Computing and Selecting ϵ-Efficient Solutions of {0,1}-Knapsack Problems

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Abstract. This work deals with the computation and the selection of approximate – or ϵ-efficient – solutions of {0, 1}-knapsack problems. By allowing approximate solutions in general a much larger variety of possibilities for the underlying problem is offered to the decision maker while the potential loss of these almost efficient solutions compared to related efficient ones can be adjusted a priori, depending on the given application. In this paper, we propose a novel population based stochastic algorithm for the computation of the entire set of ϵ-efficient solutions, state a convergence result, and address the related decision making problem. For the latter we propose an interactive selection process which is intended to help the decision maker to understand the landscape of the obtained solutions.

1 Introduction

Since Loridan ([9]) has introduced the concept of ϵ-efficiency for multi-objective optimization problems (MOPs) more than two decades ago, it has been studied and used by many researchers, e.g. to allow (or tolerate) nearly optimal solutions ([9], [19]), to approximate the set of optimal solutions ([13]), in order to discretize this set ([7], [16]), or to tackle real world problems (e.g., [20], [1], [13]). One advantage of allowing approximate solutions is that by this a larger flexibility is offered to the decision maker (DM) whos task is to select an 'adequate' solution according to the given problem (and to his or her preferences) while the possible loss on the solution quality can be adjusted a priori. In this work we aim for the numerical treatment of {0,1}-knapsack problems which have a wide range of real-world applications (e.g., captial budgeting ([11]), relocation problems ([6]), or planning remediation ([5])), and in all of them the value of ϵ has a physical meaning, and thus, the potential loss compared to possible exact solutions is calculable. The explicit computation of approximate solutions has been addressed in several studies, most of them employing scalarization methods (e.g., [19], [1], [2]). Recently, an archving strategy has been proposed to maintain the entire set of ϵ-efficient solutions (denote by E_ϵ) in the limit using stochastic search algorithms. On the basis of this work we will propose a novel population based search procedure which is designed to compute the approximate solutions of the {0,1}-knapsack problems.

Furthermore, we will propose an interactive procedure which should help the DM to explore the landscape of E_ϵ, and which should thus ease his or her task to find the 'right' solution according to the current situation.

The remainder of this paper is organized as follows: in Section 2, we give the required background for the understanding of the sequel. In Section 3 we state the problem and motivate why we have chosen to tackle it with stochastic search algorithms. In Section 4 we propose such an algorithms and give some numerical results. Section 5 proposes an interactive selection procedure, and finally we conclude in Section 6.

2 Background

In the following we consider multi-objective optimization problems

$$\min_{x\in Q}\{F(x)\}, \tag{MOP}$$

where the function F is defined as the vector of the objective functions $F : Q \to \mathbb{R}^k, \quad F(x) = (f_1(x), \ldots, f_k(x))$, and where $Q \subset \mathbb{R}^n$ is finite.

Definition 1. *(a) Let $v, w \in \mathbb{R}^k$. Then the vector v is* less than *w ($v <_p w$), if $v_i < w_i$ for all $i \in \{1, \ldots, k\}$. The relation $\leq_p$ is defined analogously.*
(b) $y \in \mathbb{R}^n$ is dominated *by a point $x \in \mathbb{R}^n$ ($x \prec y$) with respect to (MOP) if $F(x) \leq_p F(y)$ and $F(x) \neq F(y)$, else y is called nondominated by x.*
(c) $x \in \mathbb{R}^n$ is called a Pareto point *if there is no $y \in \mathbb{R}^n$ which dominates x. Denote by P_Q the set of Pareto points of a given MOP.*

Definition 2. *Let $\epsilon = (\epsilon_1, \ldots, \epsilon_k) \in \mathbb{R}^k_+$ and $x, y \in \mathbb{R}^n$.*

(a) x is said to ϵ-dominate y ($x \prec_\epsilon y$) with respect to (MOP) if $F(x) - \epsilon \leq_p F(y)$ and $F(x) - \epsilon \neq F(y)$.
(b) x is said to $-\epsilon$-dominate y ($x \prec_{-\epsilon} y$) with respect to (MOP) if $F(x) + \epsilon \leq_p F(y)$ and $F(x) + \epsilon \neq F(y)$.

