# M ergers when ..rms sell asymmetric complements 

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#### Abstract

This paper addresses merger activity when ..rms sell complements. C ontrary to previous literature, at least one of the goods can be purchased separately, and need not be combined to create a composite good. In this sense complementarity is asymmetric. Several market structures, having dixerent degrees of vertical and/ or horizontal integration, are analyzed as to their internal and external exects. We use an endogenous mechanism for coalition formation and establish that the predictable merger is not welfare maximizing.


K eywords: mergers, one-sided complements
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[^0]
## 1 Introduction

What happens to the surplus of Compaq clients when this ..rm merges with the printers' seller Hewlett-P ackard? A nd what about the surplus of competing computer producers?

The former is a good example of asymmetric complementary goods, that is, a case in which one of them does not need to be purchased in conjunction with the other (computers may operate by themselves, printers need to be complemented by computers). Other examples are mobile phones and phone calls. A $n$ increase in computer sales shifts the demand for printers outwards, as an increase in mobile phones' sales shifts demand for phone calls outwards. These two goods may be combined to create a composite good, or one of them may be consumed separately.

In this context, mergers in one market (intramarket) change equilibria in the other but in an asymmetric way. The impact of a given merger must be analyzed in both markets. M ergers across markets (intermarket) also change rivals' behavior.

Previous literature has focused mainly on perfect complements (such as hardware and software, ATMs and bank cards, train or airline travels). This is the case of Gaudet and Salant (1992), Economides and Salop (1992), and Kim and Shin (2002). They all consider price competing ..rms and look at the total price that must be paid to purchase a composite good. There may be composite substitute goods: in Economides and Salop's paper there are two dixerentiated brands of each of the two components needed to create the composite good, which may thus be combined in four dixerent ways; K im and Shin require three components for the composite good, with competition for the ..rst component only, so there are two substitute composite goods; in Gaudet and Salant there are n components and no substitutes for none of them, so the composite good is unique. All these papers consider the pro..tability and welfare exects of mergers between some of the components' producers, in order to evaluate the need for policy intervention.

Mergers between complements have "vertical" integration exects ${ }^{1}$ which push the total price downwards, because the complementarity bene.ts are internalized. In turn, when the merger also involves substitute components, the horizontal exect pushes the total price upwards, so that in most partial or full integration scenarios the overall exect is ambiguous, depending on the degree of dixerentiation, as given by the demand parameters. For example, in E conomides and Salop joint ownership (full integration) yields the lowest total price when composite goods are distant substitutes, and the highest one when they are close substitutes. In Gaudet and Salant, mergers always improve welfare, because they only have vertical exects.

Our paper resembles more Economides and Salop and Kim and Shin than Gaudet and Salant, in that there is substitutability between components. It particularly resembles Economides and Salop, because we also consider two components, with competition for each. However, as explained above, complementarity is asymmetric, which makes the markets for each component dixerent and allows us to distinguish pro..tability and welfare eaects on each of them. We look at the total price for the composite good, but also at the individual price for the good that can be consumed separately. Contrary to the literature mentioned, we consider quantity instead of price competition. Several market structures, having dixerent degrees of vertical and/ or horizontal integration, are analyzed as to their internal and external exects. This goes one step beyond Economides and Salop's paper, which only considers either "vertical" or horizontal integration. This paper also dixers from earlier work by using the core market structure, as de..ned in Horn and Persson (2001), as the equilibrium concept. This allows us to compare a given market structure with all its alternatives and not merely with those arising from unilateral deviations by a given ..rm. Kim and Shin also address merger stability, but their concept of stability is dixerent from the one we use.

The paper is structured as follows. Section 2 presents the basic model and

[^1]derives the equilibria under all possible market structures. The welfare exects of mergers when complementarity is asymmetric are studied in section 3. Section 4 ..nds out the core à la Horn and Persson (2001). Finally, the last section concludes.

## 2 The M odel

Consider two markets, $A$ and $B$. There are $n_{A}$..rms operating in the former, and $n_{B}$ in the latter. Sales in market $A$ positively in $\ddagger$ uence demand in market B. Sales in market B, however, have no impact on market A. As stated before, this may be the case, for instance, of computers and printers, respectively. So we have $P_{A}\left(Q_{A}\right)$ and $P_{B}\left(Q_{B} ; Q_{A}\right)$, where $Q_{i}={ }_{j=1}^{P_{i}} q$. In each market ..rms compete in quantities and we assume marginal costs to be zero. We study the dixerent impacts of mergers within market $A$, within market $B$, and across markets.

For the sake of simplicity and comparability with Economides and Salop (1992), we con..ne our attention to the $n_{A}=n_{B}=2$ case. Firms 1 and 2 operate in market A , while in market B ..rms are labeled 3 and 4. Consider markets $A$ and $B$ characterized by the following inverse linear demands: $P_{A}=1_{i} Q_{A}$ and $\mathrm{P}_{\mathrm{B}}=1_{\mathrm{i}} \mathrm{Q}_{\mathrm{B}}+{ }^{-} \mathrm{Q}_{\mathrm{A}}$, with $0<{ }^{-}<1$ (imperfect complements). Selling computers (good A) stimulates demand for printers (good $B)$.

