# Monitoring and forecasting hospital performance in Portugal

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## Biomedical Engineering

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# Declaration

I declare that this document is an original work of my own authorship and that it fulfills all the requirements of the Code of Conduct and Good Practices of the Universidade de Lisboa.

# Preface

The work presented in this thesis was performed at the Institute for Bioengineering and Biosciences of Instituto superior Técnico (Lisbon, Portugal), during the period February-December 2020, under the supervision of Prof. Diogo Filipe da Cunha Ferreira and co-supervision of Prof. Alexandre Morais Nunes.

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The process of writing and working on this dissertation reflected my journey at IST: not only in the hard work and persistence necessary but also because it could not have been done without the support and encouragement of several people.

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Education starts early in life, so I need to thank the people who have accompanied me through it: my dad and my sisters who are great listeners and have always helped me to never give up. A special thanks to my mother who taught me values of responsibility and hard work that will prevail even though she is not around. To the rest of my family, thank you for supporting me through this journey and showing me that with persistence and a bit of faith everything can be done. Family can be also found in friends, so to Maria and Catarina thank you for being there for me every step of the way. To my high school friends, you have helped me to keep my feet on the ground, so thank you for reminding me that there is more to life than university and everything can be done at our own pace.

My path in this university and during the completion of this work could not have been done without the support of friends that I want to acknowledge here: Leonor and Sofia for making it possible to make it to this point, especially in the first years, Margarida and Isabel for reassuring me that a just a few words before going home are enough to make it through. And to all the other friends I made during this journey, without you I don't think I could have made it through, so thank you.

**Abstract**

A great deal of resources is applied to the health sector in Portugal, with a considerable part going to public hospitals. Therefore, a lot of attention has been drawn to public hospitals' efficiency and productivity analysis. This work aims to assess and predict the performance of Portuguese public hospitals. Data Envelopment Analysis is used to calculate hospital efficiency, and the Malmquist Productivity Index to evaluate hospital productivity. A sample of 26 public hospitals and hospital centers with data from 2013 to 2017 was used for this analysis. The Malmquist Productivity Index was forecasted for 2018 and then compared, for some hospitals, with the real values. The performance of hospitals has been slowly increasing, with overall average DEA score considering CRS being 0.648 and under VRS 0.764. Hospital efficiency seems to be increasing throughout the years, as well as scale efficiency. In terms of productivity, the MPI shows seasonality, presenting high peaks in May-June. The overall average MPI is 1.049, suggesting productivity increase but not presenting a clear trend. The terms regarding changes in technology seem to influence more the MPI than the ones considering efficiency changes. The forecasted MPI suggested a very small increase for the year 2018 but the forecast did not seem to present reliable enough results.

**Keywords:** Efficiency, Productivity, Data Envelopment Analysis, Malmquist Productivity Index, Portuguese public hospitals

**Resumo**

Uma grande quantidade de recursos é aplicada ao setor saúde em Portugal, sendo uma parte considerável para hospitais públicos. Assim, bastante atenção tem sido dada à análise de eficiência e produtividade destes hospitais. Este trabalho tem como objetivo avaliar e prever o desempenho dos hospitais públicos portugueses. A técnica *Data Envelopment Analysis* é utilizada para calcular a eficiência dos hospitais e o método *Malmquist Productivity Index* para avaliar o seu desempenho. Uma amostra de 26 hospitais públicos e centros hospitalares com dados de 2013 a 2017 foi utilizada para esta análise. Foi feita a previsão do *Malmquist Productivity Index* para 2018 e depois comparado, para alguns hospitais, com os valores reais. O desempenho dos hospitais tem aumentado lentamente, sendo que o valor médio da eficiência considerando CRS foi 0,648 e considerando VRS 0,764. A eficiência hospitalar parece estar a aumentar ao longo do tempo, assim como a eficiência de escala. Em termos de produtividade, o MPI mostra sazonalidade, apresentando picos elevados em maio-junho. O MPI médio é de 1,049, sugerindo um aumento de produtividade, mas sem apresentar uma tendência clara. Os termos relativos à variação da tecnologia parecem influenciar mais o MPI do que aqueles que consideram a variação da eficiência. O MPI previsto sugeria um aumento muito pequeno para o ano de 2018, mas a previsão não parece apresentar resultados suficientemente fiáveis.

**Keywords:** Eficiência, Produtividade, Data Envelopment Analysis, Malmquist Productivity Index, Hospitais públicos portugueses

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# Acronyms

| | |
|---|---|
| **ACSS** | *Administração Central do Sistema de Saúde*. |
| **ARIMA** | Auto Regressive Integrated Moving Average. |
| **ARMA** | Auto Regressive Moving Average. |
| **CCM** | Clinical Consumption Material. |
| **CMI** | Case Mix Index. |
| **CRS** | Constant Returns to Scale. |
| **DEA** | Data Envelopment Analysis. |
| **DMU** | Decision Making Unit. |
| **DRG** | Diagnosis Related Group. |
| **EPE** | *Entidade Pública Empresarial*. |
| **ESS** | External Services and Supplies. |
| **FDH** | Free Disposal Hull. |
| **FTE** | Full-Time Equivalent. |
| **GDP** | Gross Domestic Product. |
| **INE** | *Instituto Nacional de Estatística*. |
| **IPO** | *Instituto Português de Oncologia*. |
| **MLGRT** | Maximum Legal Guaranteed Response Time. |
| **MPI** | Malmquist Productivity Index. |
| **NHS** | National Health Service. |
| **NPM** | New Public Management. |
| **OECD** | Organisation for Economic Co-operation and Development. |
| **PCA** | Principal Component Analysis. |
| **PPP** | Public–Private Partnership. |
| **RHA** | Regional Health Administration. |

**SA** *Sociedade Anónima*.
**SFA** Stochastic Frontier Analysis.
**SMI** Service Mix Index.
**SNS** *Sistema Nacional de Saúde*.
**SPA** *Setor Público Administrativo*.

**VRS** Variable Returns to Scale.

# Chapter 1

# Introduction

## 1.1 Motivation

Health is one of the most powerful factors in social integration, but also in generating wealth and well-being.

In the last years, according to Organisation for Economic Co-operation and Development (OECD)'s report "Tackling Wasteful Spending on Health", health care systems in OECD countries have been getting better at promoting health. However, this involves major commitments regarding budget that countries struggle to keep under control.[1]

The changing demographic profile, increased complexity of diseases and technological development have led to new problems being faced by the health sector of all countries.

Portuguese life expectancy has been growing, being 81.1 years in 2017 and exceeding the European Union's average growth. Together with an aging population - a million Portuguese were over 75 years in 2018 - it represents a new scenario for the delivery of health care.[2] It is an indicator of the better living conditions, improved provision of quality health care and the decreasing prevalence of some diseases. However, it leads to an increase in the demand and consumption of health resources by the elder and an increase of chronic diseases.

Moreover, with employment and wages being determinant factors, conditioning access to essential goods and services and to health and well-being, as well as increasing the quality of life, the fact that the minimum wage has been increasing the and the unemployment decreasing leads to more informed and demanding citizens.

There is also an increased complexity of diseases, as well as new treatment and diagnostic tools due to therapeutic and technological innovation, which may contribute both to the population living longer and higher expenditures for the health institutions.

These new demographics, socioeconomic conditions and technological progress generates the need for the health sector to adapt to new problems that are presented different than the ones existing before, putting a great amount of pressure on the health sector.

Portugal spent around 9.4% of the Gross Domestic Product (GDP) in health expenditure in 2018 (provisory value) and 9.4% in 2019 (preliminary value).[3] However, data are provisional for 2018 and preliminary for 2019. So, the most recent final and validated data is from 2017. In this year, 9.3% of the

[1] OECD (2017), Tackling Wasteful Spending on Health, OECD Publishing, Paris (Available at: www.OECD.org/health/tackling-wasteful-spending-on-health-9789264266414-en.htm. Accessed on: 3/12/2020

[2] *Sistema Nacional de Saúde* (SNS) - Retrato da Saúde 2018 (Available at: www.sns.gov.pt/retrato-da-saude-2018/). Accessed on: 30/11/2020

[3] Conta Satélite da Saúde 2020. Instituto Nacional de Estatística - *Instituto Nacional de Estatística* (INE) (Available at: www.gee.gov.pt/pt/indicadores-diarios/ultimos-indicadores/30399-ine-conta-satelite-da-saude). Accessed on: 10/12/2020

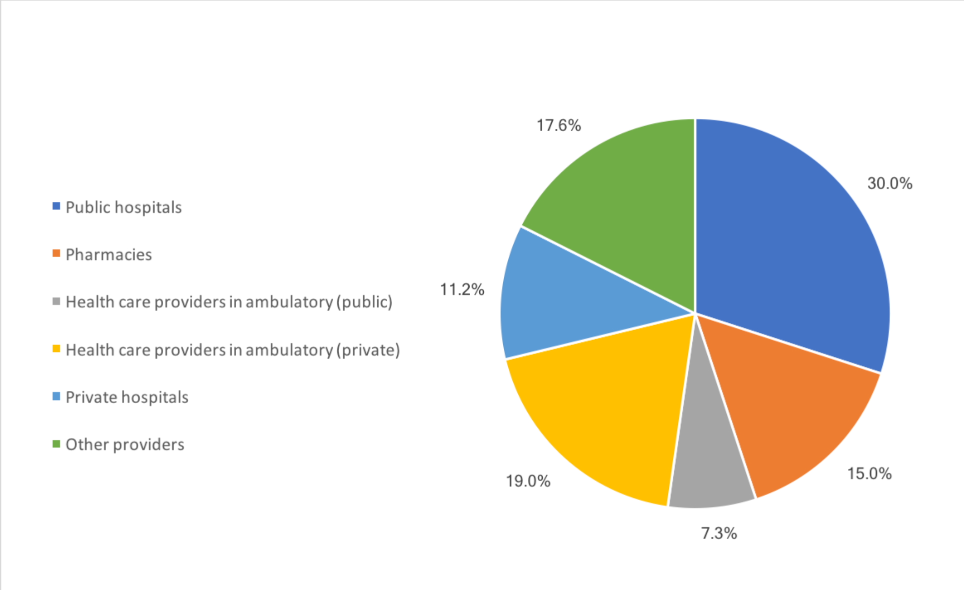


Figure 1.1: Health expenditure by provider in 2017 in Portugal.
Source: Conta Satélite da Saúde 2020.[3]

GDP was spent on health, and 30% of the health expenditure in Portugal went to public hospitals - see Figure 1.1.[3] These values draw attention to the efficiency of the management of health systems and, in particular, public hospitals.

Hospitals are crucial components of a health system, offering specialized health care that cannot be provided in other settings. However, this also means that hospitals are expensive to operate with a high number of staff, equipment and other operating costs.[1]

Health care services are mostly provided by public institution, where health care is not seen as an area to obtain profits, and is seen as priceless. Doctors and nurses and other health care providing workers aim to maximize the patients' well-being, and not optimize profits or resource utilization (Prezerakos, Maniadakis, Kaitelidou, Kotsopoulos, & Yfantopoulos, 2007). This, allied with all the changing factors stated above, may lead to health care institutions being often thought of presenting inefficiency and low productivity (Prezerakos et al., 2007).

These growing concerns and pressures have led policy makers, administrators, and clinicians to evaluate and improve health care services' efficiency (Peacock, Chris, Melvino, & Johansen, 2001), and not just its quality. Achieving value for money is an important objective in all OECD countries' health sector. With health care spending *per capita* rising by more than 70% since the 1990s, health care demand can grow to undermine public finances. Yet, the countries spending most are not necessarily the best performers in terms of health outcomes.[4] Even small improvements in the health sector can yield considerable savings of resources (Peacock et al., 2001).

Inefficient can lead to unwanted and avoidable poor outcomes for the patients, either in their health improvement or in their overall satisfaction with the health system. Inefficiency at some point in the health system can lead to treatments and health improvement being denied (Prezerakos et al., 2007). Moreover, the resources that are applied to health care are not being used elsewhere, such as education or infra structures, so there is the need and responsibility to assure that these resources are being well spent and used efficiently.

Efficiency assessment can be a useful tool for health planning and evaluation of policies, being of interest to a range of people, from the general public to hospital managers and to governmental

[4]OECD 2010, "Health care systems: Getting more value for money", OECD Economics Department Policy Notes, No. 2. (Available at: www.OECD.org/economy/growth/46508904.pdf). Accessed on: 12/12/2020

policymakers (Peacock et al., 2001).

According to the OECD, Portugal has an above average Data Envelopment Analysis (DEA) efficiency score but below average health care spending.[5] As well as more doctors, less nurses and less hospital discharges and doctor consultations, all *per capita*. Moreover, the same report says that efforts to increase consistency in the allocation of resources across government levels could contribute to raise spending efficiency.

## 1.2 Objectives

This work's objective is twofold. Firstly, it aims to assess the performance of Portuguese public hospitals. Besides this, it also intends to forecast their performance.

To assess the performance of the hospitals, two methodologies will be applied. One to evaluate the efficiency of the hospitals, using the Data Envelopment Analysis (DEA) method, identifying the hospitals that are most and less efficient, and the overall efficiency through the years 2013 to 2017. Despite the existence of values until 2019 publicly, these are not complete nor validated. Hence, the most recent year with complete and final data is 2017, thus justifying the period analyzed through this work. The other methodology, Malmquist Productivity Index (MPI), will be made use of to evaluate the productivity of these same hospitals, in order to identify the most and less productive ones, as well as to draw conclusions about their overall productivity.

The last objective, which is the forecast the MPI for one year, for the Portuguese public hospitals, will be done using a MPI decomposition presented by Daskovska, Simar, and van Bellegem (2010).

All results will be discussed and interpreted, in order to draw conclusions about which perform best and worst and its implications.

## 1.3 Outline

This dissertation is divided into seven chapters. This first chapter consists of an introduction to the work, presenting the motivation and objectives that will be addressed. The second chapter describes the Portuguese National Health System, giving context to the work. In the third chapter, a literature review is presented, showing the most used variables and methodologies for assessing hospital performance in several countries. The fourth chapter includes an overview of the existing models to evaluate efficiency and productivity, describing in detail the ones used in this work - Data Envelopment Analysis (DEA) and Malmquist Productivity Index (MPI) - as well as the forecasting methodology used. In chapter five, the specifics of the case study in question are described, including the sample and variables considered. The results obtained are presented and discussed in chapter six. Lastly, chapter seven comprises the conclusions of this dissertation, including limitations of the work and future work suggestions.

[5]Portugal: health care indicators (Available at: www.OECD.org/portugal/46507414.pdf). Accessed on 30/11/2020

# Chapter 2

# The Portuguese National Health Service

## 2.1 Overview

The Portuguese public health system's activity is mainly characterized by a National Health Service (NHS) - *"Serviço Nacional de Saúde" (SNS)* - that follows a Beveridge system, similarly to other countries such as the United Kingdom, Italy and Spain (Ferreira & Marques, 2019). Additionally, the state maintains agreements with the private and social sectors to complement the health care provision (Nunes & Ferreira, 2019a). Also, there are health subsystems (health insurance schemes associated with professional or occupational sectors), and private insurance schemes (Simões, Augusto, Fronteira, & Hernandez-Quevedo, 2017).

Health protection, provision of global health care and access for all citizens, despite their economic and social condition, are rights under the terms of the constitution.[1] Hence, through the NHS, universal (i.e. "for all citizens, regardless of their ability or willingness to pay") and general ("to all areas and needs") coverage must be guaranteed (Ferreira & Nunes, 2019; Nunes & Ferreira, 2019a). The NHS includes health promotion and surveillance, disease prevention, diagnosis and treatment of patients and medical and social rehabilitation. It holds administrative and financial autonomy and is structured in a decentralized and deconcentrated organization, comprising central, regional and local bodies.[1]

Public health care services are under the authority of the Ministry of Health, which is responsible for the development of health policies and the supervision and evaluation of their implementation (Simões et al., 2017). The management, planning and regulation of the NHS are carried out centrally by the Ministry of Health and its institutions. Thus, hospitals are not autonomous in a number of issues, such as the purchase of innovative new technologies, or the hiring of personnel (doctors, for example).

In the Portuguese NHS, four levels of care can be differentiated: (1) primary health care, in health care centers (2) secondary care, in hospital units, (3) post hospital care, involved in rehabilitation processes, and (4) palliative care for end-of-life cases (Ferreira & Nunes, 2019).

Health care management is decentralized in Regional Health Administration (RHA). Each RHA is responsible for the regional implementation of national health policies and coordination of all levels of health care. As well as the coordination of all aspects of health care provision, supervision of the hospitals and health centres' management, and articulation of agreements with the private and social sectors, and municipal councils, in its geographical area and for its population (Simões et al., 2017). RHAs' financial responsibility is limited to primary health care, since hospital budgets are defined and

[1] SNS - Portal SNS (Available at: www.sns.gov.pt/). Accessed on: 15/04/2020

distributed centrally (Simões et al., 2017).

The primary health care response consists, then, of a network distributed by all RHAs, covering health care provided out-of-hospital. Secondary healthcare, which is more specialized, is provided by public hospitals that are uniformly distributed across the country, according to the resident population, its health needs, and the existence of medical professionals (Nunes & Ferreira, 2019a). As well as singular hospitals, secondary heath care also includes hospital centers (horizontal merging), local health units (vertical merging of a singular hospital and primary health care centres), hospitals in Public–Private Partnership (PPP), oncology centers (*Instituto Português de Oncologia* (IPO)), maternity hospitals (which provide specialized Obstetrics, Gynaecology, and Paediatrics), and psychiatric hospitals. These last three represent specialized hospitals, requiring specialized physicians who may only serve in these specific specialties (Ferreira & Marques, 2020a; Ferreira, Marques, & Nunes, 2018).

Additionally to public providers and besides the private hospitals, health care in Portugal has other private providers, especially in the areas of pharmaceuticals, complementary diagnostics and therapeutics, and medical appointments (Nunes & Ferreira, 2019a).

Despite the commitment to the social state, Portugal still presents some inequalities in the access to care determined by geographic factors and demographic distribution (Ferreira, Nunes, & Marques, 2018). There is a greater difficulty in access manifested mainly in the countryside, where there is lower health literacy, lower access to information/internet, and lower average income, which restricts access to products/ drugs and services that are not covered by the NHS (Nunes & Ferreira, 2019a). With the Northern and Lisbon and Tagus Valley regions concentrating more than 70% of the health workforce (Ferreira, Nunes, & Marques, 2018), the more isolated regions of the interior have fewer physicians per inhabitant and, therefore, fewer medical specialties (Nunes & Ferreira, 2019b). Additionally, the health sector also presents some problems on high expenditures and levels of inefficiency, particularly in public hospitals (Ferreira & Nunes, 2019).

There are, currently, five RHAs in the country. Under the tutelage of each one there are primary and secondary health care facilities, encompassing health centres and hospitals, as well as continued and palliative care centres.[1] There were, in 2017, a total of 225 health facilities, with 100,147 people as hospital staff, and 34,953 hospital beds.[2] Private and public hospitals represented 51.7% and 46.5%, respectively, of total hospitals, with PPPs representing 1.7%.[2]

### 2.1.1 History and reforms

The Portuguese NHS was created in 1979 (*Diário da República*, law number 56/79).[1] After the 1974 revolution in Portugal, with which there was an end to the dictatorship regime, there was an evolution in the public health services. In 1976, the new Constitution was approved, and with it an article (64) stating every citizen has the right to health protection and the duty to defend and promote it. This was achieved by creating a NHS, universal and free, guarantying access to everyone regardless of their economic conditions and with medical and hospital coverage throughout the country.[1] After this and throughout the years, several health reforms have been introduced, to reduce operational costs and the waste of public funds, and improve the value for money, efficiency and effectiveness of healthcare providers (Ferreira & Marques, 2019).

Regarding the legal and organizational regimes of hospitals, three different periods can be set out.

In the first one, between 1979 and 2002, all public hospitals belonged to the Administrative Public Sector (*Setor Público Administrativo* (SPA)), being under the public/administrative law. These hospitals were the traditional public hospitals with limited administrative and financial autonomy, only autonomous

[2]PORDATA (Available at: www.pordata.pt/). Accessed on: 30/04/2020

regarding the human and financial resources. The Ministry of Health fiscally supervised hospitals and had full administrative authority over their management (Ferreira & Marques, 2015).

In 2002, 91% (31 out of the existing 34) of SPA hospitals were transformed into hospital enterprises (*Sociedade Anónima* (SA)), which were anonymous societies with exclusively public capital, becoming subjected to the commercial/private law (that regulates business companies) (Ferreira & Marques, 2015). SA hospitals were equivalent to private companies, being the capital shared among shareholders. Since the State was the only shareholder, it turned the hospitals into public enterprises with exclusively public capital (Ferreira & Marques, 2015). Although with more autonomy, for example in contracting or acquiring equipment, there was still regulatory intervention by the Ministries of Health and Finance.[3] This phase shows the adoption and adaptation of New Public Management (NPM) principles to the health care sector (application of private management tools to the public sector), intended to replace the traditional hierarchical management model by an innovative management model (Ferreira & Marques, 2015; Nunes & Ferreira, 2019a).

In 2005, these SA hospitals were transformed into corporate public entities (*Entidade Pública Empresarial* (EPE)). This new management scheme incorporated management efficiency and user satisfaction.[3] Since 2005, the number of EPE hospitals increased and there are nowadays 47 health centers and 53 hospitals and hospital centers from which 41 are EPEs, six belong to the Public Administrative Sector, and from the remaining six, three are Public-Private Partneships (PPP) and the other three are managed by the *Misericórdias* (social sector).[1] The autonomy of an EPE hospital is lower, given that Ministries must approve their activity reports and budgets and deal with the most important issues (Ferreira & Marques, 2015).

Another relevant period occurred between 2011 and 2015, characterized by the economic and financial crisis, which was followed by the post-crisis recovery period (Nunes & Ferreira, 2019a). During this crisis phase, the Portuguese health system underwent a reform due to the external intervention by the International Monetary Fund, the European Commission, and the European Central Bank. This lead to the implementation of a set of austerity measures. In an attempt to reduce costs and maximize efficiency, wages were reduced and the regular number of working hours increased for public health professionals. There was also a blockage on the hiring of new professionals, which led to the migration of NHS professionals to private hospitals and clinics, as well as emigration (Ferreira, Nunes, & Marques, 2018). Moreover, inequities in the access to health care increased and there was a reduction on the investment in equipment and infrastructures (Nunes & Ferreira, 2019a). On the other hand, some positive results were also obtained, such as efficiency gains, particularly in the drug market and debt reduction in the NHS (Nunes & Ferreira, 2019a).

Regarding the management model of hospitals, other noteworthy reforms were applied, such as:

- The corporatization of healthcare providers, which consisted in the transformation of traditional public hospitals (SPA hospitals) into “companies” (SA hospitals), as well as the introduction of a prospective payment system and individual labor contracts, in 2002. Later, between 2005 and 2009, SA hospitals were converted into EPE hospitals (Ferreira & Marques, 2015);
- The vertical and horizontal merging of healthcare providers, which consisted in the reorganization of services, resulting in geographically distinct production units (keeping the number of physical institutions) but with one single management unit. These mergers were horizontal - between

[3]Franca, L., Monte, A. P. (2010). Comparação entre sistemas de gestão hospitalar: SPA, SA e EPE, na perspectiva do planeamento e controlo orçamental: um estudo de caso. XIV Congreso Internacional de la Academia de Ciencias Administrativas, Monterrey. (Available at: hdl.handle.net/10198/2541). Accessed on: 30/04/2020

hospitals and resulting in hospital centers - and/or vertical – between hospitals and primary-care centers, resulting in local health units (Azevedo & Mateus, 2014);

- The introduction of public-private partnerships, which involves a temporary or partial ownership transfer to the private sector, but with the regulating aspect left to the public sector (Cruz & Marques, 2013).

Brought in by the introduction of NPM, the aforementioned mergers have changed the number of hospitals from approximately 90 to around 50 in less than a decade. Aiming for more efficient and effective communication between the different levels of healthcare and possible economies of scale, by operating efficiently at higher levels of production, there should be a decrease of the average costs, with hospitals increasing its operational size and/or the services they provide (Azevedo & Mateus, 2014). After the first ones in 1999, most of the public hospitals are now merged, vertically and/or horizontally, with only a few remaining as individual entities (Ferreira & Marques, 2019).[4]

In 2003, the primary health care network was created. Its mission being both providing health care to citizens and being in permanent communication and articulation with hospital health care and other healthcare facilities.[1] In 2006, the integrated continuing healthcare network was established, to respond to the progressive ageing population, increased life expectancy, and prevalence of incapacitating chronic diseases.[1]

### 2.1.2 Financing

Regarding its financing (Figure 2.1), the Portuguese health system has not been through significant changes since the promulgation of the Health Bases Law in 1990 (Nunes & Ferreira, 2019a). Although some entities present private management, such as PPPs, all health institutions belonging to the Portuguese NHS are public (Ferreira & Marques, 2015). Currently, all Portuguese public hospitals belong to the State Business Sector, being subject to the commercial/private law as corporate public entities (Ferreira, Nunes, & Marques, 2020). The public providers in the Portuguese NHS are financed through general taxation (Simões et al., 2017). The Government distributes the funds collected from the citizens by the different ministries, including the Ministry of Health, with which it negotiates the annual prospective budgets (Ferreira & Marques, 2019). This share of the General State Budget received by the Ministry of Health is distributed by the public providers of health care using contract payments (see 2.1.3) (Ferreira et al., 2020). The allocation of resources by provider is done prospectively and depends on a set of features: size, scope of services provided, complexity of handled patients, cost-efficiency, expected volume of services to be delivered, among others (Ferreira & Marques, 2019).

However, the NHS is also financed by other means, such as private voluntary insurances, which cover about 20% of the population, health subsystems (special, either public or private, insurance schemes associated with a set of professions), and the citizens themselves (with copayments), which represent a very small share of the revenues (Nunes & Ferreira, 2019a). These copayments, or moderating fees, have been introduced to moderate the access and to prevent abuse of the public health system (Ferreira & Nunes, 2019), with the poorest population being free of charges.[1] Hence, the NHS is not totally cost-free. However, it continues to be tendentiously free, according to the individual's ability to pay (Ferreira & Marques, 2015).

In the financial year of 2017, 18,282 million euros were dedicated to health expenses, which represents 9.3% of the GDP.[5] These expenses represent 1,774.9 euros *per capita*.[5] From the total expenses with health care, about 30% are expenses with hospitals, in 2017.[5]

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[4] *Administração Central do Sistema de Saúde* (ACSS) (Available at: www.acss.min-saude.pt/). Accessed on: 30/04/2020

[5] *INE(2020). Conta Satélite da Saúde – Base 2016*. (Available at: www.gee.gov.pt/pt/indicadores-diarios/ultimos-indicadores/30399-ine-conta-satelite-da-saude). Accessed on: 12/12/2020

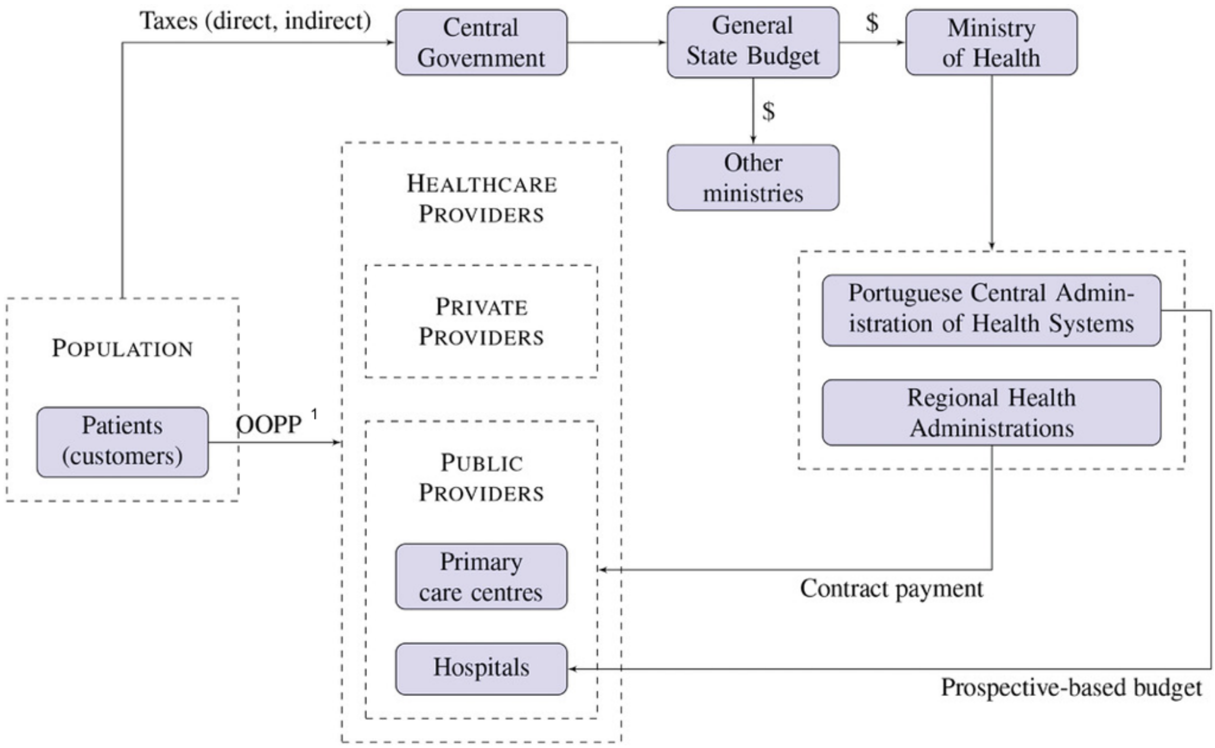


Figure 2.1: Financing in the Portuguese National Health Service. [1]OOPP: Out-of-pocket money.
Source: Ferreira et al. (2020)

### 2.1.3 Contracting

The contracting process in health care aims at establishing a mechanism for resource allocation, according to each health care service provision and the corresponding population needs, ensuring quality, efficiency, and effectiveness (Ferreira, Marques, & Nunes, 2019; Ferreira et al., 2020). In the Portuguese health care, it is possible to discriminate the different stakeholders in (a) public funding source (Portuguese state), (b) regulator (Portuguese state, through the independent Regulatory Agency for Health and the Ministry of Health), and (c) health care providers (hospitals, primary health care centers, etc.) (Ferreira et al., 2020). Currently, the contract process in Portugal integrates a triennial strategic planning process that incorporates existing forecasting documents - Business Plan, Performance Plan, and Adjustment Plan - and Financial Statements (Ferreira et al., 2020). As well as variables such as provisions type, volume and duration, referencing networks, human resources and facilities, monitoring schemes, performance prizes, and prices, contracts should also contain quality-related terms, such as penalties for poor quality (Ferreira et al., 2019; Ferreira et al., 2020). There is a budget based on the contract between the Ministry of Health and the public health care providers (Simões et al., 2017). However, budgetary constraints should not limit the health care provision ability/capacity, since one may observe the opposite of the desired effect, i.e., underfinancing generates over indebtedness (Ferreira et al., 2019; Ferreira et al., 2020).

In the past, financing was based on a retrospective model and the prices derived from each hospital's history. However, currently, financing follows a negotiation phase and the resulting contract. Each hospital-specific budget is negotiated in terms of the delivered health care services, between each hospital's Administrative Council and the Ministry of Health (Ferreira et al., 2020). Hospitals are clustered in groups with similar production technologies, but not including quality or environment characteristics (Ferreira et al., 2020). Payments, made by the Ministry of Health, are defined by averaging the unitary costs of the most efficient hospitals belonging to the same group since, theoretically, hospitals belonging to the same group have similar production technologies (Ferreira et al., 2020). Budget is negotiated in terms of production (referred in terms of DRG (Diagnosis Related Group)), services (emergencies, medical appointments, inpatient services, etc.) and quality (Ferreira et al., 2019). This current mechanism, being basically a case-based payment, has several disadvantages such as disregarded or negligently treated cases, reducing the overall quality of care, or the discouragement of introducing quality-raising technologies, that can increase cost (Ferreira et al., 2019).

### 2.1.4 Regulation

Health care providers are public though autonomous entities. Thus, there must be a monitoring model holding them responsible for, for example, their weak performance, as well as inducing transparency for the population (Ferreira et al., 2020). In 2003, the Health Regulatory Authority (ERS - *"Entidade Reguladora da Saúde"*) was created. It is an independent public entity that aims to regulate the activity of health care providers in Portugal. This entity handles every health care provider, independently of its legal nature and its private or public model, namely hospitals, clinics, health care centers, private practices, clinical analysis laboratories, equipment or any other health unit. It makes sure that legal operating requirements, citizen access, users' rights, quality and safety, legality, transparency and competition in the health sector are in order.[6]

## 2.2 National Health Service Performance

Strengthening of the resources allocated to the NHS happened during a period of economic growth that included Portugal's entry into the European Union. This lasted until the international markets' instability period and economic crisis that started in 2008 (Nunes & Ferreira, 2019b). Despite the different efficiency searching measures implemented until 2017 and the overall improvements in health outcomes, only in the austerity period the health expenditure decreased (Nunes & Ferreira, 2019a). This financial, economic, and social crisis forced the Portuguese government to reduce public financing, including in the health sector. The measures adopted aimed at resources rationalization, although they also caused an increase in the barriers to access health care, as well as divestment in equipment and infrastructures and the reduction of human resources (Nunes & Ferreira, 2019a; Nunes, Ferreira, & Fernandes, 2019). After this period of bailout, characterized by austerity measures, a new strategy (from 2016 onward) was implemented, aiming to reform the NHS (Nunes & Ferreira, 2019b; Nunes et al., 2019). This included measures to (a) improve responsiveness, by enlarging the type of delivered services in primary care centres, for example, (b) enhance access, which includes the decrease of waiting lists and times, (c) extend the supply of continuous and palliative care, and (d) enhance performance, with the creation of a transparency portal and an improvement of accountability (Nunes et al., 2019). Additionally, some methods regarding public health promotion have been implemented, to encourage healthier lifestyles. As well as measures to reduce inequities on access to health care (for example, reducing co-payments) (Nunes et al., 2019). Ensuring better access, quality, and efficiency should be enough to safeguard the NHS sustainability. Hence, this represents the main goals of the current developments in the health sector in Portugal (Nunes & Ferreira, 2019b).

Several authors have studied, in numerous ways, the performance of the Portuguese NHS, showing that, despite efforts from the different governments throughout the years, hospitals still exhibit inefficiency levels, for example as shown in Ferreira and Marques (2015). This is true for both public hospitals

[6] Entidade Reguladora da Saúde - ERS (Available at: www.ers.pt/). Accessed on: 5/5/2020

and PPPs, at least in terms of social inefficiency, according to Ferreira and Marques (2020a). Moreover, resource allocation should be careful in order to prevent the existent congestion levels (for example costs with staff and hospital days) and other inefficiencies and improve the production, as seen in Ferreira and Marques (2018).

From Ferreira, Marques, Nunes, and Figueira (2018), it was concluded that one of the criteria most valued by patients is the admissions process, namely the waiting time for medical appointments. Hence being one of the most important topics to be covered in future health policies. Also perceived as very important are the quality of the facilities, such as waiting areas, and of the clinical staff, trustworthiness and exams and treatments' waiting times.

According to Ferreira and Nunes (2019), the national efficiency (mean of the efficiency scores in the country) was 0.92. This means that, overall, hospital units have good results. However, hospitals could, generally, reduce 8% of their resources and keep the volume of delivered health care. In the regional overview, the northern region presents the highest average of the groups, followed by the Central Region, with Alentejo presenting the lowest efficiency score. The greatest differential was found in the Lisbon and Tagus Valley region, having the worst score in the country (0.65 out of 1). Additionally, more than 66% of the hospitals studied by Ferreira and Nunes (2019) presented great performance in the optimization of resources, but with waste and inefficiencies persisting in 25% of hospitals.

Regarding hospital staff, Ferreira, Nunes, and Marques (2018) studied the optimal scale size in the Portuguese public hospitals, with results pointing out to this scale being considerably below the average observed scale, with an uneven demographic distribution of clinical workforce, along the same lines as Ferreira and Nunes (2019).

Therefore, although hospitals and the whole health sector in Portugal have gone through several changes and developed through the years, they still present some room to grow and improve performance. NHS performance can be assessed through several criteria such as efficiency, quality and access.

### 2.2.1 Efficiency

Despite some arguments that health care institutions should not be expected to be efficient, there is a big interest in assessing hospital efficiency and ensure the best use of the great amount of resources that go to their funding (Jacobs, 2001). Some of the problems involving the health sector in Portugal are the high expenditures and levels of inefficiency, especially of the public hospitals (Ferreira & Nunes, 2019).

One of the many indicators to evaluate efficiency of the NHS is hospital operating expenses per standard patient. The monthly evolution of this indicator over the last two years is depicted in Figure 2.2.

Another indicator is, for example, the level of occupancy of hospitals, which represents the hospital's inpatient capacity being utilized for inpatient care. This indicator has increased in the Portuguese NHS over the last three years, as can be seen in Figure 2.3.

