TÉCNICO LISBOA

Renewable Energy Sources and Dispersed Power Generation

Chapter 4

Wind Power

Rui Castro

Edition 0 – October 2018

ABOUT THE AUTHOR

Rui Castro is a Professor at the Power Systems Section, Electrical and Computer Engineering Department of Técnico Lisboa (IST) and a researcher at INESC-ID/IST.

He lectures the IST Master's Courses on "Renewable Energy and Dispersed Power Generation" and "Economics and Energy Markets" and the PhD Course on "Renewable Energy Resources". He published two books, one on Renewable Energy and the other on Power Systems (in Portuguese). He has participated in several projects with the industry, namely with EDP group, REN (Portuguese Transmission System Operator) and ERSE (Portuguese Energy Regulator). In the last 3 years, he published more than 20 papers in top international journals, covering topics on renewable energy, impact of PV systems on the LV distribution grid, demand side management, offshore wind farms, energy resource scheduling on smart grids, water pumped storage systems, battery energy storage systems.

More details can be found at his website.

Readers are kindly asked to report any errors in this text to rcastro@tecnico.ulisboa.pt.

LIST OF ACRONYMS

cdf – cumulative distribution function

EWEA – European Wind Energy Association

LCOE – Levelized Cost of Energy

O&M – Operation and Maintenance

pdf – probability density function

RES – Renewable Energy Sources

TSR – Tip Speed Ratio

WTG – Wind Turbine Generator

INDEX

1 Introduction ... 1

2 Basic Concepts ... 4

2.1 Power in the Wind ... 4

2.2 Betz Limit ... 5

2.3 Prandtl Law ... 6

3 Wind Data Analysis ... 9

3.1 Histogram ... 9

3.2 Statistical Analysis ... 10

3.2.1 Weibull distribution ... 10

3.2.2 Rayleigh distribution ... 17

4 The Power Curve ... 19

4.1 Analytic Equations ... 20

4.2 Power Coefficient – C_P ... 21

5 Electricity Production Estimates ... 23

5.1 Complete Models ... 23

5.2 Simplified Models ... 27

5.2.1 Johnson's model ... 27

5.2.2 Fast estimate model ... 30

6 Wind Characteristics and Resources ... 34

7 Wind Turbine Generator Components ... 38

8 Some Features of Modern WTG ... 41

8.1 The Need for Variable Speed Operation ... 41

8.2 Power Control ... 42

1 INTRODUCTION

It goes without saying that wind power plays nowadays a major role in the electricity mix in many countries in Europe and in the World. For instance, in Portugal, in 2017, it accounted for roughly 25% of the electricity demand; in Denmark, for 44% and in Ireland also for 25%.

As for 2017, wind power is the second largest power generating capacity in the EU, as can be seen in Figure 1-1. Wind power capacity increased from 41 GW in 2005 to 169 GW in 2017. Natural gas, with 188 GW, leads the ranking, but it expected that this situation is reversed in the coming years.

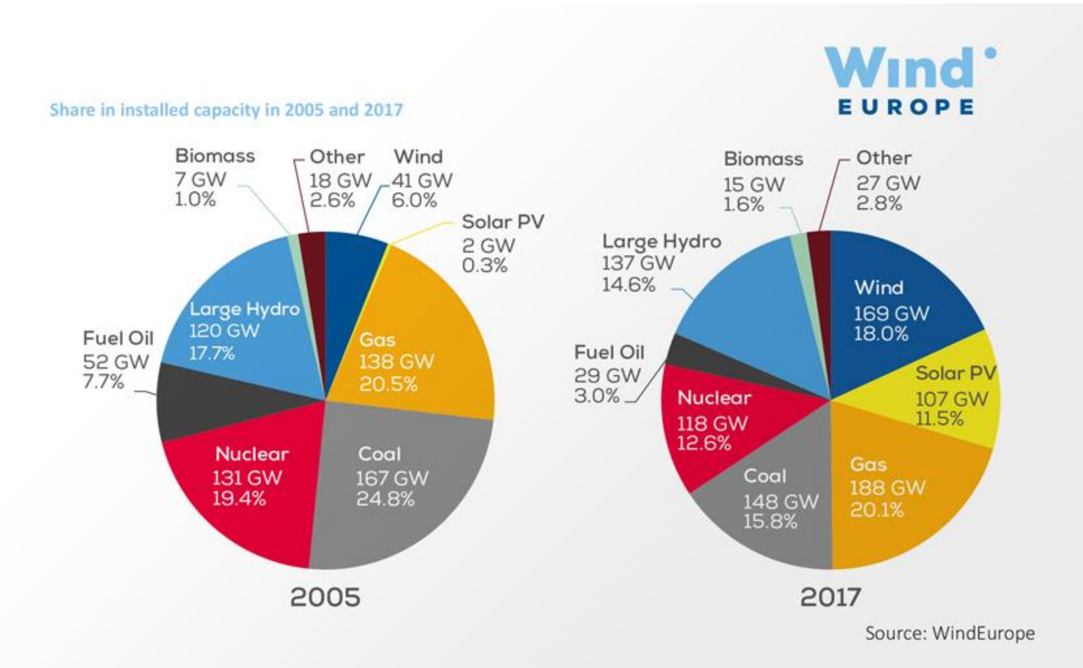

Figure 1-1: Share in installed capacity in the EU in 2005 and 2017; Source: EWEA, 2017.

Regarding 2017 year, wind power accounted for 55% of the total new installed capacity in the EU, followed by solar PV with 21%, these two Renewable Energy Sources (RES) totaling more than 75%. Complete figures are shown in Figure 1-2.

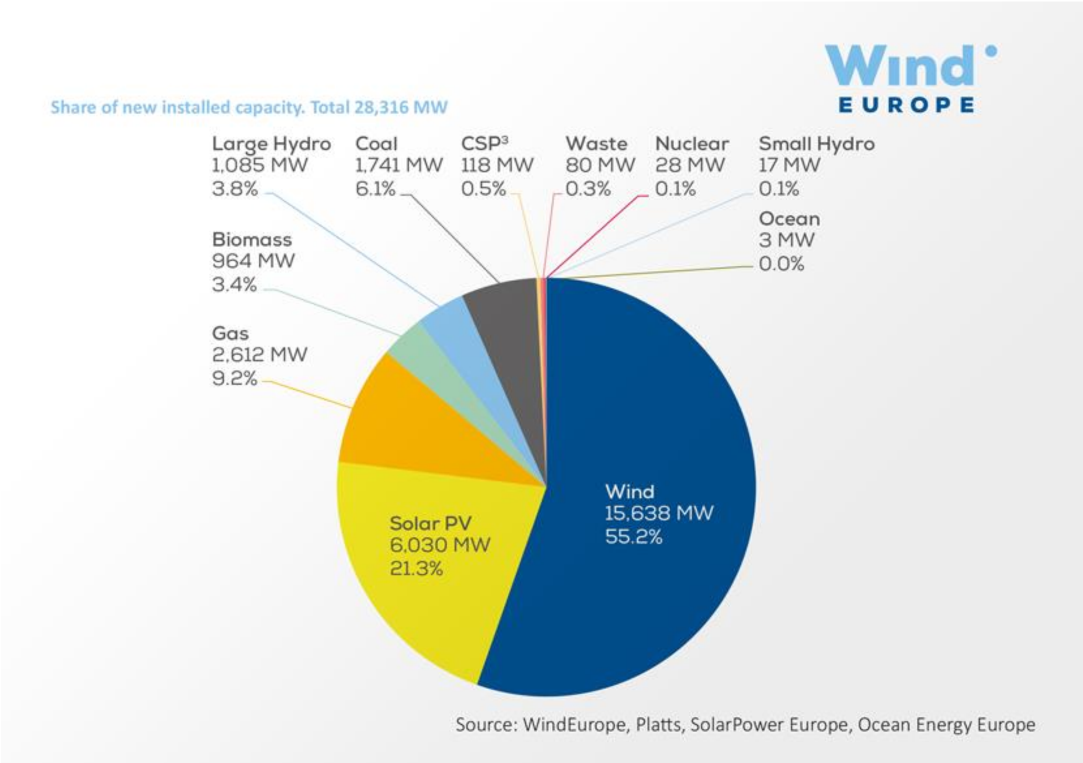

Figure 1-2: Share of new installed capacity in the EU in 2017; Source: EWEA, 2017.

These are very impressive figures, that clearly show the importance of wind power in nowadays power generating mix.

Power from the wind is captured using horizontal axis Wind Turbine Generators (WTG), usually grouped together in the popular wind parks (Figure 1-3), with tens or hundreds of installed MW. Most of the installed capacity is onshore: in the EU, from the grand total of installed 169 GW, only 16 GW (slightly less than 10%) are offshore. Offshore wind power offers tremendous potential, but the associated costs are still higher than onshore. The UK is the country with more offshore wind power installed capacity, reaching a cumulative capacity of almost 7 GW and more than 1700 offshore WTG; Germany follows with 5 GW and 1100 WTG.

WTG are as tall as more than 100 m and have a turbine composed by three 60 m long blades. The kinetic energy in the wind is used to spin the blades, which turn a shaft connected to the generator, thereby producing electricity.

Figure 1-3: Onshore wind park; Source: . https://www.power-technology.com/projects/galway-wind-park/

In Portugal, the average utilization factor usually lies between 2300 and 2500 h, which corresponds to 26% and 28% capacity factor, respectively. This means that, in average, WTG are able to produce about 27% of the maximum theoretical energy they can produce.

Many studies point to onshore wind power being nowadays the cheapest power generating technology, with a Levelized Cost of Energy (LCOE) around 45 €/MWh. This figure can be easily obtained using typical values for the current investment (about 1.1€/W), O&M (1%), lifetime (20 years), capacity factor (25%) and discount rate (5%).

WTG capacity has increased over time. In 1985, typical turbines had a rated capacity of 500 kW and a rotor diameter of 15 m. Today's new wind power projects have turbine capacities of about 2–3 MW onshore and 4–6 MW offshore, but there are already commercially available wind turbines with 8 MW capacity, with rotor diameters of up to 160 m.

2 BASIC CONCEPTS

2.1 POWER IN THE WIND

Consider a volume of air with mass, m (kg), crossing a disk of air with area, A (m^2) at constant speed, u (m/s), the thickness of the volume of air being x (m), as in Figure 2-1.

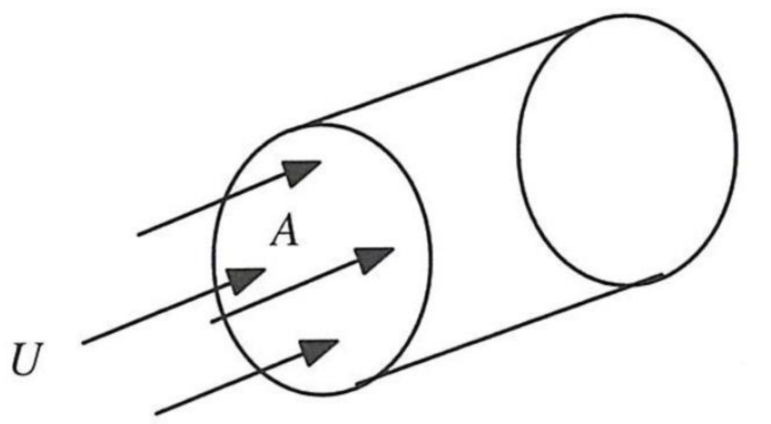

Figure 2-1: Volume of air crossing a section of an air disk at constant speed.

The associated kinetic energy is given by:

$$E_{kin} = \frac{1}{2} m u^2 = \frac{1}{2}(\rho A x) u^2 \qquad \text{equation 2.1}$$

where ρ is the air density (ρ=1.23 kg/m^3, θ=15ºC). The identity $m = \rho A x$ can be easily confirmed by a dimensions analysis.

Bearing in mind the relationship between power and energy, the available power in the wind is:

$$P_{avail} = \frac{dE_{kin}}{dt} = \frac{1}{2}\left(\rho A u^2\right)\frac{dx}{dt} \qquad \text{equation 2.2}$$

which can be written as:

$$P_{avail} = \frac{1}{2} \rho A u^3 \qquad \text{equation 2.3}$$

This result shows that the power available in the wind changes with the cube of the wind speed, which is a strong relationship. If the wind speed doubles, the available power is multiplied by 8, but if the area swept by the wind turbine doubles, the available power is also multiplied by 2. On the other side, if the wind speed is reduced to one half, the available power is only 12.5%.

2.2 BETZ LIMIT

Let us assume that the average wind speed through the rotor of a wind turbine is the average between the wind speed before the turbine (non-perturbed wind speed), u_1, and the wind speed after the turbine (wake wind speed), u_2. The mass flow rate is therefore:

$$\dot{m} = \frac{dm}{dt} = \rho A \frac{dx}{dt} = \rho A \frac{u_1 + u_2}{2} \quad \text{equation 2.4}$$

The power extracted from the wind by the turbine's rotor is:

$$P_{ext} = \frac{1}{2} \dot{m} \left(u_1^2 - u_2^2 \right) \quad \text{equation 2.5}$$

Manipulating equation 2.5 and taking into account equation 2.4, one can write:

$$P_{ext} = \frac{1}{4} \rho A u_1 \left(1 + \frac{u_2}{u_1} \right) u_1^2 \left(1 - \frac{u_2}{u_1} \right) \quad \text{equation 2.6}$$

Dividing equation 2.6 by equation 2.2, one obtains:

$$\frac{P_{ext}}{P_{avail}} = \frac{1}{2} \left(1 + \frac{u_2}{u_1} \right) \left[1 - \left(\frac{u_2}{u_1} \right)^2 \right] \quad \text{equation 2.7}$$

Figure 2-2 displays the plotting of equation 2.7, i.e. P_{ext}/P_{avail} vs. u_2/u_1. It is possible to verify that the maximum occurs at the point (1/3; 16/27), meaning that the maximum power that theoretically can be extracted from wind is 59.3% (16/27) of the available power. This is the maximum ratio between the mechanical power output at the ideal turbine's rotor and the wind power input. This value is known as Betz

Limit[1] and represents the maximum theoretical efficiency of a wind turbine. Currently, modern wind turbines can extract around 40% of the available power, which is more than 80% of the theoretical maximum.

