TÉCNICO LISBOA

Renewable Energy Sources and Dispersed Power Generation

Chapter 6

Small-Hydro Power

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List of Acronyms

FDC – Flow Duration Curve

NPV – Net Present Value

O&M – Operation and Maintenance

pu – per unit

SHP – Small Hydro Plant

RES – Renewable Energy Sources

INDEX

1 INTRODUCTION

In Portugal, the path towards the renewables was initiated in the early 80's by the Small-Hydro Plants (SHP). At that time, people begun to be aware of the finite nature of fossil fuels and their associated pollution problems. As Portugal had a valuable knowledge on large hydro plants, SHP were chosen by the investors to be the first truly renewable power installations in Portugal. In between, the interest for SHP decreased, as new economical places become hard to find and wind power begun its walking to current competitiveness with other power sources.

According to the standards, the following classifications apply:

Table 1-1: Small-Hydro Plants classifications.

	Capacity (MW)
Small-Hydro	< 10
Mini-Hydro	< 2
Micro-Hydro	< 0.5

A drawing with the basic components of a SHP is shown in Figure 1-1. Figure 1-2 shows the details of the powerhouse.

The water is taken at the intake and forwarded to the fore bay. Then, a penstock (the so-called hydraulic circuit) carries the water to the powerhouse where the potential and kinetic energy of the water is transformed into mechanical energy in a turbine and further converted into electrical energy using a generator. The water is finally returned to the river at the tail race. Not all the incoming water can be used to produce electricity, a residual flow must be left in the river, so that the river does not go completely dry.

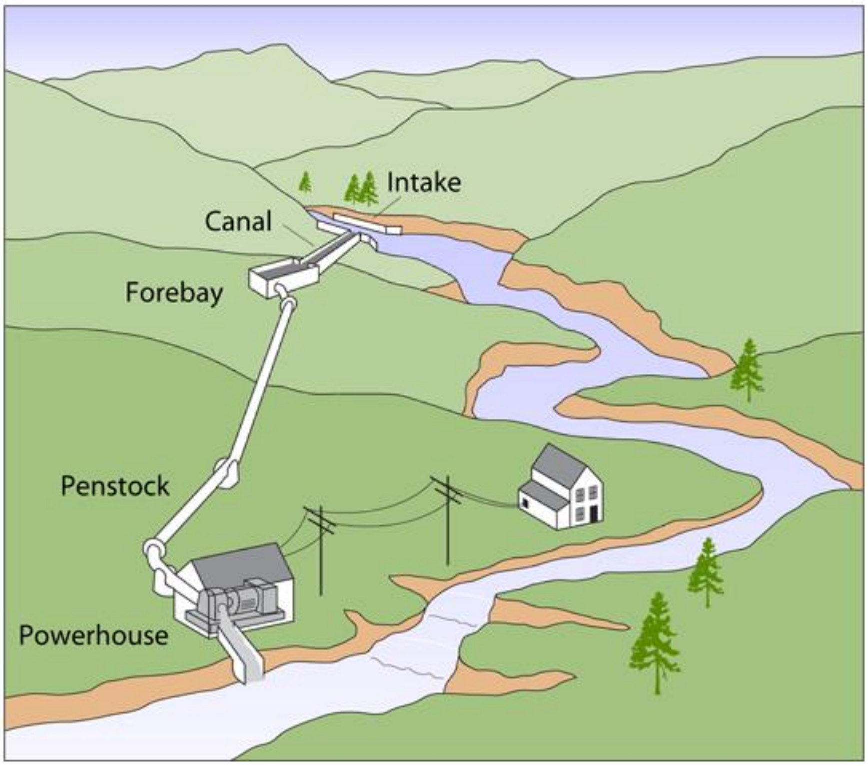


Figure 1-1: A Small-Hydro Plant.

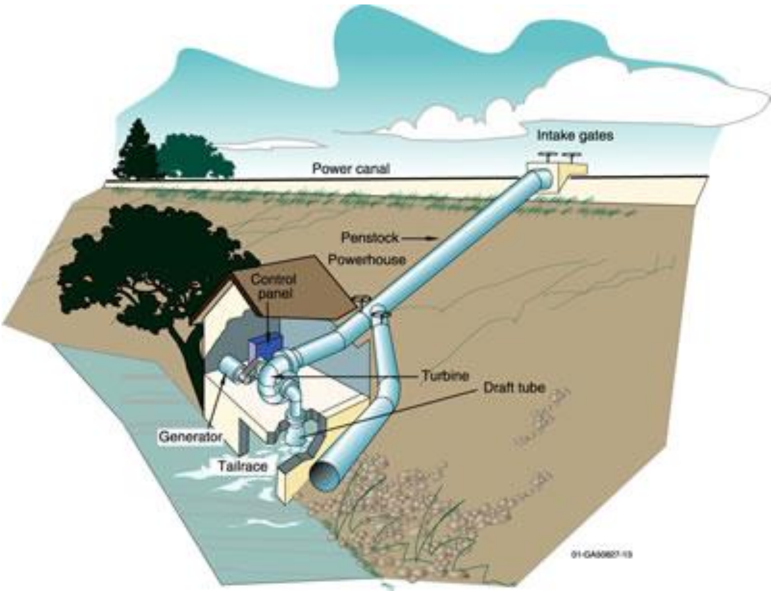


Figure 1-2: Inside the powerhouse of a Small-Hydro Plant.

2 Flow Duration Curve

Any assessment of a small-hydro project begins with information about the water flow. This information is usually gathered in the form of a chronological flow times series, an example of which is depicted in Figure 2-1 .

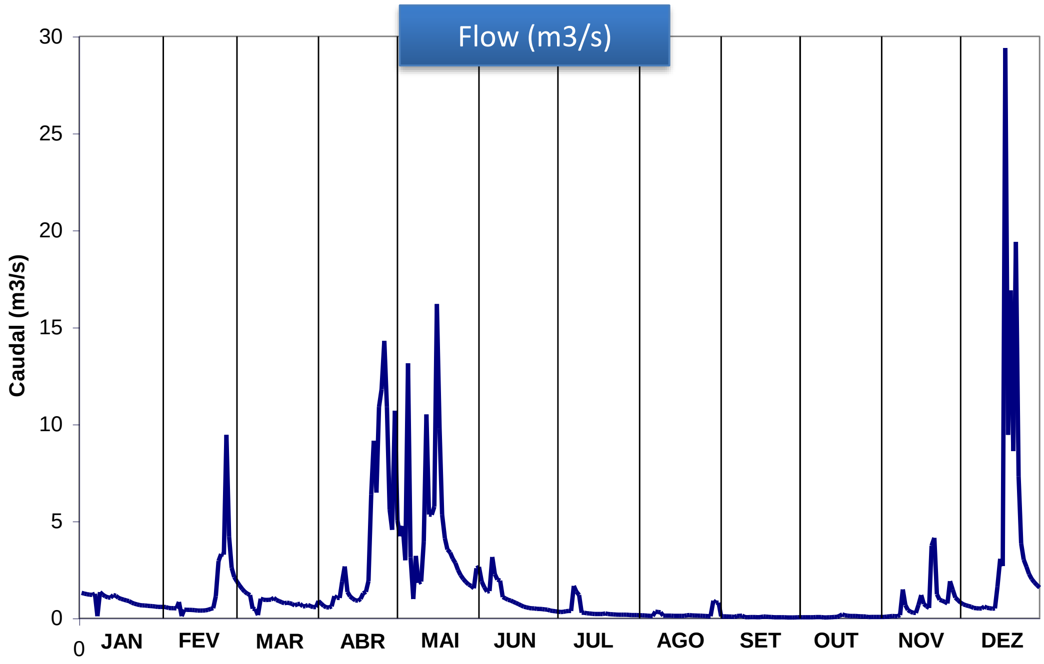


Figure 2-1: Chronological flow time series.

The base time is usually a day. As so, the chronological flow time series represents the daily average incoming flow to a particular section of a river. The daily averages can be based on the average of several years. So, for instance, the time series point corresponding to January 1st is the daily average flow on that day taken as an average of several years.

Then, the flows are ordered in a descending way, from the highest to the lowest: the first point is the maximum flow, the second point is the second highest flow, and so on. We obtain the Flow Duration Curve (FDC), as shown in Figure 2-2.

The information provided by the FDC is the number of days in the year a particular flow is exceeded. For instance, the maximum flow is exceeded 0 days in the year; the flow equal to 2.5 m^3/s is exceeded 50 days in the year.

