# Path-following and Cooperative Path-following for Underwater Snake Robots

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**Abstract**

Small, flexible, and maneuverable robots that can perform light tasks, inspection, maintenance, and access small places at low cost are a growing need for the thriving market of subsea exploration. Underwater Snake Robots have the ability and efficiency to overcome what once was, a costly operation when using Remotely Operated Vehicles (ROVs), or Autonomous Underwater Vehicles (AUVs).

Path-following is an essential problem within pipe/cable inspection as it might be required for the robot to inspect a considerable length of pipes and cables without moving away from them in the presence of external disturbances. By resorting to cooperative path-following, the inspection of the can be sped up with USR working together and synchronized.

This thesis presents two models for underwater snake robots, where the influence of constant and irrotational ocean currents is considered. An analysis of different controllers for path following is addressed. Moreover, a solution to the CPF problem concerning this type of robot will be divided into two steps.

Having as the main goal the coordination of the robots along a path, CPF can be decoupling into two sub-problems: (i) The problem of the path-following mentioned above for a single vehicle and (ii) multi-agent system (MAS) coordination. Simulations will support the work and, in the latter, will eventually tested on the real robot.

**Keywords:** Cooperative Path Following, Underwater Snake Robot, Path Following, ILOS

## 1. Introduction

In recent years, there has been a substantial increase in the number of subseacompanies. As the offshore industry's carbon footprints continue to grow, these companies are looking at ways to cut costs and reduce environmental impacts. The growth in the number of subsea production installation has created a demand for subsea Inspection, Maintenance, and Repair (IMR) operations, making it a field of technology with enormous potential for autonomous marine robotics to thrive.

This is where the Underwater Snake Robots come into the game as its shape of a biological snake makes it ideal for moving in high viscosity environments such as water. USR is an articulated structure consisting of serially connected joint modules. Although it can mimic the eel-like motion of biological snakes, this solution has its limitations. It becomes challenging to navigate in tight areas as the entire body must move to generate propulsive forces. Moving the whole body has a direct impact on tasks such as Dynamic Positioning (DP), becoming much harder to maintain its position. Adding thruster modules will open up a full new range of applications as it can achieve forward, backward, and sideways motion without performing undulatory movements.

The aim of the work using Underwater Snakes Robots is to design systems for motion control entailing system modeling and design of algorithms for path-following and cooperative path-following in the presence of unknown currents. This studies will thereby be evaluated based on system performance analysis and numerical simulations. Line-of-Sight approaches will be used in the USR for path following. An approach to Virtual Holonomic Constraitns will be carried out that will make possible to solve the maneuvering problem and therefore making the bridge needed for a solution of the Cooperative Path Following in Underwater Snake Robots.

This document is organized as follows: section 2 presents the control oriented model of the underwater snake robot under the influence of ocean currents. Section 3 presents the Maneuvering Control Problem making use of the Virtual Holonomic Constraints (VHCs) to solve it. Here the approach terrestrial snake robots in [12] is extended to the Maneuvering Problem using VHC for Underwater Snake Robots. Section 4 is the derivation of a control method for coordination control of multiple underwater snake robots. The results of the Cooperative Path Following under the influence of constant, unknown and irrotational ocean currents are presented in Section 5. Finally, in section 6 the conclusion and future work topics are presented.

### 2. Control Oriented Model for USR

Even though the control-oriented model is derived from the complex model for Underwater Snake Robots, the last model is not presented in this documented since it was not used to solve either the problem of path-following or cooperative path following. The reader is then referred to the work presented in [4] for a better insight of the complex model. In fact a simplified derivation of the control oriented model is presented here for completion.

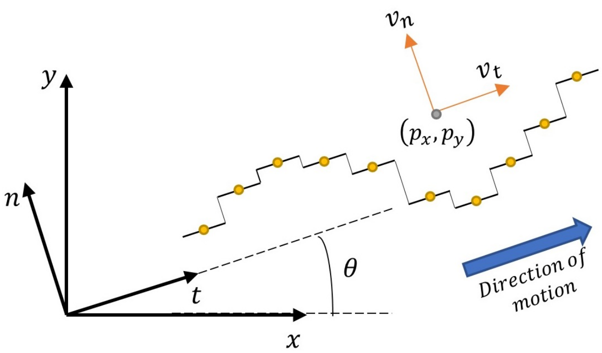


**Figure 1:** Control Oriented Model

The simplified model consists of $N$ links of length $2l$ with mass uniformly distributed in each link so as the CM is at the midpoint of each link. To derive the control-oriented model the revolute joints are modelled as prismatic joints that move transversal to the direction of movement (figure 1). This approximation is valid for sinusoidal gaits that follow the following properties [5]:

***Property* 1.** *Forward propulsion, under the assumption that $c_n > c_t$, is achieved through transversal motion of the links, where $c_n$ and $c_t$ are the drag parameters normal and tangential, respectively.*

***Property* 2.** *The motion of the links, under sinusoidal gait pattern, consists mainly of a normal displacement of the CM of the links*

The motion of the robot is defined with respect to the global fixed x-y and t-n frame where the origin of both coincide. The t-n frame is always aligned with the direction of the robot as it is seen in figure 1. The t-axis will represent the forward direction while the n-axis the normal direction. From the t-n frame the velocity components of the robot are written as forward velocity, $v_t$ and sideways velocity, $v_n$.

The angle $\theta$ is expressed with respect to the global axis with counter-clockwise positive direction. As the links of the doesn't rotate w.r.t each other they all have the same orientation, that is coincident with the t-n frame. $\theta$ can be defined as:

***Definition* 1.** The orientation of the robot is given by $\theta \in \mathbb{R}$ with counter-clockwise positive direction.