### 2.2.2 Quality

Quality assessment provides a method for evaluating health services. It is related to the value associated with different aspects of care. Quality monitoring is important for assessing health care costs and the delivery of services (McGlynn, 1997). One of the quality indicators is the number of pressure (or decubitus) ulcers, which can happen when a person is bedridden. The evolution of this indicator over time

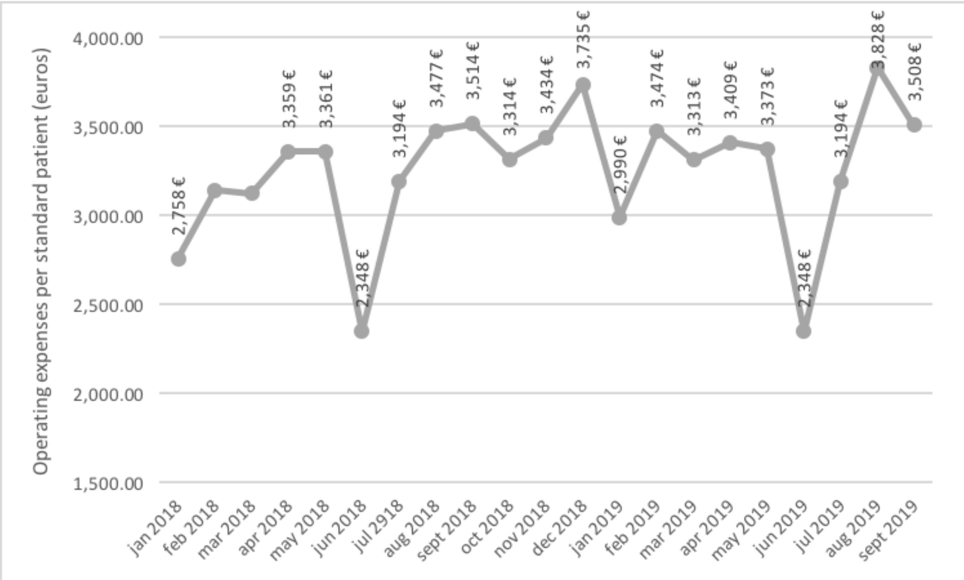


Figure 2.2: Evolution over time of the average operating expenses per standard patient. Source: https://benchmarking-acss.min-saude.pt

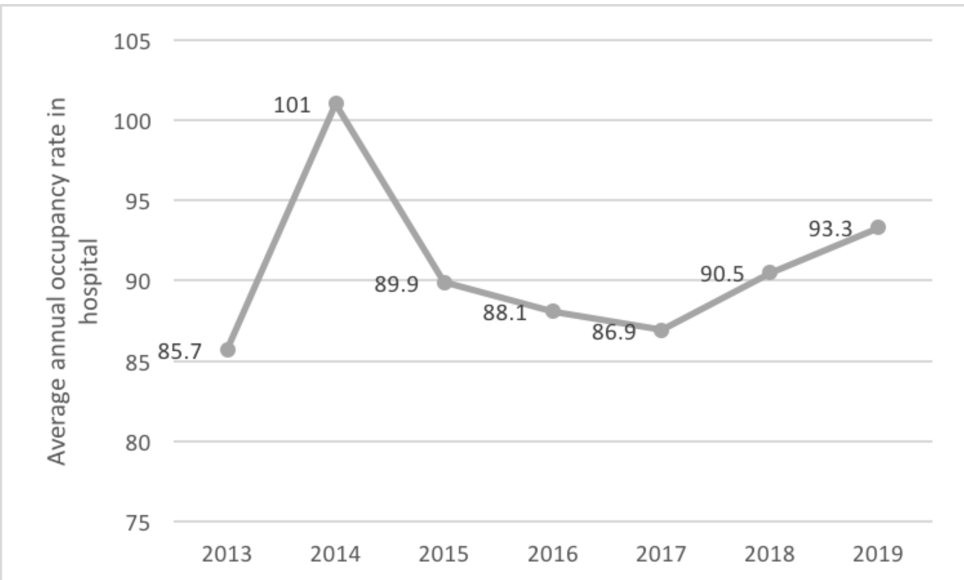


Figure 2.3: Evolution over time of hospital's occupancy rate. Source: https://www.sns.gov.pt/transparencia

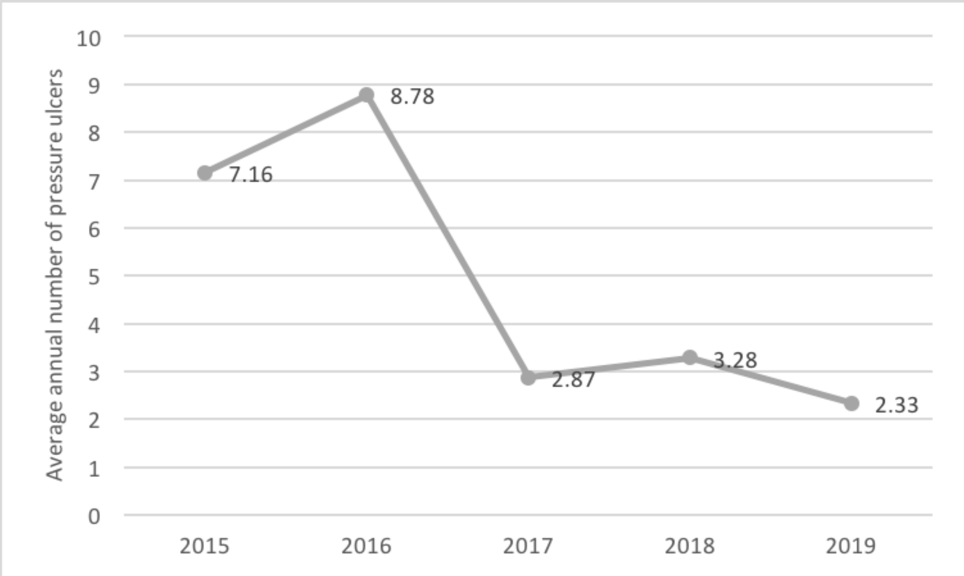


Figure 2.4: Evolution over time of the average number of pressure ulcers.
Source: https://www.sns.gov.pt/transparencia

in the Portuguese NHS is depicted in Figure 2.4. The numbers show a decreasing tendency, reaching the lowest number in 2019.

### 2.2.3 Access

Access to health care has do with the relationship between need, provision and utilization of health services (Gulliford et al., 2002). It is concerned with helping people secure appropriate health care resources to preserve or improve their health (Gulliford et al., 2002). Despite the the fact that it is hard to be considered independent of other system features (Gold, 1998), indicators can be used to assess access to health care.

One indicator to assess access to the NHS is the number of first appointments made within time. Its average, as seen in Figure 2.5, has slightly increased with time in the Portuguese NHS.

Throughout this work the focus will be on hospital efficiency, considering its indicators and existing methods to measure it.

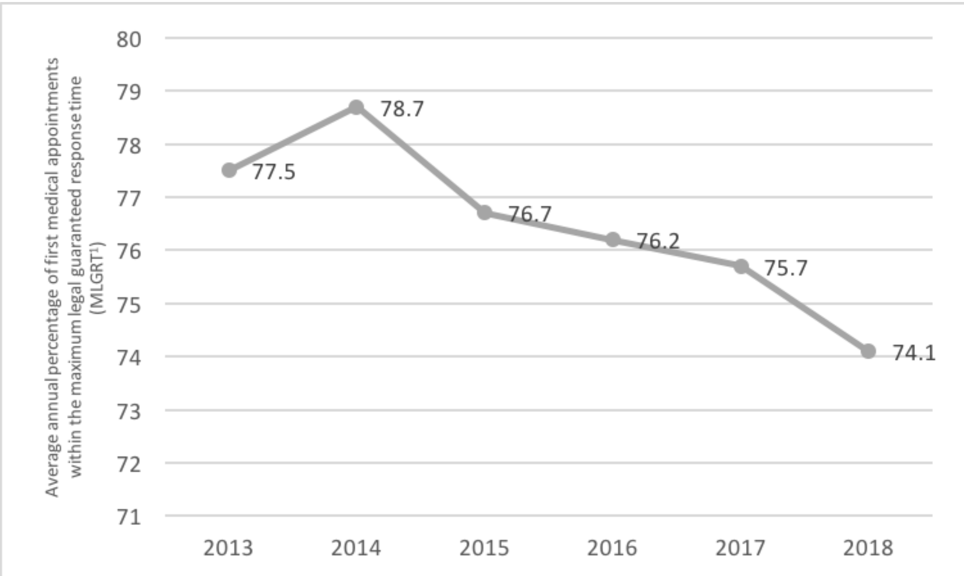


Figure 2.5: Evolution over time of the average percentage of first medical appointments within the MLGRT. [1] MLGRT: Maximum Legal Guaranteed Response Time
Source: https://www.sns.gov.pt/transparencia

# Chapter 3

# Literature Review

Several studies have been performed in order to assess hospitals' performance, making use of different variables and methodologies. These studies measure, beyond health services' efficiency and productivity, its delivered care quality and access. Since the early 1980s, efficiency analysis has been used to assess performance of health care services, including hospitals (Hollingsworth, 2008). Over these past two decades, efficiency measurement has been one of the most explored areas of research concerning health services (Moshiri, Aljunid, & Amin, 2010).

Various model can be used in hospital performance assessment, making use of different variables (Rahimi, Khammar-Nia, Kavosi, & Eslahi, 2014).

Hence, in this chapter, a literature review is performed in order to identify the most used methodologies and variables, as well as the most used variables when using specific methodologies. Moreover, it helps characterize other similar studies and the samples analyzed by those, along with possible literature gaps. For this purpose, fields such as the objectives, main conclusions and limitations are summarized. Along with these, the sample (sample type and size and years analyzed), methodologies and variables used by the analyzed studies are also presented in the following table - Table 3.1.

This table presents the main points of the literature review performed, as was done similarly, for example, in Ferreira, Marques, and Nunes (2018), Ferreira and Marques (2019) and Chowdhury and Zelenyuk (2016).

A total of 23 studies, both from Portugal and the world, were identified and analyzed. Regarding the methodologies used, 16 out of the 23 studies employed Data Envelopment Analysis (DEA) or DEA based methods, while six studies additionally used the Malmquist Index to assess productivity. Other common methods used were Order-$\alpha$, Free Disposal Hull (FDH) and Order-m, as can be seen in Figure 3.1.

Studies recurred mostly to inputs and outputs (82.6% of the studies used in/output variables - see Figure 3.2), with some considering environment or exogenous variables. Quality was also studied, as well as access. Quality was many times divided into care appropriateness and clinical safety, and access into timeliness and services availability. Moreover, most of the inputs and outputs used were adjusted with the Case-Mix Index (CMI).

The average number of variables used by authors is nine, with 39 being the maximum and the minimum only four. The variables more commonly used for in/outputs, quality and access are summarized in the Figures 3.3 to 3.7.

In terms of environment, which refers also to exogenous variables, studies used mainly the population density and purchasing power (either *per capita* or parity), as can be seen in Figure 3.5.

In terms of quality, authors used mostly the variables postoperative pulmonary embolism or deep vein thrombosis, postoperative septicemia and readmissions rate (within 30 days after discharge). Other

Table 3.1: Literature review

| Author, Year | Objectives | Sample | Variables | Methodology | Main Conclusions | Limitations |
|---|---|---|---|---|---|---|
| Ferreira et al., 2020 | provide tool to optimize payments for Portuguese public hospitals, taking into account quality and environment (+ illustration of new tool) | 27 public hospitals (single hospitals and hospital centres), Portugal, FY2016 | **Environment**: (a) population density, (b) elderly rate, (c) youth rate, (d) dependence index, (e) death rate, (f) child mortality rate, (g) elderly relative mortality rate, (h) birth rate, (i) stillbirth rate, (j) illiteracy rate, (k) complete higher education rate, (l) inhabitants per doctor, (m) inhabitants per pharmacist, (n) purchasing power per capita; **Complexity**: (a) Case Mix Index (CMI)[1] surgical (Case-Mix Index for surgical specialities), (b) CMI[1] medical (for medical specialities), (c) CMI[1] inpatients (for inpatient admissions), (d) GINI specialization index (based on data related to the Diagnosis Related Group (DRG)s[2] per hospital); **Quality** - *Clinical Safety*: (a) decubitus ulcers, (b) pulmonary embolism/deep vein thrombosis, (c) septicemia, (d) trauma on vaginal delivery (with instrumentation), (e) trauma on vaginal delivery (no instrumentation), (f) in-hospital avoidable mortality, *Care Appropriateness and Timeliness*: (a) inappropriate readmissions, (b) inpatients staying delay, (c) outpatient surgeries appropriateness, (d) hip surgeries timeliness | DEA[3] (Data Envelopment Analysis), Free Disposal Hull (FDH)[4] + GRA (Gray Relational Analysis) | "nearly €273 million of potential cost savings arising from the implementation of the proposed model" | proposed tool is semi-parametric, which leads to "curse of dimensionality" (the larger the number of variables, the lower the method's resolution power) |
| Ferreira and Marques, 2020a | find if PPPs[5] outperform public hospitals in quality and access by comparing their social performance | public hospitals (single hospitals, hospital centres, and local health units) and PPP[5] arrangements, Portugal, FY2013-FY2017 | **Quality** - *Care Appropriatness*: (a) outpatient surgeries per 100 potential outpatient procedures, (b) readmissions within 30 days after discharge per 100 discharges, (c) inpatients staying more than 30 days per 100 patients, *Clinical Safety*: (a) decubitus ulcer cases per 1000 patients, (b) catheter related bloodstream infections per 1000 inpatients, (c) postoperative pulmonary embolism/deep vein thrombosis per 100 000 inpatinets, (d) postoperative septicaemia cases per 100 000 inpatients; **Access** - *Timeliness*: (a) first nonurgent medical appointments within the maximum (legislated) guaranteed time per 100 first nonurgent medical appointments, (b) hip surgeries in the first 48h after fracture per 100 hip surgeries, (c) average delay before surgery (in days), *Services Availability*: (a) hospital beds per 100 standard patients | Benefit of Doubt + Bootstrap, Monte Carlo procedure and statistical tests (student's t and Kruskal-Wallis) | • public hospitals do not perform better socially than PPPs[5]<br>• both groups show considerable social inefficiency levels | • use of outcomes as measure of quality (external and non discretionary events dependency)<br>• only four access-related variables considered<br>• missing data |
| Weng, Wu, Blackhurst, and Mackulak, 2009 | present extended data envelopment analysis (DEA) model | 65 hospitals, Iowa (USA), 2001-2005 | **Inputs**: (a) number of beds available, (b) number of staff members in the hospital); **Outputs**: (a) average speed of treatment of acute care service, (b) average speed of treatment of swing bed service, (c) admission of acute care patients, (d) admission of swing bed patients | DEA[3], Malmquist, Panel-based Benchmarking [new proposed method] | "the proposed method can generate better benchmarks which consistently perform well over time" | • "sensitivity analysis should be performed to determine which factors have the most impact"<br>• "cluster analysis may be explored to better group homogeneous hospital types for a more robust analysis" |

[1]CMI: Case Mix Index; [2]DRG: Diagnosis Realted Group; [3]DEA: Data Envelopment Analysis; [4]FDH: Free Disposal Hull; [5]PPP: Public–Private Partnership

Table 3.2: Literature review (continued)

| Author, Year | Objectives | Sample | Variables | Methodology | Main Conclusions | Limitations |
|---|---|---|---|---|---|---|
| Ferreira and Marques, 2016b | propose new index to classify by service the sample of inpatient discharges, to substitute Case Mix Index (CMI)[1] | 49 public hospitals, Portugal, 2002-2009 | **Inputs**: (a) costs of goods sold and consumed, (b) supplies and external services, (c) staff costs, (d) other costs, (preciation and indirect costs related to the other services in the same hospital), (e) hospital days; **Outputs**: (a) inpatient discharges; **Environment**: (a) population density, (b) wealth index, (c) aging index, (d) morbidity, (e) corporatization status, (f) merging status, (g) year | Order-m and Translog functions + Nadaraya-Watson method and Bootstrap (3 different approaches developed) | • efficiencies do not change by using the CMI[1] or the Service-Mix Index (Service Mix Index (SMI)[2], new tool)<br>• inpatients adjustment influences hospital productivity<br>• severity adjustment is pointless if the model corrects for environment | • CMI[1] data is quite homogeneous<br>• no cluster comparison analysis done |
| Ferreira and Marques, 2017 | extend the order-$\alpha$ concept, with the inclusion of weight restrictions (WR), convexity and non-variable returns to scale | public hospitals (singular hospitals and hospital centers), Portugal (except Madeira and Azores), 2015 | **Resources**: (a) total operational costs, (b) excluding staff costs, (c) number of Full-Time Equivalent (FTE)[3] doctors, (d) number of FTE[3] nurses, (e) number of beds; **Quantity**: (a) number of standard (case-mix adjusted) patients, including the number of emergency episodes, inpatient discharges and medical appointments; **Quality/Clinical Outcomes**: (a) percentage of first medical appointments under the maximum guaranteed response time (by law), (b) percentage of surgeries under the maximum guaranteed response time (by law), (c) percentage of reinternments within 30 days, (d) percentage of hip surgeries (elderly) on the first 48 h, (e) percentage of trauma on vaginal births (either instrumental or not), (f) pressure ulcers in bedridden rate, (g) postoperative septicemia rate, (h) rate of pulmonary embolism/ deep venous thrombosis in the postoperative period, (i) in-hospital mortality rate, severities 1 and 2 (on 4 possible levels, being 4 assigned to the most severe/critical cases); **Environment**: (a) population density, (b) elderly rate, (c) purchasing power per capita | Order-$\alpha$ (standard and modified) | • each inefficient Decision Making Unit (DMU)[4] has only one peer benchmark<br>• "in the particular case of non convex attainable sets, unrestricted formulations and variable returns to scale assumption, the proposed procedure returns the same results as the standard order-$\alpha$"<br>• less sensitive to outliers, since frontier is partial (does not envelope all sample) | proposed algorithm(s) computationally complex and expensive |
| Ferreira and Nunes, 2019 | analyze the efficiency scores of hospital units in the 5 Portuguese health administrative regions (RHAs[5]) | 27 public hospitals (single hospitals and hospital centres, from the 5 RHAs[5]), Portugal, 2017 | **Inputs**: (a) overall costs, (b) number of beds, (c) number of FTE[3] clinical human resources (doctors and nurses); **Outputs**: (a) number of patients with hospitalization episode, (b) total number of medical appointments, (c) number of patients who went through emergency service, (d) number of surgeries | DEA[6] | • more than 66% hospital units have excellent performance in the optimization of resources<br>• waste and inefficiencies exist in 25% of hospitals, generating inequalities, with the biggest being in Alentejo and Lisbon and Tagus Valey | • does not consider environment or quality variables<br>• low number of hospitals analyzed |

[1]CMI: Case Mix Index; [2]SMI: Service Mix Index; [3]FTE: Full-Time Equivalent; [4]DMU: Decision Making Unit; [5]RHA: Regional Health Administration; [6]DEA: Data Envelopment Analysis

Table 3.3: Literature review (continued)

| Author, Year | Objectives | Sample | Variables | Methodology | Main Conclusions | Limitations |
|---|---|---|---|---|---|---|
| Ferreira and Marques, 2018 | evaluate congestion levels, sources and determinants in Portuguese Intensive Care Units (ICU), using a set of nonparametric models | 630 ICUs[1] (polyvalent, cardiology, pediatric, gynecology, obstetrics and neonatology, and surgical) of public hospitals, Portugal, 2002-2009 | **Inputs**: (a) costs of goods sold and consumed, (b) supplies and external services, (c) staff costs, (d) capital costs, (e) hospital days; **Outputs**: (a) inpatient discharges | DEA[2] (nonparametric models based on DEA)[2] + Bootstrap | • congestion and inefficiency levels identified<br>• sources of congestion: costs with staff and length of stay<br>• congestion levels dependent on: ICU specialty, ICU complexity, degree of hospital differentiation and demographic patterns of the population | • no adjustment of inpatients by their probability of death at the ICU entrance<br>• no quality data |
| Ali, Wang, Chaudhry, Geng, and Ashraf, 2017 | classify hospitals based on their relative waste management efficiencies in Gujrandwala, Pakistan | public and private hospitals, Pakistan, Nov2014-Mar2015 | **Inputs**: (a) number of beds, (b) number of admitted patients/day, (c) number of outdoor patients, (d) number sanitary workers); **Outputs**: (a) general waste, (b) biomedical waste | DEA | • hospital waste generation rate similar to other Pakistan cities, but lower than in other developing countries<br>• 75% of surveyed hospitals show scale or pure technical inefficiencies | • other healthcare facilities not included<br>• no quality or environment variables |
| Williams, Asi, Raffenaud, Bagwell, and Zeini, 2016 | understand the characteristics of top performing hospitals regarding health information technology (HIT) | 1039 hospitals, USA, 2011 | **Inputs**: (a) number of Staffed Beds, (b) FTE[3] nursing and physician staff, (c) technology inputs); **Outputs**: (a) number of 30 day hospital readmission rates, (b) number of deaths in 30 days from admission | Two-stages: DEA[2] and then Automatic Interaction Detector Analysis (AID) using decision tree regression (Dtreg) | • ”electronic access to diagnostic results systems was the most influential technological characteristic”<br>• ”organizational characteristics were more important than technological inputs”: hospital size and FTE[3] being indicative of hospital quality than HIT adoption | • limited ability to generalize<br>• data missing and with errors<br>• hospital systems are complex and technology hard to evaluate |

[1]ICU: Intensive Care Unit; [2]DEA: Data Envelopment Analysis; [3]FTE: Full-Time Equivalent

Table 3.4: Literature review (continued)

| Author, Year | Objectives | Sample | Variables | Methodology | Main Conclusions | Limitations |
|---|---|---|---|---|---|---|
| Halkos and Tzeremes, 2011 | ”measure the Greek public healthcare delivery efficiency from a regional perspective” | public hospitals and public health centres, Greece, 2005 | **Inputs**: (a) number of beds, (b) number of medical staff (doctors and nurses); **Outputs**: (a) days of inpatient care | DEA[1], FDH[2] + Bootstrap | • increased levels of GDP per capita don't ensure efficiency, having overall negative effect on delivery<br>• population density increases efficiency, indicating over-supply by urban hospitals<br>• ”poor state of public healthcare management and misallocation of healthcare resources” | • small sample (only one year of data considered)<br>• few variables<br>• no quality or access factors |
| Mitropoulos, Mitropoulos, Karanikas, and Polyzos, 2018 | ”examine the impact of the reform on the efficiency and productivity of public hospitals” | 111 public hospitals, Greece, 2009-2012 | **Inputs**: (a) number of doctors, (b) number of other personnel (nurses, administrative and support staff), (c) number of beds, (d) operating cost (annual operating expenses excluding payroll); **Outputs**: (a) inpatient discharges (annual numbers), (b) outpatient attendances | DEA[1], Malmquist + Bootstrap | • only 23 out of the 111 Greek hospitals exhibit statistically significant growth in productivity in pre reform period<br>• significant after reform productivity increase: in the 2 periods after the reform, most hospitals exhibited statistically significant productivity progress<br>• ”management activities that reduced pharmaceutical and/or medical consumables costs were found to improve the hospitals' productivity” | no quality or access factors |
| Ersoy, Kavuncubasi, Ozcan, and Harris, 1997 | examine technical efficiencies of Turkish acute general hospitals | 573 general hospitals, Turkey, 1994 | **Inputs**: (a) number of beds, (b) number of primary care physicians, (c) FTE[3] of specialists; **Outputs**: (a) inpatient discharges, (b) outpatient visits, (c) surgical operations | DEA[1] + T-tests | • 90.6% of the studied hospitals are inefficient<br>• ”on average, inefficient hospitals utilized larger numbers of specialist and primary care physicians, and over two times the number of beds” | • small sample<br>• no quality, access or environment factors |

[1]DEA: Data Envelopment Analysis; [2]FDH: Free Disposal Hull; [3]FTE: Full-Time Equivalent

Table 3.5: Literature review (continued)

| Author, Year | Objectives | Sample | Variables | Methodology | Main Conclusions | Limitations |
|---|---|---|---|---|---|---|
| Walker, 2018 | estimate the effect of Health Information Exchange (HIE) participation on efficiency | 1017 hospitals, USA, 2009-2012 | **Inputs**: (a) total licensed beds, (b) licensed nursing staff, (c) other full-time employees); **Outputs**: (a) surgical outpatient procedures, (b) medicare CMI[1] adjusted admissions, (c) average daily census, (d) emergency room visits, (e) outpatient load | Malmquist + Robustness analysis | • 51.2% of the analyzed hospitals increased their technical efficiency, 58.1% increased technological efficiency and 62.2% increased total factor productivity<br>• "any participation in HIE can improve both technical efficiency change and total factor productivity (TFP)"<br>• benefit of one and three years of participation on TFP[2] | "technical flaws in the determination of the Malmquist, making inferences derived from this approach potentially invalid due to the complicated determination of the frontier, the assumption of no measurement error, and unknown serial correlation" |
| Silwal and Ashton, 2017 | explore trends in inputs, outputs and productivity changes | 32 public hospitals, Nepal, 2011-2014 (3 fiscal years) | **Inputs**: (a) expenditure amount, (b) available beds, (c) FTE[3] human resources; **Outputs**: (a) number of outpatient visits, (b) emergency visits, (c) inpatient discharges | Malmquist, DEA[4] | • productivity of Nepalese hospitals declined over the studied years<br>• total factor productivity loss influenced by decline in technology change, despite efficiency increase | limited data: "Availability and accessibility of accurate, detailed and consistent measures of hospital inputs and outputs is a major challenge for this type of analysis" |
| Ferreira and Marques, 2015 | "investigate if the market structure reforms in the Portuguese health system have improved hospital performance and productivity" | 216 SPA[5] hospitals (2002–2009), 40 SA[6] hospitals (2003–2004), and 136 EPE[7] hospitals (2005–2009), Portugal | **Inputs**: (a) Costs of goods sold and consumed, (b) supplies and external services, (c) staff costs, (d) other costs (including depreciation and indirect costs related to the other services in the same hospital), (e) hospital days, (f) beds, (g) number of doctors, (h) number of nurses, (i) number of other staff; **Outputs**: (a) inpatient discharges, (b) number of emergency episodes treated in the same hospital, (c) number of outpatient visits; **Environment**: (a) population density, (b) wealth index (purchasing power, adjusted for inflation), (c) aging index, (d) morbidity [5 different models (regarding hospital dimension) for inputs and outputs] | Malmquist, DEA[4] + Bootstrap | • SPA[5] present the best productivity<br>• EPE[7] are more productive than SA[6]<br>• SA has highest efficiency<br>• more autonomy means leads to productivity | results may depend on several other factors, beyond the corporatization effect, for example sample size |

[1]CMI: Case Mix Index; [2]TFP: Total Factor Productivity; [3]FTE: Full-Time Equivalent; [4]DEA: Data Envelopment Analysis; [5]SPA: *Setor Público Administrativo*; [6]SA: *Sociedade Anónima*; [7]EPE: *Entidade Pública Empresarial*

Table 3.6: Literature review (continued)

| Author, Year | Objectives | Sample | Variables | Methodology | Main Conclusions | Limitations |
|---|---|---|---|---|---|---|
| Chowdhury and Zelenyuk, 2016 | "analyze production performance of hospital services in Ontario (Canada), by investigating its key determinants" | 113 acute-care hospitals, Canada, 2003-2006 | **Inputs**: (a) administrative staff hours, (b) nursing hours staffed beds, (c) medical-surgical supplies costs, (d) non-medical supplies costs, (e) equipment expenses; **Outputs**: (a) ambulatory visits, (b) case-mix weighted inpatient days | DEA[1] + Bootstrap and Sensitivity analysis | • "organizational factors such as outpatient-inpatient ratio, occupancy rate, the rate of unit producing personnel and the case-mix index are positively associated with efficiency of hospitals"<br>• rate of equipment expense is negatively associated with efficiency of hospitals | • no quality, access or environment factors<br>• few variables |
| Khushalani and Ozcan, 2017 | examine efficiency of producing quality hospital and characteristics that contribute to it | 1259 hospitals, USA, 2009-13 | **Inputs** *to medical/surgical care sub-unit*: (a) inpatient beds, (b) non-physician FTEs[2], (c) operating expenses per bed; *to quality sub-unit*: (a) hi-tech service mix, (b) registered nurses (total nurses ratio); **Outputs** *from medical/surgical care sub-unit*: (a) outpatient visits; *from quality sub-unit*: (a) % of patients who would definitely recommend the hospital, (b) % of patients who gave the hospital a rating of 9 or 10 out of 10; **Links** *from medical/surgical care sub-unit to quality sub-unit*: (a) emergency visits, (b) total surgeries, (c) case mix adjusted discharges; **Carry-overs** *from medical/surgical care sub-unit*: (a) net patient revenue per bed; *from quality sub-unit*: (a) heart attack 30 day mortality rate, (b) heart attack 30 day readmission rate, (c) heart failure 30 day mortality rate, (d) heart failure 30 day readmission rate, (e) pneumonia 30 day mortality rate, (f) pneumonia 30 day readmission rate | Dynamic Network DEA[1], Malmquist + t-tests, Pearson's correlation test, multinomial logistic regression | • "overall hospital efficiency (which incorporates quality) improved significantly between 2009 and 2013"<br>• "hospitals that showed improvement in the efficiency of both the quality sub-unit (QSU) and medical-surgical care sub unit (MCSU) are more likely to be non-teaching and rural"<br>• "improvements in efficiency of both sub-units are positively correlated" | • only a limited number of quality measures in the model<br>• unavailability of data |
| Ferreira et al., 2019 | propose tool to achieve best tariffs, with quality and financial sustainability concepts | 29 hospitals (including 2 PPPs[3], some oncology centres and psychiatric centres but not local health units), Portugal, 2016-17 | **Inputs**: (a) total operating expenses; **Outputs**: (a) number of standard patients; **Environment**: (a) population density, (b) elderly rate, (c) child mortality rate, (d) stillbirth rate, (e) illiteracy rate, (f) purchasing power, (g) CMI[6] for inpatient sercives, (h) GINI's specialization index; **Quality**: (a) nonurget (NU) first medical appointments within the maximum legal guaranteed response time (MLGRT[4]) per 100 NU first appointments, (b) NU surgeries within the MLGRT[4] per 100 NU surgeries, (c) minor surgeries per potential minor surgeries, (d) rate of readmissions within 30 days after discharge, (e) rate of bedridden cases, (f) rate of bloodstream infections resulting from central venous catheter (CVC), (g) rate of postoperative pulmonary embolism or deep vein thrombosis, (h) rate of postoperative septicaemia | Multiplicative (or log-) DEA[1] [new method] | cost savings with new approach | • unfair comparisons, due to environment and technology not considered in current approach and hospital clustering only regarding size<br>• some quality dimensions not considered |

[1]DEA: Data Envelopment Analysis; [2]FTE: Full-Time Equivalent; [3]PPP: Public–Private Partnership; [3]MLGRT: Maximum Legal Guaranteed Response Time

Table 3.7: Literature review (continued)

| Author, Year | Objectives | Sample | Variables | Methodology | Main Conclusions | Limitations |
|---|---|---|---|---|---|---|
| Ferreira, Nunes, and Marques, 2018 | analyze scale efficiency, optimal scale for hospital clinical staff, and exogenous dimensions associated | 27 public hospitals (general/acute-care hospitals and hospital centres), Portugal, 2013-2016 | **Inputs**: (a) FTE[1] doctors,(b) FTE[1] nurses, (c) other resources (beds and operational expenses); **Outputs**: (a) inpatient services, medical appointments, (b) operating theatre, emergency room; **Quality**: (a) outpatient surgeries per 100 potential outpatient procedures, (b) rate of caesarean sections per delivery, (c) readmissions within 30 days after discharge per 100 patients, (d) obstetric trauma rate (vaginal delivery with or without instrument), (e) decubitus ulcer rate, (f) postoperative septicaemia rate, (g) postoperative pulmonary embolism/ deep vein thrombosis rate, (h) mortality rate for less severe cases; **Access**: (a) number of first nonurgent medical appointments within the maximum (legislated) guaranteed time per 100 first nonurgent medical appointments, (b) hip surgeries in the first 48h after fracture per 100 hip surgeries, (c) waiting time before surgery [access variables depend on 2 models]; **Environment**: (a) number of inhabitants, (b) inhabitants per squared kilometre, (c) elderlies per 100 inhabitants, (d) youngsters per 100 inhabitants, (e) dependence index, (f) deaths per 100 inhabitants, (g) under-5-deaths per 100 live births, (h) mortality rate over 65 per 100 inhabitants, (i) births per 100 inhabitants, (j) fetal deaths per 100 under-5 deaths, (k) illiterates per 100 inhabitants, (l) secondary and tertiary education rate, (m) inhabitants per doctor, (n) inhabitants per pharmacist, (o) purchasing power parity | DEA[2] + statistical tests (2 sample t-test, Kruskal-Wallis and Kolmogorov-Smirnov) | • misdistribution of health workforce (coastline/countryside)<br>• optimal scale centered on 274 FTE[1] doctors and 475 FTE[1] nurses | • did not analyze optimal scales per medical speciality, department or ward<br>• limited number of available quality and access dimensions |
| Ferreira, Marques, Nunes, and Figueira, 2018 | evaluate the satisfaction of Portuguese patients with the NHS[3] and public hospital medical appointment services, and identify the main areas of low performance | public hospitals (medical appointments) Portugal, 2015 | **Criteria** (with subcriteria) - *Hospital image*: (a) trustworthy hospital, (b) hospital know-how/expertise, (c) hospital's concern about patients, (d) technological progress, (e) global, *Admission process*: (a) waiting time for a medical appointment, *Facilities quality*: (a) facilities cleaning, hygiene, and conservation, (b) office comfort, (c) privacy protection, (d) cleaning, hygiene, conservation and comfort of the waiting area, (e) global, *Doctors*: (a) availability and care, (b) patient's health status explanation, (c) professional competence, (d) prescriptions and diagnosis' explanations, (e) further care provided information, (f) global, *Nurses*: (a) kindness and availability, (b) professional competence, (c) global, *Diagnosis and treatments*: (a) waiting time for an exam or treatment, (b) health technicians' kindness and availability, (c) health technicians' professional competence, (d) global, *Waiting time after admission*: (a)waiting time before an appointment (after admission), *Overall satisfaction* | Multicriteria Satisfaction Analysis (MUSA) method, Kanos model + Bootstrap | • "In general, patients are satisfied with the Portuguese public health service, but some areas deserve some attention"<br>• most valued criteria: waiting time, hospital image, facilities quality | • "need for simulating patients based on frequencies of their satisfaction assessments"<br>• reduced number of satisfaction levels |
| Akkan et al., 2020 | analyze the efficiency of emergency departments of general hospitals | 7 general hospitals, Istanbul (Turkey), 1994 | **Inputs**: (a) ED[4] level category, (b) total number of beds in the ED; **Outputs**: (a) the total number of emergency patients, (b) the total number of referrals from the ED | DEA[2] + ALSCAL and PCA[5], statistical methods, regression | "less-equipped EDs[4] are supported by better equipped larger EDs[4]" | small number of hospitals |

[1]FTE: Full-Time Equivalent; [2]DEA: Data Envelopment Analysis; [3]NHS: National Health Service; [4]ED: Emergency Department; [5]PCA: Principal Component Analysis

Table 3.8: Literature review (continued)

| Author, Year | Objectives | Sample | Variables | Methodology | Main Conclusions | Limitations |
|---|---|---|---|---|---|---|
| Ferreira, Marques, and Nunes, 2018 | "identify economies of scope in Portuguese hospitals using frontier-based methods" | general hospitals and specialized hospitals (obstetrics, gynaecology, and paediatrics (OGP)[1], and psychiatric (Psy) services), Portugal, 2002-2009 | **Inputs**: (a) costs of goods sold and consumed, (b) supplies and external services, (c) staff costs (expenses with doctors, nurses and other personnel working on the analyzed services), (d) other costs (extraordinary costs and amortizations in OGP[1] and/or Psy services), (e) indirect costs (indirect expenses, related to other services in the same hospital, that can be related to the services under analysis), (f) hospital days; **Outputs**: (a) inpatient discharges, (b) emergency cases, (c) medical appointments [outputs and inputs depend on the model (different services that hospital uses)] | Order-$\alpha$ + SMI[2] (Service-Mix Index), PCA[3], Kruskal-Wallis Test | • economies and diseconomies of scope found<br>• general hospitals can exploit economies of scope<br>• diseconomies of scope more likely for larger hospitals<br>• "considerable dependence on the production line and on the merger status of the hospital" | • no quality, safety or access variables<br>• environment variables not considered (only SMI[2]) |
| Ferreira and Marques, 2019 | "study the impact of quality and access to healthcare services on their operational efficiency" | public hospitals (7 singular hospitals and 20 hospital centres), Portugal, FY2013-FY2016 | **Inputs**: (a) costs with staff, (b) operating costs, (c) costs of goods sold and cosnumed, (d) costs with outsourcing, (e) FTE[4] doctors, (f) FTE[4] nurses, (g) hospital days, (h) doctor's time ordinary, (i) doctor's time overdue, (j) nurses time ordinary, (k) nurses time overdue [3 different models for inputs]; **Outputs**: (a) inpatient discharges, (b) emergency cases, (c) first medical appointments, (d) follow-up medical appointments, (e) outpatient surgeries, (f) conventional surgeries, (g) urgent surgeries, (h) number of births; Quality - Care Appropriatness: (a) outpatient procedures on potential outpatient procedures, (b) rate of readmissions within 30 days of discharge, (c) rate of inpatients staying more than 30 days; **Quality** - *Clinical Safety*: (a) decubitus ulcer rate, (b) postoperative pulmonary embolism/deep vein trombosis rate, (c) postoperative septicaema rate, (d) obstetric trauma rate - vaginal delivery with instrument, (e) obstetric trauma rate (vaginal delivery withtout instrument), (f) inhospital death rate for low severity levels; **Access** - *Timeliness of Services*: (a) rate of first medical appointments within time, (b) rate of surgeries within time, (c) hip fracture surgery in the first 48h, (d) waiting time before surgery, *Services Availability*: (a) hospital beds per 1000 inhabitants, (b) FTE[4] nurses per 1000 standard patients, (c) FTE[4] doctors per 1000 standard patients, (d) inpatient bed occupancy rate, (e) operating theatre capacity utilization, (f) average delay in inpatient services (days per patient), *Population at Risk*: (a) purchasing power per capita, (b) illiteracy rate, (c) stillbirth rate, (d) childhood ($\leq$1 y.o.) mortality rate | Order-$\alpha$ | • potential trade-offs between clinical safety and efficiency<br>• "no meaningful link between access and technical efficiency was observed" | • "hospitals' size may have an influence on the trade-off between efficiency and quality"<br>• assumption that quality (and access) variables are independent among themselves can introduce biasing |

[1]OGP: Obstetrics, Gynaecology, and Paediatrics; [2]SMI: Service-Mix Index; [3]PCA: Principal Component Analysis; [4]FTE: Full-Time Equivalent

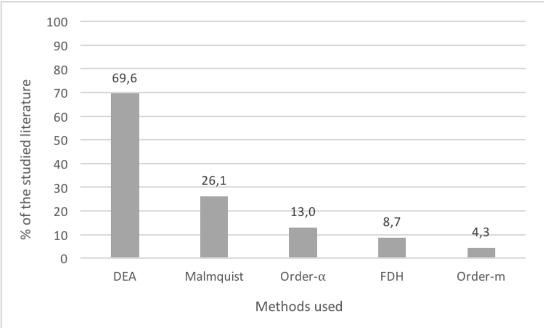


Figure 3.1: Most used methods in the studied literature

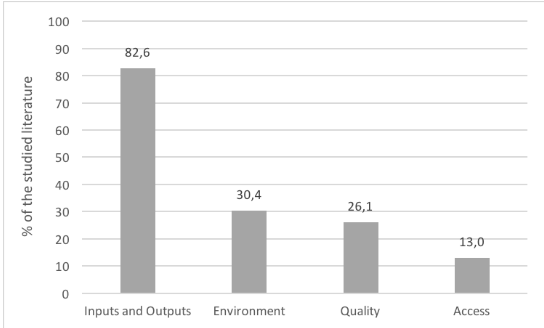


Figure 3.2: Variable types most considered in the studied literature

commonly used variables are summarized in Figure 3.6.

Regarding the studies using inputs and outputs, the variables most used for these variables are shown in Figures 3.3 and 3.4. In terms of inputs, costs are the most commonly adopted, mainly

- Costs of goods sold and consumed, which includes drugs and clinical materials expenditures;
- Supplies and external services, including expenditures with external labor outsourcing;
- Staff costs, which encompasses expenditures with staff, including salaries and bonuses to physicians, nurses and other (non-administrative) staff;
- Operating costs, which includes annual operating expenses (but excluding staff's payroll).

The number of nurses, doctors and other staff (such as sanitary workers or administrative and support staff) are also regularly employed as inputs, either in total numbers of these worker types, number per patient or inhabitant, or full time equivalent (FTE). The indicator ”beds” is also generally adopted as an input, referring to the total number of beds, number of available beds, number of licensed beds or even number of staffed beds. Beds can also be adopted to measure access, for example number of beds per patients, number of beds per inhabitants or even total number of beds in some studies.

The number of inpatient discharges (which represents the total number of patients treated in any service of the internment department) and the total number of medical appointments or outpatient visits are the most commonly used outputs. The variable ”number of hospital days” (total number of days used by all inpatients) was was also used in some cases as an input. Other most used variables are emergency cases (number of emergency episodes treated in the same hospital), ambulatory surgeries (or outpatient surgery, which does not require an overnight hospital stay) and the number of patients treated.

To analyze access, some studies made use of variables also adopted for quality or inputs and outputs by other authors, for example the number of beds (as previously mentioned), as well as the rate of surgeries within time (or waiting time before surgery or average delay before surgery) and the rate of first medical appointments within time. The number or rate of hip surgeries in the first 48h after fracture is also one of the most used criteria to assess access, as can be seen in Figure 3.7.

With the literature review performed, it is possible to understand that the most used methodology to assess performance of health care providers is DEA. When measuring productivity, the Malmquist Productivity Index is also commonly used. The most used variables for these methods are input and output variables.

