Signaling Advertising by Multiproduct Firms

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Abstract

We consider the use of advertising expenses as quality signals in multiproduct firms, extending previous results on single product firms. In our model a firm introduces sequentially two products whose qualities are positively correlated. We investigate whether there exist information spillovers from the first to the second market. We show that, when correlation is high, the equilibrium in market 2 depends on the «quality reputation» the firm has gained in market 1. Moreover, a firm with a high-quality product 1 may need to advertise a very high amount in this market in order to separate from her low-quality counterpart. By advertising such high amount, the firm is also signalling the quality of the product that she will introduce in the future. Thus advertising in the first market has information spillovers in the second market.

Keywords: Quality signaling; Advertising; Multiproduct firms

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1 Introduction

Many firms in real world produce more than one good, and the quality of these goods is often correlated.[1] However economic literature has mainly dealt with the case of single product firms. This paper extends the previous analysis on signaling advertising in monoproduct firms to multiproduct firms.

Advertising expenses may be used to indirectly signal the quality of experience goods, by spending an amount high enough to separate high-quality producers from low-quality ones. The incentive to use costly advertising signals changes if there are information spillovers across markets. Actually, signaling one good's quality reflects positively on the demand for the other goods, if their quality is believed to be correlated with the quality of the advertised good.

In this paper we consider a firm introducing sequentially two products in two distinct markets. The good introduced in each market may either be of low or high-quality, and the qualities of the two products are correlated. Advertising can be used in both markets to signal quality. However, due to quality correlation, the advertising expenditure in the first market may influence the consumers' perception of the second product's quality.

Our model is quite interesting from a game-theoretic point of view. It is a signaling game but with some special features. The sender (the firm) learns its type gradually and may send a signal each time it gets more information about its own type. Thus we have a model with a sequence of signals: a sequential signaling model.

Although there is no paper directly connected with this work, it can be related to two branches of the literature: signaling advertising and quality signaling by multiproduct firms. The idea of using advertising expenditures as a signal of product quality was first presented by Nelson (1974). He identified three effects which support his argument: efficiency, repeat-business and match-products-to-buyers. According to the efficiency effect, demand expansion is most attractive to efficient firms (the ones who offer a better quality/price ratio).[2] Thus these firms will set low prices and advertise heavily to increase demand. The second reason why high-quality firms may advertise to signal quality is because high-quality products generate repeat purchases. Hence a high-quality firm gains more in creating goodwill. The last reason for advertising to be used as signal is that the firm has more incentive to send its ads to the consumers who value its product the most.

In the last three decades many authors have developed formal models where advertising is used as signal of product quality. Kihlstrom and Riordan (1984) show that dissipative advertising

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[1] Common examples of this are car producers, soft drink producers, and others.

[2] Schmalensee (1978) argues the opposite. He defends that a high-quality firm is likely to have higher production costs. Thus, for a given price, a low-quality firm has a higher profit margin.

may be used to signal quality in a model where firms are competitive price takers, as long as marginal cost is sufficiently lower when quality is high (efficiency effect) or there exists a repeat-business effect which overwhelms an eventual marginal-cost advantage of low-quality firms (repeat-business effect). In Kihlstrom and Riordan (1984) prices are not used as signals of product quality. However, many models consider the possibility of using both advertising and prices as signals of product quality. This raises the interesting issue of whether the two signals will be used in equilibrium or not. The answer depends on whether we assume that advertising is dissipative or demand enhancing and on the presence of effects such as repeat-business and informed consumers. In static models Overgaard (1991) and Zhao (2000) show that dissipative advertising is not used as a signal (signaling is done exclusively through prices). However this result is not valid in other settings. Milgrom and Roberts (1986), who developed the first multiple signals model, consider a two period model so as to incorporate a repeat-business effect. They show that dissipative advertising may complement prices in order to achieve separation at minimal cost.[3] Linnemer (2002) obtains a similar result in a static model with a mixture of informed and uninformed consumers. Dissipative advertising may also be used to signal high quality in duopoly models, as demonstrated by Hertzendorf and Overgaard (2001) and Fluet and Garella (2002). In our model we assume that the only quality signal is dissipative advertising. The existing literature suggests that one could build a multiproduct firm model where both price and advertising are used as signals. Such a model would be necessarily complex due to the existence of multiple signals and multiple products. Our simplifying assumption allows us to focus on the effects upon advertising of having a multiproduct firm, without having to deal with multiple signals.

To our knowledge, Bagwell (1992) is the only paper sofar concerned with quality signaling in multiproduct firms. Our focus, however, is distinct from his: whereas Bagwell concentrates on differences between products of a given product line, we concentrate on differences between monoproduct and multiproduct firms. Moreover there are several important modelling differences. Bagwell (1992) considers a product line which may either be of low or high quality, implicitly assuming perfect correlation for the quality of the various products in the product line, while we only assume that products are positively correlated. In Bagwell (1992) the issue is how to use the prices of the various products in the product line to signal the quality of the all product line, whereas in our case advertising in the first market, which signals quality of the first product, may also affect the consumer's beliefs about the second product quality.

The remainder of the article is organized as follows. The next section presents the model. In

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[3] Empirical work conducted by Thomas et al (1998) on Milgrom and Roberts' findings has confirmed the use of advertising as a signal of product quality for the U.S. automobile industry.

Section 3 we discuss the results for a single product firm. The next two sections consider the case where the firm introduces sequentially two products whose qualities are correlated. In Section 4 we derive the most reasonable perfect Bayesian equilibrium when the quality of the first product is observed before the introduction of the second product. On the other hand, Section 5 presents the most reasonable equilibrium when the quality of the first product is not observed before the second product is introduced. Conclusions are summarized in the final section.

2 Model

Consider a firm producing two goods (1 and 2), whose quality is determined by Nature and can be either high (H) or low (L). Quality is observable to the firm, but not to the consumer. In this incomplete information game the firm may take one of four possible types: (H_1, H_2), (H_1, L_2), (L_1, H_2), or (L_1, L_2), where the subscripts denote the market.

Qualities are known to be positively correlated. Let ρ be the degree of quality correlation. The joint density function, which is common knowledge, is the following one:

	H_2	L_2
H_1	$p^2 + \rho p(1-p)$	$p(1-p) - \rho p(1-p)$
L_1	$p(1-p) - \rho p(1-p)$	$(1-p)^2 + \rho p(1-p)$

The prior probability for a high-quality product is $p \in (0,1)$. Once the quality of the first product is known, the posterior probability for a high-quality product 2 is $p + \rho(1-p)$ if the first product was of high-quality, and it is $p - \rho p$ if the first product was of low-quality. As expected, the probability of the second product being of high-quality is revised upwards when the quality of the first product is high and revised downwards when the quality of the first product is low. Moreover, the revision in the prior probabilities is larger when ρ is larger.

Let $\pi(q, \mu)$ denote gross profits of a firm having true quality $q \in \{H, L\}$ and perceived to be selling a high-quality product with probability μ, where $\mu \in [0,1]$. Let a denote advertising expenditures. Net profits are $\pi(q, \mu) - a$. Advertising has no direct impact on demand or gross profits, only an indirect effect through quality perception; consumers' decisions about how much to buy and their willingness to pay depend on the expectation they have about the quality of the good. It is assumed that $\pi(q, \mu)$ is increasing[4] and continuos in μ. When μ takes the value 1 (0) the good is believed to be high (low) quality. For simplicity, we assume that $\pi_1 = \pi_2 = \pi$.

For our purpose price is considered exogenous and not used as quality signal. The only signaling variable is advertising expense. This assumption is based on Milgrom and Roberts

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[4]No similar assumption is made about true quality, because production costs may be increasing in q.