The dynamic model together with the hydrodynamic model [4] leads to the complete control-oriented model and is given by the following equations:

$$\ddot{\boldsymbol{\phi}} = -\frac{c_n}{m}\dot{\boldsymbol{\phi}} + \frac{c_p}{m}v_{t,rel}\boldsymbol{A}\boldsymbol{D}^T\boldsymbol{\phi} + \frac{1}{m}\boldsymbol{D}\boldsymbol{D}^T\mathbf{u} \quad (1a)$$

$$\ddot{\theta} = -\frac{1}{\lambda_3 + 1}\left(\lambda_1\dot{\theta} - \frac{\lambda_2}{N-1}v_{t,rel}\bar{\boldsymbol{e}}^T\boldsymbol{\phi}\right) \quad (1b)$$

$$\dot{v}_t = -\frac{c_n}{m}v_{t,rel} + \frac{2c_p}{Nm}v_{n,rel}\bar{\boldsymbol{e}}^T\boldsymbol{\phi} - \frac{c_p}{Nm}\boldsymbol{\phi}^T\boldsymbol{A}\bar{\boldsymbol{D}}\dot{\boldsymbol{\phi}} \quad (1c)$$

$$\dot{v}_n = -\frac{c_n}{m}v_{n,rel} + \frac{2c_p}{Nm}v_{t,rel}\bar{\boldsymbol{e}}^T\boldsymbol{\phi} \quad (1d)$$

$$\dot{p}_x = v_t\cos\theta - v_n\sin\theta \quad (1e)$$

$$\dot{p}_y = v_t\sin\theta + v_n\cos\theta \quad (1f)$$

Since this model takes into consideration the presence of constant and irrotational ocean currents the velocities are relative in the body aligned frame:

$$\begin{bmatrix} v_{t,rel} \\ v_{n,rel} \end{bmatrix} = \begin{bmatrix} v_t \\ v_n \end{bmatrix} - R_\theta^T\boldsymbol{v_c} \quad (2)$$

with $\boldsymbol{v_c} = [V_x, V_y]^T$

### 3. Maneuvering Control in straight-line path using VHC

In the following chapter the simplified dynamic model of a planar snake is widely used in order to address the maneuvering control problem. It is considered that the robot is neutrally buoyant and that it moves with planar sinusoidal gait in the presence of constant, irrational and unknown ocean currents. Furthermore we aim to make the USR converge to and follow a desired straight line path and achieve a desired reachable speed after the convergence. Moreover, the relative forward velocity must comply to assumptions 1 and 2. This section investigates the planar maneuvering problem for the USR , studied first for marine vessels in [14].

For the resolution of this problem the ocean disturbances are contemplated for a more realistic approach and those are accounted in the guidance scheme, augmenting it with an integral action to compensate for the steady state error that arise from constant ocean currents. With all things considered, the control objective to be solved consists in making the USR converge to a desired planar path and traverse it with a desired forward velocity, $v_{t,rel} = v_d$.

The path following problem will be inserted in this section instead of dedicate a section solely to it.

However in the master thesis from which this document is based on, a dedicated chapter with simulation results is presented and the reader is refered there for better understanding of the problem.

Due to the fact that making the robot move accordingly to:

$$\phi_{i,ref} = \alpha g(i) \sin(\omega t + (i-1)\delta) + \phi_0 \quad (3)$$

in the path following problem, a different approach is needed since the parameters presented in equation (3), $\alpha, \omega, \delta$, directly affect the forward velocity of the robot. As a result it is not possible to guarantee that the forward velocity will reach the desired velocity once it converges. To solve this complication the proposed feedback control strategy enforces VHCs.

VHC will encode the gaits studied previously on the USR configuration, where, parameterized by states of dynamic compensator, they will control both orientation and forward velocity.

The maneuvering problem is divided into two fundamental tasks: ***(i)*** a geometric task, where it solves the path following problem, and in addition, ***(ii)*** a dynamic task which is responsible to keep the relative forward velocity steady and regulate the heading of the underwater snake. The proposed control algorithm will then be tested and verified by means of simulation together with the formation control in place.

### 3.1. Assumptions and Transformed Control-Oriented Model

Assuming that the unknown ocean current $v_c$ is considered, the following assumptions can be made, and the forward velocity is now considered as relative forward velocity due to ocean currents.

***Assumption*** **1.** The snake robot moving by a sinusoidal gait with a constant relative forward velocity, is bounded by $V_{max}$ and $V_{min}$, $v_{trel} \in [V_{max}, V_{min}]$, $V_{max} \geq V_{min} > 0$

Nevertheless, in order to move forward in the presence of ocean currents, the forward velocity generated by the planar sinusoidal gait has to be such that compensates for the ocean current, if not the problem of path following can't be achieved. A new assumption is made with regards to this and states the following:

***Assumption*** **2.** Under the influence of ocean current, the relative forward velocity must be large enough to compensate for this disturbance, i.e, $v_{trel} > V_{min} > V_{c,max} \geq 0$.

From the control oriented equations (1), the joint coordinates, $\phi$ are present in the equations of both the dynamics of the angular velocity, $\dot{v}_\theta$ and sideways velocity, $\dot{v}_n$. As long as the joint coordinates are considered in (1b) and in (1d) the design of the control system will complicate as the body shape changes will affect both heading and sideways motion of the robot [10]. To overcome this problem a change of coordinates is performed. It is suggested in [10] and motivated by [8, 2] that in order to get rid of the effect of $\phi$ in the sideways velocity one should move the point that defines the position of the snake by a distance $\epsilon$ from the CM of the robot along the tangential direction of the robot to a point where the joint offset $\phi_0$ generates a pure rotational motion and no sideways forces. Based on these, the following change of coordinates is defined:

$$\bar{p}_x = p_x + \epsilon \cos\theta \quad (4a)$$

$$\bar{p}_y = p_y + \epsilon \sin\theta \quad (4b)$$

$$\bar{v}_n = v_n + \epsilon v_\theta \quad (4c)$$

$$\epsilon = -\frac{2(N-1)}{\lambda_2}\frac{c_p}{Nm} \quad (4d)$$

For the chosen approach absolute velocities should be removed from the model by introducing the following relations:

$$v_t = v_{t,rel} + V_t \quad (5a)$$

$$\bar{v}_n = \bar{v}_{n,rel} + V_n \quad (5b)$$

where $V_t = V_x \cos\theta + V_y \sin\theta$ and $V_n = -V_x \sin\theta + V_y \cos\theta$ are the ocean currents expressed in the body frame of the robot. Taking the derivative of (5a) and substituting (4c) and (1c) in it, $\dot{v}_{t,rel}$ is easy to obtain and the transformed control-oriented model is given by:

$$\ddot{\boldsymbol{\phi}} = -\frac{c_n}{m}\dot{\boldsymbol{\phi}} + \frac{c_p}{m} v_{t,rel} \boldsymbol{A}\boldsymbol{D}^T \boldsymbol{\phi} + \frac{1}{m}\boldsymbol{D}\boldsymbol{D}^T \mathbf{u} \quad (6a)$$

$$\ddot{\theta} = -\lambda_1 \dot{\theta} + \frac{\lambda_2}{N-1} v_{t,rel} \bar{\boldsymbol{e}}^T \boldsymbol{\phi} \quad (6b)$$

$$\dot{\bar{p}}_y = v_{t,rel} \sin\theta + \bar{v}_{n,rel} \cos\theta + V_y \quad (6c)$$

$$\dot{\bar{v}}_{n,rel} = \left(\epsilon \left(\frac{c_n}{m} - \lambda_1\right) + V_x \cos\theta + V_y \sin\theta\right) v_\theta - \frac{c_n}{m}\bar{v}_{n,rel} \quad (6d)$$

$$\dot{v}_{t,rel} = -X_t v_{t,rel} + Y_t \bar{v}_{n,rel} - Z_t v_\theta - \frac{c_p}{m}\boldsymbol{\phi}^T A \bar{D} v_\phi \quad (6e)$$