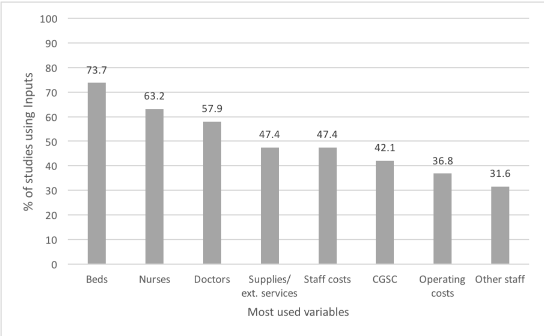


Figure 3.3: Most used variables for inputs in the studied literature.

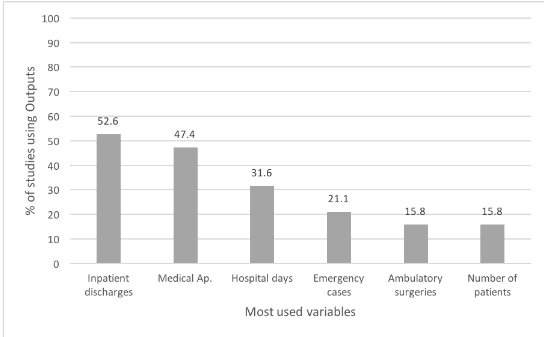


Figure 3.4: Most used variables for outputs in the studied literature.

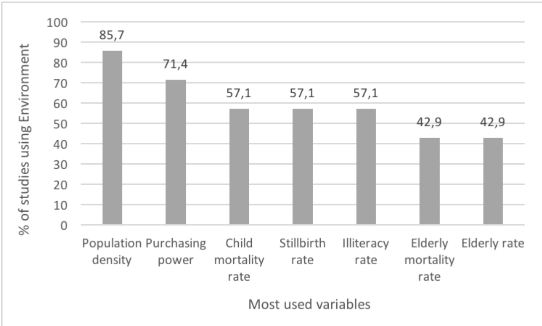


Figure 3.5: Most used environment variables in the studied literature.

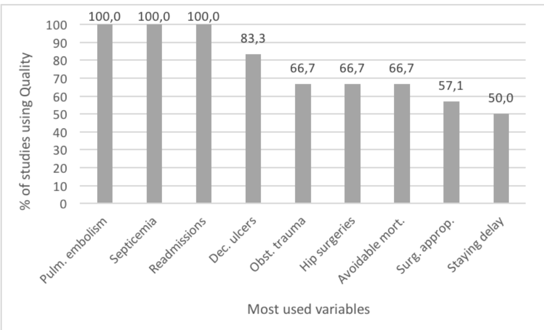


Figure 3.6: Quality variables most used in the studied literature.

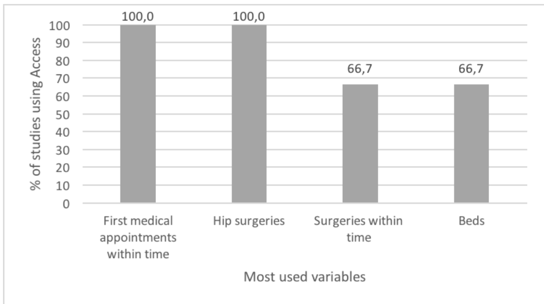


Figure 3.7: Access variables most used in the studied literature.

# Chapter 4

# Methodology

The notion of efficiency refers to an optimal situation, which may represent either the maximum output for a given level of input or the minimum input for a given level of output (Ji & Lee, 2010). Efficiency can also be measured over time since it is possible that the production frontier shifts due to technological advances (Hollingsworth, 2008). There are several tools of benchmarking used in the literature to evaluate the performance of entities, by analyzing their efficiency.

## 4.1 Overview of the existing models

Benchmarking is considered a management tool to achieve performance goals by learning from best practices and understanding their processes (Anand & Kodali, 2008). There are several definitions of benchmarking in the literature. One of the most quoted is "Benchmarking is the search for the best industry practices which will lead to exceptional performance through the implementation of these best practices" (Anand & Kodali, 2008).

Numerous different benchmarking models exist, and can be divided into frontier and non frontier methods. The non frontier methods can be further organized, as in the following flowchart (Figure 4.1), into parametric and non parametric methods.

Parametric methods assume a particular functional form (Jacobs, 2001), requiring the specifications of the frontier function technology and the inefficiency term (Murillo-Zamorano & Vega-Cervera, 2001). On the other hand, non-parametric methods do not need any specific form, since they do not require any assumption about the production frontier (Jacobs, 2001). These methods measure performance not in absolute terms but relative to each other (Stroobants & Bouckaert, 2014). These approaches determine

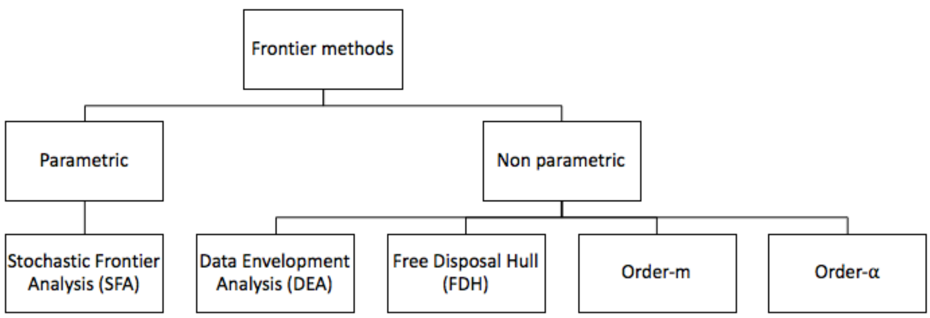


Figure 4.1: Frontier methods for benchmarking.

a frontier "enveloping" all observations, relative to which each Decision Making Unit (DMU)'s efficiency can be assessed (Bezat, 2009).

Methods can also be stochastic or deterministic. The stochastic ones make explicit assumptions about the stochastic nature of the data, while the deterministic do not (Kerstens, Borger, & Vanneste, 1994). Non-parametric methods have a deterministic nature (Murillo-Zamorano & Vega-Cervera, 2001).

Another criteria to categorize these approaches distinguishes between statistical and non-statistical. Statistical methods tend to make assumptions about the stochastic nature of the data, so they tend to be parametric. Non-statistical methods, such as DEA, tend to be non-parametric and deterministic (Jacobs, 2001).

There are several examples for the methods mentioned. Stochastic Frontier Analysis (SFA) is a stochastic and parametric method since it uses stochastic procedures to evaluate the frontier parametrically (Bezat, 2009). Being parametric, it requires the assumption of a given functional form, so the frontier is estimated with an econometric approach, such as a variant of least squares, maximum likelihood (Bezat, 2009) or regression (Jacobs, 2001). The frontier is smooth and curved (Bezat, 2009).

Data envelopment analysis is a non-parametric, deterministic method that determines a frontier "enveloping" the observations (Bezat, 2009). It is one of the most common examples of a non-statistical and non-parametric method (Jacobs, 2001), which means it estimates the efficiency frontier in an empirical way and, therefore, requires fewer hypotheses (Ferreira & Nunes, 2019). It is usually implemented as a linear programming process which examines the relationship between inputs and outputs (Jacobs, 2001). It was first introduced by Charnes, Cooper, and Rhodes (1978) and later extended by Banker, Charnes, and Cooper (1984) and can be applied to compare DMUs' performance (Homburg, 2001).

The Free Disposal Hull (FDH) is a non-parametric, non-stochastic method for evaluating efficiency (B. Lim, Lee, & Lee, 2012). It requires minimal assumptions about the production technology and, contrary to DEA, does not require convexity (Kerstens et al., 1994).

Non-parametric models are generally sensitive to outliers and extreme data points (Ferreira & Marques, 2019). This happens because they create frontiers, that can be convex or not, but that envelop the whole sample which causes the efficiency estimates to probably be biased (Ferreira & Marques, 2019).

There are, however, partial frontier methods such as Order-m and Order-$\alpha$, which envelop just a sub-sample of the observations. Order-m is a locally convex non-parametric and partial frontier method that allows the inclusion of direct environmental information (Ferreira & Marques, 2016b). It is based on benchmarking a DMU according to the expected best performance in a sample of m units (Tauchmann, 2012). Order-$\alpha$ is also a non-parametric partial frontier approach. Its efficiency score may be interpreted as "the amount of inputs that the unit must reduce to reach the quantile efficient frontier of level $\alpha$" (Ferreira & Marques, 2020b). The frontier, in this case, is estimated after the probability of observing points above it is defined (Ferreira & Marques, 2020b).

The advantages and disadvantages of parametric and non-parametric methods have been already largely discussed and some were already mentioned. Parametric approaches rely on restrictive assumptions about the functional form, and non-parametric approaches are deterministic and vulnerable to outliers and measurement error (Tauchmann, 2012). Partial frontier methods also have pros and cons of their own. They are good to avoid the curse of dimensionality and are less sensitive to outliers and extreme data, and can include direct environmental information. However, weight restrictions and non-variable returns to scale technology are not allowed in this case (Ferreira & Marques, 2020b). These advantages and disadvantages are presented in more detail on Table 4.1.

Table 4.1: Advantages and disadvantages of the different existing methods for measuring efficiency.

| Methods | Advantages | Disadvantages |
|---|---|---|
| SFA[1] | • Deviations from production function treated as both random error and inefficiency (Bezat, 2009)<br>• Exogenous aspects can be estimated as random errors (Estruch-Juan, Cabrera, Molinos-Senante, & Maziotis, 2020) | • Assumption about the functional form of frontier required *a priori* (Bezat, 2009)<br>• Delicate selection of variables (Estruch-Juan et al., 2020) |
| DEA[2] | • Able to handle multiple inputs and outputs simultaneously without assumption of functional form (Weng et al., 2009)<br>• Does not require information on prices (Mitropoulos et al., 2018)<br>• Estimates efficiency frontier in empirical way, requiring fewer hypotheses (Ferreira & Nunes, 2019)<br>• Easier to individually analyze each unit[1] | • Random error interpreted as inefficiency (Bezat, 2009)<br>• Efficiency is relative - measurements are only valid in a sample (Bezat, 2009)<br>• Sensitive to number of DMUs (as the number of DMUs included increases, efficiency of each DMU tends to decrease) (Banker et al., 1984) |
| FDH[3] | • Very weak assumptions regarding the production technology (Kerstens et al., 1994)<br>• Does not require convexity (Borger, Kerstens, Moesen, & Vanneste, 1994)<br>• Intuitive since closest to the concept of technical efficiency (Borger et al., 1994) | • Sensitive to outliers and measurement error (Tauchmann, 2012)<br>• Sensitive to number and distribution of observations in the data set and to the number of input and output dimensions (Borger et al., 1994) |
| Order-m | • Few frontier asumptions (Ferreira & Marques, 2020b)<br>• Less sensitive to outliers (Ferreira & Marques, 2016b)<br>• Allows superefficient DMUs to be located beyond the frontier (Tauchmann, 2012) | • Requires choosing parameter values, that may require trying several values (Tauchmann, 2012)<br>• Long computation time (Gnewuch & Wohlrabe, 2018) |
| Order-$\alpha$ | • Few frontier asumptions (Ferreira & Marques, 2020b)<br>• Less sensitive to outliers (Ferreira & Marques, 2016b)<br>• Faster computation when compared to order-m (Tauchmann, 2012) | • Appropriate model for efficiency assessment is needed prior (Ferreira & Marques, 2017)<br>• Computationally complex (Ferreira & Marques, 2017) |

[1]SFA: Stochastic Frontier Analysis; [2]DEA: Data Envelopment Analysis; [3]FDH: Free Disposal Hull

Because with DEA it is easier to individually analyze each unit, it makes this approach more useful when giving management information to hospital managers.[1] Moreover, the DEA methodology is the one that adapts best to the multiplicity of resources and products existing in the hospital activity, it allows the analysis of the efficiency frontier without making *a priori* assumptions, as well as not taking a very long computation time.

Productivity can be defined as the ratio of output to input usage. It possible for productivity to change over time as well, either due to shifts of the production frontier or efficiency change, i.e. firms' shifts over time relative to their frontier (Hollingsworth, 2008). More recently, productivity measures have included technological as well as efficiency changes, as opposed to only considering technical ones (Silwal & Ashton, 2017).

The Malmquist Productivity Index (MPI) is a bilateral index that compares the production technology of two economies, evaluating the efficiency change over time (Tone, 2005), and can be defined as the ratio of two input distance functions (Simar & Wilson, 1999). The MPI can be decomposed into indices describing changes in technology and efficiency (Simar & Wilson, 1999), indicating progress or regress in efficiency as well as progress or regress of the technology frontier over time (Tone, 2005).

Another productivity index is the Hicks-Moorsteen productivity index, which can be defined as the ratio of a Malmquist output over a Malmquist input quantity index in the same base period (Kerstens & Van De Woestyne, 2014). The Hicks-Moorsteen Index can be decomposed into scale efficiency change and mix efficiency change components (Kerstens & Van De Woestyne, 2014).

Even though the MPI just measures local technical change (Kerstens & Van De Woestyne, 2014) and is only unbiased if the technology exhibits Constant Returns to Scale (CRS) (Ferreira & Marques, 2016a), it allows the separation of productivity changes into efficiency and technical changes (Barros & Alves, 2004). Being estimated with DEA, it also encompasses the DEA advantages, such as no need to impose functional form to the data or to make distributional assumptions. Moreover, it is of easier interpretation when compared to the Hicks-Moorsteen Index (Kerstens & Van De Woestyne, 2014).

## 4.2 Data Envelopment Analysis

Each hospital can be considered as having its own production technology, consuming resources (inputs) to deliver health care services to the population (outputs). Since the economic theory suggests that homogeneous entities demonstrate similar production technologies, these entities can be compared against each other (Ferreira & Nunes, 2019).

As stated before, DEA is a benchmarking technique (Ji & Lee, 2010) and linear programming method used to examine the relationship between inputs and outputs of each Decision Making Unit's (DMU) production process from observed data, comparing the result with the best practice frontier (Büchner, Hinz, & Schreyögg, 2016). In a DEA model, the efficiency of a DMU is defined as the ratio of the sum of its weighted outputs (for example, number of patients treated) to the sum of its weighted inputs (for example, resources used in a hospital) (Weng et al., 2009).

This efficiency approach is based on the Pareto-Koopmans definition, which states that an input-output vector is technically efficient if none of the outputs can be increased without any other output being reduced or some input being is increased, and none of the inputs can be reduced without other input being increased or some output being reduced (Lins, Lobo, Moreira Da Silva, Fiszman, & Ribeiro, 2007).

[1] Castro, Ricardo A. S., Portela, Conceição S., Camanho, Ana S. (2020). Benchmarking dos serviços dos hospitais portugueses: uma aplicação de data envelopment analysis. Imprensa da Universidade de Coimbra (Available at: digitalis.uc.pt/handle/10316.2/35942). Acessed on: 25/10/2020

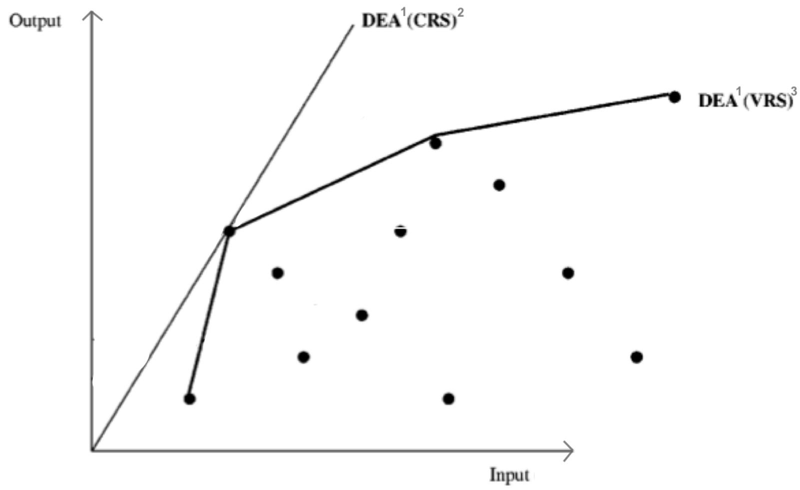


Figure 4.2: Data Envelopment Analysis productivity frontiers assuming Constant Returns to Scale and Variable Returns to Scale (each dot represents a Decision Making Unit).
[1]DEA: Data Envelopment Analysis; [2]CRS: Constant Returns to Scale; [3]VRS: Variable Returns to Scale
Figure adapted from Jacobs (2001).

DEA calculates relative efficiency, since the efficiency of each DMU is determined in relation to all other DMUs (Ersoy et al., 1997). It simultaneously analyzes each DMU's efficiency and identifies the optimal input/output combination, depicted as the "best practice frontier" (Ersoy et al., 1997). This frontier represents the production technology of the most efficient entities, with DMUs belonging to it having an efficiency score of one and being benchmarks for the other, inefficient, entities, since they can deliver the same kind of services with a more efficient use of the available resources (Ferreira & Nunes, 2019). Accordingly, DMUs operating below the frontier are assigned a score inferior to one, but greater than zero, hence being capable to improve capacity and future performance (Ersoy et al., 1997; Ji & Lee, 2010).

The modeling can be input or output oriented, depending if the objective is the reduction of resources or production increase (Lins et al., 2007). In the scope of this work, input orientation is assumed, since in hospitals there is little control of production (outputs), i.e. managers can control the inputs, such as number of hired staff or hospital costs, whereas outputs, for example number of patients treated, can be considered exogenous (Büchner et al., 2016).

Moreover, DEA can be carried out based on Constant Returns to Scale (CRS), meaning that the output will change by the same proportion as inputs are changed, or Variable Returns to Scale (VRS), which reflects that production technology may increase, decrease or maintain returns to scale. The efficiency frontier is, then, different in the two models (see Figure 4.2), with the VRS approach being considered to have the more flexible frontier (Ji & Lee, 2010; Lins et al., 2007).

Mathematically, consider a set of $j = 1, 2, ..., n$ DMUs (hospitals, in this case) that transform a vector of $i = 1, 2, ..., m$ inputs into a vector of $r = 1, 2, ..., s$ outputs. Each hospital $n$ is characterized by the vector $(x_n, y_n)$ of inputs and outputs, with $x \in R^m_+$ and $y \in R^s_+$. Let $(x^i_j, y^r_j)$ be the vector defining the DMU whose efficiency is being assessed and $\lambda_n$ the weights regarding the outputs and inputs.

The input oriented efficiency of each DMU $j$ is then calculated by solving the following linear programming problem $n$ times:

$$\min \quad \theta_j \tag{4.1}$$

$$\text{subject to} \quad \sum_n \lambda_n x_n^i \leq \theta_j x_j^i \tag{4.2}$$

$$\sum_n \lambda_n y_n^r \geq y_j^r \tag{4.3}$$

$$\lambda \geq 0. \tag{4.4}$$

in the case of assuming CRS. If it is the case of VRS then another condition is needed (Jacobs, 2001):

$$\sum_{j=1}^{n} \lambda_j = 1. \tag{4.5}$$

Calculating the efficiency scores with DEA under both CRS and VRS, it is possible to evaluate the scale efficiency, by dividing the score under CRS for the one considering VRS (Kirigia & Asbu, 2013). The maximum scale efficiency score is one, which implies that the DMU considered is operating at its optimal scale or size. If the score is less than one, the unit is either too small or too big relative to the optimal size (Kirigia & Asbu, 2013).

DEA can be combined with the Malmquist or Hicks-Moorsteen Productivity Indexes in order to assess efficiency over a period of time.

## 4.3 Malmquist Productivity Index

The Malmquist Productivity Index (MPI) is one of the most frequent productivity measures. It computes the change in productivity between two time periods with respect to a reference technology (Álvarez, Barbero, & Zofío, 2020; Mitropoulos et al., 2018).

Let's consider, as before, a DMU that produces $s$ outputs from $m$ inputs, with $x \in R_+^m$ and $y \in R_+^s$ being input and output vectors, respectively, and a production possibilities set at time $t$ denoted as (Simar & Wilson, 1999):

$$P = \{(x^t, y^t) | x^t \text{ can produce } y \text{ at time } t\}. \tag{4.6}$$

Its upper boundary can be referred as the production technology or the production frontier (Simar & Wilson, 1998). Let $(x_i^t, y_i^t)$ be the input and output vectors of production unit $i$ at time $t$.

The MPI, which measures the productivity change of the DMU under evaluation by comparing its relative performance with respect to the technologies in two different time periods ($t$ and $t+1$) (Álvarez et al., 2020), is defined as a geometrical mean of relative productivity changes from time $t$ to time $t+1$ (Daskovska et al., 2010):

$$\begin{aligned} \Pi^{t,t+1} &= \left( \frac{\theta_{CRS}^{t}(x^{t+1}, y^{t+1})}{\theta_{CRS}^{t}(x^t, y^t)} \cdot \frac{\theta_{CRS}^{t+1}(x^{t+1}, y^{t+1})}{\theta_{CRS}^{t+1}(x^t, y^t)} \right)^{1/2} = \\ &= \left( \frac{\theta_{CRS}^{t+1}(x^{t+1}, y^{t+1})}{\theta_{CRS}^{t}(x^t, y^t)} \right) \cdot \left( \frac{\theta_{CRS}^{t}(x^{t+1}, y^{t+1})}{\theta_{CRS}^{t+1}(x^{t+1}, y^{t+1})} \cdot \frac{\theta_{CRS}^{t}(x^t, y^t)}{\theta_{CRS}^{t+1}(x^t, y^t)} \right)^{1/2} = \\ &= \Delta Eff^{t,t+1} \cdot \Delta Tech^{t,t+1} \end{aligned} \tag{4.7}$$

in which the term $\Delta Eff^{t,t+1}$ "measures the change in relative efficiency (i.e., the change in how far observed production is from maximum potential production)" between times $t$ and $t+1$, and the term $\Delta Tech^{t,t+1}$ "captures the shift in technology between the two periods" evaluated in the hyperplanes where the inputs for production unit $i$ are maintained constant at times $t$ and $t+1$ (Simar & Wilson, 1998).

If $\Delta Eff^{t,t+1}>1$ or $\Delta Tech^{t,t+1}>1$, productivity change is driven by technical efficiency and technical progress. On the other hand, if $\Delta Eff^{t,t+1}<1$ or $\Delta Tech^{t,t+1}<1$, it means lower productivity due to greater inefficiency and technical regress. Values equal to one mean, logically, that the technical efficiency and the reference frontier remain unchanged (Álvarez et al., 2020).

It is also possible to decompose productivity change from an initial to a final period into consecutive sub-periods, given the transitivity property of index numbers. Thus, having a sequence of periods $t = 1, 2, 3$, for example, the Malmquist index between the initial and final periods can be defined in terms of its chain components (Álvarez et al., 2020):

$$\Pi^{1,3} = \Pi^{1,2} \times \Pi^{2,3}. \tag{4.8}$$

The MPI can also be decomposed into four terms (Simar & Wilson, 1998), which will be further described in the next subsection.

## 4.4 Forecasting the Malmquist Productivity Index

This subchapter is based on the work of Daskovska et al. (2010).

As presented before, the MPI is a bilateral index with which one can compare two economies' production technology.

In Daskovska et al. (2010), a new method for forecasting the MPI is introduced. Their motivation was the fact that there were no methods but "naive" ones, which consisted of, for example, using the geometrical mean of previous years to forecast the coming year. These past approaches were static and did not take full advantage of the information given by the productivity evolution over time. Hence, the new method developed uses a dynamical approach for the forecast that considers the productivity's behaviour over time. In order to do so, a required condition is the circularity property of the index. Therefore, because the MPI is not circular, Daskovska et al. (2010) also propose a new decomposition of the index into circular components. This framework by Daskovska et al. (2010) is the one used to forecast the MPI in this work.

The MPI decomposition previously mentioned is only capable of measuring productivity change if the underlying, true technology exhibits constant returns to scale everywhere, which is often not the case. So, another decomposition is needed, such as the one proposed by Simar and Wilson (1998).

After having defined $P^t$ in the previous subsection, now we need to define the set $V^t$ as the convex cone with vertex at the origin spanned by $P^t$, meaning $P^t \subseteq V^t$. If the technology exhibits CRS everywhere, then $P^t = V^t$.

So, both terms $\Delta Eff^{t,t+1}$ and $\Delta Tech^{t,t+1}$ (from Equation 4.7) can be further decomposed. $\Delta Eff^{t,t+1}$ can be defined as:

$$\Delta Eff^{t,t+1} = \Delta PureEff^{t,t+1} \cdot \Delta Scale^{t,t+1} \tag{4.9}$$

where

$$\Delta PureEff^{t,t+1} = \frac{\theta^{t+1}_{CRS}(x^{t+1}, y^{t+1})}{\theta^{t}_{CRS}(x^{t}, y^{t})} \tag{4.10}$$

and

$$\Delta Scale^{t,t+1} = \frac{\theta^{t+1}_{VRS}(x^{t+1}, y^{t+1})/\theta^{t+1}_{CRS}(x^{t+1}, y^{t+1})}{\theta^{t}_{VRS}(x^{t}, y^{t})/\theta^{t}_{CRS}(x^{t}, y^{t})}. \tag{4.11}$$

And $\Delta Tech^{t,t+1}$ can be decomposed as:

$$\Delta Tech^{t,t+1} = \Delta PureTech^{t,t+1} \cdot \Delta ScaleTech^{t,t+1} \tag{4.12}$$

with

$$\Delta PureTech^{t,t+1} = \left( \frac{\theta^{t}_{CRS}(x^{t+1}, y^{t+1})}{\theta^{t+1}_{CRS}(x^{t+1}, y^{t+1})} \cdot \frac{\theta^{t}_{CRS}(x^{t}, y^{t})}{\theta^{t+1}_{CRS}(x^{t}, y^{t})} \right)^{1/2} \tag{4.13}$$

and

$$\Delta ScaleTech^{t,t+1} = \left( \frac{\theta^{t}_{VRS}(x^{t+1}, y^{t+1})/\theta^{t}_{CRS}(x^{t+1}, y^{t+1})}{\theta^{t+1}_{VRS}(x^{t+1}, y^{t+1})/\theta^{t+1}_{CRS}(x^{t+1}, y^{t+1})} \cdot \frac{\theta^{t}_{VRS}(x^{t}, y^{t})/\theta^{t}_{CRS}(x^{t}, y^{t})}{\theta^{t+1}_{VRS}(x^{t}, y^{t})/\theta^{t+1}_{CRS}(x^{t}, y^{t})} \right)^{1/2}. \tag{4.14}$$

The MPI can then be defined in this new decomposition (Simar & Wilson, 1998):

$$\Pi^{t,t+1} = \Delta PureEff^{t,t+1} \cdot \Delta Scale^{t,t+1} \cdot \Delta PureTech^{t,t+1} \cdot \Delta ScaleTech^{t,t+1} \tag{4.15}$$

where $\Delta PureEff^{t,t+1}$ measures the change in relative efficiency, meaning how far production is from the maximum potential production, $\Delta Scale^{t,t+1}$ measures the changes in scale efficiency, $\Delta PureTech^{t,t+1}$ is the shift in technology, and $\Delta ScaleTech^{t,t+1}$ measures the changes in scale technology, i.e. change in the shape of the technology, which may be the flattening of techonolgy if the value obtained is smaller than one, or an increasing of the curvature or change away from CRS, if the value is bigger than one (Simar & Wilson, 1998).

### 4.4.1 Circularity and decomposition into circular components

According to Daskovska et al. (2010), right at the beginning of the forecast, two essential points must be covered: what happens to the index when time $T$ tends to infinity and what happens between two time periods.

A bilateral index $I^{t,s}$ is considered circular only if

$$I^{t,t+2} = I^{t,t+1} \cdot I^{t+1,t+2}, \forall t = 1, ..., T-2. \tag{4.16}$$

In this case, the circularity property serves the purpose of a "connector" for the indices. This means that if we can compare the productivity between times $t$ and $t+1$ and between $t+1$ and $t+2$, we should be able to compare times $t$ and $t+2$ via the time period $t+1$.

Even though circularity is a required necessity, the MPI is not circular despite some special cases such as the production unit or the production technology frontier being constant over time. Thus, the MPI needs to be decomposed into circular components. Considering the decomposition of Equation 4.15, while $\Delta PureEff^{t,t+1}$ and $\Delta Scale^{t,t+1}$ have easily demonstrable circularity, the other terms, $\Delta PureTech^{t,t+1}$ and $\Delta ScaleTech^{t,t+1}$, are not circular. Starting with $\Delta PureTech^{t,t+1}$, it is a geometric mean of two factors that represent relative changes:

$$\begin{aligned} \Delta PureTech^{t,t+1} &= \left( \frac{\theta^{t}_{CRS}(x^{t+1}, y^{t+1})}{\theta^{t+1}_{CRS}(x^{t+1}, y^{t+1})} \cdot \frac{\theta^{t}_{CRS}(x^{t}, y^{t})}{\theta^{t+1}_{CRS}(x^{t}, y^{t})} \right)^{1/2} = \\ &= \left( \Delta PT^{t+1}_{t,t+1} \cdot \Delta PT^{t}_{t,t+1} \right)^{1/2} \end{aligned} \tag{4.17}$$

in which the first factor represents the distance to the "true" frontier for a fixed point at $t+1$, and the second the "relative change of distance to the "true" frontier for a point fixed at time $t$". And, if the production unit is fixed at times $t$ or $t+1$, each of the following terms is circular:

$$\Delta PT^{t}_{t,t+2} = \Delta PT^{t}_{t,t+1} \cdot \Delta PT^{t}_{t,t+2} \tag{4.18}$$

$$\Delta PT_{t,t+2}^{t+1} = \Delta PT_{t,t+1}^{t+1} \cdot \Delta PT_{t+1,t+2}^{t+1} \quad (4.19)$$

Taking this into account, it seems possible to forecast each circular component separately. Given a production unit working at levels $(x^t, y^t)$ for different time periods $t$, with $t = 1, ..., T$, we can have: $\Delta PT_{s,s+1}^{t}$ where $s = 1, ..., T-1; t = 1, ..., T$, which can be organized in a table such as Table 4.2.

Table 4.2: Technological change forecast decomposition. Table adapted from Daskovska et al. (2010).

| Time periods' shift | $(x^1, y^1)$ fixed | $(x^2, y^2)$ fixed | ... | $(x^T, y^T)$ fixed | Forecast |
|---|---|---|---|---|---|
| 1,2 | $\Delta \hat{PT}_{1,2}^{1}$ | $\Delta \hat{PT}_{1,2}^{2}$ | ... | $\Delta \hat{PT}_{1,2}^{T}$ | $\Delta \hat{PT}_{1,2}^{T+1}$ |
| 2,3 | $\Delta \hat{PT}_{2,3}^{1}$ | $\Delta \hat{PT}_{2,3}^{2}$ | ... | $\Delta \hat{PT}_{2,3}^{T}$ | $\Delta \hat{PT}_{2,3}^{T+1}$ |
| ... | ... | ... | ... | ... | ... |
| $T-1, T$ | $\Delta \hat{PT}_{T-1,T}^{1}$ | $\Delta \hat{PT}_{T-1,T}^{2}$ | ... | $\Delta \hat{PT}_{T-1,T}^{T}$ | $\Delta \hat{PT}_{T-1,T}^{T+1}$ |
| Forecast $T, T+1$ | $\Delta \hat{PT}_{T,T+1}^{1}$ | $\Delta \hat{PT}_{T,T+1}^{2}$ | ... | $\Delta \hat{PT}_{T,T+1}^{T}$ | $\Delta \hat{PT}_{T,T+1}^{T+1}$ |

where $\Delta PT_{t,t+1}^{t}$, $\Delta PT_{t+1,t+2}^{t}$, $\Delta PT_{t+2,t+3}^{t}$, ... represent Table 4.2's columns and $\Delta PT_{t,t+1}^{t}$, $\Delta PT_{t,t+1}^{t+1}$, $\Delta PT_{t,t+1}^{t+2}$, ... Table 4.2's rows.

The term $\Delta ScaleTech^{t,t+1}$ is dealt with in the same way as $\Delta PureTech^{t,t+1}$ since it presents the same structure, and so its sequences can also be organized in a similar table.

### 4.4.2 Forecasting

The aim is to forecast, based on the data from $t = 1, ..., T$, the productivity performance of a production unit from the time period $T$ to the time period $T+1$. So, we need to forecast:

$$\Pi^{T,T+1} = \Delta PureEff^{T,T+1} \cdot \Delta Scale^{T,T+1} \cdot \Delta PureTech^{T,T+1} \cdot \Delta ScaleTech^{T,T+1}. \quad (4.20)$$

In order to do so, firstly the circular terms ($\Delta PureEff^{t,t+1}$ and $\Delta Scale^{t,t+1}$) are forecasted using the time-series method auto-regressive moving average (Auto Regressive Moving Average (ARMA)), mentioned by Daskovska et al. (2010) (see subsection below). After, the more complicated term, $\Delta PureTech^{t,t+1}$, is forecasted. This is done treating each term , $\Delta PT_{T,T+1}^{T+1}$ and $\Delta PT_{T,T+1}^{T}$, of Equation 4.17 independently, each forecasted from the estimates sequence:

$$\Delta \hat{PT}_{t,t+1}^{s} = \frac{\hat{\theta}_{CRS}^{t}(x^s, y^s)}{\hat{\theta}_{CRS}^{t+1}(x^s, y^s)}. \quad (4.21)$$

This is compiled in Table 4.2, in which the last two entries of the lower row correspond to the terms of interest, with their geometrical mean ($\Delta \hat{PT}_{T,T+1}^{T} \cdot \Delta \hat{PT}_{T,T+1}^{T+1}$) being the wanted forecast. Since every column has the circularity property, the forecasting, once again using the ARMA model, is done on every row. After this, the last row is forecasted as well giving us the two desired terms.

The forecast of the term $\Delta ScaleTech^{t,t+1}$ is obtained by the same procedure as the one used for $\Delta PureTech^{t,t+1}$, using an analogous table, since

$$\Delta ScaleTech^{t,t+1} = \left(\Delta ScT_{t,t+1}^{t+1} \cdot \Delta ScT_{t,t+1}^{t}\right)^{1/2} \quad (4.22)$$

in which each term: $\Delta ScT_{t,t+1}^{t+1} = \frac{\theta_{VRS}^{t}(x^{t+1},y^{t+1})/\theta_{CRS}^{t}(x^{t+1},y^{t+1})}{\theta_{VRS}^{t+1}(x^{t+1},y^{t+1})/\theta_{CRS}^{t+1}(x^{t+1},y^{t+1})}$ and $\Delta ScT_{t,t+1}^{t} = \frac{\theta_{VRS}^{t}(x^{t},y^{t})/\theta_{CRS}^{t}(x^{t},y^{t})}{\theta_{VRS}^{t+1}(x^{t},y^{t})/\theta_{CRS}^{t+1}(x^{t},y^{t})}$ is circular.

Finally, the MPI forecast is given by the product of all the previously forecasted indices:

$$\hat{\Pi}^{T,T+1} = \Delta \widehat{PureEff}^{T,T+1} \cdot \Delta \widehat{Scale}^{T,T+1} \cdot \Delta \widehat{PureTech}^{T,T+1} \cdot \Delta \widehat{ScaleTech}^{T,T+1} \tag{4.23}$$

### 4.4.3 Autoregressive moving average model

Time series analysis is the most popular approach in the literature when it comes to forecasting, among which there are the Auto Regressive Moving Average (ARMA) and the Auto Regressive Integrated Moving Average (ARIMA), some of the most used time series forecasting methods (Divina, Torres, Vela, & Noguera, 2019). These two methods have been commonly applied to short-term forecasting (Divina et al., 2019) and to multi-period predictions (Colak, Yesilbudak, Genc, & Bayindir, 2016). The difference between ARMA and ARIMA is that the first is applied to stationary time series data and the latter to non-stationary stochastic data (Colak et al., 2016).

As all time series forecasting methods, past observations of the same variable are analyzed in order to develop a model that describes their underlying relationship. This model will then be used to extrapolate the time series in the future (Zhang, 2003). It is useful when not very much knowledge is available about the data generating process or when there is no satisfactory explanatory model relating the prediction variable to other explanatory variables (Zhang, 2003).

ARIMA models are quite flexible since they can represent several different types of time series (Zhang, 2003) and, unlike several exponential smoothing procedures that attempt fitting the data to a particular model, ARIMA models fit various models to data (C. Lim & McAleer, 2002). However, the pre-assumed linear form of the model presents the major limitation of the model, given that the approximation of complex real problems to linear models is sometimes inadequate (Zhang, 2003).

ARMA models are especially better for short term forecasts. Also, they are easy implemented and quite robust (Karia & Bujang, 2011). However, with ARMA the non-stationarity in the series is not accounted for (Huang & Shih, 2003).

The future value of a variable, in an ARIMA/ARMA model, is assumed to be a linear function of past observations and random errors (Karia & Bujang, 2011; Zhang, 2003):

$$y_t = \theta_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + ... + \phi_p Y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - ... - \theta_q \varepsilon_{t-q} \tag{4.24}$$

where $y_t$ and $\varepsilon_t$ represent, respectively, the actual value and random error at time period t, and $\phi_p$ and $\theta_q$ are respectively, the autoregressive and moving average coefficients to be estimated, with $p$ and $q$ often referred to as orders of the model.

Firstly, the appropriate model order is determined and the parameters specified, after which the model $ARMA(p,q)$ or $ARIMA(p,d,q)$ can be used for the forecasting (Casella, Fienberg, & Olkin, 2006). Firslty, it is necessary to verify if the data is stationary, in order to determine if $d = 0$ or not. The estimation of the other two parameters involves fitting autoregressive (AR) models of order $p$ and moving average (MA) models of order $q$ (C. Lim & McAleer, 2002). Forecast accuracy measures, such as mean squared error (MSE), can be used for selecting a model for a given set of data, provided the errors are not computed from the same data as were used for model estimation (Karia & Bujang, 2011). Autocorrelation function (ACF) and partial autocorrelation function (PACF) can be used for this purpose as well (Huang & Shih, 2003).

## 4.5 Pre-processing

### 4.5.1 Filling data gaps

The data collected (see Chapter 5) presented, for most DMUs and for some months and years, sporadic data gaps. Excluding these DMUs from the analysis would result in less reliable results. For this reason, correlation between variables and linear regression were adopted to solve the problem.

First, the correlation between variables was calculated for the year and DMU for which there was missing data. After this, the missing month was estimated using linear regression with the variable that had the highest correlation and that had data recorded for the same month.

### 4.5.2 Principal Component Analysis

Principal Component Analysis (PCA) is a data reduction technique, used to reduce the dimensions of the used data (Akkan et al., 2020), reducing the number of variables describing the objects to a lower number of principal components with minimal loss of infotmation (Adler & Yazhemsky, 2010). PCA is based on the correlation matrix of the variables and principal components are linear combinations of the original variables obtained by a multidimensional method of orthogonal transformation (Domagała, 2014) that can replace all inputs and outputs or just certain groups of variables (Ueda & Hoshiai, 1997). Then, these principal components are used as the inputs and outputs for the analysis, reducing the data given to the DEA model (Ueda & Hoshiai, 1997) from $p$ original variables to $r \leq p$ unobservable and uncorrelated new variables (Domagała, 2014).

In DEA, the number of DMUs and variables being considered is of particular importance in the case of a small number of DMUs being described by a large number of variables (Domagała, 2014). If this happens, the efficiency ratios may be overestimated which weakens the discriminatory property of DEA (Domagała, 2014). According to Cooper, Seiford, and Tone (2007), the minimum number of DMUs should be:

$$n_{min} = max\{m \cdot s; 3 \cdot (m + s)\} \tag{4.25}$$

where $n$ represents the number of objects, $m$ the number of inputs and $s$ the number of outputs that describe the object.

In the case of this condition not being verified, or it being close to the limit, the use of PCA may be helpful, and it does not require the reduction of the number of variables nor the increase of the number of DMUs (Domagała, 2014).