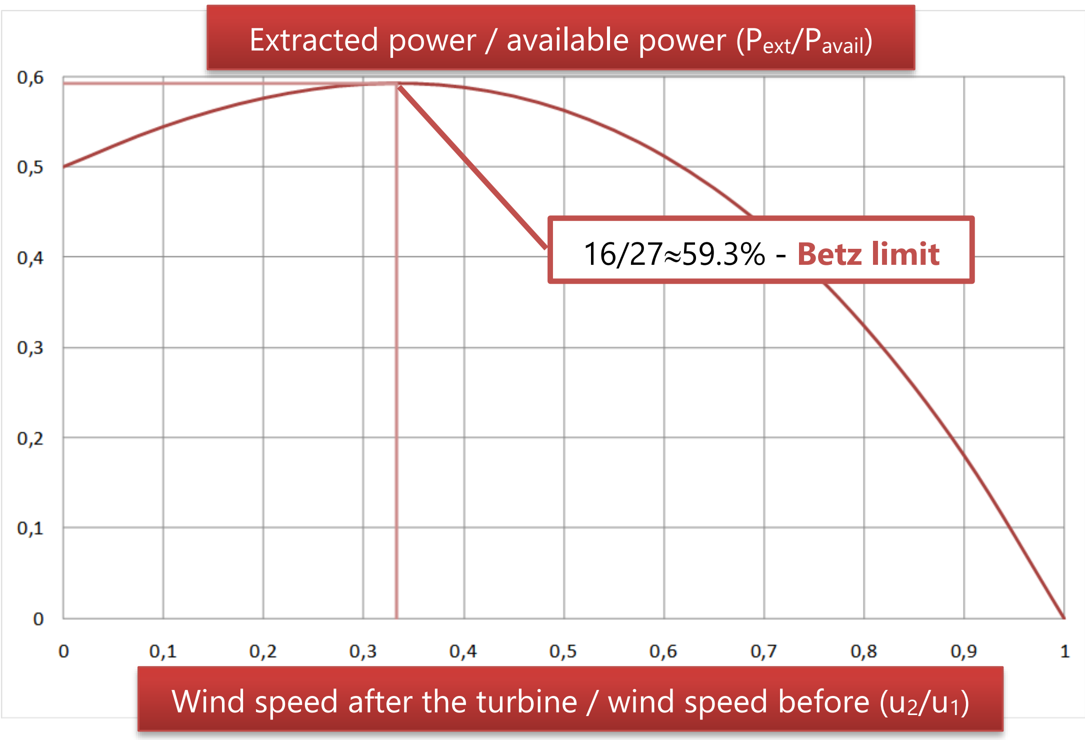

Figure 2-2: Extracted power / available power vs wind speed after the turbine / wind speed before.

2.3 Prandtl Law

The atmospheric surface layer, the lowest part of the atmospheric boundary layer, is the region of interest, as far as the wind turbines are concerned. In this region, the wind speed profile is strongly dependent on the topography of the terrain and on the roughness of the soil. The so-called log wind profile is considered to be valid in the surface layer, the wind speed at height *h*, being given by:

$$u(z) = \frac{u_*}{k} \ln\left(\frac{z}{z_0}\right) \quad \text{equation 2.8}$$

where u_* is the friction velocity (m/s), k is the Von Karman constant (~0.41), and z_0 is the surface roughness length (m), i.e. a measure of the texture of the soil.

1 It was published in 1919, by the German physicist Albert Betz.

u_* is difficult to compute, as it depends on so many factors. The common use of equation 2.8 is to be able to compute the wind speed at a certain height, z, knowing its value at a measuring height, z_R (reference height). As so, the difficulty of u_* computation disappears, as can be seen in equation 2.9 (Prandtl Law).

$$\frac{u(z)}{u(z_R)} = \frac{\ln\left(\frac{z}{z_0}\right)}{\ln\left(\frac{z_R}{z_0}\right)} \qquad \text{equation 2.9}$$

where $u(z_R)$ is the wind speed measured at a reference height.

We note that equation 2.9 is valid for flat and homogeneous lands, and do not include topographic effects, obstacles, and roughness length changes. Therefore, its application should be used with caution.

A frequent and useful application of Prandtl law occurs when one intends to determine the wind speed at the turbine's rotor height, which is essential for calculating the energy that can be captured by a wind turbine, from the records of wind speed measuring devices (called anemometers), usually installed at a lower height.

The issue with the use of equation 2.9 to this purpose is to determine the proper value of the roughness length, z_0. There are many tables with typical values of z_0. Table 2-1 illustrates one of the most used tables in wind potential assessment studies, for instance followed in the preparation of the "European Wind Atlas". As it can be seen, the land is divided into characteristic classes.

However, this is a very subjective way of working. To overcome this difficulty, two anemometers, mounted at two different heights (z_1 and z_2), can be installed. Then, the unknown is z_0 and equation 2.9 becomes:

$$\frac{u(z_1)}{u(z_2)} = \frac{\ln\left(\frac{z_1}{z_0}\right)}{\ln\left(\frac{z_2}{z_0}\right)}$$ equation 2.10

The solution of equation 2.10 is:

$$z_0 = \exp\left(\frac{u(z_1)\ln(z_2) - u(z_2)\ln(z_1)}{u(z_1) - u(z_2)}\right)$$ equation 2.11

This way, the value of z_0 can be computed.

Table 2-1: Roughness length according to land characteristics; Source: https://barani.biz/apps/wind-height/.

Roughness Class RC	Roughness Length, Z0 (m)	Energy Index (%)	Local Terrain Type, Landscape, Topography, Vegetation
0	0.0002	100	Water surface.
0.5	0.0024	73	Completely open terrain with a smooth surface, such as concrete runways in airports, mowed grass.
1	0.03	52	Open agricultural area without fences and hedgerows and very scattered buildings. Only softly rounded hills.
1.5	0.055	45	Agricultural land with some houses and 8 meter tall sheltering hedgerows within a distance of about 1250 meters.
2	0.1	39	Agricultural land with some houses and 8 meter tall sheltering hedgerows within a distance of about 500 meters.
2.5	0.2	31	Agricultural land with many houses, shrubs and plants, or 8 meters tall sheltering hedgerows within a distance of about 250 meters
3	0.4	24	Villages, small towns, agricultural land with many or tall sheltering hedgerows, forests and very rough and uneven terrain.
3.5	0.8	18	Larger cities with tall buildings.
4	1.6	13	Very large cities with tall buildings and sky scrapers.

3 WIND DATA ANALYSIS

Wind data as given by the anemometers and further extrapolated to the rotor height needs preparation to be useful in wind potential assessments. There are two methods used in wind data analysis: the histogram (or method of bins) and statistical analysis.

3.1 HISTOGRAM

Data is separated in data intervals (bins or classes) of width 1m/s, each one with f_j occurrences. The starting point is a file with 8760 values of hourly average wind speeds computed for the rotor height. Then, for instance, a wind speed of 6.5 m/s belongs to the class of 7 m/s; a wind speed of 8.38 m/s belongs to the class of 8 m/s, and so on, as seen in Table 3-1.

Table 3-1: Distribution of hourly average wind speeds by classes.

h	u	classe
8679	6,50	7
8680	8,38	8
8681	9,75	10
8682	9,76	10
8683	10,98	11
8684	10,64	11
8685	11,76	12
8686	13,72	14
8687	15,42	15
8688	16,94	17

Based on the distribution by wind speed classes, a histogram can be built, counting the number of occurrences of each class. An example of such a wind speed histogram is shown in Figure 3-1.

Referring to Figure 3-1, wind speeds laying between 7.5 and 8.4 m/s occur 16% of the year (about 1400 h). This means that the probability of the wind speeds being between 7.5 and 8.4 m/s is 16%. Nevertheless, in common language, we say that the probability of the wind speed being equal to 8 m/s is 16%. We know that this is not correct, but we often use this formulation.

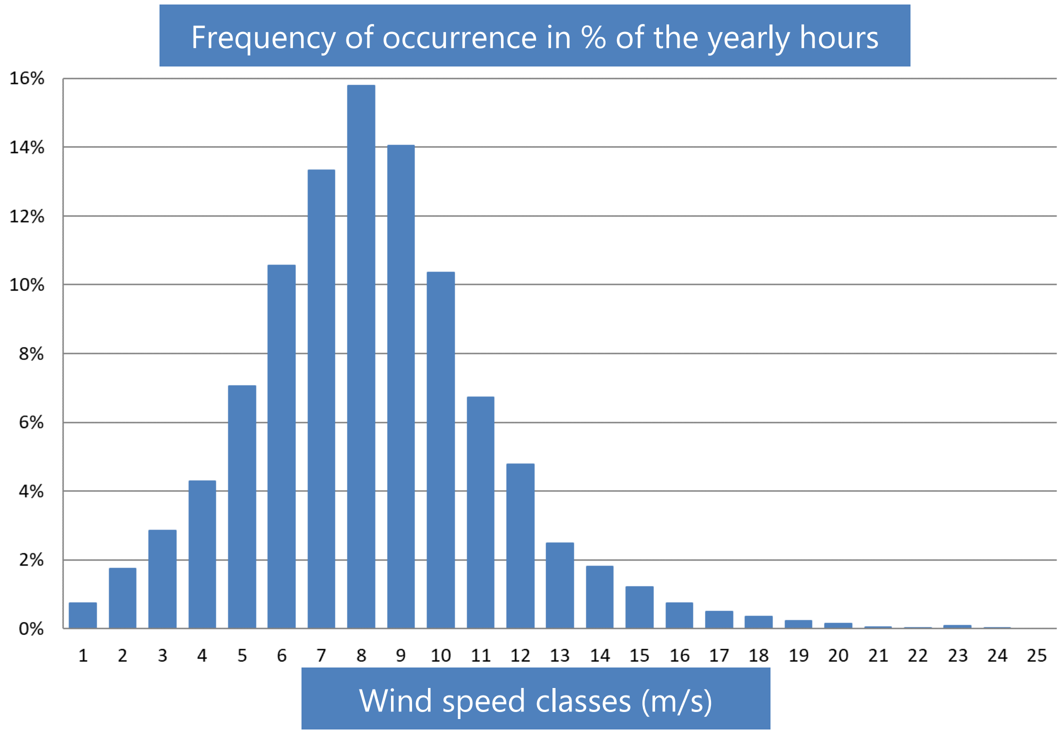

Figure 3-1: Example of a wind speed histogram.

3.2 Statistical Analysis

Raw wind speed data gain importance if it can be described by analytic functions. Approximation by analytic functions, of probability density type, allows to wide-spread the framework of the studies, or projections, or forecasts, that can be made. Weibull distribution is the probability distribution that is normally used to describe wind speed profiles. If preliminary assessment is intended, the simpler Rayleigh distribution may be used.

3.2.1 Weibull distribution

The Weibull probability density function (pdf), *f*(*u*), is given by:

$$f(u) = \frac{k}{c}\left(\frac{u}{c}\right)^{k-1} \exp\left\{-\left[\left(\frac{u}{c}\right)^{k}\right]\right\} \qquad \text{equation 3.1}$$

where u is the wind speed, c>0 (m/s) is a scale parameter and k>0 is a shape parameter.

Mean annual wind speed and variance

The mean annual wind speed is:

$$u_{ma} = \int_0^\infty u\, f(u)\, du \qquad \text{equation 3.2}$$

The mean annual wind speed and the wind variance can be related to the Weibull parameters *k* and *c* through the *Gamma* function Γ[2]:

$$u_{ma} = c\Gamma\left(1+\frac{1}{k}\right) \qquad \text{equation 3.3}$$

$$\sigma^2 = c^2\left[\Gamma\left(1+\frac{2}{k}\right) - \left(\Gamma\left(1+\frac{1}{k}\right)\right)^2\right] \qquad \text{equation 3.4}$$

Figure 3-2 shows the plotting of $\Gamma\left(1+\frac{1}{k}\right)$ as a function of Weibull *k* parameter.

When the Weibull distribution is used to describe the variation of the wind speed, the *k* parameter usually fluctuates between 1.5 and 2.5. From the plotting of Figure 3-2, one can conclude that for $k \in [1.4;3.0]$, then $\Gamma\left(1+\frac{1}{k}\right) \approx 0.9$, that is, *Gamma* function computed in the point $\left(1+\frac{1}{k}\right)$ is always close to 0.9. For $k = 2$, it is $\Gamma\left(1+\frac{1}{2}\right) = \sqrt{\frac{\pi}{4}} = 0.8862$.

Therefore, the following approximation is, in general, valid:

$$u_{ma} = 0.9c \qquad \text{equation 3.5}$$

The close relationship between mean annual wind speed and Weibull *c* parameter is apparent from equation 3.5.

[2] The *Gamma* function can be accessed in Excel® with the command EXP(GAMMALN(x)) and in MATLAB® through gamma(x).

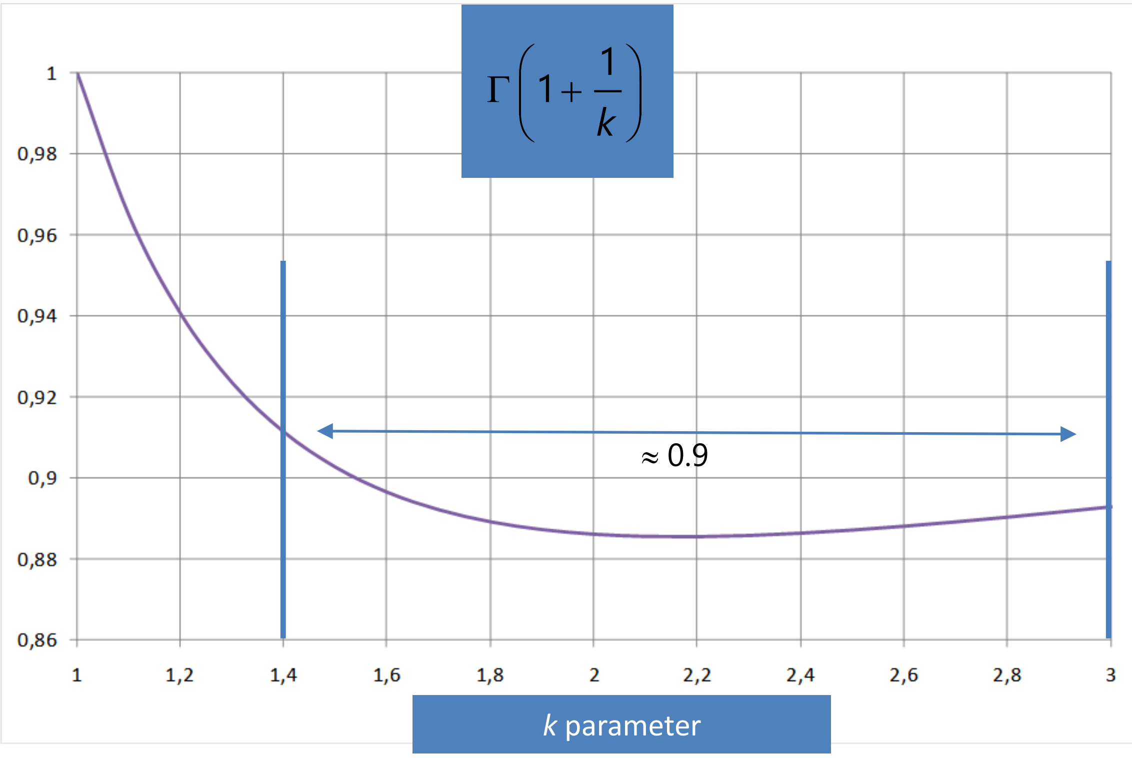

Figure 3-2: Gamma function vs Weibull k parameter.

