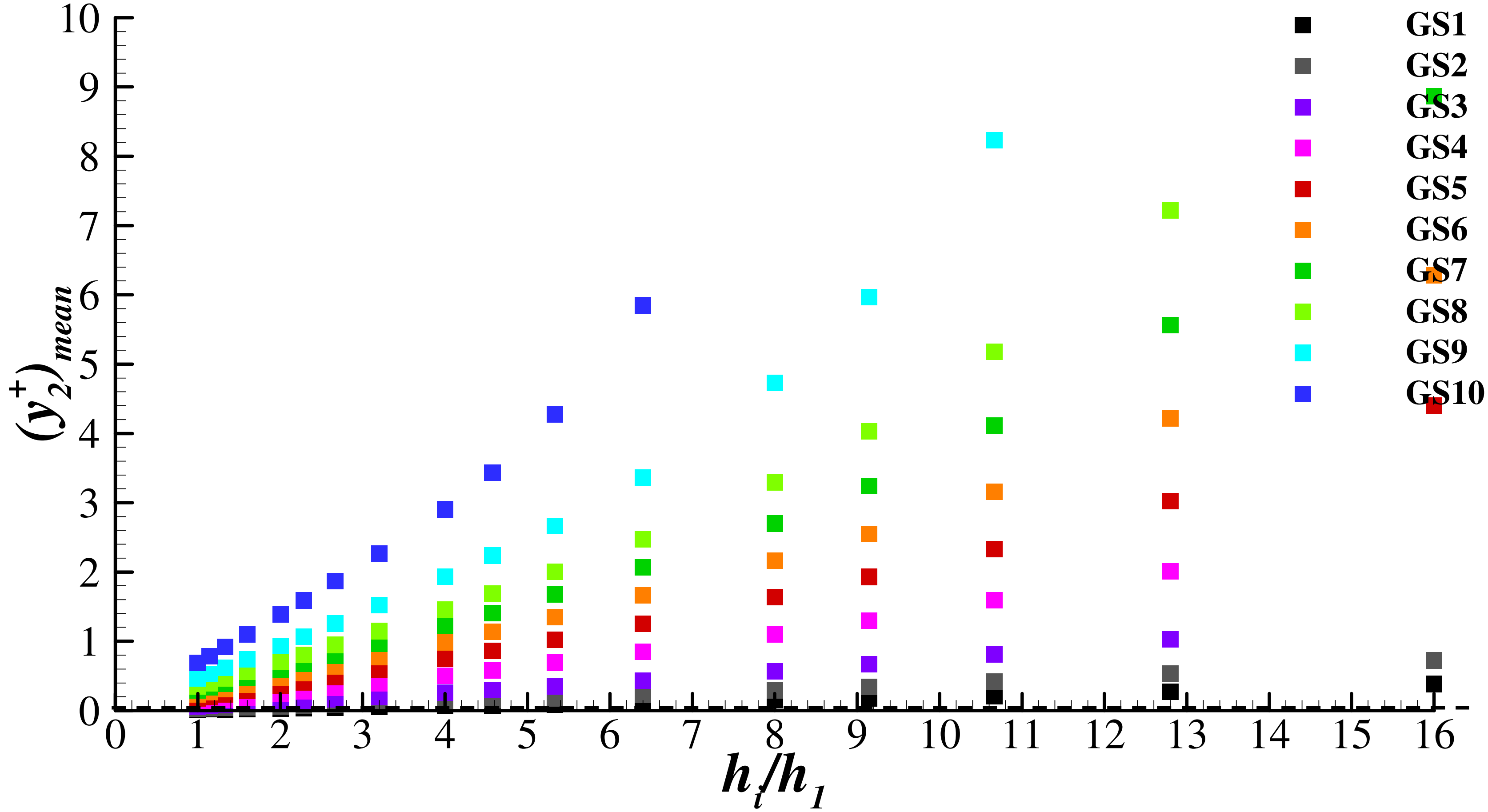
Solution Verification

- Example II
  - Flow over a flat plate at Reynolds number 10^7



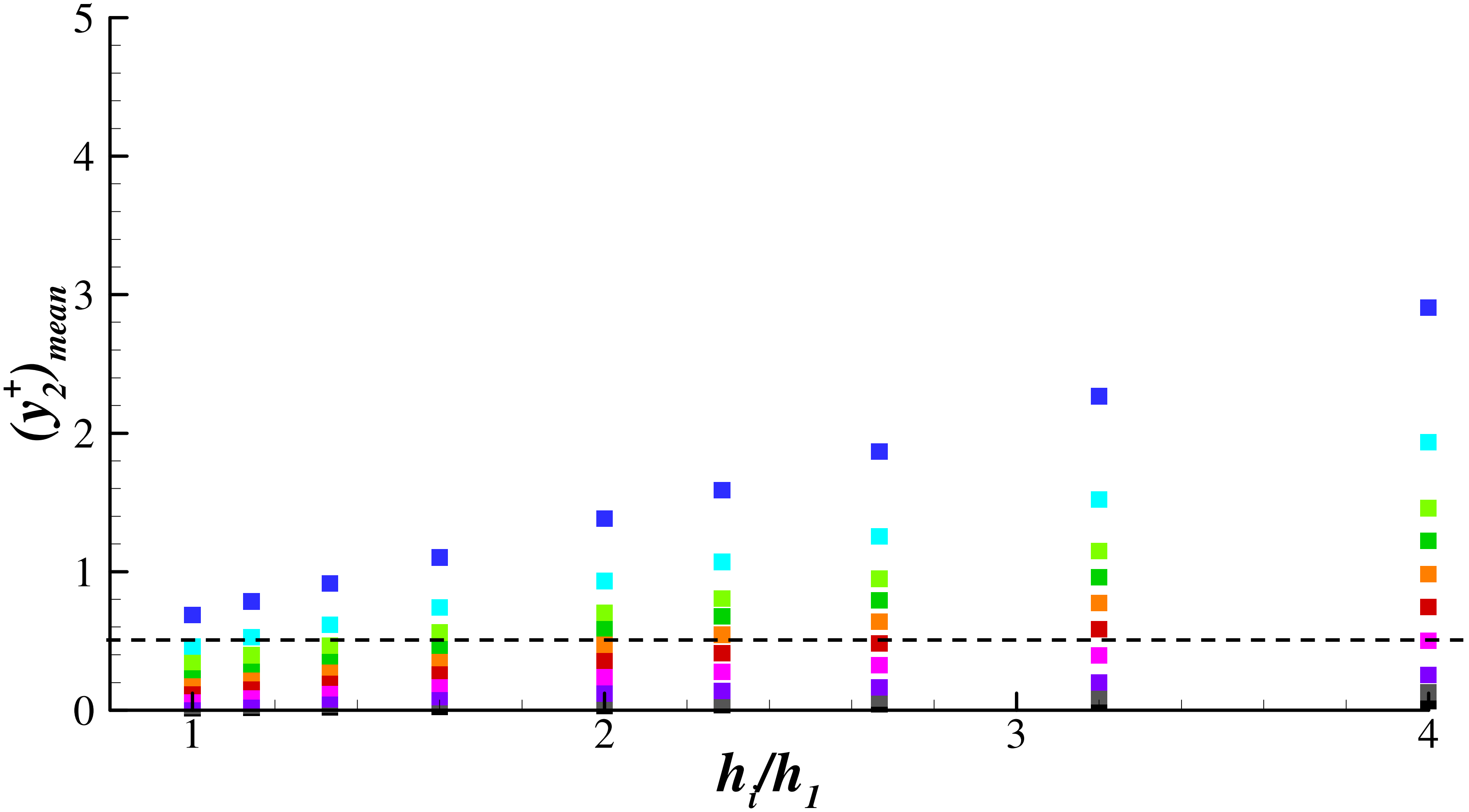
Solution Verification

- Example II
  - Flow over a flat plate at Reynolds number 10^7



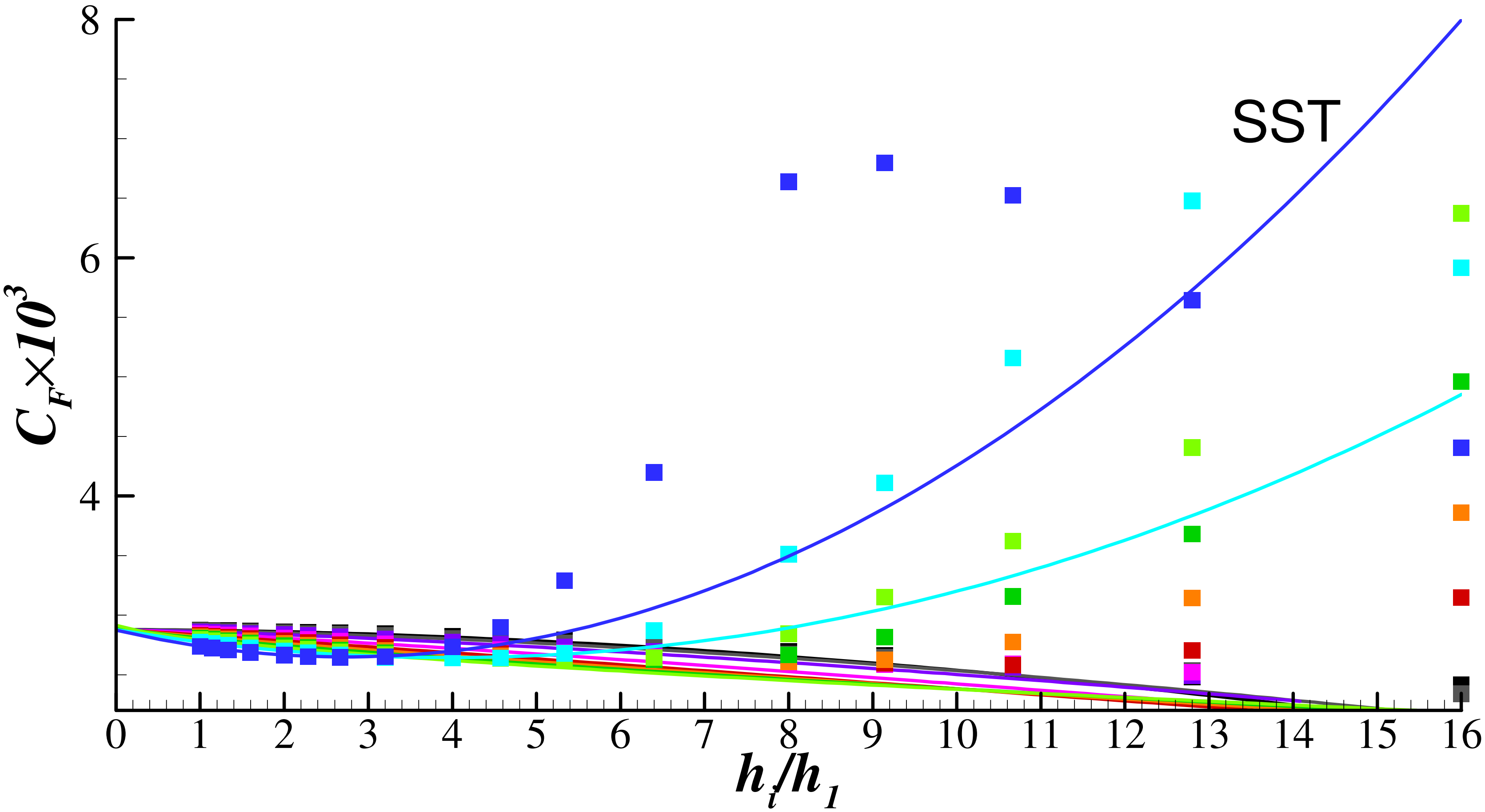
Solution Verification

- Example II
  - Flow over a flat plate at Reynolds number 10^7



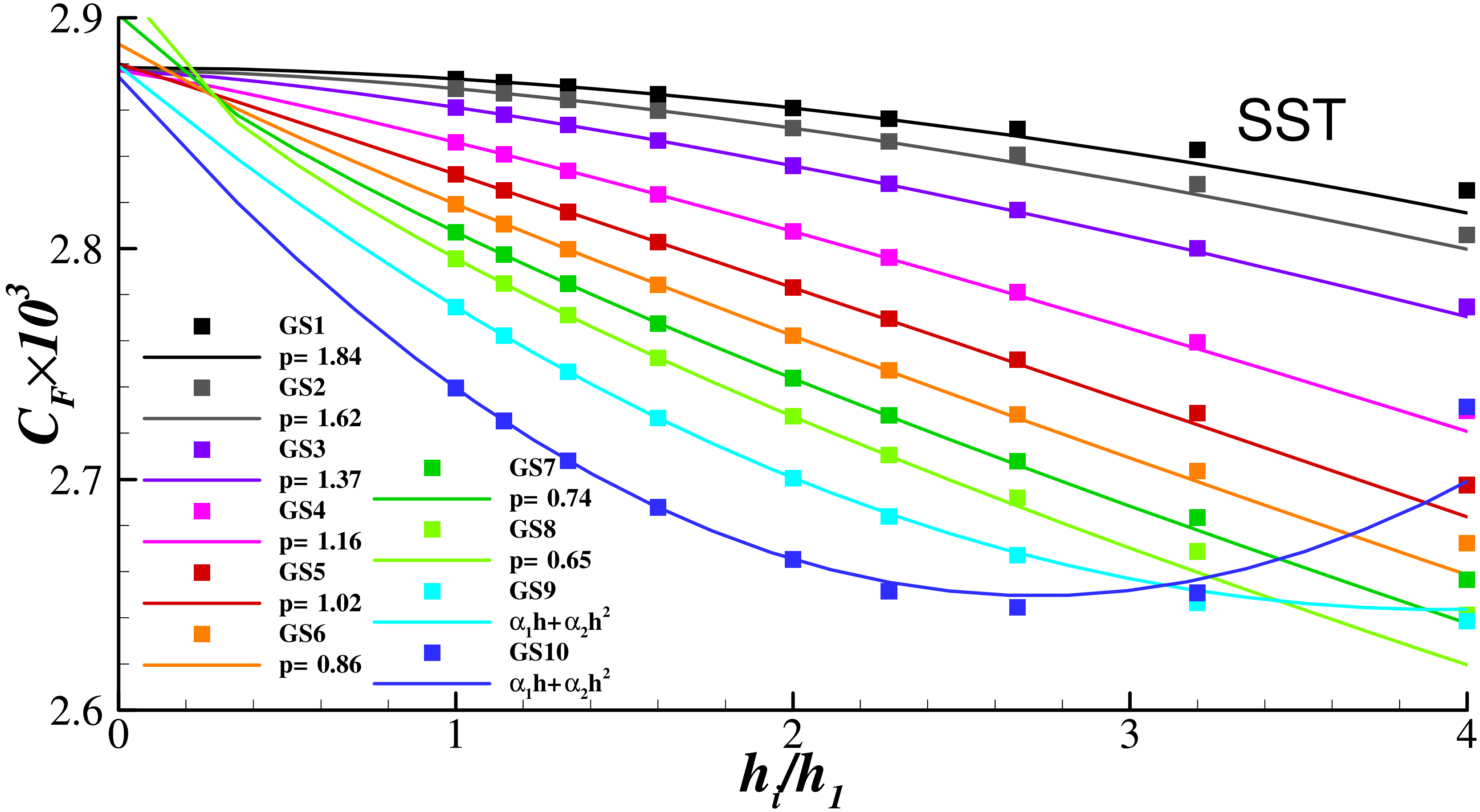
Solution Verification