In the case of this work, five inputs and three outputs were considered for the analysis with DEA. With this number of variables, the minimum of DMUs should me 24. The number studied is 26, which is near the limit. Therefore, PCA was performed, resulting in one vector for input and one for output, since the first principal component explained, in both cases, more than 90% of the total variance.

# Chapter 5

# Case Study

As mentioned before, this dissertation's objective consists of assessing and forecasting Portuguese public hospital's efficiency and productivity. This was done using DEA and MPI. For both these methods DMUs and variables are needed. In this case, as is obvious, the DMUs are Portuguese public hospitals. Data relative to this hospitals and the variables chosen were collected and analyzed.

## 5.1 Sample

The sample of DMUs adopted is here described, as well as some decisions made regarding its constitution. From all the health facilities in Portugal some were not considered in the analysis. Firstly, as this analysis is about public hospitals, local health units (*"Unidade Local de Saúde"*) were not included. Moreover, only entities with public management were of interest so private hospitals, public-private partnerships and hospitals run by the *Misericórdias* were not included. Specialized hospitals, with specific technology of production, such as maternities, oncology centers (*"Instituto Português de Oncologia (IPO)"*) and psychiatric hospitals are also rejected.

Taking this into account, from all the portuguese health facilities, besides the the local health units and IPOs, the following were not included:

- *Hospital de Magalhães Lemos, EPE*, which is a psychiatric hospital;
- *Hospital de Braga, EPE*, not included since it was under a public-private partnership during the studied years;
- *Centro Hospitalar do Oeste, EPE*, only created in 2018, so no data from the analyzed years was available;
- *Hospital da Senhora da Oliveira Guimarães, EPE*, which was part of the health center *Centro Hospitalar do Alto Ave, EPE* along with *Hospital São José* in Fafe until 2015 (year in which hospitals belonging to the *Misericórdias* were returned), not included since the data registered for the years prior and after 2015 are not consistent for a reliable analysis.[1]

So, a total of 26 hospital and hospital centers, shown in Table 5.1, were studied.

[1] Serviço Nacional de Saúde - Entidades de Saúde (Available at: www.sns.gov.pt/institucional/entidades-de-saude/). Accessed on: 1/6/2020

Table 5.1: Portuguese public hospitals analyzed.

| DMU[1] | Hospital |
|---|---|
| $H_1$ | Centro Hospitalar Barreiro/Montijo, EPE[2] |
| $H_2$ | Centro Hospitalar de Leiria, EPE[2] |
| $H_3$ | Centro Hospitalar de Lisboa Ocidental, EPE[2] |
| $H_4$ | Centro Hospitalar de Setúbal, EPE[2] |
| $H_5$ | Centro Hospitalar do Baixo Vouga, EPE[2] |
| $H_6$ | Centro Hospitalar do Médio Ave, EPE[2] |
| $H_7$ | Centro Hospitalar e Universitário de Coimbra, EPE[2] |
| $H_8$ | Centro Hospitalar Entre Douro e Vouga, EPE[2] |
| $H_9$ | Centro Hospitalar Médio Tejo, EPE[2] |
| $H_{10}$ | Centro Hospitalar Póvoa de Varzim/Vila do Conde, EPE[2] |
| $H_{11}$ | Centro Hospitalar Tâmega e Sousa, EPE[2] |
| $H_{12}$ | Centro Hospitalar Tondela-Viseu, EPE[2] |
| $H_{13}$ | Centro Hospitalar Trás-os-Montes e Alto Douro, EPE[2] |
| $H_{14}$ | Centro Hospitalar Universitário Cova da Beira, EPE[2] |
| $H_{15}$ | Centro Hospitalar Universitário de Lisboa Central, EPE[2] |
| $H_{16}$ | Centro Hospitalar Universitário de São João, EPE[2] |
| $H_{17}$ | Centro Hospitalar Universitário do Algarve, EPE[2] |
| $H_{18}$ | Centro Hospitalar Universitário do Porto, EPE[2] |
| $H_{19}$ | Centro Hospitalar Universitário Lisboa Norte, EPE[2] |
| $H_{20}$ | Centro Hospitalar Vila Nova de Gaia/Espinho, EPE[2] |
| $H_{21}$ | Hospital Distrital da Figueira da Foz, EPE[2] |
| $H_{22}$ | Hospital Distrital de Santarém, EPE[2] |
| $H_{23}$ | Hospital Espírito Santo de Évora, EPE[2] |
| $H_{24}$ | Hospital Fernando Fonseca, EPE[2] |
| $H_{25}$ | Hospital Garcia de Orta, EPE[2] |
| $H_{26}$ | Hospital Santa Maria Maior, EPE[2] |

[1]DMU: Decision Making Unit; [2]EPE: *Entidade Pública Empresarial*

All data were collected from the Portuguese Central Health System Administration (ACSS) website.[2] ACSS ensures the management of the financial and human resources of the Ministry of Health and the NHS, as well as the NHS facilities and equipment. Its benchamrking website is meant to increase the transparency of the NHS operations and improve economic performance.

This website has data from year 2012 to the present year of 2020, however data relative to year 2012 and from 2018 until 2020 presented several gaps in several hospitals. Thus, the data analyzed in this work refers to years 2013 to 2017, and is organized by month. Data was exported from the database in Excel files that were then imported to Matlab.

## 5.2 Variables

Efficiency and productivity measures require the use of variables, more commonly inputs and outputs. A literature review was performed in Chapter 3 where the most used variables were identified. The variables were then chosen according to this and to what was available on the ACSS website.[2]

Inputs are any resources, which may or not include costs, that are consumed by the organization needed for production process, in the case of an hospital it's the resources needed to provide care. Outputs are factors that describe the goods, services or other outcomes, for example health care provided,

[2]ACSS - Benchmarking Hospitais (Available at: benchmarking-ACSS.min-saude.pt). Accessed on 4/5/2020

obtained from the processing of the unit's resources.

In line with what was found in the literature review, five variables were chosen as inputs and three as outputs.

Therefore, we have as inputs:

- Costs of External Services and Supplies (ESS) per standard patient - expenditures with external labor outsourcing;
- Costs of Staff per standard patient - expenditures with staff, including salaries and bonuses to doctors, nurses and other non-administrative staff;
- Costs of Clinical Consumption Material (CCM) per standard patient - expenditures with drugs and clinical materials;
- Standard patients per doctor FTE - average number of standard patients whose direct care a doctor is responsible for;
- Standard patients per nurse FTE - average number of standard patients whose direct care a nurse is responsible for.

And, as outputs:

- Discharges per bed - total number of patients per bed that leave the hospital having been treated;
- Total number of medical appointments - total amount of medical appointments, either first or not, that occur in a month;
- Total number of emergency room visits - number of patients that went through the emergency service.

Standard patient is a measure of hospital activity that expresses, in a single unit, the quantities of the different production lines, using as a weighting criteria the price equivalence between the production line considered as the reference and the rest.[2]

Full-time equivalent, or FTE, is a unit that indicates the workload of an employee so that it is comparable among different contexts.

The variable total number of emergency room visits, despite being only the fourth most used in the literature review, is in the three chosen as outputs. This happens since the variable number of hospital days was not found in the database, so it was replaced with the next most used as output. The two input variables regarding doctors and nurses were also not exactly the ones described in the other studies but were the ones existing to take into account the medical staff.

The data's descriptive statistics are presented in Table 5.2.

Moreover, Figures 5.1 to 5.3 depict the behavior of the variables over time.

Firstly, in Table 5.2, the standard deviation of the eight variables can be observed and demonstrates the heterogeneity present in each variable. However, there is also homogeneity and correlation between variables, as it can be observed that input variables related to costs are practically all of the same order of magnitude (specifically ESS costs and costs of CCM, standard patients per doctor and standard patients per nurse, and emergency visits and medical appointments). The same can also be said about the other two input variables, and the outputs. Moreover, regarding the variables costs of ESS and CCM

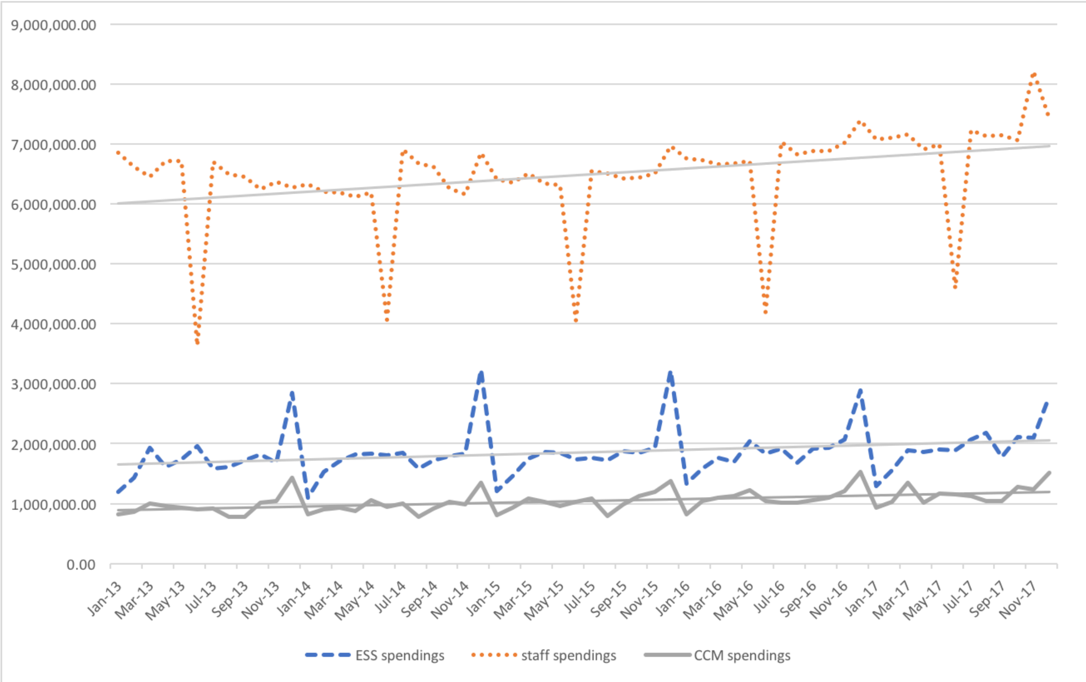


Figure 5.1: Average of the cost related input variables (in euros) through the analyzed years.
[1]ESS: External Services and Supplies; [2]CCM: Cinical Consumption Material

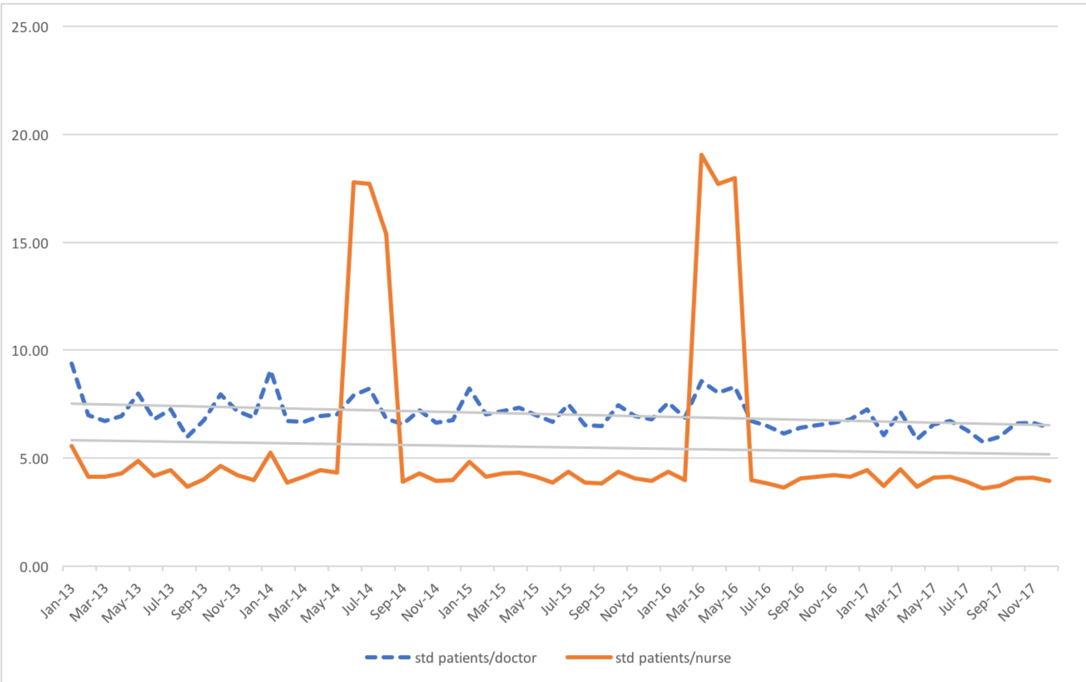


Figure 5.2: Average of the input variables standard patients per doctor and standard patients per nurse through the analyzed years.

Table 5.2: Descriptive statistics of the used variables.

| | Variables | Mean | Standard deviation | Maximum | Minimum |
|---|---|---|---|---|---|
| Inputs | ESS[1] costs (€) | 1,854,020.2 | 1,493,347.4 | 13,022,372.4 | 41,745.4 |
| | staff costs (€) | 6,490,769.6 | 4,921,307.2 | 26,046,966.8 | 157,002.8 |
| | CCM[2] costs (€) | 1,042,619.0 | 1,065,161.0 | 7,577,415.8 | 0 |
| | standard patients/doctors | 7.0 | 3.1 | 51.0 | 0.2 |
| | standard patients/nurse | 5.5 | 22.2 | 392.2 | 0.1 |
| Outputs | discharges | 1,891.2 | 1,124.1 | 10,152 | 324 |
| | medical appointments | 27,595.5 | 19,269.9 | 88,459 | 3,602 |
| | emergency room visits | 14,078.9 | 5,797.3 | 37,183 | 4,192 |

[1]ESS: External Services and Supplies; [2]CCM: Clinical Consumption Material

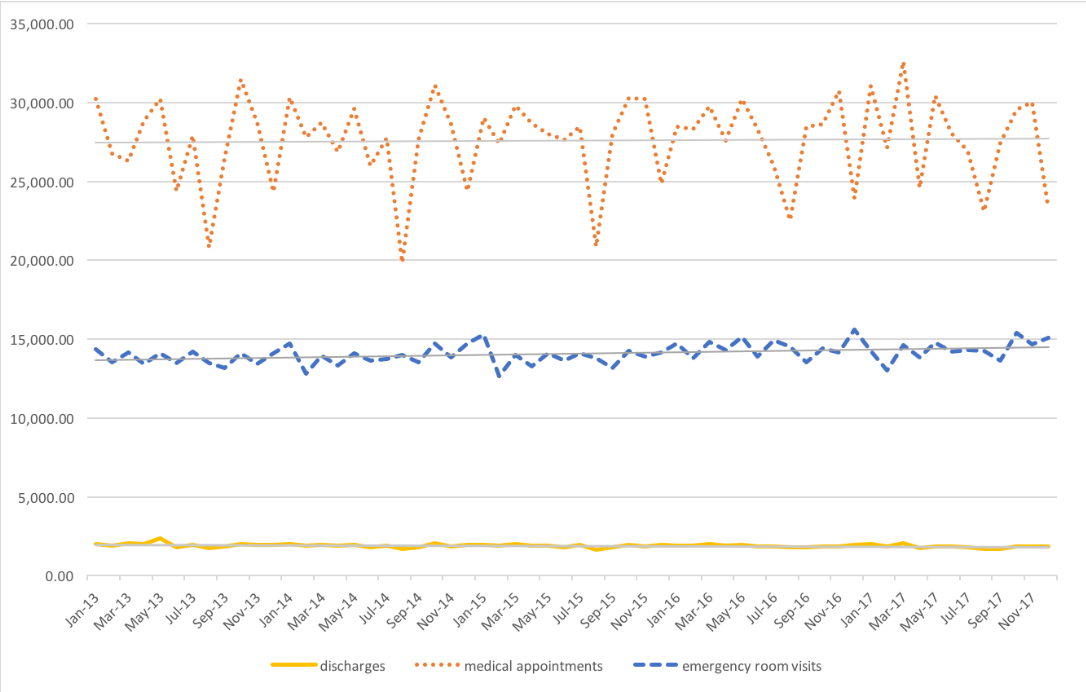


Figure 5.3: Average of the output variables through the analyzed years.

costs, they have a very similar behaviour over time. This is even more clear for the variables standard patients per doctor and standard patients per nurse, in which the curve shapes are practically the same.

ESS costs and CCM costs have peaks in December, which corresponds to the end of the year and so the time to make new contracts and purchase services and material, and since some suppliers are dealing with the balance sheets in January, the orders need to be done in December. Staff costs also have these same peaks, however smaller and less pronounced. These and the ones every July correspond to the subsidies payments. More attention should be drawn, in the case of staff costs, to the down peaks in June, which are represented because some hospitals, even though there is a small decrease overall, present a bigger decrease in the values in this month, which can be related to the end of some contracts, for example.

There can be seen a slight increase overall in all these input variables regarding costs. This is in line with the exit of the financial rescue program.

Still regarding input variables, the standard patients per doctor and per nurse present a peak, much more evident in the case of standard patients per nurse, around June-August 2014 and 2016. These outliers in the data come from values of around 300 in June, July and August of 2014 and March, April and May in 2016 of $H_{14}$.

The output variable medical appointments presents down peaks in August. This is the principal vacation month, making sense that the number of medical appointments would decrease.

In terms of the other output variables, no pattern catches the eye, and both variables (discharges and emergency room visits) have somewhat remained constant over time.

Part of these variables, more the input variables perhaps than output ones, present seasonality, displaying patterns that recur over a one-year period.

# Chapter 6

# Results and discussion

The aim of this work was to assess the performance of the Portuguese public hospitals from years 2013 to 2017 and forecast it for 2018. In this chapter, the results obtained are presented and analyzed. The complete Tables of results can be seen in the Appendix.

Firstly, the results obtained with DEA are presented. These results were obtained both considering CRS and VRS. Tables 6.1 to 6.4 present the principal statistics (mean, standard deviation, minimum and maximum) of the results obtained, organized both by hospital and time period.

In addition, Figures 6.1 and 6.2 show the distribution of the mean efficiency scores, i.e. the number of hospitals that present an average score between the shown intervals, during the studied years.

The scores vary in a considerate range, from the minimum observed 0.116 to 1.000. A score of 1.000 corresponds to an efficient unit, managing correctly their resources. In all analyzed years, both under CRS and VRS, at least one unit is considered efficient in every month, as can be seen in Tables 6.3 and 6.4, being this a benchmark for the other less efficient units. The other hospitals present inefficiency, that can come from different sources, and they could decrease their inputs to produce the same quantity of outputs. Considering CRS, the average efficiency score is 0.648 with an average standard deviation of 0.143. And when assuming VRS, the average efficiency score is 0.764 with an average standard deviation of 0.097. Hospitals have better efficiency scores when considering VRS, and the results are more homogeneous. Moreover, the number of efficient units increases when considering VRS. This can be seen in Figures 6.1 and 6.2, and Table 6.5. Observing these, it can be verified that, if assuming CRS, most hospitals do not show great efficiency scores, with most having values between 0.500 and 0.599, and with 17 between 0.500 and 0.699, which represents 65% of the analyzed hospitals. Only one has an average between 0.900 and 1.000. When assuming VRS, hospitals seem to perform better, with

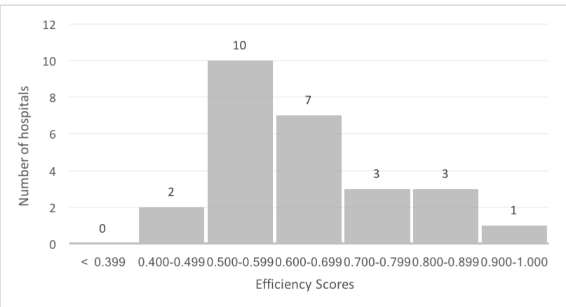


Figure 6.1: Distribution of the mean efficiency values, when considering Constant Returns to Scale.

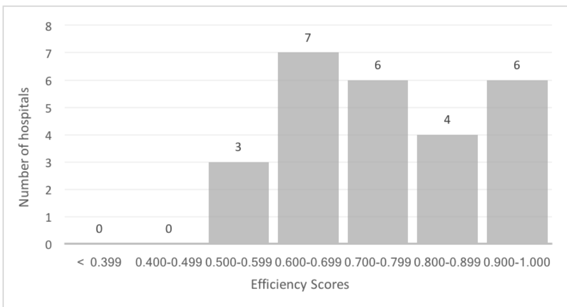


Figure 6.2: Distribution of the mean efficiency values, when considering Variable Returns to Scale.

Table 6.1: Data Envelopment Analysis results: mean, standard deviation, minimum and maximum values of efficiency scores for each of the analyzed hospitals in each of the analyzed years (2013-2017) considering constant returns to scale.

| | 2013 | | | | 2014 | | | | 2015 | | | | 2016 | | | | 2017 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Mean | Std. Dev. | Max. | Min. | Mean | Std. Dev. | Max. | Min. | Mean | Std. Dev. | Max. | Min. | Mean | Std. Dev. | Max. | Min. | Mean | Std. Dev. | Max. | Min. |
| $H_1$ | 0.441 | 0.213 | 0.665 | 0.160 | 0.627 | 0.047 | 0.721 | 0.544 | 0.634 | 0.123 | 0.783 | 0.257 | 0.640 | 0.050 | 0.703 | 0.510 | 0.657 | 0.044 | 0.721 | 0.546 |
| $H_2$ | 0.514 | 0.263 | 0.820 | 0.183 | 0.765 | 0.171 | 0.837 | 0.202 | 0.769 | 0.175 | 0.863 | 0.201 | 0.781 | 0.178 | 0.893 | 0.211 | 0.806 | 0.167 | 0.917 | 0.261 |
| $H_3$ | 0.456 | 0.224 | 0.673 | 0.154 | 0.622 | 0.058 | 0.702 | 0.475 | 0.627 | 0.071 | 0.755 | 0.472 | 0.637 | 0.065 | 0.715 | 0.448 | 0.646 | 0.062 | 0.716 | 0.495 |
| $H_4$ | 0.443 | 0.238 | 0.825 | 0.145 | 0.592 | 0.051 | 0.655 | 0.444 | 0.613 | 0.064 | 0.677 | 0.460 | 0.636 | 0.052 | 0.699 | 0.475 | 0.663 | 0.055 | 0.731 | 0.522 |
| $H_5$ | 0.499 | 0.248 | 0.772 | 0.185 | 0.707 | 0.056 | 0.789 | 0.560 | 0.690 | 0.112 | 0.762 | 0.330 | 0.709 | 0.076 | 0.774 | 0.472 | 0.730 | 0.073 | 0.834 | 0.538 |
| $H_6$ | 0.628 | 0.312 | 0.990 | 0.206 | 0.895 | 0.161 | 1.000 | 0.386 | 0.848 | 0.107 | 1.000 | 0.688 | 0.856 | 0.052 | 0.952 | 0.760 | 0.909 | 0.061 | 0.970 | 0.777 |
| $H_7$ | 0.383 | 0.187 | 0.664 | 0.148 | 0.555 | 0.130 | 0.721 | 0.186 | 0.579 | 0.141 | 0.731 | 0.174 | 0.604 | 0.145 | 0.691 | 0.162 | 0.597 | 0.128 | 0.743 | 0.249 |
| $H_8$ | 0.703 | 0.337 | 1.000 | 0.263 | 0.952 | 0.086 | 1.000 | 0.696 | 0.910 | 0.115 | 1.000 | 0.557 | 0.899 | 0.082 | 1.000 | 0.675 | 0.863 | 0.073 | 0.972 | 0.675 |
| $H_9$ | 0.382 | 0.210 | 0.652 | 0.128 | 0.567 | 0.062 | 0.662 | 0.392 | 0.588 | 0.053 | 0.638 | 0.440 | 0.623 | 0.062 | 0.708 | 0.482 | 0.621 | 0.072 | 0.688 | 0.391 |
| $H_{10}$ | 0.502 | 0.250 | 0.850 | 0.152 | 0.669 | 0.074 | 0.827 | 0.520 | 0.726 | 0.073 | 0.933 | 0.635 | 0.716 | 0.115 | 0.845 | 0.375 | 0.731 | 0.066 | 0.805 | 0.546 |
| $H_{11}$ | 0.669 | 0.329 | 0.985 | 0.218 | 0.952 | 0.083 | 1.000 | 0.694 | 0.967 | 0.088 | 1.000 | 0.683 | 0.964 | 0.115 | 1.000 | 0.584 | 0.973 | 0.046 | 1.000 | 0.841 |
| $H_{12}$ | 0.443 | 0.240 | 0.893 | 0.142 | 0.604 | 0.134 | 0.693 | 0.171 | 0.612 | 0.138 | 0.740 | 0.178 | 0.627 | 0.090 | 0.768 | 0.391 | 0.610 | 0.127 | 0.775 | 0.221 |
| $H_{13}$ | 0.471 | 0.233 | 0.724 | 0.144 | 0.638 | 0.097 | 0.723 | 0.335 | 0.637 | 0.091 | 0.761 | 0.395 | 0.647 | 0.070 | 0.726 | 0.453 | 0.663 | 0.071 | 0.735 | 0.522 |
| $H_{14}$ | 0.535 | 0.274 | 0.914 | 0.176 | 0.714 | 0.154 | 0.819 | 0.218 | 0.706 | 0.158 | 0.822 | 0.198 | 0.714 | 0.157 | 0.801 | 0.197 | 0.721 | 0.150 | 0.823 | 0.239 |
| $H_{15}$ | 0.389 | 0.192 | 0.594 | 0.132 | 0.520 | 0.043 | 0.562 | 0.389 | 0.526 | 0.044 | 0.572 | 0.410 | 0.528 | 0.052 | 0.573 | 0.369 | 0.533 | 0.058 | 0.578 | 0.349 |
| $H_{16}$ | 0.458 | 0.229 | 0.739 | 0.149 | 0.618 | 0.098 | 0.698 | 0.312 | 0.640 | 0.079 | 0.707 | 0.407 | 0.656 | 0.086 | 0.716 | 0.384 | 0.667 | 0.074 | 0.757 | 0.518 |
| $H_{17}$ | 0.347 | 0.176 | 0.597 | 0.116 | 0.453 | 0.063 | 0.595 | 0.351 | 0.477 | 0.056 | 0.599 | 0.351 | 0.502 | 0.081 | 0.621 | 0.324 | 0.479 | 0.080 | 0.644 | 0.326 |
| $H_{18}$ | 0.512 | 0.272 | 0.810 | 0.185 | 0.706 | 0.161 | 0.803 | 0.210 | 0.751 | 0.170 | 0.851 | 0.212 | 0.755 | 0.173 | 0.869 | 0.205 | 0.755 | 0.156 | 0.850 | 0.266 |
| $H_{19}$ | 0.376 | 0.201 | 0.605 | 0.140 | 0.530 | 0.123 | 0.628 | 0.139 | 0.563 | 0.130 | 0.649 | 0.158 | 0.591 | 0.138 | 0.696 | 0.152 | 0.615 | 0.133 | 0.732 | 0.193 |
| $H_{20}$ | 0.532 | 0.300 | 0.989 | 0.189 | 0.741 | 0.171 | 0.826 | 0.183 | 0.750 | 0.174 | 0.854 | 0.189 | 0.766 | 0.179 | 0.883 | 0.181 | 0.776 | 0.169 | 0.889 | 0.237 |
| $H_{21}$ | 0.566 | 0.277 | 0.836 | 0.205 | 0.782 | 0.079 | 0.857 | 0.539 | 0.817 | 0.089 | 0.922 | 0.560 | 0.858 | 0.075 | 0.933 | 0.640 | 0.905 | 0.095 | 0.988 | 0.639 |
| $H_{22}$ | 0.681 | 0.329 | 1.000 | 0.204 | 0.590 | 0.122 | 0.853 | 0.262 | 0.545 | 0.102 | 0.709 | 0.248 | 0.566 | 0.112 | 0.754 | 0.237 | 0.548 | 0.099 | 0.738 | 0.312 |
| $H_{23}$ | 0.434 | 0.208 | 0.683 | 0.159 | 0.628 | 0.122 | 0.770 | 0.341 | 0.627 | 0.121 | 0.713 | 0.342 | 0.656 | 0.155 | 0.757 | 0.151 | 0.651 | 0.142 | 0.762 | 0.197 |
| $H_{24}$ | 0.467 | 0.258 | 0.874 | 0.170 | 0.608 | 0.136 | 0.687 | 0.169 | 0.598 | 0.135 | 0.690 | 0.166 | 0.627 | 0.142 | 0.696 | 0.159 | 0.632 | 0.127 | 0.713 | 0.221 |
| $H_{25}$ | 0.433 | 0.221 | 0.654 | 0.146 | 0.572 | 0.111 | 0.648 | 0.215 | 0.584 | 0.134 | 0.683 | 0.155 | 0.598 | 0.141 | 0.754 | 0.152 | 0.614 | 0.128 | 0.721 | 0.217 |
| $H_{26}$ | 0.589 | 0.303 | 1.000 | 0.197 | 0.784 | 0.086 | 0.948 | 0.576 | 0.853 | 0.058 | 1.000 | 0.773 | 0.890 | 0.060 | 1.000 | 0.766 | 0.948 | 0.100 | 1.000 | 0.635 |

Table 6.2: Data Envelopment Analysis results: mean, standard deviation, minimum and maximum values of efficiency scores for each of the analyzed hospitals in each of the analyzed years (2013-2017) considering variable returns to scale.

| | 2013 | | | | 2014 | | | | 2015 | | | | 2016 | | | | 2017 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Mean | Std. Dev. | Max. | Min. | Mean | Std. Dev. | Max. | Min. | Mean | Std. Dev. | Max. | Min. | Mean | Std. Dev. | Max. | Min. | Mean | Std. Dev. | Max. | Min. |
| $H_1$ | 0.505 | 0.167 | 0.680 | 0.262 | 0.651 | 0.036 | 0.721 | 0.594 | 0.661 | 0.105 | 0.813 | 0.354 | 0.671 | 0.029 | 0.717 | 0.621 | 0.679 | 0.036 | 0.768 | 0.621 |
| $H_2$ | 0.678 | 0.173 | 0.909 | 0.285 | 0.778 | 0.154 | 0.849 | 0.272 | 0.786 | 0.152 | 0.871 | 0.290 | 0.794 | 0.153 | 0.904 | 0.311 | 0.825 | 0.163 | 0.988 | 0.308 |
| $H_3$ | 0.763 | 0.036 | 0.836 | 0.693 | 0.775 | 0.068 | 0.957 | 0.676 | 0.751 | 0.074 | 0.928 | 0.649 | 0.754 | 0.069 | 0.941 | 0.672 | 0.756 | 0.082 | 0.866 | 0.540 |
| $H_4$ | 0.536 | 0.155 | 0.845 | 0.351 | 0.613 | 0.032 | 0.677 | 0.553 | 0.640 | 0.041 | 0.679 | 0.530 | 0.662 | 0.019 | 0.701 | 0.624 | 0.683 | 0.045 | 0.733 | 0.546 |
| $H_5$ | 0.622 | 0.143 | 0.794 | 0.412 | 0.731 | 0.036 | 0.789 | 0.655 | 0.717 | 0.081 | 0.791 | 0.462 | 0.737 | 0.035 | 0.796 | 0.661 | 0.753 | 0.062 | 0.845 | 0.573 |
| $H_6$ | 0.707 | 0.257 | 1.000 | 0.364 | 0.925 | 0.162 | 1.000 | 0.402 | 0.897 | 0.090 | 1.000 | 0.732 | 0.904 | 0.047 | 1.000 | 0.818 | 0.942 | 0.054 | 0.988 | 0.778 |
| $H_7$ | 0.983 | 0.051 | 1.000 | 0.816 | 0.994 | 0.020 | 1.000 | 0.928 | 1.000 | 2.70E-16 | 1.000 | 1.000 | 1.000 | 3.68E-16 | 1.000 | 1.000 | 1.000 | 5.31E-16 | 1.000 | 1.000 |
| $H_8$ | 0.983 | 0.030 | 1.000 | 0.922 | 0.979 | 0.039 | 1.000 | 0.876 | 0.940 | 0.057 | 1.000 | 0.812 | 0.929 | 0.051 | 1.000 | 0.834 | 0.889 | 0.075 | 0.984 | 0.689 |
| $H_9$ | 0.430 | 0.184 | 0.660 | 0.213 | 0.588 | 0.060 | 0.686 | 0.416 | 0.619 | 0.034 | 0.654 | 0.523 | 0.653 | 0.044 | 0.718 | 0.561 | 0.641 | 0.062 | 0.697 | 0.447 |
| $H_{10}$ | 0.643 | 0.219 | 0.939 | 0.294 | 0.806 | 0.070 | 0.943 | 0.671 | 0.824 | 0.067 | 1.000 | 0.684 | 0.790 | 0.129 | 0.928 | 0.412 | 0.768 | 0.101 | 1.000 | 0.556 |
| $H_{11}$ | 0.897 | 0.079 | 1.000 | 0.749 | 0.977 | 0.029 | 1.000 | 0.908 | 1.000 | 1.43E-16 | 1.000 | 1.000 | 0.993 | 0.020 | 1.000 | 0.926 | 1.000 | 0.000 | 1.000 | 1.000 |
| $H_{12}$ | 0.573 | 0.117 | 0.895 | 0.439 | 0.613 | 0.120 | 0.701 | 0.229 | 0.627 | 0.117 | 0.746 | 0.257 | 0.650 | 0.087 | 0.774 | 0.397 | 0.625 | 0.120 | 0.777 | 0.260 |
| $H_{13}$ | 0.647 | 0.065 | 0.725 | 0.496 | 0.655 | 0.066 | 0.738 | 0.472 | 0.657 | 0.053 | 0.761 | 0.577 | 0.668 | 0.038 | 0.726 | 0.599 | 0.681 | 0.060 | 0.757 | 0.542 |
| $H_{14}$ | 0.568 | 0.276 | 0.934 | 0.178 | 0.747 | 0.163 | 0.868 | 0.219 | 0.742 | 0.148 | 0.847 | 0.265 | 0.749 | 0.149 | 0.834 | 0.260 | 0.745 | 0.150 | 0.878 | 0.267 |
| $H_{15}$ | 0.892 | 0.081 | 1.000 | 0.758 | 0.851 | 0.077 | 1.000 | 0.748 | 0.801 | 0.078 | 1.000 | 0.714 | 0.765 | 0.076 | 1.000 | 0.709 | 0.769 | 0.080 | 0.964 | 0.672 |
| $H_{16}$ | 0.944 | 0.050 | 1.000 | 0.863 | 0.936 | 0.066 | 1.000 | 0.778 | 0.927 | 0.031 | 1.000 | 0.890 | 0.936 | 0.050 | 1.000 | 0.858 | 0.950 | 0.059 | 1.000 | 0.791 |
| $H_{17}$ | 0.496 | 0.067 | 0.697 | 0.439 | 0.490 | 0.092 | 0.751 | 0.362 | 0.526 | 0.139 | 0.939 | 0.360 | 0.550 | 0.163 | 1.000 | 0.330 | 0.500 | 0.083 | 0.655 | 0.343 |
| $H_{18}$ | 0.954 | 0.144 | 1.000 | 0.478 | 0.943 | 0.143 | 1.000 | 0.491 | 0.957 | 0.143 | 1.000 | 0.481 | 0.956 | 0.134 | 1.000 | 0.513 | 0.954 | 0.137 | 1.000 | 0.501 |
| $H_{19}$ | 0.785 | 0.146 | 0.920 | 0.329 | 0.743 | 0.129 | 0.842 | 0.339 | 0.758 | 0.121 | 0.891 | 0.373 | 0.784 | 0.128 | 0.899 | 0.389 | 0.841 | 0.150 | 0.974 | 0.367 |
| $H_{20}$ | 0.888 | 0.165 | 1.000 | 0.353 | 0.905 | 0.166 | 1.000 | 0.366 | 0.888 | 0.159 | 1.000 | 0.371 | 0.896 | 0.155 | 1.000 | 0.395 | 0.916 | 0.163 | 1.000 | 0.385 |
| $H_{21}$ | 0.715 | 0.242 | 0.999 | 0.342 | 0.925 | 0.059 | 1.000 | 0.796 | 0.921 | 0.095 | 1.000 | 0.639 | 0.936 | 0.074 | 1.000 | 0.714 | 0.944 | 0.057 | 1.000 | 0.798 |
| $H_{22}$ | 0.695 | 0.318 | 1.000 | 0.208 | 0.618 | 0.126 | 0.861 | 0.267 | 0.579 | 0.089 | 0.738 | 0.332 | 0.593 | 0.100 | 0.776 | 0.309 | 0.566 | 0.088 | 0.745 | 0.351 |
| $H_{23}$ | 0.486 | 0.179 | 0.725 | 0.258 | 0.651 | 0.127 | 0.807 | 0.360 | 0.656 | 0.104 | 0.738 | 0.419 | 0.678 | 0.146 | 0.793 | 0.206 | 0.672 | 0.141 | 0.793 | 0.223 |
| $H_{24}$ | 0.668 | 0.146 | 0.876 | 0.265 | 0.624 | 0.119 | 0.708 | 0.250 | 0.615 | 0.115 | 0.713 | 0.248 | 0.657 | 0.117 | 0.718 | 0.271 | 0.667 | 0.124 | 0.803 | 0.285 |
| $H_{25}$ | 0.579 | 0.072 | 0.658 | 0.435 | 0.582 | 0.092 | 0.652 | 0.290 | 0.595 | 0.115 | 0.685 | 0.225 | 0.607 | 0.124 | 0.756 | 0.222 | 0.627 | 0.118 | 0.722 | 0.256 |
| $H_{26}$ | 0.819 | 0.223 | 1.000 | 0.427 | 1.000 | 1.28E-16 | 1.000 | 1.000 | 1.000 | 1.47E-16 | 1.000 | 1.000 | 1.000 | 1.32E-16 | 1.000 | 1.000 | 1.000 | 1.60E-16 | 1.000 | 1.000 |

Table 6.3: Data Envelopment Analysis results: mean, standard deviation, minimum and maximum values of efficiency scores for each of the months of the analyzed years (2013-2017) considering constant returns to scale.