<u>Variation of *k* and *c*</u>

Figure 3-3 displays the plotting of the Weibull pdf (equation 3.1), for 3 different *k* values, with constant *c* = 8 m/s, as shown in the table. It can be seen that the standard deviation is the quantity mainly affected by the *k* parameter variation. The standard deviation is a measure of wind speed data scattering.

Figure 3-4 displays the plotting of the Weibull pdf for 3 different *c* values, with constant *k* = 2.3, as shown in the table. It can be seen that the mean annual wind speed is greatly affected by the variation of *c* parameter, as seen before. However, the variation of the standard deviation is not to be neglected too.

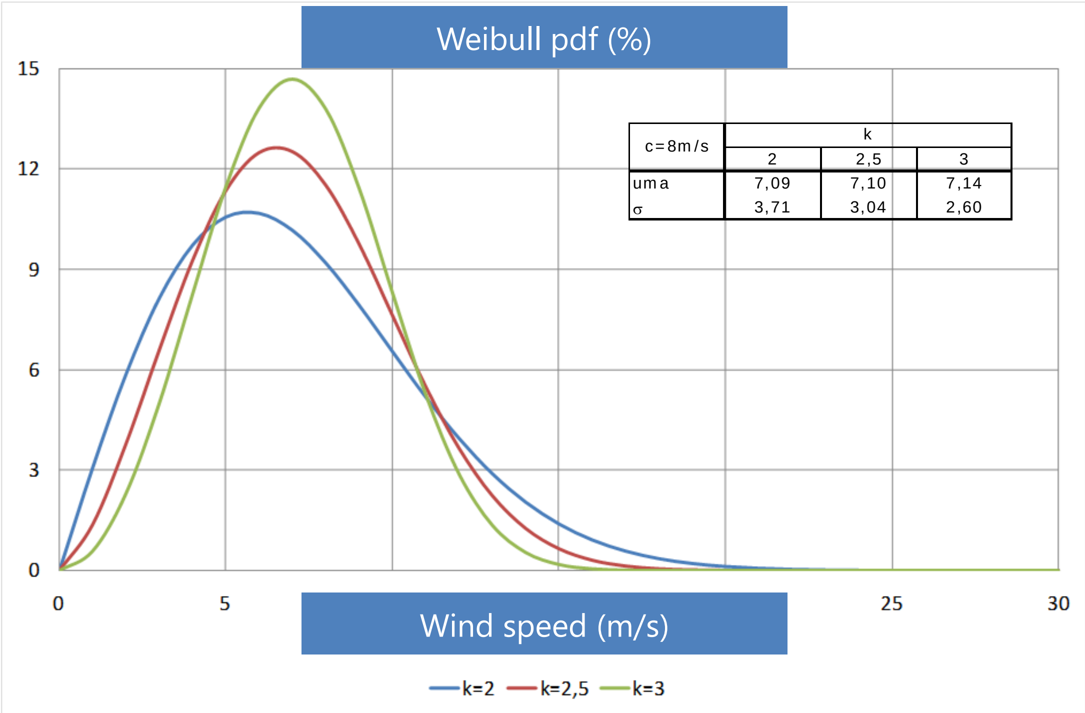

Figure 3-3: Weibull pdf vs wind speed, for changing k and constant c.

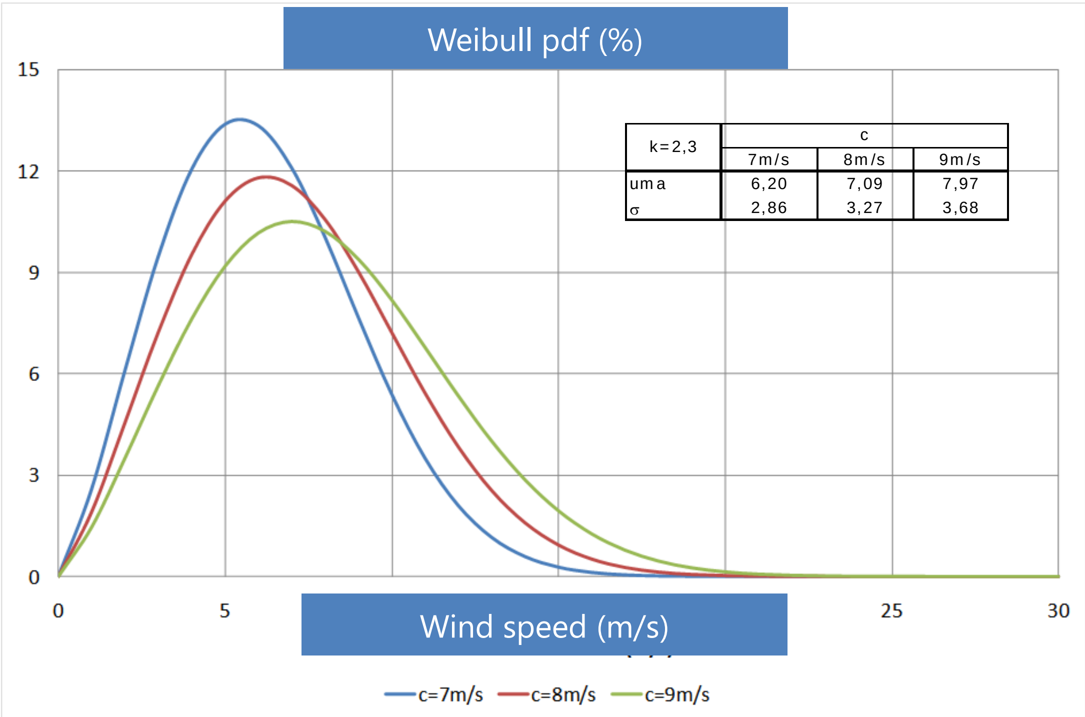

Figure 3-4: Weibull pdf vs wind speed, for changing c and constant k.

k and *c* parameters in Portugal

In Figure 3-5 we show the *c* (left) and *k* (right) parameters in Portugal, at 60 m height.

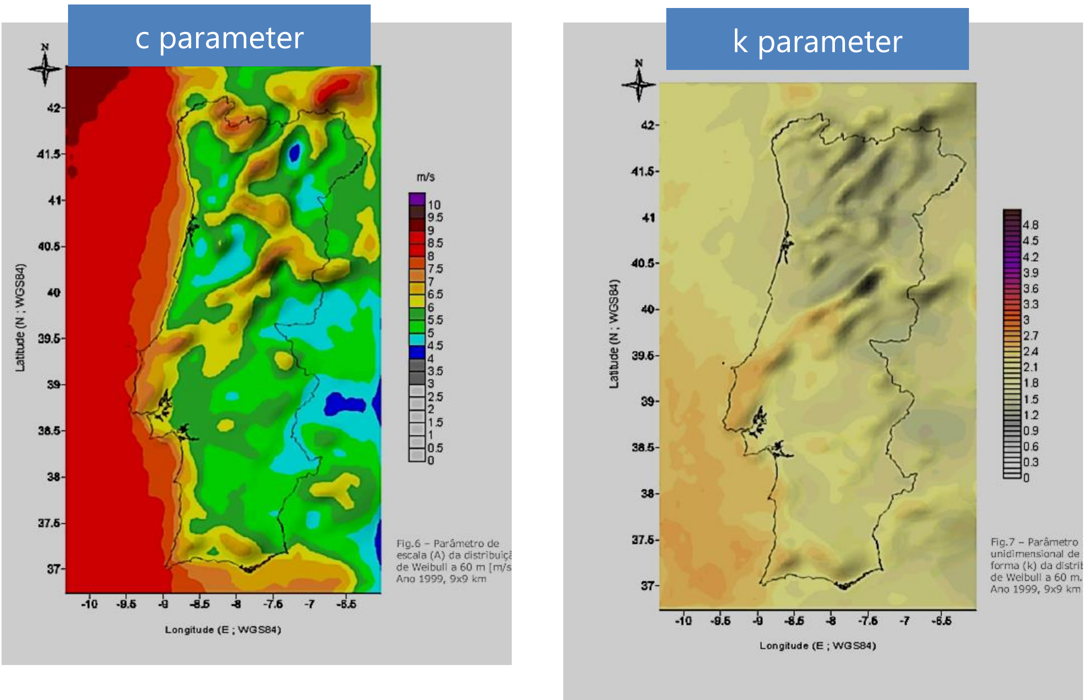

Figure 3-5: Weibull c (left) and k (right) parameters in Portugal, at 60 m height; Source: LNEG, www.lneg.pt.

As mentioned before, the *k* parameter is changing between 1.5 and 3.0, the central values being typical.

<u>Weibull cdf</u>

The cumulative distribution function, *F*(*u*), is defined, for wind power studies, as the probability of the wind speed to exceed a particular value. Therefore:

$$F(u) = 1 - \int_{-\infty}^{u} f(u)\, du = 1 - \int_{0}^{u} f(u)\, du \qquad \text{equation 3.6}$$

and:

$$f(u) = -\frac{dF(u)}{du} \qquad \text{equation 3.7}$$

The application to the Weibull distribution case leads to:

$$F(u) = \exp\left[-\left(\frac{u}{c} \right)^{k} \right] \qquad \text{equation 3.8}$$

In Figure 3-6, it is shown the Weibull pdf together with the Weibull cdf for k = 1.545 and c = 7.45 m/s. The cdf values are to be read on the left scale and the pdf values on the right one.

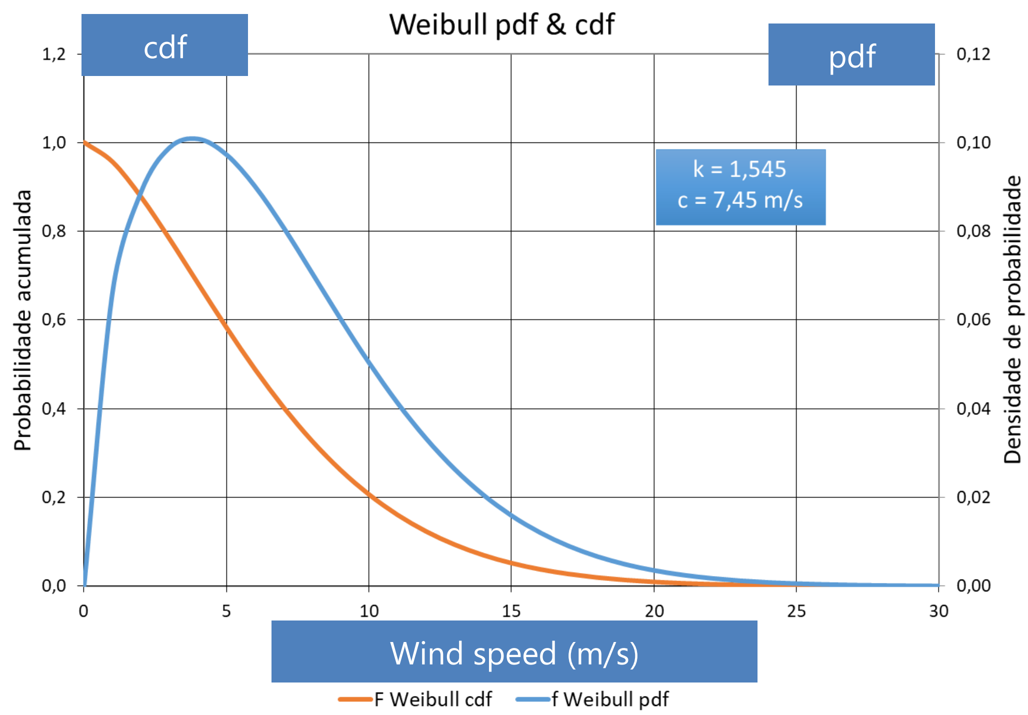

Figure 3-6: Weibull pdf (blue line; right scale) and cdf (orange line; left scale) vs wind speed; k = 1.545; c = 7.45 m/s.

Weibull distribution estimation parameters

A simple method to estimate the *k* and *c* parameters of an equivalent Weibull distribution is based on the cdf and is next presented.

Let us begin by applying twice logarithm to equation 3.8, the Weibull cdf. We obtain:

$$\ln[F(u)] = -\left(\frac{u}{c}\right)^k \qquad \text{equation 3.9}$$

$$\ln\{-\ln[F(u)]\} = \ln\left(\frac{u}{c}\right)^k = k\ln(u) - k\ln(c) \qquad \text{equation 3.10}$$

It should be noted that equation 3.10 is a linear function that can be written in the form:

$$Y = AX + B \qquad \text{equation 3.11}$$

in which:

$$\begin{aligned} Y &= \ln\{-\ln[F(u)]\} \\ X &= \ln(u) \end{aligned} \qquad \text{equation 3.12}$$

The Weibull k and c parameters are related to the slope and intercept of equation 3.11 and are given by (compare equation 3.10 and equation 3.11):

$$\begin{aligned} k &= A \\ c &= \exp\left(-\frac{B}{A}\right) \end{aligned} \qquad \text{equation 3.13}$$

A practical use of this method is when we know a wind speed cdf, $F(u)$, for a particular site, and it is not a Weibull distribution, but something else. So, we give values to the wind speed u and obtain several points (X_i,Y_i) (using equation 3.12) in the plane (X,Y). As $F(u)$ is not a Weibull cdf, a straight-line will not be obtained; if it is, off course a straight-line is directly obtained. Then, we compute the slope and intercept (using least-squares method, for instance) of the straight-line that best fit the obtained points. Finally, the parameters k and c of the Weibull distribution that best fit the original data are obtained through equation 3.13.

A more accurate method to compute the k and c parameters of the Weibull pdf that best fit a known experimental histogram is to use ExcelSolver®. This tool has a built-in optimization algorithm that seeks for the best fit k and c, by minimizing the mean square error between the iteratively computed Weibull pdf and the original histogram. A guess on the initial values is to be provided by the user.

An example of a practical result of ExcelSolver® application is depicted in Figure 3-7. The green line is the Weibull pdf that best approximates the known experimental histogram.