Let $Q_{A}=q_{1}+q_{2}$ and $Q_{B}=q_{B}+q_{A}$. Firm 1 competes with ..rm 2, and so do ..rm 3 and ..rm 4.

As Economides and Salop (1992) and K im and Shin (2002), we ..rst analyze the benchmarks corresponding to the two market structures considered by Cournot in his famous zinc, copper and brass example: independent ownership and full integration. We then consider several other intermediate market structures, which include intra or intermarket mergers or both.

### 2.1 Extreme market structures

Independent ownership

Firm $\mathrm{i}(\mathrm{i}=1 ; 2 ; 3 ; 4$ ) chooses $q$ to maximize i i , where

$$
\begin{aligned}
& \left.\right|_{1}=\left(\begin{array}{lll}
1 & q_{1} & i \\
q_{2}
\end{array}\right) q_{1} \\
& \text {; } 2=\left(\begin{array}{lll}
1 & q_{1} & i \\
q_{k}
\end{array}\right) q_{k} \\
& i_{3}=\left(1_{i} q_{B} i q_{A}+{ }^{-} Q_{A}\right) q_{B} \\
& i_{4}=\left(1_{i} q_{B} ; q_{A}+{ }^{-} Q_{A}\right) q_{A}
\end{aligned}
$$

Equilibrium outputs and corresponding pro..ts are

$$
\begin{gathered}
q_{1}=Q_{Q}=Q_{A}=2=1=3 \\
q_{B}=q_{A}=Q_{B}=2=\frac{1}{3}+\frac{2}{9}- \\
i_{1}=1_{2}=1=9 \\
1_{3}=1_{4}=\frac{1}{81}\left(3+2^{-}\right)^{2}
\end{gathered}
$$

## Full integration

In this case, all the ..rms are owned by a single decision maker, which maximizes the sum of pro..ts in both markets, $i 1234=\left(1_{i} Q_{A}\right) Q_{A}+\left(1_{i} Q_{B}+\right.$ $\left.{ }^{-} Q_{A}\right) Q_{B}$. The solution is

$$
\begin{aligned}
\mathrm{Q}_{\mathrm{A}} & \left.=\mathrm{Q}_{\mathrm{B}}=1 \neq 2 \mathrm{i}^{-}\right) \\
1234 & =\frac{1}{2 \mathrm{i}^{-}}
\end{aligned}
$$

Comparing the two extreme cases, we observe that total quantity in market $B$ is lower under full integration than under independent ownership. As this market does not imply any positive externality on the other market, full integration leads solely to the creation of a monopoly. However, total quantity in market A may be higher when the cross exect is sut ciently large, speci..cally when ${ }^{-}>1=2$. Despite having a monopoly in this market, the fact that pro..ts in market B are also considered by the ..rms in market A will lead them to increase their production. This may or may not be compensated by the fact that market A is now served by a monopoly.

The fact that $\mathrm{Q}_{\mathrm{A}}=\mathrm{Q}_{\mathrm{B}}$ may be counterintuitive given that sales in market A shift outwards the demand in market $B$. Under full integration, one
would expect $\mathrm{Q}_{\mathrm{A}}$ to be larger than $\mathrm{Q}_{\mathrm{B}}$ due to the internalization of this exect. Nonetheless, the fact that demand for good B is larger than the one for $A$ (recall also that marginal costs are the same) has the opposite impact. Due to the linearity assumptions, these two exects exactly compensate for each other.

We now turn to intermediate market structures. To begin with, we analyze equilibria with intramarket mergers.

### 2.2 Intramarket mergers

Merger in market $A$
After a merger between ..rm 1 and ..rm 2 its owner will choose $Q_{A}$ to maximize : $12=\left(1_{i} \quad Q_{A}\right) Q_{A}$, while in market $B$ both ..rms remain independent Cournot players with ..rm $j=3 ; 4$ maximizing $i_{j}=\left(1_{i} q_{B} i q_{A}+{ }^{-} Q_{A}\right) q$. The solution is

$$
\begin{gathered}
Q_{A}=1=2 \\
q_{B}=q_{A}=\left(2+^{-}\right)=6
\end{gathered}
$$

As for equilibrium pro..ts we have

$$
\begin{gathered}
\mathrm{I}_{12}=1=4 \\
\mathrm{i}_{4}=\mathrm{I}_{3}=\frac{1}{36}\left(2+^{-}\right)^{2}
\end{gathered}
$$

Naturally, this is the worst two-..rm merger from the consumers' point of view. The monopolization of market $A$ axects not only this market, but also market $B$, reducing demand for ..rms 3 and 4. Consequently, welfare decreases in both markets and there is no internalization of the positive exect that output in market $A$ has upon market $B$ :

## Merger in market $B$

The owner of ..rms 3 and 4 chooses $Q_{B}$ to maximize ${ }_{34}=\left(1 ; Q_{B}+\right.$ $\left.{ }^{-} Q_{A}\right) Q_{B}$. In market $A ; . . r m j=1 ; 2$ maximizes $\left.\right|_{j}=\left(1 ; q_{i} i q_{R}\right) q$. The
straightforward solution is

$$
\begin{aligned}
q_{1} & =q_{R}=1 \leftrightharpoons 3 \\
Q_{B} & =\left(3+2^{-}\right)=0
\end{aligned}
$$