Where $X_t = \frac{c_t}{m}$, $Y_t = \frac{2c_p}{Nm}\boldsymbol{e}^T \boldsymbol{\phi}_{ref}$ and $Z_t = Y_t \epsilon - V_x \sin\theta + V_y \cos\theta$.

### 3.2. Virtual Holonomic Constraints

The method of virtual holonomic constraints is a method used frequently to solve locomotion control problems. It was first used for Snake Robots in

[13], leaving aside the velocity control. Moreover, VHCs are a useful concept for the control of oscillations. While performing gait pattern lateral undulation all the solutions of the snake robot dynamics have inherited oscillatory behaviours, thus it can be analytically and constructively controlled based on Virtual Holonomic Constraints. With these approach the state evolution of the mechanical system is confined to an invariant constraint manifold. Those constraints are virtual because they arise from the action of a feedback controller rather than a physical connection between two variables [16]. The time dependency presented in (3) will be removed guaranteeing that the time-evolution of the state variables are confined to state-dependent constraint functions [13]

#### 3.3. Control System Design

In this section, motivated by the work done for Snake Robots in [11], a solution using the control-oriented model for the underwater snake robot, and the method of virtual holonomic constraints is presented.

The control system for the underwater snake robot can be divided into 3 main stages, first a body shape controller, second, a velocity and for last the path following controller. The control approach can be seen as an hierarchical design in a sense that has three main stages, each one with a prioritize control specifications. A bridge will be made between the stages for snake robots and underwater snake robots, where the redefinition of the control objectives is made in comparison with the last section.

#### 3.4. Control Objectives

The control objectives needed to solve the maneuvering problem are presented next. The solution for this problem is the starting point to solve the cooperative path following. The maneuvering problem, once again can be divided into two main tasks [15].

- Geometric task: The objective is to make the robot converge to and follow a desired path
- Dynamic task: Consists in satisfying dynamical constraints, where in this case is to satisfy a desired relative forward velocity along the desired path

The first control objective is to asymptotically stabilize the desired gait pattern that produces forward propulsion given by $\phi_{ref}$, such that

$$\lim_{t\to\infty} \phi(t) - \phi_{ref}(t) = 0 \tag{7}$$

For the second control objective we look for asymptotically stabilize $\theta \to \theta_{ref}$:

$$\lim_{t\to\infty} \theta(t) - \theta_{ref}(t) = 0 \tag{8}$$

Thirdly is required that the robot's position converges to the path. We can define a straight line path as $P \triangleq \{(x, y) \in \mathbb{R}^2 : y = 0\}$. In addition we consider that the global x-axis is aligned with the desired straight path as motioned in remark **??**.

For the robot to converge we want that the cross-track error, $\bar{p}_y$ goes to zero, $\bar{p}_y \to 0$. However it might be required that the robot, instead follows a path in a position different than zero. That way this control objective can be generalized in a way that the desired straight line path can be re-written as $P \triangleq \{(x, y) \in \mathbb{R}^2 : y = y^{path}_{tofollow}\}$, where $y^{path}_{tofollow} \in \mathbb{R}$ is the path that the robot should follow away from the origin of the global coordinate frame. The convergence is still achieved when the the cross track error converge to zero. To make sure that $\bar{p}_y \to 0$, a change in equation (4b) is made, taking into consideration $y^{path}_{tofollow}$. Thus, the cross-track error, $\tilde{\bar{p}}_y$, is defined as:

$$\tilde{\bar{p}}_y = p_y + \epsilon \sin\theta - y^{path}_{tofollow} \tag{9}$$

and the third control objective defined as follow:

$$\lim_{t\to\infty} \tilde{\bar{p}}_y(t) = 0 \tag{10}$$

***Remark* 1.** The control objective (10) is only achieved, under ocean currents, requiring that the desired reference forward velocity, $v_d$ lies within $[V_{max}, V_{min}]$ , $v_d \geq v_{t,rel} \geq V_{max} > V_{min} > 0$

For the last control objective, after it's convergence the robot must regulate the forward velocity along the path for a desired forward velocity profile, $v_{t,ref} > 0$. A reference position along the desired path is defined as $p_{t,ref} = \int_0^t v_{t,ref}(\tau) d\tau$. The last control objective is then defined as:

$$\lim_{t\to\infty} p_t(t) - p_{t,ref}(t) = 0 \tag{11}$$

As soon as the objective 4 is asymptotically stabilized, the robot moves accordingly to $v_{t,rel} = v_{t,ref}$

***Assumption* 3.** For the following sections, it is considered that $v_{t,rel}$ has no finite-escape time.

The next theorem states that the maneuvering controller to be defined in the following sections solves the maneuvering problem based on stability results under the constraint manifolds defined.

***Theorem* 1.** The constraint manifolds in this chapter were defined such that $\Gamma_1 \subset \Gamma_2 \subset \Gamma_3 \subset \Gamma_4 \subset \mathcal{Q}$, where $\mathcal{Q}$ stands for the configuration space. The constraint manifold $\Gamma_1$ is a compact set as all variables used to define it in (41) are bounded. Furthermore, the constraint manifold $\Gamma_i$ is asymptotically stable with respect to the constraint manifold $\Gamma_{i+1}, i = 1, ..., 3$. From here and according to

Proposition 14 presented in [3], the constraint manifold $\Gamma_1$ is asymptotically stable for the controlled system. As a consequence of this, all the solution of the controlled system remain uniformly bounded and the four control objectives defined in section 3.4 are all achieved.