The definition in (b) is of course analogous to the 'classical' ϵ-dominance relation in (a) but with a value $\tilde{\epsilon} \in \mathbb{R}^k_-$. However, we highlight it here since it will be used frequently in this work. While the ϵ-dominance is a weaker concept of dominance, $-\epsilon$-dominance is a stronger one. We now define the set of interest

Definition 3. *[15] Denote by $P_{Q,\epsilon}$ the set of points in $Q \subset \mathbb{R}^n$ which are not $-\epsilon$-dominated by any other point in Q, i.e.*

$$P_{Q,\epsilon} := \{x \in Q | \nexists y \in Q : y \prec_{-\epsilon} x\} \tag{1}$$

Algorithm 1 gives a framework of a generic stochastic multi-objective optimization algorithm, which will be considered in this work. Here, $Q \subset \mathbb{R}^n$ denotes the domain of the MOP, P_j the candidate set (or population) of the generation process at iteration step j, and A_j the corresponding archive.

Algorithm 1 Generic Stochastic Search Algorithm

$P_0 \subset Q$ drawn at random
$A_0 = ArchiveUpdate(P_0, \emptyset)$
for $j = 0, 1, 2, \dots$ **do**
  $P_{j+1} = Generate(P_j)$
  $A_{j+1} = ArchiveUpdate(P_{j+1}, A_j)$
end for

3 The Problem

In this section we present the class of MOPs which is beeing considered in this work and make some discussions on it.

In the sequel we consider bi-objective {0,1}-knapsack problems of the following form:

$$f_1, f_2 : \{0,1\}^n \to \mathbb{R},\ f_1(x) = \sum_{j=1}^{n} c_j^1 x_j,\ f_2(x) = \sum_{j=1}^{n} c_j^2 x_j \tag{2}$$

s.t.

$$\sum_{j=1}^{n} w_j x_j \leq W,\ x_j \in \{0,1\},\ j = 1, \dots, n,$$

where c_j^i represents the value of item j on criterion $i, i = 1,2$; $x_j = 1,\ j = 1, \dots, n$, if item j is included in the knapsack, else $x_j = 0$. w_j is the weight of item j, and W the overall knapsack capacity.

Here we are particularly interested in instances where the items have 'similar' values – i.e., where some c_j^i's (not necessarily all) are within a relatively small range – since in that case the set of ϵ-efficient solutions can become large, even for small values of ϵ.

Table 1 shows some results for $n = 500$ items, and where the values c_j^i are chosen within the interval $[10 - d, 10 + d],\ d = 1, 2, 3$. We can observe that the magnitudes of $\tilde{P_Q}$ – the number of nondominated solutions found by the search procedure– is nearly independent from the choice of the interval. This does not hold for the magnitudes of $\tilde{P_{Q,\epsilon}}$, i.e., the set of points which is not $-\epsilon$ dominated by any other test point. We see that $|\tilde{P_{Q,\epsilon}}|$ gets larger the closer the values of the items are, and in all cases we have $|\tilde{P_{Q,\epsilon}}| > \tilde{P_Q}$. However, in case the values of the items vary a lot, it can happen that $P_Q = P_{Q,\epsilon}$, even for large values of ϵ (see e.g. the model in [8], or [18]).

Example 1. Let a knapsack problem and a value of ϵ be given. If there exits a point $p \in P_Q$ and two indices $j_1, j_2 \in \{1, \dots, n\}$ such that

$$p_{j_1} = 1, \quad p_{j_2} = 0, \quad e_{j_2} \prec_\epsilon e_{j_1}, \quad \omega_{j_2} \leq \omega_{j_1}, \tag{3}$$

	$\lvert\tilde{P_Q}\rvert$	$\lvert\tilde{P_{Q,\epsilon}}\rvert$
$c^i_j \in [9, 11]$	8.7	144.93
$c^i_j \in [8, 12]$	8.87	42.8
$c^i_j \in [7, 13]$	9.07	26.93

