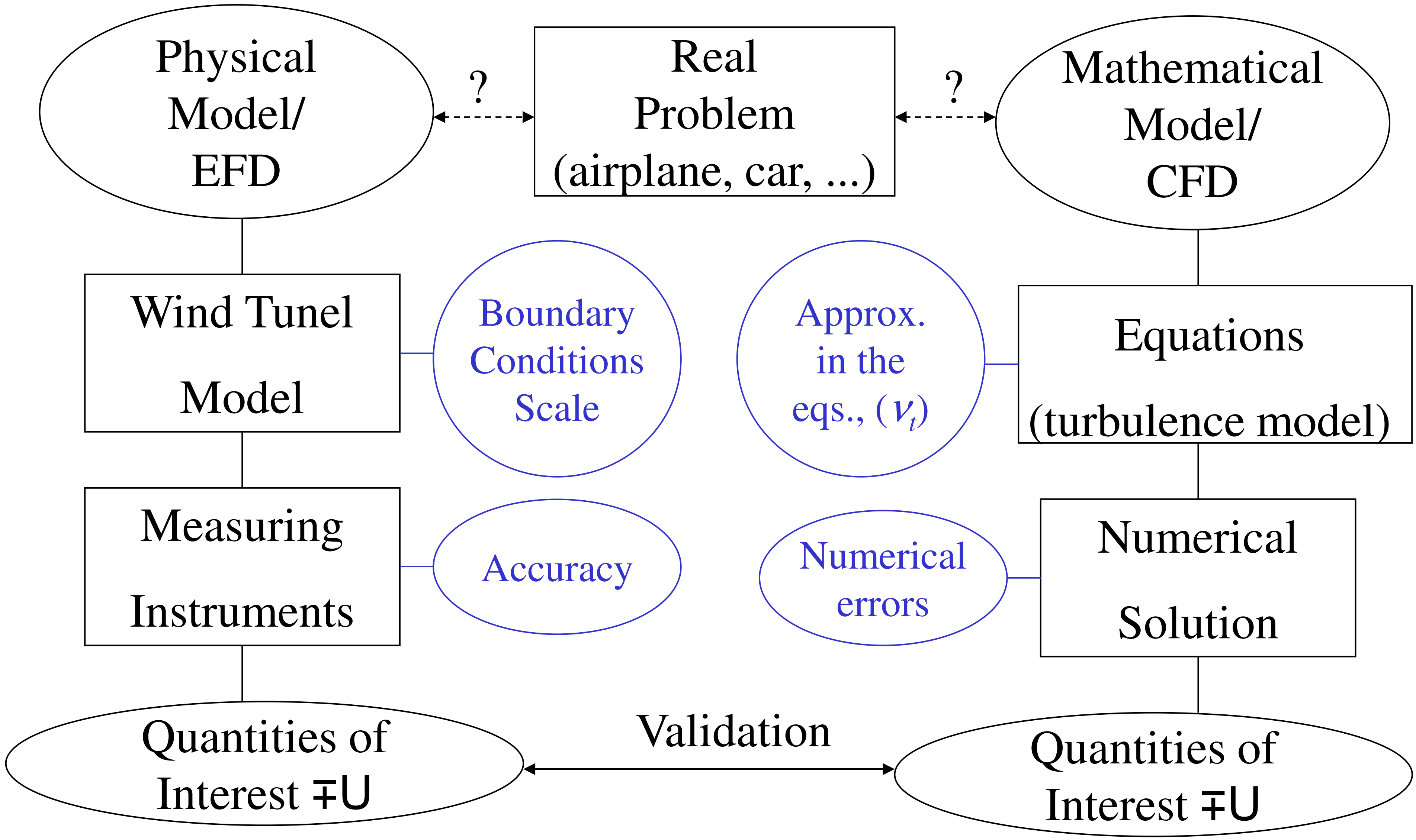
Aerodynamics



- Physical models
  - Scaling of models (Reynolds number inequality)
  - Boundary Conditions
  - Turbulent versus Laminar flow (forced transition)
- Measuring instruments
  - Accuracy
  - Post-processing

- Mathematical models

  - Modelling assumptions
    (ideal fluid, boundary-layer, Reynolds averaging,
    turbulence model,...)

