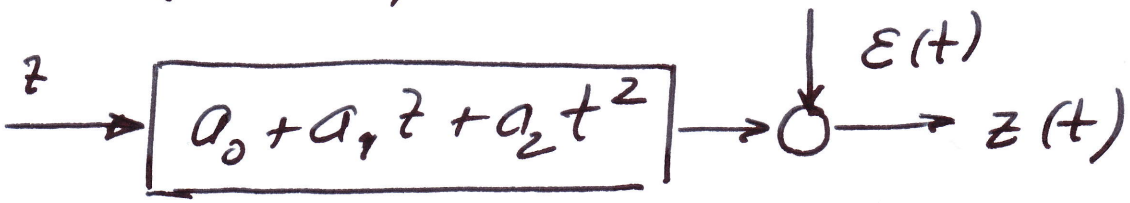
P.2 – Estimação de parâmetros

P.2.1 Modelo de geração da variável (medida) z:



Dadas medidas $z(t_i)$; $i = 1, 2, \cdots, 4$ de $z(t)$ nos instantes t_i, pretende-se determinar a_0, a_1, a_2 que "melhor explicam" os dados experimentais.

Estratégia de mínimos quadrados:

$$\min_{(a_0, a_1, a_2)} \underbrace{\sum_{i=1}^{4} \left\{ z(t_i) - \left[a_0 + a_1 t_i + a_2 t_i^2 \right] \right\}^2}_{C(a_0, a_1, a_2)}$$

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P.2.2. Condições necessárias para atingir o mínimo:

$$\frac{\partial C}{\partial a_0} = \frac{\partial C}{\partial a_1} = \frac{\partial C}{\partial a_2} = 0$$

Cálculo de $\frac{\partial C}{\partial a_0} = 0$

$$\frac{\partial}{\partial a_0} \sum_{i=1}^{4} \left\{ z(t_i) - [a_0 + a_1 t_i + a_2 t_i^2] \right\}^2$$

$$= -2 \sum_{i=1}^{4} \left\{ z(t_i) - [a_0 + a_1 t_i + a_2 t_i^2] \right\} = 0$$

$$\Leftrightarrow \quad \begin{bmatrix} 4 & \sum t_i & \sum t_i^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \sum z(t_i) \qquad (A)$$

Cálculo de $\frac{\partial C}{\partial a_1} = 0$

$$\frac{\partial}{\partial a_1} \sum_{i=1}^{4} \left\{ z(t_i) - [a_0 + a_1 t_i + a_2 t_i^2] \right\}^2$$

$$= -2 \sum_{i=1}^{4} t_i \left\{ z(t_i) - [a_0 + a_1 t_i + a_2 t_i^2] \right\}$$

$$\Rightarrow \quad \begin{bmatrix} \sum t_i & \sum t_i^2 & \sum t_i^3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \sum t_i z(t_i) \qquad (B)$$

Cálculo de $\frac{\partial C}{\partial a_2} = 0$

$$\frac{\partial}{\partial a_2} \sum_{i=1}^{4} \{ z(t_i) - [a_0 + a_1 t_i + a_2 t_i^2] \}^2$$

$$= -2 \sum_{i=1}^{4} t_i^2 \{ z(t_i) - [a_0 + a_1 t_i + a_2 t_i^2] \}$$

$$\Leftrightarrow \quad [\Sigma t_i^2 \quad \Sigma t_i^3 \quad \Sigma t_i^4] \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \Sigma t_i^2 z(t_i) \qquad (C)$$

(A), (B), (C) $\Leftrightarrow$

$$\underbrace{\begin{bmatrix} 4 & \Sigma t_i & \Sigma t_i^2 \\ \Sigma t_i & \Sigma t_i^2 & \Sigma t_i^3 \\ \Sigma t_i^2 & \Sigma t_i^3 & \Sigma t_i^4 \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}}_{a} = \underbrace{\begin{bmatrix} \Sigma z(t_i) \\ \Sigma t_i z(t_i) \\ \Sigma t_i^2 z(t_i) \end{bmatrix}}_{b}$$

$$\Rightarrow \quad a = M^{-1} b$$