(1986) results, according to which when the firm combines price and advertising it ends up using both. So, advertising is also employed even when price has already been chosen. This allows us to concentrate on the effects upon advertising choices of producing more than one good.

The sequence of the game is as follows: In the first stage Nature chooses the quality of the first product. In the second stage, after observing the quality of the first product, the firm chooses the advertising level a_1. In stage 3, after observing the advertising expenditure a_1, consumers decide on how much to buy of the first product. In the fourth stage, Nature chooses the quality of the second product. In the fifth stage, after observing the quality of the second product, the firm chooses the advertising level a_2. In stage 6, after observing the advertising expenditure a_2, consumers form their expectations about the quality of product 2 and decide on how much to buy of it. Consumers only learn the quality of the two products after stage 6.

One important feature of our model's timing is that consumers decide on how much to buy of the second product before learning the true quality of the first product. One case where this assumption is extremely reasonable is when the first product is a durable good. For a durable good it is natural to assume that quality is not known immediately after purchase. Characteristics such as durability will only be known many periods after the purchase is done. If the decision on how much to buy of the second product was taken after knowing the quality of the first product, the level of advertising for product 1, a_1, would be irrelevant in the second product purchasing decision. Since the consumers would learn if product 1 was H_1 or L_1 before the second product was introduced, the posterior beliefs would be $p + \rho(1-p)$ if H_1 was observed and $p - \rho p$ if L_1 was observed, regardless of the value of a_1. Our timing was chosen because it allows a richer informational spillovers' analysis. In our setup it may happen that a_1 affects the consumer's beliefs in market 2, thus we may have advertising spillovers. However, for comparison purposes, we will also describe in Section 4 what would happen if the consumer learns the quality of good 1 immediately after purchase.

Another interesting property of our model's timing is that the firm learns its type gradually (quality of product 1 is observed before quality of product 2) and sends signals every time it learns more about its type. Thus we have a model with a sequence of signals. To the best of our knowledge, the idea of sequential signaling has not been used before.[5] This type of model is technically more complex than a simple signaling game, but provides us interesting insights.

The firm's strategy is described by (a_1, a_2), where a_k $(k = 1, 2)$ is the advertising level in market k, a_1 being contingent on the quality of the first product (H_1 or L_1), and a_2 being

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[5]The idea of sequential signaling can be applied to any signaling model, by assuming that the sender learns his type gradually instead of knowing his type in a precise manner immediately. Moreover, the receiver has a less precise knowledge than the sender.

contingent on the firm's type, as well as on the past strategy a_1. When the firm decides a_2 the only relevant issue is how this decision affects the profit in market 2. However, when the firm decides a_1 the effect on both markets has to be taken into account. Consumers' expectations about the quality of product 1 are based on a_1. The posterior probability of good 1 being of high-quality given a_1 is denoted by $\mu_1(a_1)$. For product 2 the prior probability is revised twice, after a_1 is observed, and then after a_2 is observed. Let the first revised probability be $\mu_{21}(a_1)$ and the second one be $\mu_{22}(a_2)$. We will look for pure-strategy Perfect Bayesian Equilibria (henceforth PBE).

3 Monoproduct firm – a critical review

In a separating equilibrium the two firms choose different advertising levels $a^L \neq a^H$. Thus, when a^L or a^H are observed consumers learn the quality of the product. By contradiction, it is easy to show that $a^L = 0$ in any separating equilibrium.[6] On the other hand, a^H has to be such that a low-quality firm does not want to choose a^H even if, by doing so, it is perceived as being of high-quality:

$$\pi(L,0) \geq \pi(L,1) - a^H \quad \Leftrightarrow \quad a^H \geq \pi(L,1) - \pi(L,0)$$

Moreover, a^H has to be optimal for the high-quality producer:

$$\pi(H,1) - a^H \geq \pi(H,0) \quad \Leftrightarrow \quad a^H \leq \pi(H,1) - \pi(H,0)$$

The (sorting or single-crossing) condition for the existence of separating equilibria is that $\pi(q,\mu) - \pi(q,\mu')$, with $\mu > \mu'$, is increasing in q, which means that the high-quality firm gains more by being perceived as having higher quality than the low-quality one. When the sorting condition holds, there exists a continuum of separating equilibria[7] with $a^L = 0$ and $a^H \in [\pi(L,1) - \pi(L,0), \pi(H,1) - \pi(H,0)]$.

Notice that any level of $a \in (\pi(L,1) - \pi(L,0), \pi(H,1) - \pi(H,0)]$ is a strictly dominated strategy for the low-quality firm, but not for the high-quality one. Thus if such level of a is observed consumers should put probability zero on the firm being of low-quality (we are just using the refinement that off-the-equilibrium path beliefs should put probability zero on dominated strategies, whenever possible). However, these beliefs imply that a H firm will never spend more than $\underline{a}^H = \pi(L,1) - \pi(L,0)$. Thus $\underline{a}^H$, known as least-cost separating equilibrium,

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[6]Suppose $a^L > 0$. Then the low quality firm would gain by deviating to $a = 0$, because it would spend less and beliefs would be at least as favorable as with $a^L > 0$.

[7]The following beliefs can be used to support the separating equilibria as PBE: if $a < a^H$, the posterior belief for μ is 0; if $a \geq a^H$ the posterior belief for μ is 1. These beliefs are consistent with Bayes' rule on the equilibrium path and under the sorting condition imply that no type wants to deviate from the separating equilibrium.

is the only separating PBE which survives the domination criterion. In the least-cost separating equilibrium the amount spend by the high-quality firm is equal to the gain that a L firm would have if it pretended to be of high-quality and was perceived by consumers as such.

There is also a set of pooling equilibria $\tilde{a} \in [0, \pi(L, p) - \pi(L, 0)]$, supported by the belief that any firm deviating from $\tilde{a}$ is L with probability one. However these equilibria do not survive the Cho-Kreps (1987) intuitive criterion. To show this, consider a pooling equilibrium with advertising level $\tilde{a}$. Then there exists an advertising level $\hat{a} > \tilde{a}$ such that:

$$\pi(L, p) - \tilde{a} = \pi(L, 1) - \hat{a} \iff \hat{a} = \pi(L, 1) - \pi(L, p) + \tilde{a}.$$

Clearly any advertising level above $\hat{a}$ is an equilibrium dominated action for the low-quality firm. Moreover, the advertising level $a = \hat{a} + \varepsilon$ with ε small enough is not equilibrium dominated for the high-quality firm. Therefore, according to Cho-Kreps, if $a = \hat{a} + \varepsilon$ is observed then consumers should believe that the firm is high-quality, $\mu = 1$. But with these beliefs the high-quality firm gains by deviating from the pooling equilibrium. In fact,

$$
\begin{aligned}
\pi(H, 1) - [\hat{a} + \varepsilon] &> \pi(H, p) - \tilde{a} \iff \\
\pi(H, 1) - \pi(H, p) - \varepsilon &> \pi(L, 1) - \pi(L, p),
\end{aligned}
$$

which, for ε small enough, is true by the sorting condition. Thus the pooling equilibrium fails the intuitive criterion. Hence the least-cost separating equilibrium is the only PBE which survives the Cho-Kreps (1987) refinement.