As for equilibrium pro..ts we have

$$
\begin{gathered}
i_{1}=12=1=9 \\
34=\frac{1}{36}\left(3+2^{-}\right)^{2}
\end{gathered}
$$

As one would expect, this structure is worse (as far as welfare is concerned) than the case of four independent ..rms, but not as bad as the one resulting from a merger in market A .
$M$ ergers in markets $A$ and $B$
The owner of ..rms 1 and 2 chooses $Q_{A}$ to maximize ${ }_{12}=\left(1_{i} Q_{A}\right) Q_{A}$ while the owner of ..rms 3 and 4 chooses $Q_{B}$ to maximize ${ }_{34}=\left(1_{i} Q_{B}+{ }^{-} Q_{A}\right) Q_{B}$. The outcome is

$$
\begin{gathered}
Q_{A}=1=2 \\
Q_{B}=\left(2+^{-}\right)=4
\end{gathered}
$$

and the corresponding equilibrium pro.ts for both ..rms are

$$
\begin{gathered}
34=\frac{1}{16}\left(2+^{-}\right)^{2} \\
12=\frac{1}{4}
\end{gathered}
$$

Contrasting the case in which a merger occurs in the computers market (market A) with the case in which there is a merger in the printers market ( $B$ ), we observe that in the latter total quantity in market A remains unchanged as compared with the independent ownership case, while it is reduced in the former case (merger in market $A$ ). Accordingly, $\mathrm{Q}_{\mathrm{B}}$ is lower under the market structure $12,3,4$ than under $1,2,3,4$, and so is $\mathrm{P}_{\mathrm{B}}$. As expected, due to competition ease, $Q_{B}$ is also lower under $1,2,34$ than under $1,2,3,4$, but this fact has no impact
on $Q_{A}$. Despite the fact that $Q_{A}$ is lower in $12,3,4$ than in $1,2,34$, and hence complementarities may not be so well exploited, total consumer surplus is higher in the former case than in the latter. The reverse occurs for overall pro..ts and also for social welfare. Thus, looking just at the global surplus, society would be better ox when the printers market is monopolized than when the computers market is, even though consumers would prefer the computers market to become a monopoly. This divergence is deeper the higher ${ }^{-}$is.

Adding a merger in market $B$ to the structure 12,3,4 does not change the equilibrium in market A , but reduces quantity and increases price in market B . However, if we move from $1,2,34$ to 12,34 both quantities are reduced, while $P_{A}$ is increased and $P_{B}$ reduced.

### 2.3 Intermarket mergers

Let us now look at intermarket mergers. Note that, since there is symmetry inside markets, market structures $13,2,4,14,2,3,23,1,4$ and $24,1,3$ are all equivalent (composite goods 13 and 14, for instance, are perfect substitutes). Also, market structures 13,24 and 14,23 are equivalent.

M erger across markets
The owner of ..rms 1 and 3 will choose $q_{1}$ and $q_{\beta}$ to maximize $; 13=\left(1_{i}\right.$
 and ..rm 4 chooses $q_{A}$ to maximize $\left.\right|_{4}=\left(1_{i} q_{3} ; q_{1}+{ }^{-} q_{1}+{ }^{-} q_{k}\right) q_{q_{1}}$. The solution is

$$
\begin{aligned}
& q_{i}=\left(3+2^{-}++^{-2}\right)=\left(9 i^{-2}\right) \\
& q_{2}=\left(3 i^{-} i^{-2}\right)=\left(9 i^{-2}\right) \\
& \left.q_{B}=q_{4}=\left(3+2^{-}\right)=9 i^{-2}\right)
\end{aligned}
$$

which lead to the following equilibrium pro..ts:

$$
\begin{gathered}
1_{13}=\frac{18+15^{-}+2^{-2} i 3^{-3} i^{-4}}{\left(3 i^{-}\right)^{2}\left(3+{ }^{-}\right)^{2}} \\
i_{2}=\frac{i_{3 i} i^{-} i^{-2^{4}}}{\left(3 i^{-}\right)^{2}\left(3+^{-}\right)^{2}} \\
1_{4}=\frac{\left(3+2^{-}\right)^{2}}{\left(3 i^{-}\right)^{2}\left(3+^{-}\right)^{2}}
\end{gathered}
$$

M ergers across markets
As above, the owner of ..rms 1 and 3 chooses $q_{1}$ and $q_{\beta}$ to maximize ${ }_{13}=$ $\left(1 ; q_{1} i q_{2}\right) q_{1}+\left(1_{i} q_{B} ; q_{4}+{ }^{-} q_{1}+{ }^{-} q_{z}\right) q_{B}$. N ow the owner of ..rms 2 and 4 chooses $q_{2}$ and $q_{4}$ to maximize ${ }_{24}=\left(l_{i} q_{i} i q_{2}\right) q_{2}+\left(l_{i} q_{B i} q_{q_{4}}+{ }^{-} q_{1}+{ }^{-} q_{k}\right) q_{q_{2}}$. The result is

$$
\begin{gathered}
\left.q_{1}=q_{2}=\left(3+^{-}\right) \neq 9 ; 2^{-2}\right) \\
q_{3}=q_{A}=\left(3+2^{-}\right)=\left(9 i^{-} 2^{-2}\right) \\
\quad 124=113=\frac{2+{ }^{-}}{9 i^{-2^{\Phi}}}
\end{gathered}
$$