| | 2013 | | | | 2014 | | | | 2015 | | | | 2016 | | | | 2017 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Mean | Std. Dev. | Min. | Max. | Mean | Std. Dev. | Min. | Max. | Mean | Std. Dev. | Min. | Max. | Mean | Std. Dev. | Min. | Max. | Mean | Std. Dev. | Min. | Max. |
| Jan. | 0.674 | 0.128 | 0.483 | 1.000 | 0.697 | 0.133 | 0.428 | 1.000 | 0.717 | 0.135 | 0.474 | 1.000 | 0.728 | 0.128 | 0.424 | 1.000 | 0.748 | 0.132 | 0.490 | 1.000 |
| Feb. | 0.200 | 0.163 | 0.116 | 1.000 | 0.668 | 0.129 | 0.435 | 1.000 | 0.723 | 0.138 | 0.481 | 1.000 | 0.736 | 0.120 | 0.545 | 1.000 | 0.742 | 0.126 | 0.506 | 1.000 |
| Mar. | 0.229 | 0.159 | 0.137 | 1.000 | 0.687 | 0.128 | 0.458 | 1.000 | 0.646 | 0.144 | 0.430 | 1.000 | 0.737 | 0.121 | 0.554 | 1.000 | 0.731 | 0.136 | 0.487 | 1.000 |
| Apr. | 0.705 | 0.145 | 0.320 | 1.000 | 0.704 | 0.135 | 0.456 | 1.000 | 0.716 | 0.125 | 0.483 | 1.000 | 0.731 | 0.120 | 0.504 | 1.000 | 0.746 | 0.125 | 0.548 | 1.000 |
| May | 0.680 | 0.136 | 0.381 | 1.000 | 0.736 | 0.136 | 0.464 | 1.000 | 0.722 | 0.125 | 0.480 | 1.000 | 0.738 | 0.122 | 0.492 | 1.000 | 0.756 | 0.125 | 0.491 | 1.000 |
| Jun. | 0.530 | 0.242 | 0.191 | 1.000 | 0.389 | 0.205 | 0.139 | 1.000 | 0.389 | 0.211 | 0.155 | 1.000 | 0.406 | 0.218 | 0.151 | 1.000 | 0.466 | 0.254 | 0.193 | 1.000 |
| Jul. | 0.679 | 0.139 | 0.332 | 1.000 | 0.685 | 0.134 | 0.432 | 1.000 | 0.704 | 0.123 | 0.483 | 1.000 | 0.685 | 0.133 | 0.391 | 1.000 | 0.723 | 0.137 | 0.494 | 1.000 |
| Aug. | 0.721 | 0.137 | 0.512 | 1.000 | 0.669 | 0.156 | 0.398 | 1.000 | 0.643 | 0.136 | 0.396 | 1.000 | 0.708 | 0.125 | 0.501 | 1.000 | 0.722 | 0.141 | 0.519 | 1.000 |
| Sep. | 0.244 | 0.159 | 0.146 | 1.000 | 0.703 | 0.134 | 0.521 | 1.000 | 0.728 | 0.133 | 0.475 | 1.000 | 0.726 | 0.128 | 0.483 | 1.000 | 0.735 | 0.139 | 0.473 | 1.000 |
| Oct. | 0.275 | 0.151 | 0.169 | 1.000 | 0.694 | 0.129 | 0.466 | 1.000 | 0.711 | 0.119 | 0.463 | 1.000 | 0.705 | 0.123 | 0.455 | 1.000 | 0.773 | 0.137 | 0.494 | 1.000 |
| Nov. | 0.256 | 0.154 | 0.150 | 1.000 | 0.729 | 0.132 | 0.484 | 1.000 | 0.735 | 0.119 | 0.473 | 1.000 | 0.736 | 0.126 | 0.471 | 1.000 | 0.640 | 0.145 | 0.339 | 1.000 |
| Dec. | 0.741 | 0.174 | 0.204 | 1.000 | 0.665 | 0.141 | 0.352 | 1.000 | 0.705 | 0.140 | 0.351 | 1.000 | 0.695 | 0.146 | 0.324 | 1.000 | 0.693 | 0.141 | 0.326 | 1.000 |

Table 6.4: Data Envelopment Analysis results: mean, standard deviation, minimum and maximum values of efficiency scores for each of the months of the analyzed years (2013-2017) considering Variable Returns to Scale.

| | 2013 | | | | 2014 | | | | 2015 | | | | 2016 | | | | 2017 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Mean | Std. Dev. | Min. | Max. | Mean | Std. Dev. | Min. | Max. | Mean | Std. Dev. | Min. | Max. | Mean | Std. Dev. | Min. | Max. | Mean | Std. Dev. | Min. | Max. |
| Jan. | 0.774 | 0.160 | 0.485 | 1.000 | 0.789 | 0.154 | 0.428 | 1.000 | 0.789 | 0.148 | 0.481 | 1.000 | 0.789 | 0.146 | 0.430 | 1.000 | 0.802 | 0.138 | 0.508 | 1.000 |
| Feb. | 0.591 | 0.267 | 0.211 | 1.000 | 0.767 | 0.157 | 0.435 | 1.000 | 0.777 | 0.146 | 0.492 | 1.000 | 0.802 | 0.140 | 0.552 | 1.000 | 0.806 | 0.146 | 0.513 | 1.000 |
| Mar. | 0.624 | 0.255 | 0.275 | 1.000 | 0.786 | 0.154 | 0.464 | 1.000 | 0.752 | 0.173 | 0.478 | 1.000 | 0.804 | 0.137 | 0.609 | 1.000 | 0.795 | 0.153 | 0.489 | 1.000 |
| Apr. | 0.794 | 0.168 | 0.337 | 1.000 | 0.801 | 0.159 | 0.459 | 1.000 | 0.788 | 0.141 | 0.494 | 1.000 | 0.795 | 0.135 | 0.513 | 1.000 | 0.804 | 0.134 | 0.566 | 1.000 |
| May | 0.795 | 0.176 | 0.412 | 1.000 | 0.805 | 0.154 | 0.477 | 1.000 | 0.783 | 0.141 | 0.492 | 1.000 | 0.808 | 0.145 | 0.503 | 1.000 | 0.817 | 0.141 | 0.499 | 1.000 |
| Jun. | 0.628 | 0.266 | 0.213 | 1.000 | 0.585 | 0.283 | 0.219 | 1.000 | 0.594 | 0.274 | 0.225 | 1.000 | 0.654 | 0.288 | 0.206 | 1.000 | 0.611 | 0.280 | 0.223 | 1.000 |
| Jul. | 0.785 | 0.163 | 0.398 | 1.000 | 0.769 | 0.157 | 0.442 | 1.000 | 0.777 | 0.142 | 0.496 | 1.000 | 0.783 | 0.169 | 0.397 | 1.000 | 0.793 | 0.159 | 0.499 | 1.000 |
| Aug. | 0.819 | 0.141 | 0.615 | 1.000 | 0.765 | 0.166 | 0.402 | 1.000 | 0.764 | 0.167 | 0.419 | 1.000 | 0.798 | 0.133 | 0.634 | 1.000 | 0.803 | 0.148 | 0.552 | 1.000 |
| Sep. | 0.635 | 0.260 | 0.178 | 1.000 | 0.787 | 0.156 | 0.555 | 1.000 | 0.784 | 0.149 | 0.484 | 1.000 | 0.783 | 0.146 | 0.487 | 1.000 | 0.793 | 0.155 | 0.493 | 1.000 |
| Oct. | 0.635 | 0.230 | 0.255 | 1.000 | 0.778 | 0.151 | 0.484 | 1.000 | 0.774 | 0.140 | 0.465 | 1.000 | 0.779 | 0.147 | 0.456 | 1.000 | 0.821 | 0.142 | 0.504 | 1.000 |
| Nov. | 0.632 | 0.257 | 0.232 | 1.000 | 0.786 | 0.145 | 0.509 | 1.000 | 0.800 | 0.137 | 0.480 | 1.000 | 0.795 | 0.143 | 0.480 | 1.000 | 0.761 | 0.183 | 0.343 | 1.000 |
| Dec. | 0.806 | 0.189 | 0.208 | 1.000 | 0.752 | 0.169 | 0.362 | 1.000 | 0.780 | 0.170 | 0.360 | 1.000 | 0.785 | 0.171 | 0.330 | 1.000 | 0.807 | 0.156 | 0.352 | 1.000 |

Table 6.5: Evolution of the statistical values of the efficiency scores obtained with Data Envelopment Analysis over the analyzed years.

| | CRS[1] | | | | VRS[2] | | | |
|---|---|---|---|---|---|---|---|---|
| | Mean | Std. Deviation | Minimum | Maximum | Mean | Std. Deviation | Minimum | Maximum |
| 2013 | 0.494 | 0.220 | 0.200 | 0.741 | 0.710 | 0.087 | 0.591 | 0.819 |
| 2014 | 0.669 | 0.087 | 0.389 | 0.736 | 0.764 | 0.056 | 0.585 | 0.805 |
| 2015 | 0.678 | 0.092 | 0.389 | 0.735 | 0.763 | 0.052 | 0.594 | 0.800 |
| 2016 | 0.694 | 0.089 | 0.406 | 0.738 | 0.781 | 0.039 | 0.654 | 0.808 |
| 2017 | 0.706 | 0.079 | 0.466 | 0.773 | 0.784 | 0.054 | 0.611 | 0.821 |

[1]CRS: Constant Returns to Scale; [2]VRS: Variable Returns to Scale

most - 88%, compared to only 54% with CRS - presenting an average score of more than 0.600, and six between 0.900 and 1.000.

Considering VRS, $H_{26}$ has the best average efficiency score, being efficient every month from 2014 on, having a mean of 1.000 every year except 2013. Other two hospitals are also efficient during entire years: $H_7$ from 2015 to 2017 and $H_{11}$ in 2015 and 2017, also presenting mean of 1.000 for these years. Both results are seen in Table 6.2. This does not happen with CRS, since no hospital is efficient every month of one year, so no hospital presents a mean of 1.000. Nevertheless, $H_8$ and $H_{11}$ are the ones with the best average efficiency scores, considering CRS. Together with $H_6$, $H_{22}$ and $H_{26}$ they are the only that are benchmarks in any of the periods analyzed, under CRS, as they present maximum values of 1.000 in at least one year. Considering VRS, 13 hospitals are benchmarks in at least one of the periods considered.

$H_7$, $H_{16}$ and $H_{18}$ are the ones most consistent, presenting an average score greater than 0.9 for all years. Moreover, $H_2$, $H_6$, $H_8$, $H_{11}$, $H_{18}$, $H_{20}$, $H_{21}$ and $H_{26}$ are the ones presenting scores bigger than 0.8 for at least 40 of the 60 analyzed periods.

$H_{17}$ is always the one with the lowest average efficiency scores for all years under CRS and VRS, with the exception of 2013 under VRS. Despite presenting better results for some months (mostly June and August, under VRS), its average efficiency scores are not bigger than 0.600. These results go accordingly to what is commonly known about this hospital - *Centro Hospitalar Universitário do Algarve, EPE* - which is that it has a greater influx of people during the summer months since there is a large movement of people to this region, with this hospital center ensuring the provision of health care to a number of people that can "triple in the high season of tourism".[1] This is observable by the increase in the output number of emergency room visits during August. Since there is no significant change in the number of inputs and there is an increase in the outputs, an increase in the efficiency score is logically observed. Regarding June, it is in line with the results of the other units, which will be discussed further ahead. Hospitals $H_9$, $H_{17}$ and $H_{22}$ are the ones presenting scores lower than 0.6 more times in the analyzed periods.

In terms of time periods, June is the month which presents the lowest average efficiency scores more often, both under CRS and VRS especially from 2014 on. In 2013, the period with the lowest average is February also for both returns to scale. The periods with the best averages are more diverse, being May the one that presents more often the highest average values.

In order to analyze more carefully these hospital's efficiency through time, the plots in Figure 6.3 and Table 6.5 are presented.

As can be seen in Figure 6.3 and Table 6.5, the average efficiency scores both under CRS and VRS, has been overall increasing over time. However, its behaviour per month is not always increasing,

[1]Centro Hospitalar Universitário do Algarve (Available at: www.chualgarve.min-saude.pt/chalgarve-em-numeros/area-de-influencia/). Accessed on: 25/11/2020

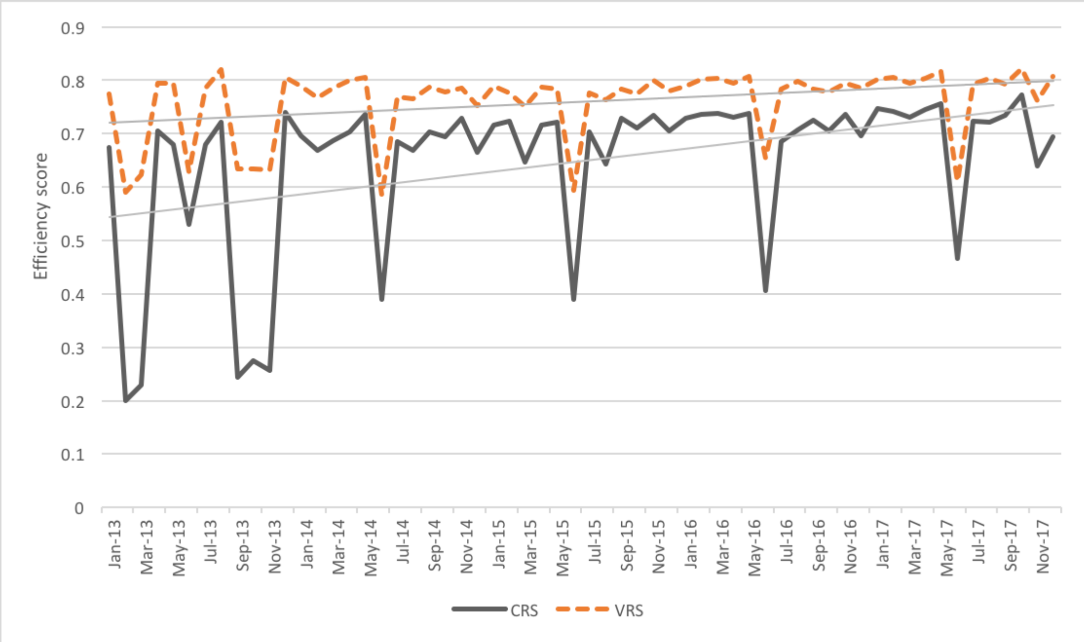


Figure 6.3: Evolution of the average efficiency scores obtained with Data Envelopment Analysis over the analyzed years. CRS: Constant Returns to Scale; VRS: Variable Returns to Scale

presenting, as was already mentioned, peaks of minimum values every June. This behaviour reflects the existence of seasonality in the results. The lower peaks correspond to a month where minimum values of efficiency occur in several hospitals. This occurs despite the input variable staff costs presenting lower values for this particular month. It is explained by the fact that, even though most units present lower values of staff costs in June, some hospitals, such as $H_{21}$, present exceptionally low values, creating the peaks observed in the plot of inputs in the previous chapter (Figure 5.1). However, June is a month where outputs also decrease overall. So, some of the hospitals with low values of staff costs are offset by higher values of the other inputs and/or also low values of outputs. And the ones that do not present the low peaks of staff costs lead to lower peaks of efficiency scores, since they are producing less outputs, perhaps even with an increase in the inputs ESS and CCM costs. Moreover, it should be noted that since the results are presented as a mean of the results for all hospitals, it represents overall tendencies and not results for the specific units.

Beyond this, in 2013, there are two lower peaks in the months of February and March and September to November. These correspond to periods in which there is an increase in the inputs and/or decrease in the outputs, meaning most hospitals either increased their costs and resources' usage, produced less with the same costs or both.

There was a bigger increase in efficiency during the analyzed years under CRS than under VRS. The values of average efficiency score are the most heterogeneous in 2013 under CRS, as can be verified by the standard deviation values. The most homogeneous values occur in 2016 under VRS.

Technical inefficiency can have as a reason the fact that the unit is not operating at its optimal scale. The scale efficiency, which consists in dividing the CRS by the VRS efficiency scores, was also analyzed. The averages per year of the results obtained are displayed in Table 6.6, organized by hospital.

Regarding scale efficiency, the results are very heterogeneous, with hospitals having efficiency scores ranging from as low as 0.174 to the maximum possible of 1.000. Moreover they show that, in every month of every analyzed year, there is at least one hospital with a scale efficiency of 1.000, which is the maximum value. These correspond to the hospitals that present the same efficiency score

Table 6.6: Average scale efficiency of each hospital in each year.

| DMU[1] | 2013 | 2014 | 2015 | 2016 | 2017 | Avg. |
|---|---|---|---|---|---|---|
| $H_1$ | 0.822 | 0.962 | 0.950 | 0.954 | 0.968 | 0.931 |
| $H_2$ | 0.740 | 0.970 | 0.963 | 0.967 | 0.971 | 0.922 |
| $H_3$ | 0.599 | 0.809 | 0.841 | 0.854 | 0.859 | 0.792 |
| $H_4$ | 0.765 | 0.965 | 0.958 | 0.962 | 0.970 | 0.924 |
| $H_5$ | 0.760 | 0.966 | 0.955 | 0.960 | 0.969 | 0.922 |
| $H_6$ | 0.832 | 0.967 | 0.945 | 0.949 | 0.966 | 0.932 |
| $H_7$ | 0.394 | 0.558 | 0.579 | 0.604 | 0.597 | 0.547 |
| $H_8$ | 0.709 | 0.973 | 0.964 | 0.970 | 0.972 | 0.918 |
| $H_9$ | 0.840 | 0.963 | 0.949 | 0.954 | 0.967 | 0.935 |
| $H_{10}$ | 0.731 | 0.831 | 0.880 | 0.907 | 0.959 | 0.862 |
| $H_{11}$ | 0.721 | 0.974 | 0.967 | 0.969 | 0.973 | 0.921 |
| $H_{12}$ | 0.730 | 0.972 | 0.963 | 0.967 | 0.971 | 0.920 |
| $H_{13}$ | 0.713 | 0.968 | 0.965 | 0.969 | 0.972 | 0.918 |
| $H_{14}$ | 0.930 | 0.958 | 0.939 | 0.942 | 0.965 | 0.947 |
| $H_{15}$ | 0.431 | 0.619 | 0.667 | 0.701 | 0.697 | 0.623 |
| $H_{16}$ | 0.478 | 0.658 | 0.692 | 0.705 | 0.703 | 0.647 |
| $H_{17}$ | 0.684 | 0.936 | 0.933 | 0.941 | 0.959 | 0.891 |
| $H_{18}$ | 0.540 | 0.734 | 0.770 | 0.774 | 0.780 | 0.720 |
| $H_{19}$ | 0.484 | 0.700 | 0.729 | 0.739 | 0.722 | 0.675 |
| $H_{20}$ | 0.600 | 0.803 | 0.829 | 0.836 | 0.836 | 0.781 |
| $H_{21}$ | 0.743 | 0.844 | 0.887 | 0.916 | 0.958 | 0.870 |
| $H_{22}$ | 0.968 | 0.954 | 0.935 | 0.946 | 0.965 | 0.953 |
| $H_{23}$ | 0.846 | 0.965 | 0.950 | 0.954 | 0.965 | 0.936 |
| $H_{24}$ | 0.699 | 0.960 | 0.956 | 0.936 | 0.942 | 0.899 |
| $H_{25}$ | 0.720 | 0.972 | 0.964 | 0.970 | 0.972 | 0.920 |
| $H_{26}$ | 0.667 | 0.784 | 0.853 | 0.890 | 0.948 | 0.828 |
| Avg. | 0.698 | 0.876 | 0.884 | 0.894 | 0.905 | 0.851 |
| Max. | 0.968 | 0.974 | 0.967 | 0.970 | 0.973 | 0.970 |
| Min. | 0.394 | 0.558 | 0.579 | 0.604 | 0.597 | 0.547 |

[1] DMU: Decision Making Unit

considering both CRS and VRS, showing scale efficiency, and meaning that the DMU is operating at the optimal scale.

$H_7$ is the one with the lowest scale efficiency scores for all analyzed years, and $H_{11}$ presents the highest average scores in most years (2014, 2015 and 2017). $H_{22}$ and both $H_8$ and $H_{25}$ present the highest scores for years 2013 and 2016, respectively. Only five hospitals present scale efficiency at some month in these five years. $H_{11}$ is the one that presents a score of one more times, being the one that is scale efficient in most months, in particular from 2015 on and being scale efficient for half of the periods studied. $H_8$ and $H_{26}$ follow this one being also scale efficient in several months. In addition to these, only $H_{22}$ and $H_6$ are also scale efficient in some period. The rest always present scale inefficiency. These hospitals are not operating at their optimum size and could benefit from an adjustment of their production capacity.

The average scale efficiency scores have been slowly increasing over the years, as well as the minimum values. However, there is not a clear increase in the maximum values. This means that, hospitals may be slowly approaching their appropriate and optimal size, in particular the ones that present the lowest scores.

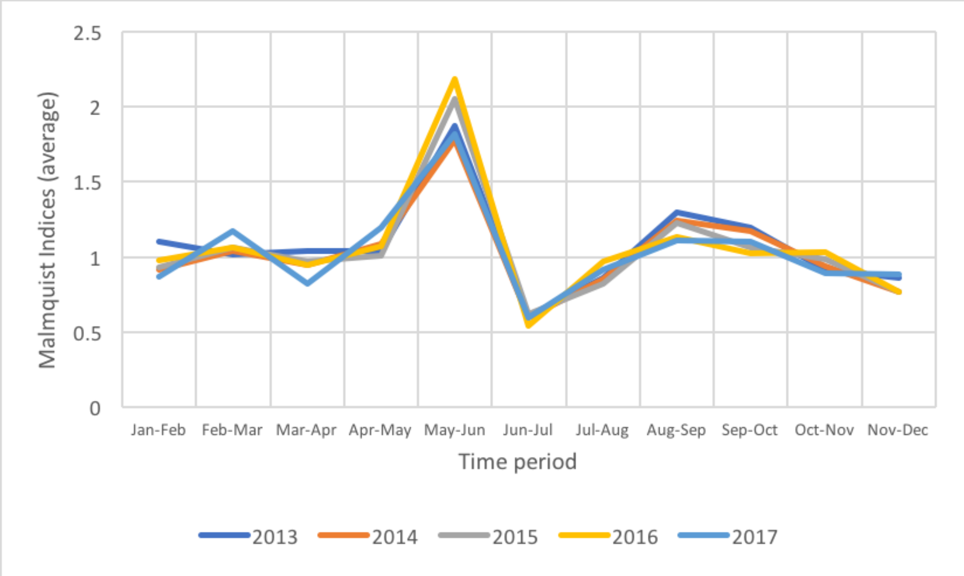


Figure 6.4: Evolution of the average Malmquist Productivity Index for all hospitals through all time periods in each year.

In terms of the MPI results, they are presented in Tables 6.7 and 6.8. In each one of these two Tables, results are displayed for each hospital and for each time period, respectively. Moreover, the plot in Figure 6.4 shows the evolution of the average MPI for all hospitals during the time periods of each year.

The MPI, which measures productivity change between time periods, presents a total average of 1.049 and standard deviation of 0.475. Its values range from 0.054 to 6.137. However, the average value of MPI for each year does not vary very much, being around 1.000 for all five analyzed years, as can be seen in Table 6.8. This suggests that the productivity of the Portuguese hospitals has not changed significantly through these years.

Even though none of the hospitals have values bigger than 1.000 in every analyzed period, 12 present an average per year that is greater than 1.000 for all years. From these, $H_{26}$ is the one that presents the biggest average MPI and is the highest in 2017. $H_6$, $H_{17}$ and $H_{22}$ present the highest indices in 2014, 2015 and 2016 and 2013, respectively. $H_{17}$ and $H_{22}$, however, are not consistently productive. Firstly, $H_{22}$ presents very good efficient scores at the end of 2013 due to a decrease in the costs and increase in the number of medical appointments, which leads to an average high MPI score. $H_{17}$ was already discussed and presents high efficiency scores when the rest of of the hospitals don't (vacation months), leading to high indexes which average to a good overall result.

There are also hospitals that never present average values in any year bigger than 1.000, suggesting they are not progressing in terms of productivity. These are $H_{18}$ and $H_{19}$. Some others have total average values smaller than 1.000 but do show, in at least one year, an average MPI of more than 1.000. Despite having an MPI bigger than 1.000 in some time periods, $H_{18}$ and $H_{19}$ do not present averages bigger than 1.000. $H_{18}$, however, is one of the most efficient, so this low average productivity indexes are justifiable since, because it performs very well in all months except the summer months, the index for these is low, leading to a low average.

Overall, the results of the MPI are very homogeneous, and there is not one hospital that clearly stands out, for example, in terms of presenting indexes bigger than 1.000 for all periods.

Even though in 2016 the average MPI was the highest, this is not verified for all months of that year and the difference is not big enough to be significant. Nevertheless, it can be seen that, regardless of the year, the period May-June presents the higher values which reflects the highest increase in productivity. On the other hand, the period June-July exhibits the lowest values, being very often and for

Table 6.7: Malmquist Productivity Index results: mean, standard deviation, minimum and maximum values of efficiency scores for each hospital of each analyzed year (2013-2017).

| | 2013 | | | | 2014 | | | | 2015 | | | | 2016 | | | | 2017 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Mean | Std. Dev. | Max. | Min. | Mean | Std. Dev. | Max. | Min. | Mean | Std. Dev. | Max. | Min. | Mean | Std. Dev. | Max. | Min. | Mean | Std. Dev. | Max. | Min. |
| $H_1$ | 1.076 | 0.439 | 2.290 | 0.386 | 1.091 | 0.526 | 2.567 | 0.299 | 0.991 | 0.217 | 1.400 | 0.673 | 1.102 | 0.598 | 2.850 | 0.275 | 1.071 | 0.468 | 2.374 | 0.367 |
| $H_2$ | 1.011 | 0.242 | 1.717 | 0.780 | 0.982 | 0.129 | 1.221 | 0.825 | 0.978 | 0.107 | 1.198 | 0.797 | 0.985 | 0.061 | 1.096 | 0.877 | 0.993 | 0.149 | 1.243 | 0.748 |
| $H_3$ | 1.054 | 0.417 | 2.190 | 0.434 | 1.077 | 0.482 | 2.407 | 0.375 | 1.105 | 0.577 | 2.783 | 0.354 | 1.085 | 0.514 | 2.534 | 0.332 | 1.068 | 0.474 | 2.360 | 0.365 |
| $H_4$ | 1.054 | 0.359 | 1.923 | 0.486 | 1.047 | 0.453 | 2.315 | 0.358 | 1.090 | 0.585 | 2.812 | 0.371 | 1.100 | 0.589 | 2.837 | 0.319 | 1.068 | 0.511 | 2.517 | 0.341 |
| $H_5$ | 0.998 | 0.179 | 1.283 | 0.665 | 1.076 | 0.563 | 2.719 | 0.351 | 1.010 | 0.299 | 1.792 | 0.587 | 1.076 | 0.529 | 2.645 | 0.389 | 1.070 | 0.508 | 2.495 | 0.370 |
| $H_6$ | 1.099 | 0.535 | 2.644 | 0.342 | 1.114 | 0.785 | 3.399 | 0.222 | 1.095 | 0.653 | 2.964 | 0.345 | 1.150 | 0.769 | 3.461 | 0.245 | 1.088 | 0.562 | 2.669 | 0.337 |
| $H_7$ | 1.018 | 0.238 | 1.559 | 0.718 | 0.987 | 0.248 | 1.488 | 0.566 | 0.994 | 0.237 | 1.582 | 0.662 | 0.984 | 0.188 | 1.476 | 0.708 | 0.999 | 0.243 | 1.499 | 0.679 |
| $H_8$ | 1.051 | 0.432 | 2.253 | 0.417 | 1.052 | 0.478 | 2.386 | 0.368 | 1.032 | 0.406 | 2.159 | 0.421 | 1.104 | 0.556 | 2.731 | 0.305 | 1.102 | 0.626 | 2.947 | 0.311 |
| $H_9$ | 1.001 | 0.094 | 1.131 | 0.858 | 1.067 | 0.424 | 2.244 | 0.392 | 1.086 | 0.576 | 2.797 | 0.353 | 1.097 | 0.641 | 2.989 | 0.289 | 1.026 | 0.322 | 1.845 | 0.478 |
| $H_{10}$ | 1.080 | 0.463 | 2.357 | 0.435 | 1.057 | 0.436 | 2.135 | 0.318 | 1.143 | 0.691 | 3.214 | 0.297 | 1.031 | 0.359 | 1.973 | 0.490 | 1.051 | 0.423 | 2.180 | 0.433 |
| $H_{11}$ | 1.071 | 0.524 | 2.581 | 0.368 | 1.056 | 0.457 | 2.358 | 0.385 | 1.068 | 0.518 | 2.596 | 0.385 | 1.053 | 0.433 | 2.293 | 0.406 | 1.070 | 0.505 | 2.499 | 0.364 |
| $H_{12}$ | 1.092 | 0.435 | 2.220 | 0.444 | 0.992 | 0.134 | 1.199 | 0.824 | 0.977 | 0.118 | 1.159 | 0.723 | 1.169 | 0.703 | 3.165 | 0.182 | 0.983 | 0.113 | 1.138 | 0.794 |
| $H_{13}$ | 1.048 | 0.413 | 2.134 | 0.474 | 0.994 | 0.273 | 1.630 | 0.543 | 1.028 | 0.416 | 2.184 | 0.444 | 1.088 | 0.596 | 2.848 | 0.311 | 1.034 | 0.412 | 2.168 | 0.390 |
| $H_{14}$ | 1.084 | 0.453 | 2.281 | 0.402 | 0.979 | 0.118 | 1.163 | 0.756 | 0.989 | 0.120 | 1.201 | 0.771 | 0.983 | 0.106 | 1.187 | 0.815 | 0.984 | 0.178 | 1.268 | 0.725 |
| $H_{15}$ | 1.064 | 0.429 | 2.269 | 0.386 | 1.067 | 0.485 | 2.420 | 0.362 | 1.099 | 0.605 | 2.882 | 0.342 | 1.077 | 0.514 | 2.551 | 0.329 | 1.028 | 0.317 | 1.842 | 0.477 |
| $H_{16}$ | 1.062 | 0.415 | 2.177 | 0.442 | 1.015 | 0.257 | 1.558 | 0.565 | 1.049 | 0.435 | 2.261 | 0.429 | 1.043 | 0.392 | 2.109 | 0.422 | 1.052 | 0.430 | 2.170 | 0.414 |
| $H_{17}$ | 1.023 | 0.368 | 2.066 | 0.508 | 1.065 | 0.529 | 2.567 | 0.329 | 1.226 | 1.137 | 4.739 | 0.212 | 1.257 | 1.165 | 4.825 | 0.194 | 1.056 | 0.575 | 2.747 | 0.334 |
| $H_{18}$ | 0.986 | 0.155 | 1.236 | 0.726 | 0.972 | 0.202 | 1.433 | 0.660 | 0.986 | 0.154 | 1.363 | 0.734 | 0.971 | 0.127 | 1.248 | 0.699 | 0.971 | 0.176 | 1.206 | 0.607 |
| $H_{19}$ | 0.995 | 0.177 | 1.325 | 0.714 | 0.994 | 0.185 | 1.404 | 0.752 | 0.998 | 0.162 | 1.376 | 0.711 | 0.993 | 0.165 | 1.373 | 0.772 | 0.988 | 0.211 | 1.339 | 0.704 |
| $H_{20}$ | 1.012 | 0.157 | 1.238 | 0.762 | 0.990 | 0.151 | 1.236 | 0.752 | 0.985 | 0.109 | 1.186 | 0.834 | 0.989 | 0.089 | 1.111 | 0.855 | 0.973 | 0.152 | 1.209 | 0.703 |
| $H_{21}$ | 1.051 | 0.460 | 2.356 | 0.377 | 1.040 | 0.403 | 2.136 | 0.393 | 1.061 | 0.498 | 2.503 | 0.374 | 1.086 | 0.557 | 2.694 | 0.320 | 1.099 | 0.621 | 2.894 | 0.289 |
| $H_{22}$ | 1.758 | 1.766 | 6.137 | 0.054 | 0.989 | 0.204 | 1.420 | 0.618 | 0.997 | 0.203 | 1.328 | 0.562 | 1.000 | 0.193 | 1.339 | 0.547 | 1.000 | 0.261 | 1.529 | 0.527 |
| $H_{23}$ | 1.037 | 0.364 | 1.922 | 0.514 | 1.059 | 0.418 | 1.959 | 0.497 | 1.049 | 0.427 | 1.905 | 0.492 | 0.982 | 0.126 | 1.186 | 0.784 | 0.994 | 0.168 | 1.263 | 0.769 |
| $H_{24}$ | 1.015 | 0.118 | 1.169 | 0.823 | 0.985 | 0.158 | 1.266 | 0.769 | 0.984 | 0.093 | 1.116 | 0.794 | 0.986 | 0.067 | 1.122 | 0.867 | 0.989 | 0.119 | 1.189 | 0.823 |
| $H_{25}$ | 1.033 | 0.347 | 1.901 | 0.484 | 0.983 | 0.183 | 1.296 | 0.718 | 0.975 | 0.130 | 1.267 | 0.720 | 0.995 | 0.137 | 1.282 | 0.726 | 0.984 | 0.164 | 1.285 | 0.722 |
| $H_{26}$ | 1.126 | 0.692 | 3.198 | 0.302 | 1.069 | 0.463 | 2.377 | 0.376 | 1.217 | 1.005 | 4.296 | 0.219 | 1.253 | 1.092 | 4.607 | 0.182 | 1.127 | 0.670 | 3.071 | 0.289 |

Table 6.8: Malmquist Productivity Index results: mean, standard deviation, minimum and maximum values of efficiency scores for each time period of the analyzed years (2013-2017).

| | 2013 | | | | 2014 | | | | 2015 | | | | 2016 | | | | 2017 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Mean | Std. Dev. | Min. | Max. | Mean | Std. Dev. | Min. | Max. | Mean | Std. Dev. | Min. | Max. | Mean | Std. Dev. | Min. | Max. | Mean | Std. Dev. | Min. | Max. |
| Jan-Feb | 1.102 | 1.008 | 0.818 | 6.137 | 0.914 | 0.036 | 0.831 | 0.969 | 0.934 | 0.037 | 0.876 | 1.012 | 0.974 | 0.065 | 0.880 | 1.232 | 0.865 | 0.047 | 0.783 | 0.965 |
| Feb-Mar | 1.016 | 0.067 | 0.869 | 1.209 | 1.041 | 0.049 | 0.951 | 1.199 | 1.060 | 0.075 | 0.868 | 1.208 | 1.061 | 0.072 | 0.891 | 1.282 | 1.173 | 0.055 | 1.028 | 1.268 |
| Mar-Apr | 1.040 | 0.211 | 0.094 | 1.396 | 0.948 | 0.059 | 0.769 | 1.085 | 0.967 | 0.099 | 0.604 | 1.137 | 0.943 | 0.068 | 0.792 | 1.185 | 0.819 | 0.081 | 0.736 | 1.088 |
| Apr-May | 1.041 | 0.097 | 0.757 | 1.274 | 1.089 | 0.112 | 0.904 | 1.420 | 1.008 | 0.092 | 0.886 | 1.292 | 1.068 | 0.084 | 0.865 | 1.339 | 1.192 | 0.108 | 0.891 | 1.529 |
| May-Jun | 1.869 | 0.667 | 0.762 | 3.198 | 1.776 | 0.759 | 0.752 | 3.399 | 2.052 | 1.053 | 0.878 | 4.739 | 0.180 | 1.137 | 0.828 | 4.825 | 1.820 | 0.792 | 0.782 | 3.071 |
| Jun-Jul | 0.602 | 0.296 | 0.302 | 1.238 | 0.596 | 0.298 | 0.222 | 1.197 | 0.619 | 0.313 | 0.212 | 1.114 | 0.545 | 0.316 | 0.182 | 1.186 | 0.596 | 0.287 | 0.289 | 1.090 |
| Jul-Aug | 0.863 | 0.303 | 0.685 | 2.354 | 0.853 | 0.120 | 0.548 | 1.203 | 0.822 | 0.092 | 0.558 | 1.002 | 0.966 | 0.148 | 0.798 | 1.545 | 0.915 | 0.081 | 0.753 | 1.054 |
| Aug-Sep | 1.296 | 0.558 | 0.859 | 3.953 | 1.243 | 0.188 | 0.897 | 1.959 | 1.226 | 0.185 | 0.942 | 1.905 | 1.130 | 0.121 | 0.851 | 1.476 | 1.114 | 0.132 | 0.913 | 1.499 |
| Sep-Oct | 1.198 | 0.140 | 0.836 | 1.524 | 1.174 | 0.065 | 1.062 | 1.346 | 1.067 | 0.058 | 0.892 | 1.181 | 1.021 | 0.045 | 0.963 | 1.149 | 1.101 | 0.061 | 0.975 | 1.285 |
| Oct-Nov | 0.915 | 0.045 | 0.796 | 1.015 | 0.935 | 0.030 | 0.887 | 1.010 | 0.986 | 0.040 | 0.896 | 1.085 | 1.035 | 0.047 | 0.912 | 1.110 | 0.890 | 0.136 | 0.680 | 1.091 |
| Nov-Dec | 0.862 | 0.189 | 0.054 | 1.196 | 0.769 | 0.123 | 0.323 | 0.907 | 0.771 | 0.081 | 0.598 | 1.042 | 0.770 | 0.094 | 0.563 | 0.980 | 0.884 | 0.141 | 0.607 | 1.260 |
| Avg. | 1.073 | 0.326 | 0.621 | 2.227 | 1.031 | 0.167 | 0.741 | 1.427 | 1.047 | 0.193 | 0.746 | 1.520 | 1.063 | 0.200 | 0.775 | 1.574 | 1.033 | 0.175 | 0.767 | 1.382 |

most hospitals less than 1.000. August-September and, in the first years, September-October are also very often, and for the majority of the hospitals, periods of increased productivity, presenting values greater than 1.000. This suggests a seasonal effect, which means, in the case of May-June, that during these periods, there is an increase in production, a decrease in spending and in the resources used, progress in the production technology or any combination of these, and the contrary in June-July. Thus, from May to June there is an increase in productivity and from June to July there is a decrease. As will be seen ahead, this has more to do with the technology change than with efficiency change. The August-September increase may be in line with the fact that during August, which is the principal vacation month, the majority of hospitals will present less amount of outputs but no change in the inputs, being less productive.

There are some values that seem a bit unreasonable, especially in the period May-June, which presents the highest values, since they stand out more than what was expected, as is possible to see from the standard deviation values in these periods of every year in Table 6.8. In 2013, some periods present values higher than expected, such as January-February, July-August and August-September, as well as values lower than expected in March-April and November-December, all for $H_{22}$. All these high values are mostly due to the also high values of $\Delta$Tech for these periods, with values between 2.4 and 3.9. The only exception to this is the $\Delta$Eff of $H_{22}$ in July-August 2013, which is exceptionally high (around 3), and the $\Delta$Tech presents an average value. The efficiency score of this unit increases from 0.332 in July 2013 to 1.000 in August of the same year, which is comprehensible given what was already mentioned about this particular unit (*Centro Hospitalar Universitário do Algarve, EPE*).

As previously mentioned, the MPI can be divided into change in efficiency ($\Delta$Eff) and technology change ($\Delta$Tech), consisting in the geometric mean of these two terms. In order to further understand and interpret the MPI results, the results obtained for the separated terms are presented in Tables 6.9 and 6.10.

$\Delta$Tech is generally higher than $\Delta$Eff, which may lead to $\Delta$Tech influencing more the total MPI. With the exception of 2013, $\Delta$Eff presents its higher values in the period June-July and $\Delta$Tech in the period May-June. It makes sense that June-July presents the highest values for $\Delta$Eff since, as was seen in the DEA results, efficiency presents its lower values in June, which leads to the biggest increase in June-July. The lowest and highest values of these two terms belong to the same periods, but are switched. Hence, when $\Delta$Eff presents its highest values, $\Delta$Tech presents its lowest and vice versa, with the exception of 2013.

In terms of $\Delta$Eff, $H_{20}$ presents the best average from 2013 to 2015. And $H_{23}$ and $H_{19}$ for 2016 and 2017, respectively. The lowest averages belong to $H_{22}$, $H_6$, $H_{17}$ and $H_9$.

The technology change - $\Delta$Tech - is the same for all hospitals in each time period. This is also the case of some examples in Lee, Leem, Lee, and Lee (2011) and Coeil, T. J., Rao, D.S.P. and Battese (2005), in which the MPI is computed with a single input and a single output, which ends up being the same case as in this work given the PCA performed, reducing the dimensions to one for both inputs and outputs.

The plot shown in Figure 6.5 represents the behaviour, during all the time periods analyzed, of the MPI: its total average values as well as its two terms - technical change ($\Delta$Tech) and efficiency change ($\Delta$Eff) - average values. The term contributing more to the MPI change seems to be $\Delta$Tech, the term regarding the technological change, as was discussed previously. It can be seen in Figure 6.5 that when the total MPI presents a peak, either low or high, it is the term $\Delta$Tech that presents a peak in the same period, even if the other term, $\Delta$Eff, presents values with the opposite trend. Thus, a hospital with a decrease in efficiency can still present an index that suggests productivity increase.