Surprisingly, the result that the pooling equilibria do not survive the intuitive criterion does not depend on the prior probability of each type. Even if the probability of the high-quality firm is arbitrarily close to 1, the result still holds. However, in this case, the pooling equilibrium with $\tilde{a} = 0$ seems more reasonable from an economic point of view than the separating equilibrium. If the consumer is almost sure that the product has high-quality, why spend money just to convince him that quality is high? Thus, the intuitive criterion may eliminate equilibria which look more reasonable than the surviving equilibrium, and one may wonder whether the intuitive criterion is a too strong refinement.[8]

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[8]The idea that if $a > \hat{a}$, then $\mu = 1$, is not absolutely compelling either. Using the Cho-Kreps speech analogy, if the high type argued «look, consumer, if I choose $a = \hat{a} + \varepsilon$ you must believe I am high-quality, because if I was low-quality I would never choose $a > \hat{a}$, since my profits would be lower than what I would get in equilibrium», then the consumer could argue: «But if I believe that $a > \hat{a}$ means a high-quality firm you would never choose $a = \tilde{a}$ if you were high-quality; then I should also believe $a = \tilde{a}$ means low-quality, but then if you were a low-quality firm you would also prefer $\hat{a}$ to $\tilde{a}$. Thus only if you advertise so high that the low-quality type does not want to imitate you, should I believe you are high quality for sure!».

Another problem with the intuitive criterion (and other refinements which are based on the concept of equilibrium dominance) is that it is based on the assumption of common knowledge of the equilibrium being played.

The elimination of the pooling equilibria is quite reasonable when $\pi(H,1) - [\pi(L,1) - \pi(L,0)] > \pi(H,p)$. Since $a > \underline{a}^H = \pi(L,1) - \pi(L,0)$ is a dominated action for type L, beliefs should be $\mu = 1$ if such level of advertising is observed. However, with this beliefs, the high-quality firm gains by deviating from the pooling advertising level $\widetilde{a}$. If the previous condition holds, the high-quality firm prefers to choose $\underline{a}^H$ if it is perceived as being of high-quality than to choose $\widetilde{a}$ and be «pooled» with L. Notice that, in this case, the elimination of the pooling equilibria is based only on the refinement that off-the-equilibrium path beliefs should put probability zero on dominated strategies, whenever possible.

However, when $\pi(H,1) - [\pi(L,1) - \pi(L,0)] < \pi(H,p)$ the elimination of the pooling equilibria cannot be based on the requirement that off-the-equilibrium path beliefs should put probability zero on dominated strategies, whenever possible. In fact, the high-quality firm prefers the pooling equilibrium payoff $\pi(H,p)$ to the payoff it would get by choosing $a > \underline{a}^H$. The elimination of the pooling equilibria using the intuitive criterion is based on the argument that off-the-equilibrium path beliefs following a slightly above $\widehat{a}$ should be $\mu = 1$. Although we agree that following $a = \widehat{a} + \varepsilon$, the probability of the product being high-quality should not be lower than at the pooling equilibrium, that is, $\mu \geq p$, it seems extreme to impose that $\mu = 1$, since $a = \widehat{a} + \varepsilon$ is not a definitive proof that the product is high-quality.

It is interesting to notice that if $\pi(H,1) - [\pi(L,1) - \pi(L,0)] < \pi(H,p)$ there exists no perfect sequential equilibrium as proposed by Grossman and Perry (1986),[9] which in our opinion is another indicator that the forward induction arguments which are implicit in refinements like the intuitive criterion and the perfect sequential equilibrium may sometimes be too restrictive.

In what follows we will only use the domination criterion to restrict the set of PBE. When there exist multiple PBE which survive the domination criterion we select the Pareto optimal equilibrium in this set (from the two types perspective). When $\pi(H,1) - [\pi(L,1) - \pi(L,0)] > \pi(H,p)$ the least-cost separating equilibrium is the unique PBE which survives the domination criterion and, consequently, it is the most reasonable equilibrium.

On the other hand, when $\pi(H,1) - [\pi(L,1) - \pi(L,0)] < \pi(H,p)$ the set of PBE which survive the domination criterion also includes a set of pooling equilibria,[10] thus we have multiple

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[9]Since a perfect sequential equilibrium is a refinement of the Cho-Kreps intuitive criterion, only the least-cost separating equilibrium is a candidate for a perfect sequential equilibrium. However one can find beliefs ($\mu = p$ if $a = \varepsilon$, with ε small), such that under them both types would want to deviate from the separating equilibrium; $\mu = p$ are thus consistent beliefs and the least-cost separating equilibrium fails to be a perfect sequential equilibrium.

[10]The reasonability of the pooling equilibria can be defended on other grounds. An interesting (and for us appealing) alternative is proposed by Eichberger and Kelsey (2000). Their starting point is that players are not expected utility maximizers. Players are uncertainty averse and have non-additive subjective beliefs. They show that if there are high degrees of uncertainty, the pooling equilibria may be stable.

surviving equilibria. However, there exists a unique Pareto optimum in this set of equilibria: the pooling equilibrium where both types choose $a = 0$. We consider this to be the most reasonable equilibrium in this case. Since consumers know that both types are better off by choosing $a = 0$ with beliefs $\mu(0) = p$, it is quite natural that they expect this pooling equilibrium to happen.[11]

Lemma 1 *There exists a unique p^* such that:*

$$\pi(H,1) - [\pi(L,1) - \pi(L,0)] = \pi(H,p^*)$$

Proof: By the sorting condition when $p = 0$, $\pi(H,1) - [\pi(L,1) - \pi(L,0)] - \pi(H,p) > 0$. In addition, when $p = 1$, $\pi(H,1) - [\pi(L,1) - \pi(L,0)] - \pi(H,p) < 0$. Since, by assumption, $\pi(H,p)$ is increasing and continuous in p, there exists a unique p^* such that $\pi(H,1) - [\pi(L,1) - \pi(L,0)] - \pi(H,p) = 0$.■

For values of p below p^* the most reasonable equilibrium is the least-cost separating equilibrium whereas for values above p^* both types «pooling» at $a = 0$ is the most reasonable equilibrium (see figure 1). The value of p^* depends on the difference between the two types, as measured by $(\pi(H,\mu) - \pi(H,\mu')) - (\pi(L,\mu) - \pi(L,\mu'))$ where $\mu > \mu'$. When the two types are very different the value of p^* is high, when they are similar the value of p^* is low.
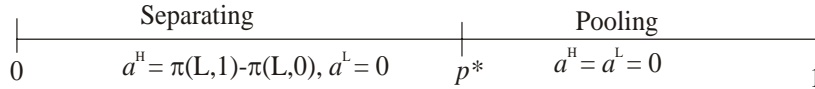

Figure 1: Most reasonable equilibrium as a function of prior beliefs.

It is worth mentioning that the characterization of the most reasonable equilibrium would be similar if we had developed a more complete model where price and advertising are both used as quality signals. Actually in these models there exist pooling equilibria which survive the intuitive criterion and the pooling equilibrium at $a = 0$ can be justified using refinements such as the one proposed by Grossman and Perry (1986). Thus we believe that the results that we derive in the next two sections would still be valid in a multi-signals model.

In the following sections we extend the monoproduct firm results for a firm producing two goods, which quality is believed to be positively correlated. The firm can influence the perception of quality in one market through the advertising amount spent in the other market.

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[11]In order to satisfy the domination criterion off-the-equilibrium path beliefs have to be such that if $a > \underline{a}^H$ then $\mu(a) = 1$. For $0 < a < \underline{a}^H$ one can assume that $p < \mu(a) < 1$, with μ increasing with a and such that no type wants to deviate from the pooling equilibrium $a = 0$.

4   Equilibria when quality of good 1 is observed

According to our assumptions, when choosing the second product advertising level the firm has already observed its first product quality, but the consumer has not. In this section we analyze what would happen if the consumer learns the quality of the first product, before observing a_2.