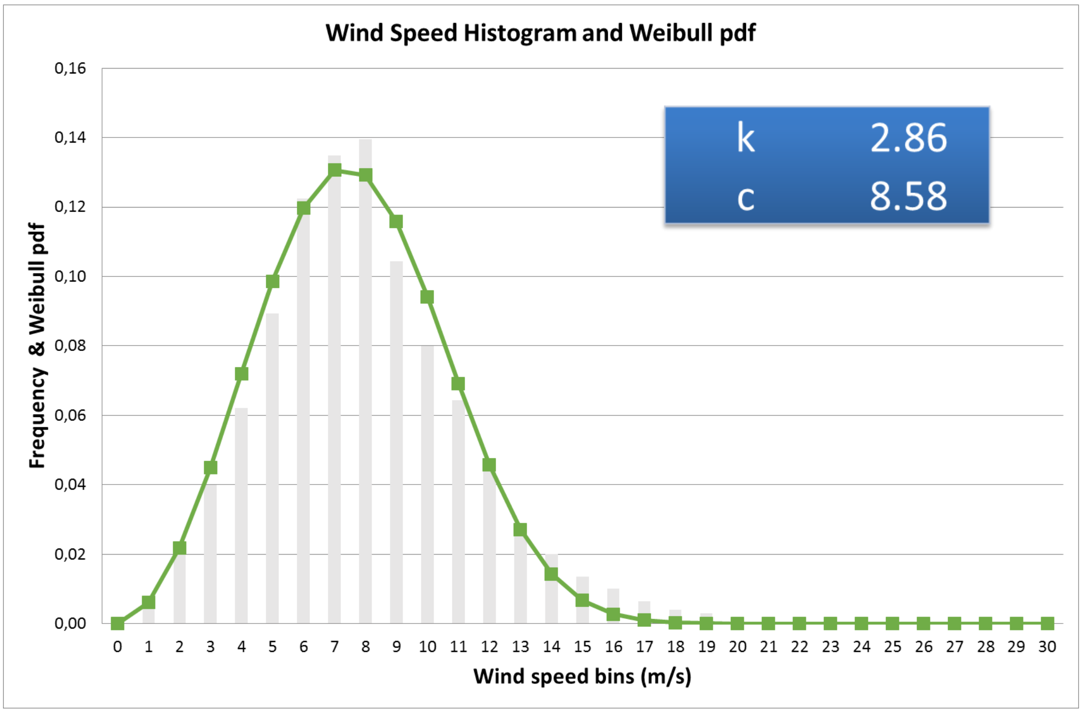

Figure 3-7: Experimental histogram (gray bars) and ExcelSolver® computed Weibull pdf approximation (green line).

3.2.2 Rayleigh distribution

To be able to obtain a Weibull distribution, a measured wind speed time series must be available, allowing for an experimental histogram to be produced. If this data is not available, the alternative is to use the simpler Rayleigh distribution that depends only on one parameter – the mean annual wind speed, as shown next.

The Rayleigh distribution is a Weibull distribution in which k = 2. So, in equation 3.1, it is:

$$f(u) = \frac{2}{c}\left(\frac{u}{c}\right)\exp\left\{-\left[\left(\frac{u}{c}\right)^2\right]\right\} \qquad \text{equation 3.14}$$

Recalling equation 3.3, for k = 2:

$$c = \frac{u_{ma}}{\Gamma\left(1+\frac{1}{2}\right)} = \frac{2}{\sqrt{\pi}}u_{ma} \qquad \text{equation 3.15}$$

Replacing equation 3.15 into equation 3.14, we finally obtain for the Rayleigh pdf:

$$f(u) = \frac{\pi}{2} \frac{u}{u_{ma}^{2}} \exp\left[-\frac{\pi}{4} \left(\frac{u}{u_{ma}} \right)^{2} \right] \qquad \text{equation 3.16}$$

As can be seen in equation 3.16, the Rayleigh distribution depends only on the mean annual speed. As so, it is appropriate to be used to describe the wind speed profile in places where the available data is scarce.

From equation 3.6, one obtains the Rayleigh cdf, as:

$$F(u) = \exp\left[-\frac{\pi}{4} \left(\frac{u}{u_{ma}} \right)^{2} \right] \qquad \text{equation 3.17}$$

Figure 3-8 displays the Rayleigh pdf (blue line; right scale) and cdf (red line; left scale).

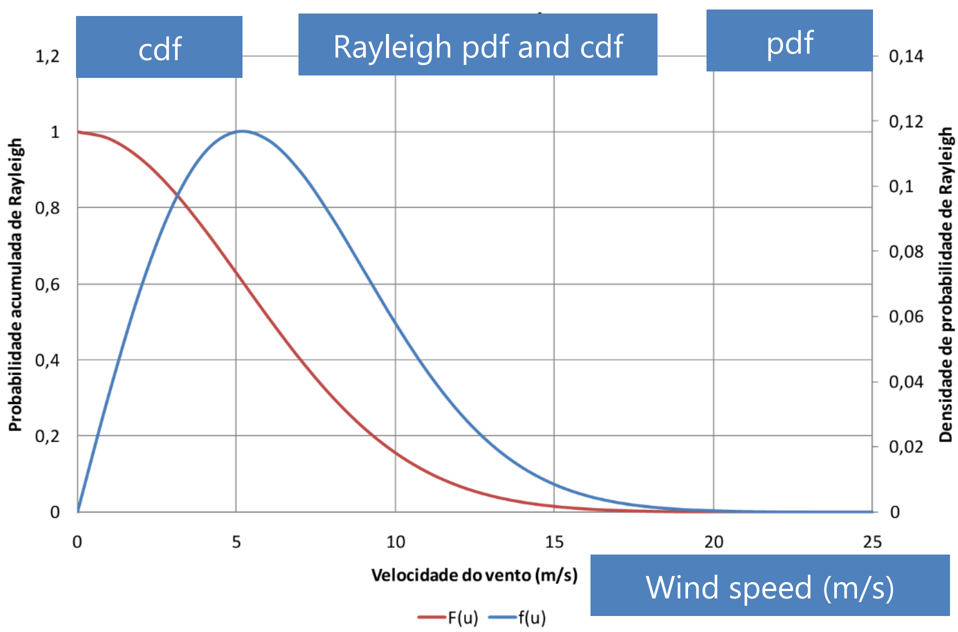

Figure 3-8: Rayleigh pdf and cdf vs wind speed; u_{ma} = 6.5 m/s.

4 The Power Curve

The output power of a WTG depends on the wind speed input following the so-called power curve. Figure 4-1 shows a typical power curve of a 2 MW WTG.

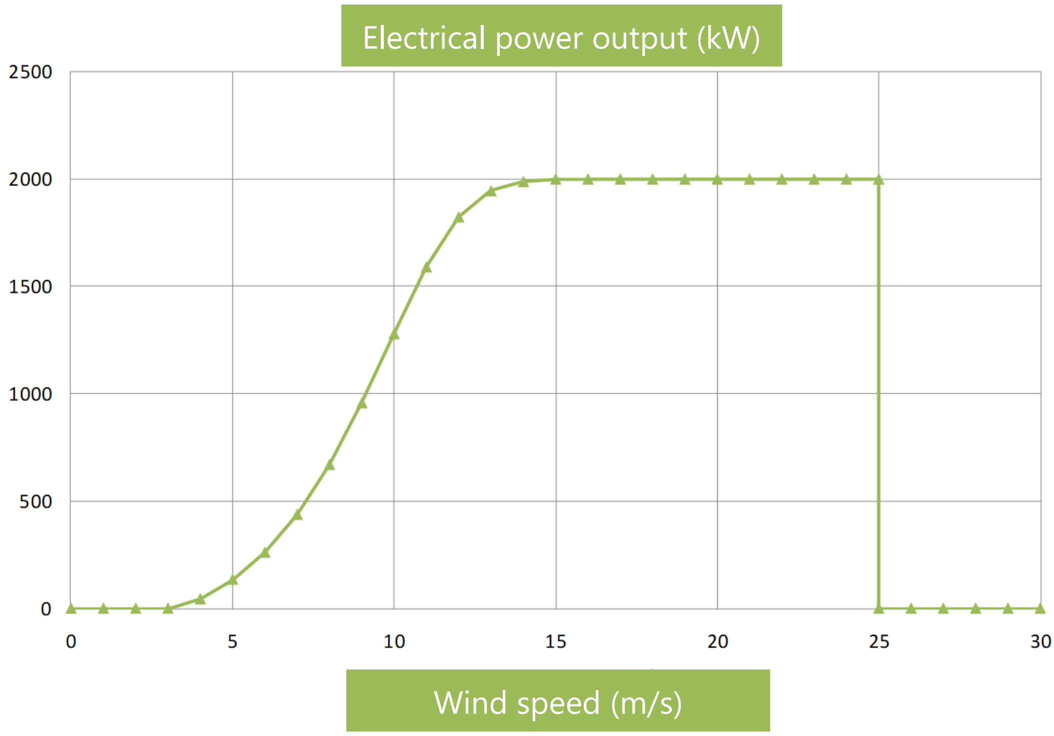

Figure 4-1: Typical power curve of a 2 MW WTG.

The WTG are designed to supply the maximum power for a certain wind speed. The maximum power is known as rated power and the corresponding wind speed is known as rated wind speed. In the case of Figure 4-1, the rated power is 2 MW and the rated wind speed is 15 m/s. There is no normalized rated wind speed; its value depends on the WTG model, values between 13 and 16 m/s being common.

Analysing the typical power curve showed in Figure 4-1, it is possible to conclude that:

- For wind speeds lower than a certain value, the so-called cut-in wind speed, the WTG is disconnected, because it is not economical to extract power from the wind, due to the cubic variation law. Usually, the cut-in wind speed is between 3 and 5 m/s, 4 m/s being the cut-in wind speed of the WTG of Figure 4-1.

- Then, there is an operating zone, in which the WTG is regulated to capture the maximum power from the wind. In this zone, that goes from the cut-in wind speed to the rated wind speed, the output changes approximately with the cube of the wind speed.
- For wind speeds higher than the rated wind speed, the WTG is regulated to operate at constant rated power, so that the maximum power is not exceeded and the WTG is not damaged. In this operating zone, the conversion efficiency is artificially lowered by the WTG control system.
- When the wind speed is dangerously high, higher than the so-called cut-off wind speed (usually between 25 and 30 m/s), the WTG is disconnected from the grid, for safety reasons.

All the WTG power curves have the same pattern as depicted in Figure 4-1. Of course, the rated power depends on the specific model. Current typical WTG are about 3 to 4 MW rated power, for onshore. For offshore, the typical rated power is higher, about the double of onshore installations. Power curves are available at the manufacturers internet sites.

4.1 Analytic Equations

It is useful to have the WTG power curve described by an analytic equation. As seen in Figure 4-1, the modelling problematic zone is the zone in which the output power changes approximately with the cube of the wind speed; for the remaining operating zones, the modelling is straightforward.

To model the power curve is the output power changing zone, a sigmoid function is often used. Therefore, the model of a power curve can be described by:

$$P_e = \begin{cases} P_e = 0 & u < u_0 \\ \dfrac{P_N}{1 + \exp\left(-\dfrac{u - c_1}{c_2}\right)} & u_0 \leq u < u_N \\ P_e = P_N & u_N \leq u \leq u_{\max} \\ P_e = 0 & u > u_{\max} \end{cases}$$ equation 4.1

In equation 4.1, it is: P_e is the WTG electrical output power, P_N is the rated power, u_0, u_N and u_{max} are the cut-in, rated, and cut-off wind speeds, respectively. c_1 and c_2 are fitting constants, that are adjusted from the experimental power curve supplied by the manufacturer.

In Figure 4-2, we show a comparison of a power curve as supplied by the manufacturer (solid line) and as approximated by a sigmoid function (dotted line).

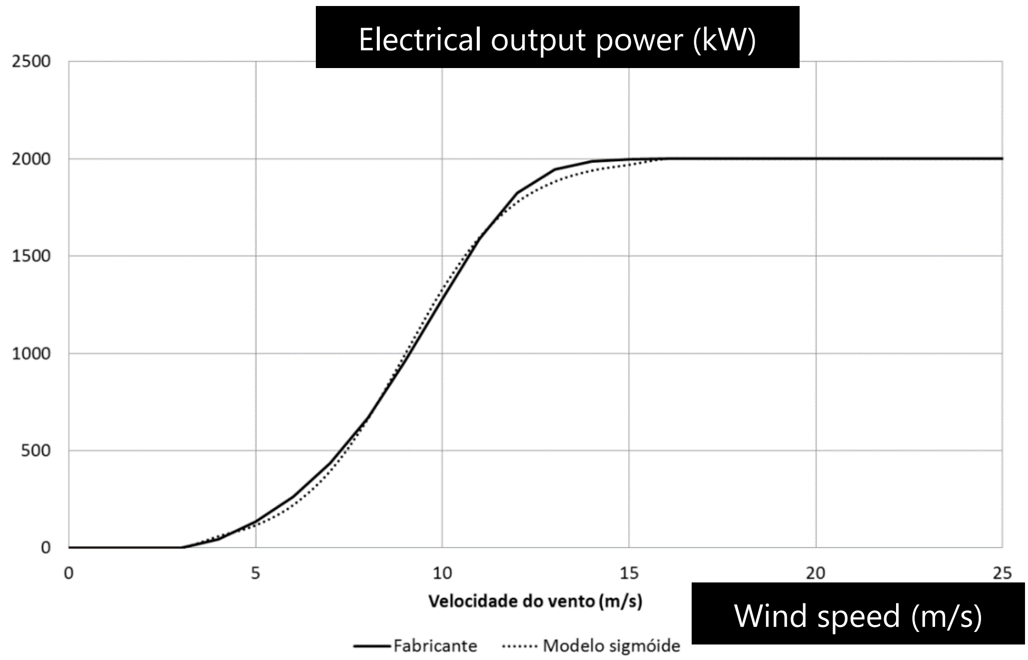

Figure 4-2: 2 MW WTG power curve: manufacturer (solid line) and sigmoid approximation (dotted line).

To find the best fitting c_1 and c_2 parameters, ExcelSolver® may be used.

4.2 POWER COEFFICIENT – C_P

The wind-electric conversion efficiency is called power coefficient and is denoted by C_P, the definition equation being:

$$C_P(u) = \frac{P_e(u)}{P_{avail}(u)} = \frac{P_e(u)}{\frac{1}{2}\rho A u^3} \qquad \text{equation 4.2}$$

The power coefficient, commonly known as C_P, is in fact the WTG efficiency, given by the ratio of the output electric power by the input available power in the wind.

As seen in equation 4.2, C_P depends on the wind speed. Given the WTG power curve is available, the power coefficient is very straightforward to compute. Figure 4-3 displays the graphic of the C_P variation as a function of the wind speed for a Vestas V90[3] WTG.