- Example II
  - Flow over a flat plate at Reynolds number 10^7



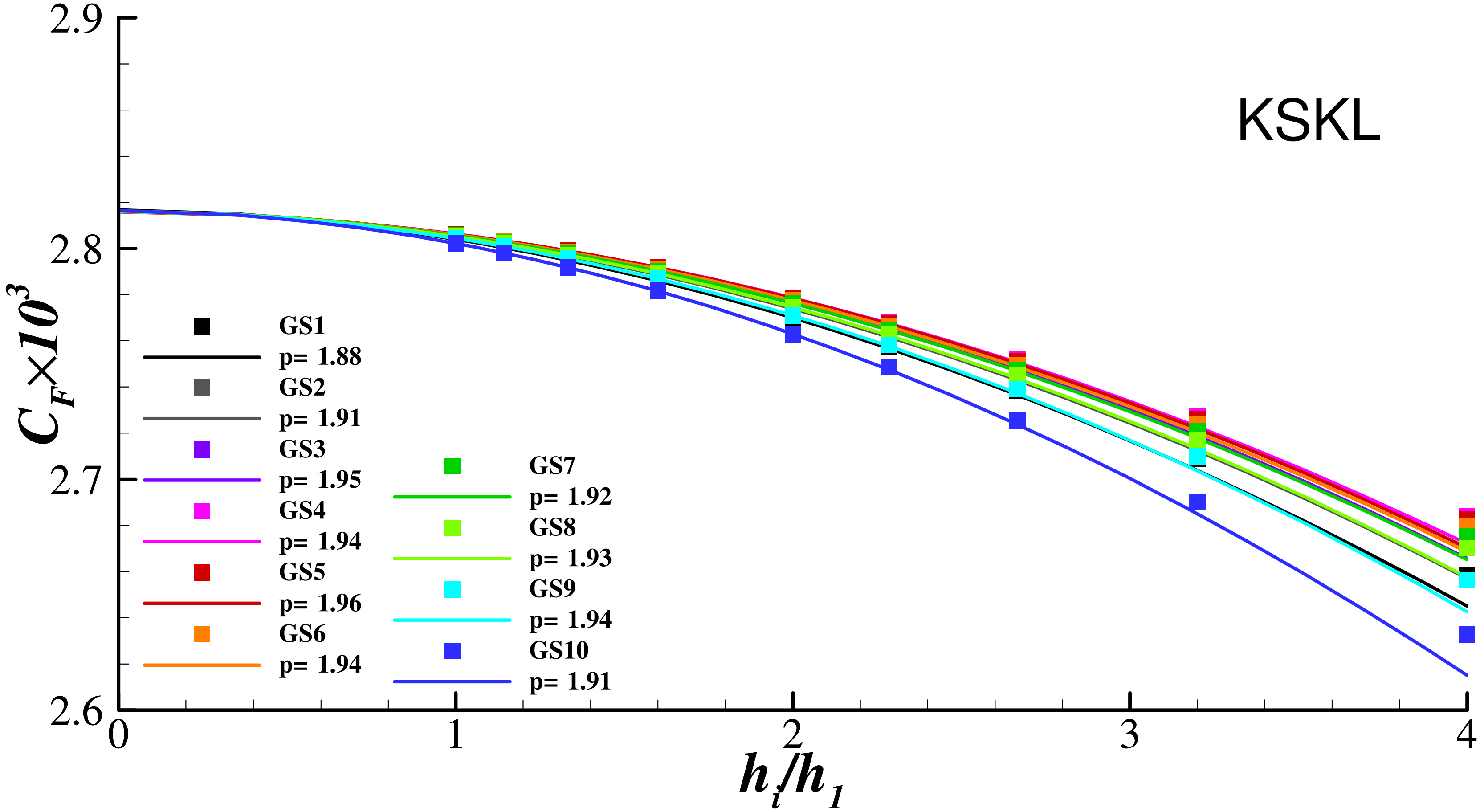
Solution Verification

- Example II
  - Flow over a flat plate at Reynolds number 10^7



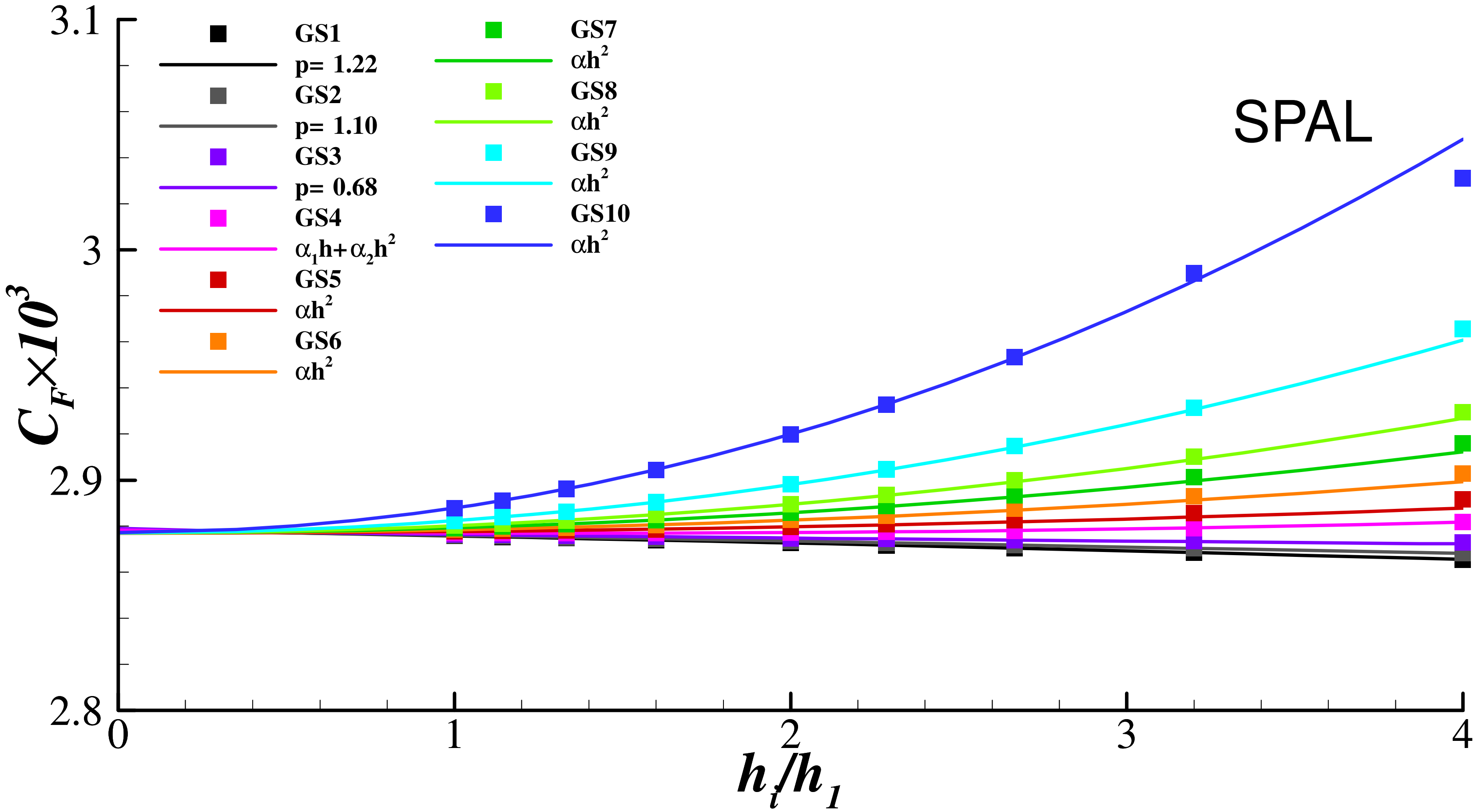
Solution Verification

- Example II
  - Flow over a flat plate at Reynolds number 10^7



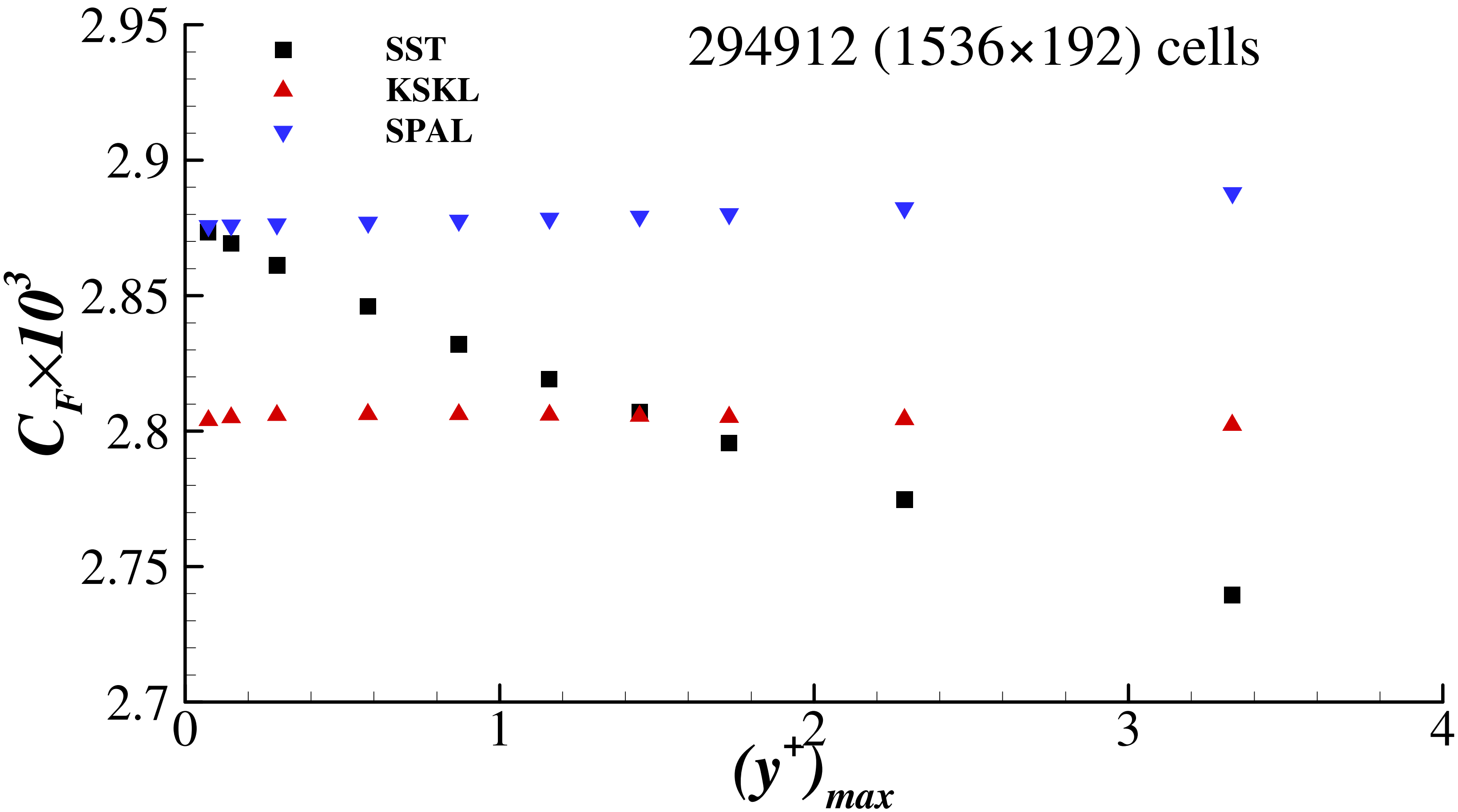
Solution Verification

- Example II
  - Flow over a flat plate at Reynolds number 10^7