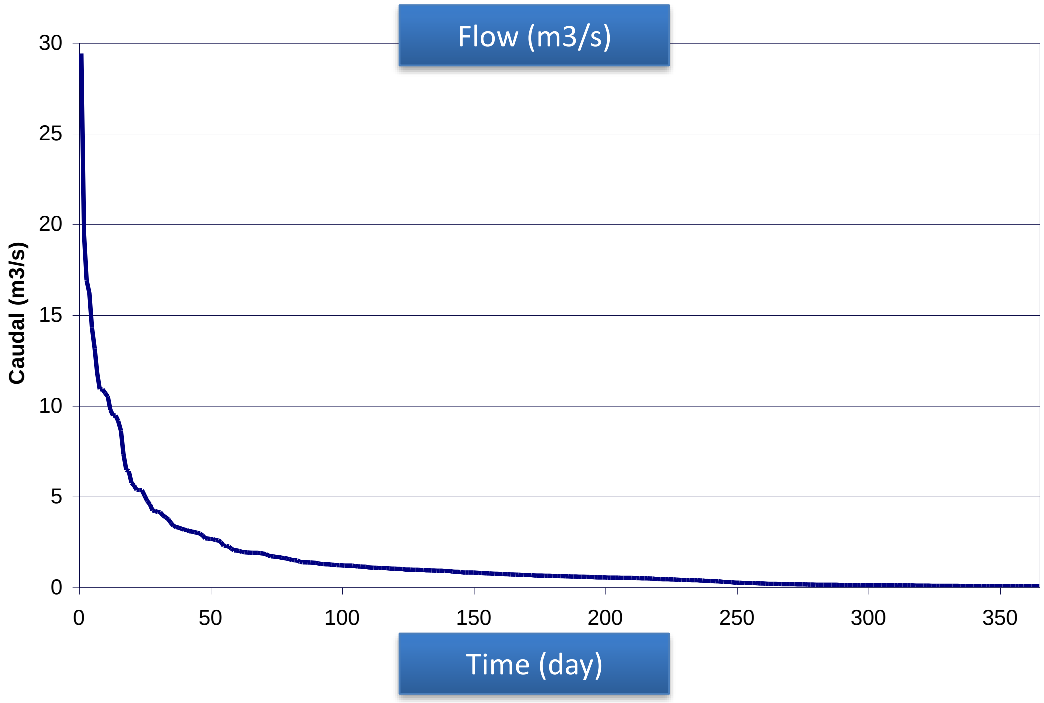


Figure 2-2: Flow Duration Curve (FDC).

If we divide all flows by the mean annual flow and the time by 365 days, we obtain the FDC in pu (per unit), as shown in Figure 2-3.

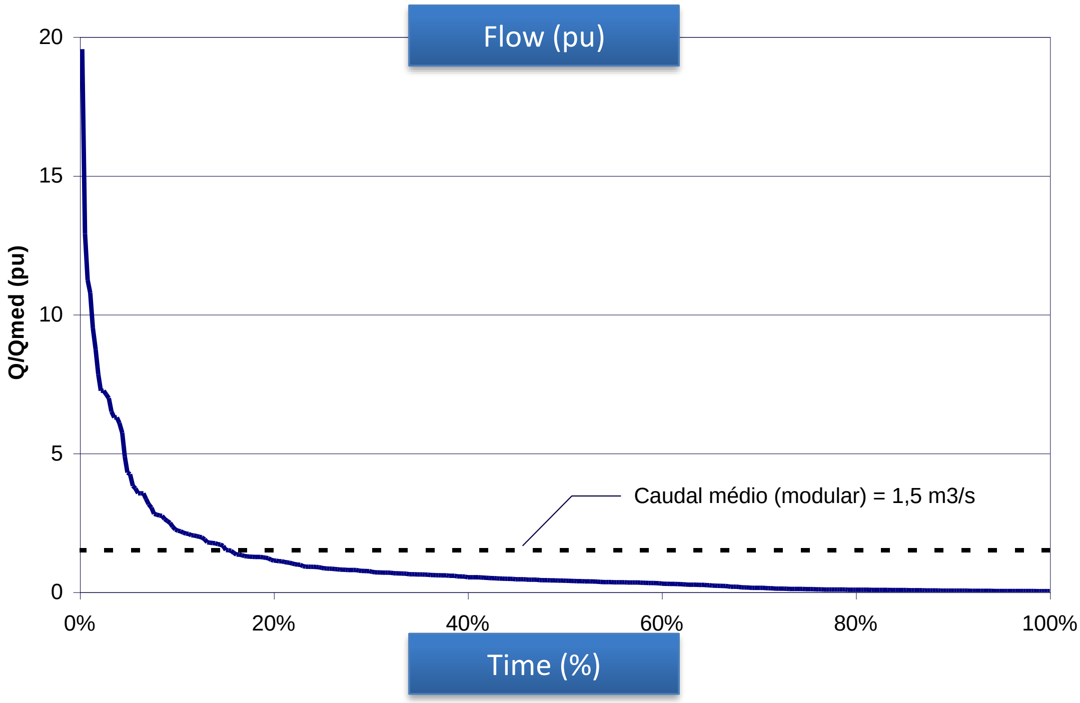


Figure 2-3: Flow Duration Curve (FDC) in pu.

3 TURBINE CHOICE

3.1 TURBINE TYPES

The detailed analysis of hydro turbines is outside the scope of this course. We offer hereafter some basics on hydro turbines. Basically, there are two types of hydro turbines: impulse turbines and reaction turbines.

In impulse turbines, the stator has nozzle jets and the rotor is a wheel with spoon-shaped buckets, at atmospheric pressure. They are used for high heads and low flows. Examples of impulse turbines are Pelton, Turgo and Banki-Mitchell hydro turbines, Pelton being the most know type. In Figure 3-1, we show the rotor and the nozzle jets of a Pelton turbine.

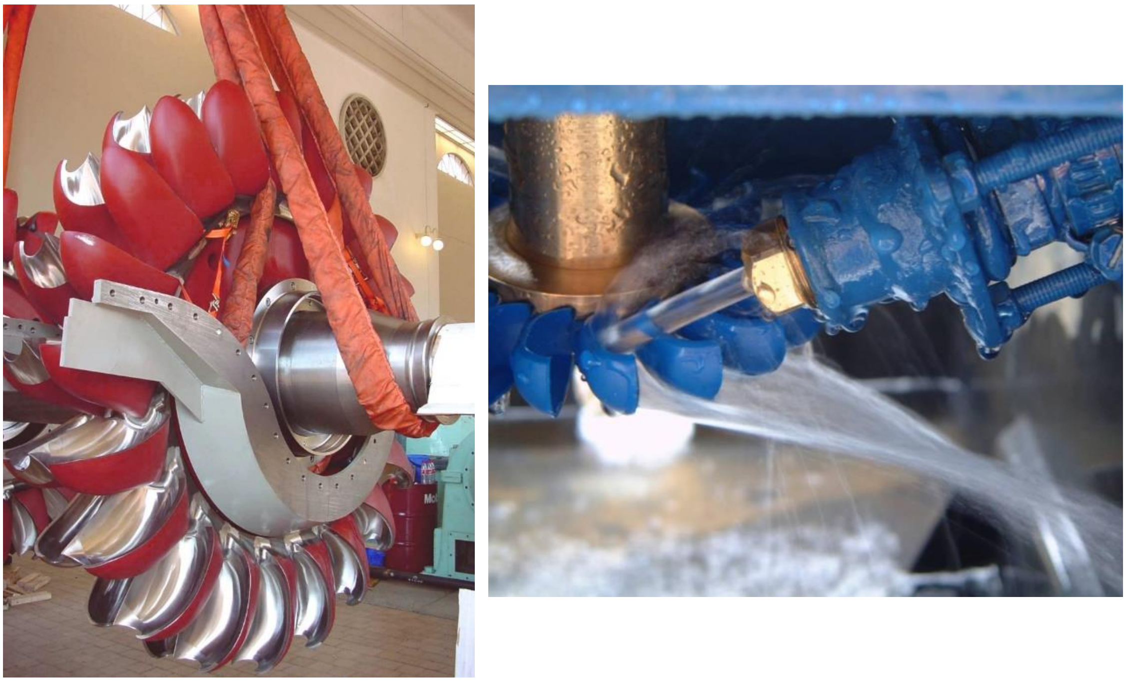


Figure 3-1: Rotor (left) and nozzle jets (right) of a Pelton turbine .

As far as reaction turbines are concerned, in the stator there is a distributor and the rotor is composed of a runner, where the pressure is not constant and the water is accelerated.

There are mainly two types of reaction turbines. Francis, which are used in intermediate heads and flows and Kaplan (or axial), used mainly in low heads and high flows. Kaplan turbines can have fixed runner blades, in this case they are called propeller; on the other hand, they can have double regulation – distributor guide vanes and blades are regulated; or single regulation – only blades are regulated. Details of a Francis turbine are shown in Figure 3-2, whereas in Figure 3-3 we present pictures showing details of Kaplan turbines.