#### 3.5. Forward propulsion VHC and Body Shape Controller

Virtual Holonomic Constraints will encode the sinusoidal gaits, studied earlier ((3)), which allow the robot to propel forward, as it has been used in [11] for snake robots. As regards to snake robots, VHC come from adopting the reference signal for the single joints as follows:

$$\phi_{i,ref}(\lambda, \phi_0) = \alpha g(i) \sin(\lambda + (i-1)\delta) + \phi_0, \quad (12)$$

where the scaling factor $g(i)$ is added for the solely purpose to achieve a more generalized class of gaits in swimming snake robots. Equation (12) is the proposed VHC, for the body shape variables of the USR, where, $\lambda$ and $\phi_0$ represents the solutions of the compensators that are defined next:

$$\ddot{\lambda} = u_\lambda \quad (13)$$

$$\ddot{\phi}_0 = u_{\phi_0} \quad (14)$$

$u_\lambda$ is used as a controller to regulate the relative forward velocity of the robot while $u_{\phi_0}$ is used as a controller to regulate the heading of the snake. This VHC will be enforced in the robot through the control input $\bar{\boldsymbol{u}}$ in $v_\phi = \bar{\boldsymbol{u}}$. This control input is used to stabilize the solutions of the joint coordinates dynamics to the constraint manifold $\Gamma_4$.

Associated with the constraint function $\phi(\lambda, \phi_0)$ is the following constraint-manifold [12].

$$\begin{aligned}\Gamma_4 =&\{(x, \dot{x}, \phi_0, \dot{\phi}_0, \lambda, \dot{\lambda}) \in \mathbb{R}^{2N+8} : \\ &\phi_i = \phi_{i,ref}(\lambda, \phi_0), \dot{\phi} = \dot{\lambda}\frac{\partial \phi_{ref}}{\partial \lambda} + \dot{\phi}_0 \frac{\partial \phi_{ref}}{\partial \phi_0}\}\end{aligned} \quad (15)$$

To globally exponentially stabilize the constraint manifold (15) induce forward propulsion, the linearizing feedback controller law, $\bar{\boldsymbol{u}}$ defined for the path following problem is used :

$$\begin{aligned}\bar{\boldsymbol{u}} =& \ddot{\phi}_{ref} - k_{v_\phi}(\dot{\phi} - \dot{\phi}_{ref}) - k_\phi(\phi - \phi_{ref}), \\ & i \in \{1, \dots, N-1\},\end{aligned} \quad (16)$$

where $k_{v_\phi}, k_\phi > 0$ are constant controller gains. Defining the following joint error vector:

$$\tilde{\phi} = [\phi_i - \phi_{ref}, \dots, \phi_N - \phi_{N,ref}] \in R^{N-1} \quad (17)$$

we can rewrite (16) as:

$$\bar{\boldsymbol{u}} = \ddot{\phi}_{ref} - k_{v_\phi}\dot{\tilde{\phi}} - k_\phi \tilde{\phi}, \quad (18)$$

Substituting (18) in $v_\phi = \bar{\boldsymbol{u}}$ the tracking error dynamics of the joint angles is written as:

$$\ddot{\tilde{\boldsymbol{\phi}}} + k_{v_\phi}\dot{\tilde{\boldsymbol{\phi}}} + k_\phi \tilde{\boldsymbol{\phi}} = 0 \quad (19)$$

which has a globally exponentially stable equilibrium at the origin $\left(\tilde{\boldsymbol{\phi}}, \dot{\tilde{\boldsymbol{\phi}}}\right) = (0_{N-1}, 0_{N-1})$, that implies that the joint coordinates error will converge exponentially to zero, that is, the constraint manifold $\Gamma_4$ is globally exponentially stable and the control objective 7 is met.

#### 3.6. Velocity Controller

The stage two unfolds into two sub-stages, the orientation controller and the speed controller, that together are responsible for the velocity controller. The speed controller is inserted in the dynamic task of the maneuvering problem while the orientation controller belongs to the geometric task.

##### 3.6.1. Basic notation for the velocity controller

To the derivation of the controllers that constitute the control system design, the following matrices and expressions are vastly used. This expressions are modified from [12] so that they take into consideration the relative velocities, the constant and irrotational ocean currents, and the the hydrodynamics of the underwater snake robot:

$$C = [\alpha \cos(\lambda), \dots, \alpha \cos(\lambda + (i-1)\delta)]^T \in \mathbb{R}^{N-1} \quad (20)$$

$$\Phi_{ref} = [\phi_{1,ref}, \dots, \phi_{N-1,ref}]^T \in \mathbb{R}^{N-1} \quad (21)$$

$$\eta = -\frac{c_p}{Nm}\Phi_{ref}^T A\bar{D} \in \mathbb{R}^{N-1} \quad (22)$$

As we use the frequency of the joint angle oscillation as an additional control term to make the relative forward velocity to follow a reference we define:

The tangential position error, $\tilde{p}_t$ and the velocity error, $\tilde{v}_{trel}$as:

$$\tilde{p}_t = p_t - p_{t,ref} \quad (23a)$$

$$\tilde{v}_{t,rel} = v_{t,rel} - v_{t,ref} \quad (23b)$$

The position error and velocity error dynamics for the Underwater snake robot evaluated in (15) are the following:

$$\dot{\tilde{p}}_t = \tilde{v}_{trel} \quad (24a)$$

$$\dot{\tilde{v}}_{trel} = -X_t v_{t,rel} + Y_t(\bar{v}_{n,rel} - Z_t) - \frac{c_p}{m}\boldsymbol{\phi}^T A\bar{D}v_\theta - \dot{v}_{tref} \quad (24b)$$

with $X_t, Y_t$and $Z_t$ defined in section 3.1

### 3.7. Orientation Controller

In the path following problem a LOS guidance law that considers an integral action to circumvent the influence of ocean disturbances was derived [4]. Following that analysis, the same guidance law can be applied here and the reference orientation for the robot, as a function of the cross-track error( equation (9)) is given by:

$$\theta_{ref} = -\arctan\left(\frac{\tilde{\bar{p}}_y + \sigma y_{int}}{\Delta}\right), \tag{25}$$

$$\dot{y}_{int} = \frac{\Delta \tilde{\bar{p}}_y}{\left(\tilde{\bar{p}}_y + \sigma y_{int}\right)^2 + \Delta^2}. \tag{26}$$

$\Delta > 0$ denote the look-ahead-distance and is used to tune the rate of convergence of the snake.