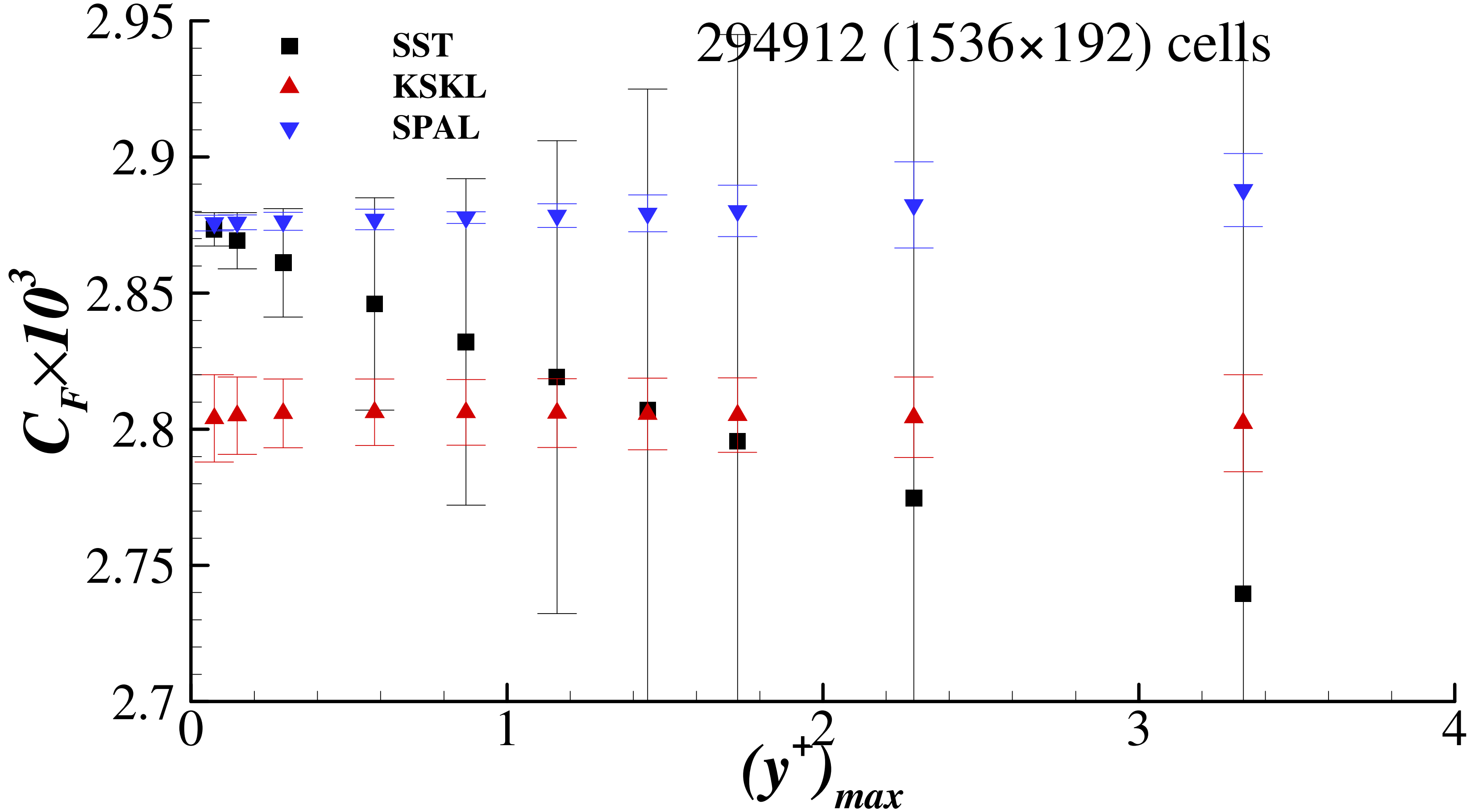
Solution Verification

- Example II
  - Flow over a flat plate at Reynolds number 10^7



Solution Verification

- Example II
  - Flow over a flat plate at Reynolds number 10^7



Validation (ASME V&V 20)

ASME V&V 20 procedure

Goal: Estimate an interval $\left[E - U_{val}, E + U_{val}\right]$
that contains the modeling error, δ_{model}
with 95% confidence

Needed:

- a numerical simulation result S with