The higher ${ }^{-}$, the higher the pro..t level of the cross market agreement. Adding another cross market merger does not change the equilibrium in market B. However, in market A price is reduced and quantity is increased, in order to stimulate even further sales in market B.

Contrasting the duopoly resulting from mergers across two markets with the double monopoly in 12,34, one can observe that even though total pro..ts are higher under 12,34 , consumers and the society as a whole are better ow under 13,24.

### 2.4 Intra and intermarket mergers

Finally, let us combine intra and intermarket mergers. Note that 123,4 is equivalent to 124,3 , and 1,234 is equivalent to 134,2 .

Merger in market A combined with across market merger
The owner of ..rms 1,2 and 3 maximizes : $123=\left(\begin{array}{lll}1 & q_{1} & q_{2}\end{array}\right) q_{1}+\left(1_{i} \quad q_{1} i\right.$ $\left.q_{2}\right) q_{2}+\left(1 ; q_{B} ; q_{4}+{ }^{-} q_{1}+{ }^{-} q_{Z}\right) q_{B}$ and ..rm 4 maximizes $i_{4}=\left(1 ; q_{B} ; q_{4}+\right.$
$\left.{ }^{-} q_{1}+{ }^{-} q_{2}\right) q_{4}$. We have

$$
\begin{gathered}
Q_{A}=\left(3+^{-}\right)=\left(6 i^{-2}\right) \\
q_{B}=q_{4}=\left(2+^{-}\right)=\left(6 i^{-2}\right)
\end{gathered}
$$

and

$$
\begin{gathered}
123=\frac{13+4^{-} i^{-2} i^{-3}}{{ }^{1} 6 i^{-2^{42}}} \\
\quad 14=\frac{\left(2+{ }^{-}\right)^{2}}{{ }^{4} i^{-2^{42}}}
\end{gathered}
$$

Merger in market $B$ combined with across market merger
The owner of ..rms 2, 3 and 4 maximizes $; 234=\left(1 ; q_{1} ; q_{2}\right) q_{2}+(1 ;$ $q_{B}$ i $\left.q_{A}+{ }^{-} q_{1}+{ }^{-} q_{R}\right) q_{B}+\left(1 ; q_{B}\right.$ i $\left.q_{A}+{ }^{-} q_{1}+{ }^{-} q_{R}\right) q_{A}$ and ..rm 1 maximizes $\left.\right|_{1}=\left(1_{i} \quad q_{1} i \quad q_{z}\right) q_{1}$. We obtain

$$
\begin{gathered}
q_{1}=\left(2 i^{-} i^{-2}\right)=\left(6 i^{-2}\right) \\
q_{Z}=\left(2+2^{-}+^{-2}\right)=\left(6 i^{-2}\right) \\
\left.Q_{B}=\left(3+2^{-}\right)=6 i^{-2}\right)
\end{gathered}
$$

and

$$
\begin{gathered}
234=\frac{13+14^{-}+2^{-2} i_{\&} 3^{-3} i^{-4}}{{ }_{i} \mathbf{i}^{-2^{42}}} \\
i_{1}=\left(2+^{-}\right)^{2} \frac{\left(1_{\mathbf{i}}-\right)^{2}}{1_{6} \mathbf{i}^{-2^{42}}}
\end{gathered}
$$

The market where rivalry subsists is the one where quantity is higher and price is lower as compared with the other intra and intermarket mergers situation. W ith the market structure $1,234 \mathrm{Q}_{\mathrm{A}}$ is higher than in the independent ownership structure, but with 123,4 it may be lower for ${ }^{-}$suф ciently small.

## 3 Welfare exects

In this section we compare welfare under the dixerent possible market structures. Table 1 below presents prices and consumer surplus for both markets as well as total welfare, under the nine dixerent market outcomes.

Table 1: Prices, consumers surplus and welfare under the dimerent market structures