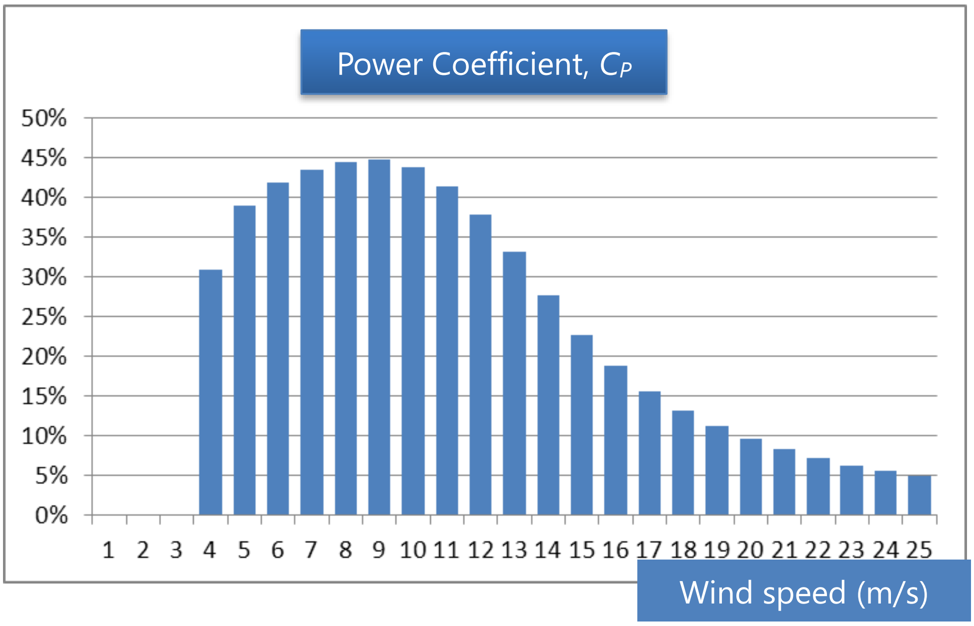

Figure 4-3: Power coefficient C_P as a function of the wind speed for a Vestas V90 model.

We note that the WTG efficiency is very low for high wind speeds. This decrease in the efficiency is on-purpose caused by the control system, so as to achieve rated power limitation for high wind speeds. We will come back to this subject later on.

[3] Vestas is a leading WTG manufacturer. Enercon, GoldWind, Siemens, GE are examples of other WTG manufacturers.

5 ELECTRICITY PRODUCTION ESTIMATES

5.1 COMPLETE MODELS

To be able of computing the electricity production of a WTG, a twofold information is required: wind speed profile at rotor height and power curve of the chosen WTG. The way these data is available implies a different appropriate model to use for the estimation of the yearly electricity production of a WTG.

Available data: analytic equations for wind speed pdf and WTG power curve

In this case, the appropriate yearly electricity production equation is:

$$E_a = 8760 P_{avg} = 8760 \int_{u_0}^{u_{\max}} f(u) P_e(u) du \qquad \text{equation 5.1}$$

where, u_0 and $u_{\max}$ are the cut-in and cut-off wind speeds, respectively, $f(u)$ is the analytic equation for the wind speed pdf, and $P_e(u)$ is the analytic equation WTG power curve. Similarly to equation 3.2, we note that the average power is given by: $\int_{u_0}^{u_{\max}} f(u) P_e(u) du$.

Available data: wind speed discrete histogram and WTG discrete power curve

If this is the case, then we are not allowed to integrate, and we must sum. Therefore, the yearly electricity production equation is:

$$E_a = \sum_{u_0}^{u_{\max}} f_r(u) P_e(u) \qquad \text{equation 5.2}$$

where $f_r(u)$ is the wind speed frequency of occurrence (as given by the histogram for each wind speed class) in h/year.

Available data: analytic equation for wind speed pdf and WTG discrete power curve

These is the data that is more often available in WTG project. In this case, the most accurate equation to compute the yearly electricity production is:

$$E_a = 8760 \sum_{u_0+1}^{u_{\max}} \left[\left(F(i-1) - F(i) \right) \frac{P_e(i-1) + P_e(i)}{2} \right] \quad \text{equation 5.3}$$

where *F*(i) is the wind speed cdf.

We highlight that, for instance, *F*(5) – *F*(6) is the probability of the wind speed being between 5 and 6 m/s and $\frac{P_e(5) + P_e(6)}{2}$ is the average output power when the wind speed is between 5 and 6 m/s.

Let us look at an example that will help in better understanding the technique.

Example 5—1

Consider a WTG with the characteristics shown in Table 5-1*:*

Table 5-1: WTG characteristics.

Rated power	*2310*	kW
Rotor diameter	*71*	m
Hub height	*80*	m

This WTG is to be deployed in a place where the Weibull parameters, at rotor height, are k = 2.86 and c = 8.58 m/s. The Weibull cdf is depicted in Figure 5-1*. The power curve is displayed in* Figure 5-2.

Compute the best estimate for the annual WTG electricity production.

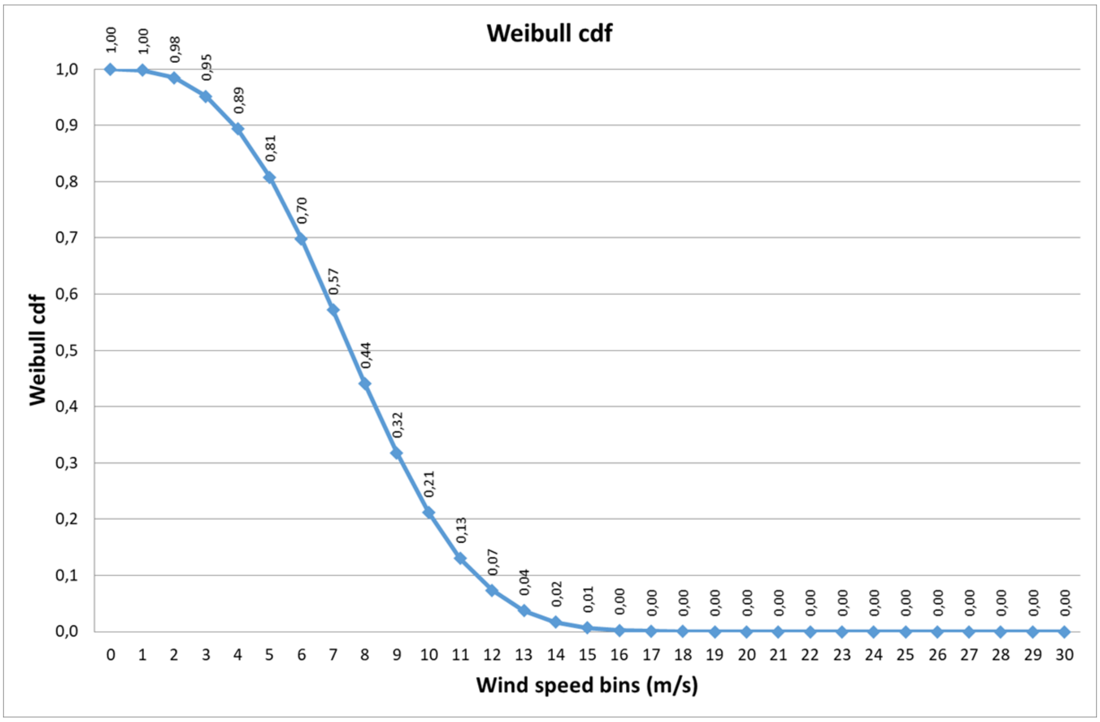

Figure 5-1: Wind speed Weibull cdf.

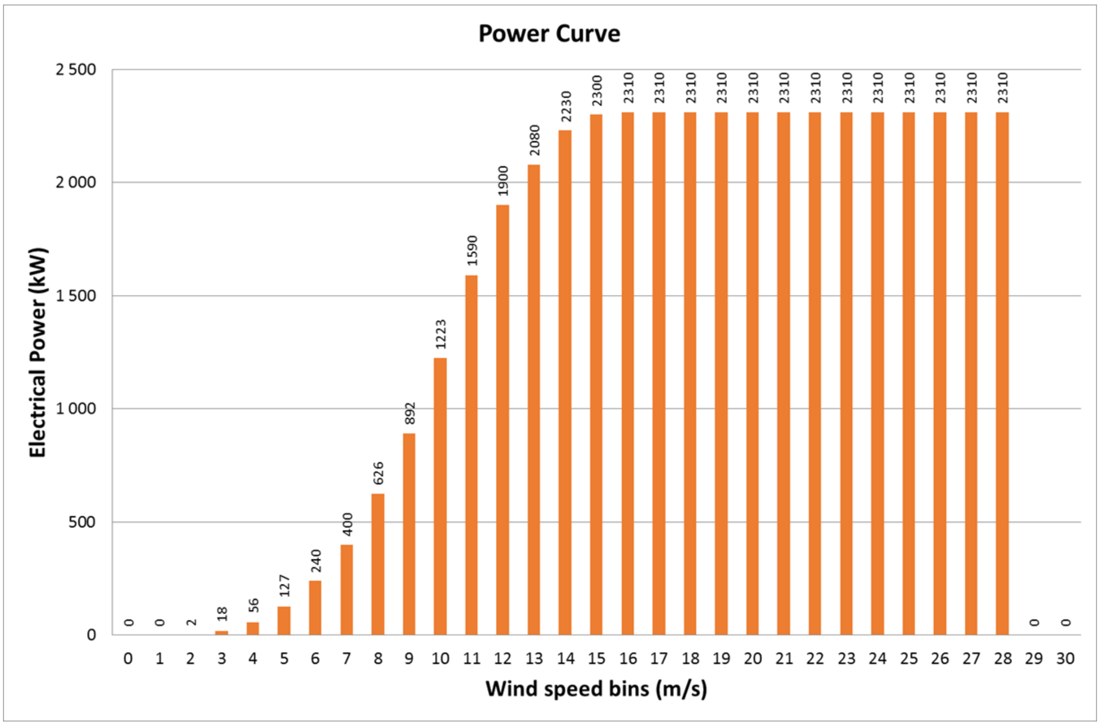

Figure 5-2: WTG power curve.

Solution: Given the type of data we have, the correct model is the one given by equation 5.3. *Applying this model, the yearly electricity production for each wind speed bin is showed in* Figure 5-3.

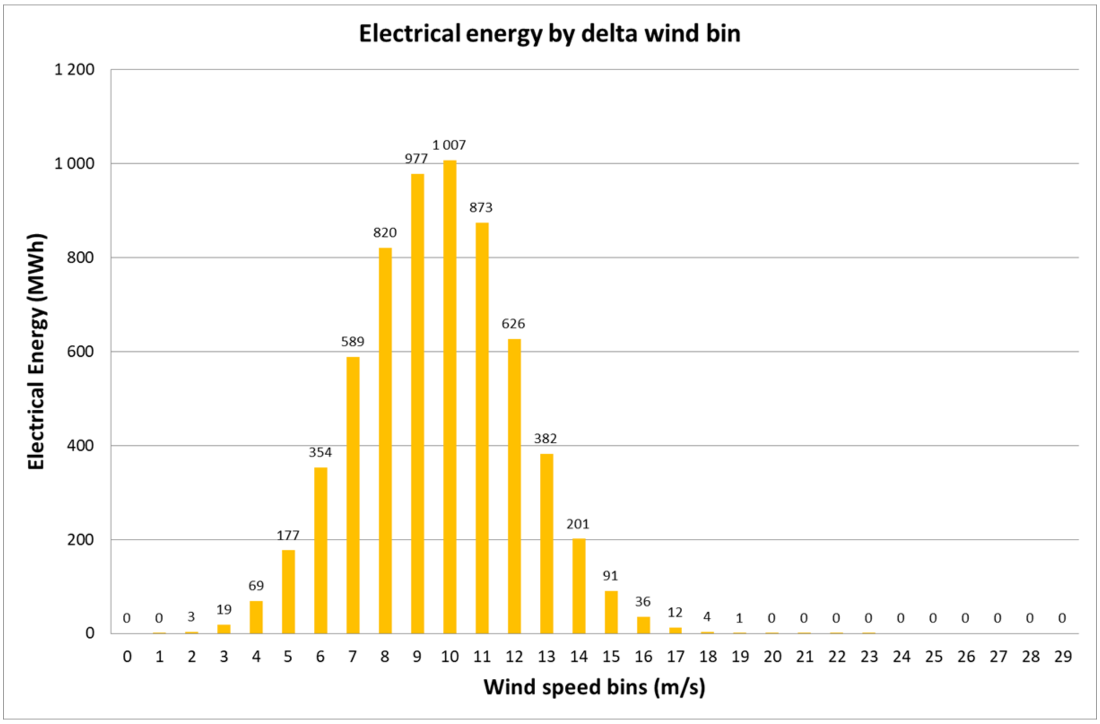

Figure 5-3: WTG yearly electricity production for each wind speed class.

The obtained results are summarized in Table 5-2.

Table 5-2: Example 5—1 results.

Annual Energy	*6 239*	MWh
Utilization factor	*2 701*	h
Average power	*712*	kW
Capacity factor	*30.83%*	

Example 5—2

Consider a 660 kW WTG, with a rotor diameter of 47 m, the rotor height being 40 m. The sigmoid parameters of the power curve are c_1=8.76 and c_2=1.48 and the rated wind speed is 15 m/s. The mean annual wind speed at the installation place and at 40 m height is 8.24 m/s.

Compute the electricity produced when the wind speed is between 9 and 11 m/s. Compare different methods.

The relevant functions are:

$$P(u) = \left(\frac{660}{1 + \exp\left(-\frac{u - 8.76}{1.48} \right)} \right) \quad u_0 < u < 15$$

$$f(u) = \frac{\pi}{2} \frac{u}{8.24^2} \exp\left[-\frac{\pi}{4} \left(\frac{u}{8.24} \right)^2 \right]$$

$$F(u) = \exp\left[-\frac{\pi}{4} \left(\frac{u}{8.24} \right)^2 \right]$$

As the only wind speed data we have is the mean annual wind speed, a Rayleigh distribution is to be used.

The accurate method is:

$$E(9 < u < 11) = 8760 \int_9^{11} P(u) f(u) du = 578.20\text{MWh} \quad (\text{step} = 0.01\text{m/s})$$

This was performed using numerical integration with a step of 0.01 m/s, using Excel®.

With the provided data, the method formulated in equation 5.3 *can also be used:*

$$E(9 < u < 11) = 8760 \left[\left(F(9) - F(10) \right) \left(\frac{P(9) + P(10)}{2} \right) + \left(F(10) - F(11) \right) \left(\frac{P(10) + P(11)}{2} \right) \right] = 574.34\text{MWh}$$

As can be seen, the accuracy of this method is good, the error being just –0.7%.

