Staff Scheduling in a hospital context: the case of Luz Saúde Hospital

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Declaration

I declare that this document is an original work of my own authorship and that it fulfills all the requirements of the Code of Conduct and Good Practices of the Universidade de Lisboa.

Preface

The work presented in this thesis was performed at the *Hospital da Luz* (Lisbon, Portugal), during the period February-December 2020, under the supervision of Dr. Daniel Santos. The thesis was co-supervised by Dr. João Colaço.

Acknowledgments

I would first like to thank my supervisor, Dr. Daniel Santos, who guided me through this work, always sharing knowledge, revealing availability and interest, proving to be an exceptional professor. All feedback and meetings were fundamental to outline the best direction. I also want to acknowledge Hospital da Luz – Lisboa, in particular Dr. João Colaço – my co-supervisor – for all his availability, support and feedback throughout the thesis. It was a pleasure to work with both.

I wanted to thank my family for giving me the opportunity to get here. They were a fundamental pillar along this journey. A very special thank you to my mother, father, brother, and sister. And to my grandmother Alice, who wherever she is, will certainly be proud.

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Abstract

This work proposes a multi-objective genetic algorithm that aims to optimize the schedule of the staff in a hospital service. Health professionals are one of the indispensable resources in the quality of services provided and represent more than half of hospitals' operating costs, which requires an endless search for methods to improve the efficiency of operations. This work is motivated by Hospital da Luz de Lisboa, which identified some needs in the planning of these resources, since its management is time consuming, mostly hand-made and by trial and error, which often causes several inconsistencies. This can be reflected in a dissatisfied staff or situations of staff shortages, overloading the remaining staff. The Intensive Care Unit (ICU) served as case study and intends to incorporate personal preferences in a balanced way, ensuring a fair schedule. The algorithm was also applied in the Imagery service. Both have points in common, mainly the emerging need to find a solution for staff scheduling. The algorithm is able to effectively find good approximations of the Pareto-front in a timely manner. Additionally, it incorporates the possibility of choosing a single solution based on weights attributed by the decision-makers. The ICU is satisfied with the results, mainly by the fact that the algorithm is able to generate a balanced schedule with the possibility of using historical records from previous months, contributing to a more effective and fair scheduling in the long term.

Keywords: Staff Scheduling, Genetic Algorithm, Fairness, Preferences, Multi-objective

Resumo

Este trabalho propõe um algoritmo genético multi-objetivo que pretende otimizar a escala do staff num serviço hospitalar. Os profissionais de saúde são um dos recursos indispensáveis na qualidade do serviço prestado e representam mais de metade dos custos operacionais de um hospital, o que requer uma busca incessante por métodos para melhorar a eficiência das operações. Este trabalho é motivado pelo Hospital da Luz de Lisboa que identificou algumas necessidades neste planeamento, uma vez que se trata de um processo moroso, feito maioritariamente à mão e por tentativa e erro, o que acarreta habitualmente diversas inconsistências. Isto pode refletir-se num *staff* insatisfeito ou em situações de escassez de pessoal, sobrecarregando o restante. A Unidade de Cuidados Intensivos (UCI) serviu como caso de estudo e pretende incorporar preferências pessoais de uma forma equilibrada, garantindo uma escala justa. O algoritmo foi também aplicado no serviço de Imagiologia. Ambos têm pontos em comum, principalmente a necessidade emergente de encontrar uma solução para a gestão do pessoal. O algoritmo é capaz de encontrar boas aproximações da fronteira de Pareto em tempo útil. Para além disso, incorpora a possibilidade de escolher uma solução baseada em pesos atribuídos pelos decisores. A UCI mostrou-se satisfeita com os resultados, principalmente pelo algoritmo ser capaz de gerar uma escala equilibrada com a possibilidade de usar informação dos meses anteriores, contribuindo para uma gestão mais eficaz e justa a longo prazo.

Keywords: Escalonamento, Algoritmo Genético, Justiça, Preferências, Multi-objetivo

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Acronyms

ADSE *Instituto de Proteção e Assistência na Doença*.

DM Decision Maker.

GA Genetic Algorithm.

HBA *Hospital Beatriz Ângelo*.

HL *Hospital da Luz*.

HLL *Hospital da Luz Lisboa*.

ICU Intensive Care Unit.

IS Imagery Service.

LS *Luz Saúde*.

MOEA Multi-Objective Evolutionary Algorithm.

MRI Magnetic Resonance Imaging.

NHS National Health Service.

NIS Negative Ideal Solution.

NSGA-II Non-dominated Sorting Genetic Algorithm II.

PIS Positive Ideal Solution.

PPP Public-Private Partnership.

SAMS *Serviços de Assistência Médico-Social*.

TOPSIS Technique for Order of Preference by Similarity to Ideal Solution.

VHI Voluntary Health Insurance.

Chapter 1

Introduction

The introduction chapter intends to present the motivation and the purpose of this dissertation, highlighting the main contributions, in Section 1.1 and Section 1.2 respectively. At the end, in Section 1.3, the thesis outline is provided.

1.1 Motivation

Over the last years, one of the biggest challenges faced by the world is the provision of high-quality healthcare with affordable expenses. This requires a never-ending search for methods to improve the efficiency of the operations. Staff-related costs represent more than half of the operating costs in hospitals (Erhard et al. 2018). Furthermore, the health workforce is an indispensable, valuable and critical resource, which with their skills, capacity and commitment, are notably determinant regarding the quality of the healthcare delivery (Ozcan & Hornby 1999). Consequently, it is crucial to find efficient and effective ways to plan and schedule the workforce.

Personnel scheduling problems have been widely discussed in the literature (Erhard et al. 2018). However, over time, the way it is approached has changed. In particular, recently, there has been a growing concern regarding employee satisfaction and preferences (Van Den Bergh et al. 2013). There is an increased interest among hospitals to satisfy these demands, not only due to the scarcity of available health staff on the job market but also the difficulty in replacing, which often leads to understaffing (Erhard et al. 2018, Simões et al. 2017). In other words, the size of the scheduled workforce in the hospital is often insufficient to match the actual demand. Hospitals operate 24 hours a day, seven days a week, every day of the year, with more than one shift per day and they are under high uncertainty and fluctuating conditions, and these issues can lead to an increase in lengths of stay as well as in waiting times, and consequently, a forfeit of quality. Additionally, to compensate this scenario it is necessary to increase the workload of the designated team, creating a further unsatisfied and unmotivated staff, and this will also have a negative impact in effectiveness as well as it can lead to an increase of absences, due to complications such as burnout (Erhard et al. 2018). The opposite scenario is also occurring, namely overstaffing, in times of less demand. This can cause extra costs and staff dissatisfaction since they cannot use their time so usefully. Any of these scenarios brings consequences to both hospitals and staff.

Currently, most part of schedules are hand-made, which often raise unfairness and violations of labour regulations. In addition, they are usually planned by a team element – who wastes valuable time. This implies extra and needless costs that, in some cases, could be prevented.

The aim of this dissertation is to develop an algorithm to support and optimize the staff scheduling of different hospital services, considering the available resources, and satisfying the rules and targets that must be met, as well as the stakeholders' perspectives. A decision support algorithm, based on Operations Research techniques, to automate and improve the hospital schedules is proposed. This follows the main steps of the Non-dominated Sorting Genetic Algorithm II (NSGA-II), a metaheuristic method based on the theory of evolution that advocates that the most fit individuals perpetuate themselves in the species. This will be made analyzing the *Hospital da Luz Lisboa* (HLL) case study. Since each service has their own requirements and resources, the solution should be analyzed individually by observing each service and their routine. Therefore, the preferences, constraints and objectives must be identified in order to automate and improve scheduling activities. At present, the staff scheduling plan is made in an *Excel* file mostly by trial and error method, which could bring inaccuracies and in some cases the waste of time by people who would be more useful in the service.

1.2 Contributions

Currently, the human resources scheduling in HLL is a time consuming, error-prone and mostly manual process, frequently leading to inconsistencies. The first step is to identify existing requirements in each of the services. This includes identifying the criteria that, according to the opinion of those targeted, contribute to a good schedule. This is evaluated by talking to different people in each service, trying to identify several points in common. Once the criteria of each of the service have been identified it is possible to start designing solutions.

The goal of the dissertation is to solve a staff scheduling problem by proposing a shift assignment determination for a given time horizon, what is referred as *rostering problem* by Erhard et al. (2018). Since this work is applied to a real-case, it is possible to test with real needs and hospital data, contributing to solutions that are more reliable and closer to what is expected in a real scenario. Finally, it will be possible to give some recommendations to the hospital.

Overall, the main contribution of this work is to improve the staff scheduling process at HLL in regards to four aspects: (i) reducing the time needed to obtain a feasible schedule; (ii) increasing the efficiency of the schedules produced; (iii) ensuring that a greater number of conditions are met; and (iv) incorporating preferences and fairness.

1.3 Thesis Outline

This thesis consists of a total of seven chapters. Firstly, in the Introduction Chapter, the motivation and the main contributions of this work are described, including a brief introduction to the case study. In Chapter 2 a description of HLL and the case study is addressed. An overview of the Portuguese Health System, *Luz Saúde* (LS) within this System, and the current situation of staff scheduling problem in services under consideration are described. Chapter 3 introduces the relevant literature according to the problem addressed. This allows to recognize the current state-of-the-art, highlighting the main features of the scheduling problem and types of objectives. Chapter 4 describes the model proposed, based on Genetic Algorithms, to solve the problem. First a general overview of the NSGA-II is provided, then the assumptions in each of the services are presented. The results obtained from the algorithm are illustrated and analyzed in Chapter 5. In Chapter 6 a model validation considering the hospital point of view is done, emphasizing the contributions made. Finally, in Chapter 7 the thesis is concluded mentioning the main objectives achieved. Furthermore, limitations to the metaheuristic are identified and future work is proposed.

Chapter 2

Problem Contextualization

Demand for healthcare services and, consequently, their expenditures, has been increasing in the past years, resulting in a never-ending search for methods to improve the efficiency of their operations (Erhard et al. 2018). One of the challenges faced by hospitals these days is to improve the efficiency of the scheduling of healthcare resources, such as schedules for physicians, nurses, rooms or equipment, in order to meet the patient needs. This is a remarkably complex, time-consuming and error-prone work that should simultaneously consider multiple criteria, from cost savings, preferences, employee and patient satisfaction to dependencies between human and material resources. Therefore, the number of conditions is high and most of the time they are variable and uncertain, thus complex to meet simultaneously. It is important to observe and analyze several hospital services, in their daily environment, in order to identify the scheduling conditions.

This chapter describes the scheduling problem at *Hospital da Luz* (HL) in Lisbon, presenting the current situation of its different services. Section 2.1 provides an overview of the Portuguese Health System and how Luz Saúde, in particular HLL, is inserted into this system. Thereafter, Section 2.2 introduces an analysis of the scheduling process implemented in Luz Hospital, highlighting the current problems, taking into account two different case studies.

2.1 Portuguese Health System

This section presents an overview of the Portuguese Health System, highlighting the way it is currently divided. Next, a brief introduction of the LS Group is provided, locating the HL within the Health System.

2.1.1 Health System overview

Portugal tends to follow the Beveridge model, which sustains the idea that healthcare is a human right, instead of a privilege. Thus, it is universal – should cover all citizens, regardless of their economic and social conditions –, it is financed by taxes, its provision is essentially public but increasingly contractual, it is usually free at the moment of access and the primary care physician are the gatekeepers of the system (Lameire et al. 1999). Nevertheless, other systems have already been implemented at this moment and they continue to be used. For this reason, nowadays, the Portuguese Health System may be divided into three coexisting systems: the universal National Health Service (NHS); health subsystems; and private Voluntary Health Insurance (VHI). Therefore, the Portuguese Health System is a combination of public and private financing (Simões et al. 2017). The Portuguese NHS was created in 1979, with a focus on providing access of every citizen's right to health, which emerged with the new democratic Constitution, through the State. The NHS provides universal coverage – it covers all residents in Portugal –, it is nearly free access at the point of use and it is mainly financed by an universal tax system (Simões et al. 2017, Oliveira & Pinto 2005). Overall, the NHS is publicly governed, through the Ministry of Health, and its provision comes from both public and private providers.

On the other hand, the health subsystems are health insurance schemes which offer, total or partial, both public or private healthcare to particular professions or sectors. An example of a public health subsystem is *Instituto de Proteção e Assistência na Doença* (ADSE), which guarantees health protection to civil servants, and a private one is *Serviços de Assistência Médico-Social* (SAMS). The health subsystems allow their beneficiaries to go to an appointment without needing referral from a primary health center and have quicker access to private providers (Barros 2017). Finally, the private VHI complements both the NHS and health subsystems, and is offered in the private market, with either individual or group policies (Barros 2017). The decision to join this system is voluntary, however, it allows faster access to hospital treatments, such as skipping waiting lists and ambulatory consultation, as well as the opportunity to choose the provider. Despite the existence of the NHS, the private market has been growing. In 2015, the VHI covered around 26% of the population (Simões et al. 2017).

The HLL, which belongs to the LS Group, is mostly part of the latter subsystem, as will be described below. To this end, a brief introduction of this group is initially presented, emphasizing its distribution in the country, its mission and values.

2.1.2 Luz Saúde Group

LS SA, formerly know as *Espírito Santo Saúde*, was founded twenty-years ago, in 2000. In the beginning, it was a division of the *Espírito Santo* Financial Group, however in 2014 the *Fidelidade* insurance Group acquired control. Currently, LS is one of the biggest groups delivering health care services in the Portuguese market. These services are provided through thirty units spread all around the country, particularly in the northern, central, and south-central regions of Continental Portugal and in the Autonomous Region of Madeira (Saúde 2018). Specifically: fourteen private hospitals, where Luz Hospital is integrated; the *Hospital Beatriz Ângelo* (HBA), which works as a Public-Private Partnership (PPP), i.e, it is a hospital which belongs to the NHS but is operated by LS; thirteen private clinics working with outpatient treatment; and, additionally, they own two senior residences. Therefore, LS has been developing a diversified business model, organized into three operating segments: (i) the private healthcare segment, which includes the main hospitals; (ii) the public healthcare segment, which corresponds to HBA, under the PPP contract; and (iii) other activities, where the two senior residences are included (Saúde 2018).

The company has their vision, mission and values clearly defined. There is a great effort to apply the best practices to achieve exceptional results, regarding the patient perspective. LS crosses the talent, innovation and dedication, in order to ensure the best, rapid and effective diagnosis and treatments. For this reason, they are committed to the seeking of technology and innovation. In addition, the fundamental values present on LS are the tireless quest for results, intellectual accuracy, ongoing learning, personal accountability, positive attitude, respect and humbleness, integrity and team spirit (Luz Saúde 2020).

The HLL is one of the aforementioned private hospitals, which belongs and is managed by LS Group. Therefore, it owns a long list of agreements with several private health insurances and health subsystems, opening the possibility for beneficiaries to take advantage of the available services.

2.2 Scheduling Problem at Luz Hospital

This work is motivated by HLL which identified some needs in human resources planning. In the following sections it will be presented how this management is currently done and which are the main inconsistencies found in the two cases-studies addresses in this work, namely the ICU in Section 2.2.1 and the Imagery Service (IS) in Section 2.2.2.

2.2.1 Intensive Care Unit Case Study

The ICU is a special unit within a hospital, which provides intensive treatment. In other words, this unit carries patients with severe or life-threatening illnesses or injuries, who require constant supervision and care. Therefore, it is crucial to have a rigorous control of life support equipment and medication, which allow normal bodily functions to work even when the patient's body cannot support them by itself. Furthermore, this implies highly trained staff. Another particular characteristic of the ICU to be taken into account is the higher staff-to-patient ratio comparing to other wards. One can easily understand that poor management in this unit may compromise the patients' lives and, consequently, increase morbidity and mortality rates. Moreover, this can bring an overworked staff, resulting in frustration and decreased

Table 2.1: Shifts existing in Hospital da Luz ICU.

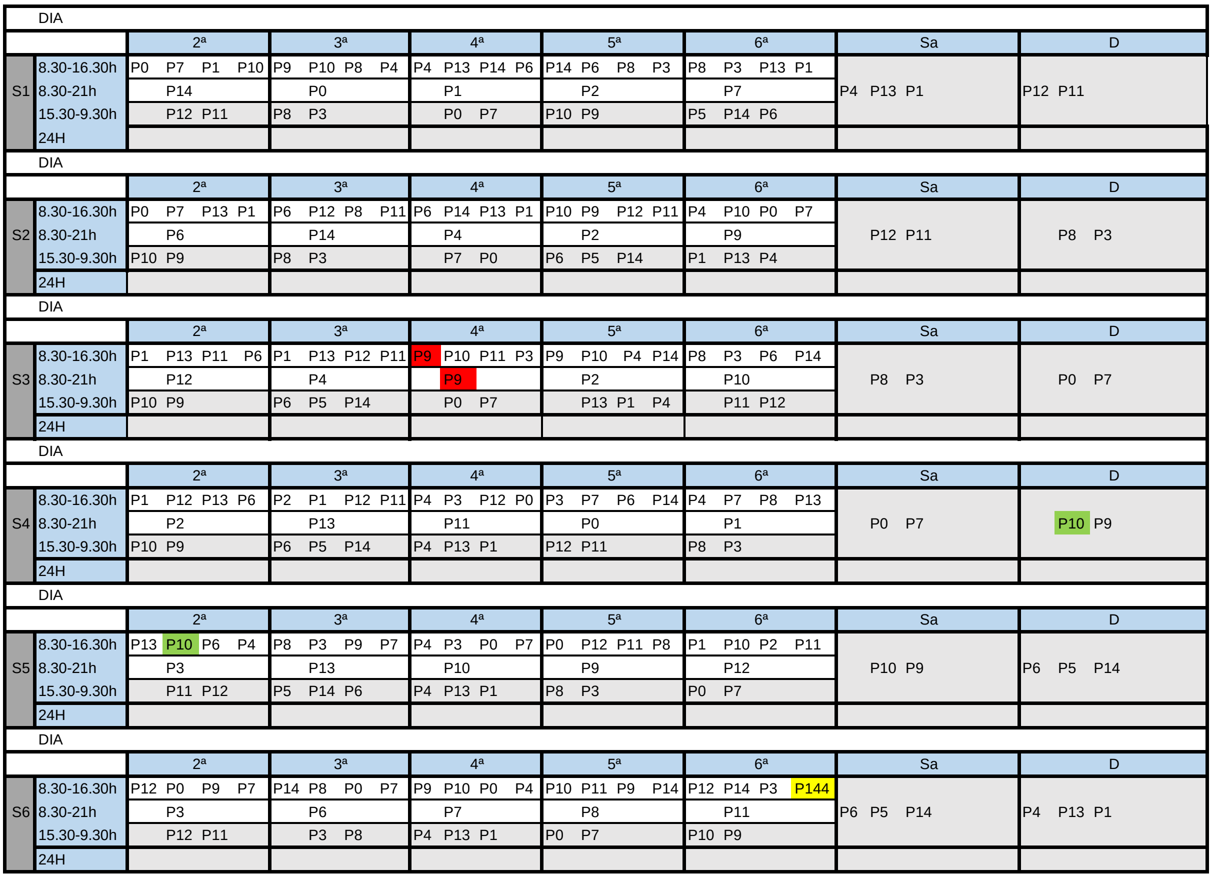
	Shifts			Number of hours
Morning	8:30 am	-	4:30 pm	9
Prolongation	8:30 am	-	9:00 pm	12,5
Night	3:30 pm	-	9:30 am	18
24-hours	8:30 am	-	8:30 am	25
Weekends	9:30 am	-	9:30 am	25

revenue. Overall, the ICU is an expensive resource in hospitals strongly affected by uncertainty and variability. All these factors contribute to an increasing need in understanding how to provide high quality care with limited capacity (Bai et al. 2018).

The HLL ICU provides assistance 24 hours a day, everyday. The team is composed by physicians, with specialization in Internal Medicine and Intensive Medicine, nurses and its own secretary, with specific training. Currently, the ICU has 8 centrally monitored beds, each in a single room. One of the rooms allows to keep patients in isolation and 3 of the beds are equipped for hemodialysis. All beds are served by monitors, ventilators, and infusion and perfusion systems, with continuous recording of monitored parameters. Works are underway to increase the size and capacity of the hospital and, as such, it is planned to build another 8 rooms with these particularities.

Similarly to several hospitals, HL has also faced shortages of medical personnel (Erhard et al. 2018). Nevertheless, recently they hired two new physicians, which are not yet enough to fill the shifts ideally. Currently, the ICU in HL is composed by 15 physicians, where one is in charge of administrative tasks. Thus, he/she works less hours and, if it is possible, he/she is neither assigned to a night shift nor a weekend shift. They are also helped by physicians from outside the ICU, but those have special restrictions, and a specialized physician should be present in each shift. Thus, one of the challenges faced by ICU in the HL is related to the physicians scheduling. Previously, the schedule was made by a single physician. But that had to be changed. This happened, mainly, because the team expanded and the scheduling complexity associated also increased exponentially, which severely affected the normal work routine of the person in charge, as he/she was losing valuable time that should be used for patients. Currently, the person responsible for the physicians scheduling is the ICU assistant, that for now does not have any experience in the process. Table 2.1 presents the shifts available in ICU.

Their main idea is to build a basic schedule for 6 weeks, roughly on a rotating basis. Afterwards, the built schedule has to be adjusted according to the needs of each month. The process has been done in an *Excel* file, which counts the total number of different physician shifts, the number of different shifts and hours per week, and the total number of hours and shifts per physician and week. The fulfillment is performed by trial-error method. Ideally, each physician should work eight morning shifts, two prolongation shifts, five night shifts and four weekend shifts, which corresponds to a total of 229 hours for six weeks. It is possible for one to be assigned to a morning shift and a night shift, forming a consecutive 24-hours shift; the opposite is not possible.



DIA								
		2ª	3ª	4ª	5ª	6ª	Sa	D
S1	8.30-16.30h	P0 P7 P1 P10	P9 P10 P8 P4	P4 P13 P14 P6	P14 P6 P8 P3	P8 P3 P13 P1	P4 P13 P1	P12 P11
	8.30-21h	P14	P0	P1	P2	P7		
	15.30-9.30h	P12 P11	P8 P3	P0 P7	P10 P9	P5 P14 P6		
	24H							
DIA								
		2ª	3ª	4ª	5ª	6ª	Sa	D
S2	8.30-16.30h	P0 P7 P13 P1	P6 P12 P8 P11	P6 P14 P13 P1	P10 P9 P12 P11	P4 P10 P0 P7	P12 P11	P8 P3
	8.30-21h	P6	P14	P4	P2	P9		
	15.30-9.30h	P10 P9	P8 P3	P7 P0	P6 P5 P14	P1 P13 P4		
	24H							
DIA								
		2ª	3ª	4ª	5ª	6ª	Sa	D
S3	8.30-16.30h	P1 P13 P11 P6	P1 P13 P12 P11	P9 P10 P11 P3	P9 P10 P4 P14	P8 P3 P6 P14	P8 P3	P0 P7
	8.30-21h	P12	P4	P9	P2	P10		
	15.30-9.30h	P10 P9	P6 P5 P14	P0 P7	P13 P1 P4	P11 P12		
	24H							
DIA								
		2ª	3ª	4ª	5ª	6ª	Sa	D
S4	8.30-16.30h	P1 P12 P13 P6	P2 P1 P12 P11	P4 P3 P12 P0	P3 P7 P6 P14	P4 P7 P8 P13	P0 P7	P10 P9
	8.30-21h	P2	P13	P11	P0	P1		
	15.30-9.30h	P10 P9	P6 P5 P14	P4 P13 P1	P12 P11	P8 P3		
	24H							
DIA								
		2ª	3ª	4ª	5ª	6ª	Sa	D
S5	8.30-16.30h	P13 P10 P6 P4	P8 P3 P9 P7	P4 P3 P0 P7	P0 P12 P11 P8	P1 P10 P2 P11	P10 P9	P6 P5 P14
	8.30-21h	P3	P13	P10	P9	P12		
	15.30-9.30h	P11 P12	P5 P14 P6	P4 P13 P1	P8 P3	P0 P7		
	24H							
DIA								
		2ª	3ª	4ª	5ª	6ª	Sa	D
S6	8.30-16.30h	P12 P0 P9 P7	P14 P8 P0 P7	P9 P10 P0 P4	P10 P11 P9 P14	P12 P14 P3 P144	P6 P5 P14	P4 P13 P1
	8.30-21h	P3	P6	P7	P8	P11		
	15.30-9.30h	P12 P11	P3 P8	P4 P13 P1	P0 P7	P10 P9		
	24H							

Figure 2.1: *Excel* file with the Basic Schedule for 6 weeks in development. Each week is represented by S_i, $i = 1, ...6$. The colored squares symbolize some inaccuracies made during the process.

The Figure 2.1 displays the basic schedule in development. It is divided into six weeks represented by S_i, $i = 1, ..., 6$, with every day of the week (Monday, represented by 2ª, to Sunday, as D). During weekdays (between Monday, 2ª and Friday, 6ª) four physicians are assigned to morning shifts and one physician to a prolongation shift; they are identified by the first letter of their name and surname, but here they will be designated as P0, P1, etc. For instance, looking at the first week (S1) on Monday (2ª) P0, P7, P1 and P10 are the physicians assigned to the morning shift. On the other hand, P14 is assigned to the prolongation shift. Regarding night shifts, there are two or three physicians assigned. Concerning the weekends (represented as *Sa* and *D*) there are two or three physicians assigned for the entire day (the same happens for night shifts, due to some lack of personnel, as previously stated). To illustrate this, one can look to Saturday (*Sa*) in S1 and observe that the physicians P4, P13 and P1 are assigned to that day, starting at 8:30am and ending the shift at 9:30am in the following day, which corresponds to a weekend shift (see Table 2.1). There is also a row dedicated to 24-hour shifts (that, basically, are a sum of a morning shift plus a night shift), which is empty during the filling, to avoid unintentional mistakes resulting in having more physicians assigned than the ones needed. In other words, looking to S5 on Wednesday (4ª), one can notice that physician P4 is assigned to both the morning and the night shifts; in theory this is a 24-hour shift, but filling in this way helps to block the leading and trailing shifts for this physician. At the end, when everything is checked, these shifts are correctly fulfilled.

Following the process, it was possible to observe that first the assistant fills all morning shifts, only then proceeds to the afternoon shifts. The night shifts were already chosen by physicians. From here, it was possible to identify several problems. Frequently, the same physician assigned to the morning or afternoon shift on Monday (2^a - which corresponds to the first column of the week) was assigned to the Sunday before (D - last column) represented with green in Figure 2.1. There are also writing mistakes, as represented with yellow, which affect the counts; in addition, sometimes the same physician is assigned in a morning and prolongation shift, which could not be possible since they are simultaneous shifts; these are represented with red in Figure 2.1.

The main aim of the ICU physician scheduling is to assign the same number of hours to all physicians during the 6 week period. Besides, whenever possible, it is taking into consideration the physicians' preferences. When it is necessary to choose one physician' preferences in detriment of other, they try to compensate this loss in the next weeks.

Physicians should work about 45 hours a week, but since they have to compensate times when there are other physicians in holidays or attending conferences – each one has the right to 5 holiday weeks plus 2 weeks to participate in conferences –, they should work, in average, for approximately 38 hours. Note that the physicians must not work less hours than the 38 established per week, even if the reduced number of patients allows it to happen, in order to respect the work regulation.

The amount of restrictions, factors and errors described shows that it is necessary to invest in an alternative to this manual solution, in order to optimize the process and for the unit to meet expectations. The same is true for IS, as it is described below.

2.2.2 Imagery Service Case Study

The IS of a hospital is equipped with image technology able to give support and find the correct diagnostic based on images of organs. This service plays an important role during the health care process from the medical communication, to education and research. The technologies have been increasing and, consequently, more and more sophisticated techniques allow to have a more detailed image of the body and tissues structures, which also requires a more experienced staff. The IS team is composed by physicians, technicians, auxiliaries and administrative workers.

There are several methodologies used, in particular, the HLL offers a high number of imaging exams from CT scans, Ultrasound, Magnetic Resonance Imaging (MRI), conventional radiology, that is, X-rays, mammography, bone densitometry, among others. In addition, it operates seven days a week. In recent years, there has been an increase in demand and in the number of tests performed in this service (Saúde 2018).

The machines are operated – and exams are provided – by technicians, which play a central role in the process. In addition, image exams are often requested because they allow more detail and faster

Table 2.2: Shifts available in technicians scheduling of IS.

Shifts							
A8	07:30 am	-	03:30 pm	**P0**	03:30 pm	-	11:30 pm
A6	07:30 am	-	01:30 pm	**P1**	04:00 pm	-	01:00 am
M8	08:00 am	-	04:00 pm	**N**	10:00 pm	-	08:00 am
M6	08:00 am	-	02:00 pm	**S**	08:00 am	-	06:00 pm
T8	02:00 pm	-	10:00 pm	**I10**	09:00 am	-	07:00 pm
T6	02:00 pm	-	08:00 pm	**MT6**	08:00 am	-	08:00 pm
T20	12:00 pm	-	08:00 pm	**MT21**	08:00 am	-	09:00 pm
T21	01:00 pm	-	09:00 pm	**MT8**	08:00 am	-	10:00 pm
P	02:00 pm	-	12:00 am				

finding of diagnosis. Nevertheless, the resources are limited and expensive. This strengthens the need to find an efficient and optimized resources management.