$\Delta$Tech has been very slowly decreasing over the years, but never reaching an average lower than

Table 6.9: Malmquist Productivity Index average results of the total Malmquist Productivity Index and its two terms efficiency change and technology change, presented by time period (months).

| | 2013 | | | 2014 | | | 2015 | | | 2016 | | | 2017 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | MPI | ΔEff | ΔTech | MPI | ΔEff | ΔTech | MPI | ΔEff | ΔTech | MPI | ΔEff | ΔTech | MPI | ΔEff | ΔTech |
| Jan-Feb | 1.102 | 0.305 | 3.620 | 0.914 | 0.959 | 0.953 | 0.934 | 1.009 | 0.926 | 0.974 | 1.016 | 0.959 | 0.865 | 0.994 | 0.870 |
| Feb-Mar | 1.016 | 1.170 | 0.869 | 1.041 | 1.030 | 1.010 | 1.060 | 0.892 | 1.189 | 1.061 | 1.004 | 1.057 | 1.173 | 0.984 | 1.192 |
| Mar-Apr | 1.040 | 3.538 | 0.294 | 0.948 | 1.025 | 0.924 | 0.967 | 1.124 | 0.860 | 0.943 | 0.993 | 0.950 | 0.819 | 1.029 | 0.796 |
| Apr-May | 1.041 | 0.973 | 1.069 | 1.089 | 1.052 | 1.035 | 1.008 | 1.012 | 0.996 | 1.068 | 1.014 | 1.053 | 1.192 | 1.018 | 1.170 |
| May-Jun | 1.869 | 0.772 | 2.421 | 1.776 | 0.523 | 3.397 | 2.052 | 0.540 | 3.801 | 2.180 | 0.555 | 3.929 | 1.820 | 0.612 | 2.973 |
| Jun-Jul | 0.602 | 1.549 | 0.388 | 0.596 | 2.228 | 0.267 | 0.619 | 2.355 | 0.263 | 0.545 | 2.296 | 0.237 | 0.596 | 1.946 | 0.306 |
| Jul-Aug | 0.863 | 1.104 | 0.782 | 0.853 | 0.977 | 0.872 | 0.822 | 0.914 | 0.900 | 0.966 | 1.049 | 0.922 | 0.915 | 1.004 | 0.911 |
| Aug-Sep | 1.296 | 0.328 | 3.953 | 1.243 | 1.073 | 1.159 | 1.226 | 1.149 | 1.066 | 1.130 | 1.032 | 1.095 | 1.114 | 1.020 | 1.092 |
| Sep-Oct | 1.198 | 1.165 | 1.028 | 1.174 | 0.989 | 1.188 | 1.067 | 0.981 | 1.088 | 1.021 | 0.972 | 1.050 | 1.101 | 1.059 | 1.040 |
| Oct-Nov | 0.915 | 0.926 | 0.988 | 0.935 | 1.052 | 0.889 | 0.986 | 1.035 | 0.953 | 1.035 | 1.046 | 0.990 | 0.890 | 0.832 | 1.069 |
| Nov-Dec | 0.862 | 3.249 | 0.265 | 0.769 | 0.920 | 0.836 | 0.771 | 0.957 | 0.806 | 0.770 | 0.943 | 0.817 | 0.884 | 1.103 | 0.801 |
| Avg. | 1.073 | 1.371 | 1.425 | 1.031 | 1.075 | 1.139 | 1.047 | 1.088 | 1.168 | 1.063 | 1.083 | 1.187 | 1.033 | 1.055 | 1.111 |
| Min. | 0.602 | 0.305 | 0.265 | 0.596 | 0.523 | 0.267 | 0.619 | 0.540 | 0.263 | 0.545 | 0.555 | 0.237 | 0.596 | 0.612 | 0.306 |
| Max. | 1.869 | 3.538 | 3.953 | 1.776 | 2.228 | 3.397 | 2.052 | 2.355 | 3.801 | 2.180 | 2.296 | 3.929 | 1.820 | 1.946 | 2.973 |

Table 6.10: Malmquist Productivity Index average results of the total Malmquist Productivity Index and its two terms efficiency change and technology change, presented by hospital.

| | 2013 | | | 2014 | | | 2015 | | | 2016 | | | 2017 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | MPI | ΔEff | ΔTech | MPI | ΔEff | ΔTech | MPI | ΔEff | ΔTech | MPI | ΔEff | ΔTech | MPI | ΔEff | ΔTech |
| $H_1$ | 1.076 | 1.326 | 1.425 | 1.091 | 1.013 | 1.139 | 0.991 | 1.087 | 1.168 | 1.102 | 0.998 | 1.187 | 1.071 | 1.012 | 1.111 |
| $H_2$ | 1.011 | 1.434 | 1.425 | 0.982 | 1.215 | 1.139 | 0.978 | 1.204 | 1.168 | 0.985 | 1.190 | 1.187 | 0.993 | 1.149 | 1.111 |
| $H_3$ | 1.054 | 1.333 | 1.425 | 1.077 | 1.026 | 1.139 | 1.105 | 1.032 | 1.168 | 1.085 | 1.017 | 1.187 | 1.068 | 1.013 | 1.111 |
| $H_4$ | 1.054 | 1.480 | 1.425 | 1.047 | 1.000 | 1.139 | 1.090 | 1.022 | 1.168 | 1.100 | 1.011 | 1.187 | 1.068 | 0.998 | 1.111 |
| $H_5$ | 0.998 | 1.342 | 1.425 | 1.076 | 1.000 | 1.139 | 1.010 | 1.056 | 1.168 | 1.076 | 1.021 | 1.187 | 1.070 | 1.009 | 1.111 |
| $H_6$ | 1.099 | 1.343 | 1.425 | 1.114 | 0.955 | 1.139 | 1.095 | 0.997 | 1.168 | 1.150 | 0.995 | 1.187 | 1.088 | 1.000 | 1.111 |
| $H_7$ | 1.018 | 1.387 | 1.425 | 0.987 | 1.118 | 1.139 | 0.994 | 1.176 | 1.168 | 0.984 | 1.166 | 1.187 | 0.999 | 1.078 | 1.111 |
| $H_8$ | 1.051 | 1.291 | 1.425 | 1.052 | 0.999 | 1.139 | 1.032 | 1.014 | 1.168 | 1.104 | 1.022 | 1.187 | 1.102 | 1.002 | 1.111 |
| $H_9$ | 1.001 | 1.477 | 1.425 | 1.067 | 1.035 | 1.139 | 1.086 | 1.012 | 1.168 | 1.097 | 0.988 | 1.187 | 1.026 | 1.021 | 1.111 |
| $H_{10}$ | 1.080 | 1.389 | 1.425 | 1.057 | 1.007 | 1.139 | 1.143 | 1.030 | 1.168 | 1.031 | 1.047 | 1.187 | 1.051 | 1.012 | 1.111 |
| $H_{11}$ | 1.071 | 1.306 | 1.425 | 1.056 | 1.014 | 1.139 | 1.068 | 1.014 | 1.168 | 1.053 | 1.026 | 1.187 | 1.070 | 0.998 | 1.111 |
| $H_{12}$ | 1.092 | 1.475 | 1.425 | 0.992 | 1.202 | 1.139 | 0.977 | 1.187 | 1.168 | 1.169 | 1.025 | 1.187 | 0.983 | 1.119 | 1.111 |
| $H_{13}$ | 1.048 | 1.339 | 1.425 | 0.994 | 1.034 | 1.139 | 1.028 | 1.018 | 1.168 | 1.088 | 0.995 | 1.187 | 1.034 | 0.998 | 1.111 |
| $H_{14}$ | 1.084 | 1.387 | 1.425 | 0.979 | 1.137 | 1.139 | 0.989 | 1.207 | 1.168 | 0.983 | 1.188 | 1.187 | 0.984 | 1.140 | 1.111 |
| $H_{15}$ | 1.064 | 1.342 | 1.425 | 1.067 | 1.011 | 1.139 | 1.099 | 1.013 | 1.168 | 1.077 | 1.008 | 1.187 | 1.028 | 1.024 | 1.111 |
| $H_{16}$ | 1.062 | 1.370 | 1.425 | 1.015 | 1.066 | 1.139 | 1.049 | 1.027 | 1.168 | 1.043 | 1.031 | 1.187 | 1.052 | 1.015 | 1.111 |
| $H_{17}$ | 1.023 | 1.327 | 1.425 | 1.065 | 0.999 | 1.139 | 1.226 | 0.983 | 1.168 | 1.257 | 0.996 | 1.187 | 1.056 | 0.978 | 1.111 |
| $H_{18}$ | 0.986 | 1.423 | 1.425 | 0.972 | 1.174 | 1.139 | 0.986 | 1.198 | 1.168 | 0.971 | 1.175 | 1.187 | 0.971 | 1.102 | 1.111 |
| $H_{19}$ | 0.995 | 1.446 | 1.425 | 0.994 | 1.214 | 1.139 | 0.998 | 1.214 | 1.168 | 0.993 | 1.217 | 1.187 | 0.988 | 1.159 | 1.111 |
| $H_{20}$ | 1.012 | 1.532 | 1.425 | 0.990 | 1.252 | 1.139 | 0.985 | 1.233 | 1.168 | 0.989 | 1.243 | 1.187 | 0.973 | 1.151 | 1.111 |
| $H_{21}$ | 1.051 | 1.292 | 1.425 | 1.040 | 1.013 | 1.139 | 1.061 | 1.011 | 1.168 | 1.086 | 1.003 | 1.187 | 1.099 | 0.995 | 1.111 |
| $H_{22}$ | 1.758 | 1.117 | 1.425 | 0.989 | 1.087 | 1.139 | 0.997 | 1.075 | 1.168 | 1.000 | 1.088 | 1.187 | 1.000 | 1.039 | 1.111 |
| $H_{23}$ | 1.037 | 1.274 | 1.425 | 1.059 | 1.074 | 1.139 | 1.049 | 1.067 | 1.168 | 0.982 | 1.281 | 1.187 | 0.994 | 1.158 | 1.111 |
| $H_{24}$ | 1.015 | 1.491 | 1.425 | 0.985 | 1.197 | 1.139 | 0.984 | 1.199 | 1.168 | 0.986 | 1.220 | 1.187 | 0.989 | 1.115 | 1.111 |
| $H_{25}$ | 1.033 | 1.358 | 1.425 | 0.983 | 1.089 | 1.139 | 0.975 | 1.202 | 1.168 | 0.995 | 1.216 | 1.187 | 0.984 | 1.115 | 1.111 |
| $H_{26}$ | 1.126 | 1.360 | 1.425 | 1.069 | 1.028 | 1.139 | 1.217 | 1.006 | 1.168 | 1.253 | 1.003 | 1.187 | 1.127 | 1.020 | 1.111 |
| Avg. | 1.073 | 1.371 | 1.425 | 1.031 | 1.075 | 1.139 | 1.047 | 1.088 | 1.168 | 1.063 | 1.083 | 1.187 | 1.033 | 1.055 | 1.111 |
| Min. | 0.986 | 1.117 | 1.425 | 0.972 | 0.955 | 1.139 | 0.975 | 0.983 | 1.168 | 0.971 | 0.988 | 1.187 | 0.971 | 0.978 | 1.111 |
| Max. | 1.758 | 1.532 | 1.425 | 1.114 | 1.252 | 1.139 | 1.226 | 1.233 | 1.168 | 1.257 | 1.281 | 1.187 | 1.127 | 1.159 | 1.111 |

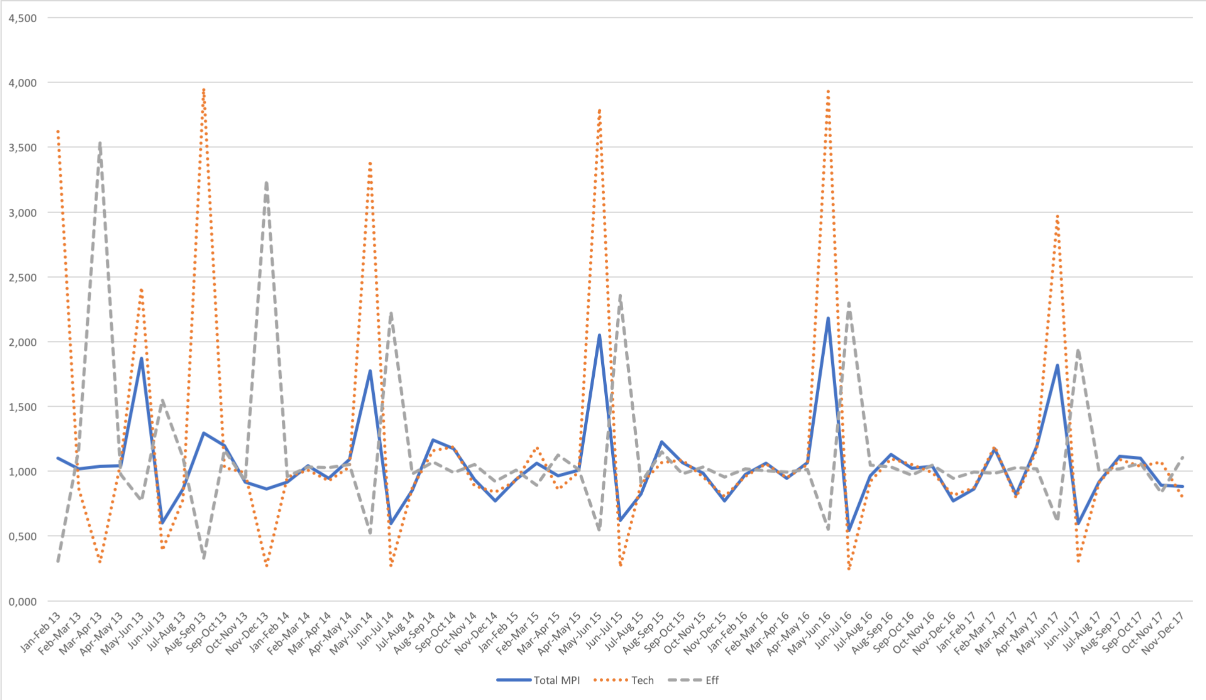


Figure 6.5: Evolution of the average total Malmquist Productivity Index and its two terms for all hospitals through all time periods in each year.

Table 6.11: Malmquist Productivity Index results for the total Malmquist Productivity Index and its terms, considering the decompositions into two and four terms.

| | MPI | ΔEff | ΔTech | MPI | ΔPureEff | ΔScale | ΔPureTech | ΔScaleTech |
|---|---|---|---|---|---|---|---|---|
| 2013 | 1.073 | 1.371 | 1.425 | 0.812 | 1.133 | 1.321 | 1.248 | 0.659 |
| 2014 | 1.031 | 1.075 | 1.139 | 0.964 | 1.053 | 1.023 | 1.102 | 0.897 |
| 2015 | 1.047 | 1.088 | 1.168 | 0.973 | 1.059 | 1.048 | 1.150 | 0.909 |
| 2016 | 1.063 | 1.083 | 1.187 | 0.985 | 1.059 | 1.048 | 1.150 | 0.909 |
| 2017 | 1.033 | 1.055 | 1.111 | 1.009 | 1.052 | 1.011 | 1.107 | 0.946 |

1.000. And the same can be said about ΔEff. However, as was seen before, DEA is increasing. So, even though the hospital's efficiency has been increasing over the studied years, the rate at which they have been becoming more efficient has been declining.

As previously seen in Chapter 4, the MPI can be further decomposed into four different terms: change in pure efficiency (ΔPureEff), change in scale efficiency (ΔScale), pure change in technology (ΔPureTech) and change in scale of technology (ΔScaleTech).

Note that the decomposition into the four terms involves the computation of efficiency assuming VRS, which may lead to no feasible point being found in the linear programming method when computing the efficiency for one time period projected into another, which then sets some hospitals' MPI results to NaN. So, these hospitals can not be considered in this MPI decomposition's results.

The average MPI for each year regarding each decomposition - into four terms or two terms - is presented in Table 6.11, as well as the average values of each term.

The MPI calculated with its decomposition in four terms presents lower values for the averages of each year, when compared to the MPI calculated with the decomposition into only two terms. And when decomposed in four terms, the MPI shows a clear increase in its average values throughout the years, as seen in Table 6.11. This can, however, be because the two hospitals that did not present the lowest

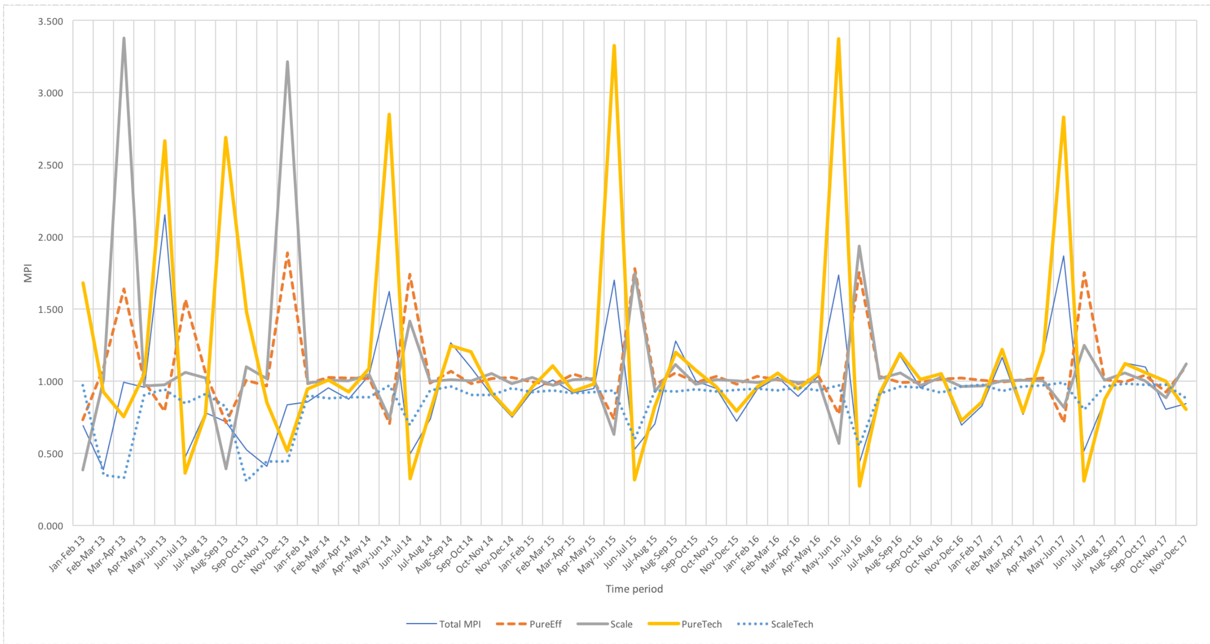


Figure 6.6: Evolution of the average total Malmquist Productivity Index and its four terms for all hospitals through all time periods in each year.

results - $H_{18}$ and $H_{19}$ - are not being considered here due to what was already mentioned about the computation of this decomposition terms.

In this case, it is again a term relative to technology, ΔPureTech, that affects more the total MPI. The upper and lower peaks more evident of the MPI overlap with upper and lower peaks of the ΔPureTech term. The maximum points reached by the MPI are smaller than the ΔPureTech ones due to the fact that ΔPureEff and ΔScale present lower peaks during the same periods. This is observable in Figure 6.6.

Lastly, the MPI for the year 2018 was forecasted. The statistics of the results obtained are presented, per hospital in Table 6.12, and per time period in Table 6.13. It should be again noted that some hospitals' results are not shown, since the MPI decomposition used in the forecasting is the one considering four terms, presenting the problem mentioned before.

These results show that, despite the values of the MPI predicted being a bit higher, they continue to be in line with the ones from previous years, which present indices around 1.000. The average MPI forecasted for 2018 is 1.229, meaning a productivity increase should be expected in 2018. All hospitals present an average index bigger than 1.000, and the same is observed for the 11 time periods forecasted. However, there are some values a bit lower than 1.000 for the first two forecasted time periods (January-February and February-March) of $H_{23}$. The two hospitals with the best average forecasted MPI are $H_{17}$ and $H_{26}$, and the ones with the lowest values are $H_{23}$ and $H_{25}$. This does not differ significantly from what could be expected, since these units are also the ones presenting some of the highest and lowest values, respectively, of MPI in some of the previous years. In another way, however, the period with the lowest forecast is May-June, which does not meet the MPI pattern from previous years. February-March presents the highest forecasted MPI.

Each forecasted MPI term can be analyzed separately, in order to possibly draw more clear conclusions. The average of the forecasted MPI terms as well as the total MPI are shown in Table 6.14.

So, starting with ΔPureEff, which represents the pure efficiency change, its average forecast is 1.054, which suggests an increase, even though small, in hospital efficiency in 2018. This somewhat makes

Table 6.12: Results of the Malmquist Productivity Index forecast: mean, standard deviation, minimum and maximum values, for each hospital.

| DMU | Mean | Std. Deviation | Min. | Max. |
|---|---|---|---|---|
| $H_1$ | 1.343 | 0.015 | 1.314 | 1.367 |
| $H_2$ | 1.174 | 0.031 | 1.153 | 1.252 |
| $H_3$ | 1.241 | 0.064 | 1.042 | 1.302 |
| $H_4$ | 1.217 | 0.036 | 1.198 | 1.326 |
| $H_5$ | 1.209 | 0.020 | 1.175 | 1.261 |
| $H_6$ | 1.217 | 0.023 | 1.193 | 1.279 |
| $H_8$ | 1.271 | 0.024 | 1.205 | 1.309 |
| $H_9$ | 1.191 | 0.018 | 1.171 | 1.227 |
| $H_{10}$ | 1.237 | 0.015 | 1.191 | 1.247 |
| $H_{11}$ | 1.111 | 0.023 | 1.083 | 1.173 |
| $H_{12}$ | 1.225 | 0.016 | 1.210 | 1.263 |
| $H_{13}$ | 1.143 | 0.017 | 1.113 | 1.181 |
| $H_{14}$ | 1.221 | 0.038 | 1.201 | 1.337 |
| $H_{17}$ | 1.472 | 0.015 | 1.453 | 1.514 |
| $H_{20}$ | 1.209 | 0.107 | 1.029 | 1.468 |
| $H_{21}$ | 1.317 | 0.020 | 1.273 | 1.343 |
| $H_{22}$ | 1.249 | 0.040 | 1.221 | 1.356 |
| $H_{23}$ | 1.030 | 0.046 | 0.893 | 1.059 |
| $H_{24}$ | 1.200 | 0.051 | 1.167 | 1.333 |
| $H_{25}$ | 1.094 | 0.014 | 1.073 | 1.119 |
| $H_{26}$ | 1.429 | 0.047 | 1.335 | 1.479 |

Table 6.13: Results of the Malmquist Productivity Index forecast: mean, standard deviation, minimum and maximum values, for each time period.

| Time period | Mean | Std. Deviation | Min. | Max. |
|---|---|---|---|---|
| Jan-Feb | 1.224 | 0.138 | 0.893 | 1.514 |
| Feb-Mar | 1.258 | 0.105 | 0.998 | 1.476 |
| Mar-Apr | 1.231 | 0.106 | 1.037 | 1.479 |
| Apr-May | 1.231 | 0.092 | 1.052 | 1.453 |
| May-Jun | 1.222 | 0.097 | 1.059 | 1.462 |
| Jun-Jul | 1.227 | 0.099 | 1.059 | 1.463 |
| Jul-Aug | 1.222 | 0.099 | 1.056 | 1.461 |
| Aug-Sep | 1.224 | 0.102 | 1.052 | 1.467 |
| Sep-Oct | 1.225 | 0.106 | 1.047 | 1.470 |
| Oct-Nov | 1.226 | 0.108 | 1.042 | 1.472 |
| Nov-Dec | 1.224 | 0.111 | 1.037 | 1.476 |

Table 6.14: Mean of the forecasted Malmquist Productivity Index terms and total Malmquist Productivity Index for each time period of 2018 and their total average.

| Time period | Total MPI | ΔPureEff | ΔScale | ΔPureTech | ΔScaleTech |
|---|---|---|---|---|---|
| Jan-Feb | 1.224 | 1.056 | 1.075 | 1.114 | 0.984 |
| Feb-Mar | 1.258 | 1.062 | 1.053 | 1.124 | 1.002 |
| Mar-Apr | 1.231 | 1.055 | 1.046 | 1.115 | 1.005 |
| Apr-May | 1.231 | 1.053 | 1.043 | 1.123 | 1.002 |
| May-Jun | 1.222 | 1.053 | 1.042 | 1.117 | 1.002 |
| Jun-Jul | 1.227 | 1.053 | 1.041 | 1.120 | 1.003 |
| Jul-Aug | 1.222 | 1.053 | 1.041 | 1.118 | 1.002 |
| Aug-Sep | 1.224 | 1.053 | 1.041 | 1.119 | 1.002 |
| Sep-Oct | 1.225 | 1.053 | 1.041 | 1.120 | 1.002 |
| Oct-Nov | 1.226 | 1.053 | 1.041 | 1.121 | 1.002 |
| Nov-Dec | 1.224 | 1.053 | 1.041 | 1.120 | 1.002 |
| Avg. | 1.229 | 1.054 | 1.046 | 1.119 | 1.001 |

sense given the DEA results obtained and the efficiency scores tendency observed. ΔScale, which measures the changes in scale efficiency of the production unit, presents a mean of 1.046, suggesting changes in the returns to scale faced by the production unit, especially an increase in scale efficiency in 2018, which means hospitals are evolving into a more ideal size, and is in line with the results obtained before regarding the scale efficiency. The changes measured by this term can be due to either changes in the shape of technology, changes in the location of the production unit in the input/output space between the time periods, or both (Simar & Wilson, 1998). ΔPureTech and ΔScaleTech present forecasted average values of 1.119 and 1.001, respectively. Both these terms indicate changes related to the technology frontier, hence meaning an increase in technology in 2018. ΔScaleTech is close to 1.000, which means the shape of technology does not change significantly.

It is also observable that, in all the four forecasted terms, the values are all tending to a certain value, which depends on the term, and some quicker than others. This is comprehensible since 11 periods are being forecasted, which is a considerate number. Besides the uncertainties regarding the future, the forecasts starting from the second step are made based on the previously forecasted values and not real ones.

The actual MPI for the year 2018 can be calculated for some (21) hospitals and some time periods. The hospitals not included did not have several variables data for the year 2018, and the time period for which there are no results presented (November-December) did not have reliable data for the year under analysis. Regardless, with the MPI calculated for these hospitals and time periods of 2018, it is possible to assess the reliability and accuracy of the forecast performed. In order to do so, the Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) were calculated. The MAE is presented in Table 6.15. The average MAE is 0.392 and the total RMSE is 0.294.

The average MAE and RMSE seem relatively good. However, if looking more closely to the values, it can be seen that some errors are bigger than what would be wanted for an accurate forecast.

In particular, the best forecast is done for hospital $H_2$. The worst is $H_{21}$. In terms of periods, the one with the worst average MAE is May-June. Taking into account the forecasted values for this period, it was already pretty obvious that this would be the period with the biggest error, as was discussed before, since it presented in the past years a peak in this period and so it would be expected that 2018 would follow the example, which was not seen in the forecast. This suggests that the forecast method is probably not the most adequate to account for seasonality in the data.

Table 6.15: Mean absolute error of the forecasted Malmquist Productivity Index.

| | Jan-Feb | Feb-Mar | Mar-Apr | Apr-May | May-Jun | Jun-Jul | Jul-Aug | Aug-Sep | Sep-Oct | Oct-Nov | Avg. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $H_2$ | 0.41 | 0.25 | 0.19 | 0.20 | 0.07 | 0.18 | 0.09 | 0.25 | 0.04 | 0.21 | 0.19 |
| $H_3$ | 0.17 | 0.30 | 0.29 | 0.26 | 1.46 | 0.88 | 0.51 | 0.03 | 0.19 | 0.66 | 0.47 |
| $H_5$ | 0.42 | 0.23 | 0.16 | 0.21 | 0.93 | 0.70 | 0.39 | 0.10 | 0.01 | 0.35 | 0.35 |
| $H_6$ | 0.34 | 0.11 | 0.31 | 0.21 | 1.95 | 0.86 | 0.32 | 0.24 | 0.04 | 0.61 | 0.50 |
| $H_9$ | 0.51 | 0.10 | 0.27 | 0.27 | 1.43 | 0.84 | 0.26 | 0.17 | 0.07 | 0.27 | 0.42 |
| $H_{10}$ | 0.35 | 0.30 | 0.31 | 0.23 | 1.10 | 0.85 | 0.35 | 0.29 | 0.21 | 0.31 | 0.43 |
| $H_{11}$ | 0.32 | 0.30 | 0.19 | 0.19 | 0.68 | 0.67 | 0.23 | 0.14 | 0.09 | 0.24 | 0.31 |
| $H_{12}$ | 0.47 | 0.06 | 0.28 | 0.20 | 0.39 | 0.15 | 0.31 | 0.26 | 0.09 | 0.39 | 0.26 |
| $H_{13}$ | 0.29 | 0.15 | 0.16 | 0.10 | 0.09 | 0.18 | 0.81 | 0.64 | 0.20 | 0.53 | 0.31 |
| $H_{14}$ | 0.39 | 0.15 | 0.34 | 0.21 | 0.25 | 0.24 | 0.32 | 0.21 | 0.02 | 0.31 | 0.24 |
| $H_{17}$ | 0.42 | 0.10 | 0.15 | 0.27 | 1.87 | 0.78 | 0.20 | 0.18 | 0.06 | 0.55 | 0.46 |
| $H_{20}$ | 0.42 | 0.22 | 0.31 | 0.14 | 0.29 | 0.13 | 0.35 | 0.14 | 0.05 | 0.31 | 0.24 |
| $H_{21}$ | 0.65 | 0.43 | 0.56 | 0.45 | 1.54 | 1.14 | 0.58 | 0.39 | 0.34 | 0.90 | 0.70 |
| $H_{22}$ | 0.12 | 0.42 | 0.12 | 0.25 | 1.46 | 0.87 | 0.28 | 0.17 | 0.08 | 0.59 | 0.44 |
| $H_{23}$ | 0.46 | 0.13 | 0.47 | 0.17 | 0.85 | 0.87 | 0.30 | 0.12 | 0.50 | 0.22 | 0.41 |
| $H_{26}$ | 0.41 | 0.16 | 0.20 | 0.42 | 2.41 | 0.82 | 0.28 | 0.07 | 0.34 | 0.46 | 0.56 |
| Avg. | 0.38 | 0.21 | 0.27 | 0.24 | 1.05 | 0.64 | 0.35 | 0.21 | 0.15 | 0.43 | 0.39 |

Table 6.16: Errors (Mean Absolute Error and Root Mean Square Error) of the Malmquist Productivity Index forecast, for the total Malmquist Productivity Index and its four different terms.

| | Total MPI | $\Delta$PureEff | $\Delta$Scale | $\Delta$PureTech | $\Delta$ScaleTech |
|---|---|---|---|---|---|
| MAE | 0.392 | 0.214 | 0.104 | 0.391 | 0.081 |
| RMSE | 0.294 | 0.171 | 0.020 | 0.444 | 0.013 |

In order to be able to further analyze the results, the average MAE and RMSE of each of the MPI terms are presented in Table 6.16.

Some terms are forecasted with better precision than others. While $\Delta$PureTech and $\Delta$PureEff have a bigger average of MAE and RMSE values, the other two terms, $\Delta$ScaleTech and $\Delta$Scale, have lower values, with $\Delta$ScaleTech having the lowest. The $\Delta$ScaleTech term presents very close values between hospitals, but without a trend as clear as other terms. For example $\Delta$PureTech, the term with the biggest error, presents a more clear trend of the data, presenting more clear ups and downs, but with values varying over a larger range.

# Chapter 7

# Conclusions

A lot of attention is drawn to the management of healthcare institutions, as has been mentioned throughout this work. The obvious importance of a good healthcare system and so the high value of resources that is put into such institutions leads to greater attention towards its efficiency and productivity. The aim of this work was to assess the efficiency and productivity of public hospitals in Portugal, as well as to forecast their productivity.

Firstly, making use of the DEA method, the efficiency of all public hospitals and hospital centers was calculated and interpreted. 26 hospitals and hospital centers were analyzed for the years 2013 to 2017. The Malmquist productivity index was then used to assess the productivity of these same hospitals, using two different decompositions based on the works of Simar and Wilson (1998), as is described in Chapter 4. The MPI was also forecasted following the theory developed by Daskovska et al. (2010). The forecast was calculated for the year 2018 since the MPI results were obtained until 2017. This allowed for the evaluation of the forecasting technique, comparing results of some hospitals, after calculating their MPI for 2018.

All the results obtained were presented and discussed in Chapter 6. The main conclusions that could be drawn from this work are mentioned below.

Overall, the hospitals that presented the best results in terms of efficiency are:

- $H_2$ - *Centro Hospitalar de Leiria, EPE*;
- $H_6$ - *Centro Hospitalar do Médio Ave, EPE*;
- $H_7$ - *Centro Hospitalar Universitário de Coimbra, EPE*;
- $H_8$ - *Centro Hospitalar Entre Douro e Vouga, EPE*;
- $H_{11}$ - *Centro Hospitalar Tâmega e Sousa, EPE*;
- $H_{18}$ - *Centro Hospitalar Universitário do Porto, EPE*;
- $H_{20}$ - *Centro Hospitalar Vila Nova de Gaia/Espinho, EPE*;
- $H_{21}$ - *Hospital Distrital da Figueira da Foz, EPE*;
- $H_{26}$ - *Hospital Santa Maria Maior, EPE*;

either because they present the best average efficiency score of a year or because they are the ones presenting good values for the majority of the analyzed time periods. On the other hand, the ones performing worst in terms of efficiency were:

- $H_9$ - *Centro Hospitalar Médio Tejo, EPE*;

- $H_{17}$ - *Centro Hospitalar Universitário do Algarve, EPE*;
- $H_{22}$ - *Hospital Distrital de Santarém, EPE*.

In terms of the MPI, the hospitals with the best performance were $H_6$ - *Centro Hospitalar do Médio Ave, EPE* and $H_{26}$ - *Hospital Santa Maria Maior, EPE*.

Overall, the performance of hospitals has been slowly increasing, with some hospitals presenting some good results. The overall average DEA score considering CRS was 0.648 and under VRS 0.764 and seems to be increasing throughout the years. Scale efficiency is also globally increasing. In terms of productivity, the MPI shows seasonality, presenting high peaks in May-June for every year between 2013 and 2017. The overall average MPI is 1.049, showing a very small productivity increase. The terms regarding changes in technology seem to influence more the MPI than the ones considering efficiency changes, in both the MPI decompositions into two and four terms.

The results obtained in this work are consistent with other studies found in the literature. For example, the hospitals that perform better all belong to more coastal areas, in line with (Ferreira, Nunes, & Marques, 2018) and not the interior of the country. Moreover, the RHA to which most belong to is the North RHA (*ARS do Norte*), in line with (Ferreira & Nunes, 2019).

Considering the second part of this work, the forecasted MPI did not present good enough results, forecasting values that are not close enough to the real ones for it to be considered a reliable forecast, which may be due to the complexity of the healthcare data as well as the method considered for the forecast. However, to the extent of the research performed, this forecast had not been applied before.

## 7.1 Limitations

Firstly, the lack of data available and the data gaps existing present one of the limitations of this work. Despite the evolution in transparency and availability of information that there's been, the data available still presents several gaps that in some cases make the analysis of some hospitals impossible. Here, this was the case of years 2018 and 2019, for which, at the date of the work, there was not enough data available to perform a reliable analysis. This brings us to another limitation which is the fact that the forecast was done for a year that has already passed and for which there was already some information, even though not enough for a complete analysis.

Still regarding the available data, the variables used may not have been the best possible. For example, the number of standard patients per nurse is probably not the best possible input variable, however it was the only one available regarding doctors and nurses.

The heterogeneity of the sample of hospitals, both as a group and individually, is also another limitation. The sample is composed of hospitals and hospital centers, which are inevitably different in dimensions and activity, since hospital centers, besides including more than one hospital, can comprise general hospitals, hospitals more specialized in certain areas (such as *Hospital Dona Estefânia*, belonging to *Centro Hospitalar de Lisboa Central, EPE* that is specialized in pediatrics) and university hospitals. This can make their comparison imprecise. Moreover, hospitals are very heterogeneous in the data, meaning that values are very disperse, and vary a lot, being difficult to model.

Through this work, efficiency scores and productivity indices were obtained. However, these consist of only quantitative results, since the source of inefficiency or unproductivity can not be known making use of the methods considered.

Moreover, no quality variables were considered, thus only evaluating quantitatively hospital performance. The complexity of the environment and patients treated is also not being considered in this work. Both these are considered limitations, making this a less reliable and detailed analysis.

The healthcare area is complex and very particular, making the services provided quite complicated to evaluate. Despite it being easy to identify and quantify inputs, such as spendings or number of staff, it is much harder to quantify outputs, since they can be varied and different, as well as presenting different quality levels. Outputs can also be very uncertain. For example, the number of hospitalizations is dependent on a number of factors that can be out of the control of hospital managers. A very clear and real example is the pandemic we are facing at this moment, that is of course leading to unexpected numbers of hospitalizations, and perhaps lower number of medical appointments and surgeries, requiring a new strategy that could not have been anticipated with a simple forecast based on previous years' data. This makes it difficult to forecast hospital performance, since data from previous years do not necessarily guarantee the results for the years following.

## 7.2 Future work

The main future work suggestions are to resolve the limitations, already stated, of this work.

Although this work focused on measuring the efficiency of Portuguese public hospitals, the quality of the services provided was not taken into account, since the efficiency measurement only allows conclusions about hospital production and allocation of resources. Hence, a future work suggestion would be to include quality factors in the analysis, to provide a more complete study. The quality of delivered healthcare services is related to its effectiveness (Ferreira & Marques, 2019). Hence, poor quality providers should not be potential benchmarks (Ferreira et al., 2019). Incorporating quality in DEA methodology can be done in several ways: either by incorporating quality variables in the DEA model (Nayar & Ozcan, 2008), calculating efficiency scores using DEA and then adapting them according to quality variables (Almeida, Frias, & Pedro Fique, 2015), imposing a threshold for the minimum acceptable level per quality dimension (Ferreira et al., 2019), developing new modified DEA methods, such as multiplicative (or log-) DEA (Ferreira et al., 2019) or congestion analysis advance (Valdmanis, Rosko, & Mutter, 2008).

Moreover, the use of exogenous variables, adjustment to environmental factors or case mix index (CMI) would also make the analysis more accurate. This would take into account the environment in which a hospitals operates, for example, considering the complexity of the patients treated, which may lead to a greater use of resources. In order to do so, it is possible to make use of the CMI to homogenize the inpatients of the hospitals considered and their complexity. Another hypothesis is the service mix index (SMI) introduced by Ferreira and Marques (2016b), which is an index of services complexity. Moreover, another possibility is to cluster hospitals into complexity groups. For example Ferreira and Marques (2016a) presents a way to calculate cluster productivity using the MPI or the Hicks-Moorsteen index and Camanho and Dyson (2006) develops a measure based on the MPI to measure inter and intra group performance. Despite most used methods including environment factors being partial frontier methods such as order-$\alpha$, the use of DEA methods is also possible, with the inclusion of environment and exogenous variables, for example (Zheng et al., 2018).

Still regarding the efficiency and productivity assessment, an analysis comprising also private hospitals and PPPs could be of interest. The sample considered in this work is believed to be representative of the Portuguese public hospitals and the Portuguese healthcare network, but the inclusion of private hospitals and PPPs would represent better the total reality of the Portuguese healthcare services. Moreover the hospitals from the autonomous regions of Açores and Madeira can also be included for a more complete analysis.