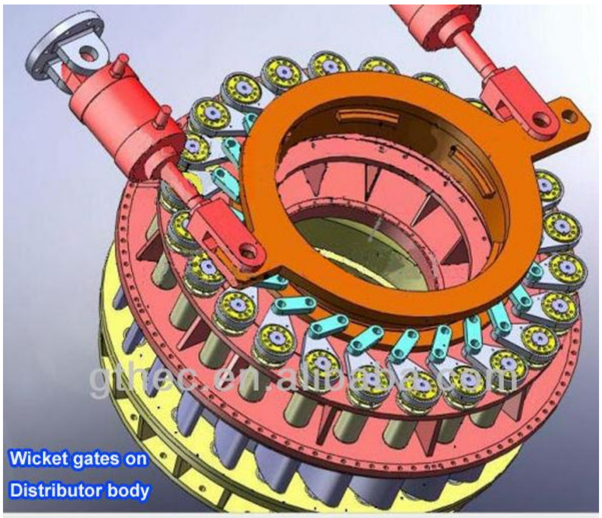
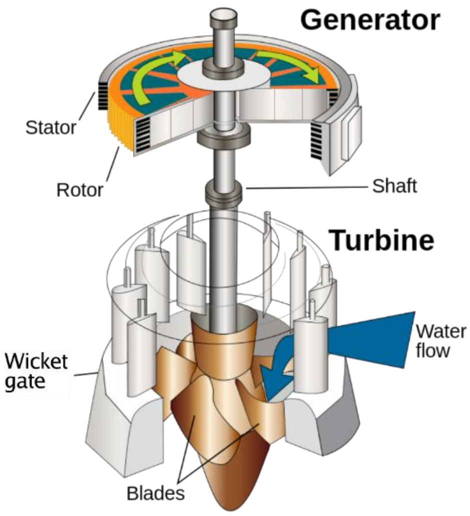


Figure 3-2: Details of a Francis turbine.



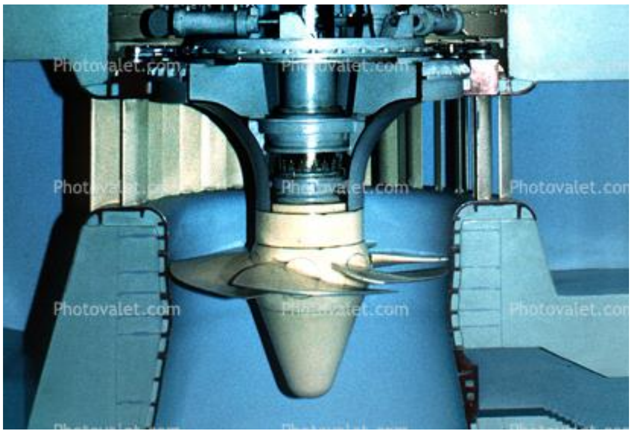


Figure 3-3: Details of Kaplan turbines.

3.2 Turbine Efficiency

An important characteristic of hydro turbines is the efficiency. The efficiency changes with the flow, the maximum efficiency being usually obtained for the rated flow[1]. The flatter the curve, the better, because it means that a high efficiency is obtained for a wide range of flow variation. Figure 3-4 shows typical efficiency curves for hydro turbines.

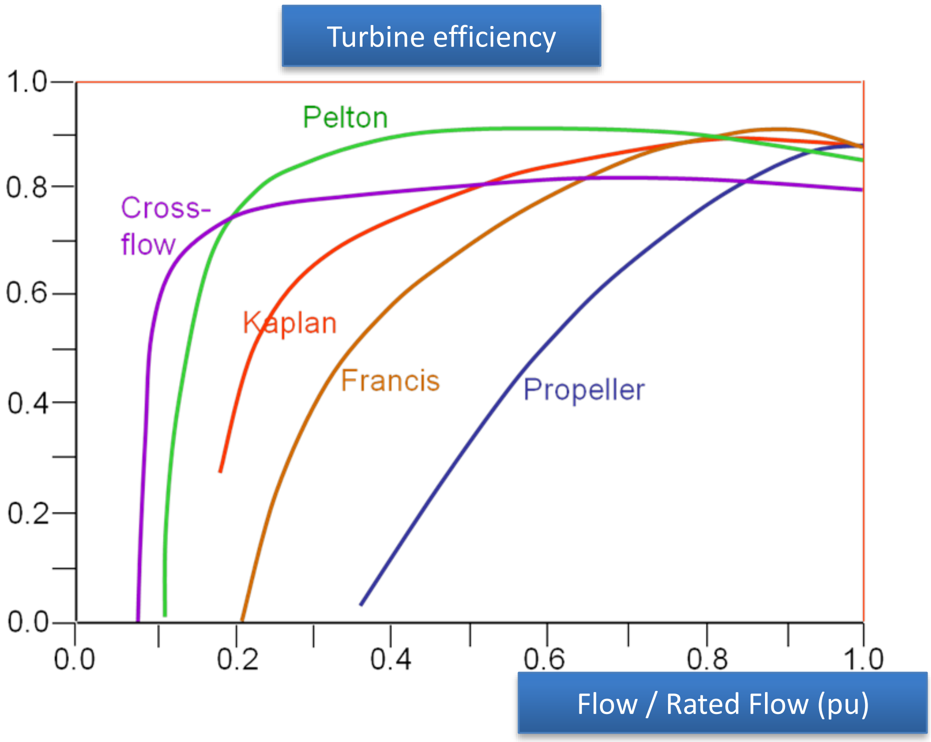


Figure 3-4: Hydro turbines typical efficiency curves.

3.3 Turbine Choice

The type of turbine depends both on the rated flow and on the head[2]. There are standard graphical tables that allow to choose the turbine type based on the knowledge of these two parameters. It goes without saying that the rated flow

[1] We will discuss later how to choose the rated flow.

[2] The head is the difference between the higher elevation and the lower elevation, i.e. the vertical change in elevation between the head (reservoir) water level and the tailwater (downstream) level.

and the head are main project parameters that must be known from the beginning. Figure 3-5 shows an example of a graphical table usually used for the turbine choice.

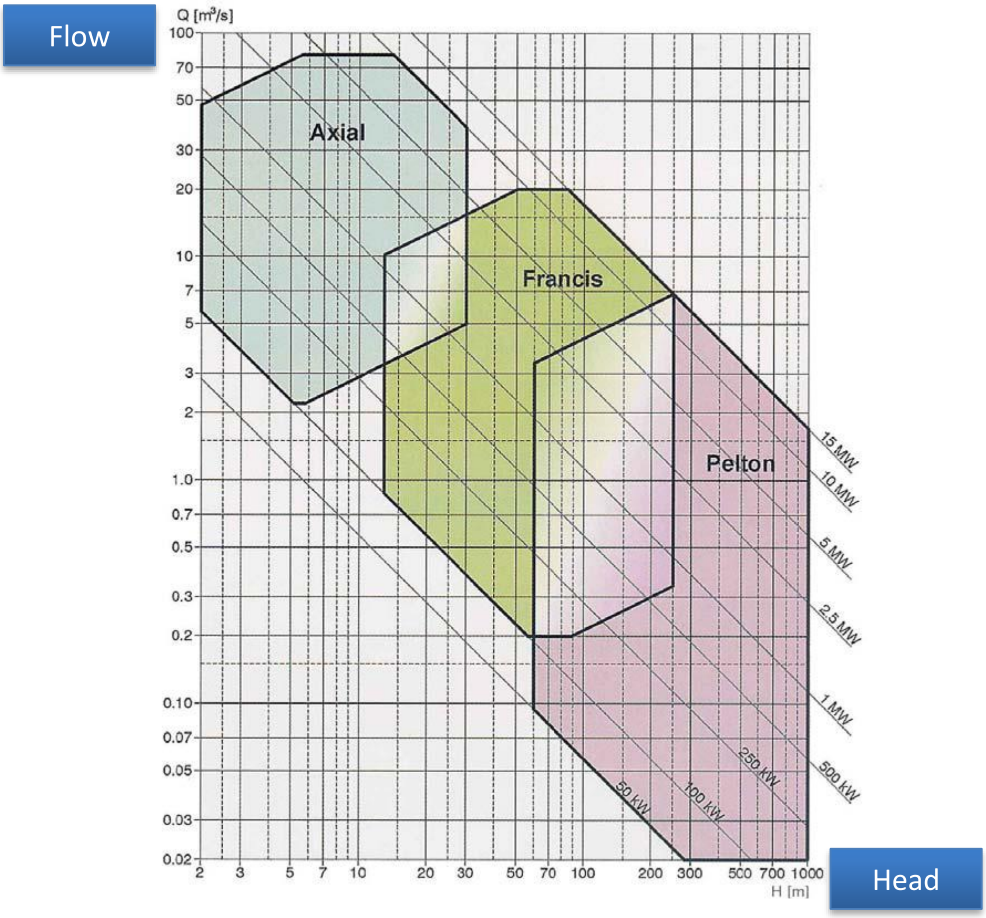


Figure 3-5: Standard graphical table for turbine choice; axial turbines are Kaplan turbines.

4 ELECTRICAL ENERGY YIELD – SIMPLIFIED MODEL

In this chapter, we are going through a simplified model to compute the electrical energy yield of a small-hydro power plant. This model is used in early stages of the project and provides an estimation of the electricity production that can be expected.