  uncertainty estimate U_{num}
- a corresponding experimental data set D
  with uncertainty estimate U_D
- the parameter uncertainty U_{input}

Validation (ASME V&V 20)

ASME V&V 20 procedure

Goal: Estimate an interval $\left[E - U_{val}, E + U_{val}\right]$
that contains the modeling error, δ_{model}
with 95% confidence

$$E - U_{val} \leq \delta_{\text{model}} \leq E + U_{val}$$

$E \rightarrow$ Comparison error,

$$E = S - D$$

$U_{val} \rightarrow$ Validation uncertainty,

$$U_{val} = \sqrt{U_D^2 + U_{num}^2 + U_{input}^2}$$

Validation (ASME V&V 20)

ASME V&V 20 procedure

Analysis of the outcome: $E - U_{val} \leq \delta_{\text{model}} \leq E + U_{val}$

$$|E| >> U_{val} \rightarrow \delta_{\text{model}} \approx |E|$$

If E too big:

"Houston, we have a problem..."

$$|E| \leq U_{val} \rightarrow$$

If U_{val} is too big:

Validation exercise must be improved.

"Noise level" is too large

Validation (ASME V&V 20)

ASME V&V 20 procedure

- Validation is not a pass or fail exercise. It is and error assessment.
- $\left|E\right| \leq U_{val}$ does not mean that $\delta_{\text{model}} \leq U_{val}$
- Decision if δ_{model} is acceptable or not or what to do next are not part of the Validation exercise

Validation (ASME V&V 20)

- Flow at the propeller plane of a tanker at model scale

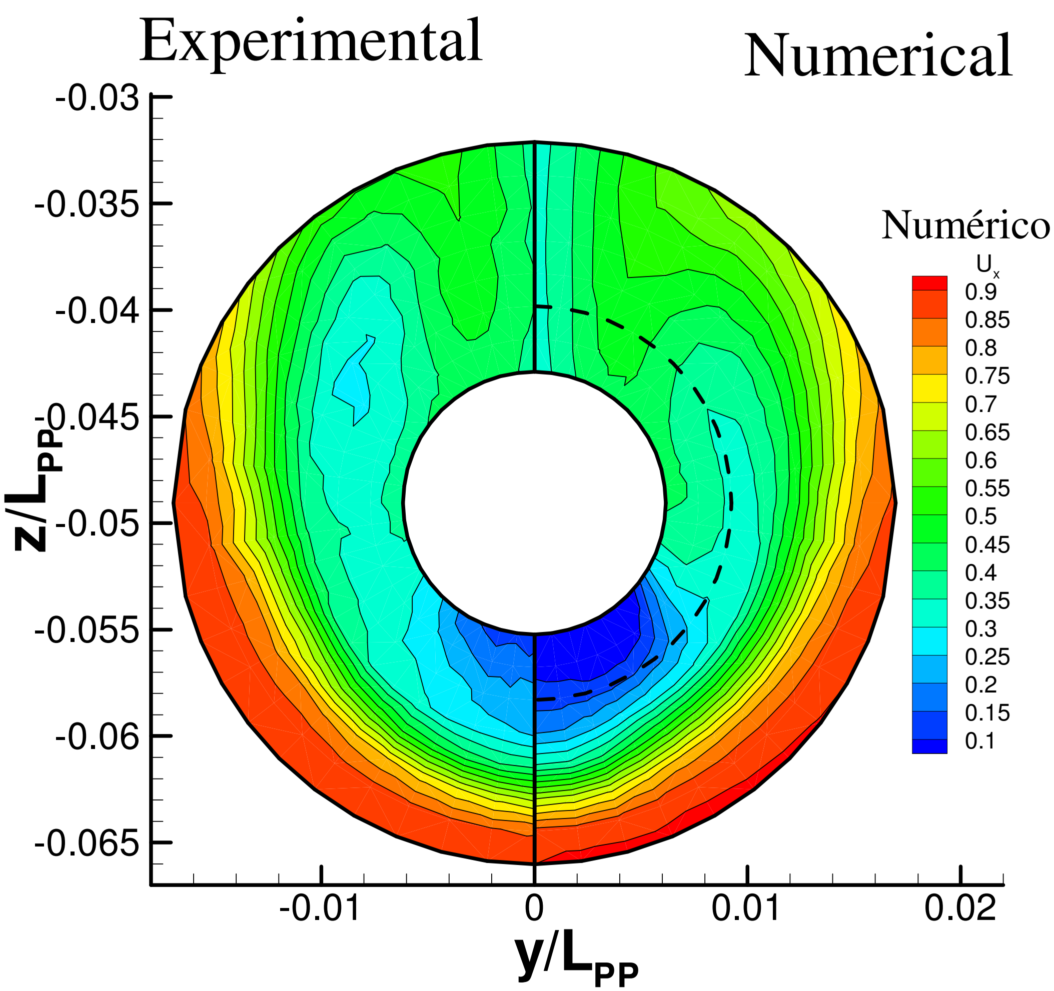
- Double body (no gravity waves)