5.2 SIMPLIFIED MODELS

5.2.1 Johnson's model

Sometimes, mainly in the early stages of a project, we do not know what specific model of a WTG will be installed. However, it is probable that we know the WTG rated capacity, the cut-in (u_0), rated (u_N) and cut-off (u_{max}) wind speeds (if we do not know these values, typical ones can be assumed) and the wind speed Weibull distribution. In this case, we can use a general-purpose WTG power curve defined as[4]:

$$\begin{cases} P_e = 0 & u < u_0 \\ P_e = a + bu^k & u_0 \le u \le u_N \\ P_e = P_N & u_N < u \le u_{\max} \\ P_e = 0 & u > u_{\max} \end{cases}$$ equation 5.4

where *k* is the Weibull parameter and *a* and *b* are two constants that can be determined from the boundary conditions:

$$\begin{aligned} u = u_0 &\Rightarrow P_e = 0 \\ u = u_N &\Rightarrow P_e = P_N \end{aligned}$$ equation 5.5

The constants *a* and *b* can therefore be computed as:

$$\begin{aligned} a &= P_N \frac{u_0{}^k}{u_0{}^k - u_N{}^k} \\ b &= P_N \frac{1}{u_N{}^k - u_0{}^k} \end{aligned}$$ equation 5.6

In Figure 5-4, we can see a plotting of the manufacturer's (blue solid line) and Johnson's (red solid line) power curves. We can see that approximation is not perfect, but it is enough to obtain a rough estimation of the annual electricity production, which is a valuable asset in a preliminary phase of the project.

Our aim is to compute an estimate for the annual energy produced by a WTG, which is given by:

$$E_a = 8760 P_{avg}$$ equation 5.7

the average power being computed through:

$$P_{avg} = \int_{u_0}^{u_{\max}} P_e(u) f(u) du = \int_{u_0}^{u_N} \left(a + bu^k\right) f(u) du + P_N \int_{u_N}^{u_{\max}} f(u) du$$ equation 5.8

where *f*(*u*) is the Weibull pdf:

[4] Gary L. Johnson, Wind Energy System, http://eece.ksu.edu/~gjohnson

$$f(u) = \frac{k}{c}\left(\frac{u}{c}\right)^{k-1} e^{-\left[\left(\frac{u}{c}\right)^{k}\right]}$$ equation 5.9

We recall that we assume we know the Weibull distribution parameters.

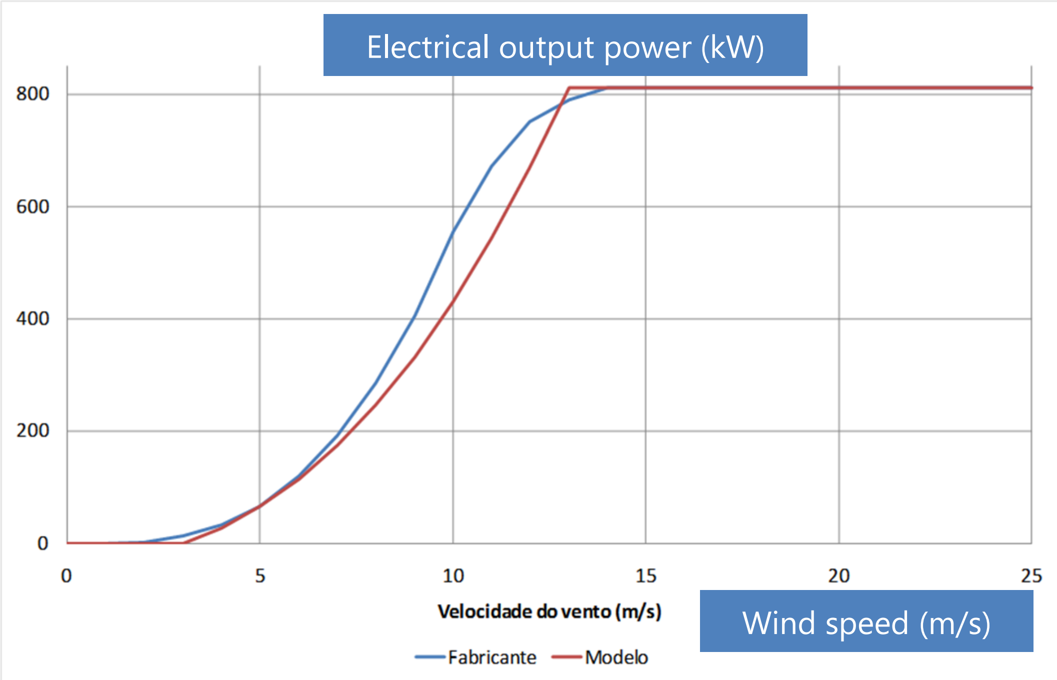

Figure 5-4: WTG power curve: Johnson approximation (red solid line) and manufacturer's (blue solid line).

In equation 5.8, there are 2 integrals that need to be solved:

$$\int f(u)du$$
$$\int u^{k} f(u)du$$ equation 5.10

Before proceeding further, we are going to make a change of variable, because this will help in solving the integrals in equation 5.10:

$$x = \left(\frac{u}{c}\right)^{k}$$ equation 5.11

and therefore:

$$dx = k\left(\frac{u}{c}\right)^{k-1} d\left(\frac{u}{c}\right)$$ equation 5.12

Under these circumstances, we have a first important result for the computation of the 1st integral in equation 5.10:

$$\int f(u)du = \int \frac{k}{c}\left(\frac{u}{c}\right)^{k-1} e^{-\left[\left(\frac{u}{c}\right)^k\right]} du = \int e^{-x}dx = -e^{-x} \qquad \text{equation 5.13}$$

Then, for the 2nd integral of equation 5.10, we write:

$$\int u^k f(u)du = \int c^k \left(\frac{u}{c}\right)^k f(u)du = c^k \int xe^{-x}dx = -c^k\left(x+1\right)e^{-x} \qquad \text{equation 5.14}$$

which portrays a 2nd important result.

Replacing equation 5.13 and equation 5.14 in equation 5.8, we finally obtain, after some manipulation:

$$P_{avg} = P_N\left(\frac{e^{-\left[\left(\frac{u_0}{c}\right)^k\right]} - e^{-\left[\left(\frac{u_N}{c}\right)^k\right]}}{\left(\frac{u_N}{c}\right)^k - \left(\frac{u_0}{c}\right)^k} - e^{-\left[\left(\frac{u_{\max}}{c}\right)^k\right]}\right) \qquad \text{equation 5.15}$$

An estimate for the annual electricity produced by a general-purpose WTG located in a place where the Weibull parameters are known is (see equation 5.7):

$$E_a = 8760P_N\left(\frac{e^{-\left[\left(\frac{u_0}{c}\right)^k\right]} - e^{-\left[\left(\frac{u_N}{c}\right)^k\right]}}{\left(\frac{u_N}{c}\right)^k - \left(\frac{u_0}{c}\right)^k} - e^{-\left[\left(\frac{u_{\max}}{c}\right)^k\right]}\right) \qquad \text{equation 5.16}$$

5.2.2 Fast estimate model

In very preliminary phases of a WTG project, when only few data are available, it is common practice to use basic models. This is the case of the fast estimate model that we present hereafter.

The annual electricity production of a WTG is given by:

$$E_a = P_{avg}8760 \approx \left(C_p\right)_{avg}\left(P_{avail}\right)_{avg}8760 \qquad \text{equation 5.17}$$

where $\left(C_p\right)_{avg}$ is the average value of the power coefficient and $\left(P_{avail}\right)_{avg}$ is the average value of the available power in the wind.

In what concerns the later, one can write:

$$\left(P_{avail}\right)_{avg} = \left(\frac{1}{2}\rho A u^3\right)_{avg} = \frac{1}{2}\rho A \left(u^3\right)_{avg} \qquad \text{equation 5.18}$$

Our attention will now be focused on the computation of the average of the cube of the wind speed, $\left(u^3\right)_{avg}$, which is different from the cube of the average wind speed $\left(u_{avg}\right)^3$.

This model assumes that the wind speed at the installation place is well represented by a Rayleigh distribution. We recall that a Rayleigh distribution is a Weibull distribution in which k=2; we also recall that the annual mean wind speed is $u_{ma} = \int_0^\infty u\, f(u)\, du$ (equation 3.2). Therefore, we can write:

$$\begin{aligned}\left(u^3\right)_{avg} &= \int_0^\infty u^3 f_{Rayl}(u) du = \\ &= \int_0^\infty u^3 \frac{2u}{c^2} \exp\left[-\left(\frac{u}{c}\right)^2\right] du = \frac{2}{c^2}\int_0^\infty u^4 \exp\left[-\left(\frac{u}{c}\right)^2\right] du\end{aligned} \qquad \text{equation 5.19}$$

An integrals table shows that:

$$\int_0^\infty x^m \exp\left(-ax^2\right) dx = \frac{\Gamma\left(\frac{m+1}{2}\right)}{2a^{\frac{m+1}{2}}} \qquad \text{equation 5.20}$$

where $\Gamma\left(\frac{m+1}{2}\right)$ is the value of the *Gamma* function in the point $\frac{m+1}{2}$.

Applying to the case of the integral we want to solve, the parameters assume the following values (compare equation 5.19 and equation 5.20):

$$m = 4;\ a = \frac{1}{c^2};\ \Gamma\left(\frac{5}{2}\right) = \frac{3}{2}\frac{\sqrt{\pi}}{2};\ a^{\frac{5}{2}} = \frac{1}{c^5} \qquad \text{equation 5.21}$$

Replacing equation 5.20 in equation 5.19 and taking into account the parameters given in equation 5.21, we obtain:

$$\left(u^3\right)_{avg} = \frac{2}{c^2}\int_0^\infty u^4 \exp\left[-\left(\frac{u}{c}\right)^2\right]du = \frac{2}{c^2}\frac{\frac{3}{4}\sqrt{\pi}}{\frac{2}{c^5}} = \frac{3}{4}c^3\sqrt{\pi} \qquad \text{equation 5.22}$$

Moreover, recalling that (equation 3.3):

$$u_{ma} = c\Gamma\left(1+\frac{1}{k}\right) \xrightarrow{k=2} u_{ma} = c\frac{\sqrt{\pi}}{2} \qquad \text{equation 5.23}$$

Replacing equation 5.23 into equation 5.22, one can write:

$$\left(u^3\right)_{avg} = \frac{3}{4}\sqrt{\pi}\left(\frac{2u_{ma}}{\sqrt{\pi}}\right)^3 = \frac{3}{4}\frac{8u_{ma}^3}{\pi} = \frac{6}{\pi}u_{ma}^3 \qquad \text{equation 5.24}$$

This is an important result, that we can use by retaking equation 5.18, thus leading to:

$$\left(P_{avail}\right)_{avg} = \frac{1}{2}\rho A\left(u^3\right)_{avg} = \rho A\frac{3}{\pi}u_{ma}^3 \qquad \text{equation 5.25}$$

We note that the annual electricity production can be written as (see equation 5.17):

$$E_a = P_{avg}8760 \approx \left(C_p\right)_{avg}\rho A\frac{3}{\pi}u_{ma}^3 8760 \qquad \text{equation 5.26}$$

As seen before, the power coefficient is not constant, it depends on the wind speed. Modern WTG show a C_P average value of $\left(C_p\right)_{avg} = 25\%$ (see, for instance, Figure 4-3). The air density in standard pressure and temperature conditions is ρ=1.23 kg/m^3. Moreover, 3/π = 0.95. Therefore, a fast estimate of the annual electricity produced by a WTG is:

$$E_a \approx 0.25\rho A\frac{3}{\pi}u_{ma}^3 8760 \approx 0.3Au_{ma}^3 8760 \qquad \text{equation 5.27}$$

where *A* is the swept area and u_{ma} is the mean annual wind speed at rotor height.

Rewriting equation 5.27 as a function of the rotor diameter, *RD*, and expressing the result in kWh, we finally have:

$$E_a \approx 2(RD)^2 u_{ma}^3 \qquad \text{equation 5.28}$$

This is a very simple equation that allows the computation of an estimate of the WTG annual electricity production, given the knowledge of readily-available data.

Example 5—3

Retake Example 5—1 data and compute the annual energy yield by using the fast estimate method.

Solution

To use the fast estimate method, we need the mean annual wind speed.

Let us recall that we have approximately: $u_{ma} = 0.9c$ *(equation 3.5). The data indicate RD = 71 m and c = 8.58 m/s, therefore* u_{ma} *= 7.72 m/s. Using equation 5.28, we obtain* E_a *= 4639 MWh, which deviates from the result obtained with the complete model in –26%.*

6 WIND CHARACTERISTICS AND RESOURCES

The mean annual wind speed at 60 m and 10 m height in Portugal is shown in Figure 6-1. It is apparent why WTG should be high enough so that to take advantage of higher wind speeds.

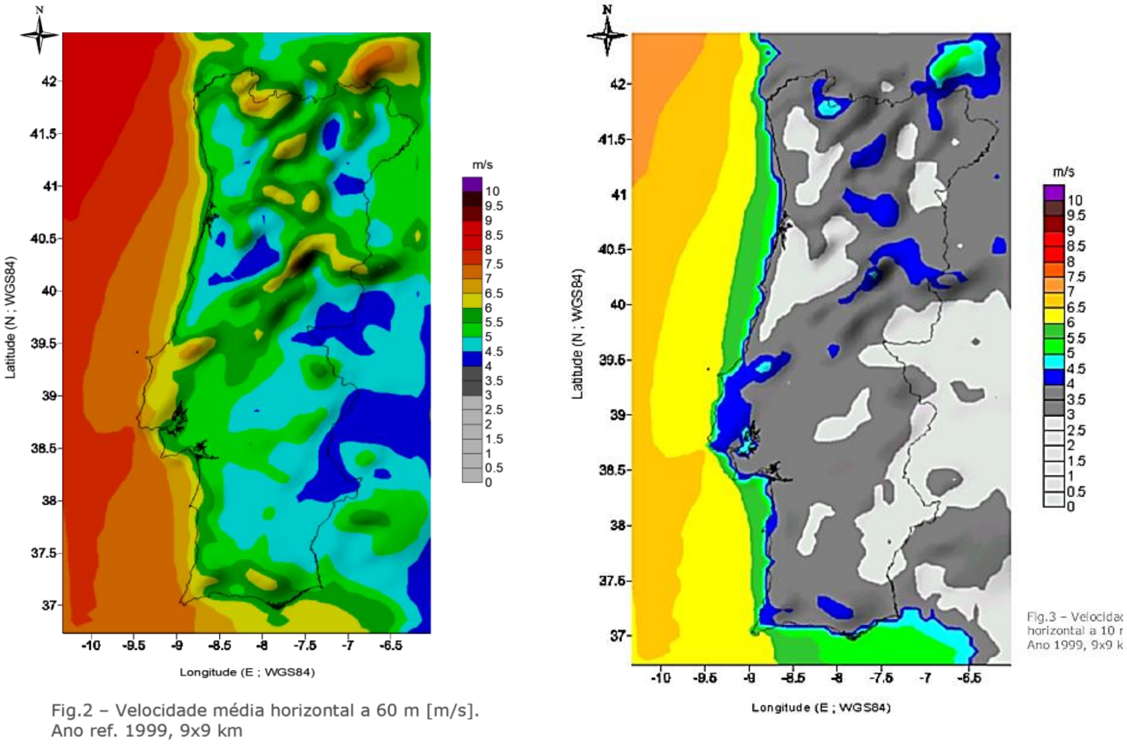

Figure 6-1: Mean annual wind speed at 60 m and 10 m height in Portugal; Source: LNEG, www.lneg.pt..