The first step is to define the rated flow. In the past, the project engineer selected the rated flow based on his own experience, his choice usually laying in a rated flow that is exceeded in 15% to 40% of the time. Nowadays, powerful computation tools are available and the rated flow is chosen as the one that maximizes an economic assessment index, the NPV, for instance. The process of selecting the rated flow is therefore an iterative process.

The first approach for the rated flow is usually to take it as the mean annual flow. Then, the electricity production is computed and a NPV is obtained. The process is repeated for different values of the rated flow, the final decision being the rated flow that maximizes the NPV. In what follows, it is assumed that the rated flow is known.

The rated capacity, P_N, that can be installed in a small-hydro plant is given by:

$$P_N = \gamma Q_N H_b \eta_c \qquad \text{equation 4.1}$$

where γ is the water specific weight ($\gamma = \rho g = 9810$ N/m^3), Q_N is the rated flow in m^3/s, H_b is the gross head in m and η_c is the overall efficiency of the power plant.

The global efficiency of a small-hydro plant is usually around 70%. Therefore, an approximate equation for easily computing the rated capacity of a small-hydro plant in kW is:

$$P_N = 7 Q_N H_b \quad \text{(kW)} \qquad \text{equation 4.2}$$

From the relationship between power and energy, we can write that the annual electricity production is:

$$E_a = \int P(t)dt = 9810\int Q(t)h_u(t)\eta(t)dt \qquad \text{equation 4.3}$$

where we assume that all quantities in equation 4.1 can change in time. h_u is the useful head (gross head minus all the hydraulic losses) and η is the combined efficiency of the turbine, generator, transformer and auxiliary equipment.

4.1 One Single Turbine

To begin with, we shall address the case of a small-hydro plant equipped with a single turbine.

We target a simplified model. So, we assume that both the head and the efficiency are constant. The head is equal to the gross head and the overall efficiency is around 70%.

We have to take a closer look at the flow, which is impossible to assume as constant, because it is apparent from the FDC that it changes. The process to take the flow variation into account will be described next.

The first step is to compute the operating area as shown in Figure 4-1.

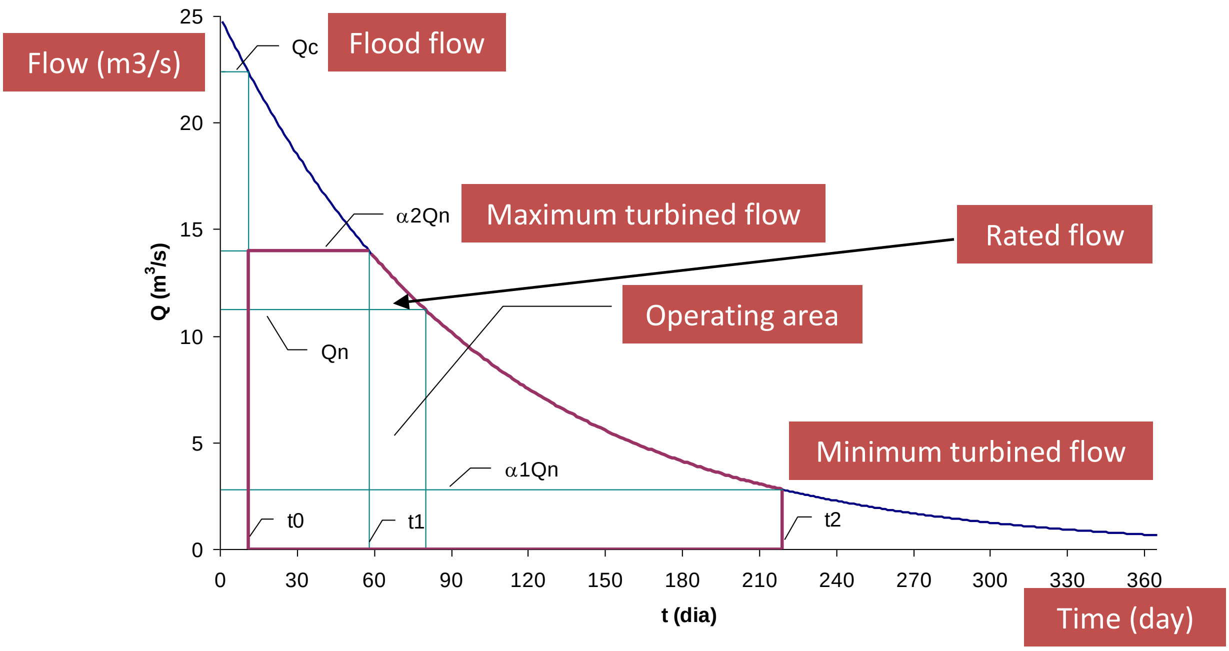


Figure 4-1: Operating area computation.

The rated flow, which is assumed to be known is marked in the FDC. Only for the purpose of this simplified model, two factors are defined for each type of hydro turbine:

$$\alpha_1 = \frac{Q_{mT}}{Q_N}$$
$$\alpha_2 = \frac{Q_{MT}}{Q_N} \qquad \text{equation 4.4}$$

where Q_{MT} is the maximum turbined flow, Q_{mT} is the minimum turbined flow and Q_N is the rated flow. The value of the two factors is defined in Table 4-1.

Table 4-1: Factors α_1 and α_2 for each type of turbine; simplified model only.

Turbine	α_1	α_2
Pelton	0.15	1.15
Francis	0.35	1.15
Double regulation Kaplan	0.25	1.25
Single regulation Kaplan	0.4	1.0
Propeller	0.75	1.0

This means that for flows higher than the maximum turbined flow, the turbine is regulated to operate at maximum turbined flow, the excess water being wasted. For flows lower than the minimum turbined flow, the turbine is disconnected, because the efficiency is too low, the remaining water being not used. In the operating area, as defined in Figure 4-1, it is assumed that the turbine's efficiency is constant (please refer to Figure 3-4). The operating area is further reduced by considering the flood flow, Q_c. For flows higher than the flood flow it is considered that the turbine is disconnected due to lack of head. The flood flow is assumed to be known. The operating area is thereby the red area in Figure 4-1. The available water outside this area is not used to produce electricity.

From the FDC, times t_0, t_1 and t_2 can be determined knowing the flood flow, the maximum turbined flow and the minimum turbined flow, respectively, as indicated in Figure 4-1 and in equation 4.5. We recall that the rated flow is supposed to be known.

$$\begin{aligned} Q(t_0) &= Q_c \\ Q(t_1) &= \alpha_2 Q_N = Q_{MT} \\ Q(t_2) &= \alpha_1 Q_N = Q_{mT} \end{aligned} \qquad \text{equation 4.5}$$

The electricity production is proportional to the operating area. As so, the electrical energy yield in kWh can be computed through:

$$E_a = 7H_b\left(\int_{t_1}^{t_2} Q(t)dt + (t_1 - t_0)\alpha_2 Q_N\right)24 \quad \text{(kWh)} \qquad \text{equation 4.6}$$

The quantity $\int_{t_1}^{t_2} Q(t)dt + (t_1 - t_0)\alpha_2 Q_N$ is the operating area. It is assumed that the global efficiency is around 70% and that the gross head, H_b, is constant. The multiplication by the 24 factor is to obtain kWh as energy unit instead of kWday.

4.2 Two Equal Turbines

Let us now address the case of a small-hydro plant equipped with two equal turbines. The objective is to find the differences relative to the one turbine case.

The turbines are equal. So, $Q_{N1} = Q_{N2} = Q_N/2$. In this situation, the clever way of operating the turbines is the following:

- For flows less than the rated flow of one turbine, let's say turbine 1, only turbine 1 is operating, turbine 2 being disconnected.
- For flows higher than the rated flow of turbine 1, the two turbines are operating, the flow being equally split by the two turbines.

Mathematically, this translates into Table 4-2.

Table 4-2: Two equal turbines operating mode.

Q_j	Turbine 1	Turbine 2
$0 \leq Q_j < Q_N/2$	Q_j	0
$Q_N/2 < Q_j \leq \alpha_2 Q_N$	$Q_j/2$	$Q_j/2$

This is clever because it is better, from an efficiency point-of-view, to have one turbine at full capacity than two turbines at half capacity.

The maximum turbined flow is unchanged relatively to the single turbine case, because:

$$Q_{MT} = \sum_{i=1}^{2} \alpha_2 Q_{Ni} = \alpha_2 Q_N \quad \text{equation 4.7}$$

As for the minimum turbined flow, it is now different (lower) from the single turbine case.

$$Q_{mT} = \alpha_1 Q_{N1} = \alpha_1 Q_{N2} < \alpha_1 Q_N \quad \text{equation 4.8}$$

In Figure 4-2, this change in the minimum turbined flow is highlighted.