In this case, the quality of the first product is perfectly known to the consumer when the second product is introduced. Consumers' beliefs after they learn the quality of product 1 but before observing a_2 are given by $\mu_2(H_1) = p + \rho(1 - p)$ if the first product is of high-quality, while $\mu_2(L_1) = p - \rho p$ if the first product is L_1. Notice that these beliefs do not depend on a_1.

The fact that a_1 cannot influence beliefs about the quality of product 2 implies that the most reasonable continuation equilibrium depends only on whether the first product is H_1 or L_1 and whether $\mu_2(H_1)$ and $\mu_2(L_1)$ are below or above p^*. Since the continuation equilibrium does not depend on a_1, the optimal a_1 for each type of product 1 is determined only by what happens in the first market.[12] Consequently, the conditions which define the separating and pooling equilibria in market 1 are precisely the same as in the monoproduct case and the most reasonable equilibrium is determined in the same manner: when $p < p^*$ the most reasonable equilibrium is the least-cost separating equilibrium, when $p > p^*$ the most reasonable equilibrium is the pooling equilibrium with $a_1 = 0$.

Depending on the prior probability and the quality correlation we may have four types of equilibria in the complete game (Figure 2 illustrates the four regions where each type of equilibrium holds):

Proposition 1 *If the consumer learns the quality of good 1 before observing a_2:*

(i) *When $p < p^*$ and $\rho < \frac{p^* - p}{1 - p}$ the most reasonable PBE is the least-cost separating equilibrium in both markets. That is, $a_2^H = \underline{a}^H$ and $a_2^L = 0$ regardless of a_1 and in equilibrium $a_1^H = \underline{a}^H$ and $a_1^L = 0$.*

(ii) *When $p < p^*$ and $\rho > \frac{p^* - p}{1 - p}$ in the most reasonable equilibrium H_1 and L_1 separate in market 1 (type H_1 just needs to advertise $a_1^{H_1} = \underline{a}^H$). If the firm is H_1, then in the second market there is pooling ($a_2^H = a_2^L = 0$). If the firm is L_1, then in the second market there is separation ($a_2^H = \underline{a}^H$ and $a_2^L = 0$).*

(iii) *When $p > p^*$ and $\rho < \frac{p - p^*}{p}$ the most reasonable equilibrium is the pooling one with $a^H = a^L = 0$ in both markets.*

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[12] To exemplify, in a separating equilibrium in a_1, the gain that a L_1 firm has by mimicking H_1 is limited to the gain in market 1. Thus, in order to separate the high-quality firm only needs to spend $a_1^{H_1} = \pi(L, 1) - \pi(L, 0)$.

(iv) *When $p > p^*$ and $\rho > \frac{p-p^*}{p}$ we have pooling in market 1. If the firm is H_1, then in the second market there is pooling ($a_2^H = a_2^L = 0$). If the firm is L_1, then in the second market there is separation ($a_2^H = \underline{a}^H$ and $a_2^L = 0$).*
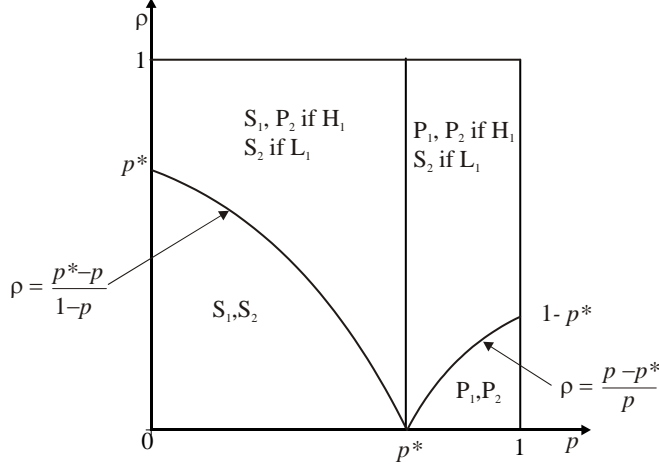

Figure 2: The most reasonable equilibrium when the consumer learns the quality of good 1 before a_2 is observed.

The previous result tells us that when ρ is low ($\rho < \frac{p^*-p}{1-p}$ for $p < p^*$ or $\rho < \frac{p-p^*}{p}$ for $p > p^*$) the multiproduct firm just replicates in each market the behavior of a single product firm: if $p < p^*$ there is separation in each market. If $p > p^*$ none of the types advertises in the first market and since correlation is low, posterior beliefs after the quality of the first product is observed do not change much, so consumers still consider that the probability of the second product being of high-quality is high, and none of the types advertises in the second market either.

On the other hand, when quality correlation is high, a firm with a high-quality first product, benefits from the consumers expectations. Since consumers learn that the first product is of high-quality and quality correlation is high, consumers attribute a very high probability to the second product being of high-quality too. Thus, in the second market the firm does not need to advertise. In the second market, the firm can exploit the reputation[13] created with the first product.

On the contrary, when quality correlation is high but the first product was of low-quality, there will be separation in the second market. In this case, the reputation of the firm after the

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[13]The reputation after the quality of the first product is observed is measured by the posterior beliefs $\mu_2(H_1)$ or $\mu_2(L_1)$.

quality of the first product is observed is low. Thus, if a firm happens to have a second product of high-quality, the firm will want to advertise high enough to credibly signal that the second product is of high-quality.

It is interesting to notice that with high correlation the behavior of a multiproduct firm is quite different from the behavior of a single product firm. In the multiproduct firm, what happens in the second market depends crucially on the quality of the first product. If the first product was of high-quality, in the second market the firm will exploit its good reputation and does not need to advertise. If the first product was of low-quality, in the second market a good quality firm will want to advertise in order to show that the second product is of high-quality.

However it should also be noted that if the quality of the first product is learned before observing a_2, then there are no signaling spillovers. The level of advertising in the first market does not affect the consumers perception of good 2's quality. Consumers perception is only influenced by whether their previous experience with product 1 was good or bad.

5   Equilibria when quality of good 1 is not observed

As usual in dynamic games, we analyze first the continuation equilibria after a_1 is observed, and then proceed backwards to derive the equilibrium levels of a_1. Notice that the level of a_1 only influences the profit in the second product through the beliefs.

Let $\mu_{21}(a_1)$ be the probability that product 2 is of high-quality given a_1, which is equal to:

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\begin{aligned}
\mu_{21}(a_1) &= \Pr(H_2|H_1) \bullet \Pr(H_1|a_1) + \Pr(H_2|L_1) \bullet \Pr(L_1|a_1) \\
&= (p + \rho(1-p))\,\mu_1(a_1) + (p - \rho p)\,(1 - \mu_1(a_1))
\end{aligned}
$$

One can interpret $\mu_{21}(a_1)$ as the beliefs before a_2 is observed. If we look to the continuation game after a_1 is observed it looks as a one product signaling game where $\mu_{21}(a_1)$ are the prior beliefs.

From the monoproduct analysis we know that we may have separating and pooling equilibria in market 2. When $\mu_{21}(a_1) < p^*$, the most reasonable continuation equilibrium is the least-cost separating one, with $a^L = 0$ and $\underline{a}^H = \pi(L, 1) - \pi(L, 0)$. On the other hand, if the previous condition fails, the most reasonable continuation equilibrium is the pooling equilibrium with $a^L = a^H = 0$.

In what follows we derive the equilibrium levels of a_1, assuming that the continuation equilibrium is the most reasonable one both on-the-equibrium path and off-the-equilibrium path. In other words, we will restrict our analysis to PBE with reasonable continuation equilibria.