We control the orientation of the robot making use of the $\ddot{\phi}_0$ as an additional control input. By defining the orientation error, $\tilde{\theta}$, the orientation error dynamics $\ddot{\tilde{\theta}}$ evaluated on the constraint manifold $\Gamma_4$ can be obtained:

$$\ddot{\tilde{\theta}} = -\lambda_1\dot{\tilde{\theta}} - \lambda_1\dot{\theta}_{ref} + \frac{\lambda_2}{N-1}v_{t,rel}\bar{\boldsymbol{e}}^T\boldsymbol{S} + \lambda_2 v_{t,rel}\phi_0 - \ddot{\theta}_{ref} \tag{27}$$

Defining the constraint manifold,

$$\Gamma_3 = \Big\{(\theta, \dot{\theta}, \phi_0, \dot{\phi}_0, v_{t,rel}, \lambda) \in \Gamma_4 : (\tilde{\theta}, \dot{\tilde{\theta}}) = (0,0), \left\|\left[\phi_0, \dot{\phi}_0\right]\right\| \leq \epsilon_{\phi_0}\Big\} \tag{28}$$

associated with the control objective (8), we want to exponentially stabilize it with respect to $\Gamma_4$ by showing that $\left(\tilde{\theta}, \dot{\tilde{\theta}}\right) = (0,0)$.

Defining $\phi_0$ as:

$$\phi_0 = -\frac{1}{\lambda_2 v_{t,rel}}\left(\frac{\lambda_2}{N-1}v_{t,rel}\bar{\boldsymbol{e}}^T\boldsymbol{S} - \lambda_1\dot{\theta}_{ref} - \ddot{\theta}_{ref} + K_\theta\tilde{\theta}\right) \tag{29}$$

Inserting $\phi_0$ into (27), the orientation error dynamics of the robot evaluated on the constraint manifold is written as:

$$\ddot{\tilde{\theta}} + \lambda_1\dot{\tilde{\theta}} + K_\theta\tilde{\theta} = 0, \tag{30}$$

which has a globally exponentially stable equilibrium point at the origin $(\tilde{\theta}, \dot{\tilde{\theta}}) = (0,0)$. This implies that the orientation errors converges exponentially to zero, i.e, the constraint manifold is globally exponentially stable, and the control objective (8) is achieved.

### 3.8. Speed Controller (Dynamic task)

In the previous sub-sections the compensator $\ddot{\phi}_0 = u_{\phi_0}$ is obtained by a low-pass filter reference model and used as a controller to regulate the heading of the snake [9]. The proposed VHC ((12)) still has another compensator, $\lambda$, which is responsible for controlling the velocity, through the control input $u_\lambda$, which will make the forward and normal velocity converge to a desired reference relative forward velocity, $v_{t,ref}$, and to a small neighbourhood around the origin, respectively. The velocity changes varying the frequency of the joint angles oscillations, induced by the dynamic compensator. A new constraint manifold that will exponentially stabilize relative to (15) is defined as following:

$$\Gamma_2 = \Big\{(\theta, \dot{\theta}, p_t, v_{t,rel}, \bar{v}_{n,rel}, \phi_0, \dot{\phi}_0, \lambda, \dot{\lambda}) \in \Gamma_4 : \left(\tilde{\theta}, \dot{\tilde{\theta}}\right) = (0,0), (\tilde{p}_t, \tilde{v}_{t,rel}) = (0,0), \|\bar{v}_{n,rel}\| \leq \epsilon_n, \left\|\left[\phi_0, \dot{\phi}_0\right]\right\| \leq \epsilon_{\phi_0}, \left\|\left[\lambda, \dot{\lambda}\right]\right\| \leq \epsilon_\lambda\Big\} \tag{31}$$

where $\epsilon_n, \epsilon_{\phi_0}$ and $\epsilon_\lambda > 0$ are constants.

The control input $u_\lambda = \ddot{\lambda}$ is used to stabilize both velocity and position at the origin $(\tilde{p}_t, \tilde{v}_{t,rel}) = (0,0)$ of $\dot{\tilde{v}}_{t,rel}$.

In order to derive the control input (13), using the techniques of back-stepping in [6] and following the analysis in [12], with the respective changes for the USR model, the Control-Lyapunov Functions are iteratively introduced, starting with a CLF for the position $\tilde{p}_t$.

$$V_1 = \frac{1}{2}\tilde{p}_t^2 \tag{32}$$

From this point on only the relevant formulas obtained from the deduction will be presented regarding the derivation of the control inputs and the back stepping. The reader is once more referred to the Master thesis for an insight on the proof of stability.

$$u_\lambda = -z_1\delta_1 + \dot{\delta}_2 - k_{z_2}z_2 \tag{33}$$

$$z_1 = v_{t,rel} - v_{t,ref} + k_{z_0}\tilde{p}_t \tag{34}$$

$$z_2 = \dot{\lambda} - \delta_2 \tag{35}$$

$$\delta_1(\phi_0, \lambda) = \eta C \tag{36}$$

$$\begin{aligned}\dot{\lambda} =& \frac{1}{\delta_1}\Big(-\tilde{p}_t + \frac{c_t}{m}v_{t,ref} - \frac{c_t}{m}k_{z_0}\tilde{p}_t - \frac{2c_p}{Nm}\bar{v}_{n,rel}\bar{\boldsymbol{e}}\boldsymbol{\phi}_{ref} \\ &- \eta\bar{\boldsymbol{e}}\dot{\phi}_0 + \dot{v}_{t,ref} + Z_t v_\theta - k_{z_0}\dot{\tilde{p}}_t - k_{z_1}z_1\Big) \\ =& \delta_2\left(\phi_0, p\dot{h}i_0, \dot{\lambda}, p_t, v_{t,rel}\right),\end{aligned} \tag{37}$$

Before proceeding to the geometric task, the highest level of hierarchy of the control system, a few remarks must be made regarding the velocity control manifold. The constraint manifold $\Gamma_3 \subset \Gamma_2$. That way stabilizing $\Gamma_2$ relative to the constraint manifold $\Gamma_4$ implies not only that the robot will follow the reference heading but also the reference velocity.