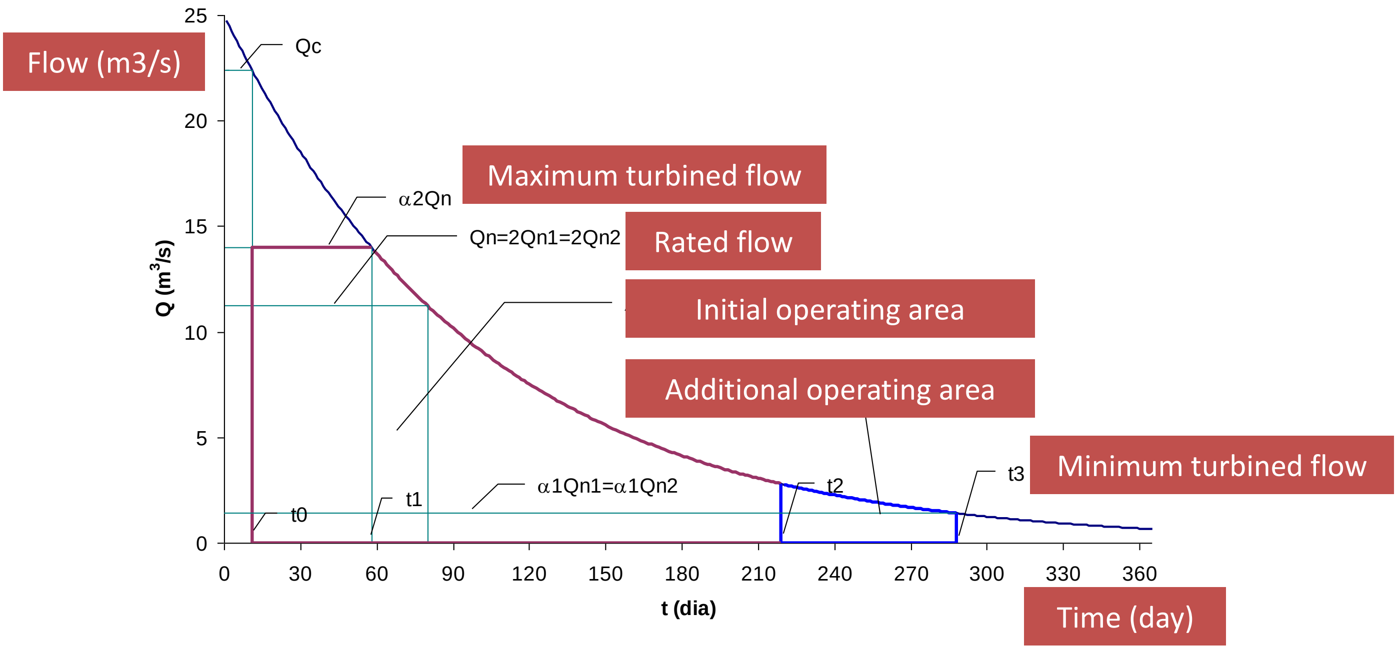


Figure 4-2: Additional operating area computation for the two equal turbines case.

A new time t_3 is now defined as:

$$Q(t_3) = \alpha_1 Q_{N1} = \alpha_1 \frac{Q_N}{2} \qquad \text{equation 4.9}$$

The total energy produced in the case the small hydro plant is equipped with two equal turbines is:

$$E_a = 7H_b\left(\int_{t_1}^{t_3} Q(t)dt + \left(t_1 - t_0\right)\alpha_2 Q_N\right)24 \quad \text{(kWh)} \qquad \text{equation 4.10}$$

Comparing equation 4.6 and equation 4.10, it is apparent that, for the same installed capacity, a small-hydro plant equipped with two turbines will produce always more energy than if it is equipped with a single turbine. This is because, a smaller turbine can pick-up lower flows. However, this does not mean that a two turbine installation is better than a single turbine one. Indeed, for the same installed capacity, the cost of two equally sized turbines, with half the rated capacity each, is higher than the cost of a full rated capacity single turbine. The same applies for the O&M costs. The best solution is provided by the NPV computation, which determines if the increased investment is compensated by increased production.

5 ELECTRICAL ENERGY YIELD – INTRODUCTION TO THE DETAILED MODEL

In a design phase, more detailed models are needed to compute the electrical energy yield of a small-hydro plant. One of these models is going to be presented next.

In real life, the FDC is not described by an analytic equation. In fact, what is usually available is a curve described by a set of discrete points (t_i, $Q(t_i)$). An example is provided in Figure 5-1, where 21 data points of the FDC are available.

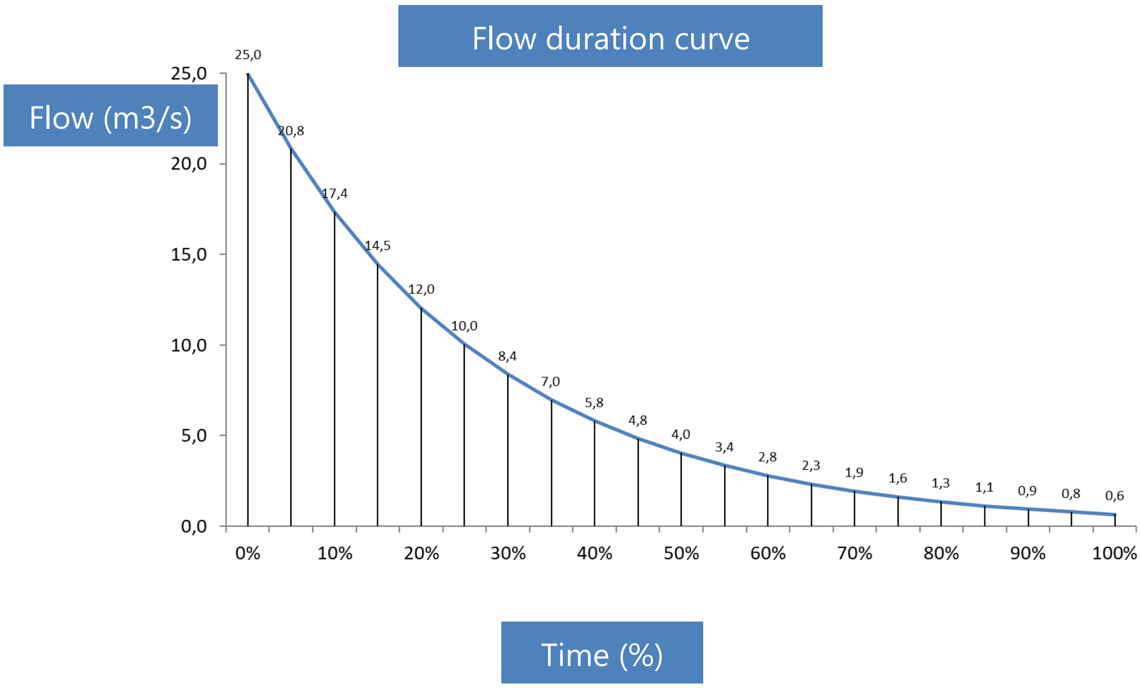


Figure 5-1: Example of a usually available discrete FDC.

The procedure in this method involves the computation of the power curve, i.e. the variation of the output power with time and then compute the energy supplied as the area below the power curve.

5.1 COMPUTING THE OUTPUT POWER AS A FUNCTION OF THE FLOW

The general equation to compute the output electrical power as a function of the incoming flow is:

$$P(Q_i) = \gamma Q_{i\_u}\left[H_b - h_{hydr}(Q_{i\_u}) - h_{flood}(Q_i)\right]\eta_t(Q_{i\_u})\eta_g\eta_{transfo}(1-p_{other}) \qquad \text{equation 5.1}$$

where:

- P is the electrical power output in W.
- Q_i is the incoming flow at point i of the FDC in m^3/s.
- γ is the water specific weight 9810 N/m^3.
- H_b is the gross head in m.
- h_{hydr} are the hydraulic circuit losses for the used flow Q_{i_u} in m.
- Q_{i_u} is the used flow in m, which is the flow that is forwarded to the turbine.
- h_{flood} are the flood losses for the incoming flow Q_i in m.
- η_t is the turbine efficiency for the used flow Q_{i_u}.
- η_g is the generator efficiency, which is assumed to be constant.
- $\eta_{transfo}$ is the transformer efficiency, which is assumed to be constant.
- p_{other} are other losses, e.g. in the auxiliary services of the power station, which are assumed to be constant.

A residual flow must be left in the river, so that the riverbed is not dry. We, therefore, define the available flow, Q'_i, as:

$$Q'_i = \max(Q_i - Q_r\ ;\ 0) \qquad \text{equation 5.2}$$

where Q_r is the residual flow. The residual flow must be known, normally it is about 3%–5% of the rated flow.