5.1 Separating equilibria in market 1

Let us start by studying the existence of PBE with separation in a_1, that is, can we have equilibria where types H_1 and L_1 choose different levels of a_1, $a_1^H \neq a_1^L$? If the two types separate, in equilibrium the consumer will learn precisely the quality of product 1 just by observing a_1. As a consequence, posterior beliefs will be as if quality of product 1 was observed, $\mu_{21}(a_1^H) = p + \rho(1-p)$ and $\mu_{21}(a_1^L) = p - \rho p$. For other values of a_1, $\mu_{21}(a_1)$ depends on off-the-equilibrium-path beliefs. For example, if other values of a_1 are interpreted as product 1 being of low-quality, $\mu_{21}(a_1) = p - \rho p$ for other values of a_1. It is very important to specify off-the-equilibrium path beliefs because the continuation equilibrium after a deviation will depend on these beliefs.

5.1.1 Low prior and low correlation: $p < p^*$ and $\rho < \frac{p^*-p}{1-p}$

When $p < p^*$ and $\rho < \frac{p^*-p}{1-p}$ we know that $p + \rho(1-p) < p^*$. From the previous discussion it is obvious that if $p + \rho(1-p) < p^*$ then the unique reasonable continuation PBE is the least-cost separating equilibrium. The most favorable beliefs occur when a_1^H is observed which reveals that good 1 is H_1 and leads consumers to update their beliefs about product 2 being of high-quality from p to $p + \rho(1-p)$. If $p + \rho(1-p) < p^*$, then $\mu_{21}(a_1) < p^*$ for all values of a_1, thus the most reasonable continuation PBE is the least-cost separating equilibrium, for all a_1.

Under these circumstances, if we proceed backwards to determine the separating equilibrium levels of a_1, we conclude that in order to have a separating equilibrium, $a_1^{H_1}$ must be such that

$$\pi(L,0) + ((1-p)+\rho p)\,\pi(L,0) + (p-\rho p)\left[\pi(H,1) - \underline{a}^H\right]$$
$$\geq \quad \pi(L,1) - a_1^{H_1} + ((1-p)+\rho p)\,\pi(L,0) + (p-\rho p)\left[\pi(H,1) - \underline{a}^H\right]$$

and

$$\pi(H,1) - a_1^{H_1} + ((1-p)-\rho(1-p))\,\pi(L,0) + (p+\rho(1-p))\left[\pi(H,1) - \underline{a}^H\right]$$
$$\geq \quad \pi(H,0) + ((1-p)-\rho(1-p))\,\pi(L,0) + (p+\rho(1-p))\left[\pi(H,1) - \underline{a}^H\right]$$

Thus, the set of separating equilibria is given by:

$$a_1^H \in [\pi(L,1) - \pi(L,0), \pi(H,1) - \pi(H,0)] \quad \text{and } a_1^{L_1} = 0$$

The least-cost separating equilibrium is:

$$a_1^{H_1} = \pi(L,1) - \pi(L,0) \text{ and } a_1^{L_1} = 0$$

But this means that the multiproduct firm just replicates in each market the single product firm. The total level of advertising that a firm of type (H_1, H_2) needs to do in order to separate itself in

both markets is the same that is needed for separation to be possible in two independent product markets (that is, with zero quality correlation, $\rho = 0$). In this case there are no information spillovers: a_1^H reveals that the firm is H_1, but this does not give any advantage to the firm in market 2. Since the prior is low and correlation is also low, the posterior beliefs are low even when the firm reveals to be H_1. But then the consumers expect the firm to advertise $\underline{a}^H$ in market 2 when product 2 is of high-quality, regardless of what happened in market 1. As a consequence advertising in market 1 does not affect the equilibrium in market 2 and the level of advertising needed to separate H_1 from L_1 is the same as for a single product firm.

5.1.2 High correlation: $\rho > \max\left[\frac{p^*-p}{1-p}, \frac{p-p^*}{p}\right]$

A more interesting case occurs when $p + \rho(1-p) > p^* > p - \rho p$ (this may happen when $p < p^*$ but $p + \rho(1-p) > p^*$ or when $p > p^*$ but $p - \rho p < p^*$, in both cases we need to have a sufficiently high correlation so that the «jump» from prior to posterior beliefs is big enough). In this case, if the firm reveals to be H_1 the most reasonable continuation equilibrium is the pooling equilibrium with $a_2 = 0$, while if the firm reveals to be L_1, only the least-cost separating equilibrium is reasonable in the second market. The continuation equilibrium if other levels of a_1 are observed depends on the off-the-equilibrium-path beliefs. We assume that if $a_1 < a_1^{H_1}$ then consumers believe that the first product is of low-quality, $\mu_1(a_1) = 0$, whereas if $a_1 \geq a_1^{H_1}$ posterior beliefs are $\mu_1(a_1) = 1$.

Considering the continuation equilibria and proceeding backwards, we conclude that, in order to separate, the H_1 firm must spend $a_1^{H_1}$ such that

$$\pi(L,0) + ((1-p) + \rho p)\,\pi(L,0) + (p - \rho p)\left[\pi(H,1) - \underline{a}^H\right]$$
$$\geq \pi(L,1) - a_1^{H_1} + ((1-p) + \rho p)\,\pi(L, p + \rho(1-p)) + (p - \rho p)\,\pi(H, p + \rho(1-p))$$

and

$$\pi(H,1) - a_1^{H_1} + ((1-p) - \rho(1-p))\,\pi(L, p + \rho(1-p)) + (p + \rho(1-p))\,\pi(H, p + \rho(1-p))$$
$$\geq \pi(H,0) + ((1-p) - \rho(1-p))\,\pi(L,0) + (p + \rho(1-p))\left[\pi(H,1) - \underline{a}^H\right]$$

This means that a H_1 firm is willing to do $a_1^{H_1}$, but the low-quality firm prefers $a_1^{L_1} = 0$ to $a_1^{H_1}$, even if by doing $a_1^{H_1}$ it is perceived as high-quality in the first market. The previous conditions take into account the expected profit in the second market, knowing the posterior probabilities and the continuation equilibrium associated with each level of a_1. From the first condition one can derive the least-cost separating equilibrium, where $a_1^{H_1}$ is given by:

$$
\begin{aligned}
\underline{a}_1^{H_1} &= \pi(L,1) - \pi(L,0) + ((1-p) + \rho p)\left[\pi(L, p + \rho(1-p)) - \pi(L,0)\right] + \\
&\quad + (p - \rho p)\left[\pi(H, p + \rho(1-p)) - \left(\pi(H,1) - \underline{a}^H\right)\right]
\end{aligned}
\tag{1}
$$

The amount that H_1 has to spend in order to separate equals the expected gain L_1 would have in both markets by pretending to be high-quality in the first and being perceived as such. This formula is a generalization of the one for a single product firm, since it includes, apart from the gain in the first market, the expected gain in the second market due to an increase in quality perception.[14]

The sorting condition stated before is sufficient for the existence of this equilibrium.

Notice that when $\rho = 1$, $\underline{a}_1^{H_1} = 2\left[\pi(L,1) - \pi(L,0)\right]$. In this case, advertising $\underline{a}_1^{H_1}$ reveals that the two products are of high-quality. The H_1 firm just advertises for the first product, but this signals high-quality in both markets.

As expected, the fact that a firm is multiproduct makes a difference. In particular, if the firm has shown to be H_1 in the first market it is more likely that the continuation equilibrium is a pooling equilibrium than in the case where markets are unrelated. Moreover, the higher is the quality correlation (the closer ρ is to 1) the higher is the likelihood of the previous result. Thus, in this case one can really speak of informational spillovers, since the advertising level in the first market has implications on the beliefs in both markets.