- Numerical Solutions

  - Numerical Errors

  - Post-processing

- Numerical Error

  - Round-off error
    Finite precision of computers

  - Iterative error
    Non linear equations, deferred corrections, iterative solvers...

  - Discretization error
    Geometrical approximations, differentiation, integration...

- Round-off error

  - It is not possible to determine the exact round-off error because that would require a machine with infinite precision

  - Comparison of single, double (or quadruple) precision gives a good estimate of the importance of the iterative error

  - Ill-conditioned problems may be dominated by the iterative error

- Round-off error

  - Polynomial interpolation of order N_{pol}

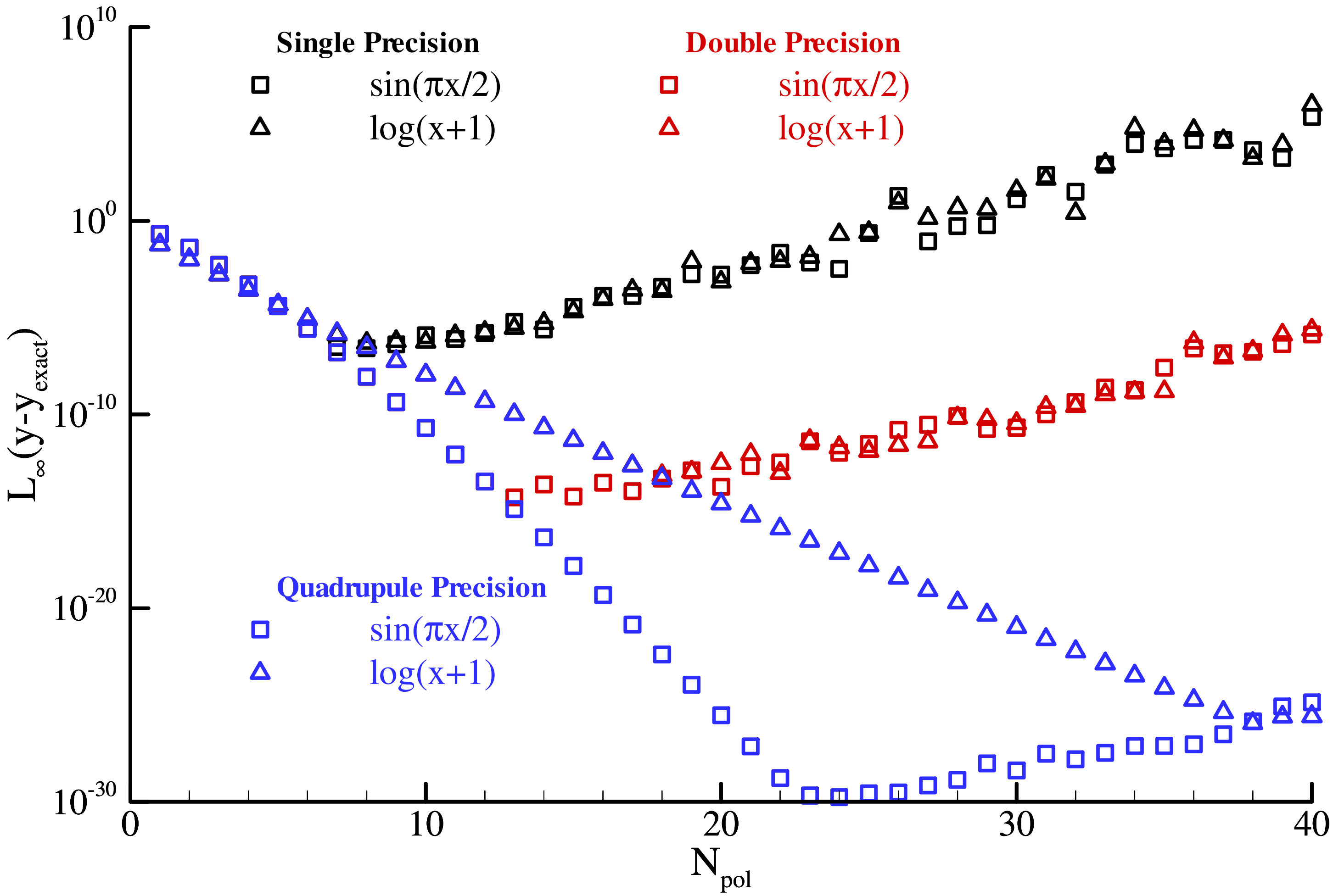
    $$y = a_1 + a_2 x + a_2 x^2 + \ldots + a_{N_{pol}+1} x^{N_{pol}}$$