In this model, it is assumed that the maximum turbined flow is equal to the rated flow. We recall that no α_1 and α_2 apply in the detailed model. Thus, the used flow, i.e. the flow that is forwarded to the turbine, is:

$$Q_{i\_u} = \min(Q'_i\ ;\ Q_N) \qquad \text{equation 5.3}$$

The hydraulic circuit losses are losses inside the pipes (penstock) that carry the water from the higher elevation to the powerhouse. The hydraulic circuit head losses are proportional to the square of the flow, in the same way the Joule losses are proportional to the square of the current, in electrical circuits. Accordingly, we can write:

$$h_{hydr} = H_b p_{hydr}^{\max} \left(\frac{Q_{i\_u}}{Q_N} \right)^2 \qquad \text{equation 5.4}$$

where $p_{hydr}^{\max}$ is the maximum value of the hydraulic circuit losses as a percentage of the gross head, H_b. Its value must be known, a common value being about 3%–5%. Of course, the hydraulic circuit losses depend upon the used flow, as this is the turbined flow, the one that travels inside the penstock. These losses are maximum for the rated flow, which is the maximum turbined flow.

The flood flow losses are not related to the used flow, but instead to the incoming flow. In fact, a flood flow causes a reduction in the available head, this reduction being accounted for in this parameter. The flood losses are maximum for the maximum incoming flow and are null for the rated flow. It makes no sense to compute the flood losses for flows lower than the rated flow, because these flows do not cause flood. Hence, the flood flow losses can be written as:

$$h_{flood} = h_{flood}^{\max} \left(\frac{Q_i - Q_N}{Q_{\max} - Q_N} \right)^2 \quad Q_i > Q_N \qquad \text{equation 5.5}$$

where $h_{flood}^{\max}$ is the maximum value of the flood losses in m, which must be known or estimated somehow. $Q_{\max}$ is the maximum incoming flow (the maximum of the FDC).

The efficiency of the turbine is specific of each turbine. Analysing several efficiency curves for the different types of turbines, as the ones showed in Figure 3-4, the following general-purpose model for the efficiency of any turbine was built:

$$\eta_t = \left\{1 - \left[\alpha\left|1 - \beta\frac{Q_{i\_u}}{Q_N}\right|^{\chi}\right]\right\}\delta$$ equation 5.6

The efficiency of the turbine depends on the used flow as would be expected. The four parameters of equation 5.6 depend on the type of turbine as shown in Table 5-1.

Table 5-1: Parameters of the efficiency model; h'u: useful head at rated flow; n: number of nozzle jets.

	Propeller	**Kaplan**	**Francis**	**Pelton**
α	1.25	3.5	1.25	$1.31+0.025n$
β	1.00	1.333	$1.1173h'^{\,0.025}_{u}$	$(0.662+0.001n)^{-1}$
χ	1.13	6.0	$3.94-11.7h'^{\,-0.5}_{u}$	$5.6+0.4n$
δ	0.905	0.905	0.919	0.864

Regarding Table 5-1, it can be seen that the Francis turbine efficiency depends on the useful head at rated flow, h'_u. The useful head, h_u, is defined as:

$$h_u = H_b - h_{hydr}(Q_{i\_u}) - h_{flood}(Q_i)$$ equation 5.7

The useful head at rated flow is:

$$\begin{aligned} h'_u &= h_u(Q_N) = H_b - h_{hydr}(Q_N) - h_{flood}(Q_N) = \\ &= H_b - p_{hydr}^{max} H_b - 0 = \\ &= H_b\left(1 - p_{hydr}^{max}\right) \end{aligned}$$ equation 5.8

When the flow reduces, the efficiency of the turbine is also reduced. As far as the detailed model is concerned, a minimum turbined flow, Q_{mT}, is defined. For used flows lower that this minimum, the turbine is disconnected, because the efficiency is too low. Table 5-2 shows the minimum turbined flow as a percentage of the rated flow.

Table 5-2: Minimum turbined flow.

Turbine type	Q_{mT}/Q_N
Propeller	65%
Kaplan	15%
Francis	30%
Pelton	10%

5.2 INSTALLED CAPACITY

To compute the installed, or rated, capacity, equation 5.1 is used as follows:

$$P_N = P(Q_N) = \gamma Q_N \left[H_b - h_{hydr}(Q_N) \right] \eta_t(Q_N) \eta_g \eta_{transfo} (1 - p_{other}) \qquad \text{equation 5.9}$$

5.3 THE POWER CURVE AND ELECTRICAL ENERGY YIELD

We recall that we assumed we had a FDC discretized in a number of points. For each point (t_i, Q_i), the output power, for each incoming flow Q_i, is computed through equation 5.1. In the FDC, to each incoming flow, Q_i, corresponds a time in which it is exceeded. Therefore, a power curve can be obtained, which plots the evolution of the output power as a function of time.

It is well-know that the area below the power curve is the electricity produced by the power plant. Assuming a FDC discretization in equal time intervals of ΔT (in %), in n points, using the trapezoidal integration method, we can write:

$$E_a = \sum_{k=1}^{n} \left(\frac{P_{k-1}(Q) + P_k(Q)}{2} \right) \Delta T (1 - p_{unav}) 8760 \qquad \text{equation 5.10}$$

where p_{unav} are the losses due to the unavailability of either the power plants (e.g. maintenance) or the grid (e.g. failures or maintenance).

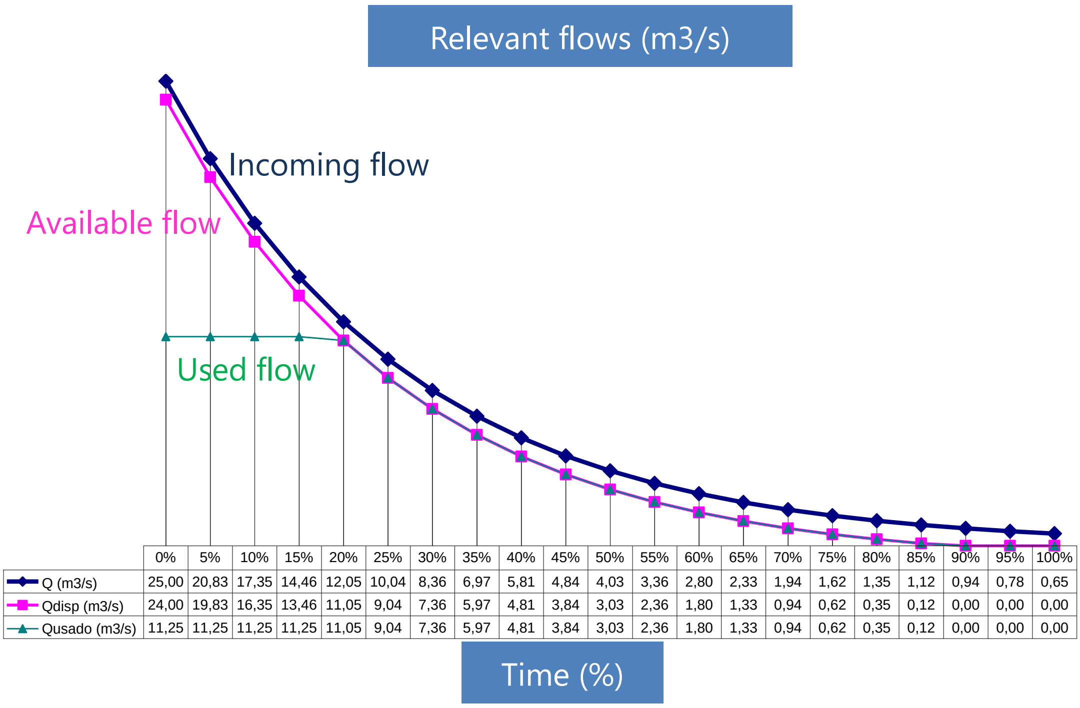
Let us look at an example that illustrates how the model works.

Example 5—1

Consider a SHP with a gross head of 6.35 m where the FDC is described by $Q(t) = 25\exp(-t/100)$ *(t in days and Q in* m^3/s*). The maximum hydraulic circuit head losses are 4% and the maximum flood losses are 6.1 m. The efficiencies of the generator and transformer are 95% and 99%, respectively, and other losses can be fixed in 2%. The unavailability losses are estimated in 4%. The SHP is equipped with a double regulation Kaplan turbine. The rated flow is 11.25* m^3/s *and the residual ecologic flow is 1* m^3/s*. Compute the annual electricity yield of the SHP.*

Solution:

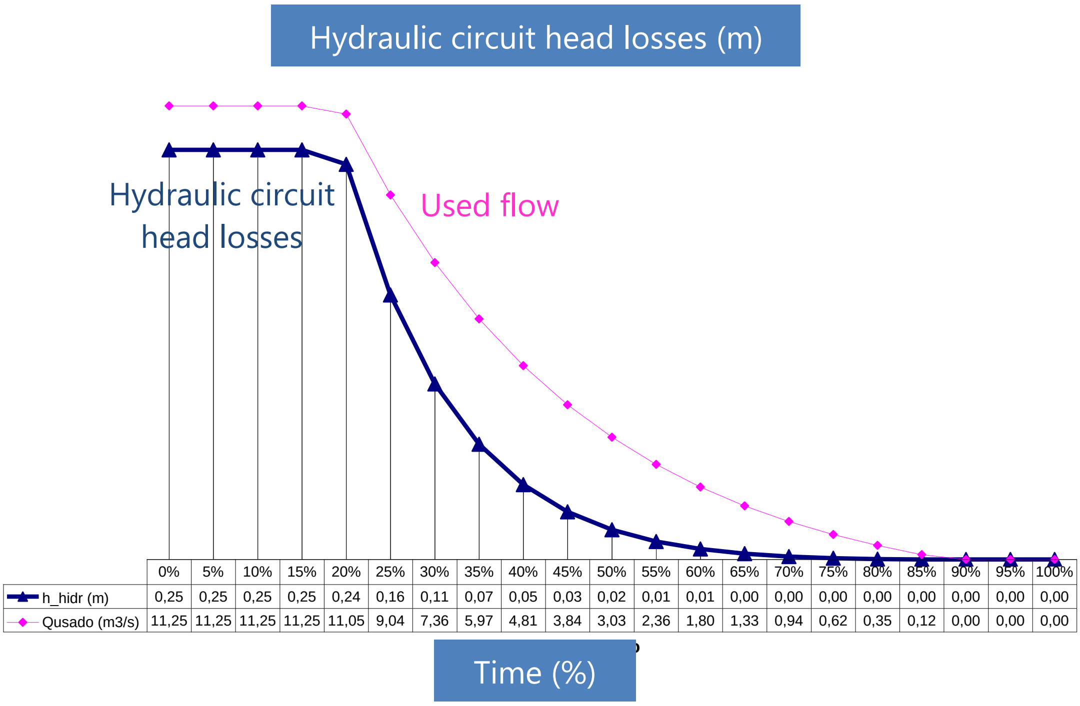
First of all, let us handle the flows: incoming flow (Q_i*), available flow (*Q'_i*) and used flow (*Q_{i_u}*). The FDC is discretized in time intervals of 5% of the number of days in one year. The results are shown in* Figure 5-2*.*



	0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%
Q (m3/s)	25,00	20,83	17,35	14,46	12,05	10,04	8,36	6,97	5,81	4,84	4,03	3,36	2,80	2,33	1,94	1,62	1,35	1,12	0,94	0,78	0,65
Qdisp (m3/s)	24,00	19,83	16,35	13,46	11,05	9,04	7,36	5,97	4,81	3,84	3,03	2,36	1,80	1,33	0,94	0,62	0,35	0,12	0,00	0,00	0,00
Qusado (m3/s)	11,25	11,25	11,25	11,25	11,05	9,04	7,36	5,97	4,81	3,84	3,03	2,36	1,80	1,33	0,94	0,62	0,35	0,12	0,00	0,00	0,00

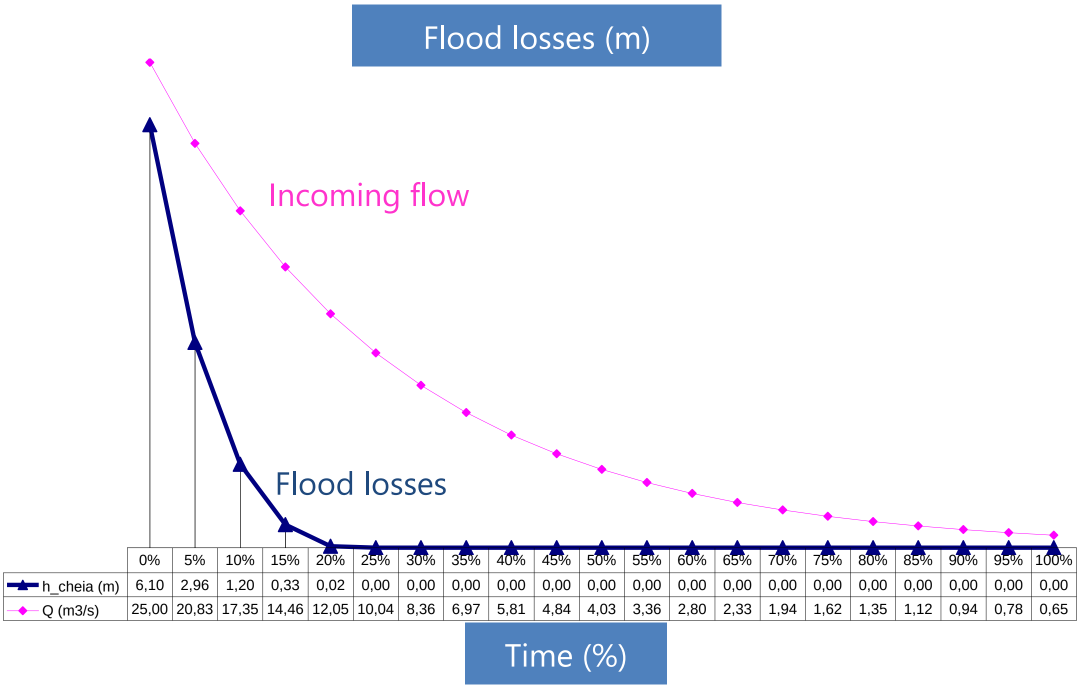
Figure 5-2: Incoming, available and used flows in a discretized FDC.

Then, let us handle the hydraulic losses – the hydraulic circuit head losses that depend upon the used flow, and the losses due to flood flows that depend on the incoming flow. We recall that it is $Q_N = 11.25\ m^3/s$ *and* $Q_{max} = 25\ m^3/s$ *(the maximum incoming flow). The results are presented in* Figure 5-3 *and* Figure 5-4*.*



	0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%
h_hidr (m)	0,25	0,25	0,25	0,25	0,24	0,16	0,11	0,07	0,05	0,03	0,02	0,01	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Qusado (m3/s)	11,25	11,25	11,25	11,25	11,05	9,04	7,36	5,97	4,81	3,84	3,03	2,36	1,80	1,33	0,94	0,62	0,35	0,12	0,00	0,00	0,00

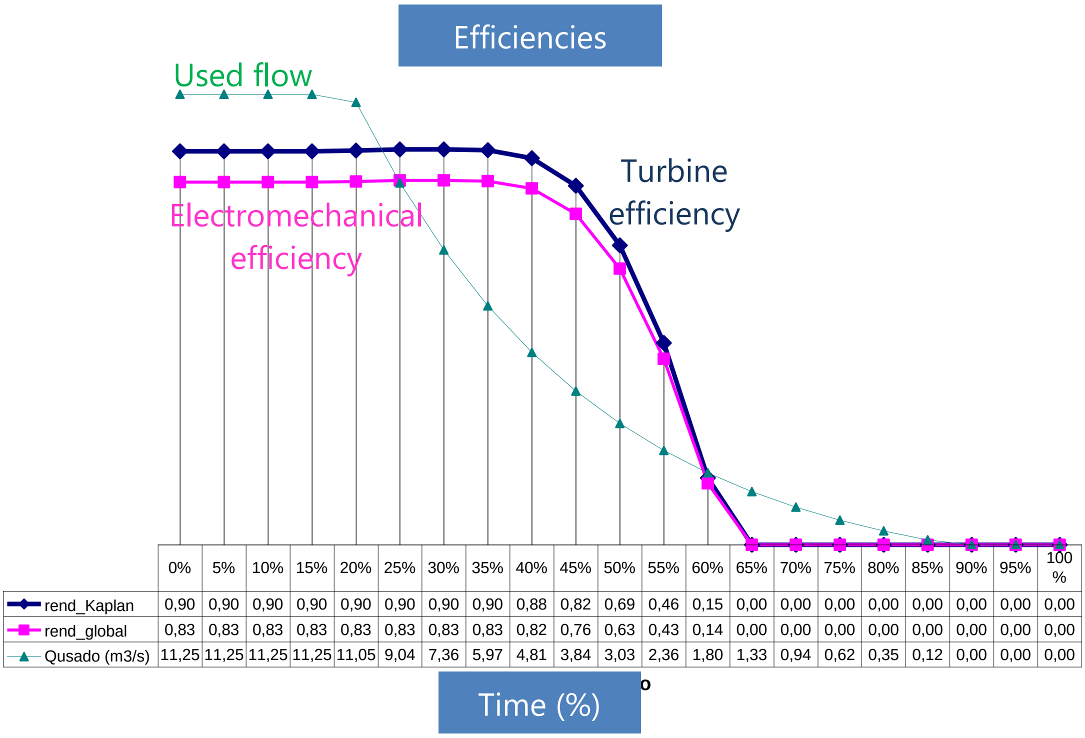
Figure 5-3: Hydraulic circuit head losses.



	0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%
h_cheia (m)	6,10	2,96	1,20	0,33	0,02	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Q (m3/s)	25,00	20,83	17,35	14,46	12,05	10,04	8,36	6,97	5,81	4,84	4,03	3,36	2,80	2,33	1,94	1,62	1,35	1,12	0,94	0,78	0,65

Figure 5-4: Flood losses.