5.1.3 High prior and low correlation: $p > p^*$ and $\rho < \frac{p - p^*}{p}$

If $p > p^*$ and $\rho < \frac{p - p^*}{p}$, we have $p - \rho p > p^*$. This means that posterior beliefs μ_{21} are above p^* even if the firm reveals to be L_1. In this case, the most reasonable continuation equilibrium is the pooling one with $a_2 = 0$, either when the firm revealed to be L_1 or when it revealed to be H_1 and also for every a_1 off-the-equilibrium path. When $a_1 = a_1^{H_1}$ the firm reveals to be of high-quality and the posterior beliefs are $\mu_{21} = p + \rho(1-p)$. When $a_1 = 0$, the consumer knows the firm is L_1 and $\mu_{21} = p - \rho p$. A separating equilibrium exists in the first product advertising level if:

$$\pi(L,0) + ((1-p) + \rho p)\,\pi(L, p - \rho p) + (p - \rho p)\,\pi(H, p - \rho p)$$
$$\geq\ \pi(L,1) - a_1^{H_1} + ((1-p) + \rho p)\,\pi(L, p + \rho(1-p)) + (p - \rho p)\,\pi(H, p + \rho(1-p))$$

and

$$\pi(H,1) - a_1^{H_1} + ((1-p) - \rho(1-p))\,\pi(L, p + \rho(1-p)) + (p + \rho(1-p))\,\pi(H, p + \rho(1-p))$$
$$\geq\ \pi(H,0) + ((1-p) - \rho(1-p))\,\pi(L, p - \rho p) + (p + \rho(1-p))\,\pi(H, p - \rho p)$$

---

[14]The expected gain is clearly positive, since we are assuming $p + \rho(1-p) > p^*$, which is equivalent to assuming:

$$\pi(H, p + \rho(1-p)) > \pi(H,1) - \underline{a}^H \quad \Leftrightarrow \quad \pi(H, p + \rho(1-p)) - \left(\pi(H,1) - \underline{a}^H\right) > 0.$$

From the first condition we get the least-cost separating advertising level for firm H_1:

$$
\begin{aligned}
\underline{a}_1^{H_1} = {} & \pi(L,1) - \pi(L,0) + ((1-p) + \rho p)\left[\pi(L, p + \rho(1-p)) - \pi(L, p - \rho p)\right] + \\
& (p - \rho p)\left[\pi(H, p + \rho(1-p)) - \pi(H, p - \rho p)\right]
\end{aligned}
\tag{2}
$$

Once again the level of advertising that the high-quality firm H_1 has to do in order to separate itself from type L_1 is greater than the level needed for separation in market 1 if the two products were independent. This level of advertising is equal to the expected gain of firm L_1 in both markets by pretending to be H_1. The gain in the first market is $\pi(L,1) - \pi(L,0)$, the expected gain in the second market is:

$$
((1-p) + \rho p)\left[\pi(L, p + \rho(1-p)) - \pi(L, p - \rho p)\right] + (p - \rho p)\left[\pi(H, p + \rho(1-p)) - \pi(H, p - \rho p)\right]
$$

and reflects the expected increase in profits through the posterior beliefs. By imitating type H_1, the low-quality firm L_1 changes beliefs from $p - \rho p$ to $p + \rho(1-p)$.

5.2 Pooling equilibria in market 1

In this section we study the existence of PBE with both types choosing the same $a_1 = \widetilde{a}_1$. In this case, the posterior beliefs after $\widetilde{a}_1$ is observed are equal to the prior beliefs, $\mu_{21}(\widetilde{a}_1) = p$. If we assume that whenever $a_1 \neq \widetilde{a}_1$ is observed the firm is perceived as having a low-quality product 1, we have $\mu_{21}(a_1) = p - \rho p$ for all other levels of a_1.

Thus the reasonable continuation PBE depends on whether $p < p^*$ is satisfied or not. When $p < p^*$ the unique reasonable continuation PBE is the least-cost separating equilibrium, for all a_1. For a pooling equilibrium to exist in the first product advertising level,

$$
\begin{aligned}
& \pi(L,p) - \widetilde{a}_1 + ((1-p) + \rho p)\,\pi(L,0) + (p - \rho p)\left[\pi(H,1) - \underline{a}^H\right] \\
\geq {} & \pi(L,0) + ((1-p) + \rho p)\,\pi(L,0) + (p - \rho p)\left[\pi(H,1) - \underline{a}^H\right]
\end{aligned}
$$

which is equivalent to

$$
\widetilde{a}_1 \leq \pi(L,p) - \pi(L,0)
$$

This is the same condition that we obtain for pooling in just one market.

When $p > p^*$ the most reasonable continuation PBE after $a_1 = \widetilde{a}_1$ is the pooling one with $a_2 = 0$. However if a type deviates and chooses $a_1 \neq \widetilde{a}_1$ it will be interpreted as a low-quality firm, thus posterior beliefs are $\mu_{21} = p - \rho p$. Following a deviation, the most reasonable continuation PBE depends on whether $p - \rho p > p^*$ holds or not. If the answer is yes the continuation equilibrium is the pooling equilibrium with $a_2 = 0$, otherwise the continuation equilibrium is

the least-cost separating equilibrium. In the first case, a pooling equilibrium with $a_1 = \tilde{a}_1$ exists as long as:

$$\tilde{a}_1 \leq \pi(L,p) - \pi(L,0) + ((1-p) + \rho p)\left[\pi(L,p) - \pi(L,p-\rho p)\right] +$$
$$(p - \rho p)\left[\pi(H,p) - \pi(H,p-\rho p)\right]$$

In the second case, a pooling equilibrium with $a_1 = \tilde{a}_1$ exists as long as:

$$\tilde{a}_1 \leq \pi(L,p) - \pi(L,0) + ((1-p) + \rho p)\left[\pi(L,p) - \pi(L,0)\right] +$$
$$(p - \rho p)\left[\pi(H,p) - \left[\pi(H,1) - \underline{a}^H\right]\right]$$

This shows that the set of pooling equilibria is larger than in a single product case. However the most reasonable pooling equilibrium continues to be $\tilde{a}_1 = 0$.

5.3 Selecting the most reasonable equilibrium

In the two previous subsections we have shown the existence of equilibria where types H_1 and L_1 separate in the first product advertising level and the existence of equilibria where these two types pool. In this subsection we analyze whether the domination criterion restricts or not the set of reasonable equilibrium to the least-cost separating equilibrium. When the least-cost separating equilibrium is the unique equilibrium which survives the domination criterion, it is selected as the most reasonable equilibrium. When there are pooling equilibria which also survive the domination criterion we select among the surviving equilibria (pooling and least-cost separating) the one which is Pareto optimal (for the two types of firms).

5.3.1 Low prior and low correlation: $p < p^*$ and $\rho < \frac{p^* - p}{1-p}$

When $p + \rho(1-p) < p^*$ the unique reasonable continuation PBE is the least-cost separating equilibrium. In this case, using the criterion that off-the-equilibrium path beliefs should put probability zero on strictly dominated strategies, we can eliminate the equilibria where the two types pool at $\tilde{a}_1 = 0$. For type L_1 choosing $a_1 > \pi(L,1) - \pi(L,0)$ is a dominated strategy, but then if such a_1 is observed consumer should put probability zero on product 1 being of low-quality, that is, $\mu_1 = 1$. However, if $\mu_1 = 1$ when $a_1 > \pi(L,1) - \pi(L,0)$, a type H_1 gains by deviating from the pooling equilibrium:

$$\pi(H,1) - \underline{a}^H + ((1-p) - \rho(1-p))\pi(L,0) + (p + \rho(1-p))\left[\pi(H,1) - \underline{a}^H\right]$$
$$\geq \pi(H,p) + ((1-p) - \rho(1-p))\pi(L,0) + (p + \rho(1-p))\left[\pi(H,1) - \underline{a}^H\right]$$

But this is equivalent to

$$\pi(H,1) - \left[\pi(L,1) - \pi(L,0)\right] > \pi(H,p)$$

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which holds for $p < p^*$. Since $p + \rho(1 - p) < p^*$ implies $p < p^*$ the previous condition is verified. Thus the pooling equilibria do not survive the domination criterion.