Future work suggestions include also the forecasting of the MPI for more recent years, perhaps for present year of 2020 since it could be interesting to compare it with actual values to assess how the COVID-19 pandemic impacted the efficiency of hospitals and in what ways. Still in the topic of forecasting, other relevant future work is the exploitation of other forecasting techniques, to assess if it is possible to obtain better results than the ones obtained in this work. For example the use of SARIMA (Seasonal Autoregressive Integrated Moving Average) method could be a good idea, since it would probably account much better for the seasonality and trends in the data. Moreover, the smooth bootstrap adaptation mentioned in Daskovska et al. (2010) can also be explored and developed in practice, to make inferences on the forecasted MPI.

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# Appendix A

# Data Envelopment Analysis results

Table A.1: Total Data Envelopment Analysis results, under Constant Returns to Scale, for 2013.

| DMU | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $H_1$ | 0.536 | 0.160 | 0.178 | 0.633 | 0.642 | 0.608 | 0.604 | 0.646 | 0.193 | 0.225 | 0.207 | 0.665 |
| $H_2$ | 0.726 | 0.183 | 0.213 | 0.801 | 0.734 | 0.280 | 0.781 | 0.779 | 0.339 | 0.275 | 0.241 | 0.820 |
| $H_3$ | 0.630 | 0.154 | 0.180 | 0.657 | 0.657 | 0.594 | 0.664 | 0.630 | 0.205 | 0.222 | 0.203 | 0.673 |
| $H_4$ | 0.602 | 0.145 | 0.174 | 0.825 | 0.585 | 0.464 | 0.581 | 0.612 | 0.177 | 0.208 | 0.192 | 0.747 |
| $H_5$ | 0.735 | 0.185 | 0.228 | 0.741 | 0.766 | 0.406 | 0.695 | 0.742 | 0.224 | 0.262 | 0.232 | 0.772 |
| $H_6$ | 0.813 | 0.206 | 0.249 | 0.853 | 0.872 | 0.952 | 0.839 | 0.898 | 0.277 | 0.306 | 0.285 | 0.990 |
| $H_7$ | 0.562 | 0.148 | 0.151 | 0.589 | 0.551 | 0.274 | 0.507 | 0.512 | 0.202 | 0.244 | 0.196 | 0.664 |
| $H_8$ | 1.000 | 0.263 | 0.304 | 1.000 | 1.000 | 0.931 | 1.000 | 0.980 | 0.300 | 0.344 | 0.320 | 1.000 |
| $H_9$ | 0.523 | 0.128 | 0.154 | 0.587 | 0.554 | 0.210 | 0.580 | 0.637 | 0.177 | 0.195 | 0.188 | 0.652 |
| $H_{10}$ | 0.668 | 0.152 | 0.211 | 0.789 | 0.632 | 0.615 | 0.688 | 0.706 | 0.234 | 0.253 | 0.231 | 0.850 |
| $H_{11}$ | 0.951 | 0.218 | 0.262 | 0.948 | 0.922 | 0.983 | 0.931 | 0.985 | 0.289 | 0.328 | 0.310 | 0.896 |
| $H_{12}$ | 0.582 | 0.142 | 0.166 | 0.616 | 0.578 | 0.530 | 0.606 | 0.618 | 0.156 | 0.229 | 0.198 | 0.893 |
| $H_{13}$ | 0.636 | 0.144 | 0.182 | 0.724 | 0.663 | 0.585 | 0.713 | 0.685 | 0.191 | 0.249 | 0.234 | 0.648 |
| $H_{14}$ | 0.698 | 0.176 | 0.226 | 0.744 | 0.739 | 0.696 | 0.721 | 0.816 | 0.177 | 0.263 | 0.248 | 0.914 |
| $H_{15}$ | 0.519 | 0.132 | 0.157 | 0.554 | 0.555 | 0.520 | 0.517 | 0.591 | 0.167 | 0.190 | 0.177 | 0.594 |
| $H_{16}$ | 0.631 | 0.149 | 0.165 | 0.653 | 0.650 | 0.584 | 0.665 | 0.616 | 0.196 | 0.224 | 0.219 | 0.739 |
| $H_{17}$ | 0.483 | 0.116 | 0.137 | 0.509 | 0.471 | 0.402 | 0.526 | 0.597 | 0.154 | 0.169 | 0.150 | 0.449 |
| $H_{18}$ | 0.791 | 0.185 | 0.215 | 0.810 | 0.788 | 0.294 | 0.800 | 0.743 | 0.232 | 0.274 | 0.246 | 0.763 |
| $H_{19}$ | 0.588 | 0.140 | 0.162 | 0.589 | 0.586 | 0.191 | 0.568 | 0.519 | 0.174 | 0.202 | 0.185 | 0.605 |
| $H_{20}$ | 0.775 | 0.189 | 0.221 | 0.798 | 0.803 | 0.253 | 0.805 | 0.789 | 0.246 | 0.268 | 0.246 | 0.989 |
| $H_{21}$ | 0.808 | 0.205 | 0.236 | 0.827 | 0.819 | 0.797 | 0.774 | 0.836 | 0.248 | 0.267 | 0.242 | 0.728 |
| $H_{22}$ | 0.590 | 1.000 | 1.000 | 0.320 | 0.381 | 0.350 | 0.332 | 1.000 | 1.000 | 1.000 | 1.000 | 0.204 |
| $H_{23}$ | 0.610 | 0.159 | 0.191 | 0.562 | 0.650 | 0.516 | 0.683 | 0.598 | 0.172 | 0.229 | 0.214 | 0.624 |
| $H_{24}$ | 0.620 | 0.170 | 0.179 | 0.694 | 0.708 | 0.241 | 0.676 | 0.734 | 0.215 | 0.244 | 0.251 | 0.874 |
| $H_{25}$ | 0.628 | 0.158 | 0.177 | 0.654 | 0.628 | 0.493 | 0.614 | 0.646 | 0.146 | 0.202 | 0.203 | 0.652 |
| $H_{26}$ | 0.813 | 0.197 | 0.226 | 0.844 | 0.757 | 1.000 | 0.778 | 0.837 | 0.243 | 0.274 | 0.246 | 0.851 |

Table A.2: Total Data Envelopment Analysis results, under Constant Returns to Scale, for 2014.

| DMU | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $H_1$ | 0.622 | 0.553 | 0.618 | 0.649 | 0.721 | 0.544 | 0.609 | 0.607 | 0.650 | 0.617 | 0.668 | 0.669 |
| $H_2$ | 0.835 | 0.773 | 0.820 | 0.833 | 0.810 | 0.202 | 0.837 | 0.791 | 0.834 | 0.803 | 0.803 | 0.836 |
| $H_3$ | 0.615 | 0.603 | 0.624 | 0.652 | 0.671 | 0.475 | 0.667 | 0.549 | 0.637 | 0.613 | 0.661 | 0.702 |
| $H_4$ | 0.655 | 0.606 | 0.571 | 0.625 | 0.651 | 0.444 | 0.595 | 0.592 | 0.600 | 0.587 | 0.607 | 0.577 |
| $H_5$ | 0.702 | 0.700 | 0.704 | 0.747 | 0.700 | 0.560 | 0.736 | 0.745 | 0.726 | 0.735 | 0.789 | 0.642 |
| $H_6$ | 0.913 | 0.920 | 0.915 | 0.915 | 0.999 | 1.000 | 0.830 | 0.926 | 0.981 | 0.955 | 1.000 | 0.386 |
| $H_7$ | 0.600 | 0.603 | 0.577 | 0.568 | 0.640 | 0.186 | 0.544 | 0.468 | 0.601 | 0.668 | 0.721 | 0.488 |
| $H_8$ | 1.000 | 1.000 | 1.000 | 1.000 | 0.991 | 0.696 | 0.959 | 0.918 | 0.982 | 1.000 | 0.999 | 0.876 |
| $H_9$ | 0.534 | 0.540 | 0.564 | 0.586 | 0.594 | 0.392 | 0.576 | 0.566 | 0.603 | 0.576 | 0.610 | 0.662 |
| $H_{10}$ | 0.742 | 0.648 | 0.655 | 0.628 | 0.827 | 0.520 | 0.619 | 0.642 | 0.624 | 0.708 | 0.738 | 0.677 |
| $H_{11}$ | 0.983 | 0.904 | 0.932 | 0.972 | 1.000 | 0.694 | 1.000 | 1.000 | 1.000 | 0.952 | 0.981 | 1.000 |
| $H_{12}$ | 0.614 | 0.584 | 0.693 | 0.618 | 0.666 | 0.171 | 0.648 | 0.636 | 0.650 | 0.608 | 0.673 | 0.684 |
| $H_{13}$ | 0.723 | 0.672 | 0.648 | 0.696 | 0.698 | 0.335 | 0.681 | 0.639 | 0.618 | 0.665 | 0.679 | 0.607 |
| $H_{14}$ | 0.743 | 0.719 | 0.767 | 0.819 | 0.781 | 0.218 | 0.669 | 0.781 | 0.784 | 0.755 | 0.803 | 0.726 |
| $H_{15}$ | 0.531 | 0.505 | 0.535 | 0.538 | 0.546 | 0.389 | 0.527 | 0.489 | 0.545 | 0.532 | 0.562 | 0.535 |
| $H_{16}$ | 0.636 | 0.616 | 0.639 | 0.661 | 0.680 | 0.312 | 0.659 | 0.570 | 0.664 | 0.624 | 0.655 | 0.698 |
| $H_{17}$ | 0.428 | 0.435 | 0.458 | 0.456 | 0.464 | 0.351 | 0.432 | 0.595 | 0.521 | 0.466 | 0.484 | 0.352 |
| $H_{18}$ | 0.800 | 0.737 | 0.743 | 0.779 | 0.803 | 0.210 | 0.803 | 0.638 | 0.789 | 0.761 | 0.784 | 0.619 |
| $H_{19}$ | 0.567 | 0.539 | 0.558 | 0.599 | 0.628 | 0.139 | 0.546 | 0.471 | 0.570 | 0.575 | 0.582 | 0.590 |
| $H_{20}$ | 0.772 | 0.729 | 0.760 | 0.819 | 0.826 | 0.183 | 0.819 | 0.766 | 0.817 | 0.771 | 0.821 | 0.811 |
| $H_{21}$ | 0.786 | 0.773 | 0.796 | 0.806 | 0.857 | 0.539 | 0.792 | 0.819 | 0.839 | 0.788 | 0.840 | 0.748 |
| $H_{22}$ | 0.628 | 0.590 | 0.586 | 0.622 | 0.853 | 0.262 | 0.606 | 0.583 | 0.582 | 0.583 | 0.592 | 0.589 |
| $H_{23}$ | 0.658 | 0.649 | 0.656 | 0.770 | 0.673 | 0.341 | 0.634 | 0.398 | 0.673 | 0.647 | 0.707 | 0.727 |
| $H_{24}$ | 0.670 | 0.625 | 0.664 | 0.553 | 0.677 | 0.169 | 0.648 | 0.656 | 0.637 | 0.645 | 0.687 | 0.664 |
| $H_{25}$ | 0.622 | 0.591 | 0.597 | 0.611 | 0.562 | 0.215 | 0.576 | 0.593 | 0.625 | 0.648 | 0.646 | 0.574 |
| $H_{26}$ | 0.742 | 0.748 | 0.772 | 0.772 | 0.823 | 0.576 | 0.810 | 0.948 | 0.734 | 0.765 | 0.869 | 0.843 |

Table A.3: Total Data Envelopment Analysis results, under Constant Returns to Scale, for 2015.

| DMU | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $H_1$ | 0.679 | 0.643 | 0.641 | 0.783 | 0.697 | 0.257 | 0.657 | 0.581 | 0.678 | 0.673 | 0.699 | 0.622 |
| $H_2$ | 0.863 | 0.828 | 0.747 | 0.851 | 0.798 | 0.201 | 0.805 | 0.764 | 0.858 | 0.824 | 0.849 | 0.839 |
| $H_3$ | 0.622 | 0.635 | 0.568 | 0.661 | 0.644 | 0.472 | 0.635 | 0.539 | 0.670 | 0.621 | 0.701 | 0.755 |
| $H_4$ | 0.619 | 0.677 | 0.509 | 0.662 | 0.622 | 0.460 | 0.649 | 0.569 | 0.634 | 0.644 | 0.664 | 0.645 |
| $H_5$ | 0.746 | 0.750 | 0.762 | 0.735 | 0.699 | 0.330 | 0.737 | 0.664 | 0.703 | 0.705 | 0.740 | 0.714 |
| $H_6$ | 0.937 | 0.995 | 1.000 | 0.702 | 0.882 | 0.688 | 0.903 | 0.780 | 0.929 | 0.762 | 0.867 | 0.725 |
| $H_7$ | 0.612 | 0.650 | 0.538 | 0.601 | 0.647 | 0.174 | 0.610 | 0.454 | 0.673 | 0.731 | 0.688 | 0.564 |
| $H_8$ | 1.000 | 0.993 | 0.908 | 0.976 | 0.980 | 0.557 | 0.892 | 0.868 | 0.936 | 0.972 | 0.958 | 0.881 |
| $H_9$ | 0.599 | 0.594 | 0.519 | 0.611 | 0.598 | 0.440 | 0.590 | 0.597 | 0.638 | 0.629 | 0.624 | 0.611 |
| $H_{10}$ | 0.725 | 0.741 | 0.646 | 0.668 | 0.751 | 0.635 | 0.716 | 0.696 | 0.750 | 0.723 | 0.722 | 0.933 |
| $H_{11}$ | 1.000 | 1.000 | 0.917 | 1.000 | 1.000 | 0.683 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| $H_{12}$ | 0.652 | 0.658 | 0.560 | 0.740 | 0.673 | 0.178 | 0.672 | 0.636 | 0.618 | 0.659 | 0.687 | 0.616 |
| $H_{13}$ | 0.698 | 0.761 | 0.555 | 0.672 | 0.687 | 0.395 | 0.667 | 0.611 | 0.650 | 0.674 | 0.699 | 0.576 |
| $H_{14}$ | 0.759 | 0.753 | 0.718 | 0.764 | 0.747 | 0.198 | 0.780 | 0.668 | 0.752 | 0.710 | 0.796 | 0.822 |
| $H_{15}$ | 0.539 | 0.538 | 0.497 | 0.553 | 0.540 | 0.410 | 0.532 | 0.473 | 0.546 | 0.548 | 0.572 | 0.565 |
| $H_{16}$ | 0.659 | 0.658 | 0.591 | 0.676 | 0.685 | 0.407 | 0.664 | 0.586 | 0.707 | 0.689 | 0.692 | 0.660 |
| $H_{17}$ | 0.474 | 0.481 | 0.430 | 0.483 | 0.480 | 0.599 | 0.483 | 0.538 | 0.475 | 0.463 | 0.473 | 0.351 |
| $H_{18}$ | 0.813 | 0.811 | 0.730 | 0.843 | 0.823 | 0.212 | 0.816 | 0.666 | 0.851 | 0.808 | 0.830 | 0.806 |
| $H_{19}$ | 0.580 | 0.580 | 0.539 | 0.621 | 0.620 | 0.158 | 0.618 | 0.489 | 0.631 | 0.628 | 0.649 | 0.647 |
| $H_{20}$ | 0.816 | 0.777 | 0.704 | 0.827 | 0.819 | 0.189 | 0.801 | 0.756 | 0.841 | 0.794 | 0.825 | 0.854 |
| $H_{21}$ | 0.840 | 0.844 | 0.754 | 0.883 | 0.850 | 0.560 | 0.797 | 0.844 | 0.922 | 0.842 | 0.881 | 0.791 |
| $H_{22}$ | 0.581 | 0.582 | 0.497 | 0.546 | 0.709 | 0.248 | 0.529 | 0.572 | 0.548 | 0.567 | 0.587 | 0.577 |
| $H_{23}$ | 0.699 | 0.672 | 0.584 | 0.703 | 0.713 | 0.342 | 0.639 | 0.396 | 0.708 | 0.692 | 0.710 | 0.662 |
| $H_{24}$ | 0.648 | 0.622 | 0.546 | 0.634 | 0.610 | 0.166 | 0.646 | 0.627 | 0.643 | 0.660 | 0.690 | 0.679 |
| $H_{25}$ | 0.642 | 0.648 | 0.552 | 0.629 | 0.615 | 0.155 | 0.627 | 0.575 | 0.683 | 0.636 | 0.654 | 0.584 |
| $H_{26}$ | 0.838 | 0.905 | 0.797 | 0.797 | 0.885 | 1.000 | 0.832 | 0.773 | 0.882 | 0.833 | 0.847 | 0.850 |

Table A.4: Total Data Envelopment Analysis results, under Constant Returns to Scale, for 2016.

| DMU | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $H_1$ | 0.661 | 0.676 | 0.660 | 0.677 | 0.703 | 0.510 | 0.592 | 0.651 | 0.630 | 0.640 | 0.678 | 0.601 |
| $H_2$ | 0.876 | 0.883 | 0.871 | 0.842 | 0.876 | 0.211 | 0.779 | 0.819 | 0.796 | 0.759 | 0.766 | 0.893 |
| $H_3$ | 0.640 | 0.677 | 0.646 | 0.638 | 0.694 | 0.448 | 0.627 | 0.591 | 0.679 | 0.644 | 0.715 | 0.647 |
| $H_4$ | 0.645 | 0.641 | 0.651 | 0.675 | 0.658 | 0.475 | 0.639 | 0.643 | 0.619 | 0.637 | 0.699 | 0.653 |
| $H_5$ | 0.754 | 0.693 | 0.706 | 0.745 | 0.701 | 0.472 | 0.774 | 0.738 | 0.752 | 0.711 | 0.707 | 0.757 |
| $H_6$ | 0.864 | 0.880 | 0.891 | 0.897 | 0.863 | 0.760 | 0.785 | 0.866 | 0.870 | 0.859 | 0.952 | 0.787 |
| $H_7$ | 0.680 | 0.691 | 0.685 | 0.674 | 0.658 | 0.162 | 0.572 | 0.501 | 0.675 | 0.689 | 0.675 | 0.585 |
| $H_8$ | 0.887 | 0.958 | 0.966 | 0.943 | 0.971 | 0.675 | 0.868 | 0.886 | 0.901 | 0.904 | 0.833 | 1.000 |
| $H_9$ | 0.688 | 0.689 | 0.619 | 0.609 | 0.634 | 0.482 | 0.586 | 0.708 | 0.621 | 0.629 | 0.673 | 0.540 |
| $H_{10}$ | 0.729 | 0.799 | 0.673 | 0.732 | 0.746 | 0.375 | 0.774 | 0.694 | 0.738 | 0.807 | 0.845 | 0.679 |
| $H_{11}$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.584 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.984 |
| $H_{12}$ | 0.659 | 0.670 | 0.616 | 0.768 | 0.631 | 0.508 | 0.391 | 0.656 | 0.641 | 0.653 | 0.660 | 0.670 |
| $H_{13}$ | 0.726 | 0.698 | 0.694 | 0.690 | 0.626 | 0.453 | 0.595 | 0.672 | 0.664 | 0.635 | 0.698 | 0.613 |
| $H_{14}$ | 0.786 | 0.770 | 0.801 | 0.740 | 0.755 | 0.197 | 0.756 | 0.720 | 0.781 | 0.746 | 0.758 | 0.756 |
| $H_{15}$ | 0.555 | 0.573 | 0.554 | 0.549 | 0.569 | 0.369 | 0.513 | 0.502 | 0.549 | 0.524 | 0.556 | 0.523 |
| $H_{16}$ | 0.694 | 0.694 | 0.711 | 0.679 | 0.716 | 0.384 | 0.684 | 0.628 | 0.691 | 0.637 | 0.689 | 0.670 |
| $H_{17}$ | 0.424 | 0.545 | 0.605 | 0.504 | 0.492 | 0.605 | 0.495 | 0.621 | 0.483 | 0.455 | 0.471 | 0.324 |
| $H_{18}$ | 0.852 | 0.869 | 0.839 | 0.834 | 0.832 | 0.205 | 0.776 | 0.735 | 0.838 | 0.781 | 0.806 | 0.690 |
| $H_{19}$ | 0.634 | 0.642 | 0.644 | 0.625 | 0.696 | 0.152 | 0.610 | 0.528 | 0.662 | 0.614 | 0.663 | 0.626 |
| $H_{20}$ | 0.841 | 0.818 | 0.860 | 0.800 | 0.833 | 0.181 | 0.785 | 0.820 | 0.808 | 0.750 | 0.816 | 0.883 |
| $H_{21}$ | 0.881 | 0.879 | 0.861 | 0.839 | 0.933 | 0.640 | 0.864 | 0.869 | 0.933 | 0.874 | 0.920 | 0.799 |
| $H_{22}$ | 0.610 | 0.584 | 0.583 | 0.593 | 0.754 | 0.237 | 0.546 | 0.614 | 0.554 | 0.560 | 0.566 | 0.594 |
| $H_{23}$ | 0.707 | 0.668 | 0.730 | 0.721 | 0.719 | 0.151 | 0.757 | 0.709 | 0.712 | 0.667 | 0.680 | 0.652 |
| $H_{24}$ | 0.675 | 0.664 | 0.666 | 0.668 | 0.664 | 0.159 | 0.657 | 0.675 | 0.692 | 0.654 | 0.655 | 0.696 |
| $H_{25}$ | 0.574 | 0.622 | 0.754 | 0.654 | 0.624 | 0.152 | 0.610 | 0.640 | 0.643 | 0.630 | 0.678 | 0.602 |
| $H_{26}$ | 0.894 | 0.850 | 0.883 | 0.898 | 0.853 | 1.000 | 0.766 | 0.913 | 0.943 | 0.865 | 0.970 | 0.850 |

Table A.5: Total Data Envelopment Analysis results, under Constant Returns to Scale, for 2017.

| DMU | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $H_1$ | 0.675 | 0.653 | 0.644 | 0.661 | 0.683 | 0.546 | 0.653 | 0.680 | 0.627 | 0.712 | 0.632 | 0.721 |
| $H_2$ | 0.877 | 0.835 | 0.852 | 0.800 | 0.850 | 0.261 | 0.853 | 0.852 | 0.863 | 0.904 | 0.809 | 0.917 |
| $H_3$ | 0.678 | 0.679 | 0.657 | 0.682 | 0.696 | 0.553 | 0.658 | 0.599 | 0.676 | 0.716 | 0.495 | 0.662 |
| $H_4$ | 0.693 | 0.682 | 0.684 | 0.687 | 0.705 | 0.597 | 0.664 | 0.731 | 0.674 | 0.697 | 0.522 | 0.623 |
| $H_5$ | 0.766 | 0.783 | 0.724 | 0.750 | 0.767 | 0.644 | 0.777 | 0.723 | 0.745 | 0.834 | 0.538 | 0.714 |
| $H_6$ | 0.966 | 0.916 | 0.970 | 0.954 | 0.953 | 0.856 | 0.940 | 0.777 | 0.895 | 0.964 | 0.812 | 0.908 |
| $H_7$ | 0.686 | 0.680 | 0.647 | 0.608 | 0.650 | 0.249 | 0.552 | 0.519 | 0.712 | 0.743 | 0.491 | 0.633 |
| $H_8$ | 0.860 | 0.925 | 0.912 | 0.894 | 0.839 | 0.832 | 0.844 | 0.972 | 0.865 | 0.933 | 0.675 | 0.805 |
| $H_9$ | 0.637 | 0.639 | 0.654 | 0.667 | 0.630 | 0.391 | 0.610 | 0.653 | 0.630 | 0.688 | 0.613 | 0.640 |
| $H_{10}$ | 0.747 | 0.805 | 0.729 | 0.674 | 0.744 | 0.546 | 0.771 | 0.698 | 0.776 | 0.794 | 0.746 | 0.743 |
| $H_{11}$ | 0.976 | 1.000 | 1.000 | 0.997 | 1.000 | 0.841 | 1.000 | 0.997 | 0.959 | 0.985 | 1.000 | 0.926 |
| $H_{12}$ | 0.652 | 0.595 | 0.568 | 0.775 | 0.677 | 0.221 | 0.632 | 0.650 | 0.619 | 0.656 | 0.641 | 0.638 |
| $H_{13}$ | 0.735 | 0.715 | 0.616 | 0.699 | 0.716 | 0.522 | 0.665 | 0.732 | 0.673 | 0.730 | 0.540 | 0.614 |
| $H_{14}$ | 0.764 | 0.745 | 0.793 | 0.781 | 0.823 | 0.239 | 0.790 | 0.721 | 0.752 | 0.805 | 0.756 | 0.685 |
| $H_{15}$ | 0.535 | 0.578 | 0.547 | 0.548 | 0.563 | 0.349 | 0.544 | 0.563 | 0.568 | 0.533 | 0.514 | 0.555 |
| $H_{16}$ | 0.686 | 0.714 | 0.697 | 0.704 | 0.709 | 0.518 | 0.700 | 0.597 | 0.757 | 0.733 | 0.531 | 0.657 |
| $H_{17}$ | 0.490 | 0.506 | 0.487 | 0.644 | 0.491 | 0.453 | 0.494 | 0.552 | 0.473 | 0.494 | 0.339 | 0.326 |
| $H_{18}$ | 0.844 | 0.800 | 0.794 | 0.816 | 0.800 | 0.266 | 0.805 | 0.763 | 0.842 | 0.833 | 0.850 | 0.644 |
| $H_{19}$ | 0.659 | 0.641 | 0.659 | 0.640 | 0.732 | 0.193 | 0.672 | 0.585 | 0.671 | 0.692 | 0.658 | 0.578 |
| $H_{20}$ | 0.858 | 0.826 | 0.782 | 0.817 | 0.844 | 0.237 | 0.843 | 0.853 | 0.856 | 0.889 | 0.805 | 0.707 |
| $H_{21}$ | 0.919 | 0.952 | 0.947 | 0.925 | 0.970 | 0.945 | 0.892 | 0.943 | 0.957 | 0.988 | 0.639 | 0.782 |
| $H_{22}$ | 0.629 | 0.594 | 0.572 | 0.565 | 0.738 | 0.312 | 0.537 | 0.551 | 0.519 | 0.589 | 0.436 | 0.540 |
| $H_{23}$ | 0.723 | 0.658 | 0.697 | 0.674 | 0.713 | 0.197 | 0.647 | 0.672 | 0.679 | 0.762 | 0.650 | 0.745 |
| $H_{24}$ | 0.662 | 0.661 | 0.659 | 0.681 | 0.664 | 0.221 | 0.617 | 0.713 | 0.669 | 0.708 | 0.640 | 0.692 |
| $H_{25}$ | 0.650 | 0.721 | 0.705 | 0.668 | 0.657 | 0.217 | 0.636 | 0.646 | 0.540 | 0.667 | 0.666 | 0.600 |
| $H_{26}$ | 1.000 | 0.900 | 0.928 | 1.000 | 0.968 | 1.000 | 0.944 | 1.000 | 1.000 | 1.000 | 0.635 | 1.000 |

Table A.6: Total Data Envelopment Analysis results, under Variable Returns to Scale, for 2013.

| DMU | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $H_1$ | 0.572 | 0.339 | 0.302 | 0.657 | 0.680 | 0.616 | 0.641 | 0.665 | 0.317 | 0.336 | 0.262 | 0.671 |
| $H_2$ | 0.746 | 0.511 | 0.506 | 0.817 | 0.761 | 0.285 | 0.803 | 0.783 | 0.909 | 0.628 | 0.568 | 0.823 |
| $H_3$ | 0.739 | 0.747 | 0.754 | 0.761 | 0.777 | 0.836 | 0.771 | 0.774 | 0.816 | 0.739 | 0.743 | 0.693 |
| $H_4$ | 0.622 | 0.351 | 0.396 | 0.845 | 0.607 | 0.472 | 0.602 | 0.616 | 0.401 | 0.397 | 0.370 | 0.751 |
| $H_5$ | 0.756 | 0.481 | 0.558 | 0.757 | 0.794 | 0.412 | 0.723 | 0.749 | 0.509 | 0.514 | 0.434 | 0.777 |
| $H_6$ | 0.855 | 0.364 | 0.429 | 0.886 | 0.930 | 0.966 | 0.890 | 0.926 | 0.428 | 0.414 | 0.399 | 1.000 |
| $H_7$ | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 0.816 | 1.000 | 1.000 | 1.000 | 1.000 |
| $H_8$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.948 | 0.926 | 0.922 | 1.000 |
| $H_9$ | 0.550 | 0.224 | 0.275 | 0.607 | 0.587 | 0.213 | 0.612 | 0.649 | 0.291 | 0.255 | 0.232 | 0.660 |
| $H_{10}$ | 0.802 | 0.294 | 0.407 | 0.909 | 0.786 | 0.618 | 0.840 | 0.805 | 0.410 | 0.465 | 0.445 | 0.939 |
| $H_{11}$ | 0.958 | 0.749 | 0.775 | 0.954 | 0.931 | 1.000 | 0.941 | 1.000 | 0.872 | 0.843 | 0.841 | 0.897 |
| $H_{12}$ | 0.591 | 0.458 | 0.464 | 0.623 | 0.590 | 0.539 | 0.616 | 0.621 | 0.439 | 0.556 | 0.482 | 0.895 |
| $H_{13}$ | 0.639 | 0.496 | 0.576 | 0.725 | 0.665 | 0.600 | 0.716 | 0.698 | 0.604 | 0.696 | 0.702 | 0.648 |
| $H_{14}$ | 0.741 | 0.211 | 0.333 | 0.777 | 0.796 | 0.706 | 0.772 | 0.841 | 0.178 | 0.267 | 0.257 | 0.934 |
| $H_{15}$ | 0.867 | 0.843 | 0.991 | 0.890 | 1.000 | 1.000 | 0.848 | 1.000 | 0.797 | 0.758 | 0.851 | 0.854 |
| $H_{16}$ | 0.932 | 0.879 | 0.909 | 0.929 | 1.000 | 1.000 | 1.000 | 0.925 | 0.897 | 0.863 | 0.996 | 1.000 |
| $H_{17}$ | 0.485 | 0.445 | 0.457 | 0.518 | 0.478 | 0.469 | 0.542 | 0.697 | 0.502 | 0.470 | 0.439 | 0.449 |
| $H_{18}$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.478 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.972 |
| $H_{19}$ | 0.874 | 0.842 | 0.890 | 0.859 | 0.920 | 0.329 | 0.805 | 0.740 | 0.781 | 0.770 | 0.816 | 0.798 |
| $H_{20}$ | 0.912 | 0.924 | 0.922 | 0.917 | 0.944 | 0.353 | 0.933 | 0.996 | 0.967 | 0.893 | 0.895 | 1.000 |
| $H_{21}$ | 0.941 | 0.342 | 0.400 | 0.938 | 0.999 | 0.801 | 0.930 | 0.950 | 0.450 | 0.521 | 0.483 | 0.821 |
| $H_{22}$ | 0.631 | 1.000 | 1.000 | 0.337 | 0.412 | 0.355 | 0.398 | 1.000 | 1.000 | 1.000 | 1.000 | 0.208 |
| $H_{23}$ | 0.646 | 0.258 | 0.306 | 0.583 | 0.688 | 0.523 | 0.725 | 0.615 | 0.267 | 0.325 | 0.258 | 0.635 |
| $H_{24}$ | 0.623 | 0.639 | 0.555 | 0.695 | 0.723 | 0.265 | 0.679 | 0.782 | 0.699 | 0.684 | 0.791 | 0.876 |
| $H_{25}$ | 0.635 | 0.531 | 0.519 | 0.658 | 0.634 | 0.502 | 0.620 | 0.653 | 0.435 | 0.518 | 0.585 | 0.655 |
| $H_{26}$ | 1.000 | 0.427 | 0.488 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.589 | 0.669 | 0.657 | 1.000 |

Table A.7: Total Data Envelopment Analysis results, under Variable Returns to Scale, for 2014.

| DMU | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $H_1$ | 0.656 | 0.594 | 0.651 | 0.684 | 0.721 | 0.595 | 0.637 | 0.614 | 0.661 | 0.645 | 0.668 | 0.690 |
| $H_2$ | 0.849 | 0.788 | 0.831 | 0.844 | 0.810 | 0.272 | 0.848 | 0.793 | 0.837 | 0.812 | 0.804 | 0.845 |
| $H_3$ | 0.718 | 0.744 | 0.767 | 0.776 | 0.784 | 0.957 | 0.775 | 0.676 | 0.745 | 0.730 | 0.778 | 0.850 |
| $H_4$ | 0.677 | 0.631 | 0.588 | 0.644 | 0.651 | 0.553 | 0.611 | 0.596 | 0.607 | 0.603 | 0.607 | 0.591 |
| $H_5$ | 0.727 | 0.728 | 0.727 | 0.769 | 0.700 | 0.684 | 0.755 | 0.750 | 0.734 | 0.753 | 0.789 | 0.655 |
| $H_6$ | 0.970 | 0.985 | 0.969 | 0.969 | 1.000 | 1.000 | 0.871 | 0.939 | 1.000 | 1.000 | 1.000 | 0.402 |
| $H_7$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.928 | 1.000 | 1.000 | 1.000 | 1.000 |
| $H_8$ | 1.000 | 1.000 | 1.000 | 1.000 | 0.998 | 1.000 | 0.960 | 0.919 | 0.996 | 1.000 | 1.000 | 0.876 |
| $H_9$ | 0.569 | 0.577 | 0.595 | 0.618 | 0.594 | 0.416 | 0.603 | 0.573 | 0.614 | 0.602 | 0.610 | 0.686 |
| $H_{10}$ | 0.908 | 0.828 | 0.800 | 0.772 | 0.943 | 0.800 | 0.726 | 0.671 | 0.776 | 0.858 | 0.811 | 0.781 |
| $H_{11}$ | 0.984 | 0.908 | 0.935 | 0.975 | 1.000 | 0.985 | 1.000 | 1.000 | 1.000 | 0.954 | 0.983 | 1.000 |
| $H_{12}$ | 0.621 | 0.593 | 0.701 | 0.626 | 0.666 | 0.229 | 0.655 | 0.638 | 0.653 | 0.615 | 0.673 | 0.689 |
| $H_{13}$ | 0.738 | 0.684 | 0.663 | 0.701 | 0.710 | 0.472 | 0.681 | 0.639 | 0.618 | 0.667 | 0.682 | 0.610 |
| $H_{14}$ | 0.798 | 0.779 | 0.818 | 0.868 | 0.790 | 0.219 | 0.708 | 0.792 | 0.816 | 0.801 | 0.809 | 0.763 |
| $H_{15}$ | 0.824 | 0.792 | 0.864 | 0.884 | 0.781 | 1.000 | 0.853 | 1.000 | 0.830 | 0.777 | 0.748 | 0.853 |
| $H_{16}$ | 0.923 | 0.923 | 0.976 | 0.996 | 0.943 | 0.778 | 1.000 | 1.000 | 0.950 | 0.880 | 0.860 | 1.000 |
| $H_{17}$ | 0.428 | 0.435 | 0.464 | 0.459 | 0.477 | 0.515 | 0.442 | 0.751 | 0.555 | 0.484 | 0.509 | 0.362 |
| $H_{18}$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.491 | 1.000 | 0.982 | 1.000 | 1.000 | 1.000 | 0.848 |
| $H_{19}$ | 0.769 | 0.774 | 0.812 | 0.842 | 0.828 | 0.339 | 0.678 | 0.742 | 0.762 | 0.798 | 0.753 | 0.824 |
| $H_{20}$ | 0.899 | 0.898 | 0.935 | 0.979 | 0.969 | 0.366 | 0.951 | 1.000 | 0.966 | 0.922 | 0.976 | 1.000 |
| $H_{21}$ | 0.966 | 0.955 | 0.957 | 0.972 | 0.973 | 0.796 | 0.919 | 0.850 | 1.000 | 0.949 | 0.916 | 0.852 |
| $H_{22}$ | 0.675 | 0.639 | 0.631 | 0.667 | 0.861 | 0.267 | 0.642 | 0.592 | 0.610 | 0.619 | 0.598 | 0.620 |
| $H_{23}$ | 0.702 | 0.688 | 0.691 | 0.807 | 0.673 | 0.360 | 0.663 | 0.402 | 0.685 | 0.675 | 0.707 | 0.754 |
| $H_{24}$ | 0.688 | 0.626 | 0.667 | 0.553 | 0.701 | 0.250 | 0.652 | 0.685 | 0.641 | 0.647 | 0.708 | 0.664 |
| $H_{25}$ | 0.629 | 0.600 | 0.603 | 0.619 | 0.562 | 0.290 | 0.584 | 0.595 | 0.627 | 0.652 | 0.647 | 0.578 |
| $H_{26}$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

Table A.8: Total Data Envelopment Analysis results, under Variable Returns to Scale, for 2015.

| DMU | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $H_1$ | 0.704 | 0.646 | 0.644 | 0.813 | 0.716 | 0.354 | 0.686 | 0.619 | 0.695 | 0.701 | 0.725 | 0.635 |
| $H_2$ | 0.871 | 0.829 | 0.795 | 0.859 | 0.805 | 0.290 | 0.816 | 0.777 | 0.863 | 0.832 | 0.855 | 0.842 |
| $H_3$ | 0.706 | 0.724 | 0.714 | 0.737 | 0.728 | 0.928 | 0.714 | 0.649 | 0.744 | 0.703 | 0.793 | 0.868 |
| $H_4$ | 0.634 | 0.679 | 0.530 | 0.676 | 0.631 | 0.650 | 0.663 | 0.589 | 0.643 | 0.656 | 0.675 | 0.652 |
| $H_5$ | 0.762 | 0.752 | 0.791 | 0.752 | 0.713 | 0.462 | 0.757 | 0.693 | 0.714 | 0.725 | 0.757 | 0.724 |
| $H_6$ | 0.976 | 1.000 | 1.000 | 0.732 | 0.908 | 0.937 | 0.947 | 0.840 | 0.958 | 0.805 | 0.911 | 0.745 |
| $H_7$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| $H_8$ | 1.000 | 0.993 | 0.986 | 0.977 | 0.983 | 0.812 | 0.898 | 0.877 | 0.938 | 0.977 | 0.960 | 0.883 |
| $H_9$ | 0.624 | 0.597 | 0.523 | 0.638 | 0.615 | 0.607 | 0.618 | 0.631 | 0.654 | 0.653 | 0.649 | 0.624 |
| $H_{10}$ | 0.837 | 0.803 | 0.785 | 0.792 | 0.827 | 0.684 | 0.826 | 0.849 | 0.824 | 0.836 | 0.828 | 1.000 |
| $H_{11}$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| $H_{12}$ | 0.656 | 0.659 | 0.601 | 0.746 | 0.676 | 0.257 | 0.679 | 0.646 | 0.623 | 0.666 | 0.691 | 0.620 |
| $H_{13}$ | 0.698 | 0.761 | 0.602 | 0.672 | 0.687 | 0.577 | 0.671 | 0.616 | 0.651 | 0.676 | 0.700 | 0.577 |
| $H_{14}$ | 0.797 | 0.761 | 0.732 | 0.804 | 0.774 | 0.265 | 0.823 | 0.729 | 0.782 | 0.758 | 0.837 | 0.847 |
| $H_{15}$ | 0.780 | 0.756 | 0.822 | 0.802 | 0.744 | 1.000 | 0.761 | 0.898 | 0.728 | 0.714 | 0.762 | 0.841 |
| $H_{16}$ | 0.914 | 0.894 | 0.923 | 0.916 | 0.920 | 0.963 | 0.915 | 1.000 | 0.929 | 0.890 | 0.905 | 0.961 |
| $H_{17}$ | 0.481 | 0.492 | 0.478 | 0.494 | 0.492 | 0.939 | 0.496 | 0.652 | 0.484 | 0.465 | 0.480 | 0.360 |
| $H_{18}$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.481 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| $H_{19}$ | 0.755 | 0.747 | 0.794 | 0.779 | 0.789 | 0.373 | 0.802 | 0.762 | 0.791 | 0.799 | 0.816 | 0.891 |
| $H_{20}$ | 0.942 | 0.891 | 0.888 | 0.927 | 0.938 | 0.371 | 0.917 | 1.000 | 0.941 | 0.911 | 0.936 | 0.992 |
| $H_{21}$ | 0.949 | 0.896 | 0.890 | 1.000 | 0.928 | 0.639 | 0.918 | 0.998 | 1.000 | 0.958 | 0.997 | 0.880 |
| $H_{22}$ | 0.612 | 0.590 | 0.517 | 0.582 | 0.738 | 0.332 | 0.566 | 0.624 | 0.571 | 0.602 | 0.618 | 0.596 |
| $H_{23}$ | 0.726 | 0.675 | 0.588 | 0.730 | 0.730 | 0.467 | 0.669 | 0.419 | 0.727 | 0.720 | 0.738 | 0.678 |
| $H_{24}$ | 0.648 | 0.622 | 0.593 | 0.634 | 0.611 | 0.248 | 0.649 | 0.646 | 0.652 | 0.668 | 0.713 | 0.692 |
| $H_{25}$ | 0.646 | 0.649 | 0.593 | 0.632 | 0.618 | 0.225 | 0.632 | 0.583 | 0.685 | 0.640 | 0.656 | 0.586 |
| $H_{26}$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