Once again, it was possible to identify a huge challenge regarding staff scheduling. In addition to the issues already mentioned in ICU, the IS endures other complex characteristics. As aforementioned, the LS Group incorporates several hospitals and clinics; for this reason, it could have some staff operating in different units. Regarding the IS technicians, they could be scheduled in Lisbon HL, Amadora and Oeiras Clinics.

Currently, there are three people, who are technicians, present during the technicians scheduling process, which is done before the end of a month, in order to schedule the next one. Two of them are in charge to do the schedule, whereas the other one is the coordinator of one clinic, who is supervising the process. Usually, the scheduling task takes almost two full time days to be concluded by them.

In total, there are 43 technicians and they are divided into two main groups: Central Group and Emergency Group. The Central Group is composed by 31 technicians; the Emergency Group is subdivided into two groups of 6 technicians: Group E1 and Group E2. The Central Group is coordinated with available physicians and is in charge of the elective exams that come mainly from appointments, while the emergency group is in charge of unpredictable situations that appear in the Emergency Room. Groups E1 and E2 differ in shift types. On the one hand the E1 group works until 1 a.m. at the most (no night shifts are assigned), however, in case of need they make night prevention, that is, they may have to be available to fill gaps; on the other hand, the E2 group works night shifts and can only do daytime prevention. In addition, there is a group of people who belong to the groups described above, that helps meeting the needs, often by working overtime.

Table 2.2 presents the shifts available in IS for technicians. It is important to note that some shifts are usually avoided, such as MP0, MP1, AT8, AT6. This is due to the fact that according to the legislation these technicians should not work more than ten hours per shift.

There are some pre-set rules, which could suffer some modifications. For instance, ideally, each technician from the Central Group should be assigned to a morning shift, usually M8 (see Table 2.2), during one consecutive week, and in the next it should be assigned to an afternoon shift, T21 shift (see Table 2.2) and so on. On the other hand, the E1 Emergency Group should work 4 consecutive days, following the shift sequence M8-M8-P0-P1. In turn, the E2 Emergency Group should follow the sequence M8-T8-T8-N. The Extra Group just fulfills the needs of the unit, thus they do not have a pre-set shift.

Firstly, during the scheduling process, the technicians begin filling the HL considering the previous rules; afterwards, they adjust with hospital needs and technicians' preferences. For instance, some technicians want to work overtime, while others do not, these aspects are taken under consideration. Once this first step is finished, they start to assign technicians to the clinics.

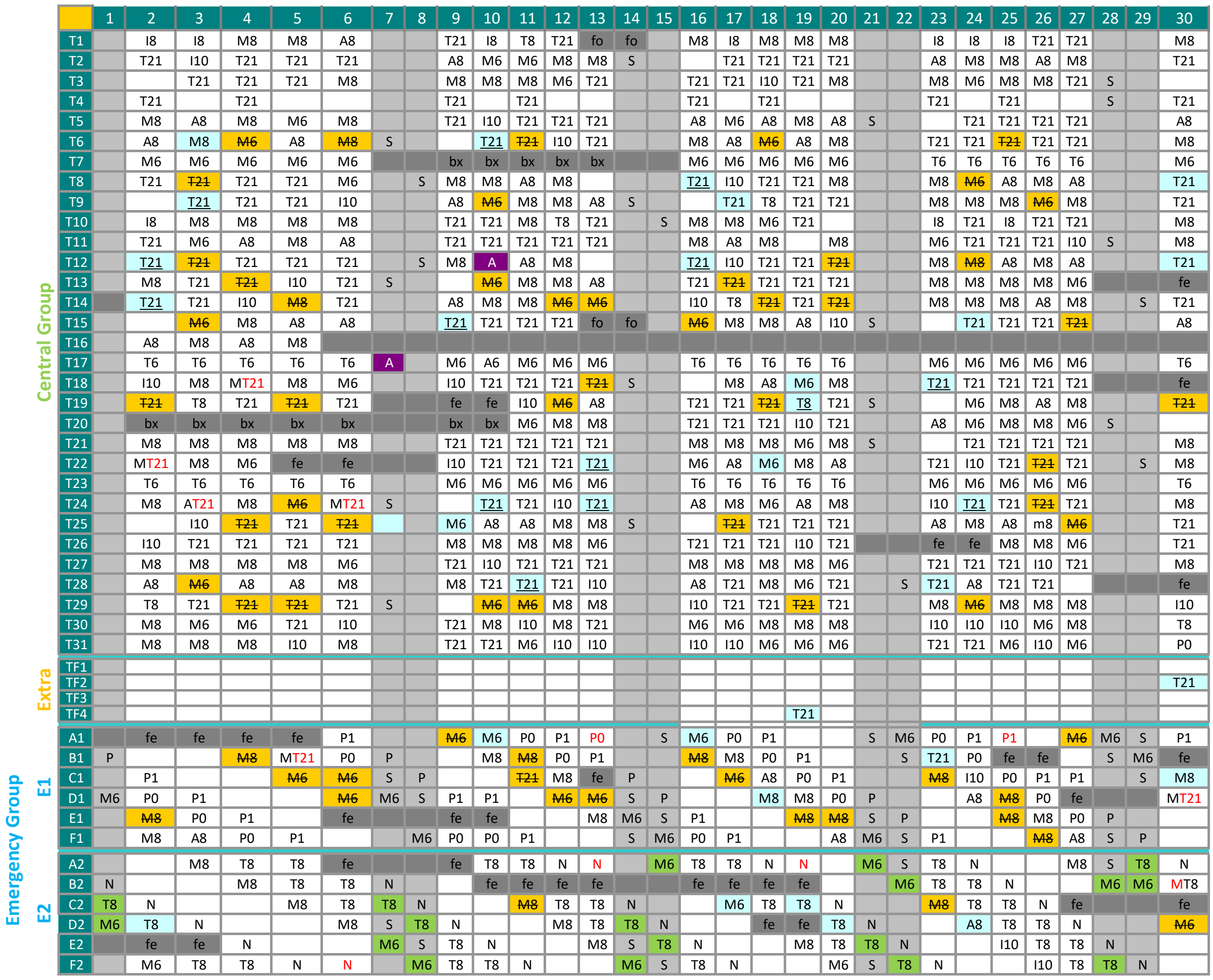
There are some restrictions that should be taken into consideration: a technician from the Central Group should only be assigned to one day of the weekend by month and the weekend shift used is the S (see table 2.2); if a technician was assigned to a weekend shift, in the next week he/she should have a day-off (if he/she worked on Saturday, he/she should have the next Monday; if he/she worked on Sunday, he/she should have the next Friday) – this could change according to the needs of the hospital – and, additionally, should work a 6-hour shift (M6 shift) in one of the days of this month, in order to make up for the 10-hour shift worked in the weekend. When a technician works in a holiday, he/she must have the double hours free; there are some technicians taking a Master Degree and usually they cannot be assigned to a weekend shift; some mothers are in breastfeeding period, and according to the regulation, they should be assigned to a 6-hour shift and should not work during the weekend; and, additionally, the equipment requires maintenance, which means that it will be unusable during this period, and there is no need to allocate technicians in this time. Another important aspect to take into account is the skills of each technician, that is, a technician follows a particular functional area that is reflected in the exams and parts of the body in which he/she is most suitable.

The clinics receive technicians from both HL and HBA. In this way, the technicians in charge of the scheduling receive the HBA technicians schedule, in order to fulfill the clinic needs according to their availability. Usually, the HL technicians are more often assigned to Oeiras Clinic while the HBA technicians should be more often assigned to Amadora Clinic.

The Oeiras Clinic employs six resident technicians working six morning shifts (M8) and six afternoon shifts (T20) in weekdays. Concerning weekends, the clinic needs four morning shifts and one afternoon shift on Saturdays, and one morning shift as well as one afternoon shift on Sundays.

The Amadora Clinic is a smaller center and employs one resident technician. The clinic needs two technicians assigned not only to a morning shift, but also to an afternoon shift. In this case, during the week it is necessary to have always a woman assigned to both morning and afternoon shifts, since she is the only one who can perform mammography exams. At the weekends, only urgent exams are performed, where mammography does not take place.

The Figure 2.2 is an *Excel* file example used in a schedule of the HLL's IS. In contrast to ICU, here the schedule is made for each technician for all month and it starts in the first day of the month. For this



		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Central Group	T1		I8	I8	M8	M8	A8			T21	I8	T8	T21	fo	fo		M8	I8	M8	M8	M8			I8	I8	I8	T21	T21			M8
	T2		T21	I10	T21	T21	T21			A8	M6	M6	M8	M8	S			T21	T21	T21	T21			A8	M8	M8	A8	M8			T21
	T3			T21	T21	T21	M8			M8	M8	M8	M6	T21			T21	T21	I10	T21	M8			M8	M6	M8	M8	T21	S		
	T4		T21		T21					T21		T21					T21		T21					T21		T21			S		T21
	T5		M8	A8	M8	M6	M8			T21	I10	T21	T21	T21			A8	M6	A8	M8	A8	S			T21	T21	T21	T21			A8
	T6		A8	M8	~~M6~~	A8	~~M8~~	S			T21	~~T21~~	I10	T21			M8	A8	~~M6~~	A8	M8			T21	T21	~~T21~~	T21	T21			M8
	T7		M6	M6	M6	M6	M6			bx	bx	bx	bx	bx			M6	M6	M6	M6	M6			T6	T6	T6	T6	T6			M6
	T8		T21	~~T21~~	T21	T21	M6		S	M8	M8	A8	M8				T21	I10	T21	T21	M8			M8	~~M6~~	A8	M8	A8			T21
	T9			T21	T21	T21	I10			A8	~~M6~~	M8	M8	A8	S			T21	T8	T21	T21			M8	M8	M8	~~M6~~	M8			T21
	T10		I8	M8	M8	M8	M8			T21	T21	M8	T8	T21		S	M8	M8	M6	T21				I8	T21	I8	T21	T21			M8
	T11		T21	M6	A8	M8	A8			T21	T21	T21	T21	T21			M8	A8	M8		M8			M6	T21	T21	T21	I10	S		M8
	T12		T21	~~T21~~	T21	T21	T21		S	M8	A	A8	M8				T21	I10	T21	T21	~~T21~~			M8	~~M8~~	A8	M8	A8			T21
	T13		M8	T21	~~T21~~	I10	T21	S			~~M6~~	M8	M8	A8			T21	~~T21~~	T21	T21	T21			M8	M8	M8	M8	M6			fe
	T14		T21	T21	I10	~~M8~~	T21			A8	M8	M8	~~M6~~	~~M6~~			I10	T8	~~T21~~	T21	~~T21~~			M8	M8	M8	A8	M8		S	T21
	T15			~~M6~~	M8	A8	A8			T21	T21	T21	T21	fo	fo		~~M6~~	M8	M8	A8	I10	S			T21	T21	T21	~~T21~~			A8
	T16		A8	M8	A8	M8																									
	T17		T6	T6	T6	T6	T6	A		M6	A6	M6	M6	M6			T6	T6	T6	T6	T6			M6	M6	M6	M6	M6			T6
	T18		I10	M8	MT21	M8	M6			I10	T21	T21	T21	~~T21~~	S			M8	A8	M6	M8			T21	T21	T21	T21	T21			fe
	T19		~~T21~~	T8	T21	~~T21~~	T21			fe	fe	I10	~~M6~~	A8			T21	T21	~~T21~~	T8	T21	S			M6	M8	A8	M8			~~T21~~
	T20		bx	bx	bx	bx	bx			bx	bx	M6	M8	M8			T21	T21	T21	I10	T21			A8	M6	M8	M8	M6	S		
	T21		M8	M8	M8	M8	M8			T21	T21	T21	T21	T21			M8	M8	M8	M6	M8	S			T21	T21	T21	T21			M8
	T22		MT21	M8	M6	fe	fe			I10	T21	T21	T21	T21			M6	A8	M6	M8	A8			T21	I10	T21	~~T21~~	T21		S	M8
	T23		T6	T6	T6	T6	T6			M6	M6	M6	M6	M6			T6	T6	T6	T6	T6			M6	M6	M6	M6	M6			T6
	T24		M8	AT21	M8	~~M6~~	MT21	S			T21	T21	I10	T21			A8	M8	M6	A8	M8			I10	T21	T21	~~T21~~	T21			M8
	T25			I10	~~T21~~	T21	~~T21~~			M6	A8	A8	M8	M8	S			~~T21~~	T21	T21	T21			A8	M8	A8	m8	~~M6~~			T21
	T26		I10	T21	T21	T21	T21			M8	M8	M8	M8	M6			T21	T21	T21	I10	T21			fe	fe	M8	M8	M6			T21
	T27		M8	M8	M8	M8	M6			T21	I10	T21	T21	T21			M8	M8	M8	M8	M6			T21	T21	T21	I10	T21			M8
	T28		A8	~~M6~~	A8	A8	M8			M8	T21	T21	T21	I10			A8	T21	M8	M6	T21		S	T21	A8	T21	T21				fe
	T29		T8	T21	~~T21~~	~~T21~~	T21	S			~~M6~~	~~M6~~	M8	M8			I10	T21	T21	~~T21~~	T21			M8	~~M6~~	M8	M8	M8			I10
	T30		M8	M6	M6	T21	I10			T21	M8	I10	M8	T21			M6	M6	M8	M8	M8			I10	I10	I10	M6	M8			T8
	T31		M8	M8	M8	I10	M8			T21	T21	M6	I10	I10			I10	I10	M6	M6	M6			T21	T21	M6	I10	M6			P0
Extra	TF1																														
	TF2																														T21
	TF3																														
	TF4																			T21											
Emergency Group E1	A1		fe	fe	fe	fe	P1			~~M6~~	M6	P0	P1	P0		S	M6	P0	P1			S	M6	P0	P1	P1		~~M6~~	M6	S	P1
	B1	P			~~M8~~	MT21	P0	P			M8	~~M8~~	P0	P1			~~M8~~	M8	P0	P1			S	T21	P0	fe	fe		S	M6	fe
	C1		P1			~~M6~~	~~M6~~	S	P			~~T21~~	M8	fe	P			~~M6~~	A8	P0	P1			~~M8~~	I10	P0	P1	P1		S	M8
	D1	M6	P0	P1			~~M6~~	M6	S	P1	P1		~~M6~~	~~M6~~	S	P			M8	M8	P0	P			A8	~~M8~~	P0	fe			MT21
	E1		~~M8~~	P0	P1		fe			fe	fe			M8	M6	S	P1			~~M8~~	~~M8~~	S	P			~~M8~~	M8	P0	P		
	F1		M8	A8	P0	P1			M6	P0	P0	P1			S	M6	P0	P1			A8	M6	S	P1			~~M8~~	A8	S	P	
Emergency Group E2	A2			M8	T8	T8	fe			fe	T8	T8	N	N		M6	T8	T8	N	N		M6	S	T8	N			M8	S	T8	N
	B2	N			M8	T8	T8	N			fe	fe	fe	fe			fe	fe	fe	fe			M6	T8	T8	N			M6	M6	MT8
	C2	T8	N			M8	T8	T8	N			~~M8~~	T8	T8	N			M6	T8	T8	N			~~M8~~	T8	T8	N	fe			fe
	D2	M6	T8	N			M8	S	T8	N			M8	T8	T8	N			fe	fe	T8	N			A8	T8	T8	N			~~M6~~
	E2		fe	fe	N			M6	S	T8	N			M8	S	T8	N			M8	T8	T8	N			I10	T8	T8	N		
	F2		M6	T8	T8	N	N		M6	T8	T8	N			M6	S	T8	N			M6	S	T8	N			I10	T8	T8	N	

Figure 2.2: *Excel* file with a schedule example of the LLH IS. The different groups (Central, Extra and Emergency) are noted.

reason, the first column corresponds to each technician, where, as previously, they were identified with the first letter of their name and surname. There are three groups and they are noted and presented in orderly manner: Central Group (the first thirty-one technicians), Extra Group (the next four technicians) and the Emergency Group (the last twelve technicians, split into two different groups: E1 and E2). Afterwards, a monthly chart is displayed. The weekends are shown in grey columns, the weekdays are in white and each entry of the table contains the shift assigned to the technician for each day. One can identify some grey rows described as *"fe"* or *"bx"*, which respectively mean holidays and sick leaves. In addition, there are some attributed shifts that are crossed out, since this shift will be performed in another unit. They are crossed out instead of just deleted for everybody to know that this technician already has a shift assigned and to count the hours. The entry with an *"A"* and purple background corresponds to the technician's birthday, when he/she is gifted with a day of absence. The red letters represent an extra shift. On the other hand, the entries with blue background represent a shift that would be assigned to one of clinics, but then it was assigned to a HL shift. Those represented in green only happen in the Emergency Group and are designated "weekend prevention", which means that if there is a need to call another member due to delays in the block or the need to perform angiographies, these people are called; in other words, they are prepared for this possibility during the time of this shift.

After attending the technicians scheduling task, it was possible to identify some difficulties. This task takes almost two days to complete and it is performed by technicians, who lose valuable time, which represents an extra-cost for the hospital and a decrease of effectiveness. The technician's preferences are reported by e-mail and by phone, often during the process, which implies excessive and spread information, potentiating inaccuracies. Additionally, the vacation map is sent by each technician rather than by each work group. The service schedule is made in a *Excel* file, as described previously, and the three units are in the same sheet, thus it is necessary to be always scrolling it when building the clinics schedule. The file counts the number of hours and the shifts assigned, in order to compare them and perceive if the needs are met. Moreover, the schedule is completed by trial-error method, being a task susceptible to inaccuracies.

Considering all the aspects, constraints, objectives and the complexity of the technicians scheduling task, one can quickly conclude that a model should be implemented, in order to save costs, maintain the impartiality and fairness and attend to both technicians and hospital needs.

2.3 Chapter Considerations

Healthcare staff are a valuable and indispensable resource for any hospital. Their ineffective management leads to several consequences, such as increasing waiting time for patients, increasing waste of valuable time and decreasing staff satisfaction. In addition, the majority of a hospital's operating costs are staff related. For this reason, it has become crucial to improve the efficiency of the scheduling of healthcare resources, namely, the personnel.

This chapter provided an overview of HLL, a contextualization of the case study, describing the current situation of the scheduling process and the problem addressed.

In the following chapter, a literature review related to staff scheduling, highlighted by healthcare units and hospitals, is presented, considering the characteristics, different objectives, and the approaches to solve the proposed problem.

Chapter 3

Literature Review

Upon the evaluation of two hospital services in their environments, the scheduling conditions were identified. The staff scheduling problem presented in this work aims at finding a way to automate and improve the schedules of those hospital resources. These problems have been widely discussed in several sectors, however, over time, the approach has changed. In particular, the importance of staff satisfaction, preferences and flexible work hours has grown (Van Den Bergh et al. 2013). For this reason, an overview of the main different goals and features will be discussed in this chapter, resorting to published studies of interest. First, in Section 3.1 an overview of the staff healthcare scheduling is performed. In Section 3.2 the main features of scheduling problems are addressed. In Section 3.3 the different objectives found in the literature are presented and briefly described. Section 3.4 summarizes and compares some of the staff scheduling studies. Finally, Section 3.5 highlights the chapter conclusions by establishing the link with the problem under study.

3.1 Staff Scheduling in Healthcare

Staff-related costs account for more than half of operating costs in hospitals (Erhard et al. 2018). Furthermore, staffing has an impact not only in costs, but also in the quality of services provided. It should also be noted that the scheduling of shifts has a significant impact on employees at physical, psychological and social levels and better schedules could prevent problems like job burnout, which is very recurring (Güler et al. 2013). Consequently, optimised staff schedules can provide substantial benefits (Ernst et al. 2004), wherefore, an endless search for methods to reduce labour costs, without compromising the healthcare quality, has been an extremely important task (Erhard et al. 2018). As a result, carefully implemented decision support tools are required, allowing to meet customer demands in a cost-effective manner while satisfying requirements as flexibility, fairness, staff preferences, among with others (Ernst et al. 2004).

In fact, there are several reasons why staff scheduling in healthcare is so difficult. The first and most obvious is due to uncertainty and high fluctuations, as demand for services varies over time, leading to under or overstaffing (Fügener et al. 2015, Erhard et al. 2018). In addition, hospitals operate 24 hours a day, 7 days a week, which implies service in off-hours - nights and weekends - and often face personnel shortage. Second, the employee preferences should be incorporated in the scheduling task in order to promote job satisfaction, because these workers are usually difficult to replace (Bodenheimer & Smith 2013). Furthermore, the individual labour regulations with the staff should be respected (Fügener et al. 2015). Currently, most of hospitals continue to use manual scheduling, a time-consuming and error-prone task, which in most cases is conducted by a highly qualified staff. Nonetheless, with the growing labour regulations this problem is becoming manually intractable, often leading to schedules with little consideration of fairness or skill levels, as well as infeasible ones with respect to those regulations (Erhard et al. 2018, Fügener et al. 2015).

According to Erhard et al. (2018) the scheduling problems can be classified into three different groups: (i) Staffing, (ii) Rostering, and (iii) Re-planning problems. Staffing problems, usually in a long-term planning horizon, belong to a strategic decision level and consist of a search for the appropriate size of a required staff. Rostering problems can include both tactical and operational offline decisions. Here remains the option to build cyclic or acyclic rosters, depending respectively on the creation of patterns or rebuilding at each planning period (Van Huele & Vanhoucke 2014). Finally, re-planning problems reside on operational online decision level and react to unforeseen or unanticipated events, such as employee absences, thus discuss short-term planning. Since the frequency of the schedule and the hierarchy level vary according to the type of problem, the approach is also different, from the objectives to the features applied, such as flexible shifts, different level skills and individual preferences (Erhard et al. 2018).

In the literature, three major problems related to the scheduling of health workforce can be found: the nurse rostering problem, the physician scheduling problem and the residency scheduling problem. However, less importance has been given to other workers, such as technicians. Although their schedules could have different approaches, they have several common features.

3.2 Features of the scheduling problem

Shift work has a detrimental influence on workers lives, affecting them psychological, physiological and on a social level. However, a shift-based work allows a permanent availability of services. For this reason, its approach is a critical aspect in order to avoid job burnout (Güler et al. 2013). The way the schedule is structured is an important aspect and influences the quality of the results, as described below.

As aforementioned, on rostering problems, scheduling can be cyclic or acyclic. Carter & Lapierre (2001) further classified the cyclical schedule into two different types: the cyclic schedule without and with rotation, depending on whether the same person always has the same pattern and repeats it over time, or the pattern is rotating for each staff team. According to Ferrand et al. (2011), finding the length of the cycle is the critical part of the process. Specifically, shorter cycles are better in the sense that they can be easily remembered by the workers, but on the other hand, these bring fewer possibilities to fulfil all the required needs. Although this kind of schedule allows to achieve predictability, it does not allow to take effective account of individual needs and preferences due to lack of flexibility (Burke et al. 2004). The legal holidays are an example of a problem in cyclic schedules. The acyclic schedules are the most common and are created from scratch every period. These schedules allow to find better quality, since individual preferences are more easily incorporated. However, they take longer to build (Carter & Lapierre 2001).

Bard & Purnomo (2006) used a cyclical schedule considering the possibility to hire additional nurses temporarily according to demand peaks, which increases the flexibility. Becker et al. (2019) also relied on a cyclic schedule, but with the goal of maximizing employee satisfaction, they achieved this by increasing the number of free weekends compared to the previous non-cyclic scheduled. In addition, they considered the cyclic schedules favorable, since these reduce planning complexity. However, this kind of schedules are only suitable in cases of continuous and homogeneous demand and staff regarding skill levels.

Demand is a determining factor in health sector, as well as volatile, uncertain and unpredictable. It is usually an input parameter that can be deterministic or stochastic, depending on whether it is a known or an uncertain value, respectively. In the literature, the majority approaches demand in a deterministic way, notwithstanding, in reality, the demand for healthcare services is most frequently stochastic and has seasonal periods. Generally, most papers that include stochastic models are found when the problem is in the emergency department, since it is prone to unexpected events (Erhard et al. 2018). Ganguly et al. (2014) built the roster of an emergency department considering the uncertain distribution of patient arrivals, whereas Fügener et al. (2017) considered the surgery duration as stochastic.

Due to the uncertain demand it may be necessary to face unbalanced distributions over time. One of the approaches used to overcome this obstacle is through the flexible modeling of shifts and inclusion of breaks (Erhard et al. 2018). In other words, in flexible shifts - or also known as implicit shifts - the start time and the duration of each shift are flexible, this is, they can vary, as long as work regulations are satisfied (Volland et al. 2017). Brunner & Edenharter (2011) relied on flexible shifts in order to better provide demand coverage and minimize the cost of hiring physicians. Brunner et al. (2009) also considered flexible shifts and incorporated breaks in addition, considering that they may start only after a minimum of hours worked, that is, the staff should work a minimum and a maximum period before and after a break. Likewise, Stolletz & Brunner (2012) included break assignments with fixed start and end times, in contrast to Brunner et al. (2009), who tried implicit modelling breaks. Despite this, the use of breaks has not been widely addressed in literature.

Nevertheless, it is more common to find predefined shifts that can be applied in different ways.

As for instance Beaulieu et al. (2000) divided each day into three periods of eight-hour shifts, without overlapping between shifts, while Puente et al. (2009) used two seven-hour shifts and one twelve-hour shift to deal with variability, however they did not use overlapping shifts as well. It is more common to find non-overlapping shifts when demand is approximately constant throughout the day, although the overlapping option offers more flexibility; namely, the latter allows having more employees present in certain peaks of demand, leading to a reduction of understaffing situations while containing costs, since hospitals are not forced to hire additional staff or schedule expensive overtime shifts. Likewise, the overstaffing situation is also avoided during low demand peaks (Van Den Bergh et al. 2013). For instance, Bard & Purnomo (2005) have taken advantage of this: according to demand periods, they used five different shifts, which overlap, in order to better deal with the demand peaks. Three are 8-hour shifts (*day, evening and night*) while the remaining are 12-hour shifts (AM and PM); the AM shift matches with the *day* shift and part of the evening shift, while the PM shift covers the remaining part of the *evening* shift and the *night* shift.

Several studies deal with different level skills, which can be evaluated from different perspectives, such as experience, degree of technical knowledge, qualifications, educational level or others. This distinction may have some consequences that influence both the hospital and the service provided. Consider the example of a higher skilled person, which may involve higher labor costs, but will most probably have a positive impact on the speed and the quality of the produced work. Other factors like efficiency, task restrictions or flexibility could arise (De Bruecker et al. 2015). One way to overcome these differences is through a cross-trained workforce, composed by the so-called *"flexible workers"* that, despite having specific skills, are allowed to perform tasks that usually are assigned to workers with other skills, in case of need. This increases flexibility and allows to cover unexpected demand peaks, avoiding the hiring of new expensive workers or even layoffs. Campbell (2011) concluded that the flexibility that results from this type of workers is better than having perfect information regarding demand. Similarly, Schoenfelder et al. (2020) found that, in fact, cross-trained nurses offer more flexibility that helps to overcome peaks in demand and, consequently, to reduce understaffing situations and patient turnaway, although in some cases this entails additional cost stemming from higher wages. Thus, the slight decrease of patient turnways may not justify the additional scheduling costs.

There are several features that can affect the staff scheduling problem. In brief, cyclic or acyclic schedules are both widely used; however, the latter is most common since fairness and individual preferences are easily incorporated. Flexible shifts and breaks allow to increase the flexibility and better fit the demand, withal they are not common in literature. Finally, the skill levels are another feature that can be taken into account, in case of the resident physicians, otherwise it is not recurrent. Proper selection of features can lead to a more effective way to achieve the objectives.

3.3 Objectives

Depending on the hospital's, healthcare professionals' or patients' point of view, a schedule may be better according to different objectives. In general, this can be divided, mainly, into financial or non-financial goals. Notwithstanding, both financial and non-financial goals can be considered simultaneously. The former will be discussed in Section 3.3.1 and the latter in Section 3.3.2, and a multi-objective perspective will be discussed in Section 3.3.3.

3.3.1 Financial Goals

Most companies want to maximize profits, increase revenue and minimize total costs. In this way, they usually apply financial goals that are expressed in monetary terms. Hospitals face enormous pressure to operate in a cost-efficient way, and in this sense, some scheduling studies try to incorporate this objective. In order to achieve their financial goals, Aickelin & Dowsland (2004) used cost as their measure, since it depends on the day of the week – for instance, in off-days (for example weekends) the costs are higher –, and given different costs deriving from skills, highly-skilled personnel demands higher spending. Brunner & Edenharter (2011) had as main objective to minimize staffing costs related to hiring physicians with different experience levels, that are needed to provide the demanded coverage. Likewise, Ganguly et al. (2014) focused on the resulting costs of a roster derived from overstaffing expenses. Alternatively, Brunner et al. (2009) and Stolletz & Brunner (2012)'s goal was to minimize the operating costs incurred due to overtime. In other words, they wanted to minimize the overtime working hours that are paid out. Schoenfelder et al. (2020) go a little further: wanting to minimize the combined costs of regular and overtime shifts assigned to nurses, they tried as well to minimize the expected value of penalty costs due to under-staffing, which consequently affects the quality of service, since it leads to turning away patients and patients transfers. Volland et al. (2017) followed another strategy to contain costs. It is known that medical staff must often have to perform tasks that are not directly related to patients or care. For instance, nurses spend approximately 10% of their time performing logistic tasks – non-patient care-related –, wasting valuable time, which is more expensive than having specific staff performing those tasks. Their goal was to find the optimum number of such employees. In addition, they found considerable potential for cost savings by scheduling shifts and tasks simultaneously.