The solution of the compensator $\lambda$, $u_\lambda$, will remain bounded, where in order to be bounded, both $\bar{v}_{n,rel}$ and $v_{t,rel}$ must be bounded, which they are proved to be in [12], without any environment disturbance. Following the same line as thought, it can be proved that both $\bar{v}_{n,rel}$ and the relative forward velocity are bounded. In that case and considering that the robot is under the influence of ocean currents, the proof goes by defining the Lyapunov function candidate and taking it's derivative,

$$V = \frac{1}{2}\bar{v}_{n,rel}^2 \tag{38}$$

$$\dot{V} = \bar{v}_{n,rel}\dot{\bar{v}}_{n,rel} \tag{39}$$

with $\dot{\bar{v}}_{n,rel}$ given by the equation (6d). For an insight on the proof the reader is referred to the chapter 6 section 4.3 of the master thesis of this document.

### 3.9. Path following controller (Geometric task)

The last step of the control system, and also the highest level of the hierarchy is the path following control, where we aim to stabilize $\bar{p}_y \rightarrow y_{tofollow}^{path}$, or in other words $\tilde{\bar{p}}_y \rightarrow 0$. Making use of the aforementioned stability results it is straightforward to prove that $\bar{p}_y$ converges to the desired path. Due to the oscillating nature of the robot, it will converge to a sufficient small neighbourhood around the desired path and the cross track error to a small neighbourhood close to zero. To stabilize $\bar{p}_y \rightarrow y_{tofollow}^{path}$ we use the definition made in (9) and in addition we define the normal velocity cross track error as:

$$\tilde{\bar{v}}_{n,ref} = \bar{v}_n - v_{n,ref}, \tag{40}$$

where $v_{n,ref} = 0$.

The manifold in which the path following problem (geometric task) is solved is defined as:

$$\Gamma_1 = \Big\{ (\theta, \dot{\theta}, p_t, v_{t,rel}, \tilde{\bar{p}}_y, \bar{v}_{n,rel}, \phi_0, \dot{\phi}_0, \lambda, \dot{\lambda}) \in \Gamma_2 : \tilde{\bar{p}}_y \leq \epsilon_p \Big\} \tag{41}$$

Considering the dynamics of the position $\dot{\bar{p}}_y$ ((6c)), the error coordinates for the position on the constraint manifold $\Gamma_2$ is written as:

The equation (**??**) evaluated on the exponentially stable manifold $\Gamma_2$ can be written as follows:

$$\dot{\tilde{\bar{p}}}_y = (\tilde{v}_{rel} + v_{t,ref}) \sin(\theta_{ref}) + \tilde{\bar{v}}_{n,ref} \cos(\theta_{ref}) + V_y \tag{42}$$

With the error dynamics of the position and by selecting a Control Lyapunov Function based on that the reader is referred to the master thesis for the proof that the position $\bar{p}_y$converges to the desired path.

## 4. Cooperative Path Following in Underwater Snake Robots

The cooperative path following problem unfolds into two important problems. A **path following problem** and a **formation control problem**.

The key idea explored here is the same as in [7], where a vehicle is elected to be the leader and the formation of the other vehicles (followers) is build around him.

With decentralized laws for each snake, the desired path for each robot in the formation is a straight line, which is achieved through the fulfillment of the geometric task in 3.9. For each snake on the path, the velocity can be controlled using the speed controller defined in last section, equation (37). By adjusting the speed of each snake along the path using the desired speed as synchronization control term, we can ensure that the desired formation pattern is achieved.

Since the problem requires multiple underwater snake robots, a way to identifying them is needed, so the superscript $j$ is used to denote the snakes number. We define $j = \{1, \dots, n\}$, where n indicates the total number of snakes.

For completeness, the objectives related to the path following problem will once again be defined, and, in addition the objectives that concerns the formation control problem will be introduced. However one is referred to the last section for the derivation of the controllers and respective stability proofs, since the focus of this chapter will be the achievement of the formation control objectives.

- Objective I - Concerns the desired gait pattern of the snake and aims to asymptotically stabilize $\phi \rightarrow \phi_{ref}$.
- Objective II - Concerns the orientation of the snake and aims to asymptotically stabilize $\theta \rightarrow \theta_{ref}$.
- Objective III - Concerns the convergence to the path and aims to asymptotically stabilize $\tilde{\bar{p}}_y \rightarrow 0$.
- Objective IV- Concerns the regulation of the forward velocity of the robot along the path to a desired velocity profile, $v_{t,ref}$. So, given a

desired velocity $v_{t,ref}$ and position $p_{t,ref}(t) = \int_0^t v_{t,ref}(\tau)d\tau$, we aim to stabilize $p_t \rightarrow p_{t,ref}$ and $v_{t,rel} \rightarrow v_{t,ref}$

The aforementioned control objectives concern the geometric and dynamic task of the maneuvering problem presented in Section 3.

For the formation control we want to make sure that all the followers follow a certain formation based on the x-axis distances between the leader and them. This can be enclosed in a matrix form, which can be called Formation Matrix with entries $d_{ji} = -d_{ij} \in \mathbb{R}$ for all $j \neq i$ with $i, j \in \{1, \ldots, Snakes\}$ such that $d_{j,i} = D_{x_j} - D_{x_i}$. The parameter $d_{j,i}$ represents the desired $x$-axis distances between the $j$-th and the $i$-th robot in the formation.

Furthermore the position of each robot is given by $p_t^j(t)$, and the control **Objective V**, that concerns the achievement of the desired formation is defined as:

$$\lim_{t\to\infty} \left[ \sum_{j=1}^{n} \sum_{i=1}^{n} \left( p_x^j - p_x^i - d_{ji} \right) \right] = 0 \tag{43}$$

Despite this being the ultimate goal of the cooperative path following, some changes are needed when deriving the controller (33) in order to achieve this last objective. The next section will address those changes and provide a solution.

### 4.1. Coordination control of multiple underwater snake robots

In previous chapters controllers were designed ((18), (29), (33)) that guarantees that each underwater snake robot converges to the desired straight line path and progresses along with a desired velocity profile. The commanded velocity provided to the robot would, after convergence, be equal to the desired velocity profile, $v_{t,ref} = v_d$. As we seek to achieve a desired formation, a control law for the commanded velocity, $v_{t,rel}^j, j = 1, \ldots, n$, must be design, satisfying the remark 1 as well as that all the snakes achieve (43). Thus, the commanded velocity will be modified to adjust the relative forward speed based on the desired velocity profile, as before, but also highly influenced by the different distance to the formation. That way, once the objective (43) is achieved, they all can tune their velocity to match the desired velocity profile $v_d$. From [1] and considering the remark 1, we can assume that the desired speed profile lies within the interval $[V_{min}, V_{max}]$, i.e, there exists an $a > 0$ so that $v_d \in [V_{min} + a, V_{max} - a]$.

