

Chapter 10

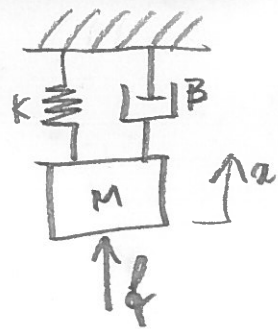
10 Os diagramas corretos podem obter-se em Matlab com o comando *bode*.

12 a) 3 s b) $-1+t+e^{-t}$ c) $-1+t$ d) 2,52 s (exige resolver uma equação não-linear, o que se consegue no Matlab)

15 As respostas corretas podem obter-se em Matlab com o comando *step*.

18 Ver página seguinte.

18



$$M\ddot{x} = f - Kx - B\dot{x}$$

$$MX\ddot{x} + BX\dot{x} + KX = F$$

$$\frac{X}{F} = \frac{1}{M\omega^2 + B\omega + K} = \frac{\frac{1}{M}}{\omega^2 + \frac{B}{M}\omega + \frac{K}{M}}$$

a)

$$\textcircled{1} y_m = \frac{1}{\frac{K}{M}} F \Rightarrow \frac{89}{K} = 3 \times 10^{-2} \Leftrightarrow K = \frac{89}{3 \times 10^{-2}} = 297 \text{ N/m}$$

$$\textcircled{2} M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \Rightarrow -\frac{\zeta\pi}{\sqrt{1-\zeta^2}} = \log M_p \Rightarrow \zeta^2\pi^2 = (1-\zeta^2)(\log M_p)^2 \Rightarrow \zeta^2(\pi^2 + (\log M_p)^2) = (\log M_p)^2$$

para ser > 0

$$\Rightarrow \zeta = \frac{-\log M_p}{\sqrt{\pi^2 + (\log M_p)^2}} \Rightarrow \zeta = \frac{-\log 0,097}{\sqrt{\pi^2 + (\log 0,097)^2}} = 0,6$$

$$\textcircled{3} t_p = \frac{\pi}{\omega_m \sqrt{1-\zeta^2}} \Rightarrow \omega_m = \frac{\pi}{t_p \sqrt{1-\zeta^2}} \Rightarrow \omega_m = \frac{\pi}{1 \sqrt{1-0,6^2}} = 3,93 \text{ rad/s}$$

$$\log_0 \frac{K}{M} = \omega_m^2 \Rightarrow M = \frac{297}{3,93^2} = 19,2 \text{ kg}$$

$$\textcircled{4} 2\zeta\omega_m = \frac{B}{M} \Rightarrow B = 2 \times 0,6 \times 3,93 \times 19,2 = 90,5 \text{ Ns/m}$$

Em resumo: $M = 19,2 \text{ kg}$; $K = 297 \text{ N/m}$; $B = 90,5 \text{ Ns/m}$

$$\frac{X(s)}{F(s)} = \frac{0,0521}{s^2 + 4,71s + 15,5}$$

b) Mesmo $y_m \Rightarrow \frac{1}{K}$ igual

Mesmo t_p (irrelevante se é 2% ou 5%) $\Rightarrow \zeta\omega_m$ igual $\Rightarrow \frac{B}{M}$ igual

$$\text{Queremos } \frac{1}{M^*s^2 + B^*s + K} = \frac{\frac{1}{M^*}}{s^2 + \frac{B^*}{M^*}s + \frac{K}{M^*}} \quad \text{com } \frac{B^*}{M^*} = \frac{B}{M} = \frac{90,5}{19,2} \text{ (K não muda)}$$

$$\text{e com } \zeta = \frac{-\log 0,0015}{\sqrt{\pi^2 + (\log 0,0015)^2}} = 0,9$$

$$\frac{B^*}{M^*} = 4,714 = 2\zeta\omega_m \Rightarrow \omega_m = \frac{4,714}{2 \times 0,9} = 2,619$$

$$\text{Ora } \frac{K}{M^*} = \omega_m^2 \Rightarrow \frac{297}{M^*} = 2,619^2 \Rightarrow M^* = \frac{297}{2,619^2} = 43,3 \text{ kg}$$

$$\text{logo } B^* = 43,3 \times 4,714 = 204 \text{ Ns/m}$$

Em resumo $M^* = 43,3 \text{ kg}$; $K = 297 \text{ N/m}$; $B^* = 204 \text{ Ns/m}$