Although financial goals are important, they do not appear regularly in the literature related to physicians scheduling, at least in an isolated way. Concerning nurses, this is usually a little more usual. It must be borne in mind that some considered objectives could affect direct or indirectly other goals. For instance, overtime hours and staff satisfaction have an indirect effect on service quality and costs (Stolletz & Brunner 2012, Erhard et al. 2018).

The next section addresses non-financial goals, which are related to individual aspects of employees and patients. These have been growing in importance in recent years.

3.3.2 Non-financial Goals

Most financial objectives are modelled on a single criterion, however this is not the case for non-financial objectives, that usually require more sophisticated measures, and where it is frequently applied a multi-criteria objective function based on multiple weighted goals, considering soft and hard constraints (Erhard et al. 2018). However, other constraint denominations are found. For instance, Ikegami & Niwa (2003) considered nurses and shift constraints, which are related to the assignment of all shifts, in order to maintain the requested level of service, while the nurse constraints correspond to the workload of each nurse (the number of shifts of a specific type, preferences for days-off, etc). These shift constraints are similar to coverage constraints present in Maenhout & Vanhoucke (2009). On the other hand, Kazemian et al. (2014) suggested required (related to organizational policy), desirable (not required but desirable, like vacations schedule) and linkage constraints (to introduce model dynamics). It is important to highlight that specific characteristics of constraints may vary depending on the hospital, needs or the state.

The following are the main focuses of the non-financial goals, these are found both from the perspective of the patient and of health professionals.

Patient-Related Goals

The non-financial goals mainly focus on individual aspects from both patients and personnel. From the patient's point of view there are several aspects that need improvement, such as the well-being and quality of care, minimizing the number of patients' hand-offs or reducing their waiting times. Kazemian et al. (2014) concentrated on aspects with respect to well-being and patients' quality of life, in particular, on minimizing the number of hand-offs. In practical terms, they developed a patient-centered model for scheduling residents and trainees, in order to minimize the number of patient hand-offs, which are strongly associated to medical errors caused by communication gaps, and may, in more serious cases, compromise the life of a patient. On the other hand, EL-Rifai et al. (2015) chose to minimize the average total patient waiting times in an emergency department. This kind of approach allows to avoid negative emotions and reduces the number of patients that leave without being consulted by a physician. Waiting times are strongly influenced by the number of staff present in the service, for that reason proper scheduling may influence and improve them. Another important aspect considered is the integration of different resources, since they impact each other - in other words, they apply their model to both nurses and physicians.

Personnel-Related Goals

Due to the scarcity of healthcare professionals, greater attention is required to aspects such as fairness and personal preferences. In fact, these factors were already considered at the time of the first computeraided physician scheduling system, as found in Ernst & Matlak (1974).

It is necessary to provide close attention when assessing individual preferences, as they can vary greatly from person to person. To illustrate this, there may be employees willing to work on weekend or night shifts, in order to receive extra money, while others prefer evenings or nights off (Gross et al. 2019). Therefore, individual preferences may include the staff requests, such as preferred days-off, duties, daytimes and so on (Erhard et al. 2018). For instance, based on interviews with physicians, Stolletz & Brunner (2012) recognized as preferences: length of the work period, days-on and days-off patterns and preferred staff working together.

There are some categories of scheduling methods, as mentioned by Bard & Purnomo (2007), that allow to better consider and understand staff preferences. One is the *self-scheduling* where the staff should sign-up for the shifts that they wish to work over the planning horizon, as long as possible these preferences should be met. Another approach is the *preference scheduling*, which through a set of constraints and a cost measure, it is intended to find a balance between employee satisfaction and outside resources used. This approach originally attended mainly to individual requests; today, the quality of the schedule and the prevention of undesirable work patterns are of great importance.

Generally, staff are not able to quantify their preferences among different solutions. For this reason, and in order to better meet and get to know the individual preferences, Cohn et al. (2009) were in constant communication with the chief of residents due to his experience, in order to improve the quality of the schedule, by satisfying all hard constraints, fairness and preferences and to identify weaknesses of each candidate schedule. However, in Bowers et al. (2016) each physician has the opportunity to assign weights to the different duties based on their personal preferences. In fact, they create two subgroups: one of them prefers a schedule based on equal workload and the other one likes the preference-based schedule. Afterwards, the proposed model creates a schedule which distributes duties between the two subgroups according to their choice.

Consequently, it is important to note that, in order to ensure staff satisfaction, individual preferences can not be considered alone, because one is at the risk of satisfying one employee at the expense of another, so this could lead to very different levels of satisfaction. To avoid this, it is becoming increasingly important to incorporate fairness aspects.

The fairness aspects are often addressed in scheduling problems and can be approached in different ways, but despite its definition, they are difficult and mostly subjective, thus should be carefully analyzed and evaluated, even if they seem equal in terms of apportioned work. This can be perceived through a simple example: suppose that all physicians have been assigned the same number of day and night shifts. Apparently this is fair, but if it is always the same employee who works every Saturday and Sunday, is not as fair as it is suggested. Hence, it is necessary to look beyond the total number of hours worked.

Fairness, or lack thereof, leads to several consequences that affect not only the employee's personal life, but also the quality of healthcare provision. Lower job performances, increased absenteeism, bickering between staff and increasing turnover rates are some of the effects produced by schedules that suggest unfairness (Bard & Purnomo 2005). Nevertheless, when fairness aspects are considered there may be a slight increase in costs as well as an increase in solution times (Stolletz & Brunner 2012).

Despite being an increasingly used aspect, its definition has varied greatly. Bard & Purnomo (2005) interpreted fairness as the equal distribution of preference fulfilment among schedule personnel. Considering the nurses' individual preferences regarding their work schedule, their model ensured that the preferred solution is one in which several nurses have a small number of violated preferences, rather than a single nurse suffering a considerable amount of violations. Often the greatest concern is to maximize the overall quality of the schedule, however this can lead to unfair individual schedules. In order to overcome this problem, Trilling et al. (2006) minimized the difference between maximum and minimum individual penalties from soft constraint violations. On the other hand, Wolbeck et al. (2020) tackled a re-scheduling problem where the main objective was to minimize the overall penalty costs, which derive from fairness violations. In this case a *fair shift change penalization scheme* was developed based on type, time and distribution of shift changes, which is able to use an accumulated penalty individual score considering previous schedules, in order to distribute fairly the shift changes among nurses overtime, whereas Gross et al. (2019) proposed a model using a satisfaction-based preference weight, where each physician has a satisfaction indicator, which measures a physician's satisfaction according to the preferences fulfilled in the roster. They also used previous planning horizons to track fairness measures. Both Wolbeck et al. (2020) and Gross et al. (2019) concluded that, using previous periods, the fairness level of schedules improves, and unfair fulfilment of preferences accumulated over time is avoided.

Another concept of fairness widely used when residents are considered in the model is equality in experience or knowledge acquired during training, as demonstrated by Smalley & Keskinocak (2016) and Kraul (2020).

Fairness is a hard concept as consequence of the associated subjectivity, and also because it depends on the circumstances in which it is applied and by whom, or in what way it is evaluated. Nevertheless, fairness does not consist only in maximizing or minimizing a particular goal, but rather about trying to find a balance for all parties, considering not only the balanced distribution of the workload, but also taking into account the individual preferences as well as particularities for each day (e.g. working on a Sunday is not the same as working on a Monday). Fairness aspects are significantly improved by automated scheduling, as demonstrated by Fügener et al. (2015).

3.3.3 Multi-objective studies

Many studies do not pursue a single goal, but rather try to consider multi-objective functions. Topaloglu (2009) developed a multi-objective programming model in order to maximize the quality of shift allocation of the residents, bearing in mind that the most important objective that should be handled is the minimization of deviations from the seniority rules. Belien & Demeulemeester (2006) also used a multi-objective function which minimizes the number of irregular assignments and the total schedule costs. In their approach they took all preferences into account, however fairness was not considered.

Although the main goal of Stolletz & Brunner (2012) was to minimize the operating costs, which incurred due to overtime, the authors also included employee preferences, fairness aspects and consistent workloads. On the other hand, Baum et al. (2014) did not only want to maximize total revenue, but also to minimize all preferences scheduling discrepancies, in order to maximize the overall workload fairness. In addition, Bruni & Detti (2014) had a bi-objective function: maximize the number of assigned shifts while minimizing the number of inconvenient assigned shifts, in order to provide fairness, which they called balance. Hamid et al. (2020) created a multi-objective model for a nurse scheduling problem by highlighting human factors, such as skills, preferences and compatibility between nurses. So, besides the goal of minimizing the total cost related to staff, it also intends to minimize incompatibilities and maximize overall satisfaction.

Van Huele & Vanhoucke (2015) incorporated in the same model the physician rostering and the surgery scheduling problems, through goal programming. Their objectives were to minimize overtime, balance workload and minimize the opening days of the operating room. Fügener et al. (2015) used a multi-criteria objective function, which maximizes the number of assignments according to labor regulations and internal department rules and minimizes costs for violating fairness goals. Likewise, Kraul (2020) created a multi-objective model which intends to fill both personnel and patient objectives. On the one hand, they want to minimize the deviations in training progress of all residents, to ensure fairness; on the other hand, they aimed at providing continuity of care for patients by reducing the total number of changes between services. In fact, the consideration of several objectives makes it possible to satisfy a greater number of requirements, despite the increase in complexity. The results were quite satisfactory, so much so that the model is currently applied in the hospital.

In recent years there has been an increase in the concern to use objectives that consider fairness and individual preferences rather than minimization of costs in order to promote job satisfaction. However, there are cases, such as in staffing problems, where the main goal is to find the optimal size of the workforce (which is still independent of each person itself), and therefore personnel aspects can hardly be considered, as happens in Brunner & Edenharter (2011), for example. The consideration of fairness and individual preferences is of great importance, as it promotes workforce satisfaction, reduces absenteeism, helps to attract new workers and also indirectly affects the quality of service and, therefore, patient satisfaction.

Table 3.1 summarizes the main goals found throughout the literature. The objectives can be financial or non-financial. The financial goals are usually measured with monetary units: minimize costs or maximize revenue. Regarding non-financial objectives, they are personnel or patient-related and take into account aspects such as fairness, individual preferences or well-being. The next section summarizes some of the staff healthcare scheduling studies mentioned previously, considering these objectives.

Table 3.1: Example goals used in scheduling problems.

Financial		**Non-financial**		
		Personnel		
Costs	**Revenue**	**Fairness**	**Preferences**	**Patients**
Minimize number of overtime hours	Maximize total revenue	Stable shift starting times	Minimize staff dissatisfaction	Minimize the waiting times
Minimize wage costs		Even distribution of working hours	Maximize number of days-off pairs	Minimize the number of patient's hand-offs
Minimize usage of outside resources		Even assignment of on-call services	Minimize deviations between scheduled and desired level	Maximize continuity of care
Minimize the number of personnel scheduled		Equal experience and knowledge acquired during training	Maximize preferences for days-off duty	
Find the optimal number of specific workers to carry out logistic tasks		Equal distribution of preference fulfilment among schedule	Willingness of working together with a specific worker	

3.4 Comparison of staff scheduling literature

A summary of the aforementioned studies found in the literature on staff healthcare scheduling is provided in this section. The studies included in this review correspond to the state-of-art in staff hospital scheduling, which is of interest to the problem presented in this work. In Table 3.2 a summary of the studies found in the literature on staff healthcare scheduling is showed. A comparison of several studies is made considering different aspects, such as the main objective(s): (i) Financial goals; (ii) Fairness (personnel-related goal); (iii) Individual Preferences (personnel-related goal); and (iv) Patient-related goal. In addition, the features found throughout the literature are also highlighted, in particular the different level skills among staff, flexible shifts, fairness aspects or individual preferences, stochastic demand, distinct professionals (for example, if both nurses and physicians are considered simultaneously in the model) and if the studies are using real-life data, that is, if the data used comes from a real problem or is just an experiment.

The current work accounts for a multi-objective approach for healthcare staff scheduling that meets the needs of the hospital, considering different skills, satisfying preferences, and incorporating fairness aspects. In addition, it will be applied to different health professionals, namely physicians and technicians, however not simultaneously. This work uses real data from the hospital.

Table 3.2: Summary of the literature on staff healthcare scheduling.

Authors	Major Objective (1)	(2)	(3)	(4)	Different Skills	Flexible Shifts	Fairness Aspects	Individual Preferences	Stochastic Demand	Distinct Professionals	Real-life Data
Brunner et al. (2009)	✓					✓		✓			✓
Topaloglu (2009)		✓	✓		✓		✓	✓			✓
Brunner & Edenharter (2011)	✓				✓	✓					✓
Ferrand et al. (2011)		✓					✓	✓			✓
Stolletz & Brunner (2012)	✓					✓	✓	✓			✓
Ganguly et al. (2014)	✓				✓	✓			✓		✓
Kazemian et al. (2014)				✓	✓		✓				✓
EL-Rifai et al. (2015)				✓		✓			✓	✓	✓
Fügener et al. (2015)		✓						✓			✓
Bowers et al. (2016)			✓					✓			
Volland et al. (2017)	✓					✓				✓	✓
Gross et al. (2019)			✓			✓	✓				✓
Kraul (2020)		✓		✓	✓		✓				✓
Schoenfelder et al. (2020)	✓				✓			✓	✓		✓
Wolbeck et al. (2020)		✓					✓				✓
Hamid et al. (2020)	✓		✓	✓		✓	✓	✓			✓
This work		✓	✓		✓		✓	✓			✓

(1) - Financial goal; (2) - Fairness (personnel-related goal); (3) - Individual preferences (personnel-related goal); (4) - Patient-related goal;

3.5 Chapter Considerations

The literature on staff scheduling in healthcare has increased in the last years, due to the impact this could have from the point of view of the hospital, the patients and the staff (Erhard et al. 2018, Wolbeck et al. 2020). Several features and objectives have been addressed over time. The correct choice of resources helps to increase flexibility as well as efficiency, allowing to achieve goals more effectively. In this chapter, relevant studies are provided and discussed in order to find the main objectives used and features that should be taken into account in healthcare staff scheduling. Most of these studies are applied to real cases, however, few are currently in use.

The purpose of this work is to solve a generic problem motivated by the case of HLL that aims to satisfy different goals. The next chapter presents the methodology followed in order to meet these goals. The proposed methodology provides a schedule for healthcare staff based on a multi-objective approach that meets the needs of the hospital, considering different skills, equating demand and supply, satisfying preferences, and without compromising fairness. The method developed is based on meta-heuristics, namely Genetic Algorithms. This allows to provide a good approximation of a Pareto-front in a short computational time.

Chapter 4

Methodology

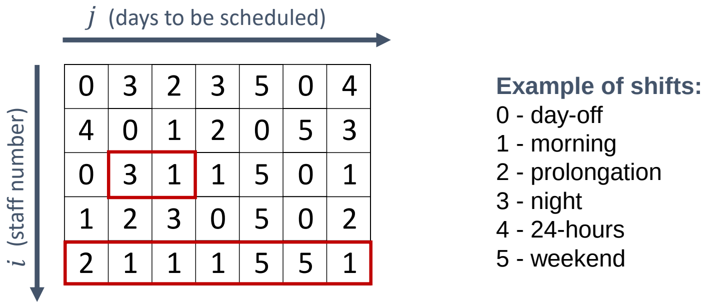
In the present chapter, the methodology based on a Multi-Objective Evolutionary Algorithm (MOEA), namely a NSGA-II, developed to solve the staff scheduling problem of HL is detailed. A Genetic Algorithm (GA) is a metaheuristic, which provides good solutions for an optimization problem, in particular, when a shorter computing time is desired. This algorithm is based on techniques inspired by Darwin's theory of natural evolution, reflecting the process of natural selection, where the fittest individuals are most likely to be chosen to originate descendants. Although the aim of this work is to build a generic algorithm, there are intrinsic particularities to each of the case studies. For this reason, Section 4.1 provides an overview of features of NSGA-II, regarding the main genetic operators, and Section 4.2 presents the main notes and assumptions considered on the algorithm construction for each case study.

4.1 Staff Scheduling: NSGA-II Algorithm

A GA is a metaheuristic that belongs to the class of Evolutionary Algorithms since it is inspired by the Evolution theory, namely the principles of variation, inheritance and natural selection, where the fittest individuals survive. In turn, the NSGA-II, first proposed by Deb et al. (2002), is a GA applied to multi-objective problems, which implies decision making by multiple criteria and consequently gives rise to optimal sets of solutions, called Pareto-fronts or non-dominated solutions (Deb et al. 2002).

This algorithm is based on three important features: (i) it considers *elitism* property, that is, it preserves the best solutions of the current generation and takes them to the next one, thus allowing good solutions not to be lost; (ii) through crowding distance, which is a specific NSGA-II operator, it is possible to preserve diversity – a fundamental aspect of GAs; and (iii) it finds non-dominated solutions.

In addition to the usual genetic operators, such as selection, crossover and mutation, two multi-objective operators are defined: non-dominated sorting and crowding distance. The former will be



0	3	2	3	5	0	4
4	0	1	2	0	5	3
0	3	1	1	5	0	1
1	2	3	0	5	0	2
2	1	1	1	5	5	1

Example of shifts:
0 - day-off
1 - morning
2 - prolongation
3 - night
4 - 24-hours
5 - weekend

Figure 4.1: Chromosome representation: $[i \times j]$ matrix, where the rows i represents each staff member and the columns j the days to be scheduled. Each entry ij represents the shift assigned to employee i in day j.

described in Section 4.1.1 and the latter in Section 4.1.2. At the end, in Section 4.1.3, the complete algorithm will be presented.

4.1.1 Usual Genetic Operators

⇒ **Chromosome:** The chromosome represents the set of variables which define a proposed solution. The chromosome designed to represent the solution of the proposed algorithm is a $[i \times j]$ matrix, where i denotes the total number of staff (physicians or technicians, depending on the case under study), and j the days that are intended to be scheduled. Each entry of the matrix indicates the shift assigned to a worker during a day. To illustrate this, Figure 4.1 shows an example of the chromosome representation for a problem with five employees and seven days to be scheduled. For instance, the first employee gets a workday off on the first scheduled day, as the entry is 0. This same employee is assigned to a night shift on the second scheduled day. The second employee has a 24-hours shift on the first day to be scheduled. Columns five and six represent a weekend, as it only contains the weekend shifts, expressed by 5. For instance, the second employee gets a workday off on Saturday while having a weekend shift the next day, the opposite is true for the first, third and fourth employee.

In order to represent a feasible solution, a chromosome must take into account the hard constraints of the underlying problem. If this is not the case, then it should either be repaired or strongly penalized. Throughout this section, the way in which this is handled in the several steps of the algorithm will be explained. The hard constraints considered are the following: (i) guarantee the coverage requirements concerning the maximum and minimum employees who must be present in each shift to satisfy the demand; (ii) do not allow an employee to work more than 6 consecutive days; and (iii) if a technician or a physician does a night shift they cannot be assigned a shift that starts in the morning of the next day. To give an illustration the red boxes in the Figure 4.1 represents examples of hard constraints that are not met. The first represent the case where a employee is assigned to a night shift followed by a morning shifts and the second the case where an employee works more than 6 consecutive days.

⇒ **Fitness Function:** Fitness function allows to evaluate the quality of a solution, and it is closely related to the objective function(s) of the problem under study. In other words, it determines the fitness of a solution allowing for the comparison of different solutions. This ensures, similarly to natural selection, that the best solutions remain in the population, considering that better fitness scores are more likely to be chosen. Since the case-studies under analysis are multi-objective in nature, more than one fitness function will be used.

⇒ **Population:** The population is a collection of individuals, which are the chromosomes and, consequently, the candidate solutions for the problem. The set of chromosomes from the initial population can be randomly generated, or can be known or predetermined feasible solutions. It is also possible to use both methods at the same time, that is to create some individuals using one method and generate the remaining using the other one.

In this dissertation, the initial population is generated randomly, although not entirely. In other words, the generation process must respect certain conditions that will be described for each case study. After the random chromosome generation, it is verified if the requirements regarding the minimum number of workers that must be present in each shift are satisfied. For this purpose, all the workers assigned to each shift of each type are counted and compared to the hospital needs. In cases where there is a lower number than the one intended, new shifts are added until the minimum staff number is satisfied. In addition, a repair is made if someone is working for more than six consecutive days (see Algorithm 5 and 7). The goal is to start with only feasible solutions, satisfying all the hard constraints.

The population size (N) is a genetic algorithm parameter, as it can be modified. Nonetheless, it is known from the literature that it should not be a very small number, since in that case the diversity is reduced, which can compromise the quality solutions, conditioning the research in the space of solutions. On the other hand, it should not be too high either, as it impairs the efficiency of the algorithm (Gendreau et al. 2010).

⇒ **Selection:** Once there is a population, the natural selection process can occur. As in the natural world, regardless of the type of selection chosen, the objective will be to maintain the good characteristics of the best solutions in the population. To this end, it is necessary to find mechanisms that give preference to more suitable solutions. Each chromosome has on associated fitness value that makes it possible to perceive whether it is a good solution compared to the rest of the population or not and, as such, if it will likely perpetuate itself or not in the following generations.

For the construction of the algorithm, tournament selection will be used. In this type of selection, sets of T individual are randomly chosen, allowing to adjust the pressure entered into the selection. The most fit individuals in each group, that is with the best values for the fitness functions, are selected and will be the parents. That is, the more fit an individual is, the greater the probability of winning the tournament and being chosen. Tournament selection takes place as often as necessary until the desired number of individuals (r) is reached and the value of T can vary. The evaluation for the best individuals will be

Algorithm 1 Tournament Selection pseudo-code

```
Require: Population $P$, Pareto-fronts $F_i$, Fitness Functions values $f_m$, $r$ and $T$
for each $i$ in 1, ..., $r$ do
    Choose $T$ individuals from the population $P$ at random
    Rank the individuals by determining to which $F_i$ each element of $T$ element belongs, where $front_b$ is the best ranked front
    if number of individuals in $front_b$ = $1$ then
        Choose the individual in the $front_b$
    else
        Calculate $Crowding\ Distance$
        Choose the individual with best $Crowding\ Distance$
    end if
    Append the best individual in parents list
    Remove the individual from the Population, in order not to be chosen in the next tournament
end for
```

described below. The tournament selection pseudo-code is presented in Algorithm 1 (note that some concepts in the pseudo-code are only defined further on).

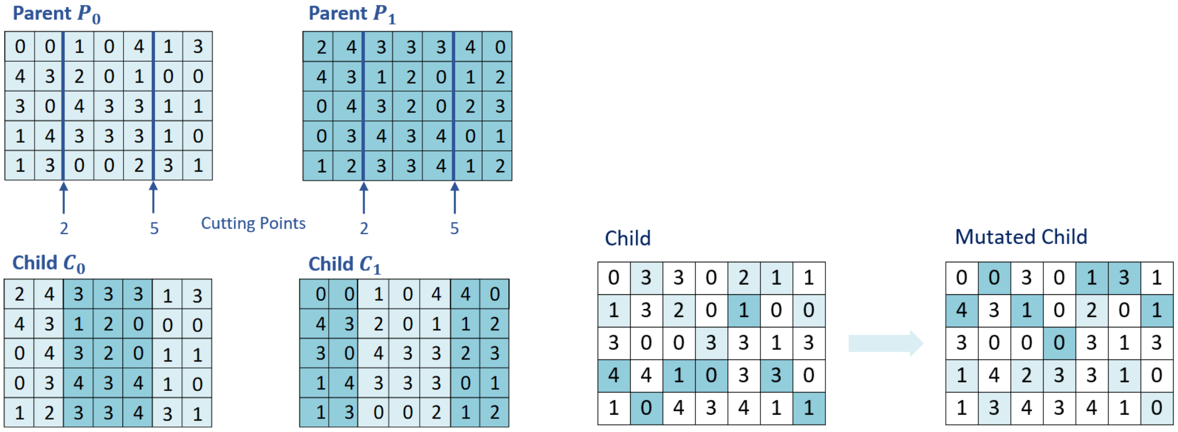
⇒ **Crossover:** One of the principles of evolution is the inheritance property in which some genes are passed on from parents to off-springs. Through the crossover the diversity is ensured. As a result of the selection, two parents are chosen from the population, and their genes will be crossed over and originate off-springs.

There are several types of crossover. In this case, the two-point crossover is applied in the proposed algorithm. As the name suggests, two points are chosen at random, between 1 and $j - 1$ (days to be scheduled), in order not to risk choosing one of the end columns and applying an one-point crossover instead of a two-point crossover. In addition, care is also taken in order not to choose two consecutive points. As soon as one has the cutting points, the chromosome is sectioned in that interval. In other words, and as represented in Figure 4.2a, the first columns from the parent P_0 to the first cutting point will pass to the child C_0, the portion between the cutting points will be inherited from the parent P_1, and the remainder from the second point to the end (j) will come from the first parent. On the other hand, the child C_1 receives the first columns (up to the cutting point) from the parent P_1, the portion between the cutting points from the parent P_0 and the remainder from P_1 again.

However the needs defined in the initial solution are still respected here, as columns are exchanged in the same position between parents, so that the needs previously guaranteed will remain. This results in a strong penalization in the fitness functions, as will be described for each case study, and, therefore, the individual will never be considered as a parent, eventually disappearing from the population. In order to not violate the remaining hard constraints, a repair function was created, which is described later.

⇒ **Mutation:** Once the off-springs are generated, they may mutate. This operator allows periodically and randomly to introduce changes into the chromosomes, thus increasing diversity and explore new areas of the solution space.

Mutation does not always occur for all chromosomes. There is a probability of mutation (p_m), which



Parent P_0

0	0	1	0	4	1	3
4	3	2	0	1	0	0
3	0	4	3	3	1	1
1	4	3	3	3	1	0
1	3	0	0	2	3	1

Parent P_1

2	4	3	3	3	4	0
4	3	1	2	0	1	2
0	4	3	2	0	2	3
0	3	4	3	4	0	1
1	2	3	3	4	1	2

Child C_0

2	4	3	3	3	1	3
4	3	1	2	0	0	0
0	4	3	2	0	1	1
0	3	4	3	4	1	0
1	2	3	3	4	3	1

Child C_1

0	0	1	0	4	4	0
4	3	2	0	1	1	2
3	0	4	3	3	2	3
1	4	3	3	3	0	1
1	3	0	0	2	1	2

(a) Crossover

Child

0	3	3	0	2	1	1
1	3	2	0	1	0	0
3	0	0	3	3	1	3
4	4	1	0	3	3	0
1	0	4	3	4	1	1

Mutated Child

0	0	3	0	1	3	1
4	3	1	0	2	0	1
3	0	0	0	3	1	3
1	4	2	3	3	1	0
1	3	4	3	4	1	0

(b) Mutation

Figure 4.2: (a) Two-point crossover operation. Two random points are generated and then the columns are cross over between parents; (b) Mutation operation. For each column, two random points are selected and are exchanged. The process is repeated for all columns.

points the occurrence of this mechanism. This probability should not be too high and can be modified throughout the process. The mutation operator inserts small changes in the chromosome. In this case, for each column j, two rows are chosen at random, i_1 and i_2. If the entries corresponding to these choices ($i_1 j$ and $i_2 j$) are different, they are exchanged. Otherwise, a new choice is made. This process is repeated for all columns of the chromosome, as illustrated by the Figure 4.2b, where the entries chosen are represented in different colors and, as it is possible to notice, they are exchanged in the mutated chromosome.

This kind of mutation allows the coverage requirements to continue to be satisfied, as there are no different shifts than those that existed, but rather an exchange of shifts between workers, so the shifts initially assigned to each day are still the same. In relation to the other hard constraints, it may not be possible to avoid and for this purpose a function has been created that repairs such situations.

Figure 4.3 illustrates the steps of the GA and operators described so far. In short, chromosomes are initially created randomly, and they will constitute the initial population. Afterwards, sets of T individuals are randomly chosen, as many as the parents desired (represented by r). Once the parents are selected the crossover takes place and originates off-springs. The way the crossover was designed allows two parents to have two children. Each child can then undergo a mutation that inserts slight changes in the chromosome. Some of these generated children will be part of the next generation.

4.1.2 Multi-objective Operators

⇒ **Non-Dominated sorting:** This is a distinctive operator of the NSGA-II algorithm which aims to find dominant solutions in order to obtain the different Pareto-fronts. In other words, the fitness of the chromosomes is evaluated on the basis of non-dominated sorting. Each solution is compared to all other

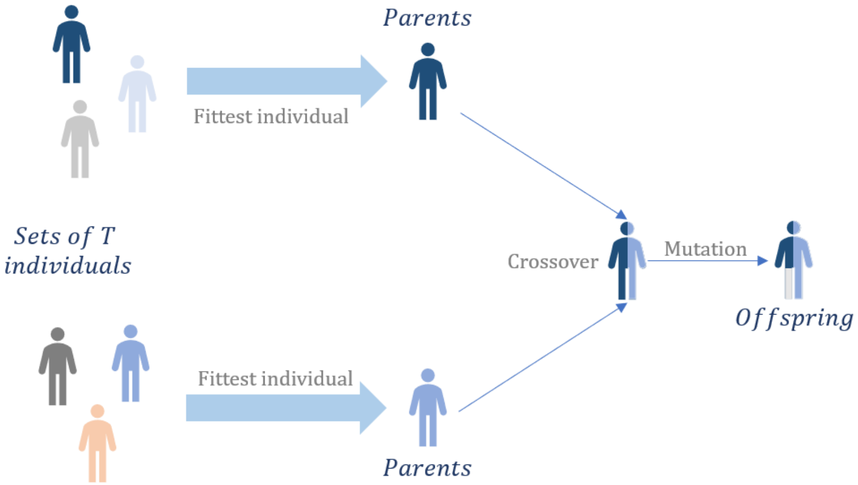


Figure 4.3: Tournament Selection representation. From the population, sets of T individuals are randomly chosen. The fittest individual is selected and will be one of the parents. The parents are crossed over and two children are developed.

solutions in the population in order to find if it dominates or it is dominated. Note that a solution x_1 dominates a solution x_2 if: (i) for all objective functions, x_1 is not worse than x_2; and (ii) for at least one of the objective functions, x_1 is better than x_2.