|  | $\mathrm{P}_{\mathrm{A}}$ | $\mathrm{P}_{\mathrm{B}}$ | $\mathrm{CS}_{\text {A }}$ | $\mathrm{CS}_{\text {B }}$ | W |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1,2,3,4 | $\frac{1}{3}$ | $\frac{1}{3}+\frac{2}{9}^{-}$ | $3 \frac{2}{9}$, | $\frac{1}{2}{ }^{1} \frac{2}{3}+\frac{4}{9}^{-}{ }^{-4}$ | $\frac{8}{9}+\frac{16}{27}^{-}+\frac{16}{81}^{-2}$ |
| 1234 | $\frac{1 i^{-}}{2{ }^{-}}$ | $\frac{1}{2 i}$ | $\frac{1}{2} \frac{1}{2 i}$ | $\frac{1}{2} \frac{1}{2 i}$ | $\frac{3 i^{-}}{\left(2 i^{-}\right)^{2}}$ |
| 12,3,4 | $\frac{1}{2}$ | $\frac{2+{ }^{-}}{6}$ | $\frac{1}{8}$ | $\frac{\left(2+^{-}\right)^{2}}{18}$ | $\frac{59}{72}+\frac{4}{9}^{-}+\frac{1}{9}^{-2}$ |
| 1,2,34 | $\frac{1}{3}$ | $\frac{3+2^{-}}{6}$ | $\frac{2}{9}$ | $\frac{\left(3+2^{-}\right)^{2}}{72}$ | $\frac{59}{72}+\frac{1}{2}^{-}+\frac{1}{6}^{-2}$ |
| 12,34 | $\frac{1}{2}$ | $\frac{2+{ }^{-}}{4}$ | $\frac{1}{8}$ | $\frac{(2+-)^{2}}{32}$ | $\frac{3}{4}+\frac{3}{8}^{-}+\frac{3}{32}^{-2}$ |
| 13,2,4 |  | $\frac{3+2^{-}}{9 i^{-2}}$ | $\frac{1}{2} \frac{\left.(6+)^{-}\right)^{2}}{\left.(3 i-)^{2}(3+)^{-}\right)^{2}}$ | $\frac{2\left(3+2^{-}\right)^{2}}{\left.\left(3 i^{-}\right)^{2}(3+)^{-}\right)^{2}}$ | $\frac{144+102^{-}+19^{-2} i 2^{-3}}{\left.2(3 i)^{-}\right)^{2}(3+)^{-}}$ |
| 13,24 | $\frac{3 i}{} 2^{-}+2^{-2}$ | $3+2^{-}$ |  |  | $\underline{2\left(36+27^{-2}+{ }^{-2} ; 2^{-3}\right)}$ |
|  | $\frac{9+22^{-2}}{}$ | $9{ }^{\text {i }} 2^{-2}$ | $\frac{\left(9 i 2^{-2}\right)^{2}}{}$ | $\frac{\left(9 i 2^{-2}\right)^{2}}{}$ |  |
| 123,4 | $\frac{3 i-2}{6 i-2}$ | $\frac{2+}{6 i^{-2}}$ | $\frac{1}{2} \frac{(3+)^{2}}{\left(6 i{ }^{-2}\right)^{2}}$ | $\frac{2(2+-)^{2}}{\left(6 i^{-2}\right)^{2}}$ | $\frac{59+38^{-}+{ }^{-2} i_{j} 2^{-3}}{2\left(66_{i}^{-}\right)^{2}}$ |
| 1,234 | $\frac{2 i^{-} i^{-2}}{6 i^{-2}}$ | $\frac{3+2^{-}}{6 i^{-2}}$ | $\frac{1}{2} \frac{\left(4+^{-}\right)^{2}}{\left(6 i^{-2}\right)^{2}}$ | $\frac{1}{2} \frac{\left(3+2^{-}\right)^{2}}{\left(6_{i}-2\right)^{2}}$ | $\frac{59+40^{-}+3^{-2} ; 2^{-3}}{2\left(6 i^{-2}\right)^{2}}$ |

The following propositions allow us to partially rank the post merger welfare (measured as consumer surplus plus industry pro..ts in both markets).

Proposition 1 The following ranking of welfare level can be established for all $0<{ }^{-}<1: W^{13 ; 24}>W^{13 ; 2 ; 4}>W^{1 ; 2 ; 3 ; 4}>W^{1 ; 2 ; 34}>W^{12 ; 34}$

Proof. We will start by showing that two intermarket mergers are better than just one, that is, $W^{13 ; 24}>W^{13 ; 2 ; 4}$ :

$$
\begin{gathered}
W^{13 ; 24}>W^{13 ; 2 ; 4}, \\
2 \frac{\left(36+27^{-}+{ }^{-2} i 2^{-3}\right)}{{ }^{1} 9 \mathrm{i}^{-2^{42}}}>\frac{1}{2} \frac{144+102^{-}+19^{-2} i 2^{-3^{2}}}{\left.(3 \mathrm{i})^{-}\right)^{2}\left(3+^{-}\right)^{2}}, \\
i 162 \mathrm{i} 459^{-} \mathrm{i} 414^{-2} \mathrm{i} 60^{-3}+76^{-4}+24^{-5}<0
\end{gathered},
$$

This structure maintains the initial duopoly in both markets while at the same time allows a positive externality to be captured by both ..rms in market A.