The unique equilibrium which survives the domination equilibrium is the least-cost separating equilibrium in both markets. This equilibrium just replicates the single product least-cost separating equilibrium in each market (see subsection 5.1.1).

5.3.2 Low prior and high correlation: $p < p^*$ and $\rho > \frac{p^* - p}{1 - p}$

When $p < p^*$ and $\rho > \frac{p^* - p}{1 - p}$, we know that $p + \rho(1 - p) > p^* > p$. This implies that when the firm reveals to be H_1 the most reasonable continuation equilibrium is the pooling equilibrium with $a_2 = 0$. On the other hand, if the firm reveals to be L_1 the most reasonable continuation PBE is the least-cost separating equilibrium. When firms pool in a_1 the unique reasonable PBE is the least-cost separating equilibrium.

Will we be able to eliminate the pooling equilibrium where firms pools at $\widetilde{a}_1 = 0$ using the domination criterion? For type L_1 choosing a_1 larger than $\underline{a}_1^{H_1}$ defined in equation (1) is a dominated strategy, but then if such a_1 is observed posterior beliefs should be $\mu_1 = 1$. With these beliefs a type H_1 gains by deviating from the pooling equilibrium as long as

$$\pi(H, 1) - \underline{a}_1^{H_1} + ((1 - p) - \rho(1 - p))\,\pi(L, p + \rho(1 - p)) + (p + \rho(1 - p))\,\pi(H, p + \rho(1 - p))$$
$$\geq \quad \pi(H, p) + ((1 - p) - \rho(1 - p))\,\pi(L, 0) + (p + \rho(1 - p))\left[\pi(H, 1) - \underline{a}^H\right]$$

Substituting $\underline{a}_1^{H_1}$ in the previous expression and simplifying we get:

$$\pi(H, 1) - \underline{a}^H + \rho\left[\pi(H, p + \rho(1 - p)) - \left(\pi(H, 1) - \underline{a}^H\right) - (\pi(L, p + \rho(1 - p)) - \pi(L, 0))\right] \geq \pi(H, p)$$

Since $\underline{a}^H = \pi(L, 1) - \pi(L, 0)$, this is equivalent to:

$$\pi(H, 1) - \underline{a}^H - \rho\left[(\pi(H, 1) - \pi(H, p + \rho(1 - p))) - (\pi(L, 1) - \pi(L, p + \rho(1 - p)))\right] \geq \pi(H, p)$$

Therefore when

$$\pi(H, 1) - \underline{a}^H > \pi(H, p) + \rho\left[(\pi(H, 1) - \pi(H, p + \rho(1 - p))) - (\pi(L, 1) - \pi(L, p + \rho(1 - p)))\right] \quad (3)$$

in the unique reasonable equilibrium L_1 does not advertise and H_1 chooses $\underline{a}_1^{H_1}$ defined in equation (1). The continuation equilibrium is the least-cost separating equilibrium if the firm has revealed to be L_1 and it is a pooling with $a_2 = 0$ if it has shown to be H_1.

Notice that, by the sorting condition,

$$\left[(\pi(H, 1) - \pi(H, p + \rho(1 - p))) - (\pi(L, 1) - \pi(L, p + \rho(1 - p)))\right] > 0$$

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This implies that the condition for the least-cost separating equilibrium to be the most reasonable one is stricter than if the two markets were independent. The intuition is that with information spillovers the amount that H_1 has to advertise in order to separate himself from his low-quality counterpart is higher, thus it is more difficult for H_1 to gain by deviating from the pooling equilibrium with $\widetilde{a}_1 = 0$.

If condition (3) is not satisfied the pooling equilibrium with $\widetilde{a}_1 = 0$ survives the domination criterion and it is better for both types (L_1 and H_1) than the least-cost separating equilibrium. Thus using Pareto optimality to select the most reasonable equilibrium, the pooling equilibrium $\widetilde{a}_1 = 0$ is the most reasonable one. In this case, the continuation equilibrium is the separating one.

5.3.3 High prior and high correlation: $p > p^*$ and $\rho > \frac{p-p^*}{p}$

When $p > p^*$ and $\rho > \frac{p-p^*}{p}$ we have $p - \rho p < p^*$. In this case, the most reasonable continuation equilibrium if the firm revealed to be H_1 is the pooling one while if it revealed to be L_1 it is the least-cost separating equilibrium. Moreover, if firms pool at $\widetilde{a}_1 = 0$, the most reasonable continuation equilibrium is the pooling equilibrium with $a_2^P = 0$.

Let us check if it possible to eliminate the pooling equilibrium at $\widetilde{a}_1 = 0$, using the domination criterion. We know that for type L_1 advertising more than $\underline{a}_1^{H_1}$ defined in equation (1) is a dominated strategy, but then if such a_1 is observed posterior beliefs should be $\mu_1 = 1$. However with these beliefs a type H_1 gains by deviating from the pooling equilibrium as long as:

$$\pi(H,1) - \underline{a}_1^{H_1} + ((1-p) - \rho(1-p))\,\pi(L, p + \rho(1-p)) + (p + \rho(1-p))\,\pi(H, p + \rho(1-p))$$
$$\geq\quad \pi(H,p) + ((1-p) - \rho(1-p))\,\pi(L,p) + (p + \rho(1-p))\,[\pi(H,p)]$$

Substituting $\underline{a}_1^{H_1}$ defined in equation (1), one can show that the previous condition is equivalent to the next expression being negative:

$$(1 + p(1-\rho))\left[\pi(H,p) - (\pi(H,1) - \underline{a}^H)\right] + ((1-p) + \rho p)\left[\pi(L,p) - \pi(L,0)\right] \qquad (4)$$
$$-\rho\left[(\pi(H, p + \rho(1-p)) - \pi(H,p)) - (\pi(L, p + \rho(1-p)) - \pi(L,p))\right]$$

In this expression, the first two terms are clearly positive whereas the third term is negative. In the third term, the largest value that the expression inside parentheses can take is:

$$(\pi(H,1) - \pi(H,p)) - (\pi(L,1) - \pi(L,p)) = -\left[\pi(H,p) - \left(\pi(H,1) - \underline{a}^H\right)\right] + (\pi(L,p) - \pi(L,0))\,.$$

Thus, a lower bound for expression (4) is:

$$(1 + p + \rho(1-p)\left[\pi(H,p) - (\pi(H,1) - \underline{a}^H)\right] + (1-p)(1-\rho)\left[\pi(L,p) - \pi(L,0)\right]$$

But, for $p > p^*$ this is for sure positive. Thus condition (4) can never be negative. Consequently, the pooling equilibrium with $\widetilde{a}_1 = 0$ survives the domination criterion. In addition it is Pareto optimal, so it is the most reasonable equilibrium.

5.3.4 High prior and low correlation: $p > p^*$ and $\rho < \frac{p-p^*}{p}$

When $p > p^*$ and $\rho < \frac{p-p^*}{p}$ we know that $p - \rho p > p^*$. In this case the most reasonable continuation equilibrium is always the pooling equilibrium with $a_2 = 0$, regardless of what happened in the first market.