For this purpose, similarly to Deb et al. (2002), two entities are calculated for each solution: (i) n_p, which is a domination count that counts the number of solutions that dominate the solution p; and (ii) S_p which contains a set of solutions that solution p dominates. As an example, all the population solutions which belong to the first non-dominated front (that is, first Pareto-front) will have $n_p = 0$, since no solution can dominate the solution p. For each solution p that dominates each q, this q is inserted in the S_p list (set of solutions that solution p dominates), and then, n_q domination count of q will be incremented (since q is dominated). Then, for each of the p solutions with the domination counter n_p equal to zero, each member q in the list S_p will have their domination account reduced by one. If any of these q elements reaches the zero value in the domination counter, the element q will be inserted in a Q list. The elements of this list will constitute the second Pareto-front, since the elements in Q are those that are left with the zero n_p counter equal to zero, or, in other words, any of the solutions dominate the solutions in Q.

Figure 4.4 represents a framework of the non-dominated sorting approach. The process is initiated with the whole population P and through solution to solution comparison, the non-dominated solutions are determined. The first non-dominated solutions have the n_p counter equal to zero and, as such, enter the first Pareto-front (F_1) since they are not dominated by any solution in the entire population. These non-dominated solutions (dark blue) will be the first Pareto-front and the portion dominated will be part of S_p. The solutions in F_1 will be removed and the remaining will be evaluated. For each S_p element q, the n_q will be reduced by one until the solutions that reach n_q equal to zero are found. These will constitute the non-dominated solutions of the next front (the ones in list Q). The procedure is repeated, until all fronts are found and all solutions have been compared and the non-dominated are found. The figure also

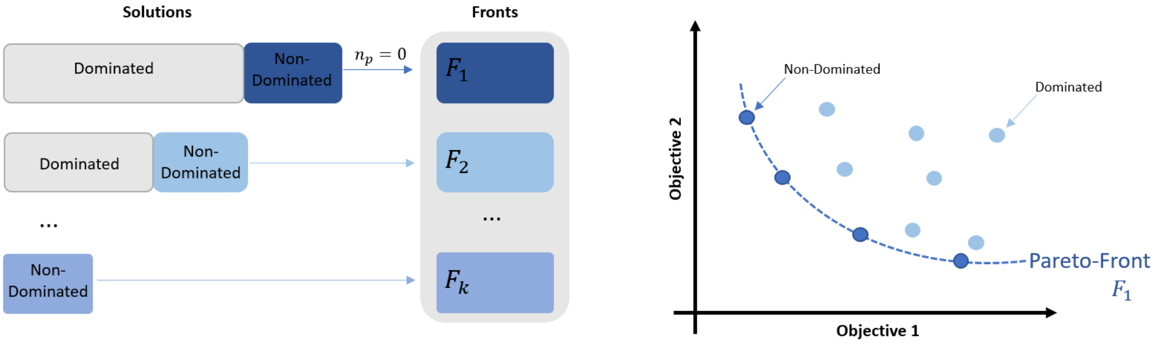


Figure 4.4: Framework of the non-dominated sorting approach. The right side shows an example of a plot with a Pareto-front where the fitness of the solutions is evaluated with two (minimization) objectives.

illustrates an example of a Pareto-front plot (for a bi-objective minimization), where the non-dominated solutions provide a suitable adjustment between all objectives. Algorithm 2 presents the pseudo-code of the main steps for non-dominated sorting according to Deb et al. (2002), previously described.

$\Rightarrow$ **Crowding Distance:** This operator allows to choose solutions when they are within the same Front. It is related to the density of solutions around each solution. It turns out that the higher the density, the less variable are the solutions. It is important to note that one of the principles of a genetic algorithm is to maintain variability, therefore, lower densities are preferred.

To determine the crowding distances, for the case of minimization objectives, we start by sorting all I solutions in a Pareto-front according to each objective function (or fitness function) value (f_m) in ascending order. The boundary solutions, this is the solutions, within the front, with highest, d_1 and smallest function values, d_l are assigned with an infinite distance value ($CD_{(d_{1,m})} = \infty$ and $CD_{(d_{l,m})} = \infty$) and for the remaining solutions i the crowding distance should be computed according to the Equation 4.1.

$$CD_{i,m} = \frac{f_{m(x_{i+1})} - f_{m(x_{i-1})}}{f_{m(x_{max})} - f_{m(x_{min})}} \tag{4.1}$$

Once the Equation 4.1 is determined for each objective function (m) it is possible to find the crowding distance of solution i as the sum of the individual crowding distance values for each objective, as represented in Equation 4.2 (Deb et al. 2002).

$$CD_i = \sum_{m=1}^{M} CD_{im} \tag{4.2}$$

Thus, given two solutions i and j, which belong to the same front, solution i is preferable if it has a larger crowding distance than solution j. In general, solution i is preferable to a solution j, if it meets one of the following conditions:

$$(F_i < F_j) \text{ or } (F_i = F_j \text{ and } CD_i > CD_j) \tag{4.3}$$

Algorithm 2 The main steps for Non-Dominated Sorting

```
Require: population $P$
for each $p \in P$ do
    $S_p = \emptyset$
    $n_p = 0$
    for each $q \in P$ do
        if $p$ dominates $q$ then
            $S_p = S_p \cup \{q\}$
        else
            $n_p = n_p + 1$
        end if
    end for
    if $n_p = 0$ then
        $F_1 = F_1 \cup \{p\}$
    end if
end for
$i = 1$
while $F_i \neq \emptyset$ do
    $Q = \emptyset$
    for each $p \in F_i$ do
        for each $q \in S_p$ do
            $n_q = n_q - 1$
            if $n_q = 0$ then
                $Q = Q \cup \{q\}$
            end if
        end for
        $i = i + 1$
        $F_i = Q$
    end for
end while
```

4.1.3 Complete Algorithm

The step-by-step procedure is shown in Algorithm 3, which presents the NSGA-II algorithm pseudo-code. The value t denotes the number of the generation, while P_t and Q_t represent, respectively, the parents and offspring population in the t^{th} generation. It should be noted that equal chromosomes within each generation will be replaced. As defined in line 6, there are two stopping criteria in the algorithm: one in which the maximum number of generations desired, and another based on how long the algorithm should run, stopping after the defined time limit.

Once the operators of the algorithm are built in a generic way, it is possible to make the necessary adjustments for each of the case studies. This description is provided in the following section.

4.2 Hospital da Luz's Algorithm

In this section, the algorithm will be introduced from the perspective of the hospital. Initially, in Section 4.2.1, an overview of the algorithm is provided. Next, the characteristics that have to be adapted for the application of the algorithm in the ICU and IS are presented, in Sections 4.2.2 and 4.2.3, respectively.

Algorithm 3 NSGA-II algorithm.

```
Initialize random Population
Evaluate Individual Fitness ($f_m$)
Non-dominated sorting
Calculate Crowding Distance
Tournament Selection to select $P_t$
while (generation number $<$ maximum generation) $or$ timer $<$ time limit do
    Generate Children Population ($Q_t$)
    for $i <$ Parents size do
        Crossover
        Mutation
    end for
    $R_t = P_t \cup Q_t$
    $F$ = Non-Dominated sort ($R_t$)
    Crowding Distance
    Select N individuals
end while
```

4.2.1 Algorithm Overview

In both services, the algorithm receives several common data as input, such as the size of the staff to be scheduled, the year, the month, the employees on vacation or sick leave during the month, and other restrictions, – which can be of three different kinds: medical (for instance, a person in breastfeeding period that should consequently work less hours a day), one-off (in which, for some reason, the employee cannot work in a specific day or in a certain shift) and permanent (when an employee can never work in a particular shift). Furthermore, the algorithm receives the needs for the scheduled period, which may vary for each day of the week, and may be subsequently changed by the person responsible for the schedule. There is also the possibility of receiving previous schedules or information from these. This is important to know the shifts worked by each employee so hard constraints are not violated. For instance, assigning a morning shift on the first day to whom worked one night on the last day of the previous month, it should be avoided. In addition, GA specific parameters are also required, such as the population size (N), mutation probability (p_m), the number of chromosomes in the Tournament Selection (T), the number of parents desired (r) and the number of generations ($nGen$).

The algorithm aims to start the population with feasible solutions, for this is required to satisfy the hard constraints, which will be described for each case study. Although the soft constraints do not lead to infeasible solutions, they should be satisfied whenever possible to generate better schedules. Those that do not satisfy the soft constraints will be penalized.

The algorithm is able to distinguish weekends from weekdays in every month under construction, and the Portuguese national holidays, both fixed and those that vary every year (like Easter Sunday), are also included. This is important to adjust the needs correctly, since these days are considered differently from the general working days.

The flowchart of the algorithm is similar for both cases under study. Initially a random population is generated, then each individual is evaluated and distributed across the different Pareto-fronts, where the non-dominated solutions are on the first front. Afterwards a tournament selection is performed to choose the parents that will crossover and originate offspring, which may or may not mutate. The parents and children are combined in a large pool from which N individuals are chosen, its number being equal to the population size defined as input, according to the front they belong to. Crowding distance is used if it is necessary to choose individuals from the same front. This process is repeated as many times as needed until the defined stop criterion is reached.

The last step of the algorithm consists in determining the final list of non-dominated solutions, as a subset of the first Pareto-front (F_1) of all generations. In each generation the list with chromosomes belonging to the front of non-dominated solutions is kept. In the end, they all come together and non-dominant sorting is applied as they may no longer be all non-dominated. Then, the duplicates are removed and the best solution can be obtained from the different alternatives. For this purpose, the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method, a multi-criteria decision method, is applied. Here, it is necessary to highlight and distinguish two terms: alternatives and criteria. Alternatives are the different options which need to be evaluated in order to select the best one or, in other words, alternatives are the different chromosomes that result in schedules. Meanwhile the criteria, also known as attributes, will impact the selection of alternatives. Three characteristics must be ensured: completeness – to guarantee that all the important criteria are included –; exclusiveness – to avoid redundant criteria – and operationality – each alternative should include all the criteria, for the alternatives to be confronted with each other according to each criterion (Shih et al. 2007).

Once these terms are defined for the problem, the relative importance of each criterion must be estimated, that is, given a weight. At least two roles are needed in this process, one or more Decision Maker (DM)s and a facilitator. They should work together, each performing their own functions, in order to find the solution that best suits the problem. The DM is responsible for weighing each criterion, whereas the facilitator should ask clear and well defined questions without ever influencing the DM, not giving his opinion or expressing his concerns.

A set of alternatives is compared according to weights assigned to each criterion, the normalized scores and the geometric distance between each alternative and the ideal one. To apply the method, a decision matrix, the weights of the criteria defined by the DM and the type of criterion – which can be cost (lower scores are desired) or benefit (higher scores are desired) – are required. This is a useful technique, which allows to organize the problem and ranking to select the best alternative through distance measures. For that purpose, two alternatives are defined: a positive ideal alternative - with the best attribute values (which is 0 for cost criteria and 1 for benefit) and a negative ideal alternative (worst criterion values - minimum benefit values and maximum cost values). The alternative selected by TOPSIS method is the one that comes closest to the ideal solution (one with the shortest geometric distance from the Positive Ideal Solution (PIS)) and the one that moves farther away from the Negative Ideal Solution (NIS) (Shih et al. 2007). The steps performed during the application of the TOPSIS method are as follows:

1. Create a decision matrix with m alternatives and n criteria: $D = (x_{ij})_{m \times n}$;
2. Normalize each entry of the decision matrix, creating a normalized decision matrix: $R = (r_{ij})_{m \times n}$
   $r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{k=1}^{m}(x_{kj})^2}}$, i =1,2,...,*m*, j=1,2,...,*n*
3. Calculate the weighted normalized decision matrix: $t_{ij} = r_{ij} \cdot w_j$
4. Calculate the PIS and NIS;
5. Calculate the Euclidean distance of each alternative to the PIS and NIS;
6. Rank the alternatives according to the distance.

As for the output, the best schedule will be obtained, selected through the TOPSIS method, which is applied to all the best individuals generated during the entire process. In addition, the number of hours worked by each employee and the fitness values of the best solution are also obtained. This output will also serve as input to generate the next schedule.

Figure 4.5 describes the main components and steps of the algorithm and Table 4.1 summarizes the parameters used.

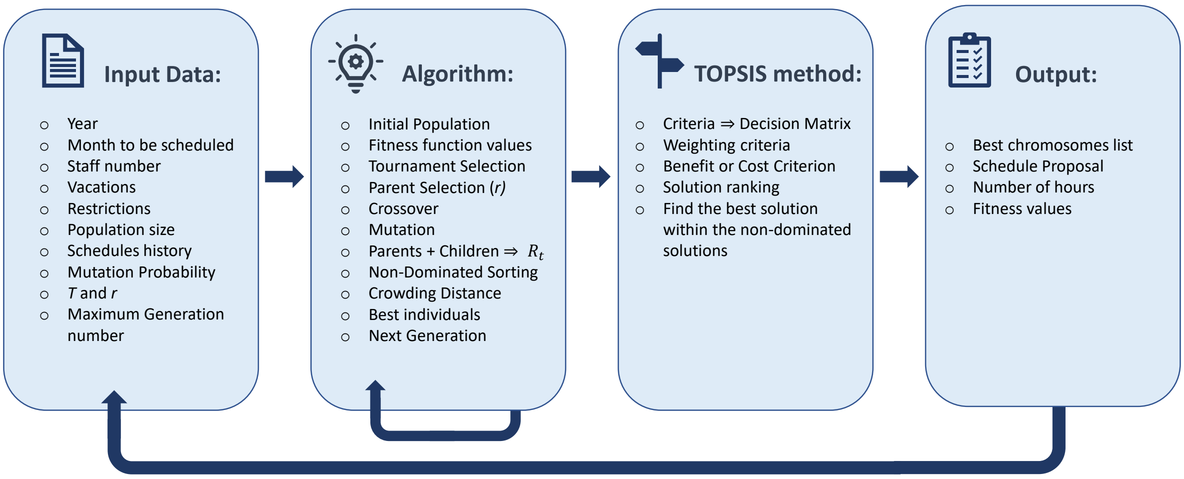


Figure 4.5: Algorithm's Flowchart.

Table 4.1: Algorithm parameters.

Parameters	
N	Population size
nGen	Number of Generations
P_t	Parents Population
Q_t	Children Population
R_t	Population $P_t \cup Q_t$
T	Number of individuals participating in the Tournament Selection
r	Number of parents resulting from the Tournament
F_i	Pareto-front i
p_m	Mutation Probability
n_s	Needs of shift s

In the following sections, important aspects that constitute the construction of the algorithm will be described. In particular the construction of the chromosome, the slight changes to the crossover and mutation operators, the fitness functions and the way in which the best solution is found, using TOPSIS method. In the case of ICU, the survey developed and answered by the ICU's physicians is also presented for a more real definition of the criteria.

4.2.2 ICU: notes and assumptions

ICU's planning is done for an entire month. For each shift in the day, an ideal number of physicians to be allocated is defined. However, in this case, it is not always possible to meet those needs, because the lack of physicians is a recognized problem. Thus, the defined weekly needs are a target demand. For that, an ideal limit is defined – that is, the ideal number of physicians that must be assigned in each shift –, and a minimum limit that needs to be reached, however it should not be less than this value, since it becomes an infeasible solution. This means that schedules with insufficient coverage (i.e., less than the ideal number) may arise as in Gross et al. (2019), without making the solution infeasible, but considering that this is a less desired option. Furthermore, there are some constraints to take into account when creating the solutions. These can be either hard or soft constraints.

Hard Constraints:

- If a physician was assigned to a night shift, she/he cannot be assigned to the morning, prolongation or night shift the next day.
- If a physician is away, for instance on vacation or participating in a congress, she/he cannot be allocated to a shift.
- It is not possible to assign two consecutive weekend days, since on weekends there are only 24-hour shifts.
- The minimum coverage limit considered for the number of physicians in each shift must be met.

Soft Constraints:

- There is the possibility to assign the same physician to more than one shift in a day, as long as they are not simultaneous. In other words, someone assigned to a morning shift can be assigned to the night shift in the same day, which is equivalent to a 24-hour shift. Nonetheless, this is a condition less desired by the majority, so this occurrence should be penalized.
- Two consecutive prolongation shifts may be attributed, however it is not recommended, because it is a 12 hour shift.
- Shifts may not be filled in an ideal way due to the lack of physicians, as long as the minimum limit is met.

In addition to the input data mentioned above, in this case there was still the opportunity to receive information regarding the previous months' history, such as the number of hours worked by each physician, the number of holidays and the type of shifts worked. This may allow physicians who were affected in the previous months to be compensated now, through a accumulated penalty score for each physician, similar to the one used by Wolbeck et al. (2020).

In this case, the DM is the person currently responsible for the schedule. This person already knows the needs of the physicians and has experience in the task of decision making and therefore, will be faced with different alternatives in order to understand what the option would be taken in certain situations. However, all opinions of each physician are taken into consideration in order to find solutions that appeal to a greater number of people.

In the following sections, the main characteristics considered for the operation of the algorithm will be presented, from operators such as mutation and crossover, to the creation of the initial population and the fitness functions. The results of a survey completed by some physicians will also be presented.

Chromosome

The chromosome is represented as described in Figure 4.1. Initially, a matrix of dimensions equal to the number of physicians and the days to be scheduled is created. It is necessary to bear in mind the possibility of having external physicians, at times, doing specific shifts. In these cases, the number of physicians varies and, therefore, the chromosome size is also variable. Typically, they only are assigned to night and weekend shifts – those that are less desired and therefore more difficult to fill. There may be more than one extra physician per month, however it is not advisable for them to take shifts at the same time and this excludes the option of selecting more than one availability for the same day, at least for the same shift. If two indicate the same availability, the one that was offered first is chosen, so these options are not included in the algorithm. Once these choices have been analyzed, they can be immediately inserted in the last rows of the chromosome with the shifts they are willing to do in the hospital, and these cannot be changed from here. In addition, on the days they are scheduled, the needs are reduced accordingly. This is the ICU chromosome. Its variable characteristics will allow to generate different individuals that will integrate the initial population, as will be described below.

Initial Population

ICU's of the chromosome has been described, now the variable characteristics that allow to create different chromosomes at random, increasing the diversity, are presented.

The needs are distributed for each shift: for example, there are needs linked to the morning, prolongation, night and weekend shifts and they are the same every week. However, the type of shifts used on weekdays and weekends or National holidays are different, although these last two are modelled in the same way. Unfortunately, the needs at night and in weekends or holidays cannot always be completely fulfilled so, in this case, there is a lower and upper limit so that the hospital does not get into a critical situation. Commonly, the ideal is to have n physicians; a not so ideal but feasible solution is $n-1$. A greater difference makes the solution infeasible. For these cases, during the chromosome construction of the initial population, the number of physicians who will be part of these shifts is randomly chosen. Subsequently, all this will be considered in order to adjust it in relation to the number of hours worked, since it is not acceptable to satisfy all shifts ideally and allow, for example, a physician to work 300 hours in a month to make this happen.

In case that the day being scheduled is a weekend or a National holiday – given that they behave the same way –, the algorithm first checks if there are external physicians on a weekend shift that day. If this is the case, the needs in each of the limits are subtracted by one, in other words, the ideal limit becomes $n-1$ physicians and the minimum is $n-2$ physicians. Consequently, it is chosen randomly if the shift is filled with the ideal number of physicians ($n_w - 1$, w index, because one is evaluating weekends) or the minimum number ($n_w - 2$). Once the number of physicians in the shift (ideal or minimum) has been chosen, one can choose this amount from the available physicians. The selection of available physicians is also made randomly. If there are no external physicians on this weekend, there should be chosen n_w or $n_w - 1$ available physicians.

On the other hand, if the day to be scheduled is a weekday and there are external physicians available, the first one identifies the shift he/she is willing to do. Looking firstly at the case where he/she is assigned to a night shift, it is already known that the needs of this shift will be reduced. First, n_p available physicians are chosen to fulfill the needs of the prolongation shift; once they do, they must be removed from the available physicians list.

From the remaining physicians n_m is chosen for the morning shift and $n_n - 1$ or $n_n - 2$ for the night shift. In that event, the same physicians cannot be selected; if so, a 24-hour shift must be assigned and removed from the morning and night shifts. If the external physician's shift is a 24-hour (which is equivalent to a morning and night shift), the process is the same, only with both night and morning needs reduced by one.

Finally, if no external physicians are available on this day of the week, the procedure is the same but with increased needs. This is summarized in Algorithm A.4. Once the needs are satisfied, considering that the objective is to obtain feasible solutions, the chromosome goes through a repair process. In this step, it is checked if there are any consecutive nights, 24-hour shifts or weekend shifts, and if a person who takes one of these shifts does not have a morning or prolongation shift the next day, as described in Algorithm A.5. If any of these alternatives occurs, the algorithm runs a repair by assigning the inappropriate shift to another physician, who has the necessary conditions to respect the restrictions. In particular, whether it is a night, 24-hour or weekend shift, the person chosen cannot have any of the shifts the next day, that is he/she must have a day-off. For the previous day they may have morning or prolongation shift, but none of the others.

Another important aspect to prevent another hard constraint from being infringed, is again to be aware of the shifts to which the physicians were assigned on the last day of the previous month. For example, in order not to run the risk of assigning a morning or prolongation shift to someone who was assigned to a night or 24-hour shift on that last day.

The way the chromosome is designed allows to avoid one of the problems that have occurred over time in hand-made scheduling, where sometimes the same physician was assigned to simultaneous shifts (see Figure 2.1, physician P9). In this case, each entry of the chromosome corresponds to one physician and one day and there is only space for one shift.

Mutation

The mutation occurs as described in the previous section with slight adjustments, since there are situations that do not make sense to be changed. The following situations can be highlighted: (i) a physician who can only do a specific shift on a specific day, should not receive other shifts, and this includes the external physicians; (ii) for a physician who is on vacation, exchanging holidays with a shift is not justified. Therefore, if any of these alternatives arise, new entries are randomly generated.

Fitness Function Fairness

One of the objectives is to minimize the penalties resulting from fairness deviations in the chromosome under analysis. The definition of fairness is difficult, because it can depend of several circumstances – individual perception, time horizon, the factors under analysis, etc. –, varying greatly. So it must be evaluated carefully. Note that, at first, one can consider a fair solution two physicians working the same 160 hours in a month. However, looking at the schedule in detail, while one was assigned to only one weekend, the other worked three weekends – the solution is no longer as fair as it seemed. Similarly, working one night or a 24-hour shift is not equivalent to working a morning shift. That is why, besides the number of hours worked, it is also necessary to compare the types of shifts and the days on which they happen, only then more fair solutions will be achieved. It was noticed through conversations with DM and one of the ICU physicians that the degree of hardship increases as follows: night, then 24-hour and then weekend or national holidays shifts. All of this must be weighted and, in this way, a degree of hardship similar to the used in remuneration will be applied. The nights are worth 50% more and the 24-hour shifts 75% more, while the weekends and holidays are worth 100% more. These penalty costs were also agreed by the physicians.

Additionally, proportions must be considered since a physician can be on vacation during a part of the current month, and, consequently, work less hours. This should be adjusted so that he/she is not forced to work more hours in the following month. The same happens in cases where physicians are doing internships, as they have the right to do less hours. In order not to ignore it, because it would also influence fitness levels in the wrong way, a proportion is made between the total number of absent days (vacation, weekends, etc) and the total worked days. This is estimated in the same way as it is in the ICU for those cases where past values are used, in order to maintain consistency. In particular, they only count working days and multiply each of those by 9 hours.

Once these considerations have been measured, the fitness function can be determined. For this purpose, the number of hours that each physician worked over the month is calculated. Next, the objective is to compare each physician with the average number of hours worked in the month under analysis. This comparison is the difference in absolute value of the average number of hours worked and of the number of hours actually worked by each physician. Then, the sum of all differences is the fitness value of the solution under analysis. The objective function minimizes this value, in order to have the smallest deviations and, consequently, a fairer work-balance.

With respect to the number of weekend days the physician works, the number of days-off they have and the nights they were assigned, the reasoning is similar to the one described previously. The total number of days worked over all physician are compared against each other. The greater the sum resulting from all the differences, the less fair is the schedule and, as such, the less fit the solution is.

Since it will not always be possible for everyone to work exactly the same number of hours, and having access to information related to previous months, a cumulative scoring system was created. These scores provide an overview of the physicians most affected over time, through the evaluation of the following parameters: total number of hours worked and number of nights, weekends and 24-hour shifts completed. These scores are assigned to each physician as follows: first they are organized in descending order in relation to each of the parameters. A score equal to the position of the physician in the ordered list is given, that is, the score of number of physicians is assigned to the first one on the list, that number minus one to the second on the list, and so on. The procedure is repeated for each of the parameters, however, as previously mentioned, the penalty is different for each of the shifts and this is also reflected in the scores. In the night shifts, the assigned score is multiplied by 1.50, since it is worth 50% more than others, while the weekend and 24-hour shifts are multiplied by 2 and 1.75. This system is cumulative, because it allows the use of the previous data and thus achieves a more balanced and fair score. Its use is expected to increase the long-term balance result, trying to decrease the score of physicians who have scored higher in the past. In addition to compensating those who were previously affected, the goal is also that the difference in scores between physicians is as small as possible.

Table 4.2 presents an example of how each physician's score is found. There are five physicians, therefore the score attributed to the less benefited physician is five, represented in dark blue, and this is reduced by one until the least affected, shown in light blue, who has a score of 1. If two physicians have the same value in any of the parameters, they will have the same rank and the same score which is the biggest value between the two or more scores is considered for all of those tied; the remaining physicians keep the rank they would have if there were no repeated elements. The results of the table also confirm something previously described, namely that the physician with the biggest number of hours

Table 4.2: Example of how each physician's score is found.

	Number of hours	Score	Number of night shifts	Score	Number of 24-hour shifts	Score	Number of weekend shifts	Score	Total Score
P0	160	3	3	4 x 1.5 = 6	6	5 x 1.75 = 8.75	2	2 x 2 = 4	21.75
P1	180	5	0	1 x 1.5 = 1.5	4	3 x 1.75 = 5.25	2	2 x 2 = 4	15.75
P2	145	1	4	5 x 1.5 = 7.5	5	4 x 1.75 = 7	1	1 x 2 = 2	17.5
P3	176	4	1	2 x 1.5 = 3	1	1 x 1.75 = 1.75	2	2 x 2 = 4	12.75
P4	156	2	2	3x 1.5 = 4.5	3	2 x 1.75 = 3.5	3	5 x 2 = 10	20

is not necessarily the one with the worst score performance. In other words, it is not the one that has a more unfair schedule according to the scoring system proposed.

The algorithm is equipped with the option to use historical information, so the score can be calculated in three different scenarios: (i) without considering any information from the past, that is, the score exactly matches the data for the month in question; (ii) considering only the information related to the previous month, so the score will be the result of the sum of the current month's score and the previous month's score; and finally (iii) considering the previous information in full, so the score will be the result of all previous scores plus the current score. In fact, the historical record can be crucial during the balance of the scores, mainly due to the lack of staff that makes harder the process of finding a fair number of hours, since overtime work is more frequent. This allows to compensate physicians who were less benefit in the previous month, increasing fairness.

Finally, it is assessed whether any of the physicians works more than 220 hours per month. If this occurs, a penalty is also attributed according to the number of hours worked above that level (see Table 4.3). The ideal number defined by the ICU is an average of 180 hours per month, which is equivalent to an average of 9 hours a day during the working days of the month. Up to 220 hours does not exceed 11 hours a day, which is still considered admissible in specific situations. From that value, it is considered to have a negative impact due to the wear and tear it causes on employees, besides starting to be much more expensive for the hospital due to the excess of overtime. The total number of hours worked during the month is also averaged and if it exceeds 180 hours (which is the equivalent of working nine hours over twenty working days, which happens on average over a month) the chromosome is also penalized according to the deviation from this value (see Table 4.3).

Fitness Function Preferences

The other objective tries to meet the individual preferences of each physician as much as possible. In conversation with a physician from the service and the DM, it was possible to identify some of those preferences, some more personal and others general, that is, of interest to all physicians.

First, it is ensured that all the hard constraints, such as shifts that physicians cannot work on specific days, are met. This is necessary since they are only guaranteed in the initial population, but not during the crossover or the mutation. The same happens to all the constraints related to consecutive shifts:

a bad example in this case would be assigning two consecutive weekend shifts to the same physician. If a situation like this happens, a high penalty cost is attributed, which will probably make the solution less fit to the algorithm, perhaps ending up being eliminated. The penalty cost assigned is 3000 for each violated constraint. Although it is possible to have two consecutive prolongation shifts, it is not recommended, thus, the appearance of this case is be penalized as well (see Table 4.3).