    Functions $y = \sin(\pi x/2)$ and $y = \ln(x+1)$ in the interval $0 < x < 1$

  - $N_{pol}+1$ points with equidistant grid nodes define a system of $N_{pol}+1$ algebraic equations

  - Maximum interpolation error calculated in the middle of the points that define the polynomial

- Round-off error



- Iterative error

  - Non-linearity of the equations

  - Segregated solutions (turbulence model solved separately from momentum balance)

  - Defered corrections in the discretization procedure (first-order upwind implicit and second-order corrections in the right-hand side)

  - Iterative method in the solution of the linear systems of equations

- Iterative error

  - Minimum level of iterative error is the round-off error

  - A good approximation of the iterative error may be obtained if a given solution is converged to machine accuracy (exact solution for the determination of the iterative error changes with grid refinement)

  - Iterative error is not necessarily equal to change between consecutive iterations or (normalized) residuals

- Iterative error

  - Example for the solution of two non-linear equations:

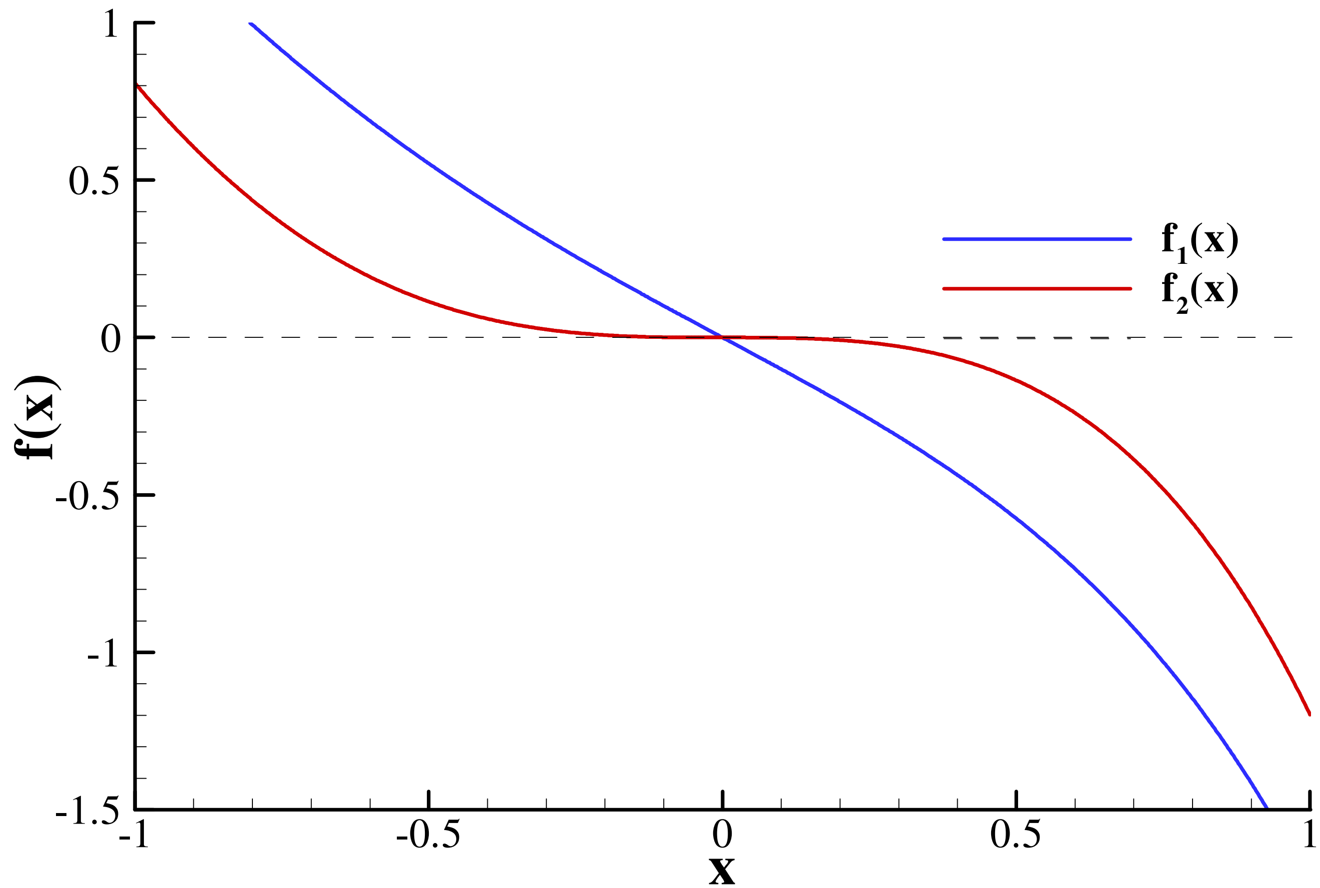
$$f_1(x) = \ln(x^2+1)\sqrt{x^2+1} - x\exp(x) = 0$$

$$f_2(x) = \ln(x^2+1)\sqrt{x^2+1} - x\exp(x) + x\cos(x) = 0$$

  - Both equations have an exact solution at $x = 0$

  - Solutions obtained with an initial guess $x_0 = 1$ with two methods:
    a) Newton-Raphson
    b) Fixed point iteration

- Iterative error



- Iterative error

  - Newton-Rahpson

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)},\ |\Delta x| = \left|\frac{f(x_i)}{f'(x_i)}\right|,\ \mathrm{Res} = |f(x_{i+1})|,\ e_{it} = |x_{i+1}|$$