Let us check whether we can eliminate the pooling equilibrium at $\widetilde{a}_1 = 0$. Any level of $a_1 > \underline{a}_1^{H_1}$ defined by equation (2) is a dominated strategy for type L_1, thus $\mu_1 = 1$ following any $a_1 > a_1^{H_1}$. With these beliefs a type H_1 would want to deviate from $\widetilde{a}_1 = 0$ as long as:

$$
\begin{aligned}
& \pi(H,1) - \underline{a}_1^{H_1} + ((1-p) - \rho(1-p))\,\pi(L, p + \rho(1-p)) + (p + \rho(1-p))\,\pi(H, p + \rho(1-p)) \\
\geq\ & \pi(H,p) + ((1-p) - \rho(1-p))\,\pi(L,p) + (p + \rho(1-p))\,\pi(H,p)
\end{aligned}
$$

As in the previous case, this condition can never hold. Consequently, the pooling equilibrium with $\widetilde{a}_1 = 0$ survives the domination criterion. In addition it is Pareto optimal, so it is the most reasonable equilibrium.

5.3.5 Summary of results

The next proposition summarizes the results on the most reasonable equilibrium. These results are illustrated in Figure 3.

Proposition 2 *If the consumer does not learn the quality of good 1 before observing a_2:*

(i) *When $p < p^*$ and $\rho < \frac{p^*-p}{1-p}$ the most reasonable PBE is the least-cost separating equilibrium in both markets. That is, $a_2^H = \underline{a}^H$ and $a_2^L = 0$ regardless of a_1 and in equilibrium $a_1^H = \underline{a}^H$ and $a_1^L = 0$.*

(ii) *When $p < p^*$ and $\rho > \frac{p^*-p}{1-p}$, if condition (3) holds then the most reasonable equilibrium involves separation in market 1, with $a_1^L = 0$ and a_1^H defined by equation (1). If $a_1 \geq a_1^H$ then in the second market there is pooling, $a_2^H = a_2^L = 0$. If $a_1 = 0$, then in the second market there is separation ($a_2^H = \underline{a}^H$ and $a_2^L = 0$).*

(iii) *When $p < p^*$ and $\rho > \frac{p^*-p}{1-p}$, if condition (3) does not hold, then the most reasonable equilibrium involves pooling in market 1 ($a_1^H = a_1^L = 0$) and separation in market 2 ($a_2^H = \underline{a}^H$ and $a_2^L = 0$)*

(iv) *When $p > p^*$, the most reasonable equilibrium is pooling in both markets ($a_1^H = a_1^L = 0$ and $a_2^H = a_2^L = 0$).*
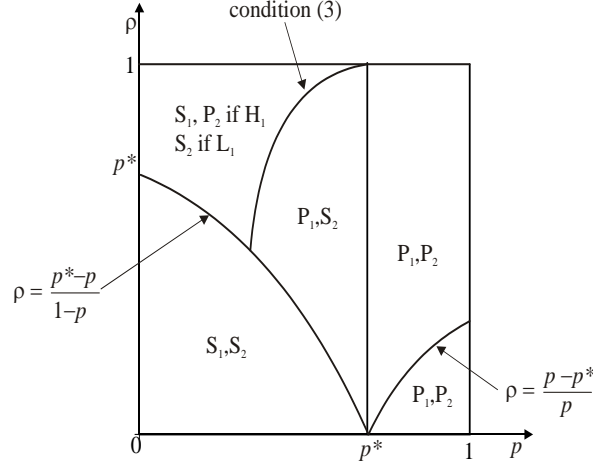

Figure 3: The most reasonable equilibrium when the consumer does not learn the quality of good 1 before a_2 is observed.

When ρ is low we get precisely the same results than in the case where the quality of the first product is observed. That is, the multiproduct firm just replicates in each market the behavior of a single product firm.

When ρ is high ($\rho > \frac{p^*-p}{1-p}$ for $p < p^*$ or $\rho > \frac{p-p^*}{p}$ for $p > p^*$), there are some similarities but there are also important differences with respect to the case where the quality of the first product is observed. As in the case where quality of the first product is observed, when ρ is high and p is sufficiently low the most reasonable equilibrium still involves separation in the first market, whereas the continuation equilibrium in the second market depends on whether the firm revealed to be H_1 or L_1. Like before, a firm who advertises high enough to credibly signal that it has a high-quality first product will exploit in market 2 its reputation. On the other hand, if a firm revealed to be L_1 and consequently has a bad reputation, then if the firm has a high-quality second product, it will advertise high enough in order to demonstrate to the consumer that the second product is in fact of high-quality H_2.

However, the amount that a H_1 firm has to advertise in order to separate itself from a L_1 firm is much higher than in the case where the quality of good 1 is observed. What happens is that a firm with a low-quality first product has more incentives to mimic the behavior of a H_1 firm, because when a firm reveals to be H_1 that benefits the firm in the first market but it also benefits the firm in the second market through the improvement in the quality expectations of good 2. But then a H_1 firm needs to advertise an higher amount in order to credibly signal

that its first product is of high-quality. Here advertising in the first market has information spillovers in the second market and to create a reputation of being H_1, a firm has to advertise a sufficiently high amount.

In addition, when $p > p^*$ and $\rho > \frac{p - p^*}{p}$ in the most reasonable equilibrium none of the types advertises in both markets and consequently the consumers posterior beliefs are equal to prior beliefs, p. This contrast with case where the quality of the first product is observed, where there was no advertising in market 1, but consumers revised their beliefs about product 2, because they learned the quality of good 1 immediately after consumption.

Finally, for high ρ and intermediate p, we now have firms pooling in the first market and separating in the second market. This may happen because separation in the first market is «too expensive» and both types, L_1 and H_1, are better off by not advertising even if doing so leads consumers not to change their beliefs regarding product 1.

6 Conclusion

This paper considered a model where a firm introduces sequentially two products with positively correlated qualities. Signaling advertising may be used in both markets. In this model we investigate whether there exist information spillovers from the first to the second market.

When the correlation between the two products is low, we have shown that the multiproduct firm just replicates in each market the behavior of a single product firm. In this case, knowing the quality of good 1 has a very small influence on the consumers' perception of product 2's quality, hence it does not affect the equilibrium advertising levels in market 2.

When the correlation between the two products is high, the equilibrium in market 2 depends a lot on what consumers have learned about product 1 prior to the introduction of the second product. If consumers have learned that product 1 is of high quality then, in the second market, the firm will exploit its reputation and will not advertise. On the contrary, if a firm has shown to have a low-quality product 1 and hence has acquired a bad reputation, then if the firm has a high-quality second product, it will advertise high enough in order to demonstrate to the consumer that the second product is in fact of high-quality. Thus, if quality correlation is high, when the firm chooses its advertising level in the first market it has to take into account the impact of its decision in market 2.

In terms of advertising expenses in the first market, the results depend crucially on whether the quality of the first product is observed by the consumers before the second product is introduced or not. If the quality of the first product is observed, the consumers learn the true quality of product 1 regardless of the advertising level in market 1. Hence the advertising level

in market 1 does not have signaling spillovers in market 2 and, consequently, the equilibrium in market 1 is the same as in a single product firm.

On the other hand, when the quality of the first product is not observed, the advertising level in the first market influences beliefs about the second product quality. In this case, if a firm with a high-quality first product whishes to separate from its low-quality counterpart, it will need to advertise a higher amount than if the qualities of the two products were unrelated. This advertising level signals not only high-quality in the first market, it also signals that it is very likely that product 2 will be of high-quality. In the particular case where the two products are perfectly correlated, a high-quality firm ends up advertising only in the first market, but the amount spent in this market is the same as what is needed for separation in two independent markets.

In reality we observe at times levels of advertising for one product which seem excessive, given the demand for that good. Our model provides an explanation for this phenomena. It may well be that, by advertising a very high amount, the firm is also signaling the quality of the products that she will introduce in the future.

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