Despite the fact that sometimes shifts are not ideally completed, due to the lack of physicians, it is in their interest that this is accomplished whenever possible. Two types of penalties are considered, one severe and the other lenient. The latter is used on any occasion the ideal number is not reached but the minimum limit is not exceeded. If the minimum number of employees is not reached, a serious penalty, much higher than the previous one, is attributed (see Table 4.3). However, in this case it is necessary to pay attention to one more aspect. As already mentioned, situations of understaffing overload those who are present and can lead to decreasing the quality of service provided and situations of exhaustion, which consequently can result in sick leaves. This was an important criterion identified during the conversations with the DM and it is therefore penalized whenever it is reached. Although it is important to fill all shifts ideally, physicians cannot be overburdened with too many overtime hours, as avoiding exhaustion on the one hand can create it on the other. This is compensated by penalizing the average number of hours worked in the other fitness function (see Table 4.3), described above.

Another aspect evaluated is the preferences in relation to the team allocated to each shift, especially in less preferable shifts, as is the case of nights and weekends. In other words, there are physicians with more affinity, in terms of level of experience, who should be together whenever possible. For this they are grouped in pairs, where a physician can be in more than one pair. The pairs considered in the ICU are: (P0, P7), (P1, P4), (P8, P3), (P6, P5), (P13, P1), (P13, P5), (P9, P10), (P11, P12), (P14, P5), (P14, P6). For example, an ideal weekend shift would be the one that had physician P5, P6 and P14 simultaneously. If this does not happen, a cost penalty of 1 is assigned, which is incremented each time they are not together on a weekend or night shift.

Finally, additional individual preferences are evaluated, namely, physician P2 does not want to be assigned to night shifts, since he/she is responsible for some administrative matters, and P4 prefers to be assigned to 24-hour shifts instead of isolated nights and mornings. The penalty is awarded according to the number of unsatisfied preferences. For example, for the first case, the number of night shifts assigned to physician P2 are counted and for the second, it is verified whether the number of nights is greater than that of 24-hour shifts and the penalty cost is equal to the difference. In addition, another preference identified and not used in the service is to maximize the number of days-off after a 24-hour shift. In this case, the penalty cost is higher, the lower is the average number of days-off. In particular, if the average is over 5 days-off after a 24-hour shift there is no penalty cost, if this value is between 4 and 5 the penalty cost is equal to the average, if it is between 3 and 4 it will be the average times 4. Between 2 and 3 is the average times 6, if the average is less than 2, this average is times 8 (see Table 4.3).

Table 4.3 compiles the main components used to calculate each fitness value. The penalty costs

Table 4.3: Fitness Function Components and respective cost penalties.

	Fitness Functions Components	Evaluation Process	Goal	Penalty Cost
Fitness 1	Number of hours	Difference in absolute value between the hours worked by each physician and the average hours worked. Sum of the differences.	Minimize the sum of differences.	Equal to the result of the sum.
	Number of 24-hour shifts	Difference in absolute value between the number of 24-hour shifts worked by each physician and the average. Sum of the differences.	Minimize the sum of differences.	Equal to the result of the sum x 1.75
	Number of weekends	Difference in absolute value between the number of weekends shifts worked by each physician and the average. Sum of the differences.	Minimize the sum of differences.	Equal to the result of the sum x 2
	Number of nights	Differences in absolute value between the number of nights worked by each physician and the average. Sum of the differences.	Minimize the sum of differences.	Equal to the result of the sum x 1.5
	Cumulative Penalty Score	Difference in absolute value between each individual score and the average of scores. Sum of differences.	Minimize the sum of differences.	Equal to the result of the sum.
	Average hours worked	Check if the average hours worked in the month is greater than 180 hours.	Minimize the average hours worked.	(Average hours worked - 180) x 50
	Maximum hours worked	Check if any physician worked more than 220 hours.	Do not overload a physician too much.	(Hours worked - 180) x 20
Fitness 2	Hard Constraints violated	If any of the constraints are being violated.	Do not have infeasible solutions.	Number of constraints violated x 3000
	Work consecutive prolongation shift	Count the number of times that there are consecutive prolongation shifts for the same physician.	Avoid assigning two consecutive prolongations shifts.	Equal to the number of consecutive prolongation shifts
	Filling shifts ideally	If the number is equal to the ideal number minus one Otherwise, if the number is equal to the ideal number minus two or more	Maximize the number of shifts filled ideally.	Number of times this is filled in this way Number of times this is filled in this way x 1000
	Physician Pairs	Count the number of shifts in which there are no satisfied pairs.	Maximize the number of satisfied pairs.	Equal to the number of pairs not satisfied.
	Individual Preferences	Confirm that preferences are being met.	Maximize the number of individual preferences satisfied.	Equal to the number of preferences not satisfied.
	Average number of days-off after a 24-hour shift	If the average is between 4 and 5 If the average is between 3 and 4 If the average is between 2 and 3 If the average is lower than 2	Maximize the average number of days-off after a 24 - hour shift	Equal to the average. Equal to average x 4 Equal to average x 6 Equal to average x 8

presented resulted from a fine tuning in order to obtain the most balanced solution possible according to the identified objectives. This resulted not only from conversations with physicians but also from successive attempts, where it was realized that some aspects weighted more than others. For example, initially it was considered more important to fill shifts ideally than the number of hours, however, it happened that the average hours became very high and this makes the solution less acceptable. The violation of hard constraints is something rare, however, it can happen, and this should be highly penalized in order to discard it as much as possible. It was also realized that, for most people, working a weekend is worst than working a night and even more than a prolongation, as opposed to what was considered in the earliest versions of the algorithm, where these shifts were treated equally.

In the end, the objective is to minimize both fitness function 1, related to fairness, and fitness function 2, which mainly evaluates preferences. Although this was created independently considering the literature and based on some conversations with one of the physicians, the best solution will be chosen according to a study conducted to most of the physicians, as described below. This had to be the procedure since the access to the hospital was restricted and therefore it was more difficult to collect information from healthcare professionals due to the pandemic we lived through.

Survey Results

The approval of the schedule by most of the physicians is essential to improve the decision quality, the accuracy and the degree of acceptance (Robbins & Langton 2004). In this way, a study was carried out through a survey. This had as main goal to understand which criteria gives rise to a good solution and the degree of importance for each one of them, in order to assign them the right weights or, at least, to rank them accordingly. Different situations were exposed, two by two, so that the physician would choose the one he/she preferred. There was also the possibility of being indifferent to a given situation. There is at least one question that confronts two criteria. Most of the questions required direct and multiple-choice answers; there were only 2 open-ended but short and non-mandatory questions. The survey was sent to all physicians in the ICU and eight responses were obtained, which means more than half of the ICU's physicians answered the survey.

In collaboration with the DM, the most important criteria were identified. These will be the focus of this analysis. This identification was achieved through the analysis of different solutions, where improvements requested were mainly related to these criteria. A common ground in the opinions from all contributors was that a good schedule should balance the number of hours among physicians but also the number of weekends. These criteria are assessed by the sum of deviations of each physician in relation to the average. In addition, it is also considered the cumulative scoring system as an important criterion, since it contributes to a fairer schedule. The difference in the number of nights or 24-hour shifts is not considered as there are some physicians who prefer to do them at expense of others, so although this may contribute to a fairer schedule it would at the same time promote dissatisfaction and its consideration could bring redundancy. Thus, these can be incorporated as preferences. They are a quantitative criterion: to determine these values, the number of satisfied preferences during the month for each physician, the number of days-off after a shift that lasts at least 24 hours and the satisfaction of the pairs of physicians are evaluated. In addition, the ideally filling of shifts was found to be important, because it helps to avoid the overburden of physicians and it increases satisfaction levels. In short, the determined criteria were: (i) the difference in the number of hours worked between physicians; (ii) the difference in the number of weekends worked between physicians; (iii) the compensation of the less benefited physicians over time (assessed using cumulative scores); (iv) preferences; and (v) shifts ideally filled.

The first question asked directly was which of the two options best described this situation. 62.50% of survey respondents consider that a good schedule should meet the hospital's needs, satisfy the maximum number of preferences, and compensate all the physicians who were affected in the previous month. None considered it essential for everyone to work the same number of weekends. Figure 4.6 shows these results. As expected, 16 responses were obtained, since each of the 8 physicians was asked to choose two options. From here it can be seen that the most important characteristics are the hospital's needs, preferences, and fairness concerning the physicians who are most affected in previous months. This meets the defined objectives for fitness functions.

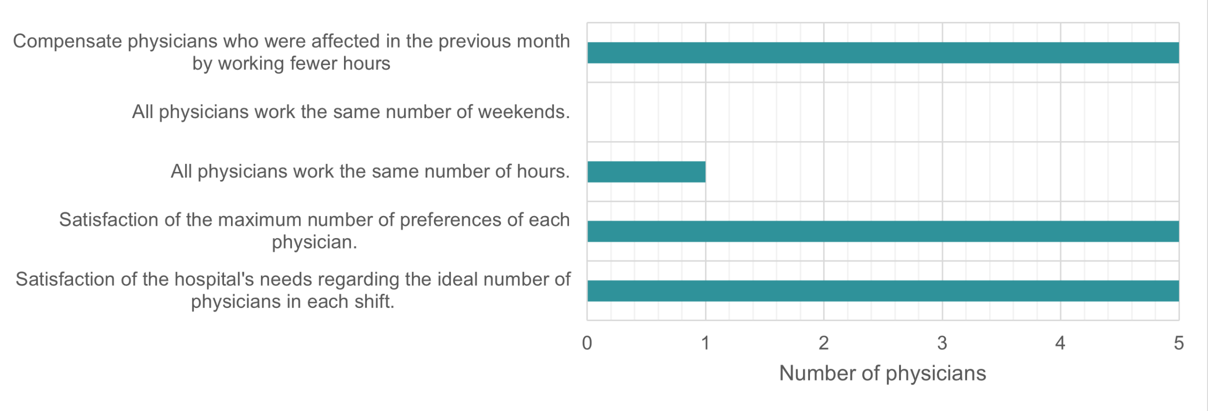
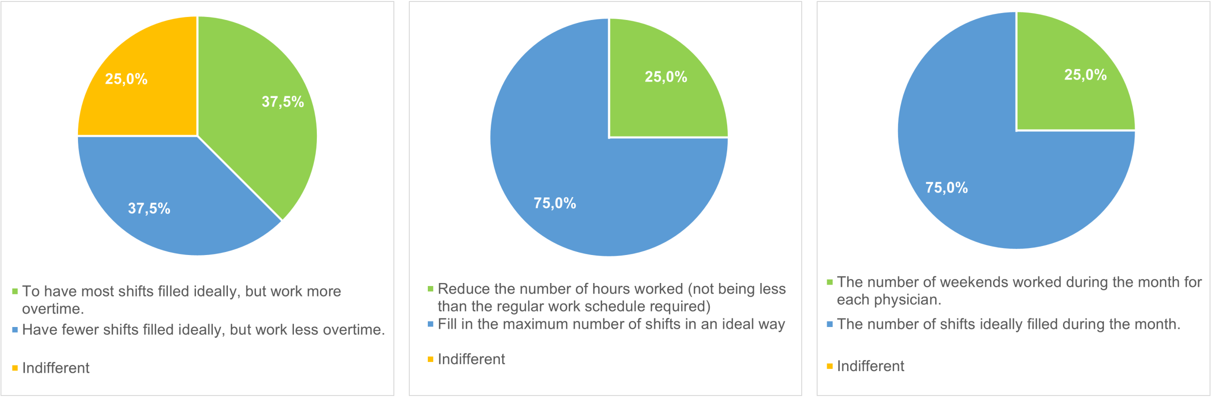


Figure 4.6: Survey' first question results. The respondents selected the two options that best reflected a good schedule.

Another question described a hypothetical situation: ”One of physicians P1 or P2 has to work a certain shift. Physician P1 wants the day-off because he/she has an appointment. However, it is physician P2's day-off and he/she had none of his preferences recently met. The algorithm chooses in favor of physician P2 to ensure that days-off are evenly distributed among all”. Each physician had to choose whether he/she agrees with the option decided by the algorithm or not. The purpose of this question was to evaluate the perception of fairness, checking whether all physicians should have the same benefits, being then more important to compensate a physician who has been affected, or whether priority should be given to occasional preferences and needs. 62.50% agreed with the option taken by the system, while the remaining 37.50% did not agree. The purpose of this question was to evaluate the perception of fairness, checking whether all physicians have the same benefits, or whether priority should be given to occasional preferences. The results show that the preferred solution is one that treats all physicians the same, always trying to choose the fairest option.

One of the questions was intended to perceive if physicians are more interested in working overtime to get more shifts ideally or, otherwise, working the normal schedule but having most shifts filled with the minimum required number. In this case, the number of answers obtained in the first option is equal to the number of answers in the second, with 3 responses in each (see Figure 4.7a). One could conclude both options are equally important but similar to this question, another one was raised to reinforce this idea. This one had as options: (i) ”Reduce the number of hours worked (not being less than the regular work schedule required)” and (ii) ”Fill in the maximum number of shifts in an ideal way”. 75% chose the second option, while the remaining 25% chose the first (see Figure 4.7b). This indicates that the number of hours is less important than the shifts ideally filled. Note that, in this case, the difference in hours worked between physicians is not evaluated, but rather the availability to work overtime. This is important because in the fitness function assessment it was considered that they would be available to work overtime, as long as the average additional time among physicians is below 40 hours per month. In fact, most are available to work overtime, and not only in weekdays. Considering the same proposal for weekends, 75% of the respondents showed a preference for shifts ideally filled throughout the month, while the remaining 25% considered the number of weekends worked more important (see Figure 4.7c).

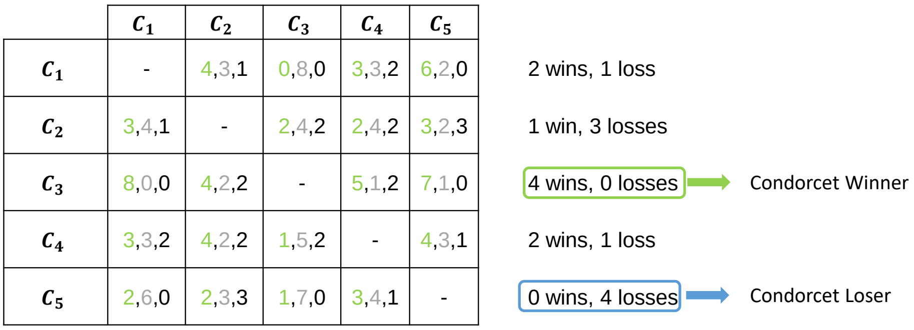


(a) (b) (c)

Figure 4.7: (a) and (b) Shifts ideally filled and overtime; (c) Weekends and shifts ideally filled.

In the remaining questions, two alternatives were given (plus the indifferent option) so that the preferred option was chosen. Through the choices it is possible to evaluate the criteria and understand the degree of importance of each one concerning the others. The questions and answer percentages can be found in Table A.1. For this purpose, the Condorcet method will be used to adequately assess this pairwise comparison. Through Condorcet ranking one can establish an order of candidates where the Condorcet winner (if it exists) comes first and the Condorcet loser (if it exists) comes last. A choice that is preferred in every pairwise confrontation will be the winner, in particular the Condorcet Winner. On the other hand, a criterion that is not preferred over any of the others is a Condorcet loser.

Figure 4.8 represents the preferences matrix regarding pairwise confrontation. This shows an election with 8 voters. There are three numbers in each cell according to the vote options (For, Against, and Neutral), and these are always related to the candidate on the left column when confronted with the candidate in the top row. To illustrate it better, four physicians preferred C_1 over C_2, three voters preferred C_2 over C_1, and one physician had no preference between the two. Note that C_x represents the different criterion: C_1 is the absolute value difference in the number of hours worked between physicians; C_2 is the absolute value difference of weekends number; C_3 is the absolute value difference of cumulative scores; C_4 is related to preferences; and C_5 to the number of shifts ideally filled. From the very beginning it is possible to identify the Condorcet winner and loser, identified in Figure 4.8. Criterion C_3 was always



	C_1	C_2	C_3	C_4	C_5		
C_1	-	4,3,1	0,8,0	3,3,2	6,2,0	2 wins, 1 loss	
C_2	3,4,1	-	2,4,2	2,4,2	3,2,3	1 win, 3 losses	
C_3	8,0,0	4,2,2	-	5,1,2	7,1,0	4 wins, 0 losses	Condorcet Winner
C_4	3,3,2	4,2,2	1,5,2	-	4,3,1	2 wins, 1 loss	
C_5	2,6,0	2,3,3	1,7,0	3,4,1	-	0 wins, 4 losses	Condorcet Loser

Legend: For, Against, Neutral

Figure 4.8: Preference matrix with 8 voters, highlighting the Condorcet Winner and Condorcet Loser.

chosen over others, so it is the preferred among all confrontations, having won them all. On the other hand, criterion C_5 is never chosen in relation to others, having lost all confrontations. Therefore, the first and last places in the ranking are already known. There is a tie between criteria C_1 and C_4. In this way, the rank, which satisfied the greatest number of physicians, obtained through the survey is: 1^{st} balance between physicians (C_3), 2^{nd} difference in absolute value between hours worked and preferences (C_1 and C_4), 3^{rd} difference in absolute value between weekends worked (C_2) and lastly the filling of shifts ideally (C_5).

Once the rank has been determined, it is possible to assign the weights according to the relative importance of each criterion. In this case, the approach applied is one of the *ranking methods*, in particular the rank sum. In this approach weights are computed from the individual ranks. The normalized weight can be expressed as:

$$w_j = \frac{n - p_j + 1}{\sum_{k=1}^{n} n - p_k + 1} \tag{4.4}$$

where n is the number of criteria ($n = 5$) and p_j is the rank of the $j - th$ criterion (Odu 2019). Table 4.4 provides the weights obtained according to the ranking defined through the Condorcet method.

One of the disadvantages of using online surveys is people's perception, as options can be interpreted differently by each respondent. It is possible to verify that the answers to the first question go against the ranking established at the end. The following considerations can be the reason for this inconsistency: in the first question the physicians looked in a general way, so they considered the fact that some physicians have different work contracts; in this way it would not be fair to give them the same number of weekends or number of hours, at least not in the way the question was asked. The physicians' perception of ideal needs was also not interpreted in the way intended, although the vision was given at the beginning of the survey. Naturally, the hospital's needs must be guaranteed for the proper operation of the hospital and the quality of the services provided, but there is the possibility to guarantee the minimum limit when the ideal is not possible. On the other hand, the last questions seem to reflect personal preferences more, as they were made in a way physicians would be confronted with their own choices. Therefore, this last ranking can be considered more adequate and closer to everyone's preferences.

These results demonstrate that, although the fitness functions were defined before the survey results, the options taken are in accordance with those preferred by physicians. This shows that the attempt to build an independent model, based on facts from the literature, has proved effective and successful.

Table 4.4: Computing the weights using the Ranking method.

	Rank	Weight (n-p_j+1)	Normalized Weight
C_1	3	3	0.21
C_2	4	2	0.15
C_3	1	5	0.36
C_4	3	3	0.21
C_5	5	1	0.07
	Sum	14	1

Once the criteria are ranked and computed, one can choose the best solution. For this purpose, it will be used TOPSIS method as described below.

TOPSIS method

The purpose of TOPSIS is to support the choice of the best solution among a set of non-dominated solutions. In each generation, all non-dominated solutions are kept. In the end, they all come together, and a non-dominated sorting is performed again, to exclude those that are no longer part of the first Pareto-front and to exclude the repeated chromosomes, because over the generations the best ones can be perpetuated without changes, due to elitism property.

Afterwards, it is necessary to define criteria and assign the respective weights, as already described previously. In addition to the weights of each criterion, it is necessary to know the type of criterion, that is, whether it is a cost or benefit criterion.

Concerning the differences in absolute value between the number of hours worked, the number of weekends worked among physicians, and the differences in absolute value between the cumulative scores, the objective is to minimize the value of these criteria. Hence, they are cost criteria. On the other hand, preferences can be classified as a benefit criterion: the more preferences are fulfilled in general, the better the plan and the more satisfied the physicians will be. Regarding the ideally filling, in the decision matrix this is evaluated with the quantity of shifts ideally filled, the higher the better. It is also a quantitative and a benefit criterion.

The first step is to create a decision matrix with m alternatives (each one of the best solutions) and n criteria. Each row gives the value for each criterion in the solution under analysis. Once the decision matrix is constructed it is possible to follow the next steps (described in Section 4.2.2), and thus obtain the best solution according to the established criteria.

As a consequence, the final goal is achieved, which is to present the best solution according to the features that physicians value the most. Next, the methodology used to reach the same goal in the case of IS will be described.

4.2.3 Imagery Service: notes and assumptions

In this section the application of the algorithm to the IS is discussed, describing the main features and operators.

The IS's scheduling horizon spans over an entire calendar month, starting on the first day and ending on the last one. On each day and for each shift there are specific needs that should, whenever possible, be met. The shifts are divided into four classes: morning, afternoon, all-day and night. Some shifts can be part of more than one class. For example, the MT21 shift (see Table 2.2) is considered both a morning and afternoon shift by the IS. This is a shift used in cases where there is no one else free to fill in a morning or an afternoon, i.e. one chooses someone who was already assigned to morning or afternoon and assign them what is missing, and as such it does not fill all-day needs. N (see table 2.2) is the only night shift that exists and is usually only used in group E2. The S and I10 shifts are the most used to meet the needs of the all day shift class. The rest are morning or afternoon categorized, depending on the time it starts.

The needs, which are referred to as week needs, are given for each day of the week for all different shifts categories (morning, afternoon, all-day and night). Usually, week needs are the same for the whole month, with slight differences in cases where equipment is under maintenance. Weekends have fewer needs than weekdays. The fulfillment of these needs is considered a hard constraint, so they must be filled out. If for some reason the available technicians cannot meet the required week needs, there is the possibility of bringing outside technicians to cover these gaps, however this is a more expensive option.

Other constraints to take into account when building solutions are:

Hard Constraints:

- A technician cannot be assigned to more than one shift within a day.
- If a technician is in breastfeeding period she must take shifts of 6 hours.
- The hospital needs should be satisfied.
- According to legislation, a technician cannot work more than 6 consecutive days.
- If a technician was assigned to a night shift, he/she cannot be assigned to a morning or an afternoon shift in the next day.

Soft Constraints:

- The technician should not work more than 10 consecutive hours.
- The rest time between two shifts should be 11 hours.
- The most part of technicians have a certain functional area or skills.
- Sometimes the equipment has scheduled maintenance, so it is not necessary to allocate technicians to them, the number of technicians maybe lower.
- On their birthday they are given a day-off.
- Ideally, the technician should just work a day in a weekend during the entire month.

Chromosome

The HL's IS consists of two groups, the central group and the Emergency group, which is subdivided into two others, E1 and E2. Although the chromosome design followed is similar to the one depicted in the Figure 4.1, each of these groups has specific characteristics and needs. Thus, in an initial population, three partial chromosomes were created in an isolated way in order to try to meet the intrinsic specificities. Then these will all be together in just one chromosome.

Initial Population

The partial chromosomes are created considering the base rotation designed by the service. In particular, it is defined that a technician of the central group should, whenever possible, do the morning shift (M8, see Table 2.2) for one week, an afternoon shift (T21, see Table 2.2) all days the next week and so on. In this way, the chromosome of the central group is created initially filling all weekends and national holidays with zeros, which corresponds to days-off. Afterwards, the shift is randomly chosen for the first week for each technician, and from there it is alternately filled. Here, some characteristics are taken into account, namely:

- If a technician is breastfeeding, the M8 and T21 shifts are replaced by M6 and T6 shifts, that have two hours less than the previous ones. In addition, these technicians cannot be assigned to weekends.
- Every Tuesday, except for national holidays, technician T1 must have the I8 shift. This shift is used to solve administrative matters. So, it does not count for the purposes of satisfying the needs of that day, but it does as working hours.
- Technician T1 and T9 also handle administrative matters together. The first working day of the month is destined for that, but if this day is a Tuesday, it will be moved to the following Wednesday. In addition, these two technicians must have two days reserved for I8 shifts between 21st and the 25th of the month, with the condition that it cannot be a weekend or a Tuesday. In this case, the days that fit these conditions are found and two are chosen at random.
- Every day of the week three A8 shifts are needed (which is also a morning shift but starts earlier), as initially the algorithm is assigned only with the M8 shift, three of these will be replaced.
- Each weekend day a technician is inserted at random, as long as he/she is not unavailable.

Regarding the E1 group, the rotation pursued for each technician is: M8-M8-P0-P1 (see Table 2.2) followed by two days-off. The idea is that one technician starts on day one with the first M8 shift and follows the rotation, the next technician will start the shift one (first M8) on day two and so on. Since this group contains six technicians, this allows to have always in each day the four shifts required for this group in order to satisfy the hospital needs. After some conversations with the DM, it was realized that the purpose is to have certain shifts covered every day and not necessarily the rotation. In this case, the idea is to ensure that every day there are two technicians doing the M8 shifts, one in P0 shift and the other in P1 shift, with slight changes on the weekend. If M8 shifts lie in a weekend, they are replaced on Saturdays by a S shift and on Sundays by a M6 shift, in order to compensate the two extra hours worked on Saturday (since the S is a 10 hours shift while M6 is a 6 hours shift). In addition, the P0 and

P1 shifts are going to be replaced. Basically, at weekends, the goal is having two technicians assigned to a S shift, one to a morning shift and the other to a P shift. This was thought of this way since shifts P0, P1 and P end after 1 a.m., therefore cannot be followed by M8 shifts, because in this situation the technician does not have the 11 hours of rest. In addition, whenever possible, two consecutive days-off should be given in order to increase job satisfaction.

These shifts are only assigned to technicians of group E1 and as such if someone is absent or on vacation it is by another technician of this group that the shift should be ensured. In this sense, it is necessary to take some restrictions into account:

- If the technician's absent shift was a P0, P1 or P shift, one will first try to assign it to someone who has a free day and in the next day is not assigned to a M8, M6 or S shift. Otherwise, one of the constraints is being violated.
- If the technician's absent shift was an S shift, a technician with a day-off, who also has a day-off the previous day, is chosen.
- If the shift is an M8 or an M6 shift, a technician with a free day that in the previous day was not assigned with a P0, P1 or P shift, is chosen.

Finally, for the E2 group, in order to satisfy the hospital needs it is necessary to have one person per day in the M8 shift, two in the T8 shift and one assigned to a night shift (N). On weekends it is necessary to have technicians assigned to the S shift - usually the T8 shifts are the ones replaced.

To guarantee the success of the schedule, it is also necessary to know the shifts of each technician on the last day of the previous month to avoid starting this new month in an infeasible way. For instance, in the case of group E1, assigning an M8 or an M6 shift to a technician who on that last day was assigned to a P shift. The same can happen in the E2 group with night shifts.

In the end, all these three parts will come together and form a single chromosome. Once they are together, holidays or sick leave issues are inserted, the week needs are verified and, if necessary, they are adjusted. The detailed process is as follows (in order to simplify the text the main steps can be find in Algorithm A.6): for each day, the number of technicians in each class of shifts will be counted and will be compared to the week needs. If the number of mornings counted is less than the hospital's needs, several conditions will be analyzed. The first check performed is whether that day is both missing morning and afternoon shifts; in that case, the missing morning is inserted in some technician who is doing an afternoon shift - if for instance, the technician j was planned to do a T21 shift (an afternoon type shift) he/she is then assigned to a MT21 shift (morning and afternoon shift), so the afternoon needs do not change and one morning is added. On the other hand, if there are too many people doing afternoon shifts on that day, what is done is to change one of those afternoons and assign this technician a morning shift. If none of these conditions is sufficient to satisfy the needs and if the number of afternoons on that day is satisfied, the missing morning will be assigned to some technician who was on a day-off, as there is no chance that a planned worker for that day can absorb the hours left to attribute. Finally, if there is no one with a day-off, and there is also lack of afternoons, someone who was assigned to an afternoon will be assigned a morning shift, ending with a MT21 shift on that day.

The needs check process is done for the mornings, and the needs will then be checked for the afternoons. This part is analogous to the mornings check, only with less conditions: it is firstly verified if there are too many mornings to be filled, in which case a technician that was initially assigned to a morning is chosen at random, and assigned to an afternoon. In case a surplus of morning shifts is not available, someone who is assigned to a morning will randomly be assigned with an extra afternoon, i.e. a MT21 shift.

Regarding all-day shifts, there are two different cases, for weekends and weekdays. Concerning weekends, the all day shift is S (see Table 2.2), and given that an all-day is intended, it is necessary to pick one completely free technician. Initially it is checked if there are free technicians on the weekend day under analysis, and one of these technicians is chosen at random and is then assigned the S shift. Note that if there are no fully available technicians on that day, the needs for this type of shift will be left unsolved and the chromosome will be an infeasible solution, which will be penalized in its fitness. Considering the extension of the population and the number of generations that can be created, the probability of infeasible solutions reaching higher stages of the algorithmic process is low.