- Comparison of axial velocity at the propeller plane

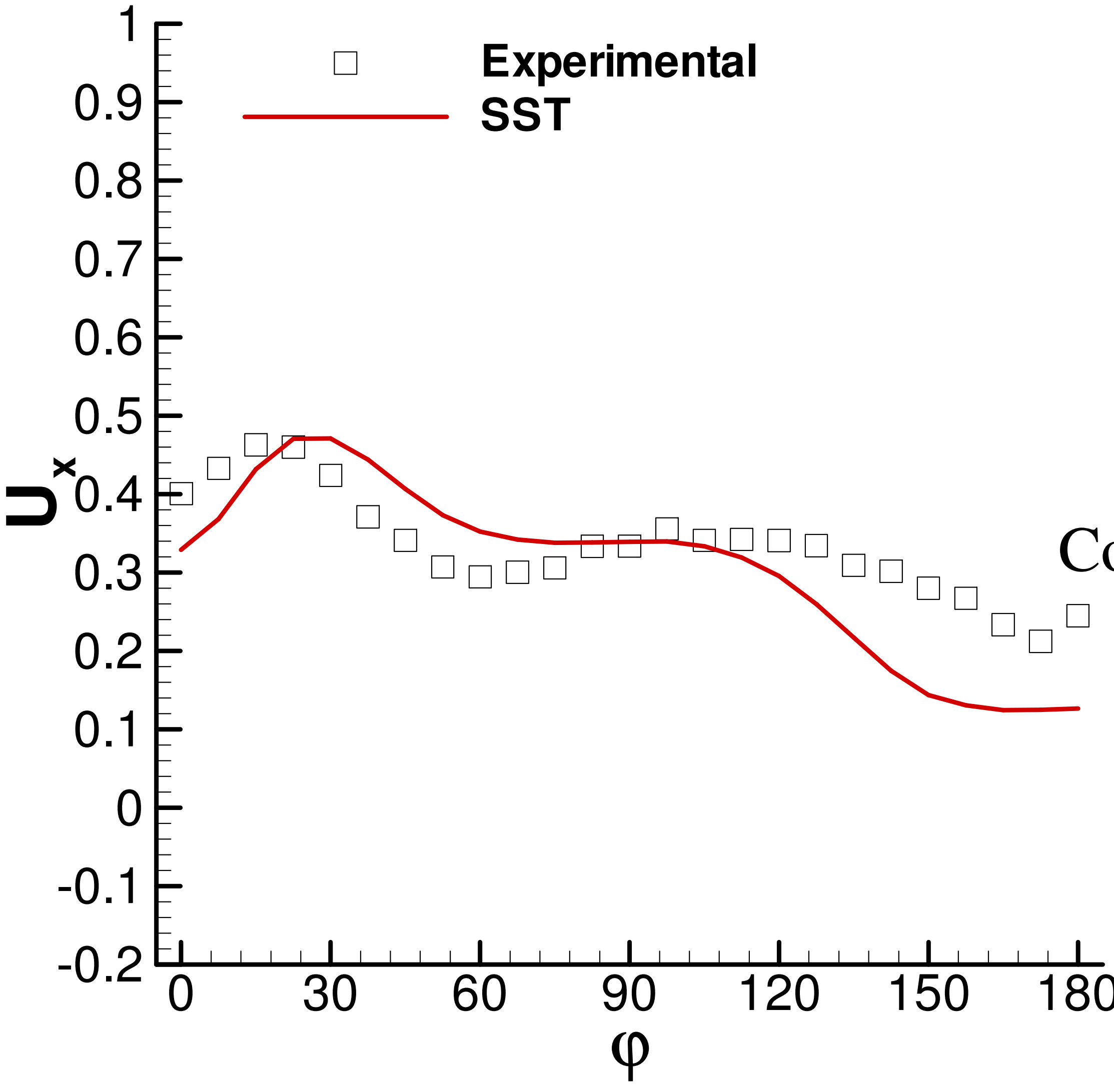
- RANS equations with an eddy-viscosity model

- Parameter uncertainty equal to zero

Validation (ASME V&V 20)