It is common knowledge that wind speed changes a lot with time. In Figure 6-2, we can observe the time variation of the wind speed in 4 days, during a period of 3 hours.

Wind speed suffers inter-annual (30 years are needed to determine long term values of weather and climate; one year produces long term seasonal mean wind speeds with 10% accuracy and 90 % confidence level), annual (significant variation in seasonal or monthly averaged wind speeds), diurnal (differential heating of the earth surface during the daily radiation cycle) and short-term (turbulence and gusts – variations over intervals of 10 min or less) variations.

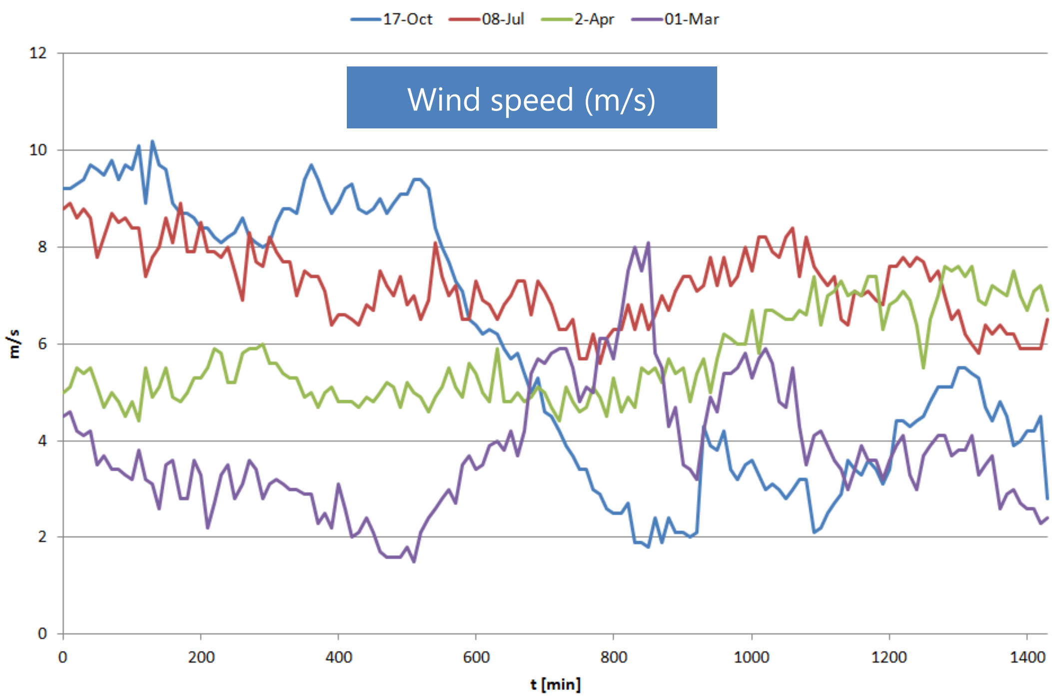

Figure 6-2: Wind speed variation with time in 4 different days, during a period of 3 hours.

The effect of obstacles in the wind flow is very significant in reducing the wind speed and thus the wind power. This can be seen in Figure 6-3, where it is shown that only after 20 times the height of the obstacle (h_s), the wind flow recovers its original free characteristics.

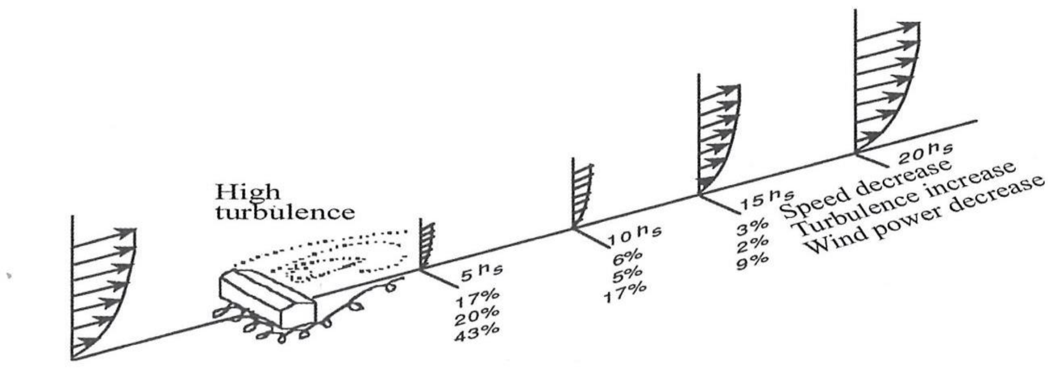

Figure 6-3: Effect of obstacles in wins speed and wind power decrease; h_s: obstacle height.

In a wind park, each WTG is an obstacle to the other ones, due to the downstream wake, portraying what is known by the wake effect. A WTG downwind of another WTG has to cope with wakes with both slower wind speeds and increased turbulence. Figure 6-4 shows the wake effect in an offshore wind park.

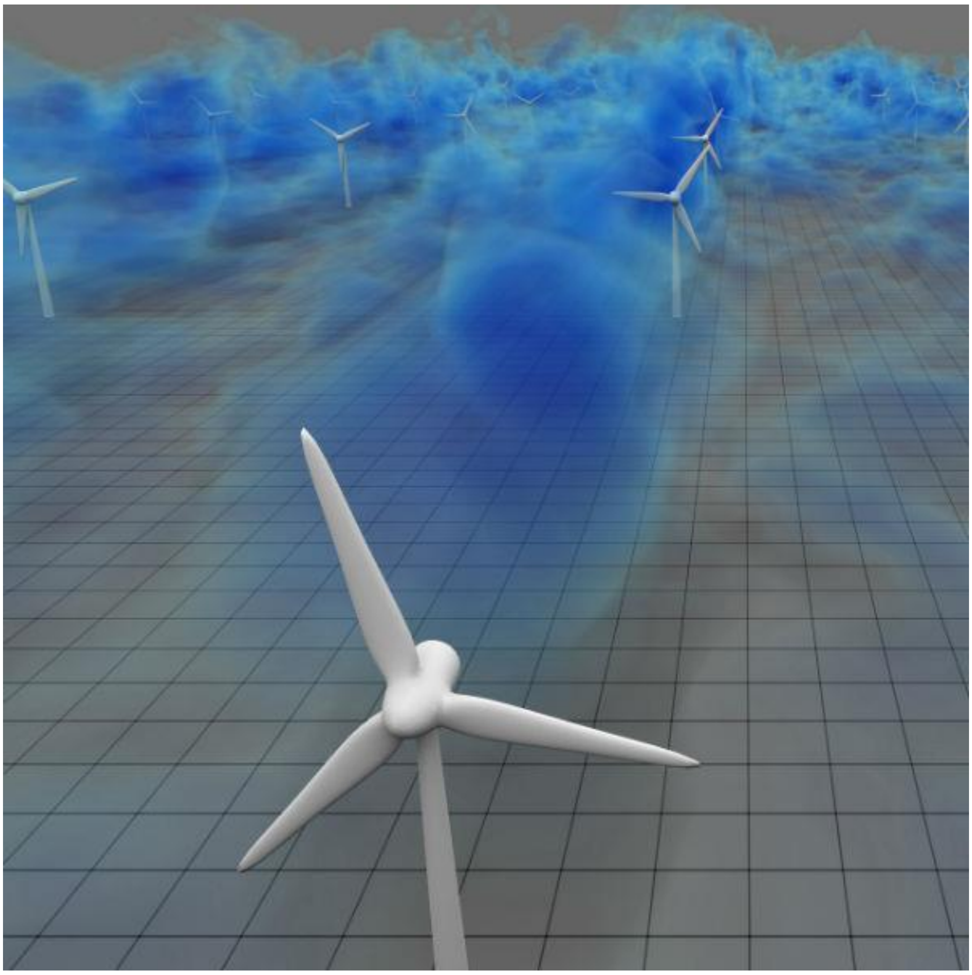

Figure 6-4: 3D visualization of the wake effect in an offshore wind park; Source: http://www.windpower-international.com/features/featurethe-wake-effect-inside-the-offshore-wind-accelerator-programme-4302542/

Wake effect causes the wind park aggregate production to be lower than the number of WTG times the production of one WTG. Therefore, wake effect should be minimized by conveniently spacing the WTG apart in a wind park. Recommended practices point to keeping 7 rotor diameters in the wind-stream direction and 4 rotor diameters in the perpendicular direction (see Figure 6-5). Even if this layout is followed, a loss of 5% due to wake effect is to be expected.

WTG should be placed so that the prevalent wind-stream is perpendicular to the rotor blades rotational plane. To cope with different wind directions, WTG are equipped with a yaw mechanism that automatically turns the nacelle to face the wind flow in the correct direction.

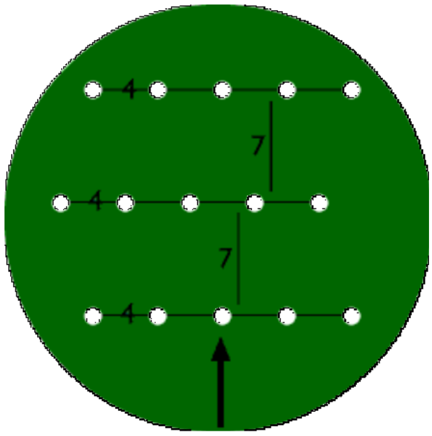

Figure 6-5: Recommended practice to space WTG apart inside a wind park; scale: rotor diameters.

Wind speed is measured using anemometers and wind direction using wind vanes. Cup anemometers, as illustrated in Figure 6-6, are the most used.

Figure 6-6: Cup anemometer and wind vane.

7 WIND TURBINE GENERATOR COMPONENTS

Most of the manufacturers offer horizontal axis wind turbines, in which the axis of rotation is parallel to the ground. Such WTG may be divided in 3 main parts: rotor, nacelle and tower.

The rotor is composed of the blades which are fixed to the nacelle in a hub. The rotor may be located upwind, if the wind turns the blades and then impacts the tower, i.e. the rotor faces the wind, or downwind, if the wind comes from the back, i.e. impacts the tower and then turns the blades. Nowadays, all WTG are upwind, as the tower introduces turbulence to the wind-stream, which is much undesirable. Also, most of the WTG offered in the market are three-bladed, as this was found to be the number of blades that maximizes the efficiency. The blades are flexible and are made of fibre-reinforced polymers (FRP), such as, glass and carbon fibre reinforced plastics (GFRP and CFRP). Figure 7-1 shows a picture of a 49 m blade from a 2.3 MW WTG.

Figure 7-1: GFRP blade of a Siemens SWT-2.3-101 WTG.

The nacelle is the place where the WTG components are installed. The main components inside the nacelle are shown in Figure 7-2. We can mention the low-speed shaft (about 15 rpm) connected to the rotor blades and the high-speed shaft (about 1500 rpm) connected to the electrical generator. In between there is a gearbox (there are WTG models that do not have a gearbox). Also, we can see the yaw motor that drives the nacelle to be always facing the wind stream, when the direction changes, as given by the wind vane. The pitch system is used to limit the output power to the rated power for high wind speeds (as given by the anemometer) and will be discussed later.

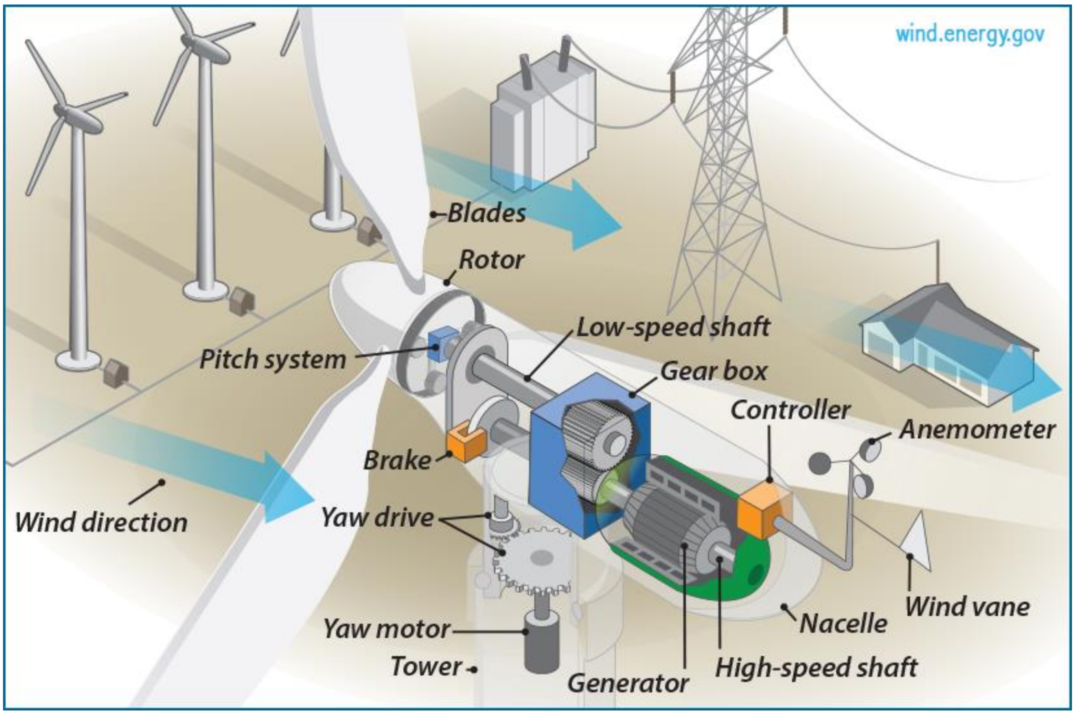

Figure 7-2: Main components inside the nacelle of a WTG; Source: https://www.ourworldofenergy.com/vignettes.php?type=wind-power&id=2

Currently, the majority of wind turbines are supported by conical tubular steel towers, as seen in Figure 7-3 and Figure 7-4.

Figure 7-3: WTG towers; Source: https://www.paintsquare.com/news/?fuseaction=view&id=15796.