Finally, in order to obtain a greater number of feasible solutions in the initial population, a repair function is applied (see Algorithm A.7). This checks if any of the hard constraints have been violated and corrects them. For instance, if someone has shifts for more than six consecutive days, one of these days is chosen and a day-off is assigned. The shift that had been assigned that day passes to another technician. It also checks for consecutive shifts with less than 11 hours of rest between them, for example a T21 followed by an A8 or an N followed by M8. It is also repaired if one of the technicians does not have a day-off or an M6 shift for having worked a weekend shift, or in the case where there are no three A8 each day.

Crossover

As mentioned above, each of the IS groups has specific properties and certain shifts that are only assigned to them. Thus, the crossover represented in the Figure 4.2a had to undergo some changes in order not to exchange specific needs.

Regarding the central group, the crossover occurs as shown and described in Figure 4.2a, with slight changes. In the central group, columns are exchanged between parents while in the emergency group, rows are exchanged. In particular, consider that two columns (j_1 and j_2) are randomly chosen on the chromosome. From the first technician to the last one in the central group, the columns from the parent P_0 between day 1 and j_1 will be transferred to child C_0, between j_1 and j_2 will be transferred to child C_1 and the rest will be passed to child C_0 again. On the contrary, the columns from the other parent (P_1)

between day 1 and j_1, as well as between j_2 and the last day will be transferred to child C_1 and the rest will be passed to child C_0 again. Furthermore, the last rows, belonging to the Emergency Group will be exchanged between parent P_0 and parent P_1.

Mutation

The mutation occurs as described in Section 4.1.1 (see Figure 4.2b), however it has slight restrictions since there are cases that should not change from what has already been defined, so it makes no sense to allow mutation. These cases are highlighted below.

- The I8 shift is only assigned to technician T1 and sometimes to T9, so it makes no sense to change these shifts during the mutation;
- Technician T4 only does afternoons on Mondays and Wednesdays, thus he/she should not be assigned another shift, so the mutation here should also not occur.
- Breastfeeding technicians do not do weekends and only do M6 and T6 shifts, so these rows are also not chosen.
- On national holidays, the technicians of the central group do not work, so they must also not take a shift through mutation.

Fitness Function for Hospital Preferences

As in the case of the ICU, there is also a DM here that supports the definition of the fitness functions. In this case, the DM is one of the technicians responsible for building the schedule every month. Unfortunately, there was no opportunity to gather more opinions, as happened at the ICU. These would help to define the weights of the criteria, important to select the best option through the TOPSIS method, as was described later.

After some conversations with the DM, it was realized that there are two fundamental aspects when creating the schedule. The first is related to the needs and preferences of the hospital while the other to the individual preferences of each technician. In fact, initially it had been thought that one of the objectives would be fairness regarding the amount of hours worked, since the DM wanted everyone to work the same number of hours, as happens in ICU. However, this turned out not to be the case, because there are technicians who are more predisposed to work overtime than others, thus trying to balance the number of hours would lead to dissatisfaction in the staff. This is a clear example that the perception of fairness is relative and can be interpreted in different ways, because at ICU the goal was that everyone worked the same number of hours, without overloading others. In IS the goal is that everyone should work 40 hours and overtime is only assigned to those who have this interest. However, here will also be introduced the notion of fairness but regarding the differences between individual preferences.

Technicians can perform all types of exams, however, some of them are specialized and are more able and qualified to perform a certain set of exams according to their functional area. In the case of HL it is possible to identify at least six functional areas: angiography, cardiology, neurology, musculoskeletal system, mammography and X-ray exams. In this sense, it is in the hospital's interest to meet the needs in relation to each functional area. In addition, this can ensure a higher quality of service provided, thus increasing patient satisfaction. These needs are defined for each month and vary depending on the available physicians and possible equipment maintenance. In this way, this meets the hospital' preferences and as such, it is used to assess the fitness of each solution. On the other hand, the hospital also wants to avoid over-staffing, because, besides being an extra burden and increasing expenses, it also does not promote staff satisfaction. Despite this, it is necessary to ensure that each technician, with a full-time contract, works at least 40 hours per week, unless they are on vacation, on sick leave or during breastfeeding period.

Therefore, in order to calculate this fitness function, associated mainly with the preferences and needs of the hospital, one begins by checking and comparing the technicians that are available each day and the needs in relation to each functional area. Whenever the needs are not met, that is, when there are fewer technicians in a certain functional area than required, a penalty cost is assigned (see Table 4.5). There are specific days that require a greater need in relation to filling the functional area due to the performance of differentiated exams requested by physicians. This usually happens especially on Monday afternoons, Thursdays mornings and Fridays mornings, when some neurology technicians must be present. Thus, in these situations the absence of neurology technicians has a greater penalty cost.

The needs for each functional area vary between morning and afternoon shifts for each day of the week. However, during the weekend there is no specific need. Generally, the plan of the first week is the same as the third week and the second week is the same as the fourth week, with slight changes between them. In other words, needs are repeated every two weeks, with some exceptions, for example when equipment maintenance is scheduled.

First it is necessary to check the hospital needs concerning functional area and confirm if they are adjusted, in excess or missing. To do this, one starts by checking which week of the year is in scheduling and if it is even or odd. If the week is odd it means that the functional area needs to correspond to the first week and if it is even they correspond to the second week of functional needs. When there are more technicians in a certain functional area than would be needed, no penalty is given, since there are other needs that do not specify the functional area and therefore can be met by anyone. However, if there are more technicians in total than required, a penalty is incurred. On the other hand, it is not so detrimental to have these extra needs, since there are technicians who have to go to the other two clinics (namely, Oeiras Clinic and Amadora Clinic, as mentioned in Section 2.2.2) and then those will be the option. It should be noted that if the needs are not fully met, a comparatively high penalty cost is attributed, to make the solution less interesting for the algorithm. Regarding the extra needs, the average and the standard deviation are calculated. The average indicates how many more technicians there are and the

Table 4.5: Fitness Evaluation concerning Hospital Preferences.

Hospital Preferences			
Skills evaluation	If all needs are met	No penalty cost is assigned.	
	If any need has not been met	If a need for neurology is not met on: - Monday afternoon - Thursday morning - Friday morning	Penalty Cost = 15 x number of unfilled skills
		Otherwise	Penalty Cost = 7.5 x number of unfilled skills
Hospital needs	If all needs are met	No penalty cost is assigned.	
	Otherwise	A high penalty cost is assigned:	Penalty Cost = 1000 x number of needs not met
Needs variation	Average of needs for each day and shift type	Penalty Cost = 10 x average/day/shift	Sum penalty costs Objective: reduce the result
	Standard deviation of needs for each day and shift type	Penalty Cost = 4 x std/day/shift	
Technicians work 40 hours/week (Note that if there is at least one holiday during the week, they should work fewer hours)	All technicians work the minimum number of hours.	No penalty cost is assigned.	
	Otherwise	Penalty Cost = |40 hours - number of hours worked| x 15	

standard deviation indicates the variability of these values. The higher each one, the greater the penalty. The goal is to minimize both values and thus have a more balanced result.

As stated earlier, it is in line with the hospital's preferences that each technician works approximately 40 hours a week. It should be noted that if there is a holiday on a working day of the week, this number of weekly hours is reduced by eight hours times the number of holidays of the week. At that time, while there is no schedule history, only whole weeks are counted within the schedule. For instance, if the month starts on a Wednesday, that week is discarded and only the next one begins to count. The same is true if the month ends in the middle of a week. The penalty is awarded according to the difference in absolute value between the 40 hours and those actually assigned (see Table 4.5). If the previous schedule is known, it is possible to identify the shifts worked in the last days of the month and calculate the number of hours worked in the first week. Once again, the objective is to minimize this difference. Table 4.5 summarizes all aspects assessed and how to obtain a fitness value.

Fitness Function for Technicians' Preferences

One of the hospital's ambitions is to offer outstanding health care, in addition to innovation and equipment, which is possible through a satisfied staff. As such, one of the objectives is to meet individual needs, whenever possible and fairly. Individual preferences will influence aspects such as fairness, personal satisfaction and quality of healthcare. Therefore, after some conversations with the DM, we realized that following aspects should be considered to promote the technicians' satisfaction.

First it is checked whether all hard constraints are being satisfied. Otherwise there will be a penalty cost associated (see Table 4.6).

When a technician works on Saturday or Sunday the usual pair of consecutive days free is lost. In order to compensate this, technicians who work on Saturdays should have the following Monday off. On the other hand, technicians who work on Sunday should have the previous Friday free. In this way, the objective is to maximize the number of pairs of days-off. More generally, a technician must have a number of pairs of days-off equal to the number of weekends in a month and, if it is lower, it is penalized (see Table 4.6). It is necessary to take into account that a balance must be found among all technicians so that all have, more or less, the same number of days-off pairs. To do this, one calculates the average of days-off of all technicians and then the difference in absolute value of each technician in relation to the average. The fitness value will be evaluated through the sum of the differences that should be as small as possible.

When a technician does a weekend shift (S) or a all-day shift (I10), which are 10-hour shifts, he/she must be assigned to an M6 shift – which is a 6-hour shift – on a day of the month, in order to compensate the two extra hours worked. If the number of weekend shifts (S) and I10 shifts is greater than the number of M6 shifts, a penalty is also added. The same happens if the number of days-off, besides weekends, is not equal to weekend days worked, since for each weekend shift worked the technician must receive a day-off.

Another aspect assessed is individual preferences. There are technicians who do not mind working overtime, while others do not want to do it at all. Therefore, a penalty cost (see Table 4.6) is added whenever someone who does not want to is assigned to an extra shift. In addition, technicians should have a day-off on their birthday. In case this does not happen, a penalty is also given (see Table 4.6).

Finally, since in the IS have implemented the rotation and prefer to keep it whenever possible, and in case there is access to the schedule of the previous month to find out in which rotation the month ended, the fitness of the chromosome can be penalized in the cases where the rotation does not follow the intended one (see Table 4.6).

Table 4.6 summarizes the penalty cost of each parameter in order to find the fitness value concerning Technicians' Preferences.

TOPSIS method

In contrast to what happened in ICU case, in the IS there was no opportunity to collect information from different technicians. Therefore, the objectives, criteria and respective weights were found only with the help of one of the technicians (the DM), who is one of those responsible for the schedule each month.

According to the DM, a good schedule must meet both personal and hospital preferences. One of the most important factors in relation to hospital preferences is concerning the number of functional areas satisfied. That is, although the technicians are able to perform any type of exam, they may be specialized in a given functional area. This can be reflected in the quality of the exam provided, which

Table 4.6: Fitness Evaluation concerning Technicians Preferences.

Individual Preferences		
Hard Constraints	If any hard constraint is violated	Penalty Cost = 1000 x number of hard constraints violated.
Pairs of days-off	If pairs = number of weekends If pairs $<$ number of weekends	No penalty cost is assigned. Penalty Cost = 30 x $\lvert$number of weekends - number of pairs$\rvert$
Sum of differences between technicians regarding pairs	Penalty Cost = sum of differences	
Technicians working more than 6 consecutive days	A high penalty cost is assigned.	Penalty Cost = Number of consecutive days worked x 1000
M6 shifts after a weekend shift	If number of M6/shifts = number of weekends worked If number of M6/shifts $<$ number of weekends worked	No penalty cost is assigned. Penalty Cost = 10 x $\lvert$number of weekends worked - number of M6 shifts$\rvert$
Days-off after a weekend shift	If number of days-off out of weekend = weekend shifts If number of days-off out of weekend $<$ weekend shifts	No penalty cost is assigned. Penalty Cost = 10 x $\lvert$number of weekends worked - number of days-off$\rvert$
Birthday	If a technician does not have a day-off on the birthday.	Penalty Cost = number of times this happens x 15
Individual Preferences	If a technician works an extra shift and does not desire it	Penalty Cost = number of extra hours x 50
Rotation desired	If the previous rotation is not maintained	Penalty Cost = number of rotations not respected x 100

will be greater if the technician is specialized in that area, and therefore it can also be reflected in greater patients satisfaction. This criterion is assessed by the number of functional areas filled, according to the needs, the more the better, consequently this is a benefit criterion.

Regarding individual preferences, the DM highlights the importance of having at least two consecutive days-off, as happens on weekends. This is important when a technician is assigned to a shift on Saturday or Sunday. Supposedly one loses the pair of days-off, however the day-off to which one is entitled must be inserted in order to recover the pair (for example, if he/she does Saturday he/she must have the day-off on Monday, so he/she has Sunday and Monday in a row). This is also a benefit criterion, since we intend to maximize it. On the other hand, there should be no big differences between the pairs of days-off between the technicians in order not to cause situations of unfairness. In this case it is to reduce the differences, so it is a cost criterion.

Another criterion considered important is the number of hours worked by each technician. This is important both from the point of view of the hospital and the staff. It is not in the hospital's interest that the technicians work fewer hours than those defined in the contract. On the other hand, technicians do not intend to work more hours than those defined, with the exceptions of technicians who have expressed interest in working overtime. Each technician must work 40 hours a week. This criterion is evaluated according to the number of technicians who do not work 40 hours and it is intended to minimize this number, so it is a cost criterion.

Finally, the DM considers important to satisfy personal preferences in order to promote personal satisfaction. It is expected to maximize the number of preferences satisfied, being a benefit criterion.

In brief, the criteria that define the quality of a schedule are: (i) the number of functional area needs filled; (ii) the number of pairs of days-off; (iii) the difference in absolute value between the number of pairs of days-off between technicians; (iv) the number of hours worked during the month by each technician, and (v) the individual preferences.

Over time some proposals were presented to the DM. These allowed us to identify the relative importance of each criterion. The fulfillment of needs concerning each functional area was identified as the most important criterion, because, in addition to promoting a better quality of the services provided, it also increases the satisfaction of patients and technicians. Then the criteria of the number of pairs of days-off and the number of hours seem to have the same importance, in fact these can be related, because if a technician is assigned to a weekend he/she will do more hours that week if he/she is not assigned the day-off pair. Then fairness regarding the number of days-off proved less important than the last two but more important than individual preferences. Therefore, the defined weights in accordance with the DM were: (i) 30% for the skills; (ii) 25% for the average number of pairs; (iii) 15% for the sum of differences concerning pairs; (iv) 25% for the minimum number of hours worked and (v) 5% for the individual preferences.

4.3 Chapter Considerations

This chapter specifies the features and particularities of the IS and ICU schedules, in order to adopt the correct adjustments. The proposed Multi-objective Genetic Algorithm is presented. The algorithm can be applied in different cases, despite having specific features for each case-study.

The NSGA-II and the features of each service are described. For each case, the algorithm minimizes two objectives. Fairness and preferences for the ICU and hospital and individual preferences for the IS.

In the following chapter, computational experiments are provided in order to validate the algorithm. The results and respective discussion are presented.

Chapter 5

Computational Experiments

In the present chapter an evaluation of the GA developed will be provided through computational experiments, using instances associated with the case studies. Additionally, the results obtained will be discussed. Section 5.1 provides the instances' descriptions. Section 5.2 presents some interesting aspects that were noticed throughout the computational experiments leading to certain choices in the algorithm construction, namely in crossover and mutation operators. In Section 5.3 an analysis is conducted to understand the influence of some criteria on the algorithm and results.

The algorithm was coded in Python. The tests were performed on a computer with an Intel Core i7-8565U processor (4 cores/8 threads each with a base frequency of 1.80 GHz) and 8 GB of RAM.

5.1 Instances' Description

In an effort to validate the algorithm, different computational experiments are performed to understand the influence of the parameters and criteria used. In this section, the instances used will be described.

First, in Section 5.3.1, an analysis of the values of the fitness function and the computational time. In this case, the December conditions were considered as input (see Table B.1), but without considering the historical data available. Under these conditions different times were used as stopping criteria. Afterwards, the average of the values of both fitness 1 and fitness 2 of the set of solutions non-dominated are calculated.

Secondly, in Section 5.3.2, a study of the impact of using historical data is conducted. For this purpose, a comparative study of the schedules was made, considering three different scenarios. First, schedules for all months of the year were created with the algorithm (scenario 0+12), only taking into account the same conditions that existed in reality, that is, considering absences, holidays, the presence of external physicians and constraints (this information can be found in Table B.1). Note that the month of

January was created without considering previous information, while the remaining months were created considering the information accumulated over the previous months, namely the number of hours and scores. Second, historical information provided by the ICU in the first half of the year was used, and the remaining months were generated by the algorithm considering that previous information and the real conditions (scenario 6+6). Finally, the same process was repeated, but this time using the historical records of the first eleven months and using the algorithm to create the last month of the year (scenario 11+1). The solutions used throughout the analysis are those that were selected by the TOPSIS method. In all three scenarios, all other conditions used were the same (see Table B.1).

Finally, in Section 5.3.3, also using the December conditions, a TOPSIS sensitivity analysis was carried out. As was explained, the algorithm is capable of generating several non-dominated solutions in each generation. In the end, the non-dominated solutions of all generations are grouped and through TOPSIS the best one according to the defined criteria is chosen. The goal is to evaluate how changes in the weight of the criteria would interfere with the solution resulting from TOPSIS.

The next section presents some details that were adjusted according to the construction of the algorithm, which proved important to improve the results of the algorithm.

5.2 Algorithm Details

This section briefly describes some details of the algorithm construction and testing, namely with respect to choice of operators and parameter tuning.

Initially, a one-point-crossover was chosen, however, it was concluded that this did not provide as much variability as expected and, as such, it resulted in a great proportion of equal individuals in the population. In this crossover, only one point on the chromosome was chosen at random. From position 0 to the chosen point, information was received from one parent and the rest from the other. Due to elitism, both parents and children can enter the next generation. Therefore, it could be the case that a father and an offspring cross and generate equal children, because they would be exchanging information that had already been exchanged. This led to a two-point crossover to be chosen instead, as described in previous chapter. Whereas the first option generates descendants with two portions exactly equal to their parents and only varies the point where the change occurs, in two-point crossover there are two cut-off points and thus a smaller portion exactly equal to each of the parents at a time. For this reason, this proves to be a more advantageous operator, since it inserts more variability and prevents equal solutions from perpetuating themselves.

The mutation also had to be redesigned. Initially, the mutation changed only two entries in the entire matrix, which introduced little variability. This alternative mutation consisted of choosing a random column and then two random rows. These two entries would then switch positions. This change proved not to be very significant, hence the choice of the mutation previously described.

Table 5.1: Considered values for various parameters.

Parameters	Values
N	160
T	4
r	80
p_m	0.08

Through some preliminary experiments, the best combination of parameters was found, presented in Table 5.1. This is used throughout the computational experiments. The parameters' designation can be found in the Table 4.1.

The next section presents the computational experiments carried out for the ICU according to the parameters and features found so far.

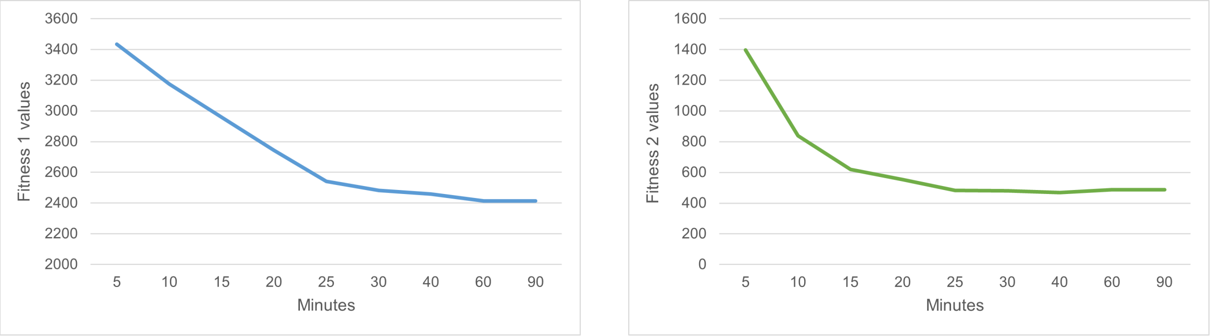
5.3 ICU

In this section, we present some computational experiments carried out with the algorithm using instances of the case study of the ICU. Initially, in Section 5.3.1, the convergence conditions of the algorithm are sought, in order to find the most suitable conditions to perform the following experiments. Afterwards, in Section 5.3.2, an exploratory study is carried out in order to understand the behavior of cumulative scores and average number of hours using historical records. The results presented are followed by a discussion. Finally, in Section 5.3.3, a sensitivity analysis to the TOPSIS method is provided.

5.3.1 Fitness Functions over Time

The first analysis performed regards the behavior of the fitness function values over time. For this purpose, the parameters defined in Section 5.1 will be used. Time was applied as a stop criterion: the experiment was performed for 5, 10, 15, 20, 25, 30, 40, 60 and 90 minutes, and for each of them the average values of fitness 1 and fitness 2 were extracted concerning the solutions resulting from the non-dominated sorting off all non-dominated solutions of all generations. Figure 5.1 presents the results obtained. Figure 5.1a presents the results obtained for fitness 1 (fairness) while 5.1b presents fitness 2 (preferences) values over time.

After analysis of the figures it can be seen that the values of fitness 2 stabilize faster than those of fitness 1. Fitness 2 values remain more or less constant after 25 minutes, while fitness 1 values only after 60 minutes have passed. Nevertheless, the values of fitness 1 decrease much slower after 30 minutes than in first 30 minutes. This allows us to conclude that using 30 minutes as stopping criterion is enough to obtain good solutions for these particular test instances. During half an hour, about 90 generations were created, so the algorithm evaluated 14400 solutions in a widely constrained



(a) (b)

Figure 5.1: (a) Fitness 1 values over time; (b) Fitness 2 values over time.

environment (vacations, unavailability, etc.).

Also for December, with a smaller number of constraints – without considering vacations, unavailability constraints, Christmas rotation – it was able to create up to 970 generations, which means 155200 solutions could be evaluated.

In the following section, where the impact of using schedule history as an algorithm input is evaluated, these experiments will be carried out using 30 minutes as a stopping criteria.

5.3.2 Evaluating the impact of Historical Records

The ICU service provided a historical record with information on all schedules, number of hours, absences and holidays for the year 2020. In this way, computational experiments were conducted to understand the influence of these records in the results.

This analysis starts by evaluating and comparing the results of the sum of the differences in cumulative scores, which represents the number of hours, weekends, night and 24-hour shifts, between the three scenarios described in Section 5.1. To obtain this value the deviation in absolute value of the score of each physician in relation to the average of the accumulated scores in the month under study is determined, and then the differences of each physician are summed. Lower values in the sum of the differences are reflected in closer values between physicians and, consequently, are considered fairer.

Figure 5.2 depicts the sum of the differences in cumulative scores over the months for the three scenarios and with the values obtained through the real historical data. The light gray bars represent the case that was created with the algorithm only, without considering historical information (scenario 0+12). The dark gray bars represent the case where the algorithm generated schedules for half a year, considering the first semester data (scenario 6+6). The light blue bars represent the scenario in which only the December schedule was generated, using all the information from previous months (scenario 11+1). Finally, the dark blue bars represent the values with respect to the historical data.

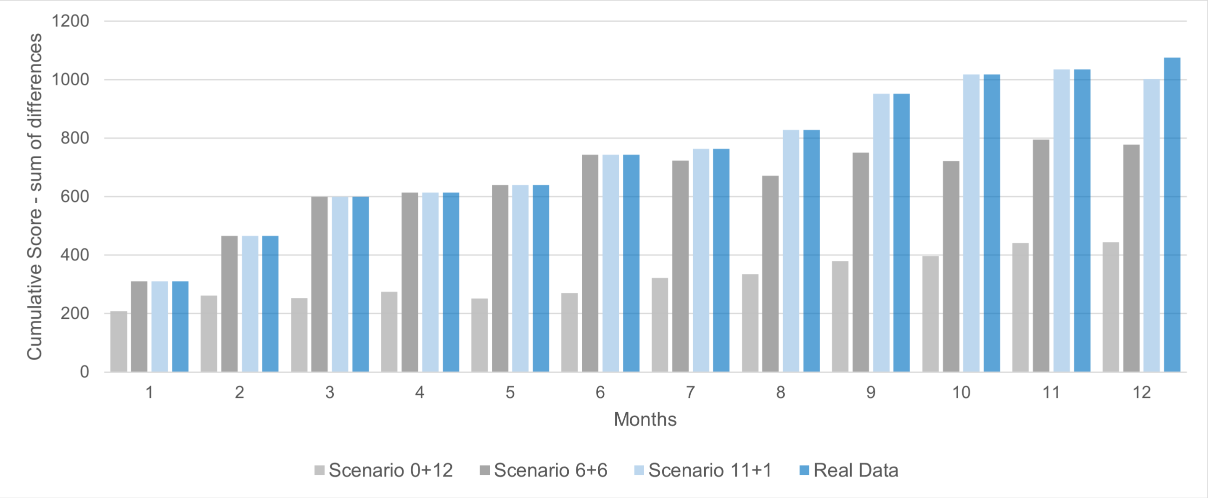


Figure 5.2: Result of the sum of the cumulative scores differences.

As one would expect, the dark blue and dark gray bars have the same height for the first six months, since the input information is the real historical data. Similarly, the dark blue bars coincide with the values of light blue bars for all months, except for December. It is possible to verify that the real data scores tend to increase steadily over the months. In contrast, the algorithm's scores increase more slowly in general, and can also decrease in some cases. This means that the variability between physicians tends to be lower when using the algorithm, which is synonymous with fairer results.

For the scenario 11+1 (light blue), the results show that for December there is a decrease in the sum of the differences in relation to what happens on the real data (dark blue), meaning the algorithm tends to counteract the almost linear rise in reality, in order to find fairer solutions.

In scenario 0+12 (light gray), the results show that there is less variation from a few months to the others – in contrast to what happens in real scenario (dark blue) – and beyond that, the sum of the differences is always smaller. This means that differences between physicians are less significant and therefore, that the algorithm is capable of generating fairer solutions. This is also the case for the half-year scenario (dark gray). The algorithm counteracts the rise in scores, and they remain more constant and at lower levels. Therefore, there is a tendency for the algorithm to find an equilibrium.

To look in more detail at the previous results, it was verified what happened to the average number of hours worked in each month and also to the number of shifts filled in an ideal way.

Figure 5.3 presents the average hours worked by physicians each month. Again, this information is represented for the scenario where no previous information is considered (light gray color), for the case where half a year is considered (dark gray color) and for the real data provided (dark blue color). Table 5.2 presents the average number of hours per month in each case.

The highest points are observed, for the three situations, in months that register the greatest number of physicians on vacation. This happens in July, August and September, where there were about sixty

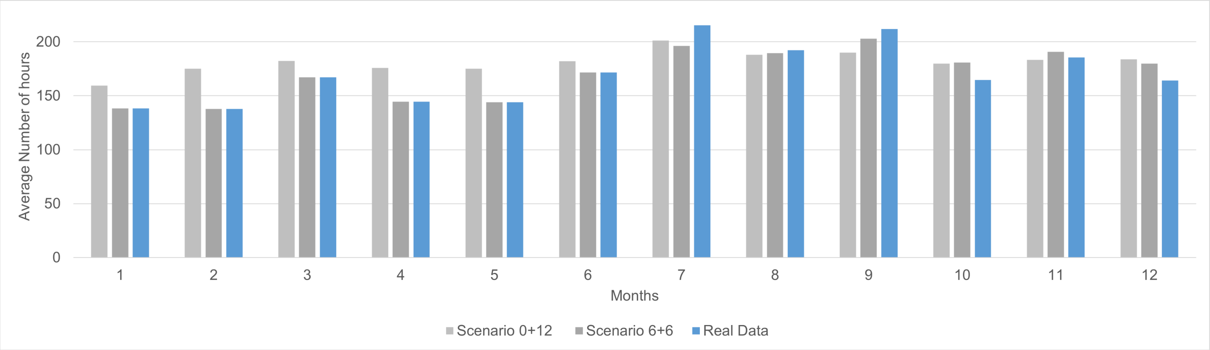


Figure 5.3: Average number of hours per month.

Table 5.2: Average number of hours per month.

	1	2	3	4	5	6	7	8	9	10	11	12
Scenario 0 + 12	159,53	175,00	182,33	175,83	175,20	182,07	201,23	188,03	190,00	179,90	183,30	183,87
Scenario 6 + 6	138,20	137,73	167,10	144,40	144,07	171,52	203,97	181,90	185,50	181,97	178,97	180,87
Real Data	138,20	137,73	167,10	144,40	144,07	171,52	215,42	192,35	211,80	164,65	185,48	164,20

days of vacations scheduled. Since the number of hours are presented in proportion (that is, for the total hours worked by the physician in the month, if a physician is on vacation, the number of days on vacation is added as a working day), this means that holidays do not affect the average, but they do increase it these cases, since there are physicians who will have to work more to compensate for these absences.

It is important to note that in January there were three physicians absent and one of them continued until the end of March, which may justify the high number of hours in the algorithm scenario in relation to the real one, because while the algorithm managed to ideally fill at least every morning, this did not happen in reality. Even so, the algorithm does not exceed the number of hours considered ideal, namely around 180 hours per month. In addition, it turns out that the algorithm starts with higher average values but, in total, it presents a smaller variation and thus, a better balance among the physicians. For example, the maximum worked time by a physician in the December schedule provided by the ICU is 215.23 hours, while the algorithm maximum for the same month is about 14 hours less. On the other hand, the minimum recorded by the algorithm is slightly higher. Still, the amplitude of the real data is higher. The 6+6 case, in which half a year of real data and half a year of schedules generated by the algorithm are used, is observed to be very similar to the case in which only data from the algorithm is used in the second half of the year. This further suggests that the algorithm is able to properly balance previously unbalanced situations. In addition, it is verified through Figure 5.4 that the high number of hours is justified with the number of shifts ideally filled.