Table 1. Some numerical results for MOP (2) with $n = 500$, averaged over 30 test runs. We have taken the algorithm described in Section 4 using a population of 100 individuals ,with a number of 10.000 generations. $\tilde{P_Q}$ denotes the set of nondominated solutions and $\tilde{P_{Q,\epsilon}}$ the set of points which is not $-\epsilon$ dominated by any other test point generated by the algorithm.

where e_j denotes the j-th unit vector, then for the point $\tilde{p}$ which is defined as

$$\tilde{p} := \begin{cases} p_i & \text{if } i \notin \{j_1, j_2\} \\ 0 & \text{if } i = j_1 \\ 1 & \text{if } i = j_2 \end{cases}, \tag{4}$$

the following holds:

$$\tilde{p} \notin P_Q \quad \text{and} \quad \tilde{p} \prec_\epsilon p, \tag{5}$$

and the Hamming distance of these two points is given by 2.
As an numerical example we consider $n = 4$, $\omega = 1$, $W = 2$, and the weights

$$\begin{aligned} c^1 &= (10, 10, 5, 12) \\ c^2 &= (10, 9, 12, 7) \end{aligned} \tag{6}$$

The Pareto set is given by the following three elements:

x	F(x)
$x_1 := (1, 1, 0, 0)$	$(20, 19)$
$x_2 := (1, 0, 1, 0)$	$(15, 22)$
$x_3 := (1, 0, 0, 1)$	$(22, 17)$

Further we have $x_4 := (0, 1, 1, 0)$ with $F(x_4) = (15, 21)$ and $x_5 := (0, 1, 0, 1)$ with $F(x_5) = (22, 16)$. When chosing $\epsilon = (1, 1)$ it holds that

$$x_4 \prec_\epsilon x_2 \quad \text{and} \quad x_5 \prec_\epsilon x_3,$$

and in both cases the Hamming distance is 2.
In the continuous case (i.e., continuous objectives defined on a continuous domain) there are always ϵ-efficient points in the neighborhood of a Pareto point. To be more precise, for every Pareto point p there exists a neigborhood U_p of p such that *every* point $x \in U_p$ is ϵ-dominating p (and thus, $P_{Q,\epsilon}$ forms an n-dimensional set while P_Q is typically $(k-1)$-dimensional). This is of course not the case for combinatorial problems and for 'sufficiently small' values of ϵ, and hence in this case we expect that the magnitude of $P_{Q,\epsilon}$ is of the same order as the magnitude of P_Q.

The next example shows that $P_{Q,\epsilon}$ can be highly disconnected, which motivates to tackle such problems with stochastic search algorithms since 'classical' exact methods designed to locate P_Q and which utilize the locality of such MOPs, can probably not be easily tuned in order to solve the problem adequately (however, the authors do not foreclose that such algorithms will not exist in future).

Example 2. For this example, we consider $n = 6$, $w = 1$, $W = 3$, and the costs

$$\begin{aligned} c^1 &= (95, 120, 80, 98, 105, 87) \\ c^2 &= (107, 75, 115, 97, 90, 108) \end{aligned} \tag{7}$$

Here, P_Q consists of 18 points- two pairs of solutions having the same values in the image space - including $x_1 = (1, 1, 1, 0, 0, 0)$ with $F(x_1) = (295, 297)$ (see Figure 1). When choosing $\epsilon = (5, 5)$ – the value of $\epsilon_i = 5$ relates to approximately 5 percent of the average weight of *one* item – we see that $x_2 = (0, 0, 0, 1, 1, 1)$ with $F(x_2) = (290, 295)$ is an ϵ-efficient solution since it is ϵ-dominating x_1 (and only this point). The Hamming distance is 6, thus the maximal possible value. There exists also $x_4 = (0, 1, 1, 1, 0, 0)$ which is an ϵ-efficient solution since it is ϵ-dominating $x_3 = (1, 0, 0, 1, 1, 0)$, with $F(x_3) = (298, 294)$, $x_3 \in P_Q$. The Hamming distance between x_3 and x_4 is 4.