  - Fixed point iteration

$$f_1(x) = 0 \Rightarrow x_{i+1} = \ln(x_i^2 + 1)\sqrt{x_i^2 + 1} / \exp(x_i)$$

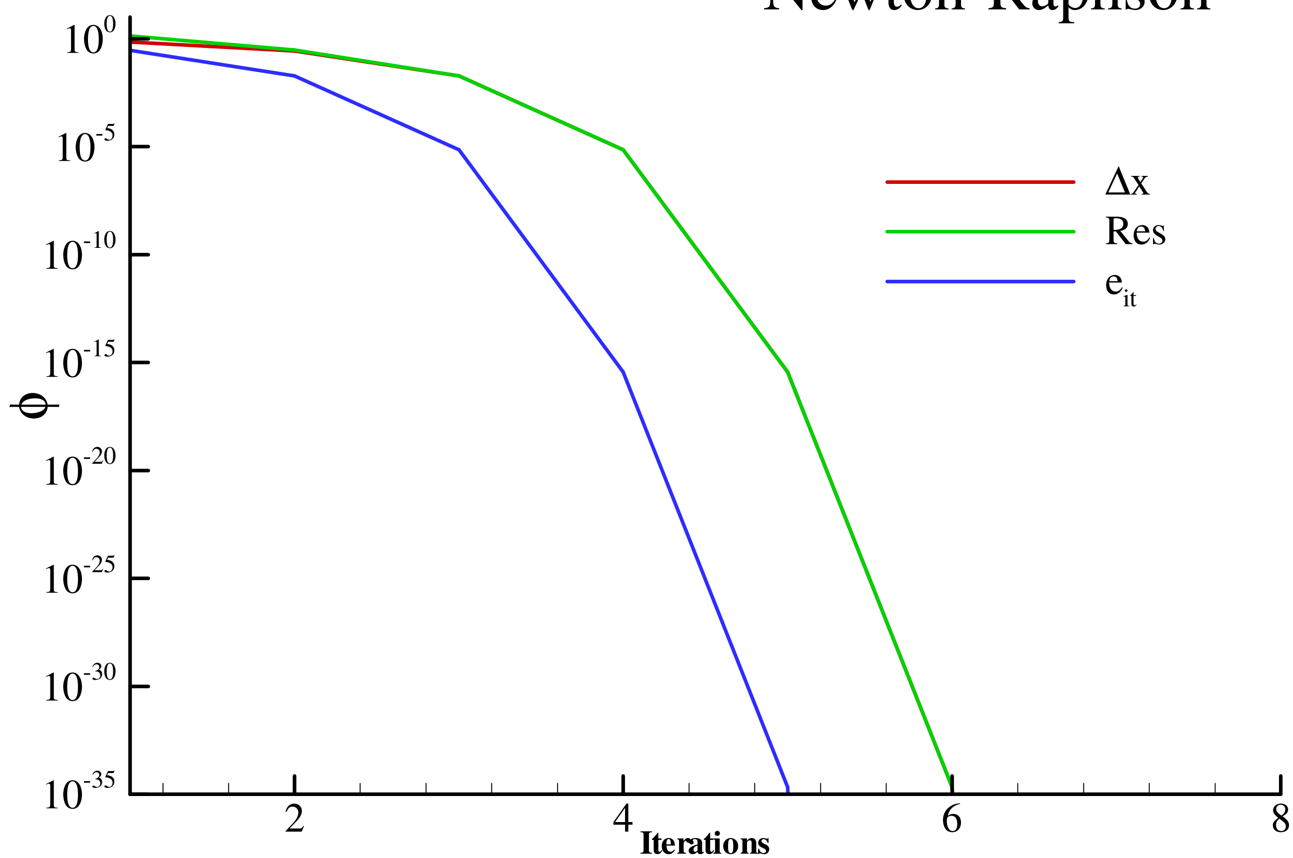
$$f_2(x) = 0 \Rightarrow x_{i+1} = \left(\ln(x_i^2 + 1)\sqrt{x_i^2 + 1} + x_i \cos(x_i)\right) / \exp(x_i)$$

  - Monitoring iterative convergence

$$|\Delta x| = |x_{i+1} - x_i|,\ \mathrm{Res} = |f(x_{i+1})|,\ e_{it} = |x_{i+1}|$$