It can be seen that the algorithm was always closer to the ideal number of shifts than in reality. This shows that there is a tendency to satisfy this need of the hospital but that, despite this, there is not an exaggerated number of hours worked on average. This said, it is possible to state that the algorithm tries to find the best balance between the number of hours, the ideally filled shifts and the fairness within the schedule.

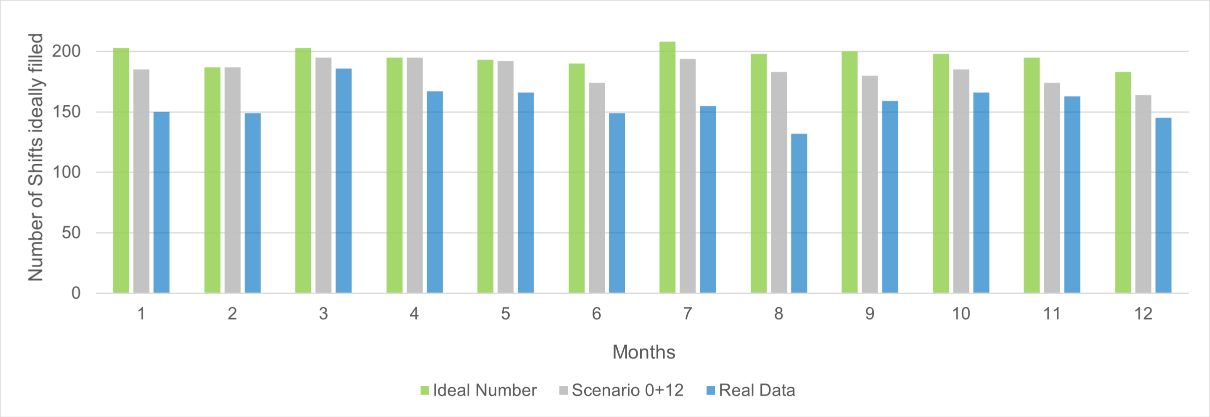


Figure 5.4: Number of shifts ideally filled.

Through all the described above, it is possible to notice that the algorithm always tends to stabilize, increasing the number of shifts ideally completed but without compromising the average number of hours. In addition, it is still possible to validate that the algorithm adapts to all months of the year, identifying holidays and weekends, adapting to a variable number of physicians, both internal and external. For example, at the beginning of the year there were three fewer internal physicians than at present, and there were months with no external physicians while in others four were available (this information is detailed in Table B.1).

5.3.3 TOPSIS Sensitivity Analysis

Any decision support system that depends on personal and qualitative judgments may be subject to uncertainty due to the inherent subjectivity. Accordingly, there is an associated error margin that may vary for each criterion. As such, it is important to perform a sensitivity analysis, which allows to evaluate how the variation of the criteria weights can influence the results. As stated earlier, selection is made through the TOPSIS method. The goal is to evaluate how changes in the weight of criteria would interfere with the solution resulting from TOPSIS. The criteria and respective weights were described in Section 4.2.2: (C_1) is the absolute value difference in the number of hours number; (C_2) the absolute value difference in the number of weekends; (C_3) is the absolute value difference of cumulative scores; (C_4) is related to preferences; and (C_5) to the number of shifts ideally filled.

In TOPSIS, the weights are normalized to a sum of 1, which means that if the weight of one criterion changes, the weight of other criteria changes accordingly. In order to carry out this analysis, different settings will be created where different rankings may arise according to the changed weights. The work of Alinezhad & Amini (2011) is followed. When one of the weights is changed, the others vary according to the following equation:

$$w'_h = \frac{1 - w'_j}{1 - w_j} w_h, \qquad h \neq j \tag{5.1}$$

where the weight of the j^{th} criterion changes from w_j to w'_j and w'_h will be the weight of attribute h.

Table 5.3: Range of criterion weights that do not affect the current solutions.

	Current Rank	Current Weight (w_j)	Weights' range for which the best solution does not change
C_1	3	0.210	[0.000, 0.299]
C_2	4	0.140	[0.100, 0.729]
C_3	1	0.360	[0.000, 0.909]
C_4	3	0.210	[0.111, 0.849]
C_5	5	0.070	[0.065, 0.159]

This sensitivity analysis was performed using the instances described in Section 5.1. Once the best non-dominated individuals were selected, TOPSIS was applied to choose the best solution. This is where the weight variation is applied. In total, 25 non-dominated solutions were obtained, and one of them was considered the best according to the previously defined weights.

Table 5.3 presents the ranges of criterion weights that do not affect the current solutions, – which means that do not change the current best solution. For each criterion the weight was varied until a new solution was obtained.

Based on the results presented in Table 5.3, one can notice that even small variations of the weights may give rise to different solutions. For example, for C_5 only a 0.005 decrease or a 0.09 increase in weight already results in a different solution. This shows that this criterion is very sensitive. Also criterion C_2 shows that a decrease of 0.04 in its weight in relation to what is currently defined results in a different solution, however, a much steeper increased is needed to find a new solution. This shows that the defined ranking is not very stable. On the other hand, the criterion C_3 presents a large range where no changes are verified, a large variation is necessary to obtain a new solution. Moreover, no matter how much the weight is reduced, it will not find a new best solution. Criterion C_5 is the most sensitive, because it is the one that has a smaller weight range for changes to occur, followed by C_1, that although has a range of 0.3 to undergo changes, only needs an increase of 0.09 over the current weight for the

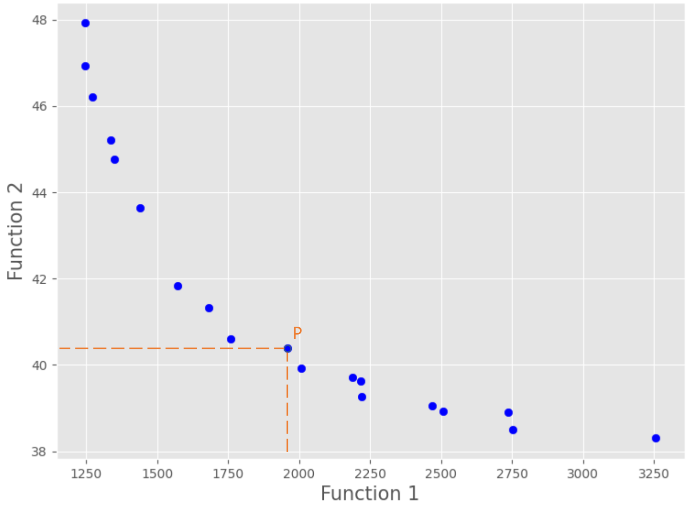


Figure 5.5: Representation of a Pareto-front obtained on February schedule with two (minimization) fitness functions.

best solution to be different. The remaining criteria present the same solution for a longer interval, so they are less susceptible to changes.

Figure 5.5 represents an example of an approximate Pareto-front obtained in February for scenario 0+12. The plot was generated from the best non-dominated solutions obtained over all generations. Each point represents different solutions and each of these solutions is not dominated by any other. This multi-objective problem is intended to minimize the two fitness functions, so it has the convex shape that one would expect. The point P represents the solution chosen through the TOPSIS method.

5.4 Chapter Considerations

This chapter presents the application of the algorithm to the ICU case study, for which a whole year of real data was provided. First an evaluation of the fitness values over time was made in order to determine an appropriate time limit. Afterwards, a study was performed to realize the influence of historical data. Finally, a sensitivity analysis of the weights used in the TOPSIS method was also performed. The results were discussed throughout the presentation of the solutions.

The algorithm presents feasible solutions and adapt to different conditions, from the month to be scheduled, the staff number, vacations, national holidays, to constraints. It can do this in a timely manner despite the very constrained problems at hand.

The next chapter will provide the validation of the algorithm solutions and feedback from the part of HL. A comparative study will be provided in order to evaluate the applicability of the GA.

Chapter 6

Validation with Hospital da Luz

From the algorithm and real-instances used it was possible to present comparable, reality-designed solutions to the hospital and receive some feedback. This will be presented throughout this chapter.

First, Section 6.1 provides a comparison between the schedule proposed by the algorithm for December of 2020 and the one used by HL, for both case studies. Section 6.2 presents the feedback and validation received from HL. In the ICU, the proposed schedule was evaluated by the DM, whereas in the IS, feedback was received from the technician responsible for the schedule creation, the IS DM.

6.1 Comparison with HL of December 2020 schedules

As the aim of the algorithm proposed in this dissertation is to solve the problems identified at HL, a comparison between the hospital schedule for December 2020 and that of the algorithm is provided. For this purpose, the same characteristics that were used in the real case were considered for the algorithm, such as vacation periods, physicians' unavailability and staff numbers. In Section 6.1.1 the comparison of December 2020 in ICU is provided and in Section 6.1.2 the comparison for IS.

6.1.1 ICU

A potential proposal for the December 2020 schedule is detailed below, using the current characteristics provided by the person responsible for its in reality. Currently there are 15 permanent physicians in the ICU. Usually, there is the possibility of incorporating external physicians, however, these did not manifest availability during December. This month has special characteristics due to Christmas, therefore it is a very requested season concerning vacations. In order to avoid conflicts, a specific rotation is already used in 24th, 25th and 31st of December that should not be changed. This information, along with

Table 6.1: Considerations about December Schedule.

	Holidays			Constraints		Christmas Rotation	Team
	Start		End	Day that cannot be assigned	Shift that cannot be assigned		
P0	05/12	-	13/12				P7
P1						24/12 and 31/12	P4, P13
P2				Tuesday Thursday	Prolongation Prolongation		
P3	19/12	-	31/12				P8
P4	01/12	-	13/12	Monday Tuesday	Night All day		P1, P13
P5				All days	Morning and prolongation		P6, P14
P6							P5, P14
P7	01/12	-	08/12	Tuesday	Prolongation and night		P0
P8				Wednesday Thursday	Night, 24-hour Morning and Prolongation	31/12	P3
P9	01/12	-	13/12			24/12	P10
P10							P9
P11						24/12	P12
P12	19/12	-	31/12				P11
P13	01/12	-	06/12			25/12	P1, P4
P14						25/12	P5, P6

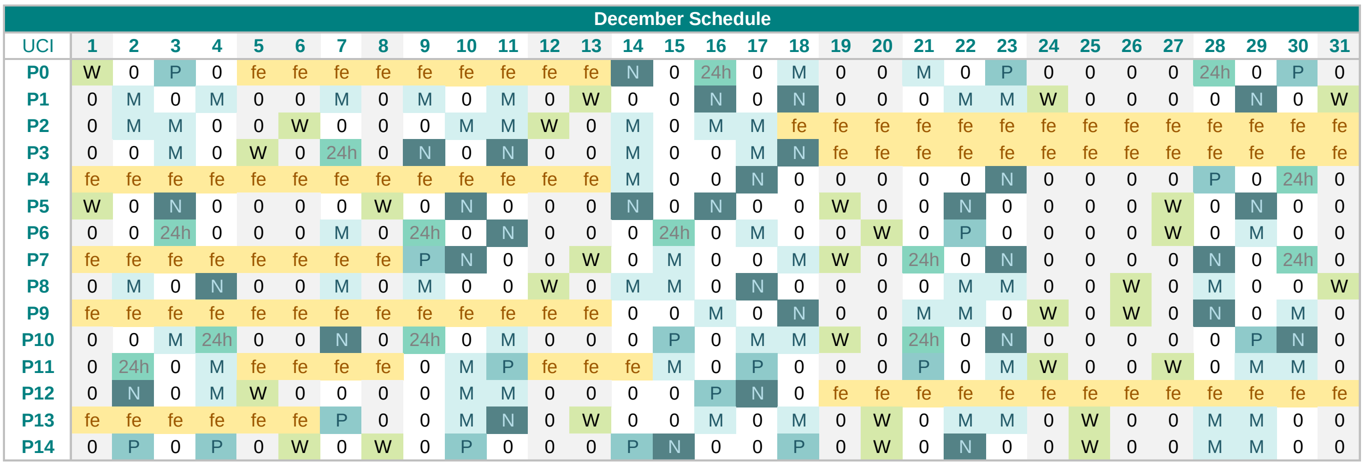
individual considerations of each physician regarding the vacation period, the availability constraints and the elements with greater affinity, are presented in the Table 6.1.

The solution used was the one following scenario 11+1, which is the one where it is used the historical records of the first eleven months and the algorithm is applied to create the last month of the year, as detailed in Section 5.3.

Currently, the ICU designs a 4-week schedule with a slightly different layout than the one presented by the algorithm. The schedule built by the DM is divided over the several weeks of the month. That is, the rows correspond to the weeks and each column of that week corresponds to one day. Each entry is divided into 4 possible shifts (morning, prolongation, night and 24-hour shifts) and the physicians are assigned there (see Figure B.1). This has proved to be a more inconsistency prone disposition as described in Section 2.2.1 than the one used. In the algorithm the schedule has the shape of the aforementioned chromosome (Figure 4.1), which is a matrix where each row i corresponds to a physician, each column j to a day of the month and each entry ij to the shift assigned to physician i in day j. Figure 6.1 presents the December 2020 schedule generated by the algorithm, whose output is an *Excel* file with the schedule representation. The letters represent each shift as follows: *W* is a weekend shift, *M* is a morning shift, *P* is a prolongation shift, *24h* is a 24-hour shift and *fe* represents the vacation periods. Columns corresponding to weekends and national holidays are gray-shaded (recall that in the ICU the 24th of December is considered in the same way as weekends).

Table 6.2: Number of physicians assigned in each shift each day.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**
Morning	0	3	3	3	0	0	3	0	2	4	4	0	0	4	3	3	4	4	0	0	2	4	4	0	0	0	0	3	4	2	0
Prolongation	0	1	1	1	0	0	1	0	1	1	1	0	0	1	1	1	1	1	0	0	1	1	1	0	0	0	0	1	1	1	0
Night	0	1	1	1	0	0	1	0	1	2	3	0	0	2	1	2	3	3	0	0	0	2	3	0	0	0	0	2	2	1	0
24-hours	0	1	1	1	0	0	1	0	2	0	0	0	0	0	1	1	0	0	0	0	2	0	0	0	0	0	0	1	0	2	0
Weekend	2	0	0	0	2	2	0	2	0	0	0	2	3	0	0	0	0	0	3	3	0	0	0	3	2	2	3	0	0	0	2
24h+night+morning	0	6	6	6	0	0	6	0	7	6	7	0	0	6	6	7	7	7	0	0	6	6	7	0	0	0	0	7	6	7	0



December Schedule																															
UCI	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**
P0	W	0	P	0	fe	fe	fe	fe	fe	fe	fe	fe	fe	N	0	24h	0	M	0	0	M	0	P	0	0	0	0	24h	0	P	0
P1	0	M	0	M	0	0	M	0	M	0	M	0	W	0	0	N	0	N	0	0	0	M	M	W	0	0	0	0	N	0	W
P2	0	M	M	0	0	W	0	0	0	M	M	W	0	M	0	M	M	fe	fe	fe	fe	fe	fe	fe	fe	fe	fe	fe	fe	fe	fe
P3	0	0	M	0	W	0	24h	0	N	0	N	0	0	M	0	0	M	N	fe	fe	fe	fe	fe	fe	fe	fe	fe	fe	fe	fe	fe
P4	fe	fe	fe	fe	fe	fe	fe	fe	fe	fe	fe	fe	fe	M	0	0	N	0	0	0	0	0	N	0	0	0	0	P	0	24h	0
P5	W	0	N	0	0	0	0	W	0	N	0	0	0	N	0	N	0	0	W	0	0	N	0	0	0	0	W	0	N	0	0
P6	0	0	24h	0	0	0	M	0	24h	0	N	0	0	0	24h	0	M	0	0	W	0	P	0	0	0	0	W	0	M	0	0
P7	fe	fe	fe	fe	fe	fe	fe	fe	P	N	0	0	W	0	M	0	0	M	W	0	24h	0	N	0	0	0	0	N	0	24h	0
P8	0	M	0	N	0	0	M	0	M	0	0	W	0	M	M	0	N	0	0	0	0	M	M	0	0	W	0	M	0	0	W
P9	fe	fe	fe	fe	fe	fe	fe	fe	fe	fe	fe	fe	fe	0	0	M	0	N	0	0	M	M	0	W	0	W	0	N	0	M	0
P10	0	0	M	24h	0	0	N	0	24h	0	M	0	0	0	P	0	M	M	W	0	24h	0	N	0	0	0	0	0	P	N	0
P11	0	24h	0	M	fe	fe	fe	fe	0	M	P	fe	fe	fe	M	0	P	0	0	0	P	0	M	W	0	0	W	0	M	M	0
P12	0	N	0	M	W	0	0	0	0	M	M	0	0	0	0	P	N	0	fe	fe	fe	fe	fe	fe	fe	fe	fe	fe	fe	fe	fe
P13	fe	fe	fe	fe	fe	fe	P	0	0	M	N	0	W	0	0	M	0	M	0	W	0	M	M	0	W	0	0	M	M	0	0
P14	0	P	0	P	0	W	0	W	0	P	0	0	0	P	N	0	0	P	0	W	0	N	0	0	W	0	0	M	M	0	0

Figure 6.1: December Schedule generated by the algorithm.

Table 6.2 supports the schedule information regarding the number of physicians assigned each day in each shift. The number of physicians considered ideal on weekend shifts is 3 (in turn, the minimum limit is 2), in the mornings it is 4 (minimum limit is 3), in prolongations it is 1 (the minimum limit coincides with the ideal) and at nights it is 3 (minimum limit is 2). The last row of the table represents the sum of morning, night and 24-hour shifts (which is equivalent to morning and night combined), which should be of 7 physicians ideally, but not less than 6.

After looking in detail it is possible to identify some points that validate the solution, namely the compliance of all hard constraints. These are described below:

- There are no night, 24-hour or weekends shifts followed by any other shift (a day-off is attributed instead).
- The vacation periods were correctly placed according to Table 6.1.
- The Christmas rotation has not been changed.
- No physicians were assigned with two consecutive prolongation shifts.
- The minimum coverage limit is ensured in all shifts, as it can be verified in Table 6.2.
- 18 of the 183 shifts were not filled ideally (presented in blue in Table 6.2), although two of these shifts with the minimum number of physicians assigned are part of the Christmas rotation.
- All of Table 6.1's constraints are respected.

Taking into account these results, it is possible to conduce that the algorithm is capable of generating feasible solutions respecting all hard constraints and trying whenever possible to reduce penalties associated with the lack of achievement of soft constraints.

Table 6.3 provides a comparison between the results of the real schedule and the algorithm schedule. The algorithm presents worst values regarding the average number of hours worked than the real schedule, however this can be explained through the number of shifts ideally filled, which is about 20% higher than in the schedule built in the service. Even so, the average number of hours is acceptable and within the defined limit of a maximum of 180 hours, as mentioned above.

Table 6.3: Comparison between real schedule and algorithm schedule.

	Real Schedule	Algorithm Schedule
Ideal Shifts Filled	79.00%	90.16%
Team affinity (night and weekends)	6	9
Average number of hours	138.40	160.93
Cumulative Score	1076.00	987.33

The option to maximize the average number of days-off after a 24-hour shift was inserted in the algorithm, however, it is verified through this solution that there was no significant increase. This may be justified by the fact that several physicians are on vacations at the same time, so there was less room for this choice to be enabled.

Another point to highlight is the difference in the cumulative scores between physicians, which decreases in relation to the real schedule. In this case, namely scenario 11+1, the algorithm used the information history (of the remaining 11 months of the year) and still managed to obtain better results, having generated a fairer schedule that compensates the physicians affected in previous months.

In conclusion, the algorithm achieves good results in relation to those found in the real schedule, with no record of inconsistencies or hard constraints violations. Additionally, it did so in a very short time, contrary to what happens in reality. These results were presented to the ICU. In Section 6.2.1 the feedback received will be presented.

6.1.2 Imagery Service

The algorithm was also applied to the IS case study. This section presents what was considered the best schedule for December 2020 resulting from the instances described in Section 5.1.

In this case, only the historical November schedule was available, so this schedule was used as an input for the algorithm to identify with which rotation the month ended and consequently with which rotation the next month should begin. In addition, the information from the previous month is also important for counting the number of hours of the week if the month to be scheduled starts in the middle of the week, which is the case of December that starts on a Tuesday. The solution obtained is shown in Figure B.2.

After analyzing the schedule obtained, it is possible to conclude that no technicians work more than six consecutive days, nor does any technician in the central group work on holidays (1, 8 and 25 December). Furthermore, none of the breastfeeding technicians (T7, T17 and T23) have weekends assigned and all shifts assigned to them are 6 hours long (M6, A6 or T6). The hospital needs are all satisfied regarding the number of technicians assigned to each shift type. Nobody works a T21 shift followed by an A8 shift, nor does any technician work any P, P0, P1 or N shift followed by a morning shift. In short, it can be concluded that no hard constraint is violated.

The solution distributes the vacation period correctly and also allocates a day-off on birthdays (represented by an A). Most technicians only work one weekend a month. There are 2 technicians that work two at most – one of them is from another clinic but expressed this availability, while the other is willing to work overtime. In this case, the algorithm is able to insert a day-off and a 6-hour shift for those who took a weekend shift. However, there are still two cases in which the number of 6-hour shifts is lower than the number of 10-hour shifts worked (I10/S), which is a problem because they are not been compensated for the two extra hours worked.

Concerning rotation, it appears that it is lost in some cases. Since this is not considered a hard constraint, it was not included in the repair function and it is very likely to be lost during the crossover. Failure to maintain rotation is penalized in the fitness function 2, which suggests the penalty was not sufficient for a better overall schedule to be found.

In general, the algorithm is already capable of generating feasible solutions, however some adjustments are still necessary, according to the DM, as will be further detailed in Section 6.2.2.

6.2 HL feedback

The results obtained were presented to the hospital in order to assess the usability of the solutions obtained by the proposed algorithm. In this section, the feedback received by the HL will be presented, highlighting the strengths and weaknesses of the algorithm. For both ICU and IS this analysis was made by the respective DM.

6.2.1 ICU Feedback

The solutions were presented and discussed with HL, in order to assess the applicability of the algorithm and the quality of the solutions obtained. This discussion was held with one of the ICU physicians (the DM), who recognizes well the present problem and has the necessary knowledge about the schedule features to make a fair assessment.

During the discussion it was possible to identify the strengths of the solutions generated and to realize which aspects should be improved, or others that should be incorporated, to increase the applicability of the solutions.

Overall, the physician is satisfied with the solutions presented and the feedback was positive. According to him/her, the proposed solution is already sufficiently balanced between elements. He/she highlights the fact that it takes into account not only the month under construction, but also the previous ones. He/she considers this is a very important aspect, as it contributes to long-term fairness and allows to compensate less benefited physicians in previous schedules.

The physician found the idea of trying to maximize the number of days-off after a 24-hour shift (one of the aspects validated in preferences) interesting, considering that it can increase the personal satisfaction of each physician. Another strength is the possibility of fixing the elements on certain days and rebalancing the schedule, as it happens, for example, in the Christmas rotation or with external physicians. More importantly, in addition to these aspects, the algorithm can present good solutions within a reasonable time.

Some aspects to consider are the construction of a method that allows the schedule to be generated by a user other than the programmer. This could be achieved by means of creating a practical interface, capable of being used by those responsible to develop the schedule. Another suggestion is to receive not only the schedule as output, but also a report containing the main characteristics used: vacation days, unavailability of physicians, etc. The physician considers that this last point allows for a better understanding of the results by those who analyze the schedule, avoiding doubts and mistrust.

The algorithm shows a lot of potential in developing feasible solutions and manages to solve some of the verified problems, such as the time spent in the scheduling task, the appearance of inconsistencies and the incorporation of preferences in a balanced way.

6.2.2 IS Feedback

The (preliminary) results obtained were also presented to the IS. Here the assessment was made by the technician responsible for administrative services (the DM), namely the construction of the schedule every month.

Some schedule proposals were presented to the IS, however, some characteristics are incomplete and could not be concluded, namely the schedule of the extra clinics, because information from the other complementary hospital HBA was not available.

In general, the technician confirmed that the scheduling task is a problem in his/her service, and that it allocates two technicians at least two days a month just for its construction. In a final analysis, and concerning the results presented in the previous section, the technician highlighted some strengths, namely:

- The algorithm makes this task practically automatic and takes into account the major objectives of the service.
- It presents feasible solutions in a timely manner.
- It can assign the number of mornings and afternoons correctly. Those that sometimes are allocated in excess can be used for allocation in other clinics, in which they are needed.

However, in this case there are still some points to be improved and suggestions were also given. The technician suggested that extra shifts should be highlighted in another color. For example a person who was initially assigned to an M8 shift and then re-assigned an MT21 shift, due to the hospital needs requiring an extra shift. A weak point also identified is in the rotations, which are quite changed for some technicians. They should be as preserved as most as possible and not be so random; changes must be seldom and more equitable.

6.3 Chapter considerations

In this chapter, a comparative study was performed between the solutions obtained by the algorithm and those built by the scheduler in HL. Several benefits are identified in the application of the present algorithm. It is also possible to adjust the algorithm to a different service simply by changing its objectives.

This study was presented to elements of the hospital who showed their satisfaction, although there are still aspects to improve, in particular the creation of an interface that allows the algorithm to be manipulated by usual schedulers. The main advantages highlighted are: (i) the opportunity of greatly reducing the time used in the execution of the scheduling task – a task that could take about two days, is reduced to some minutes, and the only effort needed is the insertion of month characteristics; (ii) the increase of personnel satisfaction – not only it incorporates aspects such as fairness and preferences, but also impartial solutions are presented; and (iii) the decrease in the number of inconsistencies.

In the next chapter, the main conclusions of this work will be addressed, namely its strengths and limitations, and an overview of future work will be provided.

Chapter 7

Conclusions and Future Work

This chapter provides the conclusions and main accomplishments of this thesis and the discussion of the limitations found, as well as the future work suggestions.

7.1 Conclusions

Staff-related costs represent more than half of hospitals' operating costs, besides being a scarce resource. This implies the search for strategies to improve its efficiency. This thesis addressed a scheduling problem in a hospital context, focused mainly on the ICU of HL, although it has also been implemented for the IS.

The main goal of this thesis was to develop, propose and validate an algorithm that was able to optimize the human resources scheduling in a timely manner, without inconsistencies, respecting all constraints and incorporating the main objectives of each service. In the case of the ICU these were to find a balance in fairness and satisfaction of preferences, while in the IS the biggest focus was the preference satisfaction, both individual and the hospital's. A general decision support system, based on NSGA-II, was developed, incorporating as many factors as possible. For this purpose, the criteria and needs of each service were identified. A common factor was the satisfaction of the hospital's needs, which is essential to achieve feasible solutions. The algorithm was able to respect the needs and hard constraints and shows a tendency to decrease the violation of soft constraints. It is capable of presenting several non-dominated solutions and choosing one according to the defined ranking. In the ICU this was achieved through the results of a survey made to the physicians. This revealed that the factors they considered most important are fairness (that is, working the same number of hours and weekends but also compensating physicians who have been affected in previous months), preferences and, finally, the ideal filling of shifts.

Overall, the solutions generated for the ICU by the algorithm showed better results for most criteria, except in relation to the average number of hours worked, however this can be justified by the increase in the number of shifts ideally filled, and the values found are not higher than those considered feasible. In addition, it was still possible to validate that the algorithm adapts to all months of the year, identifying holidays and weekends, adjusting its results to a variable number of physicians, both internal and external.

Through the results it was concluded that, regardless of using information from the past or not, the algorithm tends to find a balance between physicians concerning the previous months and the number of hours and types of shifts worked among them. This is a very relevant factor because it can contribute to increased satisfaction, which leads to a greater performance, avoids conflicts between colleagues, reduces the number of absences, and results in a lower turnover rate.

The feedback and validation from the HL was positive, both strengths and weaknesses were highlighted. The most appreciated contribution was to reduce the time of a task that took two days to a few minutes while improving some results, incorporating fairness and preferences. There are still some aspects to improve (addressed in the next section), however they can be overcome.

7.2 Limitations and Future Work

The strongest limiting factor for this work came with the COVID-19 pandemic, which is still being experienced and firstly hit Portugal around the time this work was starting. Like most connections and relationships, the remote circumstances in which this thesis was developed led to different approaches to the work performed, since it was more difficult to collect information and the exchange of ideas ended up being slower than if the contacts were made in person. Access to the hospital was restricted, which did not allow, for example, a closer contact with healthcare providers at an early stage. Due to this situation, some of the assumptions had to be based on the literature and on the opinion of a single person. However, it was found through the survey carried out later that these met the interests of the ICU staff.

In the future, it will be important to validate the satisfaction of all personnel with the algorithm schedules. Afterwards, the implementation of a proper user interface will bring added value. At the moment the algorithm is able to produce good solutions, but a programmer is required. In particular, the input concerning vacations, availability restrictions, etc., is done by hand and a way to do this through the import of files would greatly improve the usability of the solution method proposed.

Finally, the ICU has started receiving resident physicians over the last months, when this work was already underway. Thus, in the future, it would be interesting to incorporate their scheduling, which is a less discussed topic in literature. As for the IS, it would be important to incorporate a feature that allows the schedules of other clinics to be generated. It was not possible to achieve this contribution due to lack of information.