Table A.9: Total Data Envelopment Analysis results, under Variable Returns to Scale, for 2016.

| DMU | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $H_1$ | 0.679 | 0.701 | 0.679 | 0.693 | 0.717 | 0.695 | 0.624 | 0.665 | 0.640 | 0.661 | 0.682 | 0.621 |
| $H_2$ | 0.880 | 0.888 | 0.877 | 0.847 | 0.879 | 0.311 | 0.788 | 0.823 | 0.798 | 0.768 | 0.767 | 0.904 |
| $H_3$ | 0.704 | 0.739 | 0.723 | 0.711 | 0.786 | 0.941 | 0.727 | 0.672 | 0.759 | 0.751 | 0.829 | 0.708 |
| $H_4$ | 0.654 | 0.655 | 0.660 | 0.683 | 0.665 | 0.679 | 0.656 | 0.649 | 0.624 | 0.648 | 0.701 | 0.664 |
| $H_5$ | 0.766 | 0.711 | 0.720 | 0.757 | 0.712 | 0.661 | 0.796 | 0.748 | 0.758 | 0.726 | 0.710 | 0.775 |
| $H_6$ | 0.896 | 0.924 | 0.926 | 0.927 | 0.887 | 1.000 | 0.841 | 0.890 | 0.886 | 0.892 | 0.959 | 0.818 |
| $H_7$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| $H_8$ | 0.892 | 0.962 | 0.967 | 0.946 | 0.972 | 1.000 | 0.877 | 0.888 | 0.902 | 0.908 | 0.834 | 1.000 |
| $H_9$ | 0.705 | 0.715 | 0.637 | 0.625 | 0.647 | 0.661 | 0.616 | 0.718 | 0.631 | 0.648 | 0.677 | 0.561 |
| $H_{10}$ | 0.800 | 0.916 | 0.756 | 0.801 | 0.818 | 0.412 | 0.928 | 0.751 | 0.775 | 0.894 | 0.863 | 0.769 |
| $H_{11}$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.926 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.989 |
| $H_{12}$ | 0.663 | 0.675 | 0.620 | 0.774 | 0.634 | 0.741 | 0.397 | 0.659 | 0.645 | 0.658 | 0.661 | 0.678 |
| $H_{13}$ | 0.726 | 0.698 | 0.699 | 0.691 | 0.626 | 0.689 | 0.599 | 0.673 | 0.665 | 0.637 | 0.698 | 0.616 |
| $H_{14}$ | 0.814 | 0.814 | 0.834 | 0.769 | 0.782 | 0.260 | 0.812 | 0.746 | 0.799 | 0.784 | 0.765 | 0.807 |
| $H_{15}$ | 0.722 | 0.739 | 0.729 | 0.728 | 0.764 | 1.000 | 0.730 | 0.791 | 0.717 | 0.709 | 0.748 | 0.802 |
| $H_{16}$ | 0.899 | 0.877 | 0.918 | 0.896 | 0.951 | 1.000 | 1.000 | 1.000 | 0.908 | 0.858 | 0.925 | 1.000 |
| $H_{17}$ | 0.430 | 0.552 | 0.616 | 0.513 | 0.503 | 1.000 | 0.522 | 0.714 | 0.487 | 0.456 | 0.480 | 0.330 |
| $H_{18}$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.513 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.955 |
| $H_{19}$ | 0.779 | 0.784 | 0.815 | 0.794 | 0.885 | 0.389 | 0.832 | 0.722 | 0.835 | 0.810 | 0.863 | 0.899 |
| $H_{20}$ | 0.937 | 0.901 | 0.976 | 0.909 | 0.950 | 0.395 | 0.932 | 1.000 | 0.914 | 0.889 | 0.948 | 1.000 |
| $H_{21}$ | 0.956 | 0.996 | 0.949 | 0.912 | 1.000 | 0.714 | 1.000 | 0.931 | 0.975 | 0.967 | 0.938 | 0.897 |
| $H_{22}$ | 0.632 | 0.618 | 0.609 | 0.614 | 0.776 | 0.309 | 0.583 | 0.634 | 0.566 | 0.586 | 0.571 | 0.621 |
| $H_{23}$ | 0.726 | 0.694 | 0.752 | 0.738 | 0.734 | 0.206 | 0.793 | 0.724 | 0.721 | 0.689 | 0.684 | 0.678 |
| $H_{24}$ | 0.688 | 0.676 | 0.684 | 0.682 | 0.688 | 0.271 | 0.687 | 0.717 | 0.716 | 0.681 | 0.677 | 0.718 |
| $H_{25}$ | 0.577 | 0.626 | 0.756 | 0.656 | 0.625 | 0.222 | 0.617 | 0.640 | 0.645 | 0.632 | 0.680 | 0.605 |
| $H_{26}$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

Table A.10: Total Data Envelopment Analysis results, under Variable Returns to Scale, for 2017.

| DMU | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $H_1$ | 0.688 | 0.663 | 0.655 | 0.662 | 0.687 | 0.621 | 0.662 | 0.681 | 0.647 | 0.721 | 0.693 | 0.768 |
| $H_2$ | 0.898 | 0.839 | 0.855 | 0.802 | 0.851 | 0.308 | 0.856 | 0.854 | 0.899 | 0.918 | 0.833 | 0.988 |
| $H_3$ | 0.759 | 0.776 | 0.756 | 0.773 | 0.802 | 0.848 | 0.743 | 0.661 | 0.762 | 0.791 | 0.540 | 0.866 |
| $H_4$ | 0.708 | 0.687 | 0.689 | 0.688 | 0.707 | 0.698 | 0.669 | 0.733 | 0.700 | 0.707 | 0.546 | 0.668 |
| $H_5$ | 0.783 | 0.790 | 0.732 | 0.752 | 0.770 | 0.748 | 0.783 | 0.724 | 0.772 | 0.845 | 0.573 | 0.764 |
| $H_6$ | 0.982 | 0.934 | 0.988 | 0.956 | 0.960 | 0.972 | 0.955 | 0.778 | 0.923 | 0.975 | 0.915 | 0.964 |
| $H_7$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| $H_8$ | 0.880 | 0.928 | 0.914 | 0.897 | 0.840 | 0.984 | 0.847 | 0.974 | 0.900 | 0.947 | 0.689 | 0.868 |
| $H_9$ | 0.649 | 0.649 | 0.664 | 0.668 | 0.634 | 0.447 | 0.618 | 0.654 | 0.651 | 0.697 | 0.672 | 0.682 |
| $H_{10}$ | 0.750 | 0.856 | 0.772 | 0.674 | 0.762 | 0.556 | 0.805 | 0.698 | 0.781 | 0.798 | 1.000 | 0.760 |
| $H_{11}$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| $H_{12}$ | 0.667 | 0.598 | 0.571 | 0.777 | 0.678 | 0.260 | 0.635 | 0.651 | 0.643 | 0.665 | 0.667 | 0.685 |
| $H_{13}$ | 0.757 | 0.715 | 0.617 | 0.701 | 0.716 | 0.619 | 0.665 | 0.739 | 0.701 | 0.741 | 0.542 | 0.663 |
| $H_{14}$ | 0.775 | 0.767 | 0.812 | 0.782 | 0.831 | 0.267 | 0.806 | 0.722 | 0.772 | 0.813 | 0.878 | 0.720 |
| $H_{15}$ | 0.680 | 0.791 | 0.749 | 0.780 | 0.774 | 0.672 | 0.757 | 0.964 | 0.716 | 0.672 | 0.825 | 0.845 |
| $H_{16}$ | 0.889 | 0.964 | 0.936 | 1.000 | 0.964 | 1.000 | 1.000 | 0.981 | 0.967 | 0.918 | 0.791 | 0.997 |
| $H_{17}$ | 0.508 | 0.513 | 0.489 | 0.655 | 0.499 | 0.546 | 0.499 | 0.601 | 0.493 | 0.504 | 0.343 | 0.352 |
| $H_{18}$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.501 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.951 |
| $H_{19}$ | 0.831 | 0.835 | 0.882 | 0.863 | 0.974 | 0.367 | 0.945 | 0.877 | 0.839 | 0.864 | 0.950 | 0.863 |
| $H_{20}$ | 0.979 | 0.966 | 0.922 | 0.938 | 0.988 | 0.385 | 0.981 | 1.000 | 0.977 | 1.000 | 0.898 | 0.960 |
| $H_{21}$ | 0.925 | 1.000 | 0.994 | 0.925 | 0.990 | 0.980 | 0.930 | 0.944 | 0.968 | 0.993 | 0.879 | 0.798 |
| $H_{22}$ | 0.640 | 0.607 | 0.585 | 0.566 | 0.745 | 0.351 | 0.547 | 0.552 | 0.533 | 0.595 | 0.500 | 0.570 |
| $H_{23}$ | 0.736 | 0.669 | 0.709 | 0.675 | 0.718 | 0.223 | 0.657 | 0.673 | 0.700 | 0.772 | 0.735 | 0.793 |
| $H_{24}$ | 0.694 | 0.674 | 0.675 | 0.710 | 0.685 | 0.285 | 0.617 | 0.760 | 0.710 | 0.737 | 0.648 | 0.803 |
| $H_{25}$ | 0.666 | 0.722 | 0.706 | 0.670 | 0.658 | 0.256 | 0.637 | 0.648 | 0.563 | 0.678 | 0.675 | 0.647 |
| $H_{26}$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

## Appendix B

# Malmquist Productivity Index results

Table B.1: Total results of the Malmquist Productivity Index for 2013.

| DMU | Jan-Feb | Feb-Mar | Mar-Apr | Apr-May | May-Jun | Jun-Jul | Jul-Aug | Aug-Sep | Sep-Oct | Oct-Nov | Nov-Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $H_1$ | 1.082 | 0.965 | 1.045 | 1.085 | 2.290 | 0.386 | 0.837 | 1.178 | 1.200 | 0.912 | 0.851 |
| $H_2$ | 0.915 | 1.011 | 1.103 | 0.980 | 0.924 | 1.082 | 0.780 | 1.717 | 0.836 | 0.864 | 0.903 |
| $H_3$ | 0.882 | 1.019 | 1.070 | 1.070 | 2.190 | 0.434 | 0.742 | 1.290 | 1.110 | 0.905 | 0.879 |
| $H_4$ | 0.874 | 1.039 | 1.396 | 0.757 | 1.923 | 0.486 | 0.824 | 1.146 | 1.205 | 0.911 | 1.034 |
| $H_5$ | 0.914 | 1.068 | 0.954 | 1.106 | 1.283 | 0.665 | 0.834 | 1.194 | 1.202 | 0.876 | 0.883 |
| $H_6$ | 0.918 | 1.048 | 1.007 | 1.092 | 2.644 | 0.342 | 0.837 | 1.217 | 1.138 | 0.921 | 0.922 |
| $H_7$ | 0.952 | 0.889 | 1.145 | 1.001 | 1.205 | 0.718 | 0.789 | 1.559 | 1.242 | 0.796 | 0.897 |
| $H_8$ | 0.951 | 1.007 | 0.965 | 1.069 | 2.253 | 0.417 | 0.766 | 1.209 | 1.179 | 0.921 | 0.829 |
| $H_9$ | 0.886 | 1.043 | 1.122 | 1.008 | 0.919 | 1.072 | 0.858 | 1.100 | 1.131 | 0.955 | 0.918 |
| $H_{10}$ | 0.821 | 1.209 | 1.100 | 0.856 | 2.357 | 0.435 | 0.802 | 1.307 | 1.114 | 0.903 | 0.975 |
| $H_{11}$ | 0.830 | 1.042 | 1.064 | 1.040 | 2.581 | 0.368 | 0.828 | 1.160 | 1.167 | 0.933 | 0.768 |
| $H_{12}$ | 0.882 | 1.018 | 1.087 | 1.003 | 2.220 | 0.444 | 0.797 | 1.001 | 1.508 | 0.854 | 1.196 |
| $H_{13}$ | 0.818 | 1.103 | 1.166 | 0.980 | 2.134 | 0.474 | 0.751 | 1.105 | 1.339 | 0.929 | 0.734 |
| $H_{14}$ | 0.913 | 1.114 | 0.967 | 1.062 | 2.281 | 0.402 | 0.886 | 0.859 | 1.524 | 0.932 | 0.978 |
| $H_{15}$ | 0.921 | 1.031 | 1.038 | 1.072 | 2.269 | 0.386 | 0.893 | 1.117 | 1.170 | 0.919 | 0.892 |
| $H_{16}$ | 0.855 | 0.961 | 1.163 | 1.064 | 2.177 | 0.442 | 0.724 | 1.261 | 1.173 | 0.966 | 0.896 |
| $H_{17}$ | 0.871 | 1.024 | 1.092 | 0.990 | 2.066 | 0.508 | 0.887 | 1.018 | 1.131 | 0.878 | 0.794 |
| $H_{18}$ | 0.847 | 1.011 | 1.105 | 1.040 | 0.902 | 1.058 | 0.726 | 1.236 | 1.211 | 0.889 | 0.823 |
| $H_{19}$ | 0.864 | 1.004 | 1.067 | 1.063 | 0.791 | 1.152 | 0.714 | 1.325 | 1.194 | 0.908 | 0.866 |
| $H_{20}$ | 0.883 | 1.015 | 1.062 | 1.075 | 0.762 | 1.238 | 0.766 | 1.233 | 1.118 | 0.909 | 1.067 |
| $H_{21}$ | 0.920 | 0.998 | 1.029 | 1.059 | 2.356 | 0.377 | 0.845 | 1.173 | 1.108 | 0.895 | 0.799 |
| $H_{22}$ | 6.137 | 0.869 | 0.094 | 1.274 | 2.222 | 0.369 | 2.354 | 3.953 | 1.028 | 0.988 | 0.054 |
| $H_{23}$ | 0.945 | 1.044 | 0.862 | 1.236 | 1.922 | 0.514 | 0.685 | 1.138 | 1.366 | 0.925 | 0.773 |
| $H_{24}$ | 0.989 | 0.917 | 1.141 | 1.090 | 0.823 | 1.091 | 0.849 | 1.157 | 1.169 | 1.015 | 0.924 |
| $H_{25}$ | 0.911 | 0.971 | 1.088 | 1.027 | 1.901 | 0.484 | 0.822 | 0.894 | 1.419 | 0.995 | 0.853 |
| $H_{26}$ | 0.878 | 0.997 | 1.095 | 0.959 | 3.198 | 0.302 | 0.841 | 1.147 | 1.161 | 0.887 | 0.917 |

Table B.2: Total results of the Malmquist Productivity Index for 2014.

| DMU | Jan-Feb | Feb-Mar | Mar-Apr | Apr-May | May-Jun | Jun-Jul | Jul-Aug | Aug-Sep | Sep-Oct | Oct-Nov | Nov-Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $H_1$ | 0.847 | 1.129 | 0.970 | 1.149 | 2.567 | 0.299 | 0.869 | 1.241 | 1.129 | 0.962 | 0.837 |
| $H_2$ | 0.883 | 1.072 | 0.939 | 1.006 | 0.849 | 1.106 | 0.825 | 1.221 | 1.143 | 0.890 | 0.871 |
| $H_3$ | 0.935 | 1.044 | 0.966 | 1.065 | 2.407 | 0.375 | 0.718 | 1.345 | 1.143 | 0.960 | 0.887 |
| $H_4$ | 0.883 | 0.951 | 1.012 | 1.078 | 2.315 | 0.358 | 0.868 | 1.175 | 1.162 | 0.919 | 0.796 |
| $H_5$ | 0.949 | 1.017 | 0.980 | 0.969 | 2.719 | 0.351 | 0.883 | 1.128 | 1.203 | 0.955 | 0.680 |
| $H_6$ | 0.960 | 1.004 | 0.924 | 1.130 | 3.399 | 0.222 | 0.974 | 1.227 | 1.156 | 0.931 | 0.323 |
| $H_7$ | 0.958 | 0.967 | 0.910 | 1.166 | 0.988 | 0.781 | 0.751 | 1.488 | 1.321 | 0.960 | 0.566 |
| $H_8$ | 0.953 | 1.010 | 0.924 | 1.025 | 2.386 | 0.368 | 0.835 | 1.239 | 1.209 | 0.888 | 0.733 |
| $H_9$ | 0.964 | 1.054 | 0.961 | 1.049 | 2.244 | 0.392 | 0.857 | 1.234 | 1.135 | 0.943 | 0.907 |
| $H_{10}$ | 0.831 | 1.021 | 0.887 | 1.362 | 2.135 | 0.318 | 0.905 | 1.127 | 1.346 | 0.927 | 0.768 |
| $H_{11}$ | 0.877 | 1.041 | 0.963 | 1.065 | 2.358 | 0.385 | 0.872 | 1.159 | 1.130 | 0.917 | 0.852 |
| $H_{12}$ | 0.906 | 1.199 | 0.824 | 1.116 | 0.871 | 1.014 | 0.856 | 1.184 | 1.110 | 0.984 | 0.850 |
| $H_{13}$ | 0.886 | 0.974 | 0.992 | 1.038 | 1.630 | 0.543 | 0.819 | 1.120 | 1.278 | 0.908 | 0.747 |
| $H_{14}$ | 0.923 | 1.077 | 0.987 | 0.987 | 0.949 | 0.819 | 1.019 | 1.163 | 1.144 | 0.946 | 0.756 |
| $H_{15}$ | 0.906 | 1.070 | 0.930 | 1.050 | 2.420 | 0.362 | 0.810 | 1.291 | 1.158 | 0.940 | 0.797 |
| $H_{16}$ | 0.922 | 1.048 | 0.956 | 1.064 | 1.558 | 0.565 | 0.754 | 1.351 | 1.116 | 0.934 | 0.892 |
| $H_{17}$ | 0.969 | 1.062 | 0.920 | 1.055 | 2.567 | 0.329 | 1.203 | 1.015 | 1.062 | 0.925 | 0.607 |
| $H_{18}$ | 0.879 | 1.018 | 0.969 | 1.067 | 0.886 | 1.025 | 0.693 | 1.433 | 1.146 | 0.916 | 0.660 |
| $H_{19}$ | 0.906 | 1.045 | 0.992 | 1.085 | 0.754 | 1.047 | 0.752 | 1.404 | 1.198 | 0.900 | 0.847 |
| $H_{20}$ | 0.900 | 1.053 | 0.995 | 1.043 | 0.752 | 1.197 | 0.816 | 1.236 | 1.120 | 0.947 | 0.826 |
| $H_{21}$ | 0.938 | 1.040 | 0.936 | 1.100 | 2.136 | 0.393 | 0.902 | 1.186 | 1.115 | 0.948 | 0.745 |
| $H_{22}$ | 0.896 | 1.003 | 0.980 | 1.420 | 1.045 | 0.618 | 0.838 | 1.157 | 1.189 | 0.904 | 0.832 |
| $H_{23}$ | 0.940 | 1.021 | 1.085 | 0.904 | 1.719 | 0.497 | 0.548 | 1.959 | 1.143 | 0.971 | 0.860 |
| $H_{24}$ | 0.889 | 1.074 | 0.769 | 1.266 | 0.847 | 1.026 | 0.884 | 1.125 | 1.203 | 0.947 | 0.808 |
| $H_{25}$ | 0.907 | 1.020 | 0.946 | 0.952 | 1.296 | 0.718 | 0.898 | 1.221 | 1.231 | 0.887 | 0.743 |
| $H_{26}$ | 0.960 | 1.042 | 0.924 | 1.104 | 2.377 | 0.376 | 1.021 | 0.897 | 1.238 | 1.010 | 0.811 |

Table B.3: Total results of the Malmquist Productivity Index for 2015.

| DMU | Jan-Feb | Feb-Mar | Mar-Apr | Apr-May | May-Jun | Jun-Jul | Jul-Aug | Aug-Sep | Sep-Oct | Oct-Nov | Nov-Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $H_1$ | 0.876 | 1.186 | 1.051 | 0.886 | 1.400 | 0.673 | 0.795 | 1.245 | 1.079 | 0.990 | 0.717 |
| $H_2$ | 0.889 | 1.072 | 0.980 | 0.934 | 0.960 | 1.051 | 0.853 | 1.198 | 1.044 | 0.982 | 0.797 |
| $H_3$ | 0.945 | 1.064 | 1.001 | 0.971 | 2.783 | 0.354 | 0.764 | 1.325 | 1.007 | 1.076 | 0.869 |
| $H_4$ | 1.012 | 0.894 | 1.120 | 0.935 | 2.812 | 0.371 | 0.789 | 1.188 | 1.106 | 0.983 | 0.783 |
| $H_5$ | 0.931 | 1.208 | 0.829 | 0.948 | 1.792 | 0.587 | 0.811 | 1.128 | 1.091 | 1.001 | 0.778 |
| $H_6$ | 0.984 | 1.195 | 0.604 | 1.251 | 2.964 | 0.345 | 0.777 | 1.270 | 0.892 | 1.085 | 0.674 |
| $H_7$ | 0.984 | 0.985 | 0.961 | 1.072 | 1.020 | 0.923 | 0.669 | 1.582 | 1.181 | 0.896 | 0.662 |
| $H_8$ | 0.920 | 1.087 | 0.925 | 1.000 | 2.159 | 0.421 | 0.876 | 1.150 | 1.129 | 0.939 | 0.741 |
| $H_9$ | 0.918 | 1.039 | 1.013 | 0.974 | 2.797 | 0.353 | 0.910 | 1.139 | 1.072 | 0.947 | 0.789 |
| $H_{10}$ | 0.947 | 1.037 | 0.890 | 1.120 | 3.214 | 0.297 | 0.874 | 1.149 | 1.049 | 0.952 | 1.042 |
| $H_{11}$ | 0.926 | 1.091 | 0.938 | 0.996 | 2.596 | 0.385 | 0.900 | 1.066 | 1.088 | 0.953 | 0.806 |
| $H_{12}$ | 0.934 | 1.011 | 1.137 | 0.906 | 1.006 | 0.993 | 0.851 | 1.036 | 1.159 | 0.994 | 0.723 |
| $H_{13}$ | 1.010 | 0.868 | 1.041 | 1.018 | 2.184 | 0.444 | 0.823 | 1.135 | 1.128 | 0.989 | 0.664 |
| $H_{14}$ | 0.918 | 1.134 | 0.915 | 0.975 | 1.007 | 1.036 | 0.771 | 1.201 | 1.026 | 1.069 | 0.833 |
| $H_{15}$ | 0.924 | 1.100 | 0.957 | 0.973 | 2.882 | 0.342 | 0.800 | 1.230 | 1.092 | 0.994 | 0.797 |
| $H_{16}$ | 0.925 | 1.068 | 0.983 | 1.009 | 2.261 | 0.429 | 0.793 | 1.288 | 1.060 | 0.958 | 0.769 |
| $H_{17}$ | 0.939 | 1.063 | 0.967 | 0.991 | 4.739 | 0.212 | 1.002 | 0.942 | 1.059 | 0.975 | 0.598 |
| $H_{18}$ | 0.923 | 1.071 | 0.993 | 0.973 | 0.979 | 1.011 | 0.734 | 1.363 | 1.032 | 0.979 | 0.783 |
| $H_{19}$ | 0.926 | 1.104 | 0.993 | 0.994 | 0.968 | 1.029 | 0.711 | 1.376 | 1.083 | 0.986 | 0.804 |
| $H_{20}$ | 0.881 | 1.078 | 1.010 | 0.987 | 0.878 | 1.114 | 0.849 | 1.186 | 1.027 | 0.990 | 0.834 |
| $H_{21}$ | 0.931 | 1.062 | 1.007 | 0.958 | 2.503 | 0.374 | 0.954 | 1.164 | 0.993 | 0.998 | 0.725 |
| $H_{22}$ | 0.928 | 1.016 | 0.945 | 1.292 | 1.328 | 0.562 | 0.973 | 1.022 | 1.125 | 0.986 | 0.793 |
| $H_{23}$ | 0.891 | 1.033 | 1.036 | 1.010 | 1.823 | 0.492 | 0.558 | 1.905 | 1.064 | 0.978 | 0.751 |
| $H_{24}$ | 0.889 | 1.044 | 1.000 | 0.958 | 1.031 | 1.026 | 0.873 | 1.094 | 1.116 | 0.996 | 0.794 |
| $H_{25}$ | 0.934 | 1.014 | 0.980 | 0.973 | 0.961 | 1.061 | 0.825 | 1.267 | 1.012 | 0.981 | 0.720 |
| $H_{26}$ | 0.999 | 1.048 | 0.860 | 1.106 | 4.296 | 0.219 | 0.835 | 1.218 | 1.026 | 0.970 | 0.809 |

Table B.4: Total results of the Malmquist Productivity Index for 2016.

| DMU | Jan-Feb | Feb-Mar | Mar-Apr | Apr-May | May-Jun | Jun-Jul | Jul-Aug | Aug-Sep | Sep-Oct | Oct-Nov | Nov-Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $H_1$ | 0.981 | 1.032 | 0.976 | 1.094 | 2.850 | 0.275 | 1.013 | 1.060 | 1.066 | 1.050 | 0.724 |
| $H_2$ | 0.966 | 1.043 | 0.919 | 1.096 | 0.945 | 0.877 | 0.970 | 1.063 | 1.001 | 1.000 | 0.953 |
| $H_3$ | 1.014 | 1.007 | 0.939 | 1.146 | 2.534 | 0.332 | 0.869 | 1.258 | 0.996 | 1.099 | 0.739 |
| $H_4$ | 0.953 | 1.072 | 0.986 | 1.026 | 2.837 | 0.319 | 0.927 | 1.054 | 1.081 | 1.086 | 0.763 |
| $H_5$ | 0.880 | 1.077 | 1.002 | 0.991 | 2.645 | 0.389 | 0.879 | 1.115 | 0.993 | 0.985 | 0.875 |
| $H_6$ | 0.976 | 1.071 | 0.956 | 1.013 | 3.461 | 0.245 | 1.017 | 1.100 | 1.037 | 1.098 | 0.675 |
| $H_7$ | 0.975 | 1.047 | 0.936 | 1.028 | 0.970 | 0.835 | 0.806 | 1.476 | 1.073 | 0.970 | 0.708 |
| $H_8$ | 1.036 | 1.064 | 0.929 | 1.084 | 2.731 | 0.305 | 0.941 | 1.114 | 1.054 | 0.912 | 0.980 |
| $H_9$ | 0.961 | 0.949 | 0.934 | 1.096 | 2.989 | 0.289 | 1.113 | 0.961 | 1.063 | 1.060 | 0.656 |
| $H_{10}$ | 1.051 | 0.891 | 1.033 | 1.074 | 1.973 | 0.490 | 0.826 | 1.164 | 1.149 | 1.036 | 0.656 |
| $H_{11}$ | 0.959 | 1.057 | 0.950 | 1.053 | 2.293 | 0.406 | 0.922 | 1.095 | 1.050 | 0.990 | 0.804 |
| $H_{12}$ | 0.975 | 0.971 | 1.185 | 0.865 | 3.165 | 0.182 | 1.545 | 1.070 | 1.069 | 1.001 | 0.830 |
| $H_{13}$ | 0.921 | 1.050 | 0.944 | 0.955 | 2.848 | 0.311 | 1.041 | 1.082 | 1.004 | 1.088 | 0.717 |
| $H_{14}$ | 0.939 | 1.100 | 0.878 | 1.074 | 1.024 | 0.911 | 0.878 | 1.187 | 1.002 | 1.006 | 0.815 |
| $H_{15}$ | 0.990 | 1.022 | 0.941 | 1.091 | 2.551 | 0.329 | 0.903 | 1.196 | 1.003 | 1.051 | 0.768 |
| $H_{16}$ | 0.959 | 1.083 | 0.907 | 1.110 | 2.109 | 0.422 | 0.845 | 1.205 | 0.969 | 1.071 | 0.793 |
| $H_{17}$ | 1.232 | 1.173 | 0.792 | 1.028 | 4.825 | 0.194 | 1.156 | 0.851 | 0.991 | 1.024 | 0.563 |
| $H_{18}$ | 0.978 | 1.019 | 0.945 | 1.051 | 0.967 | 0.899 | 0.873 | 1.248 | 0.979 | 1.022 | 0.699 |
| $H_{19}$ | 0.970 | 1.061 | 0.921 | 1.174 | 0.861 | 0.949 | 0.798 | 1.373 | 0.973 | 1.069 | 0.772 |
| $H_{20}$ | 0.932 | 1.111 | 0.884 | 1.097 | 0.855 | 1.027 | 0.963 | 1.079 | 0.975 | 1.077 | 0.884 |
| $H_{21}$ | 0.957 | 1.035 | 0.927 | 1.171 | 2.694 | 0.320 | 0.926 | 1.176 | 0.983 | 1.043 | 0.710 |
| $H_{22}$ | 0.917 | 1.054 | 0.967 | 1.339 | 1.233 | 0.547 | 1.036 | 0.989 | 1.062 | 1.001 | 0.856 |
| $H_{23}$ | 0.906 | 1.156 | 0.938 | 1.050 | 0.828 | 1.186 | 0.863 | 1.099 | 0.985 | 1.009 | 0.784 |
| $H_{24}$ | 0.942 | 1.061 | 0.952 | 1.048 | 0.939 | 0.982 | 0.947 | 1.122 | 0.992 | 0.992 | 0.867 |
| $H_{25}$ | 1.037 | 1.282 | 0.824 | 1.004 | 0.956 | 0.954 | 0.966 | 1.101 | 1.028 | 1.065 | 0.726 |
| $H_{26}$ | 0.911 | 1.098 | 0.966 | 1.001 | 4.607 | 0.182 | 1.098 | 1.131 | 0.963 | 1.110 | 0.716 |

Table B.5: Total results of the Malmquist Productivity Index for 2017.

| DMU | Jan-Feb | Feb-Mar | Mar-Apr | Apr-May | May-Jun | Jun-Jul | Jul-Aug | Aug-Sep | Sep-Oct | Oct-Nov | Nov-Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $H_1$ | 0.842 | 1.176 | 0.817 | 1.210 | 2.374 | 0.367 | 0.949 | 1.006 | 1.181 | 0.950 | 0.913 |
| $H_2$ | 0.828 | 1.216 | 0.748 | 1.243 | 0.913 | 1.002 | 0.910 | 1.106 | 1.088 | 0.957 | 0.907 |
| $H_3$ | 0.872 | 1.154 | 0.826 | 1.194 | 2.360 | 0.365 | 0.830 | 1.233 | 1.101 | 0.740 | 1.070 |
| $H_4$ | 0.857 | 1.195 | 0.800 | 1.202 | 2.517 | 0.341 | 1.003 | 1.007 | 1.074 | 0.801 | 0.956 |
| $H_5$ | 0.889 | 1.103 | 0.824 | 1.198 | 2.495 | 0.370 | 0.848 | 1.125 | 1.164 | 0.690 | 1.063 |
| $H_6$ | 0.826 | 1.261 | 0.784 | 1.169 | 2.669 | 0.337 | 0.753 | 1.259 | 1.119 | 0.901 | 0.895 |
| $H_7$ | 0.863 | 1.134 | 0.748 | 1.251 | 1.140 | 0.679 | 0.857 | 1.499 | 1.084 | 0.708 | 1.031 |
| $H_8$ | 0.936 | 1.175 | 0.781 | 1.098 | 2.947 | 0.311 | 1.049 | 0.972 | 1.122 | 0.774 | 0.954 |
| $H_9$ | 0.872 | 1.220 | 0.812 | 1.106 | 1.845 | 0.478 | 0.976 | 1.053 | 1.136 | 0.953 | 0.837 |
| $H_{10}$ | 0.937 | 1.080 | 0.736 | 1.293 | 2.180 | 0.433 | 0.825 | 1.213 | 1.064 | 1.004 | 0.798 |
| $H_{11}$ | 0.891 | 1.192 | 0.794 | 1.173 | 2.499 | 0.364 | 0.909 | 1.050 | 1.068 | 1.086 | 0.741 |
| $H_{12}$ | 0.794 | 1.138 | 1.088 | 1.022 | 0.970 | 0.876 | 0.937 | 1.040 | 1.102 | 1.045 | 0.797 |
| $H_{13}$ | 0.846 | 1.028 | 0.904 | 1.199 | 2.168 | 0.390 | 1.003 | 1.004 | 1.128 | 0.791 | 0.911 |
| $H_{14}$ | 0.849 | 1.268 | 0.784 | 1.234 | 0.864 | 1.011 | 0.832 | 1.140 | 1.112 | 1.005 | 0.725 |
| $H_{15}$ | 0.941 | 1.127 | 0.799 | 1.202 | 1.842 | 0.477 | 0.945 | 1.102 | 0.975 | 1.031 | 0.865 |
| $H_{16}$ | 0.906 | 1.162 | 0.805 | 1.180 | 2.170 | 0.414 | 0.777 | 1.385 | 1.007 | 0.775 | 0.989 |
| $H_{17}$ | 0.900 | 1.145 | 1.054 | 0.891 | 2.747 | 0.334 | 1.018 | 0.936 | 1.085 | 0.735 | 0.769 |
| $H_{18}$ | 0.824 | 1.184 | 0.819 | 1.147 | 0.989 | 0.927 | 0.863 | 1.206 | 1.028 | 1.091 | 0.607 |
| $H_{19}$ | 0.846 | 1.225 | 0.773 | 1.339 | 0.782 | 1.070 | 0.794 | 1.252 | 1.072 | 1.016 | 0.704 |
| $H_{20}$ | 0.838 | 1.129 | 0.832 | 1.209 | 0.834 | 1.090 | 0.923 | 1.096 | 1.079 | 0.969 | 0.703 |
| $H_{21}$ | 0.901 | 1.186 | 0.778 | 1.228 | 2.894 | 0.289 | 0.964 | 1.108 | 1.073 | 0.692 | 0.979 |
| $H_{22}$ | 0.822 | 1.149 | 0.787 | 1.529 | 1.257 | 0.527 | 0.936 | 1.028 | 1.179 | 0.792 | 0.992 |
| $H_{23}$ | 0.792 | 1.263 | 0.769 | 1.239 | 0.819 | 1.009 | 0.946 | 1.103 | 1.168 | 0.911 | 0.918 |
| $H_{24}$ | 0.869 | 1.189 | 0.823 | 1.141 | 0.992 | 0.854 | 1.054 | 1.024 | 1.100 | 0.968 | 0.865 |
| $H_{25}$ | 0.965 | 1.166 | 0.755 | 1.152 | 0.980 | 0.899 | 0.926 | 0.913 | 1.285 | 1.067 | 0.722 |
| $H_{26}$ | 0.783 | 1.229 | 0.858 | 1.133 | 3.071 | 0.289 | 0.965 | 1.092 | 1.040 | 0.680 | 1.260 |

## Appendix C

# Malmquist Productivity Index forecast results

Table C.1: Total results of the forecasted Malmquist Productivity Index for 2018.

| DMU | Jan-Feb | Feb-Mar | Mar-Apr | Apr-May | May-Jun | Jun-Jul | Jul-Aug | Aug-Sep | Sep-Oct | Oct-Nov | Nov-Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $H_1$ | 1.353 | 1.326 | 1.314 | 1.330 | 1.334 | 1.339 | 1.345 | 1.350 | 1.356 | 1.361 | 1.367 |
| $H_2$ | 1.252 | 1.218 | 1.165 | 1.179 | 1.155 | 1.167 | 1.156 | 1.158 | 1.155 | 1.154 | 1.153 |
| $H_3$ | 1.042 | 1.302 | 1.260 | 1.264 | 1.254 | 1.255 | 1.255 | 1.254 | 1.254 | 1.254 | 1.254 |
| $H_4$ | 1.326 | 1.202 | 1.236 | 1.203 | 1.200 | 1.199 | 1.198 | 1.198 | 1.199 | 1.213 | 1.213 |
| $H_5$ | 1.261 | 1.175 | 1.213 | 1.193 | 1.206 | 1.206 | 1.207 | 1.208 | 1.209 | 1.210 | 1.211 |
| $H_6$ | 1.279 | 1.218 | 1.241 | 1.214 | 1.207 | 1.219 | 1.207 | 1.200 | 1.208 | 1.199 | 1.193 |
| $H_8$ | 1.205 | 1.309 | 1.263 | 1.272 | 1.271 | 1.271 | 1.273 | 1.275 | 1.277 | 1.279 | 1.281 |
| $H_9$ | 1.216 | 1.227 | 1.210 | 1.187 | 1.181 | 1.188 | 1.171 | 1.177 | 1.187 | 1.185 | 1.172 |
| $H_{10}$ | 1.191 | 1.243 | 1.245 | 1.225 | 1.241 | 1.242 | 1.238 | 1.243 | 1.244 | 1.245 | 1.247 |
| $H_{11}$ | 1.083 | 1.173 | 1.122 | 1.123 | 1.114 | 1.107 | 1.107 | 1.103 | 1.099 | 1.097 | 1.094 |
| $H_{12}$ | 1.251 | 1.263 | 1.229 | 1.223 | 1.223 | 1.220 | 1.218 | 1.216 | 1.213 | 1.212 | 1.210 |
| $H_{13}$ | 1.113 | 1.181 | 1.161 | 1.152 | 1.148 | 1.142 | 1.140 | 1.137 | 1.134 | 1.132 | 1.129 |
| $H_{14}$ | 1.240 | 1.337 | 1.215 | 1.218 | 1.208 | 1.204 | 1.203 | 1.203 | 1.203 | 1.202 | 1.201 |
| $H_{17}$ | 1.514 | 1.476 | 1.477 | 1.453 | 1.462 | 1.463 | 1.461 | 1.467 | 1.470 | 1.472 | 1.476 |
| $H_{20}$ | 1.029 | 1.468 | 1.089 | 1.303 | 1.157 | 1.238 | 1.192 | 1.215 | 1.200 | 1.207 | 1.199 |
| $H_{21}$ | 1.340 | 1.273 | 1.327 | 1.310 | 1.302 | 1.304 | 1.307 | 1.311 | 1.334 | 1.338 | 1.343 |
| $H_{22}$ | 1.356 | 1.299 | 1.257 | 1.249 | 1.222 | 1.221 | 1.235 | 1.224 | 1.223 | 1.229 | 1.229 |
| $H_{23}$ | 0.893 | 0.998 | 1.037 | 1.052 | 1.059 | 1.059 | 1.056 | 1.052 | 1.047 | 1.042 | 1.037 |
| $H_{24}$ | 1.333 | 1.273 | 1.195 | 1.188 | 1.182 | 1.176 | 1.174 | 1.173 | 1.170 | 1.169 | 1.167 |
| $H_{25}$ | 1.099 | 1.119 | 1.110 | 1.108 | 1.099 | 1.092 | 1.088 | 1.086 | 1.081 | 1.076 | 1.073 |
| $H_{26}$ | 1.335 | 1.342 | 1.479 | 1.401 | 1.438 | 1.451 | 1.439 | 1.449 | 1.458 | 1.460 | 1.466 |