We now proceed to the computation of the efficiencies of the SHP components – the Kaplan turbine, which depends on the used flow (the parameters are: $\alpha = 3.5; \beta = 1.333; \chi = 6.0; \delta = 0.905$ *) and the generator, transformer and other losses, which are constant.* Figure 5-5 *displays the obtained results.*



	0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100 %
rend_Kaplan	0,90	0,90	0,90	0,90	0,90	0,90	0,90	0,90	0,88	0,82	0,69	0,46	0,15	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
rend_global	0,83	0,83	0,83	0,83	0,83	0,83	0,83	0,83	0,82	0,76	0,63	0,43	0,14	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Qusado (m3/s)	11,25	11,25	11,25	11,25	11,05	9,04	7,36	5,97	4,81	3,84	3,03	2,36	1,80	1,33	0,94	0,62	0,35	0,12	0,00	0,00	0,00

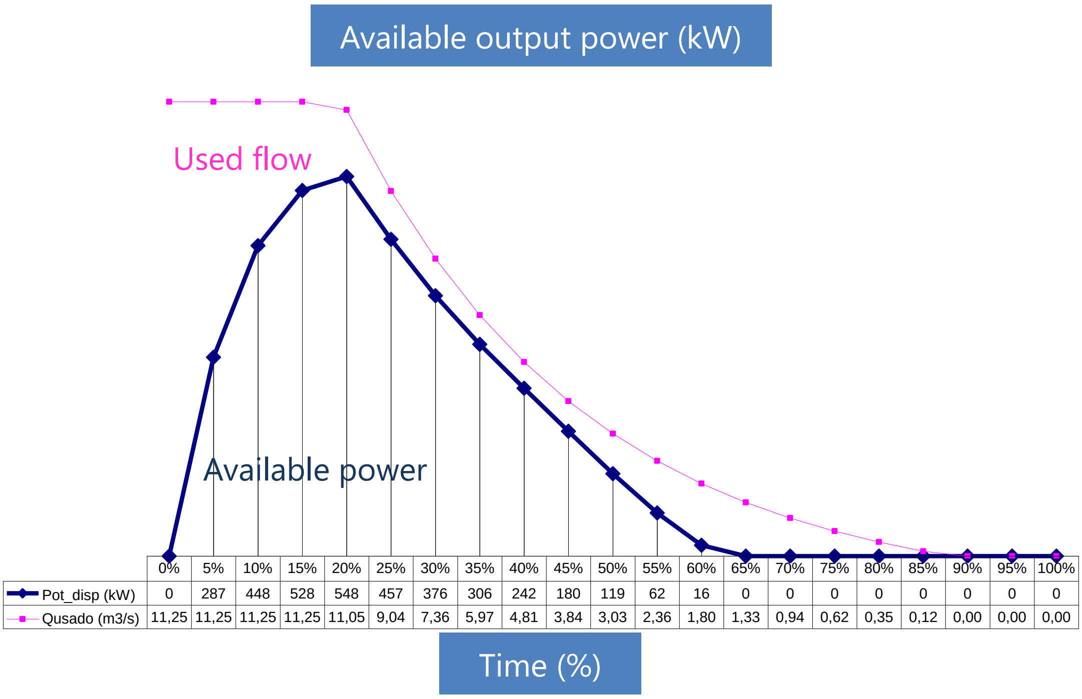
Figure 5-5: Efficiencies of the components.

We note that for used flows lower than 15% of the rated flow, the Kaplan turbine is disconnected, because the minimum turbined flow is attained.

Now, for each point, we can build the power curve using equation 5.1*, as depicted in Figure 5-6.*

The annual electrical energy yield is now straightforward computed through the area bellow the power curve, using equation 5.10*. The obtained result is 1502 MWh.*

The rated (installed) capacity of the SHP is determined through equation 5.9*, the result being 558.5 kW.*



	0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%
Pot_disp (kW)	0	287	448	528	548	457	376	306	242	180	119	62	16	0	0	0	0	0	0	0	0
Qusado (m3/s)	11,25	11,25	11,25	11,25	11,05	9,04	7,36	5,97	4,81	3,84	3,03	2,36	1,80	1,33	0,94	0,62	0,35	0,12	0,00	0,00	0,00

Figure 5-6: Output power curve.

5.4 THE TWO TURBINE-GENERATOR CASE

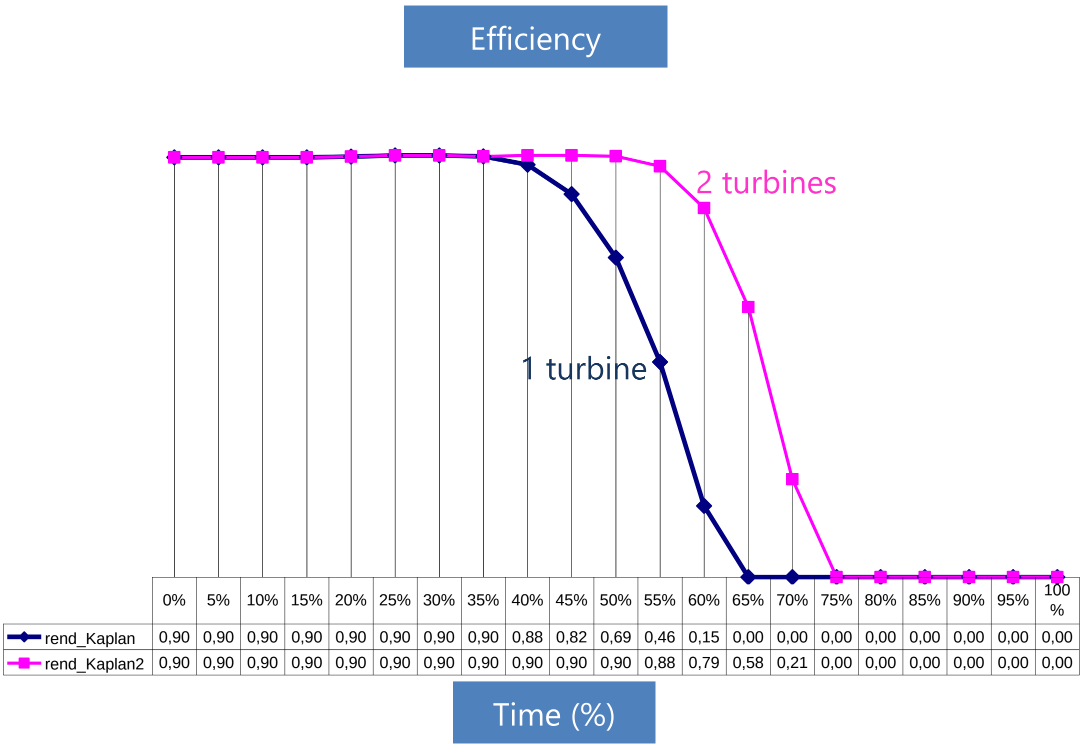
Let us now analyse the case in which the SHP is equipped with two turbine-generator groups, the installed capacity of each one being one half of the total installed capacity. We will take Example 5—1 for reference and will compare the case of one Kaplan turbine (as in Example 5—1) versus two Kaplan turbines with half rated flow. Let us focus, for example, at the point in which the used flow is Q_{50} = 3.03 m^3/s. If one Kaplan turbine is installed, the efficiency would be:

$$\eta_{t1} = \left\{1 - \left[3.5\left|1 - 1.333\frac{3.03}{11.25}\right|^6\right]\right\}0.905 = 0.686 \qquad \text{equation 5.11}$$

If two half-sized Kaplan turbines are installed, the rated flow of each one is $Q_{N1} = Q_{N2} = 5.625\,\text{m}^3/\text{s}$. For flows lower than this, only one turbine is operating, its efficiency being:

$$\eta_{t2} = \left\{1 - \left[3.5\left|1 - 1.333\frac{3.03}{5.625}\right|^6\right]\right\}0.905 = 0.903 \qquad \text{equation 5.12}$$

As one can see, the efficiency is much higher in the second case than in the first. If we repeat the very same process for the other flows, we would obtain the graphic depicted in Figure 5-7.



	0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100 %
rend_Kaplan	0,90	0,90	0,90	0,90	0,90	0,90	0,90	0,90	0,88	0,82	0,69	0,46	0,15	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
rend_Kaplan2	0,90	0,90	0,90	0,90	0,90	0,90	0,90	0,90	0,90	0,90	0,90	0,88	0,79	0,58	0,21	0,00	0,00	0,00	0,00	0,00	0,00

Figure 5-7: Efficiency of one turbine versus two turbines.

For flows larger than half the rated flow, the efficiency is equal, whether we have one or two turbines. For lower flows, only one of the two turbines is operating and with a much higher efficiency than in the case only a single turbine is installed.

Furthermore, more water is used to produce energy because the one of the two turbines that is operating is disconnected for lower flows (its rated flow is lower). Of course, this will increase the annual energy yield both because of this and because the global efficiency is higher. In the case we have been studying, the annual produced electricity increases from 1502 MWh to 1602 MWh, the total installed capacity remaining the same. This increase in the annual energy yield must be confronted with the increase on investment and O&M costs: two half-sized turbines are more expensive than a single full-rated turbine.