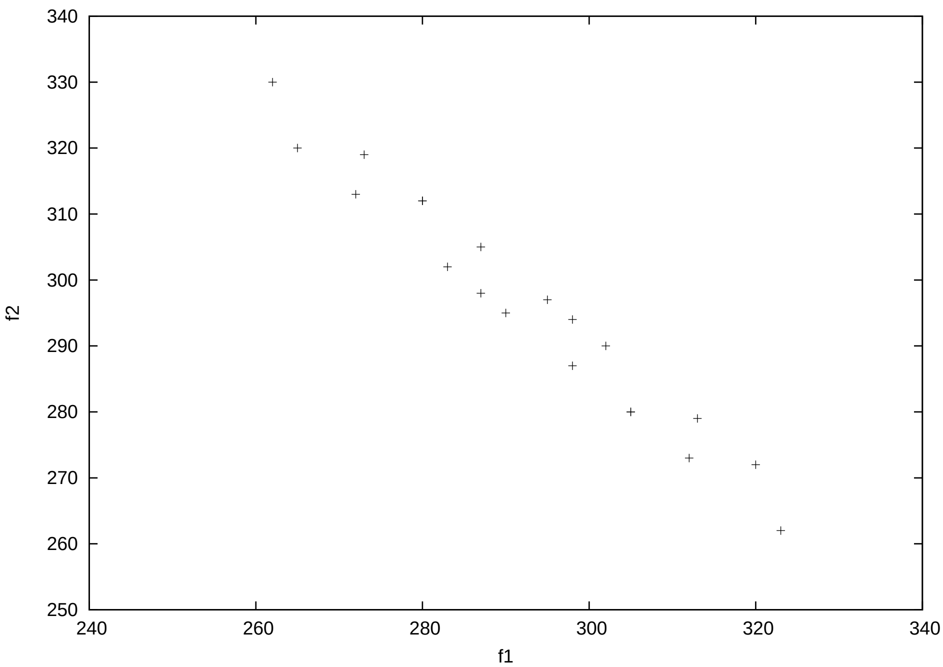

Fig. 1. The set of feasible solutions for the considered example. It can be observed that $P_{Q,\epsilon}$ contains one solution with $d_H = 6$ and several solutions with $d_H = 4$.

Further instances with larger distances of approximate solutions to P_Q can be constructed, see also the example in Section 5 or [18].

4 A Stochastic Search Algorithm

4.1 The Algorithm

The algorithm is a population based evolutionary technique intended for providing $P_{Q,\epsilon}$ approximation sets. It is intended as the first component of an exploratory process which offers to the users information about $P_{Q,\epsilon}$ approximation sets, required in studying the solutions landscape. The idea behind the exploration strategy is to improve the performances obtained using the archiving strategy by adapting the ϵ values to the feasible solutions landscape.

In [3], a basic structure of adaptative ϵ based exploration is proposed for {0,1}- multiple knapsacks problems in the context of the computation of P_Q. There, the value of ϵ is decreased by 1 each time the number of consecutive generations without improvement attains a specific value.

In this paper we propose general assumptions requisite in obtaining the convergence toward $P_{Q,\epsilon}$. These assumptions are taken into account in constructing a specific decrease function together with a way of considering the distance criterion between two consecutive archives in the adaptive process. For simplicity, we assume that all components of ϵ are identical, i.e., $\epsilon = (\epsilon^*, \dots, \epsilon^*)$. The value of ϵ^* is specified by the user, as well as a maximal starting value, $\epsilon_{\max}$. The maximal value can be deduced for specific problems by performing bounds computations for the objective functions.

Algorithm 2 exposes the generic components of the proposed adaptive ϵ-approximate searching strategy. The technique facilitates the reduction of the number of generations required in order to attain a P_{Q,ϵ^*} final set.

As regards the termination criterion, for a given t, the two conditions $t \leq MaxNoGenerations$ and $\epsilon_t \geq \epsilon_{\min}$ must be met in order to continue the loop. This implies that in the worst case the algorithm will terminate after the maximal number of generations by employing the $Decrease(t)$ adaptation for ϵ_t.

4.2 Discussion and Analysis

Adaptation of ϵ The distance criterion consists in computing a comparative metric between A_{t+1} and A_t - $dist(A_{t+1}, A_t) = Metric(A_{t+1}, A_t)$. If the value of the improvement falls bellow a specified threshold - denoted as *MinimalQualityIncrease* - the length of the step used in decreasing the value of ϵ increases by Δ. For our purposes the C-metric, proposed in [21], is computed between A_{t+1} and A_t. It was chosen by its ability of providing the percent of solutions from A_{t+1} which are dominating the ones in A_t. Also it has the advantage of being computed independently, without considering external factors, as a specified point. Other comparative metrics can be similarly employed.