We now show that total welfare increases with one intermarket merger, that is, $W^{13 ; 2 ; 4}>W^{12 ; 2 ; 34}$ :

$$
\begin{gathered}
W^{13 ; 2 ; 4}>W^{1 ; 2 ; ; 3 ; 4}, \\
\frac{1}{2} \frac{144+102^{-}+19^{-2} ; 2^{-3}}{\left.(3 i-)^{2}(3+)^{-}\right)^{2}}>\frac{8}{9}+\frac{16-}{27}+\frac{16-2}{81}, \\
i 486 ; 1539^{-} ; 1566^{-2} ; 432^{-3}+96^{-4}+32^{-5}<0
\end{gathered}
$$

which is trivially true as $0<{ }^{-}<1$ :
It is also easy to check that the intramarket merger in market B lowers welfare as compared with the independent ownership case. This is a trivial point given that the merged ..rm does not internalize any positive externality on the other market. Such merger merely creates a monopoly in market B :

$$
\begin{gathered}
W^{1 ; 2 ; 3 ; 4}>W^{1 ; 2 ; 34}, \\
\frac{8}{9}+\frac{16}{27}+\frac{16-2}{81}>\frac{59}{72}^{27}+\frac{1}{2}-+\frac{1}{6}-2 \\
\frac{5}{72}+\frac{5}{54}-+\frac{5}{162}-2>0
\end{gathered}
$$

An additional intramarket merger (this time in market A) will further reduce welfare. A gain, as the mergers do not involve ..rms in dixerent markets there is no incentive to expand production in order to increase demand for a partner ..rm.

$$
\begin{gathered}
W^{1 ; 2 ; 34}>W^{12 ; 34} \\
\frac{59}{72}+\frac{1}{2}-+\frac{1}{6}-2>\frac{3}{4}+\frac{3}{8}-+\frac{3}{32}-2 \\
\frac{5}{72}+\frac{1}{8}-+\frac{7}{96}^{-2}>0
\end{gathered}
$$

As $^{-}>0$ the last inequality holds trivially.
Proposition 2 The following ranking of welfare level can be established for all $0<{ }^{-}<1: W^{1 ; 234}>\mathrm{W}^{123 ; 4}>\mathrm{W}^{12 ; 34}>\mathrm{W}^{12 ; 3 ; 4}>\mathrm{W}^{12 ; 34}$

Proof. First of all note that if ${ }^{-}=0$ there is no connection between markets, and mergers between 234 or 123 would yield the same outcome. However, when

- > 0 these two mergers lead to dixerent outcomes: the former leads to a duopoly in market A where one of the duopolists owns market B. The incentives to increase production in market $A$ are thus enormous. Not only are we in the presence of a duopoly but, additionally, one of the duopolists will also own a monopoly that will bene..t from a positive externality. On the other hand, under the structure 123,4 we have a monopoly in market A that will not be the sole recipient of the externality gains (that is, the externality is not fully internalized). Thus production will be lower. Hence, the ..rst inequality is as expected

$$
\begin{gathered}
W^{1 ; 234}>W^{123 ; 4} \\
\frac{1}{2} \frac{59+40^{-}+3^{-2} ; 2^{-3}}{{ }^{1} 6 i^{-2^{42}}}>\frac{1}{2} \frac{59+38^{-}+{ }^{-2} ; 2^{-3}}{{ }^{6} 6 i^{-2^{42}}} \\
2^{-}+2^{-2}>0
\end{gathered}
$$

The following inequality is also quite intuitive. The merger between 123 will lead to a monopoly that internalizes part of a positive externality while the 34 merger leads to the creation of a monopoly in a larger market that internalizes nothing and thus has no incentive to increase production.

$$
\begin{gathered}
W^{123 ; 4}>W^{1 ; 2 ; 34} \\
\frac{1}{2} \frac{59+38^{-}+{ }^{-2} \mathrm{i}^{-3} 2^{-3}>\frac{59}{72}+\frac{1}{2}+\frac{1}{6}{ }^{-2^{42}}}{\text { i } 72 \text { i } 312^{-} \text {i } 360^{-2} \mathrm{i} 85^{-3}+36^{-4}+12^{-5}<0}
\end{gathered}
$$

which is true as $0<{ }^{-}<1$ :
It is also trivial to explain that $\mathrm{W}^{1 ; 2 ; 34}>\mathrm{W}^{12 ; 3 ; 4}$ : in the ..rst case the monopoly has its exect in an independent market. In the second merger the negative effects will axect both markets, contracting demand in market B: Note that in the ${ }^{-}=0$ case both mergers have the same eaect:

$$
\begin{gathered}
W^{1 ; 2 ; 34}>W^{12 ; 3 ; 4} \\
\frac{59}{72}+\frac{1}{2}^{-}+\frac{1}{6}-2>\frac{59}{72}^{-}+\frac{4}{9}^{-}+\frac{1}{9}-2 \\
\frac{1}{18}-\left(1+^{-}\right)>0
\end{gathered}
$$

Finally, starting from the structure 12,3,4 a merger between 3 and 4 will not increase welfare. This is a mere monopolization of market $B$ with no other consequences.

$$
\begin{gathered}
W^{12 ; 3 ; 4}>W^{12 ; 34} \\
\frac{59}{72}+\frac{4}{9}+\frac{1}{9}-2>\frac{3}{4}+\frac{3}{8}+\frac{3}{32}-2 \\
\frac{5}{288}\left(2+^{-}\right)^{2}>0
\end{gathered}
$$

which is trivially true as $0^{-}<1$ :
Proposition 3 The following ranking of welfare level can be established for all $0<{ }^{-}<1: W^{13 ; 2 ; 4}>W^{1 ; 234}>W^{1234}>W^{12 ; 34}$