Figure 7-4: WTG towers; Source: http://www.steelwindtower.com/different-types-of-wind-turbine-tower-internals/.

Taller towers for wind turbines make sense. For instance, an 80 m tower can let 2 to 3 MW WTG produce more power, and enough to justify the additional cost of 20-m more, than if installed at 60 m.

A very popular WTG model is the Vestas V90. The rated power is 3 MW, it has a rotor diameter of 90 m and a hub height that can vary between 80 and 105 m, depending on the specifics of the installation location. The nacelle alone weighs more than 75 tons, the blade assembly weighs more than 40 tons, and the tower itself weighs about 152 tons, for a total weight of 267 tons.

8 Some Features of Modern WTG

8.1 The Need for Variable Speed Operation

Let us define the quantity Tip Speed Ratio as:

$$TSR = \lambda = \frac{\omega_T R}{u} \qquad \text{equation 8.1}$$

where ω_T is the angular speed of the rotor blades in rad/s, R is the radius of the circle defined by the rotation of the blades (equal to the blade's length), in m, and u is the wind speed in m/s.

A typical graphics of the variation of the power coefficient (WTG efficiency) with TSR is illustrated in Figure 8-1.

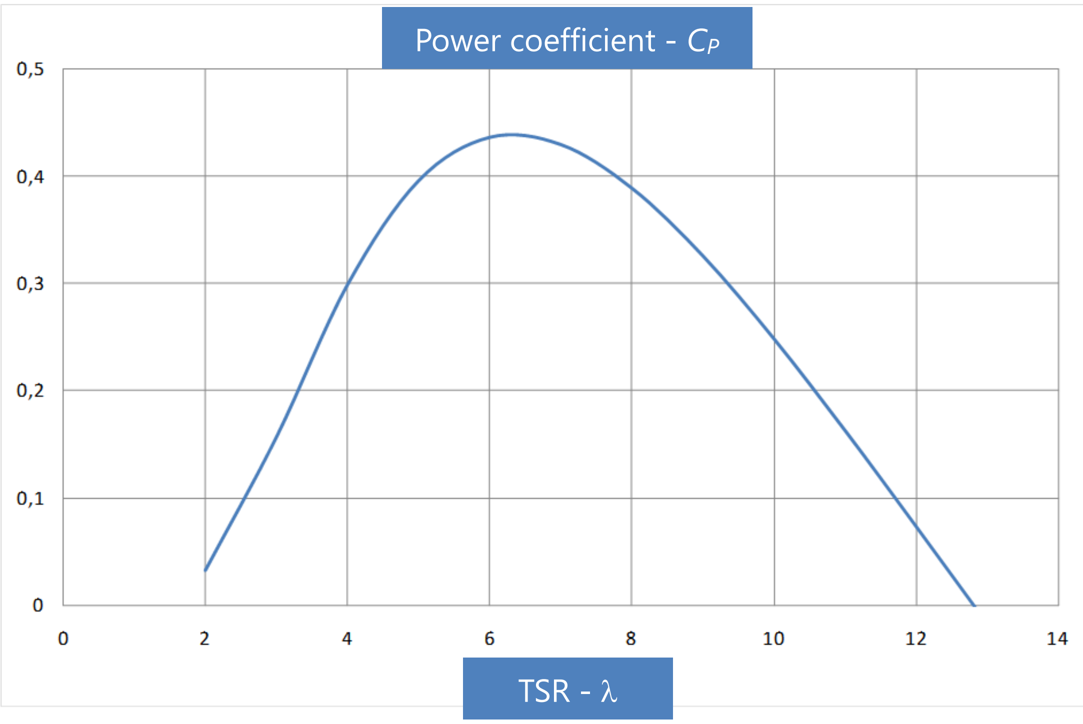

Figure 8-1: Typical variation of the power coefficient (C_P) with the TSR.

An analytic equation for the curve in Figure 8-1 is:

$$C_P = 0.22\left[116\left(\frac{1}{\lambda} - 0.035\right) - 5\right]\exp\left[-12.5\left(\frac{1}{\lambda} - 0.035\right)\right] \qquad \text{equation 8.2}$$

As can be seen from Figure 8-1, if the angular speed of the turbine is kept constant, or nearly constant, either the wind speed increases (TSR decreases) or decreases (TSR increases), the WTG efficiency, C_P, is not at its maximum value, and therefore there is a particular TSR that maximizes C_P.

This operation mode is undesirable, because in the power curve zone where the output power increases with the wind speed, we want maximum efficiency, so that to extract the maximum possible power from the wind.

The way to overcome this situation is to let the angular speed to change in pace with the wind speed. For instance, when wind speed increases, we want the turbine to rotate faster, so that TSR is at its optimal value and the maximum efficiency is obtained.

Let us remember that the turbine angular speed is an image of the generator angular speed, if a gearbox exists, or it is equal to the generator angular speed in gearless WTG. In one way or another, electrical generators that support variable speed are required.

Conventional synchronous generators operate at fixed speed and as so are not adequate; conventional asynchronous generators operate at nearly fixed speed and are not adequate either. Modern WTG electrical generators use power electronics to achieve variable speed, as it is the case of Double Fed Induction Generators (DFIG), in WTG with gearbox, or Direct Driven DC-Link Synchronous Generator, in gearless WTG.

8.2 Power Control

Let us recall a typical WTG power curve, that we repeat hereafter in Figure 8-2 for convenience.

For wind speeds higher than the rated wind speed, the output power is regulated to the WTG rated power, because rated power cannot be exceeded, or the WTG would be damaged.

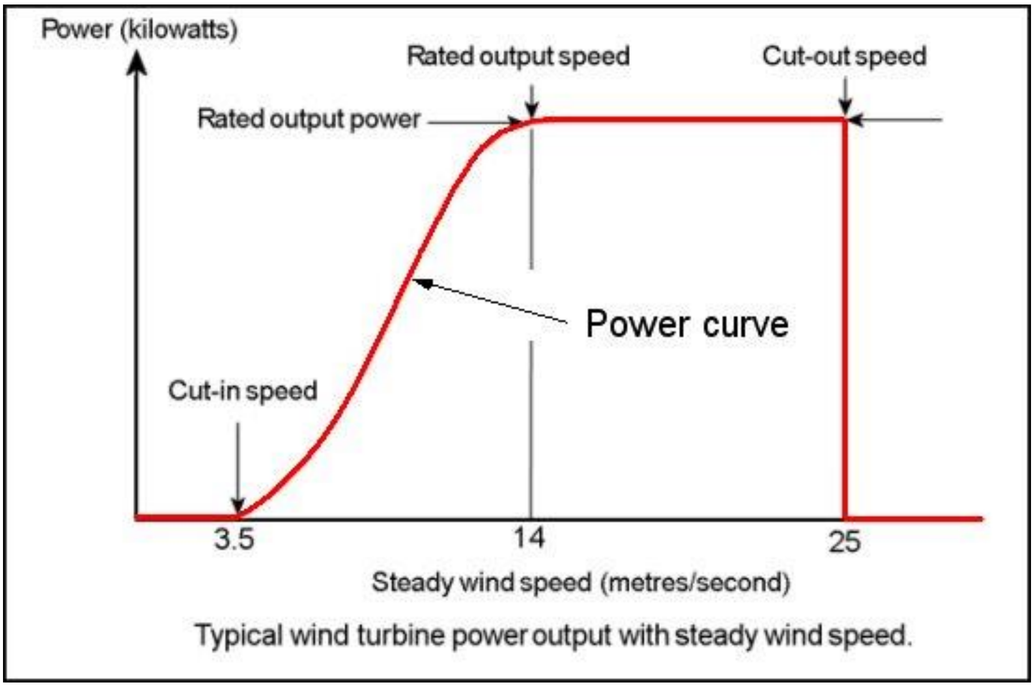

Figure 8-2: Typical WTG power curve; Source: https://www.wind-power-program.com/popups/powercurve.htm.

How is this achieved? Power control is achieved by pitching the blades, as depicted in Figure 8-3.

Figure 8-3: Pitch control; Source: http://usuaris.tinet.cat/zefir/pitch.htm.

When the wind speed, as measured by a dedicated anemometer installed on the top of the nacelle, is higher than the rated wind speed, as defined by the manufacturer, an order is sent to the blade pitch mechanism, which pitches (turns) the rotor blades slightly out of the wind. The system is in general either made up by electric motors and gears, or hydraulic cylinders and a power supply system.

By pitching the blades (increase pitch angle), the C_P is decreased, as seen in Figure 8-4, where the C_P as a function of the TSR is depicted for different pitch angles (beta). Also, recall Figure 4-3.

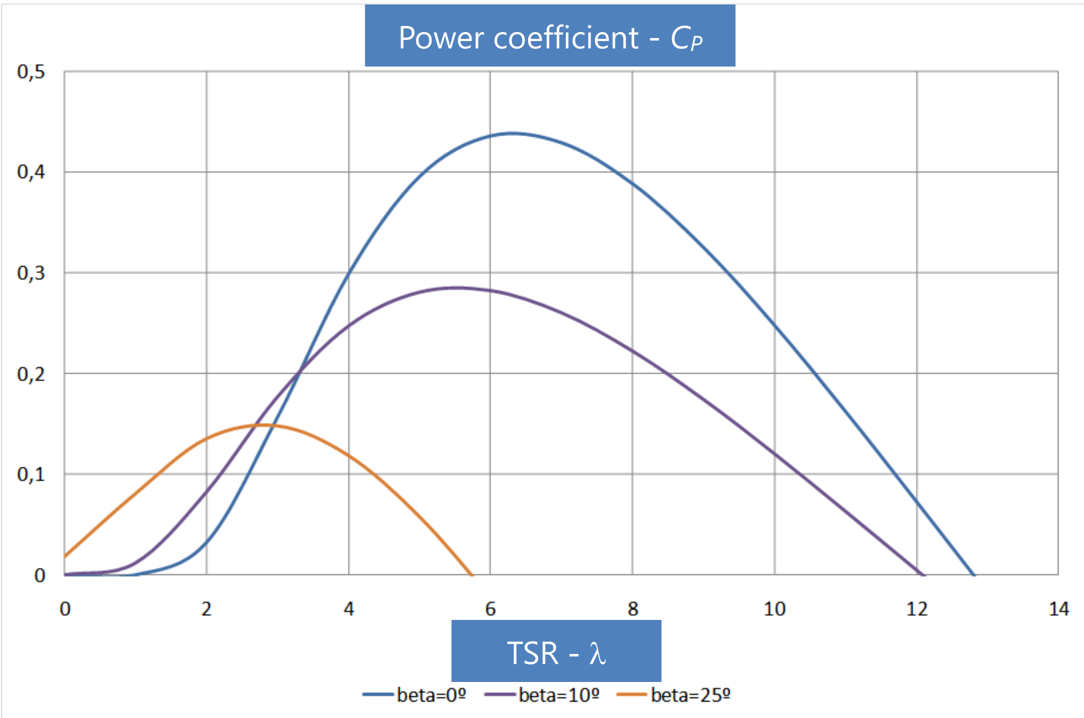

Figure 8-4: Typical variation of the power coefficient (C_P) with the TSR, for different pitch angles (beta).

We can see that the WTG efficiency decreases as the beta angle increases, as desired; maximum efficiency is obtained for beta=0 (no pitch). When the efficiency decreases, the output power also decreases. In this case, the pitch angle is controlled, so that a constant output power is obtained, through proper variation of the efficiency.

An analytic equation for obtaining the curves of Figure 8-4 is:

$$C_p = 0.22\left(\frac{116}{\lambda_i} - 0.4\beta - 5\right)\exp\left(-\frac{12.5}{\lambda_i}\right) \qquad \text{equation 8.3}$$

where:

$$\lambda_i = \frac{1}{\frac{1}{\lambda + 0.08\beta} - \frac{0.035}{\beta^3 + 1}} \qquad \text{equation 8.4}$$

The typical variation of the pitch angle with the wind speed is shown in Figure 8-5. For wind speeds lower than the rated wind speed, the pitch angle is zero, because, in this power curve zone, the objective is to maximize the efficiency. For wind speeds higher than the rated wind speed, the pitch angle continuously increases, so that to keep the output power constant, by constantly decreasing the efficiency.

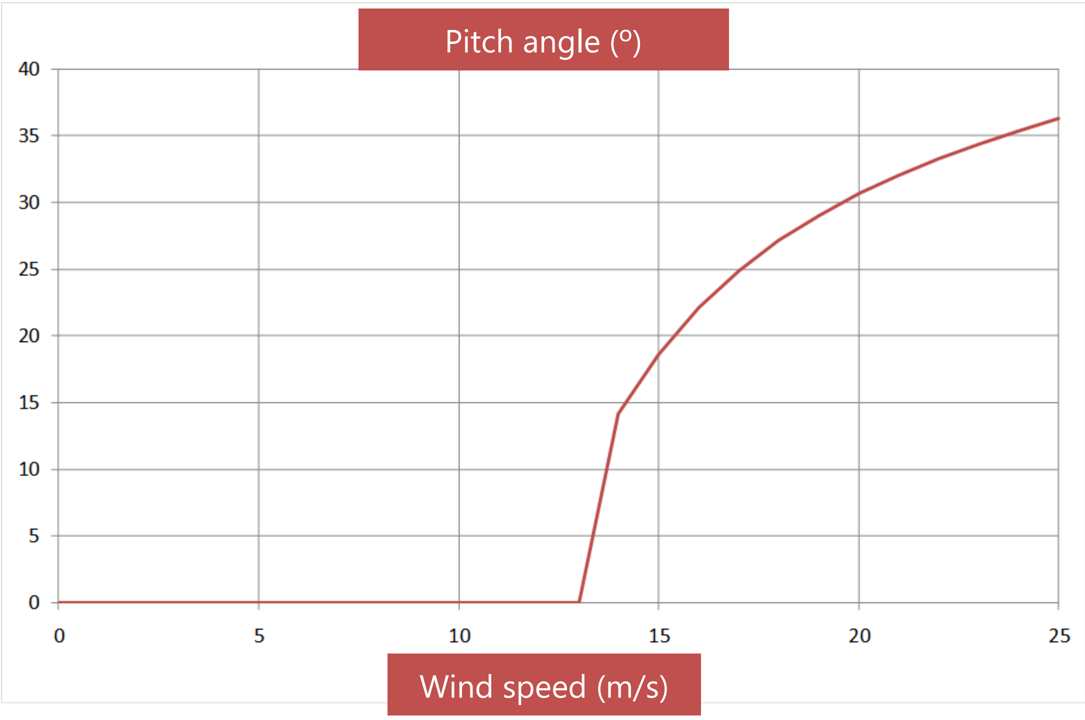

Figure 8-5: Typical variation of the pitch angle with the wind speed.