Let $Decrease : \mathbb{N} \to [\epsilon^*, \epsilon_{max}]$ be a monotonically decreasing function which defines the value of ϵ in the adaptive process. The following assumption on *Decrease* is necessary in order to ensure convergence in the limit toward $P_{Q,\epsilon}$:

$$\exists t_0 \in N : \epsilon(t) = \epsilon^*, \forall t \geq t_0. \tag{8}$$

Algorithm 2 Generic Adaptive ϵ-Approximation Search

$t = 0$;
$\epsilon_0 = \epsilon_{\max}$
$A_0 = \emptyset$
$P_0 \subset Q$ drawn at random
$dist = 0$
while $\neg$ Termination_Criterion(P_t) **do**
  P_{t+1} = Generate(A_t,P_t);
  Evaluate(P_{t+1});
  $A_{t+1} = ArchiveUpdate(\epsilon_t, P_t, A_t)$;
  $\Delta = dist(A_{t+1}, A_t)$;
  if $\Delta < MinimalQualityIncrease$ **then**
    $\epsilon_{t+1} = Decrease(t + Increase(\Delta))$
  else
    $\epsilon_{t+1} = min(\epsilon_t, Decrease(t))$;
  end if
  $t = t + 1$;
end while

For our computations we have used the following functions:

$$D(t) := \epsilon_{\max} - \exp^{-\gamma\left(\frac{\beta}{MaxNoGenerations} t\right)^2} * (\epsilon_{\max} - \epsilon^*), \quad \text{for } t \leq t_0, \tag{9}$$

where β represents an arbitrarily large value.

ArchiveUpdate Here we use the archiving strategy proposed in [15] and which was designed to maintain the entire set of ϵ-efficient solutions with generic stochastic search algorithms. The archiving strategy is simply the one which keeps all obtained points which are not $-\epsilon$-dominated by any other test point, i.e.

$$ArchiveUpdateP_{Q,\epsilon}(\epsilon, P, A) := \{x \in P \cup A : y \not\prec_{-\epsilon} x \; \forall y \in P \cup A\}, \tag{10}$$

The following theorem states a result on the underlying abstract algorithm of the procedure proposed above.

Theorem 1. *Let an MOP of the form (2) be given and* $\epsilon \in \mathbb{R}^k_+$. *Further let*

$$\forall x \in \{0,1\}^n : \quad P(\exists l \in \mathbb{N} \; : \; x \in P_l) = 1 \tag{11}$$

Then an application of Algorithm 1, where $ArchiveUpdateP_{Q,\epsilon}()$ *is used to update the archive, leads to a sequence of archives* $A_l, l \in \mathbb{N}$, *with*

$$\lim_{l \to \infty} d_H(P_{Q,\epsilon}, A_l) = 0, \quad \textit{with probability one,} \tag{12}$$

where d_H *denotes the Hausdorff distance.*

Proof. This is a direct consequence of a result from ([14]), which holds for the continuous case.

Remark 1. **a)** The crucial assumption required to obtain convergence is (11). This is e.g. fulfilled if the sequence $(P_t)_{t\geq 0}$ of candidate sets obtained by *Generate()* is a homogeneous finite Markov chain with irreducible transition matrix ([12], [4]).

b) By (8) it is assured that $P_{Q,\epsilon}$ is computed in the limit. In the first steps, where larger values of ϵ_i are used (in order to increase the performance of the algorithm), outer approximations of $P_{Q,\epsilon}$ are generated since for all $\epsilon_1, \epsilon_2 \in \mathbb{R}^k_+$ with $\epsilon_1 \leq_p \epsilon_2$ it follows that $P_{Q,\epsilon_1} \subset P_{Q,\epsilon_2}$. Condition (8) has to be added for theoretical purposes since the function $f : \mathbb{R}^+_0 \to \mathbb{R}^+_0$,

$$f(\Delta) = dist(P_{Q,\epsilon+1\Delta}, P_{Q,\epsilon}) = \sup_{p\in P_{Q,\epsilon+1\Delta}} \inf_{q\in P_{Q,\epsilon}} \|p - q\|, \tag{13}$$

does not have to be continuous (e.g., if $F(Q)$ is not convex).