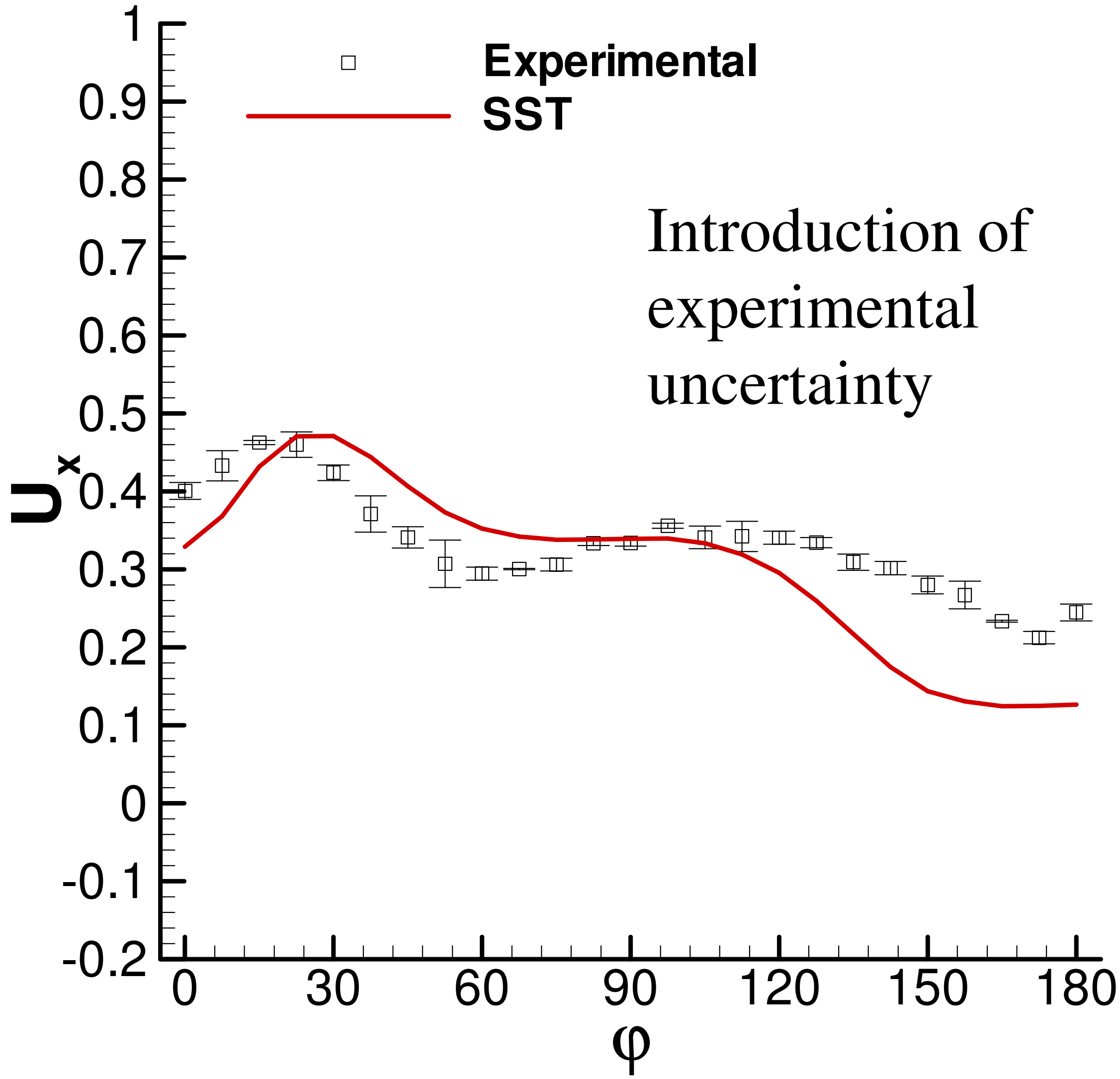


Validation (ASME V&V 20)



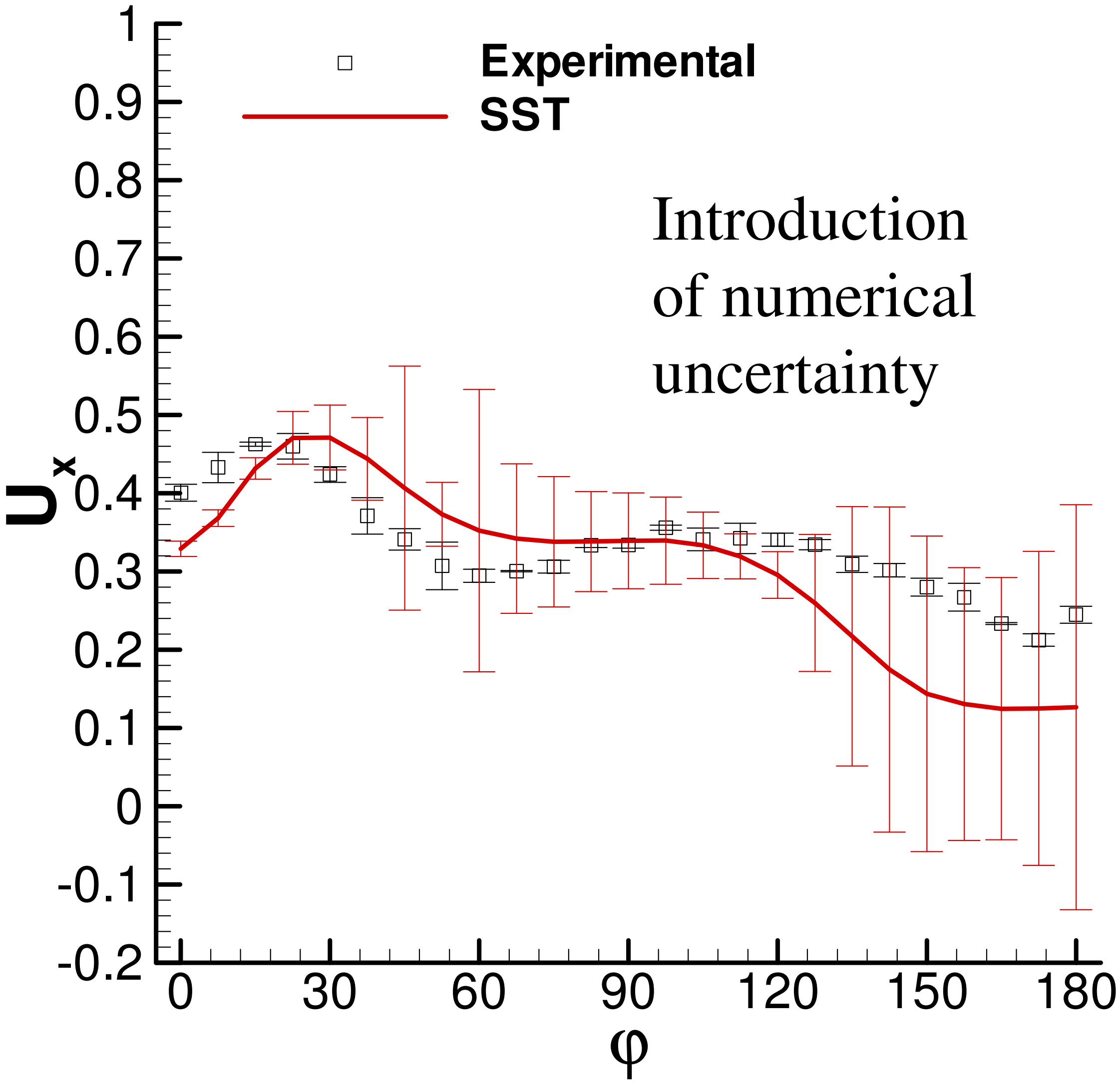
Usual Comparison

Validation (ASME V&V 20)



Introduction of experimental uncertainty

Validation (ASME V&V 20)



Validation (ASME V&V 20)