Proof. Starting from a market structure with an intermarket merger 13, adding a further ..rm (..rm 4) to the coalition will decrease welfare. There are two opposite eaects here: one the one hand market B becomes a monopoly but, on the other hand, ..rm 1 will have an extra incentive to over produce in the sense that it will be the sole bene..ciary of the increase in the demand in market B. It turns out that the ..rst exect is stronger.

$$
\begin{gathered}
W^{13 ; 2 ; 4}>W^{1 ; 234}\left(=W^{134 ; 2}\right) \\
\frac{1}{2} \frac{144+102^{-}+19^{-2} i 2^{-3}}{\left(3 i^{-}\right)^{2}\left(3++^{-}\right)^{2}}>\frac{1}{2} \frac{59+40^{-}+3^{-2} i^{-3}}{6 i^{-2} 2^{42}} \\
\frac{1}{2}\left(3+2^{-}\right) \frac{135+54^{-} i 111^{-2} i 64^{-3}+13^{-4}+8^{-5}}{\left(3 i^{-}\right)^{2}\left(3+^{-}\right)^{2 i} 6 i^{-2^{42}}}>0
\end{gathered}
$$

which is trivially true as $0<{ }^{-}<1$
Starting from structure 1,234, the move towards full integration will decrease
welfare because, instead of a duopoly where one of the ..rms will bene..t from the externality of increasing production, we will now have a monopoly with the same incentive. However, the monopoly will not produce as much as the duopoly.

$$
\begin{gathered}
W^{1 ; 234}>W^{1234} \\
\frac{1}{2} \frac{59+40^{-}+3^{-2} i 2^{-3}}{{ }^{1} 6 i^{-2^{4}}}>\frac{3 i^{-}}{\left(2 i^{-}\right)^{2}} \\
i 20 i 16^{-}+{ }^{-2}+5^{-3}<0
\end{gathered}
$$

which is true for all $0<{ }^{-}<1$ :
Finally, it is trivially better to have the full integration outcome than to have a monopoly in each market because, provided that ${ }^{-}>0$; the externality is fully internalized, which somewhat mitigates the absence of competition.

$$
\begin{gathered}
W^{1234}>W^{12 ; 34} \\
\frac{3 i^{-}}{\left(2 i^{-}\right)^{2}}>\frac{3}{4}+\frac{3}{8}^{-}+\frac{3}{32}-2 \\
i^{-16} \mathrm{i}^{12^{-}}+3^{-3}<0
\end{gathered}
$$

$\mathrm{As}^{-}<1$ the inequality above is always veri..ed.
Summarizing,
Proposition 4 A duopoly with cross-market mergers $(13,24)$ is the best structure in terms of welfare. In turn, the double monopoly $(12,34)$ is the worst. W hen competition subsists in only one market, society is better ox when it subsists in the market which drives the other $\left(\mathrm{W}^{1 ; 234}>\mathrm{W}^{123 ; 4}\right)$.

## 4 The core

Given the results obtained in section 2 for the various market structures, we now look for the core structure making use of the dominance concept de..ned in Horn and Persson (2001). We are then able to evaluate this structure in terms of welfare, using the results obtained in section 3.

We admit that full monopolization (1234) will not be allowed by the regulatory authority. Further, monopolization in both markets $(12,34)$ is also not permitted (actually this structure is even worse than 1234, as far as total welfare is concerned, as we saw above).

Proposition 5 De..ning the core as in Horn and Persson (2001) and assuming that the structures 1234 and 12,34 will never be allowed by the authorities, then the only market structure in the core is 1,234 .

Proof. To show that 1,234 is in the core we must show that this structure is not dominated by any other alternative. Recall that a market structure A is dominated by another market structure $B$ in the sense of Horn and Persson (2001) if there exists a group of decisive owners that has higher aggregate pro..ts under $B$ than under $A$. To show that a structure is undominated it must therefore be compared against all other conceivable and admissible market structures (in this case there are nine diperent market structures, two of them considered inadmissible).

1,234 vs $1,2,3,4$ ( $2,3,4$ are the decisive owners)

$$
\begin{aligned}
& \frac{13+14^{-}+2^{-2} i 3^{-3} i^{-4}}{\mathrm{i}_{\mathrm{i}}^{-2^{42}}>\frac{1}{9}+\frac{2}{81}\left(3+2^{-}\right)^{2},} \\
& \frac{81+270^{-}+198^{-2} \mathrm{i} 45^{-3} \mathrm{i} 12^{-4} \mathrm{i} 24^{-5} \mathrm{i} 8^{-6}}{81^{i} 6 i^{-2} \$_{2}}>0
\end{aligned}
$$

1,234 vs $1,2,34$ ( $2,3,4$ are the decisive owners)

$$
\begin{array}{r}
\frac{13+14^{-}+2^{-2} i 3^{-3} i^{-4}}{{ }^{\prime} 6 i^{-2^{42}}}>\frac{1}{36}\left(3+2^{-}\right)^{2}+\frac{1}{9} \\
-\left(3+2^{-}\right) \frac{24+12^{-}+4^{-2} i 3^{-3} i 2^{-4}}{36^{1} 6 i^{-4^{4}}}>0
\end{array}
$$