The change of velocity of each snake is made under the velocity controller defined in (33), to which the velocity dynamics of each j-th robot is given by:

$$v_{t,rel}^j = v_{t,ref}^j - k_{z_0} \tilde{p}_t^j \tag{44}$$

From the results in [1], the control law for the commanded velocity is tuned through the reference velocity for each snake [12]:

$$v_{t,ref}^j = v_d - g\left( \sum_{i=1}^{n} \gamma_{ji} \left( p_x^j - p_x^i - d_{j,i} \right) \right), \tag{45}$$

where $d_{ji}$ was defined above and linkage parameters $\gamma_{ji}$ are nonnegative and satisfy $\gamma_{ji} = \gamma_{ij}$, $\gamma_{ji} = 0$ for $i = j$.

The function $g$, makes one underwater snake robot move slower or faster than the others to compensate for their different distance to the formation through adding or decreasing the speed of each USR in the formation. It is a continuously differentiable non-decreasing function with bounded derivative satisfying $g'(0) > 0, g(0) = 0$ and $g(x) \in (-a, a)$ with $a$ being the parameter defined above. By choosing the function $g(x)$ equal to $g(x) = \frac{2a}{\pi} \arctan(x)$ it is certain bounded and for the problem in question converges to zero as soon as the desired formation is achieved, i.e, for all snake robots, $p_{x_i} - p_{x_j} - d_{ji}$ will be zero. Afterwards the snakes will move according to the desired velocity profile $v_d$.

Substituting (45) in (44) the velocity dynamics is presented as follows [12]:

$$v_{t,rel}^j = v_d - g\left( \sum_{i=1}^{n} \gamma_{ji} \left( p_x^j - p_x^i - d_{j,i} \right) \right) - k_{z_0} \tilde{p}_t^j \tag{46}$$

From the following change of coordinates $\hat{p}_x^j = p_x^j - D_{x_j} - \int_0^t v_d(\tau)d\tau$:

$$\dot{\hat{p}}_x^j = -g\left( \sum_{i=1}^{n} \gamma_{ji} \left( \hat{p}_x^j - \hat{p}_x^i \right) \right) - k_{z_0} \tilde{p}_t^j \tag{47}$$

With the following notations $\hat{p}_x = \left[ \hat{p}_x^1, \ldots \hat{p}_x^n \right]^T$, $g(\hat{p}_x) = \left[ g(\hat{p}_x^1), \ldots g(\hat{p}_x^n) \right]^T$, and $\tilde{p}_t = \left[ \tilde{p}_t^1, \ldots, \tilde{p}_t^n \right]^T$, we have that:

$$\dot{\hat{p}}_x = -g(\Gamma \hat{p}_x) - k_{z_0} \tilde{p}_t^j \tag{48}$$

where the matrix $\Gamma$ is given by [1]:

$$\Gamma = \begin{bmatrix} \sum_{j=1}^{n} \gamma_{1j} & -\gamma_{12} & \cdots & -\gamma_{1n} \\ -\gamma_{21} & \sum_{j=1}^{n} \gamma_{2j} & \cdots & -\gamma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -\gamma_{n1} & -\gamma_{n2} & \cdots & \sum_{j=1}^{n} \gamma_{nj} \end{bmatrix} \tag{49}$$

The matrix $\Gamma$ holds the property $\Gamma v_1 = 0$, where $v_1 = [1, \ldots, 1]^T$. This property implies that $\Gamma$ has a zero eigenvalue, where $v_1$ express the corresponding eigenvector [1]. Therefore the formation control goal for the system (48) is equivalent to:

$$\lim_{t \to \infty} \hat{p}_x(t) - cv_1 = 0, \tag{50}$$

where $c$ denotes a positive constant. From the results obtained in [1], for the system 48 coupled with the error dynamics of every snake robot through the term $k_{z_0}\tilde{p}_t^j$, assuming that the zero eigenvalue of matrix $\Gamma$ has multiplicity one and that Theorem 1 holds for every snake in the formation, 50 will be achieved exponentially. The above statement can be proved based on cascade systems theory and reader is referred to [1] where a similar proof is presented.

**5. Results**

In this section results regarding the Formation control using the control system enforcing VHC from Section 3 are presented for underwater snake robots under the influence of ocean currents. It is once again used the control oriented model defined in Chapter 2. The dynamics of the system were computed and implemented using Matlab R2019b and **ode45** solver, with absolute and relative tolerance equals to $10^{-6}$.

| | Initial positions | | Path to follow |
|---|---|---|---|
| USR | $\bar{p}_x$ | $\bar{p}_y$ | $y^{path}_{tofollow}$ |
| *snake* 1 | 3 | 2 | 1 |
| *snake* 2 | 0 | 0 | 0 |
| *snake* 3 | 2 | 0 | $-1$ |
| *snake* 4 | 0 | 4 | 0 |
| *snake* 5 | 6 | 0 | $-0.5$ |

**Table 1:** Initial conditions Formation Control, and desired path for each robot. At yellow is highlighted the leader of the formation.

An experience using lateral undulation under the influence of the ocean currents for 5 USR was performed. For the guidance system parameters, so as for the values of the controller gains and as well for the gait parameters, ocean current and desired reference forward velocity the reader is referred to the master thesis mo specifically to table 6.1. Since this chapter requires more than one robot, a table with the initial positions and the path to follow is defined next, with an highlight for the leader as the formation is defined with respect to him. The remaining initial conditions are the same ones defined in section 6.5 of the master thesis.

The Formation matrix parameters are defined as: $d_{1,2} = -3$, $d_{3,2} = -5$, $d_{4,2} = -10$, $d_{5,2} = -20$.