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Appendix A

Appendix chapter

Algorithm 4 provides the main steps for a generation of the ICU chromosome. In Algorithm 5 repair function of the chromosomes is presented.

Table A.1 provides the answers obtained from the survey by each physician. Each criterion was compared with all the others. Once 5 criteria were defined, 10 comparisons had to be performed.

In Algorithm 6 the main steps of generation of IS chromosomes for the initial population are provided and Algorithm 7 provides the repair procedure applied after the chromosome construction.

Algorithm 4 Generation of chromosomes for the initial population.

```
Create an empty chromosome of dimensions: number of physicians $\times$ days to be scheduled
for each i in 0,...,days to be scheduled do
  if day is weekend day or National holiday then
    if an external physician is available then
      Randomly choose $n_w - 1$ or $n_w - 2$ physicians and assign them to a weekend shift
    else
      Randomly choose $n_w$ or $n_w - 1$ physicians and assign them to a weekend shift
    end if
  else
    if an external physician is available then
      if external physician is assigned to a night shift then
        Randomly choose $n_p$ available physician to assign a prolongation shift
        Remove chosen physician from available physicians
        Randomly choose $n_m$ available physicians to assign morning shifts
        Randomly choose $n_n - 1$ or $n_n - 2$ available physician to assign a night shift
        if one of the physicians in the morning is equal to the one in the night shift then
          Remove from night and morning shift and assign him/her a 24-hour shift
        end if
      else if external physician is assigned to a 24-hour shift then
        Randomly choose $n_p$ available physician to assign a prolongation shift
        Remove chosen physician from available physicians
        Randomly choose $n_m - 1$ available physicians to assign morning shifts
        Randomly choose $n_n - 1$ or $n_n - 2$ available physician to assign a night shift
        if one of the physicians in the morning is equal to the one in the night shift then
          Remove from night and morning shift and assign him a 24-hour shift
        end if
      end if
    else
      Randomly choose $n_p$ available physician to assign a prolongation shift
      Remove chosen physician from available physicians
      Randomly choose $n_m$ available physicians to assign morning shifts
      Randomly choose $n_n$ or $n_n - 1$ available physician to assign a night shift
      if one of the physicians in the morning is equal to the one in the night shift then
        Remove from night and morning shift and assign him a 24-hour shift
      end if
    end if
  end if
end for
if a hard constraint is violated then
  Repair the chromosome
end if
```

Algorithm 5 Repair of ICU chromosomes at the end of their creation.

```
for each in in 0,..., physicians do
  for each j in 0, ..., days to be scheduled $-$ 1 do
    if in chromosome[$ij$] the shift is a weekend or a 24-hour shift and the next day has a shift assigned then
      Search for a physician who has a day-off on day $j$ and $j+1$
      if there is a physician in these conditions then
        Assign him/her randomly the weekend or 24-hour shift in day $j$
        Assign to physician $i$ the day-off shift
      else
        Search for a physician assigned to a morning or prolongation shift with a day-off in the next day
        Assign him the weekend or 24-hours shift
        Assign to physician $i$ the morning or prolongation shift
      end if
    else if in chromosome[$ij$] the shift is a night and the next day has a shift assigned then
      Search for a physician who has a day-off on day $j$ and $j+1$
      if there is a physician in these conditions then
        Assign him the night shift in day $j$
        Assign to physician $i$ a day-off
      else
        Search randomly a physician assigned to a morning or prolongation shift with a day-off in the next day
        Assign him the night shift
        Assign to physician $i$ the morning or prolongation shift
      end if
    end if
  end for
end for
```

Table A.1: Percentage of each answer to the questions that confront criteria in the survey.

	Options	Answers
(i)	Reduce the difference in the number of hours worked between physicians.	37.5%
	Increase the number of preferences satisfied for each physician.	37.5%
	Indifferent.	25.0%
(ii)	Fill in the maximum number of shifts in an ideal way.	25.0%
	Reduce the differences between the number of hours worked among physicians.	75.0%
	Indifferent	00.0%
(iii)	There is less time difference worked among physicians.	50.0%
	There is less difference between the number of weekends worked.	37.5%
	Indifferent	12.5%
(iv)	A physician who has been most affected in the previous months should be compensated, working less hours.	100.0%
	All physicians work the same number of hours (including which was prejudiced in the previous month).	00.0%
	Indifferent	00.0%
(v)	A physician who has been most affected in the previous months should be compensated, working less hours.	87.5%
	Maximize the number of shifts ideally filled.	12.5%
	Indifferent.	00.0%
(vi)	Compensate affected physicians in the previous month.	62.5%
	Satisfy the preferences of each physician.	12.5%
	Indifferent.	25.0%
(vii)	Assign fewer weekends to a physician who has been less benefited.	50.0%
	All physicians work the same number of weekends.	25.0%
	Indifferent.	25.0%
(viii)	Minimize the differences of number of weekends between physicians.	25.0%
	Maximize the number of preferences satisfied.	50.0%
	Indifferent.	25.0%
(ix)	To satisfy the maximum of physicians’ preferences.	50.0%
	Fill in the maximum number of shifts in an ideal way.	37.5%
	Indifferent	12.5%
(x)	Reduce the differences of number of weekends between physicians.	37.5%
	Maximize the number of shifts ideally filled.	25.0%
	Indifferent.	37.5%

Algorithm 6 Generation of IS chromosomes for the initial population.

```
Join the three partial chromosomes
for each i in 0,...,days to be scheduled do
    Count the number of technicians in morning, afternoon and all-day shifts
    if number of mornings $<$ hospital's needs then
        if number of afternoons $<$ hospital's needs then
            Insert a MT21 shift to a random technician
        else if number of afternoons $>$ hospital's needs then
            Select a random technician assigned to an afternoon shift, and replace the shift with a morning shift
        else if there is a technician with a day-off on day $i$ then
            Assign a morning shift to this technician
            Attribute the day-off in other day where the needs can be fulfilled
        else
            Choose a random technician, and assign a MT21 shift
        end if
    else if number of afternoons shifts $<$ hospital's needs then
        if number of mornings $>$ hospital's needs then
            Select a random technician assigned to a morning shift, and replace the shift with a afternoon shift
        else
            Insert a MT21 shift to a random technician (previous assigned to a morning shift)
        end if
    else if number of all-day shifts $<$ hospital's needs then
        if $i$ is a weekday then
            if there is a technician with a day-off in day $i$ then
                Select a random technician with a day-off, assign him/her an all-day shift (I10)
                Insert to this technician a M6 shift to another day
            else
                A warning to hire someone external will arise
            end if
        else
            if there is a technician with a day-off in day $i$ then
                Select a random technician with a day-off, assign him/her a weekend shift
                Insert to this technician a day-off and a M6 shift to another day
            else
                A warning to hire someone external will arise
            end if
        end if
    end if
end for
if a hard constraint is violated then
    Repair the chromosome
end if
```

Algorithm 7 Repair of IS chromosomes at the end of their creation.

```
for each $i$ in 0,..., technician do
    if technician $i$ works more than 6 consecutive days then
        Select a random day within those days
        Assign a day-off on the selected day
        Assign the substituted shift to another available technician
    end if
    for each $j$ in 0, ..., days to be scheduled $-$ 1 do
        if end time of shift[$ij$] - start time of shift[$ij + 1$] $<$ 11 hours then
            Find the technicians with end time of shift [$ij + 1$] $>$ start time
            Select one of them randomly
            Assign him/her the shift [$ij + 1$]
            Assign to technician $i$ the substituted shift
        end if
    end for
    Count the number of days-off (besides weekends)
    Count the number of weekends worked
    Count the number of M6 shifts
    if Number of M6 shifts $<$ Number of weekend shifts then
        Select a random M8 shift in $i$
        Replace the shift with M6
    end if
    if Number of days-off $<$ Number of weekend shifts then
        Select a random day
        Assign a day-off
        Check needs
    end if
end for
for each $j$ in 0, ..., days to be scheduled do
    Count the daily number of A8 shifts
    while count $< 3$ do
        Select a random M8 shift in $j$
        Replace the shift with A8
    end while
end for
```

Appendix B

Appendix chapter

Table B.1 provides the necessary information used to perform the results presented in Section 5.3.

Table B.1: Real characteristics used in the construction of the schedules of each month.

	Holidays	Absences	Restrictions			External Physicians
			Days		Not available for shifts	
January	P10 - 1 day	P5, P7, P14 all month	Monday:	P4	Night	Not assigned
			Tuesday:	P2	Prolongation	
				P7	Prolongation and night	
				P4	all day	
			Thursday	P2	Prolongation	
			Every day	P5	Morning and prolongation	
February	P4 - 2 days P10 - 2 days	P7 all month	Same as last month			Not assigned
March	P9 - 5 days	P7 all month	Same as last month			PE1: Night shift (12) Weekend shift (22)
April	-	-	Same as last month			PE1: Night shifts (1,9,17,21,29) PE2: Night shifts (2) PE3: Night shifts (27, 30) PE4: Night Shifts (23) Weekend Shifts (18)
May	-	-	Same as last month			PE1: Night shifts (7,15,19,27) PE2: Night shifts (4,20,28)

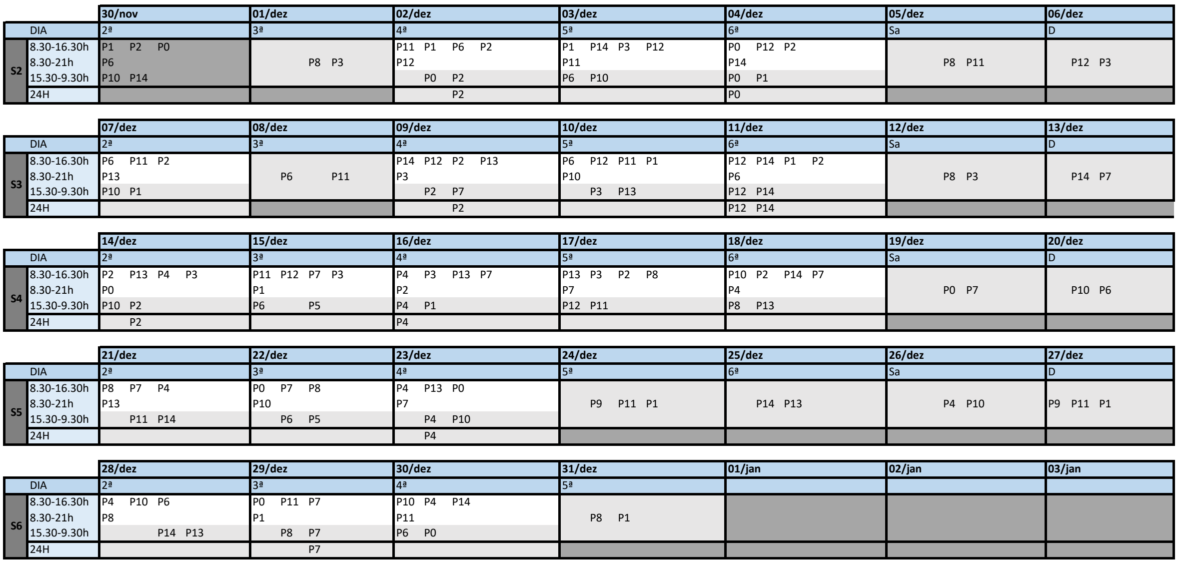
Table B.1 continued from previous page

June	P0 - 8 days P1 - 2 days P2 - 3 days P4 - 3 days P5 - 2 days P9 - 7 days P10 - 2 days P12 - 3 days P14 - 5 days	-	Same as last month			PE1: Night shifts (4,12,23)
July	P1 - 3 days P6 - 13 days P7 - 5 days P8 - 15 days P9 - 3 days P11 - 11 days	-	Same as last month			PE1: Weekend shifts (18)
August	P2 - 6 days P3 - 12 days P4 - 11 days P6 - 10 days P10 - 8 days P11 - 10 days P12 - 1 day P13 - 3 days	-	Monday:	P4	Night	Not assigned
			Tuesday:	P2	Prolongation	
				P7	Prolongation and night	
				P4	all day	
			Wednesday	P8	Night	
				P14	Prolongation and night	
			Thursday	P2	Prolongation	
				P8	Morning and Prolongation	
				P14	Prolongation	
			Every day	P5	Morning and prolongation	
September	P1 - 3 days P2 - 9 days P4 - 4 days P7 - 10 days P8 - 2 days P10 - 1 day P11 - 5 days P12 - 9 days P13 - 15 days P14 - 2 days	-	Monday:	P4	Night	Not assigned
			Tuesday:	P2	Prolongation	
				P7	Prolongation and night	
				P4	all day	
			Wednesday	P8	Night	
			Thursday	P2	Prolongation	
				P8	Morning and Prolongation	
			Every day	P5	Morning and prolongation	
October	P1 - 2 days P8 - 2 days P9 - 4 days P10 - 5 days P11 - 2 days P14 - 5 days	-	Same as last month			PE2: Night shift (16)

Table B.1 continued from previous page

November	P0 - 5 days P1 - 10 days P4 - 1 day P7 - 6 days P9 - 1 day P11 - 1 day P12 - 2 days P13 - 1 day P14 - 11 days	-	Same as last month	Not assigned
December	P0 - 4 days P3 - 8 days P4 - 7 days P7 - 4 days P9 - 8 days P12 - 8 days P13 - 4 days	-	Same as last month	Not assigned

Figure B.1 presents the December schedule made in the ICU. This schedule served as a comparison for the validation of the algorithm solution, as described in Section 6.1.1.



		30/nov	**01/dez**	**02/dez**	**03/dez**	**04/dez**	**05/dez**	**06/dez**
	DIA	2ª	3ª	4ª	5ª	6ª	Sa	D
S2	8.30-16.30h	P1 P2 P0	P8 P3	P11 P1 P6 P2	P1 P14 P3 P12	P0 P12 P2	P8 P11	P12 P3
	8.30-21h	P6		P12	P11	P14		
	15.30-9.30h	P10 P14		P0 P2	P6 P10	P0 P1		
	24H			P2		P0		

		07/dez	**08/dez**	**09/dez**	**10/dez**	**11/dez**	**12/dez**	**13/dez**
	DIA	2ª	3ª	4ª	5ª	6ª	Sa	D
S3	8.30-16.30h	P6 P11 P2	P6 P11	P14 P12 P2 P13	P6 P12 P11 P1	P12 P14 P1 P2	P8 P3	P14 P7
	8.30-21h	P13		P3	P10	P6		
	15.30-9.30h	P10 P1		P2 P7	P3 P13	P12 P14		
	24H			P2		P12 P14		

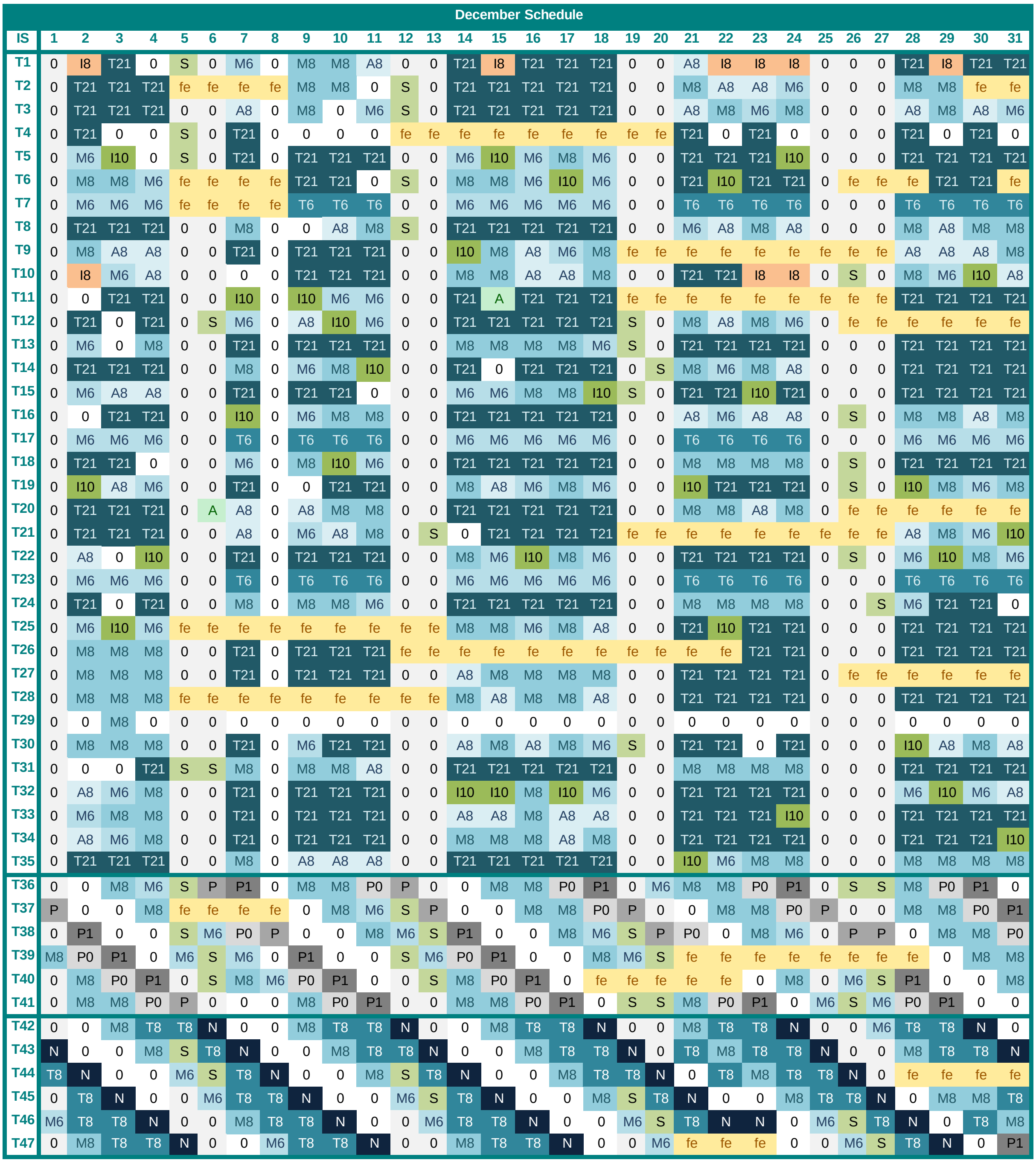
		14/dez	**15/dez**	**16/dez**	**17/dez**	**18/dez**	**19/dez**	**20/dez**
	DIA	2ª	3ª	4ª	5ª	6ª	Sa	D
S4	8.30-16.30h	P2 P13 P4 P3	P11 P12 P7 P3	P4 P3 P13 P7	P13 P3 P2 P8	P10 P2 P14 P7	P0 P7	P10 P6
	8.30-21h	P0	P1	P2	P7	P4		
	15.30-9.30h	P10 P2	P6 P5	P4 P1	P12 P11	P8 P13		
	24H	P2		P4				

		21/dez	**22/dez**	**23/dez**	**24/dez**	**25/dez**	**26/dez**	**27/dez**
	DIA	2ª	3ª	4ª	5ª	6ª	Sa	D
S5	8.30-16.30h	P8 P7 P4	P0 P7 P8	P4 P13 P0	P9 P11 P1	P14 P13	P4 P10	P9 P11 P1
	8.30-21h	P13	P10	P7				
	15.30-9.30h	P11 P14	P6 P5	P4 P10				
	24H			P4				

		28/dez	**29/dez**	**30/dez**	**31/dez**	**01/jan**	**02/jan**	**03/jan**
	DIA	2ª	3ª	4ª	5ª			
S6	8.30-16.30h	P4 P10 P6	P0 P11 P7	P10 P4 P14	P8 P1			
	8.30-21h	P8	P1	P11				
	15.30-9.30h	P14 P13	P8 P7	P6 P0				
	24H		P7					

Figure B.1: Schedule built by DM in ICU: December

Figure B.2 presents the December schedule generated by the proposed algorithm.



December Schedule

IS	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
T1	0	I8	T21	0	S	0	M6	0	M8	M8	A8	0	0	T21	I8	T21	T21	T21	0	0	A8	I8	I8	I8	0	0	0	T21	I8	T21	T21
T2	0	T21	T21	T21	fe	fe	fe	fe	M8	M8	0	S	0	T21	T21	T21	T21	T21	0	0	M8	A8	A8	M6	0	0	0	M8	M8	fe	fe
T3	0	T21	T21	T21	0	0	A8	0	M8	0	M6	S	0	T21	T21	T21	T21	T21	0	0	A8	M8	M6	M8	0	0	0	A8	M8	A8	M6
T4	0	T21	0	0	S	0	T21	0	0	0	0	fe	fe	fe	fe	fe	fe	fe	fe	fe	T21	0	T21	0	0	0	0	T21	0	T21	0
T5	0	M6	I10	0	S	0	T21	0	T21	T21	T21	0	0	M6	I10	M6	M8	M6	0	0	T21	T21	T21	I10	0	0	0	T21	T21	T21	T21
T6	0	M8	M8	M6	fe	fe	fe	fe	T21	T21	0	S	0	M8	M8	M6	I10	M6	0	0	T21	I10	T21	T21	0	fe	fe	fe	T21	T21	fe
T7	0	M6	M6	M6	fe	fe	fe	fe	T6	T6	T6	0	0	M6	M6	M6	M6	M6	0	0	T6	T6	T6	T6	0	0	0	T6	T6	T6	T6
T8	0	T21	T21	T21	0	0	M8	0	0	A8	M8	S	0	T21	T21	T21	T21	T21	0	0	M6	A8	M8	A8	0	0	0	M8	A8	M8	M8
T9	0	M8	A8	A8	0	0	T21	0	T21	T21	T21	0	0	I10	M8	A8	M6	M8	fe	fe	fe	fe	fe	fe	fe	fe	fe	A8	A8	A8	M8
T10	0	I8	M6	A8	0	0	0	0	T21	T21	T21	0	0	M8	M8	A8	A8	M8	0	0	T21	T21	I8	I8	0	S	0	M8	M6	I10	A8
T11	0	0	T21	T21	0	0	I10	0	I10	M6	M6	0	0	T21	A	T21	T21	T21	fe	fe	fe	fe	fe	fe	fe	fe	fe	T21	T21	T21	T21
T12	0	T21	0	T21	0	S	M6	0	A8	I10	M6	0	0	T21	T21	T21	T21	T21	S	0	M8	A8	M8	M6	0	fe	fe	fe	fe	fe	fe
T13	0	M6	0	M8	0	0	T21	0	T21	T21	T21	0	0	M8	M8	M8	M8	M6	S	0	T21	T21	T21	T21	0	0	0	T21	T21	T21	T21
T14	0	T21	T21	T21	0	0	M8	0	M6	M8	I10	0	0	T21	0	T21	T21	T21	0	S	M8	M6	M8	A8	0	0	0	T21	T21	T21	T21
T15	0	M6	A8	A8	0	0	T21	0	T21	T21	0	0	0	M6	M6	M8	M8	I10	S	0	T21	T21	I10	T21	0	0	0	T21	T21	T21	T21
T16	0	0	T21	T21	0	0	I10	0	M6	M8	M8	0	0	T21	T21	T21	T21	T21	0	0	A8	M6	A8	A8	0	S	0	M8	M8	A8	M8
T17	0	M6	M6	M6	0	0	T6	0	T6	T6	T6	0	0	M6	M6	M6	M6	M6	0	0	T6	T6	T6	T6	0	0	0	M6	M6	M6	M6
T18	0	T21	T21	0	0	0	M6	0	M8	I10	M6	0	0	T21	T21	T21	T21	T21	0	0	M8	M8	M8	M8	0	S	0	T21	T21	T21	T21
T19	0	I10	A8	M6	0	0	T21	0	0	T21	T21	0	0	M8	A8	M6	M8	M6	0	0	I10	T21	T21	T21	0	S	0	I10	M8	M6	M8
T20	0	T21	T21	T21	0	A	A8	0	A8	M8	M8	0	0	T21	T21	T21	T21	T21	0	0	M8	M8	A8	M8	0	fe	fe	fe	fe	fe	fe
T21	0	T21	T21	T21	0	0	A8	0	M6	A8	M8	0	S	0	T21	T21	T21	T21	fe	fe	fe	fe	fe	fe	fe	fe	fe	A8	M8	M6	I10
T22	0	A8	0	I10	0	0	T21	0	T21	T21	T21	0	0	M8	M6	I10	M8	M6	0	0	T21	T21	T21	T21	0	S	0	M6	I10	M8	M6
T23	0	M6	M6	M6	0	0	T6	0	T6	T6	T6	0	0	M6	M6	M6	M6	M6	0	0	T6	T6	T6	T6	0	0	0	T6	T6	T6	T6
T24	0	T21	0	T21	0	0	M8	0	M8	M8	M6	0	0	T21	T21	T21	T21	T21	0	0	M8	M8	M8	M8	0	0	S	M6	T21	T21	0
T25	0	M6	I10	M6	fe	fe	fe	fe	fe	fe	fe	fe	fe	M8	M8	M6	M8	A8	0	0	T21	I10	T21	T21	0	0	0	T21	T21	T21	T21
T26	0	M8	M8	M8	0	0	T21	0	T21	T21	T21	fe	fe	fe	fe	fe	fe	fe	fe	fe	fe	fe	T21	T21	0	0	0	T21	T21	T21	T21
T27	0	M8	M8	M8	0	0	T21	0	T21	T21	T21	0	0	A8	M8	M8	M8	M8	0	0	T21	T21	T21	T21	0	fe	fe	fe	fe	fe	fe
T28	0	M8	M8	M8	fe	fe	fe	fe	fe	fe	fe	fe	fe	M8	A8	M8	M8	A8	0	0	T21	T21	T21	T21	0	0	0	T21	T21	T21	T21
T29	0	0	M8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
T30	0	M8	M8	M8	0	0	T21	0	M6	T21	T21	0	0	A8	M8	A8	M8	M6	S	0	T21	T21	0	T21	0	0	0	I10	A8	M8	A8
T31	0	0	0	T21	S	S	M8	0	M8	M8	A8	0	0	T21	T21	T21	T21	T21	0	0	M8	M8	M8	M8	0	0	0	T21	T21	T21	T21
T32	0	A8	M6	M8	0	0	T21	0	T21	T21	T21	0	0	I10	I10	M8	I10	M6	0	0	T21	T21	T21	T21	0	0	0	M6	I10	M6	A8
T33	0	M6	M8	M8	0	0	T21	0	T21	T21	T21	0	0	A8	A8	M8	A8	A8	0	0	T21	T21	T21	I10	0	0	0	T21	T21	T21	T21
T34	0	A8	M6	M8	0	0	T21	0	T21	T21	T21	0	0	M8	M8	M8	A8	M8	0	0	T21	T21	T21	T21	0	0	0	T21	T21	T21	I10
T35	0	T21	T21	T21	0	0	M8	0	A8	A8	A8	0	0	T21	T21	T21	T21	T21	0	0	I10	M6	M8	M8	0	0	0	M8	M8	M8	M8
T36	0	0	M8	M6	S	P	P1	0	M8	M8	P0	P	0	0	M8	M8	P0	P1	0	M6	M8	M8	P0	P1	0	S	S	M8	P0	P1	0
T37	P	0	0	M8	fe	fe	fe	fe	0	M8	M6	S	P	0	0	M8	M8	P0	P	0	0	M8	M8	P0	P	0	0	M8	M8	P0	P1
T38	0	P1	0	0	S	M6	P0	P	0	0	M8	M6	S	P1	0	0	M8	M6	S	P	P0	0	M8	M6	0	P	P	0	M8	M8	P0
T39	M8	P0	P1	0	M6	S	M6	0	P1	0	0	S	M6	P0	P1	0	0	M8	M6	S	fe	fe	fe	fe	fe	fe	fe	fe	0	M8	M8
T40	0	M8	P0	P1	0	S	M8	M6	P0	P1	0	0	S	M8	P0	P1	0	fe	fe	fe	fe	fe	0	M8	0	M6	S	P1	0	0	M8
T41	0	M8	M8	P0	P	0	0	0	M8	P0	P1	0	0	M8	M8	P0	P1	0	S	S	M8	P0	P1	0	M6	S	M6	P0	P1	0	0
T42	0	0	M8	T8	T8	N	0	0	M8	T8	T8	N	0	0	M8	T8	T8	N	0	0	M8	T8	T8	N	0	0	M6	T8	T8	N	0
T43	N	0	0	M8	S	T8	N	0	0	M8	T8	T8	N	0	0	M8	T8	T8	N	0	T8	M8	T8	T8	N	0	0	M8	T8	T8	N
T44	T8	N	0	0	M6	S	T8	N	0	0	M8	S	T8	N	0	0	M8	T8	T8	N	0	T8	M8	T8	T8	N	0	fe	fe	fe	fe
T45	0	T8	N	0	0	M6	T8	T8	N	0	0	M6	S	T8	N	0	0	M8	S	T8	N	0	0	M8	T8	T8	N	0	M8	M8	T8
T46	M6	T8	T8	N	0	0	M8	T8	T8	N	0	0	M6	T8	T8	N	0	0	M6	S	T8	N	N	0	M6	S	T8	N	0	T8	M8
T47	0	M8	T8	T8	N	0	0	M6	T8	T8	N	0	0	M8	T8	T8	N	0	0	M6	fe	fe	fe	0	0	M6	S	T8	N	0	P1

Figure B.2: IS December Schedule generated by the algorithm.