4.3 Numerical results

A comparative study was entailed between applying classical archive which stores all the nondominated solutions $ArchiveUpdateND$ [16] based on the non-dominance relation and adaptive ϵ-approximation search based on the use of $ArchiveUpdate_{P_{Q,\epsilon}}$.

For both algorithms a comparable number of evaluations has been performed, each of the algorithms being executed having the same maximal number of generations, namely 10,000 generations. The size of the population has been set to 100 individuals. It can be observed from Figure 2 that the solutions obtained by the adaptive technique include all the solutions provided by the $ArchiveUpdateND$. For the adaptive process we used $\epsilon_{max} = 5$ and $\epsilon^* = 2$ (and thus $\epsilon = (2, 2)$).

5 Interactive Selection Method

Having computed an approximation of $P_{Q,\epsilon}$ (denote by $\tilde{P_{Q,\epsilon}}$), the question naturally arises how to select a suitable point out of this (large) set according to the given application. The scope of this section is to propose such a selection mechanism.

The selection mechanism is intended as the second step of the exploration process. The target users are developers that want to gather knowledge about the topology of the landscape described by subsets of $P_{Q,\epsilon}$ and of $F(P_{Q,\epsilon})$ sets. We provide tools that allow focusing on specific regions.

In this section we assess the performance of interactive methods in guiding the user through the selection process. The main steps of the Interactive Selection Method are exposed in the followings. The selection mechanism starts in the image space due to its low dimensionality. Given an approximation of $P_{Q,\epsilon}$, computed using the Algorithm 2. We present to the user a filtered front - further denoted as $\mathcal{F}$ - composed only of the Pareto non-dominated solutions from the $P_{Q,\epsilon}$. The user specifies a region by employing graphical tools and/or by specifying a tolerance value for ϵ. The solutions from $P_{Q,\epsilon}$ contained in the specified interest region, R - further denoted as $P_{Q,\epsilon}(R)$ - are graphically depicted in both the objective and the decisional space.

Figure 3 captures the main steps of the interactive EMO technique applied for the knapsack instance presented in Figure 2.

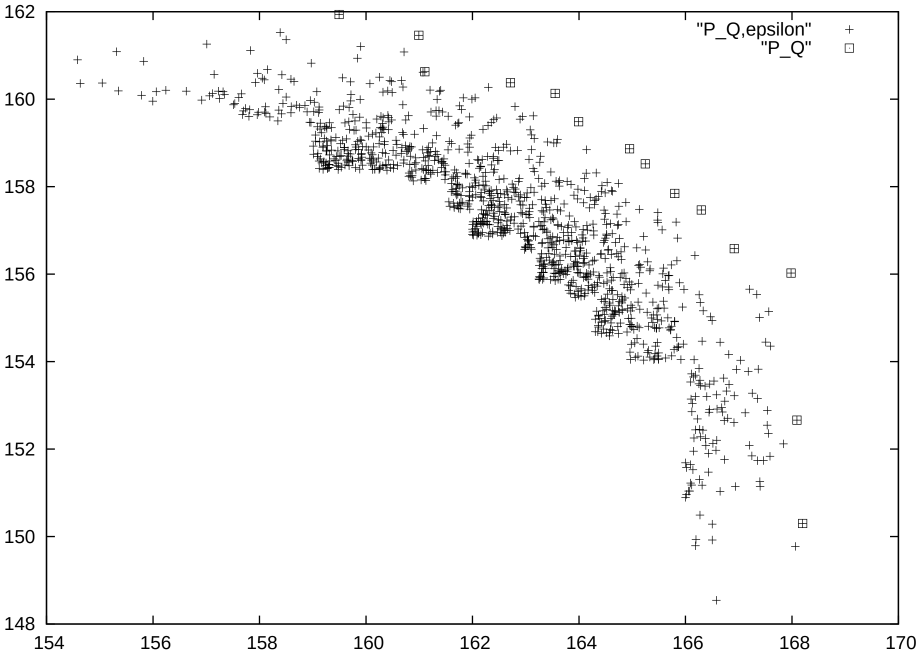

Fig. 2. Approximations of P_Q and $P_{Q,\epsilon}$ for an instance with $n = 30$ and for $\epsilon = (2, 2)$. Though ϵ is relatively small, the set of approximate solutions is much larger than the set of nondominated points, offering thus more possibilities for the DM.