1,234 vs $1,23,4$ ( $2,3,4$ are the decisive owners)

$$
\begin{gathered}
\frac{13+14^{-}+2^{-2} i 3^{-3} i^{-4}}{{ }_{6} i^{-2^{4}}}>\frac{18+15^{-}+2^{-2} i 3^{-3} i^{-4}}{{ }^{1} 9 i^{-2^{42}}}+\frac{\left(2^{-}+3\right)^{2}}{1} 9 i^{-2^{42}} \\
\left(3+2^{-}\right) \frac{3+4^{-} i^{-2} i^{-3}}{9 i^{-2^{4 i}} 6 i^{-2^{42}}}>0
\end{gathered}
$$

1,234 vs 123,4 (all are decisive owners)

$$
\begin{aligned}
& -\frac{2+{ }^{-}}{{ }_{6 i}{ }^{-2^{2}}}>0
\end{aligned}
$$

1,234 vs $12,3,4$ (all are decisive owners)

$$
\begin{gathered}
\left(2+^{-}\right)^{2} \frac{\left(1_{i}{ }^{-}\right)^{2}}{6_{i} 2^{\Phi_{2}}}+\frac{13+14^{-}+2^{-2} i_{i} 3^{-3} i^{-4}}{6 i^{-} 2^{4}}>\frac{1}{4}+\frac{2}{36}\left(2+{ }^{-}\right)^{2}, \\
{ }^{-}\left(2++^{-}\right) \frac{36+30^{-}+15^{-2} i 4^{-3} i 2^{-4}}{36^{1} i_{i} 2^{\phi_{2}}}>0
\end{gathered}
$$

1,234 vs 13,24 (all are decisive owners)

$$
\begin{aligned}
& \frac{9+18^{-}+5^{-2} i^{-3} 5^{-3} 2^{-4}}{{ }^{\prime} 6_{i}{ }^{-2^{42} i} 9 ; 2^{-2^{42}}}>0
\end{aligned}
$$

1,234 vs 12,34 (all are decisive owners)

$$
\left(2+^{-}\right)^{2} \frac{\left.\left(1_{i}\right)^{-}\right)^{2}}{1_{i}-^{42}}+\frac{13+14^{-}+2^{-2} i^{4^{-3}} i^{-4}}{{ }^{-4} i_{i}^{-2^{42}}}>\frac{1}{16}\left(2+^{-}\right)^{2}+\frac{1}{4}
$$

i $\frac{1}{16}\left(2+^{-}\right)^{i} 2+2^{-}+{ }^{-} 2^{\text {¢ }} \frac{4 i 10^{-}++^{-3}}{{ }_{6 i}{ }^{-2^{42}}}>0$ which is true if $^{-}$is large enough
$f\left({ }^{-}\right)=4 i^{-1} \quad 0^{-}+{ }^{-3}<0$ for ${ }^{-}$large enough
$\mathrm{f}(0)=4>0 ; \quad \mathrm{f}(1)=\mathrm{i} 5<0 ; \quad \frac{@}{@}=\mathrm{i} 10+3^{-2}<0$
1,234 vs 1234 (all are decisive owners)

$$
\left(2+^{-}\right)^{2} \frac{\left(1_{i}^{-}\right)^{2}}{6_{i} 2^{42}}+\frac{13+14^{-}+2^{-2} i_{i} 3^{-3} i^{-4}}{{ }_{6 i} i^{-2^{42}}}>\frac{1}{2 i^{-}}
$$

Note that this inequality is false, but merger 1234 is assumed not to be allowed by the authorities.

We thus conclude that

Proposition 6 The welfare-maximizing market structure $(13,24)$ is not in the core. The core is the third (or fourth, depending on ${ }^{-}$) best structure in terms of welfare $(1,234)$, or, if ${ }^{-}$is low enough and monopolization in both markets is allowed, the worst $(12,34)$.

## 5 Conclusion

In this paper we address mergers when markets are interrelated by asymmetric complementarity. Increasing sales in one market expands demand in the other, but not the reverse. This means that the two goods need not be purchased together, at least one of them can be consumed separately. We consider several types of mergers, namely intramaket, across markets, and combinations of both. The welfare exects of all these structures are analyzed, and it is found that a combination of two cross-market mergers yields the highest gain for the society as a whole. The core structure (according to the de..nition employed by Horn and Persson, 2001), however, does not coincide with this one, as it involves monopolization of the driven market together with one cross-market merger (the third or fourth best structure in terms of global welfare). A double monopoly is the worst solution for the society, even worse than full monopolization. We are currently using alternative endogenous mechanisms to predict which will be the equilibrium market structure. We also intend to check the robustness of our results to a more general framework (such as other functional forms for demand and a higher number of ..rms).

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[^1]:    ${ }^{1}$ The expression "vertical" is not intended to capture any upstream or downstream relationship among ..rms.