- Iterative error

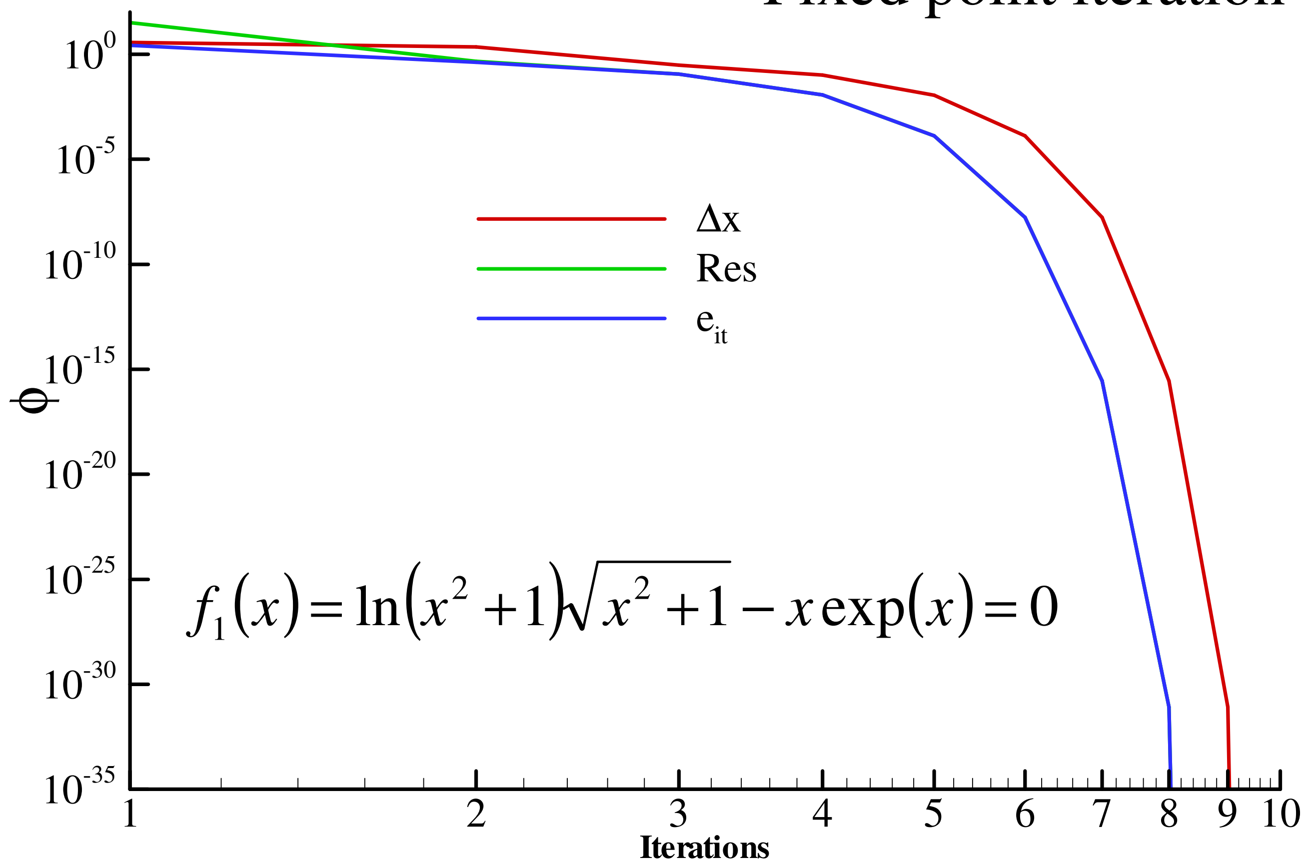
Newton-Raphson



$$f_1(x) = \ln(x^2+1)\sqrt{x^2+1} - x\exp(x) = 0$$

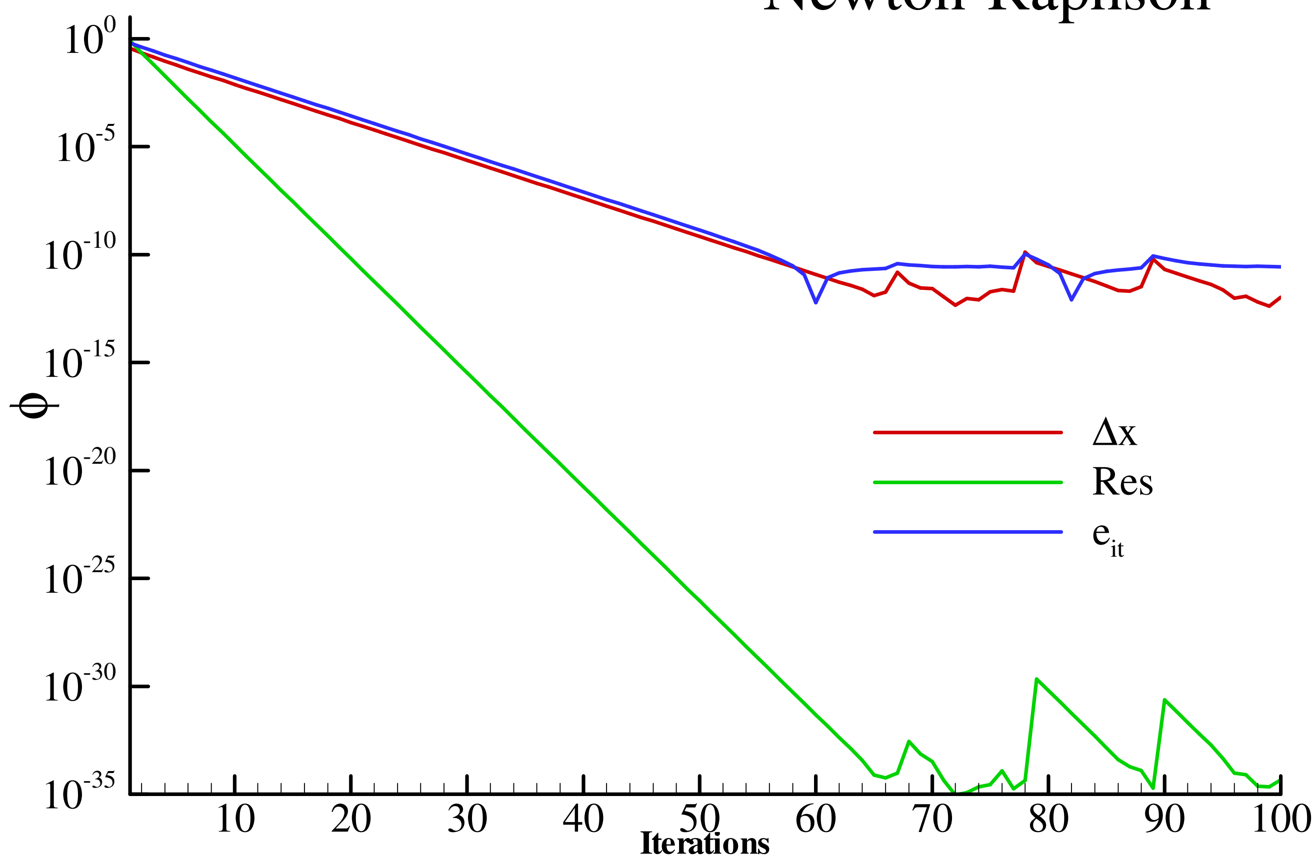
- Iterative error

Fixed point iteration



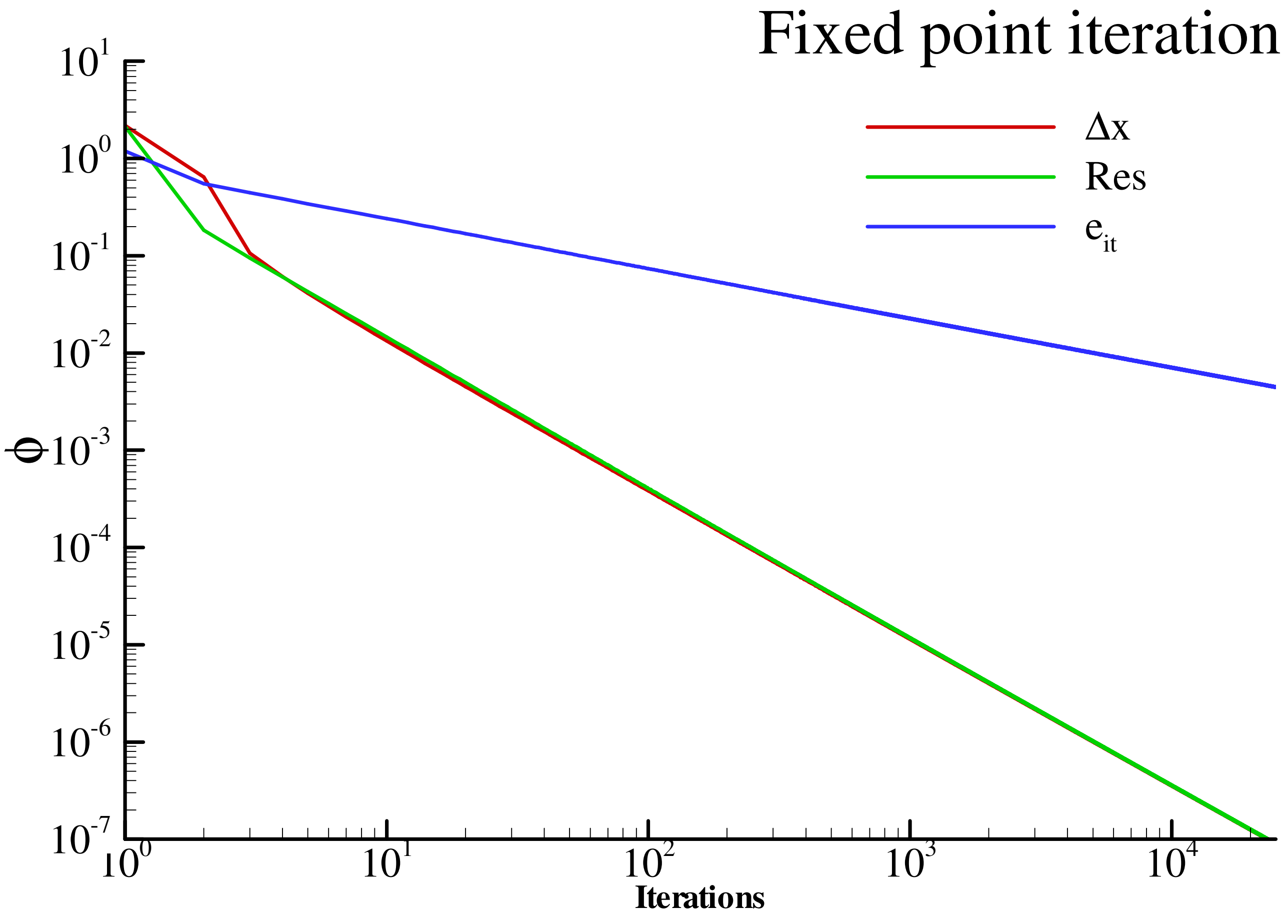
- Iterative error

Newton-Raphson



$$f_2(x) = \ln\left(x^2+1\right)\sqrt{x^2+1} - x\exp(x) + x\cos(x) = 0$$

- Iterative error



$$f_2(x)=\ln\left(x^2+1\right)\sqrt{x^2+1}-x\exp(x)+x\cos(x)=0$$

- Discretization error

  - Consequence of the transformation of the continuum equation(s) into a system of algebraic equations

  - It may have a geometric component, which may even be the dominant contribution in domains bounded by surfaces with high curvature

  - Usually, it is the main contribution to the numerical error

- Discretization error

  - Can only be determined with the knowledge of the exact solution

  - Tends to diminish with the increase of the number of degrees of freedom (grid refinement)

  - Estimate of the discretization error may be obtained from grid refinement studies

  - Iterative and round-off errors should be negligible when compared to the discretization error