Algorithm 3 Interactive component

$\mathcal{F}$:= nondominated solutions of $P_{\tilde{Q}}, \epsilon$
while user ¬ satisfied **do**
    R = User Input Interest Region($\mathcal{F}$)
    ObjectiveSpaceDisplay ($P_{Q,\epsilon}(R), P^f_{Q,\epsilon}$)
    DecisionalSpaceDisplay ($P_{Q,\epsilon}(R)$)
end while

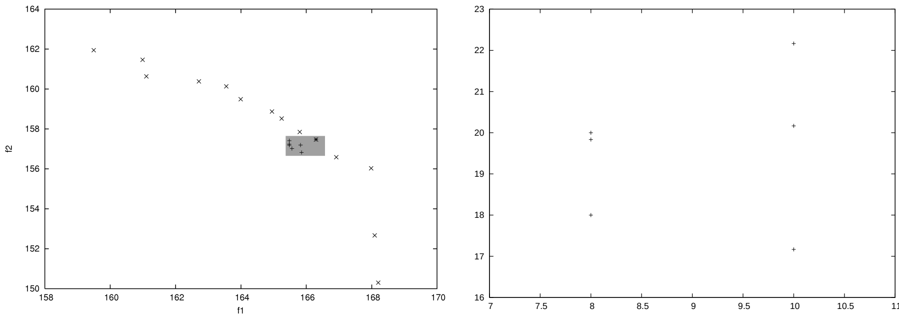

Fig. 3. Visualization of the objective space (left) and of the decision space(right) for a typical run of the proposed interactive EMO on the knapsack problem depicted in Section 3 for a 30 objects instance.

Association coefficients were chosen as distance metrics for representing ($P_{Q,\epsilon}(R)$) in the decisional space. For every Pareto point from the interest region an associated plot is presented to the user.

On the first axis of the plot, the Hamming distance is computed between the chosen Pareto point and the points in the interest region. This allows for an initial clustering of the data.

Regarding the second axis, the context is changed from the chosen Pareto point. A measure of the correlation between the solutions in the specified region is chosen. To this aim, an average of the Hamming distances of every point as regards the other points in the interest region is presented to the user. As an example, when a point has a large value on the second axis - as compared to other points on the first coordinate- this means that there are a small number of similarities between them and the rest of the points in the specified neighborhood. An in-depth study of distance measures for binary variables can be found in [10].

For the given example we obtain in the chosen interest region three solutions with Hamming distance 8 and other three solutions with Hamming distance 10. The two different perspectives - decisional and objective space - allow a better understanding of the solutions landscape topology. As shown in Example 2, providing the decisional space perspective is crucial. Two neighbor solutions, in the objective space, can represent very different solutions in the decisional space - if not completely different.

In the real life production process it is possible that the values of the objective functions are satisfactory to the user, while the proposed associated configuration cannot be applied. Thus, analyzing in the decisional space solutions from a tolerance interval can resolve the problem. Totally different solutions can lay on the tolerance interval and thus satisfy the end-user needs.

6 Conclusions

As shown by the examples in Section 3, the objective space perspective alone can be misleading in choosing the desired approximate solutions. The presence of totally different solution configurations is tackled by the proposed adaptive technique. The chances of obtaining a solution that fits the user demands are thus increased. The interactive techniques offers the last step needed in efficiently making use of the captured information.

Among the extensions envisaged, a specific comparison metric that ensures the speed-up of the adaptive process is considered. The interactive selection process can also be integrated in the search procedure as part of an interactive evolutionary multi-objective technique, as depicted in [17].

The hybridization between the algorithmic and the interactive part presented in this paper induces results with provable performance guarantee. It also allows a more complex evaluation of the solutions landscape structure.

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