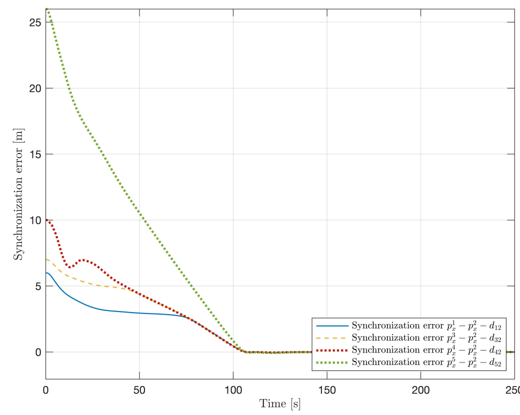


**Figure 2:** Synchronization error for each snake

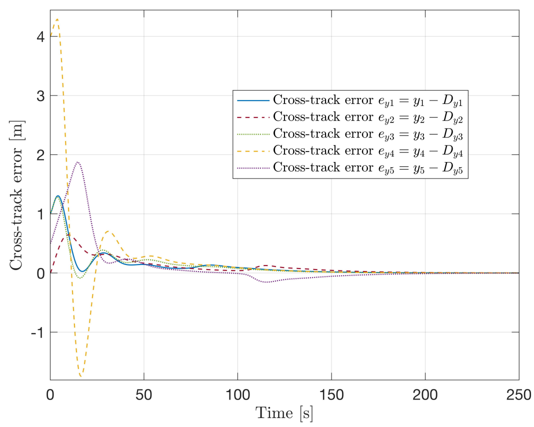


**Figure 3:** Cross track error for each snake

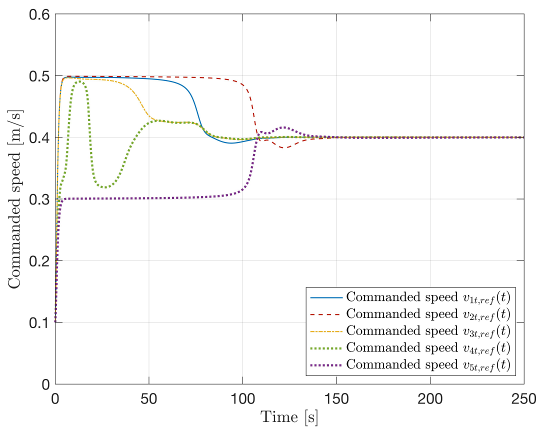


**Figure 4:** Reference velocities converge to desired speed $v_d = 0.4m/s$ after compensating the formation error

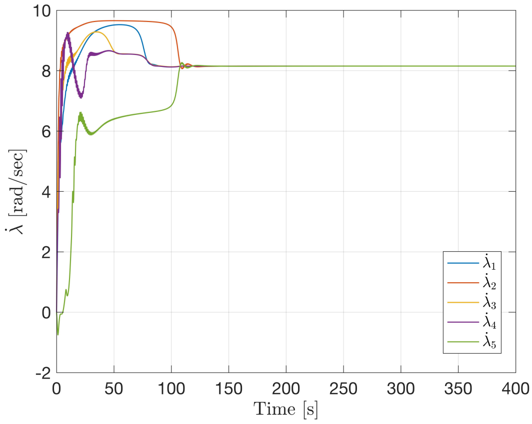


**Figure 5:** Joint Oscillation Frequency for each snake converges to a positive constant

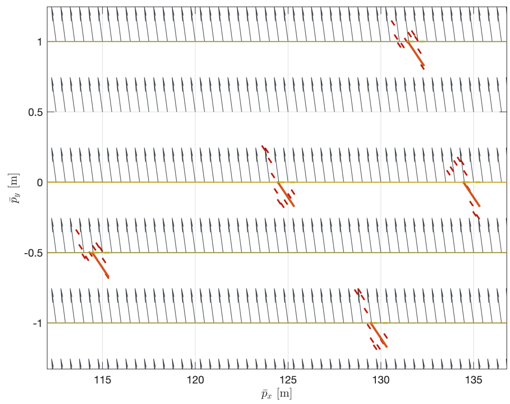


**Figure 6:** Formation after convergence for the path and formation

The simulation results of the Cooperative path following is presented in the figures above. They enclose the results for every one of the five robots when it's appropriated and will be analyzed as a whole, instead of analysing the results of each robot individually. In the Section Chapter was shown that the snake robots, individually converge to the path and progress along with the desired reference velocity. However, aside the fact that we want the system to work in a similar way, in addition it's wanted that all robots maintain a desired geometric formation. From figure 6, all five underwater snake robots converge to the path, but is not possible to attest if the desired formation was indeed met while also maintaining the desired velocity, $v_{t,ref}$. The velocity of all robots is showcased in figure 4 as well as the joint oscillation frequency 5, which is intrinsically linked to the increase or decrease of the velocity of the robots.

All the velocities converge to the desired reference velocity of the formation after compensating the formation error. Before they converge, they change over time, in order to compensate and adjust to the desired formation. On the other hand we have the oscillation frequency, which changes almost in the same way as the velocity, converging to a positive constant. This constant is the same in all robots and is the frequency of oscillation that makes all the robots move with velocity $v_{t,rel} = 0.4$ $m/s$ while overcoming to the ocean currents.

The most important part is still the achievement of the control objective (43). This can be seen through the convergence of all the synchronization errors, between leader and followers, from figure 2 to zero. Is important to refer that all robots converged to their respective paths in the presence of ocean currents as the cross track errors goes to zero 3, meaning that the Orientation controller still compensates for it when the formation is sought. A representation of the underwater snake robots moving in the desired geometric formation can be seen in figure 6. Based on all this results we can state that the control method for coordination control of multiple underwater snake robots meets all control objectives from section 4 and therefore the desired formation is achieved with all the robots moving on their paths with the desired velocity defined a posterior.

Some results for a sinusoidal path are as well presented in the master thesis. This simulation results shows that the apporach used for the CPF not only work for a straight-line path but as well as for a sinusoidal one.

### 6. Conclusion and Future Work

This document puts together the simplified model for the USR and the maneuvering formation control problem making use of the virtual holonomic constraints to solve it. The solution is divided in 2 fundamental steps: (i) a body shape controller that controls the input so that the robot has a desired gait pattern and (ii) making use of the gait parameters (compensators) to control both orientation and position of the robot. Latter a maneuvering controller was used to synchronize the velocities of the USRs to reach a desired formation and solve the CPF problem. Simulation results were presented in the end sustaining that the approach works under ocean currents.

For future work 3 points are proposed, being them (i) Expand the CPF for more generic planar curve. (ii) Implement the controllers in real USRs. (iii) Generalize the idea for a 3D space.

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